MA - 2019

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1) Let $1 \leq p < q < \infty$. Consider the followings statements:
   I. $l^p \subset l^q$
   II. $L^p[0,1] \subset L^q[0,1]$,
   where $l^p = \{(x_1, x_2, \dots) : x_i \in \mathbb{R}, \sum_{i=1}^{\infty} |x_i|^p < \infty\}$ and
$$L^p[0,1] = \left\{ f : [0,1] \to \mathbb{R} : f \text{ is } \mu - \text{measurable}, \right.$$
$$\left. \int_{[0,1]} |f|^p \, d\mu < \infty, \text{where } \mu \text{ is the Lebesgue measure} \right\}$$
   ($\mathbb{R}$ is the set of all real numbers)
   Which of the following statements is/are TRUE?
   a) Both I and II
   b) I only
   c) II only
   d) Neither I nor II
2) Consider the differential equation
$$t\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + ty = 0, t > 0, y(0+) = 1, \left(\frac{dy}{dt}\right)_{t=0+} = 0$$
   If $Y(s)$ is the Laplace transform of $y(t)$, then the value of $Y(1)$ is ____ (round off to 2 places of decimal).
   (Here, the inverse of trigonometric functions assume principal values only)
3) Let R be the region in the xy-plane bounded by the curves $y = x^2$, $y = 4x^2$, $xy = 1$ and $xy = 5$.
   Then the value of the integral $\int\int_R \frac{y^2}{x} dydx$ is equal to ____.
4) Let V be the vector space of all 3×3 matrices with complex entries over the real field. If
$$W_1 = \left\{A \in V : A = \bar{A}^T\right\} \text{ and } W_2 = \{A \in V : \text{trace of } A = 0\},$$
   then the dimension of $W_1 + W_2$ is equal to ____.
   ($\bar{A}^T$ denotes the conjugate transpose of A)
5) The number of elements of order 15 in the additive group $\mathbb{Z}_{60} \times \mathbb{Z}_{50}$ is ____.
   ($\mathbb{Z}_n$ denotes the group of integers modulo n, under the operation of addition modulo n, for any positive integer n)
6) Consider the following cost matrix of assigning four jobs to four persons:
   Then the minimum cost of the assignment problem subject to the constraint that job J_4 is assigned to the person P_2, is ____.

		Jobs			
		J_1	J_2	J_3	J_4
Persons	P_1	5	8	6	10
	P_2	2	5	4	8
	P_3	6	7	6	9
	P_4	6	9	8	10

TABLE 6

7) Let $y : [-1, 1] \to \mathbb{R}$ with $y(1) = 1$ satisfy the Legendre differential equation

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0 \text{ for } |x| < 1.$$

Then the value of $\int_{-1}^{1} y(x)\left(x + x^2\right)dx$ is equal to ____ (round off to 2 places of decimal).

8) Let $\mathbb{Z}_{125}$ be the ring of integers modulo 125 under the operations of addition modulo 125 and multiplication modulo 125. If m is the number of maximal ideals of $\mathbb{Z}_{125}$ and n is the number of non-units of $\mathbb{Z}_{125}$, then $m + n$ is equal to ____.

9) The maximum value of the error term of the composite Trapezoidal rule when it is used to evaluate the definite integral

$$\int_{0.2}^{1.4} (\sin x - \log_e x)\, dx$$

with 12 sub-intervals of equal length, is equal to ____ (round off to 3 places of decimal).

10) By the Simplex method, the optimal table of the linear programming problem:

$$\begin{aligned} \text{Maximize } Z &= \alpha x_1 + 3x_2 \\ \text{subject to } \beta x_1 + x_2 + x_3 &= 8, \\ 2x_1 + x_2 + x_4 &= \gamma \\ x_1, x_2, x_3, x_4 &\geq 0, \end{aligned}$$

where α, β, γ are real constants, is

$C_j \to$	α	3	0	0	
Basic Variable	x_1	x_2	x_3	x_4	Solution
x_2	1	0	2	-1	6
x_1	0	1	-1	1	2
$z_j - c_j$	0	0	2	1	-

TABLE 10

Then the value of $\alpha + \beta + \gamma$ is ____.

11) Consider the inner product space P_2 of all polynomials of degree at most 2 over the field of real numbers with the inner product $\langle f, g\rangle = \int_0^1 f(t)\, g(t)\, dt$ for $f, g \in P_2$.

Let $\{f_0, f_1, f_2\}$ be an orthogonal set in P_2, where $f_0 = 1, f_1 = t + c_1, f_2 = t^2 + c_2 f_1 + c_3$ and c_1, c_2, c_3 are real constants. Then the value of $2c_1 + c_2 + 3c_3$ is equal to ____.

12) Consider the system of linear differential equations

$$\frac{dx_1}{dt} = 5x_1 - 2x_2,$$
$$\frac{dx_2}{dt} = 4x_1 - x_2,$$

with the initial conditions $x_1(0) = 0, x_2(0) = 1$.
Then $\log_e(x_2(2) - x_1(2))$ is equal to ____.

13) Consider the differential equation

$$x\left(1 + x^2\right)\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 7y = 0.$$

The sum of the roots of the indicial equation of the Frobenius series solution for the above differential equation in a neighborhood of $x = 0$ is equal to ____.