Hartley Oscillator

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1. Find the frequency of oscillation for given hartley circuit and also find condition on g_m. Below is the figure, Fig 1

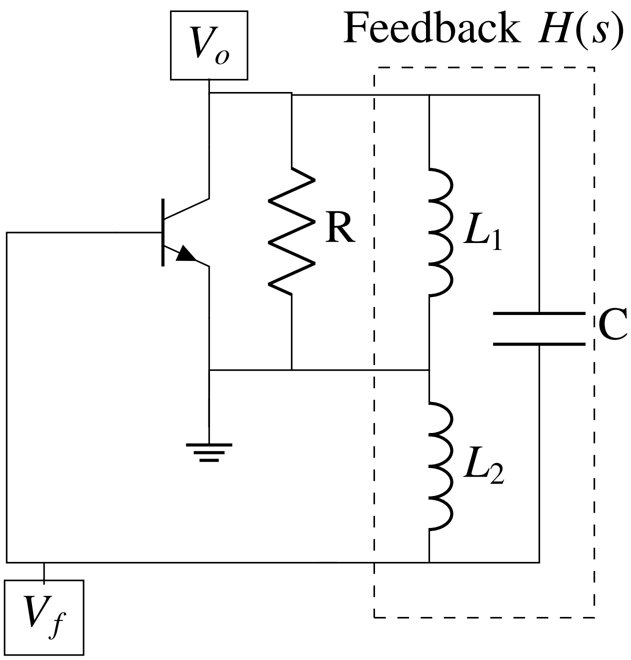

Fig. 1: Hartley oscillator

Solution: We will first draw an equivalent circuit for the above circuit.
To draw an equivalent block diagram we will draw small signal model for transistor Fig.1

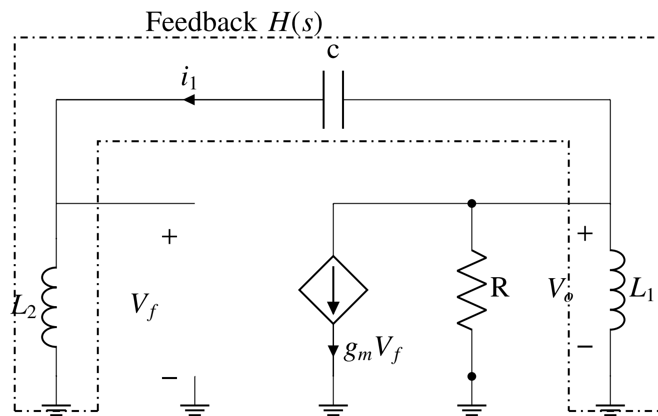

Fig. 1: Small signal model

And, its block diagram is as follows:

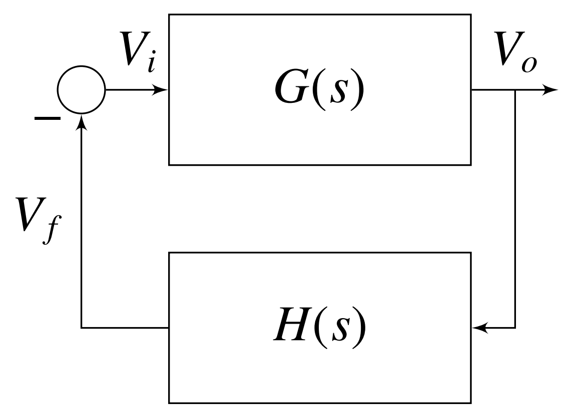

Fig. 1: Block diagram

Here G(s) is the amplification gain, and H(s) is the feedback gain.
G(s) small signal circuit is given as follows:

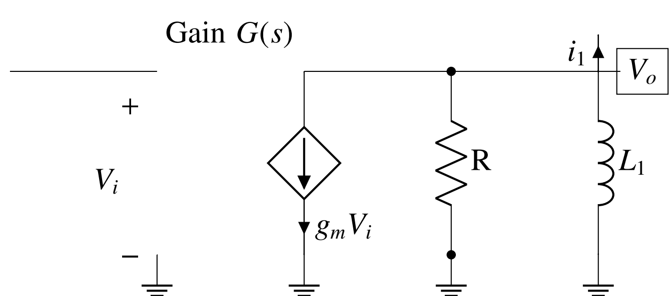

Fig. 1: Small signal G(s)

