Matrices and Determinants

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Section-A — JEE Advanced/ IIT-JEE

3) Fill in the Blanks:

a) How many 3×3 matrices M with entries from $(0, 1, 2)$ are there, for which the sum of the diagonal entries of M^TM is 5?

i) 126
ii) 198
iii) 162
iv) 135

($JEEAdv.2017$)

b) Let $M = \begin{vmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{vmatrix} = \alpha I + \beta M^{-1}$
Where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If a^* is the minimum of the set $(\alpha(\theta) : \theta \in [0, 2\pi))$ and b^* is the minimum of the set $(\beta(\theta) : \theta \in [0, 2\pi))$. Then the value of $a^* + b^*$ is

i) $-\frac{31}{16}$
ii) $-\frac{17}{16}$
iii) $-\frac{37}{16}$
iv) $-\frac{29}{16}$

($JEEAdv.2019$)

4) MCQs with More than One Correct

a) The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ is equal to zero, if

i) a, b, c are in A.P.
ii) a, b, c are in G.P.
iii) a, b, c are in H.P.
iv) α is a root of the equation $ax^2+bx+c = 0$
v) $(x - \alpha)$ is a factor of $ax^2 + bx + c$

($1986 - 2Marks$)

b) If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

i) $x = 3, y = 1$
ii) $x = 1, y = 3$
iii) $x = 0, y = 3$
iv) $x = 0, y = 0$

($1998 - 2Marks$)

c) Let M and N be two 3×3 non-singulr skew-symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2\left(M^TN^{-1}\right)^{-1}\left(MN^{-1}\right)^T$ is equal to

i) M^2
ii) $-N^2$
iii) $-M^2$
iv) MN

(2011)

d) If the adjoint of a 3×3 matrix P is $\begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$, then the possible value(s) of the determinant of P is (are)

i) -2
ii) -1
iii) 1
iv) 2

(2012)

e) For 3×3 matrices M and N, which of the following statement(s) is (are) NOT correct?

i) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric
ii) MN-NM is skew symmetric for all matrices M and N.
iii) MN is symmetric for all symmetric matrices M and N.
iv) $(adjM)(adjN) = adj(MN)$ for all invertible matrices M and N.

($JEEAdv.2013$)

f) Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = p_{ij}$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $p^2 \neq 0$, when $n =$

i) 57
ii) 55
iii) 58
iv) 56

(*JEEAdv*.2013)

g) Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if
   i) The first column of M is the transpose of the second row of M
   ii) The second row of M is the transpose of the first column of M
   iii) M is a diagonal matrix with non-zero entries in the main diagonal
   iv) The product of entries in the main:wq diagonal of M is not the square of an integer

(*JEEAdv*.2014)

h) Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then
   i) determinant of $\left(M^2 + N^2\right)$ is 0
   ii) there is 3×3 non-zero matrix U such that $\left(M^2 + MN^2\right)U$ is the zero matrix
   iii) determinant of $\left(M^2 + MN^2\right) \geq 1$
   iv) determinant of $\left(M^2 + MN^2\right)U$ equals the zero matrix then U is the zero matrix

(*JEEAdv*.2014)

i) Which of the following values of α satisfy the equation $\begin{vmatrix}(1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2\\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2\\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2\end{vmatrix} = -648\alpha$?
   i) -4
   ii) 9
   iii) -9
   iv) 4

(*JEEAdv*.2015)

j) Let X and Y be two arbitrary, 3×3, non-zero, skew-symmetric matrices and Z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is (*are*) skew symmetric?
   i) $Y^3Z^4 - Z^4Y^3$
   ii) $X^{44} + Y^{44}$
   iii) $X^4Z^3 - Z^3X^4$
   iv) $X^{23} + Y^{23}$

(*JEEAdv*.2015)

k) Let $\mathrm{P} = \begin{vmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{vmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $\mathrm{Q} = [q_{ij}]$ is a matrix such that PQ=kI, where $k \in \setminus$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then
   i) $a = 0, k = 8$
   ii) $4a - k + 8 = 0$
   iii) $\det(\mathrm{Padj}(\mathrm{Q})) = 2^9$
   iv) $\det(\mathrm{Qadj}(\mathrm{P})) = 2^{13}$

(*JEEAdv*.2016)

l) Let $a, \lambda, \mu, \in \mathbb{R}$. Consider the system of linear equations

$$ax + 2y = \lambda$$

$$3x - 2y = \mu$$

Which of the following statement(*s*) is (*are*) correct?
   i) If $a = -3$, then the system has infinitely many solutions for all value of λ and μ.
   ii) If $a \neq -3$, then the system has unique solution for all values of λ and μ.
   iii) If $\lambda+\mu = 0$, then the system has infinitely many solutions for a = -3.
   iv) If $\lambda + \mu \neq 0$, then the system has no solution for a = -3

(*JEEAdv*.2016)

m) Which of the following is (*are*) not the square of a 3×3 matrix with real entries?
   i) $\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}$
   ii) $\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{vmatrix}$
   iii) $\begin{vmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{vmatrix}$
   iv) $\begin{vmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{vmatrix}$

(*JEEAdv*.2017)