

Control Systems

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Abstract—The objective of this manual is to introduce control system design at an elementary level.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/ketan/codes>

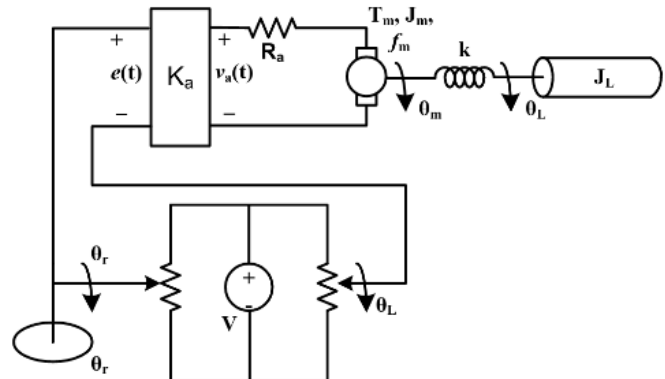


Fig. 1.1

1 DESIGNING A CONTROL SYSTEM

1.1 Example

1.1. A position control system is shown in the figure below. The proportional controller used presently (k_a) is not providing satisfactory performance, Design appropriate controller to achieve maximum overshoot less than 15%. Further, the load should be positioned with 1% accuracy. For the motor to be used, load ad torque-speed curve is shown in figure below. System parameters are $J_m = 2 \text{ kg-m}^2$, $J_L = 10 \text{ kg-m}^2$, $f_m = 2 \text{ N-m-s/rad}$, $k = 12 \text{ N-m/rad}$, sensitivity of error detector is $1/\pi$

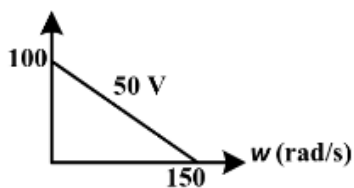


Fig. 1.1

Solution: The following paragraph tells us about the components in the given position control system.

We have,

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Input, it gives constant input $= \theta_r$

Plant, Our original system

Potentiometer, It convert position input and output to voltage.

Controller, to get desired output.

Below is the figure showing them,

Below is the block diagram, for the equivalent

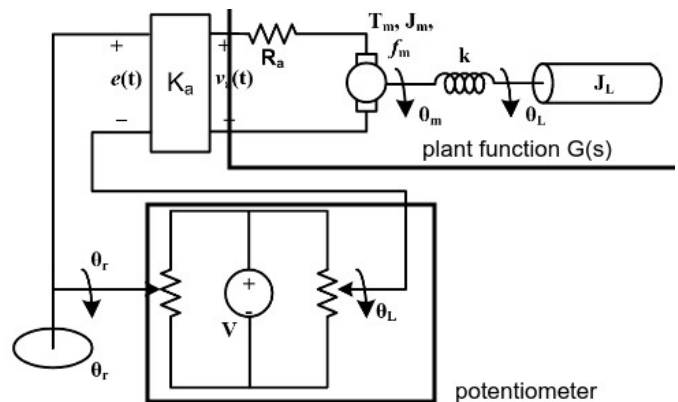


Fig. 1.1

position control system.

Clearly from the question its negative feedback system and given sensitivity of system, it basically gets multiplied after error ($e(t)$), thus giving us a block of gain $\frac{1}{\pi}$

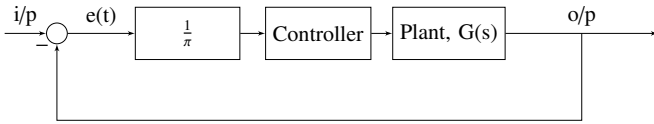


Fig. 1.1: Block Circuit diagram

Calculating plant transfer function G(s):

$V_a(t)$ is the input to the plant and output is θ_L

Parameter	Description
T_m	Torque applied on motor due to armature voltage
R_a	Motor parameter
K_t	Motor parameter
K_b	Motor parameter
V_a	Armature voltage of DC Motor
J_m	Moment of inertia of motor
J_L	Moment of inertia of load
θ_m	Angular position of motor
θ_L	Angular position of Load
f_m	Viscous friction on motor
k	Angular spring constant

