Tutorial - 2

Name - Harsh Agarwal

Section - F

Roll no - 59

Univ. Roll no - 2016757

Q1 What is the time complexity of below code?

```
void f (int n)
{   int j=1, i=0;
    while (i<n){
      i+=j;
      j++;
    }
}
```

→ $j=1$ $\quad i=1$

$j=2$ $\quad i=1+2$

$j=3$ $\quad i=1+2+3$

for (i)

$\because$ $1+2+3+\dots+<n$

$\because$ $1+2+3+m<n$

$\because$ $\frac{m(m+1)}{2}<n$

$m \simeq \sqrt{n}$

by summation method

$\Rightarrow \sum_{i=1}^{m} \Rightarrow 1+1+\dots+\sqrt{n}$ times

$T(n)=\sqrt{n}$

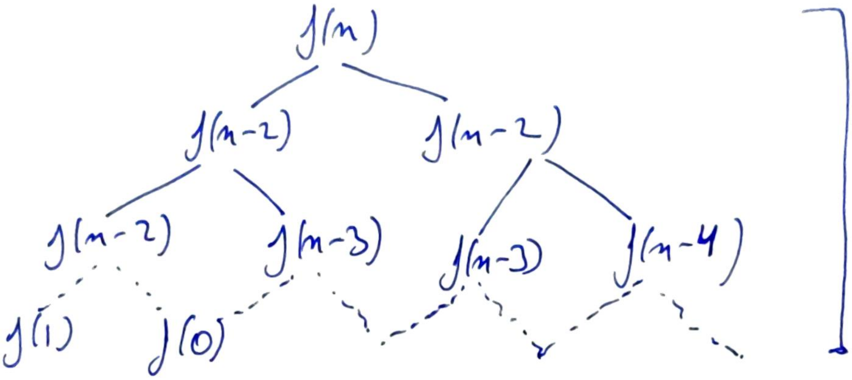
Q2: Write recurrence relation for function that prints Fibonacci series. Solve it to get the time complexity. What will be the space complexity & why?

→ For Fibonacci series

$f(n) = f(n-1) + f(n-2)$

$f(0) = 0$
$f(1) = 1$

By forming a tree



∵ At every function call we get 2 function calls

∴ for n levels
we have $= 2 \times 2 \ldots$ n times

∴ $\underline{T(n) = 2^n}$

Maximum Space Considering Recursive

Stack:
no. of calls maximum = n
For each call we have space complexity $O(1)$.
∴ $T(n) = O(n)$

without considering Recursive stack:
each call we have time complexity $O(1)$
∴ $T(n) = \underline{O(1)}$

Q3: Write programs which have complexity: $n(\log n)$, n^3, $\log(\log n)$

1) $n \log n$ → Quick sort

```
void quicksort (int arr[], int low, int high)
{   if (low < high)
    {
        int pi = partition (arr, low, high);
        quicksort (arr, low, pi-1)
        quicksort (arr, pi+1, high);

int partition (int arr [], int low, int high)
{
    int pivot = arr[high];
    int i = (low -1);
    for (int j = low; j <= high -1; j++)
    {
        if (arr[i] < pivot)
        {
            i++;
            Swap (& arr[i], & arr[j]);
        }
    }
```

2) n^3 → Multiplication of 2 square matrix:

```
for (i = 0; i < r1; i++)
    for (j = 0; j < c2; j++)
        for (k = 0; k < c1; k++)
            res[i][j] += a[i][k] * b[k][j];
```

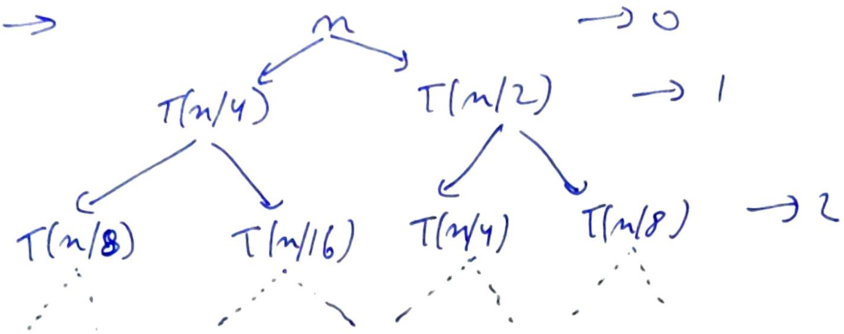
3) $\log(\log n)$

```
for (i = 2; i < n; i = i*i)
    count++;
```

