5th September 2020 Shift 1

AI24BTECH11015 - Harshvardhan Patidar

1) If $3^{2\sin 2\alpha-1}$, 14, and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P. is:
   a) 66
   b) 65
   c) 81
   d) 78

2) If the function
$$f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \le \pi \\ k_2 \cos x, & x > \pi \end{cases}$$
is twice differentiable, then the ordered pair (k_1, k_2) is equal to:
   a) $\left(\frac{1}{2}, 1\right)$
   b) $(1, 1)$
   c) $\left(\frac{1}{2}, -1\right)$
   d) $(1, 0)$

3) If the common tangent to the parabolas $y^2 = 4x$ and $x^2 = 4y$ also touches the circle $x^2 + y^2 = c^2$, then c is equal to:
   a) $\frac{1}{2}$
   b) $\frac{1}{2\sqrt{2}}$
   c) $\frac{1}{\sqrt{2}}$
   d) $\frac{1}{4}$

4) The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to:
   a) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$
   b) $(x \wedge \sim y) \vee (\sim x \wedge y)$
   c) $(x \wedge y) \vee (\sim x \wedge \sim y)$
   d) $(x \wedge y) \wedge (\sim x \vee \sim y)$

5) If the volume of a parallelepiped whose coterminus edges are given by vectors $\mathbf{a} = \hat{i} + \hat{j} + n\hat{k}$, $\mathbf{b} = 2\hat{i} + 4\hat{j} + n\hat{k}$, $\mathbf{c} = \hat{i} + n\hat{j} + 3\hat{k}, (n \ge 0)$ is 158 cu. units, then:
   a) $\mathbf{a} \cdot \mathbf{c} = 17$
   b) $\mathbf{b} \cdot \mathbf{c} = 10$
   c) $n = 7$
   d) $n = 9$

6) If $y = y(x)$ is the solution of the differential equation $\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} + e^x = 0$ satisfying $y(0) = 1$, then a value of $y(\log_e 13)$ is:
   a) 1
   b) -1
   c) 2

d) 0

7) A survey shows that 73% of the people working in an office like coffee, whereas 65% like tea. If x denotes the percentage of people who like both coffee and tea, then x cannot be:

a) 63
b) 38
c) 54
d) 36

8) The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$ is:

a) $\frac{25}{9}$
b) $\frac{25}{81}$
c) $\frac{5}{27}$
d) $\frac{5}{9}$

9) If $\int \left(e^{2x} + 2e^{x} - e^{-x} - 1\right) e^{\left(e^{x}+e^{-x}\right)} dx = g(x)\, e^{\left(e^{x}+e^{-x}\right)} + c$, where c is the constant of integration, then $g(0)$ is equal to:

a) 2
b) e^2
c) e
d) 1

10) If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \to \mathbb{R}$, defined by:

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

are m and M respectively, then the ordered pair (m, M) is equal to:

a) $(0, 4)$
b) $(-4, 4)$
c) $\left(0, 2\sqrt{2}\right)$
d) $(-4, 0)$

11) Let $\lambda \in \mathbb{R}$. The system of linear equations:

$$\begin{aligned} 2x_1 - 4x_2 + \lambda x_3 &= 1 \\ x_1 - 6x_2 + x_3 &= 2 \\ \lambda x_1 - 10x_2 + 4x_3 &= 3 \end{aligned}$$

is inconsistent for:

a) exactly one negative value of λ
b) exactly one positive value of λ
c) every value of λ
d) exactly two values of λ

12) If S is the sum of the first 10 terms of the series $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}+\tan^{-1}\frac{1}{21}+\ldots$ then $\tan(S)$ is equal to:

a) $\frac{5}{11}$
b) $-\frac{6}{5}$
c) $\frac{10}{11}$
d) $\frac{5}{6}$

13) If four complex numbers $z, \bar{z}, \bar{z}-2\mathrm{Re}(\bar{z})$ and $z-2\mathrm{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to:

a) 4
b) 2
c) $4\sqrt{2}$
d) $2\sqrt{2}$

14) If the point P on the curve $4x^2+5y^2=20$ is farthest from the point $Q(0,-4)$, then PQ^2 is equal to:

a) 21
b) 36
c) 48
d) 29

15) The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, and 14, then the absolute difference of the remaining two observations is:

a) 2
b) 4
c) 3
d) 1