TABLE 1.1: parameters description

We know that,

The relation between armature voltage and torque T_m

directly taking result from book control systems engineering by norman-nise, 6th edition. Equation no. 2.156 (on page number 82)

$$T_m(s) \frac{R_a}{K_t} + K_b s \theta_m(s) = V_a(s) \quad (1.1.1)$$

Considering free-body diagram of motor we get,

T_m is counter balanced by

- 1) inertial forces(due to J_m)
- 2) viscous frictional force(f_m)
- 3) the spring force($k\theta_m$),

but also the counter spring force is exerted on motor(in the same direction as T_m) due to presence of finite load ($k\theta_L$)

Below is the equation:

$$(J_m s^2 + f_m s + k) \theta_m(s) - k \theta_L(s) = T_m(s) \quad (1.1.2)$$

Now, considering free body diagram on the load, we get

$K\theta_m$ is the driving force

Inertial force(J_L) and opposing spring force ($K\theta_L$) are the opposing forces.

Below is the equation:

$$(J_L s^2 + k) \theta_L(s) = k \theta_m(s) \quad (1.1.3)$$

Where,

From the graph given in question:

V is 50 V

T_{stall} is torque when $\omega = 0$, i.e. = 100

$\omega_{no-load}$ is ω when T is 0, i.e. = 150

$$\frac{R_a}{K_t} = \frac{V}{T_{stall}} = \frac{50}{100} = \frac{1}{2} \quad (1.1.4)$$

$$k_b = \frac{V}{\omega_{no-load}} = \frac{50}{150} = \frac{1}{3} \quad (1.1.5)$$

On Solving we get,

$$G(s) = \frac{\theta_L(s)}{V_a(s)} \quad (1.1.6)$$

$$G(s) = \frac{1}{\frac{5}{6}s^4 + \frac{10}{9}s^3 + 6s^2 + \frac{4}{3}s} \quad (1.1.7)$$

Now, lets design a controller for the following plant function to get desired output.

- 1) Steady state error is 1%
- 2) Peak overshoot is less than 15%

Steady state error condition:

Considering the closed-loop gain of plant function.

Steady state error is given by,

$$= \frac{G(s)}{1 + G(s)} \quad (1.1.8)$$

We know that, steady state output is given by

$$\lim_{s \rightarrow 0} sR(s) \left(\frac{G(s)}{1+G(s)} \right) \quad (1.1.9)$$

$$R(s) = \frac{1}{s} \quad (1.1.10)$$

$$\text{steady state output} = 1 \quad (1.1.11)$$

Given we have 1% error,

Steady state error is given by,

$$e_{ss} = \lim_{s \rightarrow 0} \left(\frac{sR(s)}{1+G(s)} \right) \quad (1.1.12)$$

Here, as we see we get 0 steady state error, therefore to make it finite we cascade a differentiator to the plant function

Below is the block representation

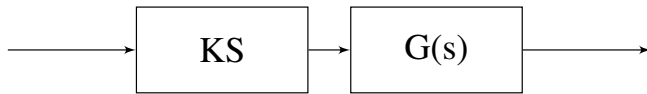


Fig. 1.1: Cascading a differentiator

As, $\frac{1}{\pi}$ is cascade to our plant, so directly multiplying for sake of simplicity.

$$\lim_{s \rightarrow 0} \left(\frac{sR(s)}{1+G'(s)} \right) = 0.01 \quad (1.1.13)$$

$$\lim_{s \rightarrow 0} \frac{1}{1 + \frac{1}{\pi}(KS)G(s)} = 0.01 \quad (1.1.14)$$

Solving,

$$K = 132\pi \quad (1.1.15)$$

Therefore, the overall system as can be written as,

$$G(s) = \frac{132}{\frac{5}{6}s^3 + \frac{10}{9}s^2 + 6s + \frac{4}{3}} \quad (1.1.16)$$

Peak overshoot:

Lets use a PID-controller for this purpose, So, as we want peak overshoot to be less than 15%.

So, this is the final block diagram

General expression for PID-controller given

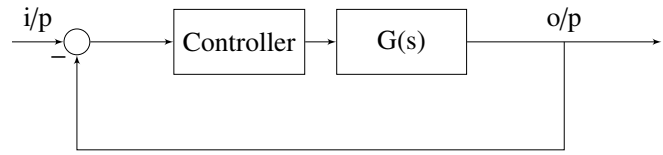


Fig. 1.1: Block diagram so far

by:

$$K_p \left(1 + T_d s + \frac{1}{T_i s} \right) \quad (1.1.17)$$

For it to satisfy our steady state condition, we will take $K_p = 1$.

And also **cascading it with a differentiator of gain = T_i** , to remove the pole and thus maintaining or steady state value.

