

# Matrices and Determinants

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Section-A — JEE Advanced/ IIT-JEE

## I. FILL IN THE BLANKS:

- 1) How many  $3 \times 3$  matrices  $M$  with entries from  $(0, 1, 2)$  are there, for which the sum of the diagonal entries of  $M^T M$  is 5?

- a) 126
- b) 198
- c) 162
- d) 135

(JEE Adv. 2017)

- 2) Let  $M = \begin{vmatrix} \sin^4 \theta & -1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{vmatrix} = \alpha I + \beta M^{-1}$

Where  $\alpha = \alpha(\theta)$  and  $\beta = \beta(\theta)$  are real numbers, and  $I$  is the  $2 \times 2$  identity matrix. If  $a^*$  is the minimum of the set  $(\alpha(\theta) : \theta \in [0, 2\pi))$  and  $b^*$  is the minimum of the set  $(\beta(\theta) : \theta \in [0, 2\pi))$ . Then the value of  $a^* + b^*$  is

- a)  $-\frac{31}{16}$
- b)  $-\frac{17}{16}$
- c)  $-\frac{37}{16}$
- d)  $-\frac{29}{16}$

(JEE Adv. 2019)

## II. MCQs WITH MORE THAN ONE CORRECT

- 1) The determinant  $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$  is equal to zero, if

- a)  $a, b, c$  are in A.P.
- b)  $a, b, c$  are in G.P.
- c)  $a, b, c$  are in H.P.
- d)  $\alpha$  is a root of the equation  $ax^2 + bx + c = 0$
- e)  $(x - \alpha)$  is a factor of  $ax^2 + bx + c$

(1986-2 Marks)

- 2) If  $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$ , then

- a)  $x = 3, y = 1$
- b)  $x = 1, y = 3$
- c)  $x = 0, y = 3$
- d)  $x = 0, y = 0$

(1998-2 Marks)

- 3) Let  $M$  and  $N$  be two  $3 \times 3$  non-singular skew-symmetric matrices such that  $MN = NM$ . If  $P^T$  denotes the transpose of  $P$ , then  $M^2 N^2 (M^T N^{-1})^{-1} (MN^{-1})^T$  is equal to

- a)  $M^2$
- b)  $-N^2$
- c)  $-M^2$
- d)  $MN$

(2011)

- 4) If the adjoint of a  $3 \times 3$  matrix  $P$  is  $\begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$ , then the possible value(s) of the determinant of  $P$  is (are)

- a)  $-2$
- b)  $-1$
- c)  $1$
- d)  $2$

(2012)

- 5) For  $3 \times 3$  matrices  $M$  and  $N$ , which of the following statement(s) is (are) NOT correct?

- a)  $N^T M N$  is symmetric or skew symmetric, according as  $M$  is symmetric or skew symmetric
- b)  $MN - NM$  is skew symmetric for all matrices  $M$  and  $N$ .
- c)  $MN$  is symmetric for all symmetric matrices  $M$  and  $N$ .
- d)  $(\text{adj} M)(\text{adj} N) = \text{adj}(MN)$  for all invertible matrices  $M$  and  $N$ .

(JEE Adv. 2013)

- 6) Let  $\omega$  be a complex cube root of unity with  $\omega \neq 1$  and  $P = p_{ij}$  be a  $n \times n$  matrix with

$p_{ij} = \omega^{i+j}$ . Then  $p^2 \neq 0$ , when  $n =$

- a) 57
- b) 55
- c) 58
- d) 56

(JEE Adv. 2013)

7) Let  $M$  be a  $2 \times 2$  symmetric matrix with integer entries. Then  $M$  is invertible if

- a) The first column of  $M$  is the transpose of the second row of  $M$
- b) The second row of  $M$  is the transpose of the first column of  $M$
- c)  $M$  is a diagonal matrix with non-zero entries in the main diagonal
- d) The product of entries in the main diagonal of  $M$  is not the square of an integer

(JEE Adv. 2014)

8) Let  $M$  and  $N$  be two  $3 \times 3$  matrices such that  $MN = NM$ . Further, if  $M \neq N^2$  and  $M^2 = N^4$ , then

- a) determinant of  $(M^2 + N^2)$  is 0
- b) there is  $3 \times 3$  non-zero matrix  $U$  such that  $(M^2 + MN^2)U$  is the zero matrix
- c) determinant of  $(M^2 + MN^2) \geq 1$
- d) determinant of  $(M^2 + MN^2)U$  equals the zero matrix then  $U$  is the zero matrix

(JEE Adv. 2014)

9) Which of the following values of  $\alpha$  satisfy

the equation  $\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$  ?

- a) -4
- b) 9
- c) -9
- d) 4

(JEE Adv. 2015)

10) Let  $X$  and  $Y$  be two arbitrary,  $3 \times 3$ , non-zero, skew-symmetric matrices and  $Z$  be an arbitrary  $3 \times 3$ , non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

- a)  $Y^3Z^4 - Z^4Y^3$
- b)  $X^{44} + Y^{44}$

c)  $X^4Z^3 - Z^3X^4$

d)  $X^{23} + Y^{23}$

(JEE Adv. 2015)

11) Let  $P = \begin{vmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{vmatrix}$ , where  $\alpha \in \mathbb{R}$ . Suppose

$Q = [q_{ij}]$  is a matrix such that  $PQ = kI$ , where  $k \in \mathbb{R}, k \neq 0$  and  $I$  is the identity matrix of order 3. If  $q_{23} = -\frac{k}{8}$  and  $\det(Q) = \frac{k^2}{2}$ , then

- a)  $a = 0, k = 8$
- b)  $4a - k + 8 = 0$
- c)  $\det(P \text{adj}(Q)) = 2^9$
- d)  $\det(Q \text{adj}(P)) = 2^{13}$

(JEE Adv. 2016)

12) Let  $a, \lambda, \mu \in \mathbb{R}$ . Consider the system of linear equations

$$ax + 2y = \lambda$$

$$3x - 2y = \mu$$

Which of the following statement(s) is (are) correct?

- a) If  $a = -3$ , then the system has infinitely many solutions for all value of  $\lambda$  and  $\mu$ .
- b) If  $a \neq -3$ , then the system has unique solution for all values of  $\lambda$  and  $\mu$ .
- c) If  $\lambda + \mu = 0$ , then the system has infinitely many solutions for  $a = -3$ .
- d) If  $\lambda + \mu \neq 0$ , then the system has no solution for  $a = -3$

(JEE Adv. 2016)

13) Which of the following is (are) not the square of a  $3 \times 3$  matrix with real entries?

- a)  $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$
- b)  $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$
- c)  $\begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$
- d)  $\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$

(JEE Adv. 2017)