

MA - 2021

AI24BTECH11015 - Harshvardhan Patidar

- 1) Let $\langle \cdot, \cdot \rangle : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ be an inner product on the vector space \mathbb{R}^n over \mathbb{R} . Consider the following statements:

P: $|\langle u, v \rangle| \leq \frac{1}{2} (\langle u, u \rangle + \langle v, v \rangle)$ for all $u, v \in \mathbb{R}^n$.

Q: If $\langle u, v \rangle = \langle 2u, -v \rangle$ for all $v \in \mathbb{R}^n$, then $u = 0$.

Then

- a) both P and Q are TRUE
- b) P is TRUE and Q is FALSE
- c) P is FALSE and Q is TRUE
- d) both P and Q is FALSE

Q.2 - Q.13 Numerical Answer Type (NAT), carry TWO mark each (no negative marks).

- 2) Let G be a group of order 5^4 with center having 5^2 elements. Then the number of conjugacy classes in G is _____.
- 3) Let F be a finite field and F^\times be the group of all nonzero elements of F under multiplication. If F^\times has a subgroup of order 17, then the smallest possible order of the field F is _____.
- 4) Let $R = \{z = x + iy \in \mathbb{C} : 0 < x < 1\}$ and $-11\pi < y < 11\pi$ and Γ be the positively oriented boundary of R . Then the value of the integral

$$\frac{1}{2\pi i} \int_{\Gamma} \frac{e^z dz}{e^z - 2}$$

is _____.

- 5) Let $D = \{z \in \mathbb{C} : |z| < 2\pi\}$ and $f : D \rightarrow \mathbb{C}$ be the function defined by

$$f(z) = \begin{cases} \frac{3z^2}{(1-\cos z)} & \text{if } z \neq 0 \\ 6 & \text{if } z = 0 \end{cases}$$

If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ for $z \in D$, then $6a_2 =$ _____.

- 6) The number of zeroes (counting multiplicity) of $P(z) = 3z^5 + 2iz^2 + 7iz + 1$ in the annular region $\{z \in \mathbb{C} : 1 < |z| < 7\}$ is _____.

- 7) Let A be a square matrix such that $\det(xI - A) = x^4(x-1)^2(x-2)^3$, where $\det(M)$ denotes the determinant of a square matrix M .

If $\text{rank}(A^2) < \text{rank}(A^3) = \text{rank}(A^4)$, then the geometric multiplicity of the eigenvalue 0 of A is _____.

- 8) If $y = \sum_{k=0}^{\infty} a_k x^k$, ($a_0 \neq 0$) is the power series solution of the differential equation $\frac{d^2 y}{dx^2} - 24x^2 y = 0$, then $\frac{a_4}{a_0} =$ _____.

9) If $u(x, t) = Ae^{-t} \sin x$ solves the following initial boundary value problem

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}, 0 < x < \pi, & t > 0, \\ u(0, t) &= u(\pi, t) = 0, & t > 0, \\ u(x, 0) &= \begin{cases} 60, & 0 < x \leq \frac{\pi}{2}, \\ 40, & \frac{\pi}{2} < x < \pi, \end{cases}\end{aligned}$$

then $\pi A =$ _____.

10) Let $V = \{p : p(x) = a_0 + a_1x + a_2x^2, a_0, a_1, a_2 \in \mathbb{R}\}$ be the vector space of all polynomials of degree at most 2 over the real field \mathbb{R} . Let $T : V \rightarrow V$ be the linear operator given by

$$T(p) = (p(0) - p(1)) + (p(0) + p(1))x + p(0)x^2.$$

Then the sum of the eigenvalues of T is _____.

11) The quadrature formula

$$\int_0^2 xf(x) dx \approx \alpha f(0) + \beta f(1) + \gamma f(2)$$

is exact for all polynomials of degree ≤ 2 . Then $2\beta - \gamma =$ _____.

12) For each $x \in (0, 1]$, consider the decimal representation $x = .d_1d_2d_3 \cdots d_n \cdots$. Define $f : (0, 1] \rightarrow \mathbb{R}$ by $f(x) = 0$ if x is rational and $f(x) = 18n$ if x is irrational, where n is the number of zeroes immediately after the decimal point up to the first nonzero digit in the decimal representation of x . Then the Lebesgue integral $\int_0^1 f(x) dx =$

_____.
13) Let $\tilde{x} = \begin{pmatrix} 11/3 \\ 2/3 \\ 0 \end{pmatrix}$ be an optimal solution of the following Linear Programming Problem

$P :$

$$\text{Maximize } 4x_1 + x_2 - 3x_3$$

subject to

$$2x_1 + 4x_2 + ax_3 \leq 10,$$

$$x_1 - x_2 + bx_3 \leq 3,$$

$$2x_1 + 3x_2 + 53x_3 \leq 11,$$

$$x_1 \geq 0, x_2 \geq 0 \text{ and } x_3 \geq 0, \text{ where } a, b \text{ are real numbers.}$$

If $\tilde{y} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$ is an optimal solution of the dual of P , then $p + q + r =$ _____. (round off to two decimal places.)