Final transfer function is given by,

$$= \frac{132 \left(s + T_d s^2 + \frac{1}{T_i} \right) T_i}{\frac{5}{6}s^3 + \frac{10}{9}s^2 + 6s + \frac{4}{3}} \quad (1.1.18)$$

Considering 2nd order approximation.

Relation between %OS = 15% and Damping ratio

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{(\pi)^2 + (\ln(\%OS/100))^2}} \quad (1.1.19)$$

$$\Rightarrow \zeta = 0.517 \quad (1.1.20)$$

Phase Margin for a Damping ratio is given by Eq (1.1.21)

$$\phi_m = 90^\circ - \arctan \left(\frac{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}{2\zeta} \right) \quad (1.1.21)$$

$$\Rightarrow \phi_m = 53.17^\circ \quad (1.1.22)$$

Adding a correction factor, to compensate for the phase gain, to our phase margin we get,

$$\phi_m = 70^\circ \quad (1.1.23)$$

Now, solving for the values,

$$|G(j\omega)| = \frac{132T_i \sqrt{(1/T_i - T_d\omega^2)^2 + \omega^2}}{\sqrt{(4/3 - 10\omega^2/9)^2 + (6\omega^2 - 5\omega^3/6)}} \quad (1.1.24)$$

$$= 1 \quad (1.1.25)$$

And,

$$\arctan\left(\frac{\omega}{1/T_i - T_d\omega^2}\right) - \arctan\left(\frac{6\omega^2 - 5\omega^3/6}{4/3 - 10\omega^2/9}\right) \quad (1.1.26)$$

$$= 70^\circ - 180^\circ = -110^\circ \quad (1.1.27)$$

Solving one the best fits we get is,

$$T_i = \frac{100}{132} \quad (1.1.28)$$

$$T_d = 0.1 \left(\frac{132}{100} \right) \quad (1.1.29)$$

$$\omega = 17.07 \quad (1.1.30)$$

Final transfer function is given by:

$$= \frac{13.2s^2 + 100s + 132}{5/6s^3 + 10/9s^2 + 6s + 4/3} \quad (1.1.31)$$

Below is the figure for the magnitude and phase plot of the function

Following's the code:

```
codes/ee18btech11019_1.py
```

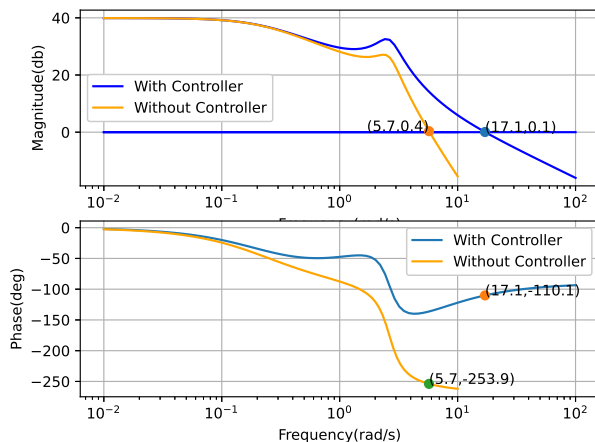


Fig. 1.1: Mag and phase plot comparison

Below is the figure for the output comparison in time domain for unit step input:

Following's the code:

```
codes/ee18btech11019_2.py
```

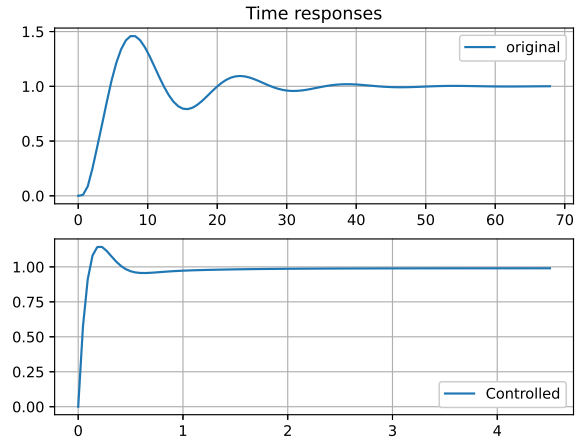


Fig. 1.1: Time response

Here, is the table of expected value and obtained value:

Originally, we had around 50% peak overshoot, without controller

Specification	Expected	Obtained
OS%	15%	14.25%
Steady state error	1%	1.007%

TABLE 1.1: Comparing the expected and Obtained results