As, $V_f = V_i$
and G(s) is given by, $\frac{V_o}{V_i}$
Here,

$$i_1 = \frac{V_i}{sL_2} \tag{1.1}$$

$$V_o = I(sL_1 \parallel R) \tag{1.2}$$

$$I = i_1 + g_m V_i \tag{1.3}$$

Solving these equations we get,

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$$\frac{V_o}{V_i} = G(s) = \left(g_m + \frac{1}{sL_2}\right)\left(\frac{RsL_1}{R + sL_1}\right) \quad (1.4)$$

Now, solving for H(s)
From the small signal model feedback, we know that H(s) is output/input,
Where,

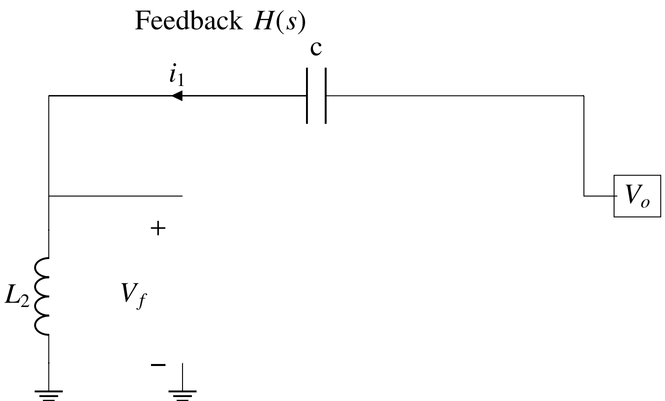

Fig. 1: Small signal H(s)

Output is V_f
Input is V_o

$$H(s) = \frac{V_f}{V_o} \quad (1.5)$$

$$V_0 = V_f + i_1 \times \frac{1}{sC} \quad (1.6)$$

$$i_1 = \frac{V_f}{sL_2} \quad (1.7)$$

$$(1.8)$$

Solving,

$$H(s) = \left(\frac{s^2CL_2}{s^2CL_2 + 1}\right) \quad (1.9)$$

Characteristic equation is given by:

$$1 + G(s)H(s) = 0 \quad (1.10)$$

Substituting the values and simplifying, we get

$$s^3(g_mCL_1L_2 + CL_1L_2) + s^2(RCL_1 + RCL_2) + sL_1 + R = 0 \quad (1.11)$$

Now, for it to oscillate, roots of the equation should lie on imaginary axis, therefore jω should be a solution
Substituting that, we get

$$(R - \omega^2(RC(L_1 + L_2)) + j(\omega L_1 - \omega^3(g_mR + 1)CL_1L_2) = 0 \quad (1.12)$$

Equating Real part to 0

$$\omega^2(RC(L_1 + L_2) = R \quad (1.13)$$

$$\omega = \frac{1}{\sqrt{C(L_1 + L_2)}} \quad (1.14)$$

Equating Imaginary part to 0

$$g_mR + 1 = \frac{C(L_1 + L_2)}{CL_2} \quad (1.15)$$

$$g_mR = \frac{L_1}{L_2} \quad (1.16)$$

Therefore to have stable oscillations, we need $g_mR >= \frac{L_1}{L_2}$

Simulation
For simulation more elaborate circuit of Hartley oscillator was used, i.e. more passive components like capacitors and resistors, so as the the oscillations don't die out quickly, and a voltage source so as the oscillations start.

