Assignment-5 Terence Tao: AI & Mathematics

Harshvardhan Patidar

Department of Artificial Intelligence Indian Institute of Technology Hyderabad ai24btech11015@iith.ac.in

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1 Terence Tao

He is one of the most influential mathematicians. He first went to the International Mathematics Olympiad (IMO) at the age of 11 and won a bronze medal. Next year he won a silver medal, and finally at the age of 13 years, he won a gold medal in the IMO. He didn't think of it as a competition but as a fun activity.

2 AI & Mathematics

Machines have been put to assist humans in mathematics for a long time. They have also used human computers, consisting of several humans, mostly women, sitting together and performing the specific calculations they have been asked to do. It enabled many people to work simultaneously on the same problem. These computers were mostly used to prepare the tables such as the logarithm and trigonometric tables.

Nowadays, such modern tables are very useful, such as the Online Encyclopedia of Integer Sequence (OEIS), has mostly all of the discovered sequences, Terence Tao thinks of it as a great tool to check if the sequence you're working on is already found by someone to ease your work. These large databases can also be used as input for training the AI using ML algorithms. The computers have also played a significant role in development of prime number conjectures.

He remarks that Modern computer algebra systems such as Magma, Maple etc. and some general-purpose programming languages are now being routinely used in mathematics. There are also new systems like the Satisfiability (SAT) Solver and Satisfiability modulo theories (SMT) solvers, which are capable of performing complex logical deductions from certain restricted sets of hypotheses and generate proof for them. He talks of one such result, the "Boolean Pythagorean triples theorem", which required 4 CPU-years of computation and generated 200-terabytes of data.

He talks about how Proof assistant helped the researchers to prove the Kepler Conjecture which said that the hexagonal close packing was the densest packing in 3D. But there were uncertainties regarding this proof. So, in order to dispel the doubts, Hales, along with 21 collaborators, proved the conjecture in a proof assistant and verified it using a computer itself rather than human. It used Isabelle and HOL light. Another such example is the verification of liquid tensor experiment of Peter Scholze, which required adding some more foundational mathematical theories to lean's mathlib library.

He also talked about the programs which help to convert these formal proofs to human readable form. He is positive regarding the future of such programs and also believes that future textbooks

will also be more innovative, in which you would click and know more about a particular topic right away. He also talks about the fact that in such proof assistants, you can have a lot of contributors without worrying about their mathematical skills. Even if they supply something bad, the program won't compile and throw some error. It doesn't even require all of them to have full knowledge of the subject, they can independently work on their part.

He then talks about Machine Learning (ML), and the Knot Theory. He says that by training a neural network on data of over 2 million knots, they found that the network then could predict the signature from the hyperbolic invariants with high accuracy. Then he finally brings LLM into the discussion. He talks of the capability of GPT-4 to solve a question that appeared in 2022 IMO. He talks about how sometimes it worked like a charm but some other times just acted crap. He talks of ways to improve the LLM in Mathematics by combining it with classical tools like Wolfram Alpha, and using the feedback from a proof assistant to reinforce the training model.

He also talks about Github Co-Pilot, which he personally uses. He says that the github co-pilot works accurately 20% of the time and is a great support. He is for now little unsure if the computer can themselves be able to resolve the major mathematical theorems on their own. But he is certain of their progress as they are increasingly being used to assist humans in various ways, be it brute-forcing or computation.

References

[1] Terence Tao. Ai and mathematics - terence tao at imo 2024, 2023. watch.