Q4. Solve the foll. recurrence relation:

$$T(n) = T(n/4) + T(n/2) + Cn^2$$

→



At level

$0 \rightarrow Cn^2$

$1 \rightarrow \frac{n^2}{4^2} + \frac{n^2}{2^2} = \frac{C5n^2}{16}$

$2 \rightarrow \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2 C$

⋮

max level $= \frac{n}{2^k} = 1$

$= k = \log_2 n$

$$T(n) = C\left(n^2 + \left(\frac{5}{16}\right)n^2 + \left(\frac{5}{16}\right)^2 n^2 + \dots + (5/16)^{\log n} n^2\right)$$

$$T(n) = Cn^2\left[1 + \left(\frac{5}{16}\right) + \left(\frac{5}{16}\right)^2 + \dots + \left(\frac{5}{16}\right)^{\log n}\right]$$

$$T(n) = Cn^2 \times 1 \times \left(\frac{1 - (5/16)^{\log n}}{1 - (5/16)}\right)$$

$$T(n) = Cn^2 \times \frac{11}{5} \times \left(1 - \left(\frac{5}{16}\right)^{\log n}\right)$$

$$T(n) = O(n^2C)$$

T.C $= O(Cn^2)$

Q5: What is the time complexity of following fun()?

```
int fun (int n) {
for (int i = 1; i <= n; i++) {
 for (int j = 1; j < n; j += i) {
  // anything of O(1) task
}}}
```

→ for

i	j
1	1
2	1+3+5
3	1+4+7
⋮	
n	1+5+9

$j = (n-1)/i$ times

$$\sum_{i=1}^{n} \frac{(n-1)}{i}$$

$$\therefore\ T(n) = \frac{(n-1)}{1} + \frac{(n-1)}{2} + \frac{(n-1)}{3} + \cdots + \frac{(n-1)}{n}$$

$$T(n) = n\,[1 + 1/2 + 1/3 + \cdots + 1/n] - 1 \times [1 + 1/2 + 1/3 + \cdots 1/n]$$

$$= n\log n - \log n$$

$$T(n) = O(n \log n)$$

Q6: What should be time complexity of

```
for (int i = 2; i < n; i = pow(i, k))
{
    // some O(1)
}
```

where k is a constant

→ for

i
2^1
2^k
2^{k^2}
2^{k^3}
⋮
2^{k^m}

where

$2^{k^m} <= n$

$k^m = \log_2 n$

$m = \log_k \log_2 n$

$\therefore \sum_{i=1}^{m} 1$

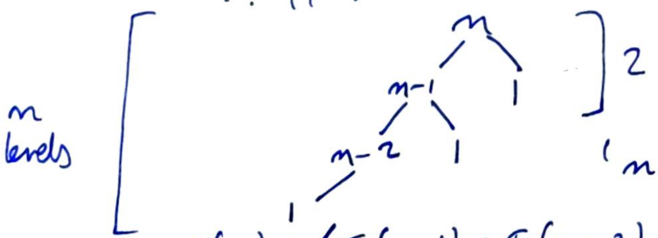
$1+1+1 \ldots \ldots m$ times

$T(n) = O(\log_k \log n)$

Q7. Write a recurrence relation when quick sort repeatedly divides array into 2 parts of 99% and 1%. Derive time complexity in this case. Show the recurrence tree while deriving time complexity & find difference in heights of both extreme parts. What do you understand by this analysis?

→ Given algorithm divides array in 99% and 1% part.

$\therefore T(n) = T(n-1) + O(1)$



'n' work is done at each level.

$T(n) = (T(n-1) + T(n-2) + \cdots + T(1) + O(1)) \times n$

$= n \times n$

$\therefore T(n) = O(n^2)$

Lowest height $= 2$, heighest height $= n$

$\therefore$ difference $= n-2$ $\quad n > 1$

The given algorithm produces linear result.

Q8. Arrange the foll. in increasing order of rate of growth.

a) $n, n!, \log n, \log\log n$, root$(n)$, $\log(n!), n\log n, \log^2(n)$, $2^{2^n}, 4^n, n^2, 100$.

$\rightarrow 100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log(n!) <$
$n^2 < 2^n < 4^n < 2^{2^n}$

b) $2(2^n), 4n, 2n, 1, \log(n), \log(\log(n)), \sqrt{\log(n)}, \log 2n$,
$2\log(n), n, \log(n!), n!, n^2, n\log(n)$

$\rightarrow 1 < \log\log n < \sqrt{\log n} < \cancel{\log n} < \log n < \log 2n < 2\log n <$
$n < n\log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{2^n}$

c) $8^{2n}, \log_2(n), n\log_6(n), n\log_2(n), \log(n!), n!$,
$\log_8(n), 96, 8n^2, 7n^3, 5n$.

$\rightarrow 96 < \log_8 n < \log 2n < 5n < n\log_6(n) < n\log_2 n <$
$\log(n!) < 8n^2 < 7n^3 < n! < 8^{2n}$.