Below is the circuit which was used 1
Table for parameter values taken Verifying

Parameter	Value
R_1	$1.2k\Omega$
R_2	$50.5k\Omega$
R_3	$10.5k\Omega$
R_3	298Ω
C_1	$22\mu F$
C_2	$22\mu F$
C	$1.1\mu F$
L_1	$1mH$
L_2	$1mH$

TABLE 1

the output:
Plot generated from transfer function, taking impulse response
?? Taking an equivalent R

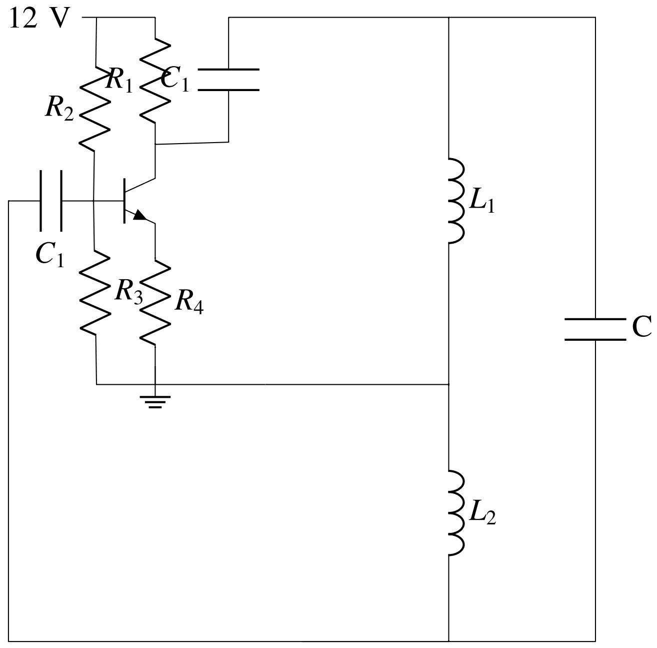

Fig. 1: Simulation circuit

$R = L_1$ and $g_m R = \frac{L_1}{L_2}$

Code for generating impulse response

codes/ee18btech11019_1.py

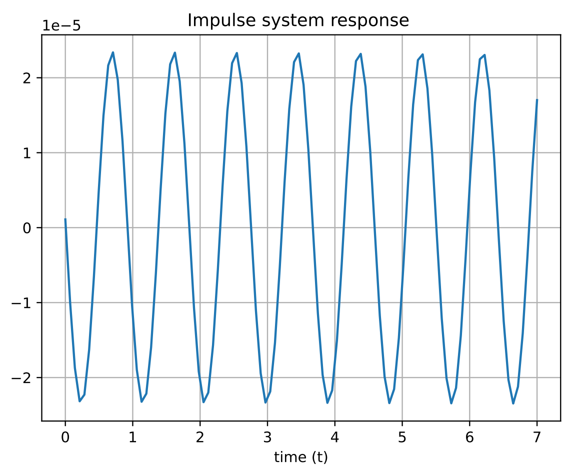

Fig. 1: Output when taken from transfer function

Actual simulation results,
Running ngspice netlist file, we produce dat file. From that data we get plot and frequency from python script found in

spice/ee18btech11019_2.py

Plot:

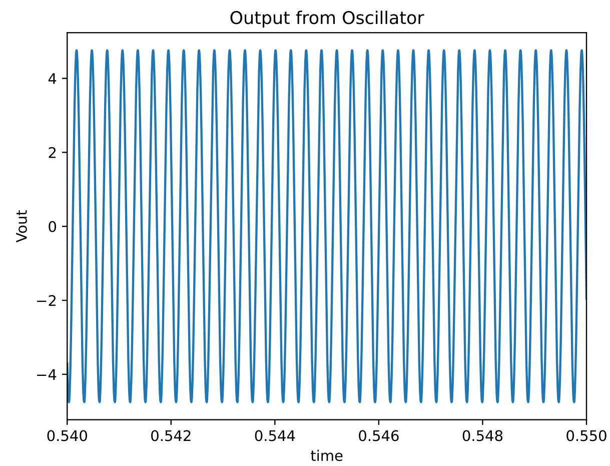

Fig. 1: Simulation result

Frequency obtained is **3384 Hz**
Actual expected frequency is:

$$= \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}} \tag{1.17}$$

$$= 3394Hz \tag{1.18}$$