Hartley Oscillator

Harsh kabra *

1. Find the frequency of oscillation for given hartley circuit and also find condition on g_m. Below is the figure, Fig 1

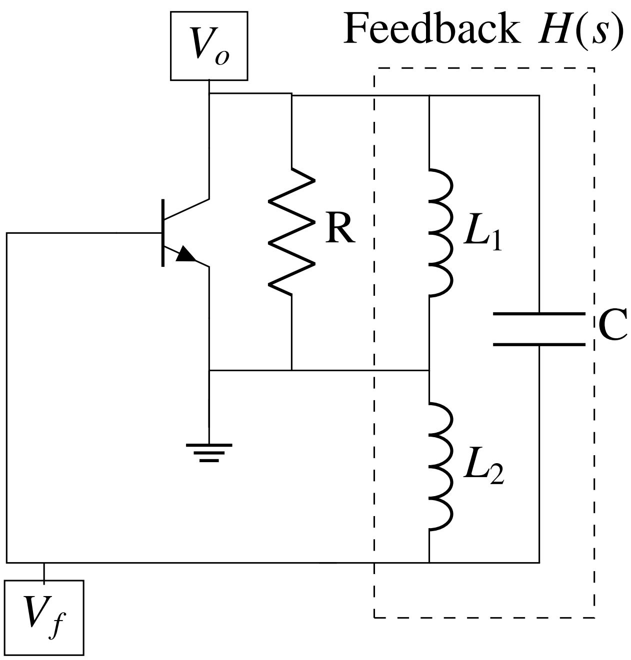

Fig. 1: Hartley oscillator

Solution: We will first draw an equivalent circuit for the above circuit.
To draw an equivalent block diagram we will draw small signal model for transistor.1 And, its block diagram is as follows:

Here G(s) is the amplification gain, and H(s) is the feedback gain.
and G(s) is given by, $\frac{V_o}{V_f}$

$$V_o = I(sL_1 \parallel R) \tag{1.1}$$

$$I = i_1 + g_m V_f \tag{1.2}$$

$$i_1 = \frac{V_\pi}{sL_2} \tag{1.3}$$

$$\tag{1.4}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India. All content in this manual is released under GNU GPL. Free and open source.

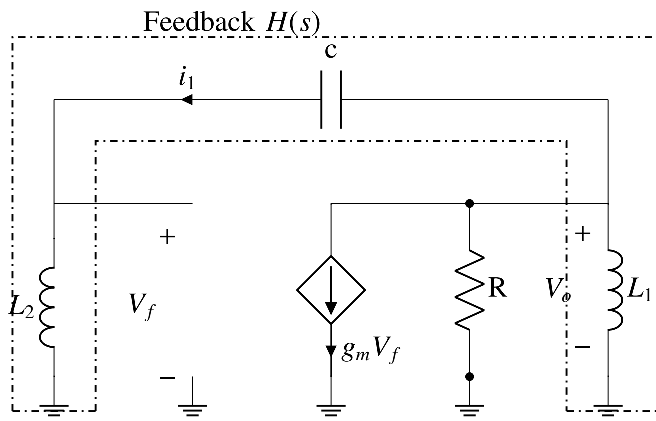

Fig. 1: Small signal model

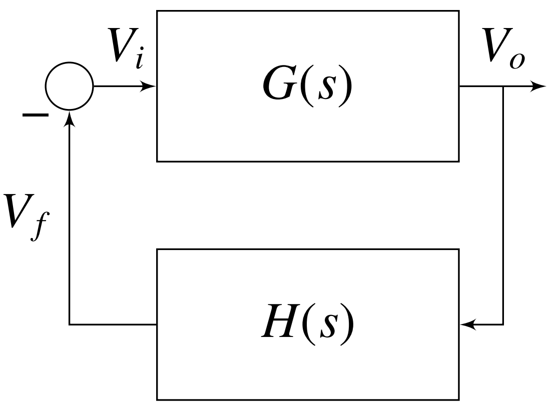

Fig. 1: Block diagram

Solving these equations we get,

$$\frac{V_o}{V_f} = G(s) = \left(g_m + \frac{1}{sL_2}\right)\left(\frac{RsL_1}{R + sL_1}\right) \tag{1.5}$$

Now, solving for H(s)
From the small signal model feedback, we know that H(s) is output/input,
Where,
Output is V_f
Input is V_o

$$H(s) = \frac{V_f}{V_o} \quad (1.6)$$

$$V_0 = V_f + i_1 \times \frac{1}{sC} \quad (1.7)$$

$$i_1 = \frac{V_f}{sL_2} \quad (1.8)$$

$$(1.9)$$

Solving,

$$H(s) = \left(\frac{s^2 CL_2}{s^2 CL_2 + 1} \right) \quad (1.10)$$

Characteristic equation is given by:

$$1 + G(s)H(s) = 0 \quad (1.11)$$

Substituting the values and simplifying, we get

$$s^3(g_m CL_1 L_2 + CL_1 L_2) + s^2(RCL_1 + RCL_2) + sL_1 + R = 0 \quad (1.12)$$

Now, for it to oscillate, roots of the equation should lie on imaginary axis, therefore jω should be a solution
Substituting that, we get

$$(R - \omega^2(RC(L_1 + L_2)) + j(\omega L_1 - \omega^3(g_m R + 1)CL_1 L_2) = 0 \quad (1.13)$$

