Matrices and Determinants

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Section-A — JEE Advanced/ IIT-JEE

1 Fill in the Blanks:

1) How many 3×3 matrices M with entries from $(0, 1, 2)$ are there, for which the sum of the diagonal entries of $M^T M$ is 5?

a) 126
b) 198
c) 162
d) 135

(JEE Adv. 2017)

2) Let $M = \begin{vmatrix} \sin^4(\theta) & -1 - \sin^2(\theta) \\ 1 + \cos^2(\theta) & \cos^4(\theta) \end{vmatrix} = \alpha I + \beta M^{-1}$
Where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If a^* is the minimum of the set $(\alpha(\theta) : \theta \in [0, 2\pi))$ and b^* is the minimum of the set $(\beta(\theta) : \theta \in [0, 2\pi))$. Then the value of $a^* + b^*$ is

a) $-\frac{31}{16}$
b) $-\frac{17}{16}$
c) $-\frac{37}{16}$
d) $-\frac{29}{16}$

(JEE Adv. 2019)

2 MCQs with More than One Correct

1) The determinant $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$ is equal to zero, if

a) a, b, c are in A.P.
b) a, b, c are in G.P.
c) a, b, c are in H.P.
d) α is a root of the equation $ax^2 + bx + c = 0$
e) $(x - \alpha)$ is a factor of $ax^2 + bx + c$

(1986-2 Marks)

2) If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

a) $x = 3, y = 1$
b) $x = 1, y = 3$

c) $x = 0, y = 3$
d) $x = 0, y = 0$

(1998-2 Marks)

3) Let M and N be two 3×3 non-singulr skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P, then $M^2N^2\left(M^TN^{-1}\right)^{-1}\left(MN^{-1}\right)^T$ is equal to
a) M^2
b) $-N^2$
c) $-M^2$
d) MN

(2011)

4) If the adjoint of a 3×3 matrix P is $\begin{pmatrix}1 & 4 & 4\\ 2 & 1 & 7\\ 1 & 1 & 3\end{pmatrix}$, then the possible value(s) of the determinant of P is (are)
a) -2
b) -1
c) 1
d) 2

(2012)

5) For 3×3 matrices M and N, which of the following statement(s) is (are) NOT correct?
a) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric
b) $MN - NM$ is skew symmetric for all matrices M and N.
c) MN is symmetric for all symmetric matrices M and N.
d) $(\text{adj}M)(\text{adj}N) = \text{adj}(MN)$ for all invertible matrices M and N.

(JEE Adv. 2013)

6) Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = p_{ij}$ be a $n\times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $p^2 \neq 0$, when $n =$
a) 57
b) 55
c) 58
d) 56

(JEE Adv. 2013)

7) Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if
a) The first column of M is the transpose of the second row of M
b) The second row of M is the transpose of the first column of M
c) M is a diagonal matrix with non-zero entries in the main diagonal
d) The product of entries in the main diagonal of M is not the square of an integer

(JEE Adv. 2014)

8) Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then
a) determinant of $(M^2 + N^2)$ is 0
b) there is 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

c) determinant of $(M^2 + MN^2) \geq 1$
d) determinant of $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

(JEE Adv. 2014)

9) Which of the following values of α satisfy the equation $\begin{vmatrix}(1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2\\(2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2\\(3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2\end{vmatrix} = -648\alpha$?
a) -4
b) 9
c) -9
d) 4

(JEE Adv. 2015)

10) Let X and Y be two arbitrary, 3×3, non-zero, skew-symmetric matrices and Z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?
a) $Y^3Z^4 - Z^4Y^3$
b) $X^{44} + Y^{44}$
c) $X^4Z^3 - Z^3X^4$
d) $X^{23} + Y^{23}$

(JEE Adv. 2015)

11) Let $P = \begin{pmatrix}3 & -1 & -2\\2 & 0 & \alpha\\3 & -5 & 0\end{pmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = \left[q_{ij}\right]$ is a matrix such that $PQ = kI$, where $k \in \mathbb{R}, k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and det(Q)= $\frac{k^2}{2}$, then
a) $a = 0, k = 8$
b) $4a - k + 8 = 0$
c) det $(Padj(Q)) = 2^9$
d) det $(Qadj(P)) = 2^{13}$

(JEE Adv. 2016)

12) Let $a, \lambda, \mu, \in \mathbb{R}$. Consider the system of linear equations

$$ax + 2y = \lambda$$

$$3x - 2y = \mu$$

Which of the following statement(s) is (are) correct?
a) If $a = -3$, then the system has infinitely many solutions for all value of λ and μ.
b) If $a \neq -3$, then the system has unique solution for all values of λ and μ.
c) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$.
d) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$

(JEE Adv. 2016)

13) Which of the following is (are) not the square of a 3×3 matrix with real entries?

a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

d) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(JEE Adv. 2017)