Tutorial-4

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Q1. $T(n) = 3T(n/2) + n^2$

$\rightarrow T(n) = aT(n/b) + f(n^2)$

$\rightarrow a \geq 1, b \geq 1$

on comparing,

$a = 3, b = 2, f(n) = n^2$

Now, $c = \log_b a = \log_2 3 = 1.584$

$n^2 = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = O(n^2)$

Q2. $T(n) = 4T(n/2) + n^2$

$\rightarrow a \geq 1, b > 1$

$a = 4, b = 2, f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

Q3. $T(n) = T(n/2) + 2^n$

$\rightarrow a = 1$

$b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 c = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

Q4. $T(n) = 2^n T(n/2) + n^n$

$\rightarrow a = 2^n$

$b = 2, f(n) = n^2$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c \Rightarrow n^n$

$f(n) = n^c$

$f(n) = \Theta(n^2 \log_2 n)$

Q5. $T(n) = 16T(n/4) + n$

$\rightarrow a = 16, b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 4 = 2\log_4 4$

$= 2^4$

$n^c \Rightarrow n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

Q6. $T(n) = 2T(n/2) + n \log n$

$\rightarrow a = 2, b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n > n$

$f(n) > n^c$

$T(n) = \Theta(n \log n)$

Mark

$T(n) = 2T(n/2) + n/\log n$

$\rightarrow a=2, b=2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\therefore \frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

Q8: $T(n) = 2T(n/4) + n^{0.51}$

$\rightarrow a=2, b=4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$\because n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

Q9: $T(n) = 0.5T(n/2) + 1/n$

$\rightarrow a = 0.5, b=2$

$a \geq 1$ but here a is 0.5 so we cannot apply Master's Theorem.

Q10: $T(n) = 16T(n/4) + n!$

$\rightarrow a=16, b=4, f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

Q11: $4T(n/2) + \log n$

$\rightarrow a=4, b=2, f(n) = \log n$

$c = \log_b a = \log_2 n = 2$

$n^c = n^2$

$f(n) = \log n$

$\because \log n < n^2$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$= \Theta(n^2)$

Q12: $T(n) = \text{sqrt}(n)\,T(n/2) + \log n$

$\rightarrow a = \sqrt{n}, b=2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) > n^c$

$T(n) = \Theta(f(n))$

$= \Theta(\log(n))$

Q13: $T(n) = 3T(n/2) + n$

$\rightarrow a=3; b=2; f(n) = n$

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584}$

$n < n^{1.584}$

$\Rightarrow f(n) = n^c$

$T(n) = \Theta(n^{1.584})$

Q14: $T(n) = 3T(n/3) + \text{sqrt}(n)$

$\rightarrow a=3, b=3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As $\text{sqrt}(n) < n$

$f(n) \leq n^c$

$T(n) = \Theta(n)$

$a = 4, \; b = 2$

$c = \log_b a = \log_2 4 = 2$

$n < n^2$ (for any constant)

$f(n) < n^c$

$f(n) = O(n^c)$

Q16. $a = 3, \; b = 4, \; f(n) = n \log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n \log n$

$T(n) = O(n \log n)$

Q17. $a = 3 \quad b = 3$

$c = \log_b a = \log_3 3 = 1$

~~$c = \log_b$~~ $f(n) = n/2$

$\therefore n/2 < n$

$f(n) < n^c$

$\therefore T(n) = \Theta(n)$

Q18. $a = 6, \; b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

$n^{1.6309} < n^2 \log n$

$\therefore T(n) = O(n^2 \log n)$

Q19.

→ $a = 4,\ b = 2\ \ f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \Theta(n^2)$

Q20.

→ $a = 64,\ b = 2$

$c = \log_b a = \log_2 64 = \log_2 (2)^2 = 2$

$c = 2$

$n^c = n^2$

$n^2 \log n > n^2$

$T(n) = \Theta(n^2 \log n)$

Q21.

→ $a = 7,\ b = 3,\ f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.712$

$n^c = n^{1.712}$

$n^{1.712} < n^2$

$\therefore T(n) = \Theta(n^2)$

Q22.

→ $a = 1,\ b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos x) > n^c$

$T(n) = \Theta(n(2 - \cos x))$

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