Equating Real part to 0

$$\omega^2(RC(L_1 + L_2) = R \quad (1.14)$$

$$\omega = \frac{1}{\sqrt{C(L_1 + L_2}} \quad (1.15)$$

Equating Imaginary part to 0

$$g_m R + 1 = \frac{C(L_1 + L_2)}{CL_2} \quad (1.16)$$

$$g_m R = \frac{L_1}{L_2} \quad (1.17)$$

Therefore to have stable oscillations, we need $g_m R >= \frac{L_1}{L_2}$

Simulation
For simulation more elaborate circuit of Hartley oscillator was used, i.e. more passive components like capacitors and resistors, so as the the oscillations don't die out quickly, and a voltage source so as the oscillations start.

Below is the circuit which was used 1

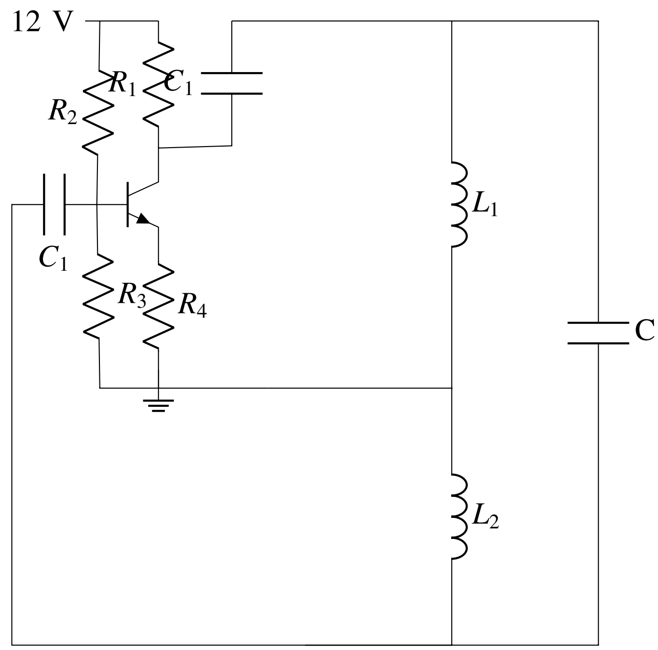

Fig. 1: Simulation circuit

Table for parameter values taken so, while

Parameter	Value
R_1	$1.2k\Omega$
R_2	$50.5k\Omega$
R_3	$10.5k\Omega$
R_3	298Ω
C_1	$22\mu F$
C_2	$22\mu F$
C	$1.1\mu F$
L_1	$1mH$
L_2	$1mH$

TABLE 1

calculating C , it equivalently becomes $45.1\mu F$ in this case
Verifying the output:
Plot generated from transfer function, taking impulse response
?? Taking an equivalent R

$R = L_1$ and $g_m R = \frac{L_1}{L_2}$

Code for generating impulse response

codes/ee18btech11019_1.py

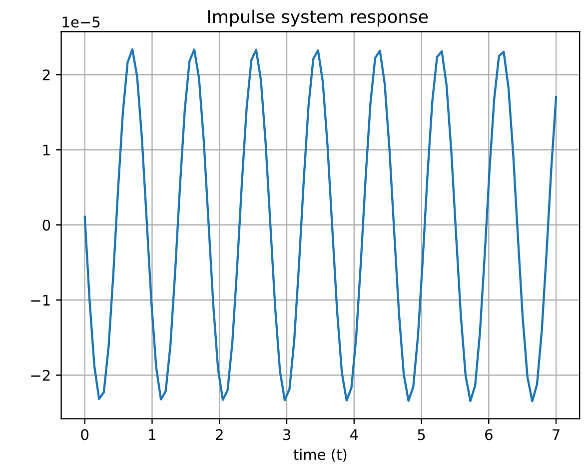

Fig. 1: Output when taken from transfer function

Actual simulation results,
Running ngspice netlist file, we produce dat file. From that data we get plot and frequency from python script found in

spice/ee18btech11019_2.py

Plot:

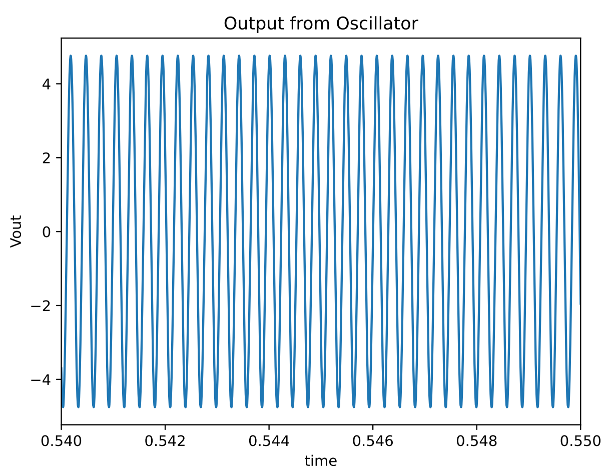

Fig. 1: Simulation result

Frequency obtained is **3384 Hz**
Actual expected frequency is:

$$= \frac{1}{\sqrt{C(L_1 + L_2)}} \tag{1.18}$$

$$= 3333Hz \tag{1.19}$$