

Definite Integrals and Applications of Integrals

AI24BTECH11015 - Harshvardhan Patidar

Section-B — JEE Main / AIEEE

1. $\int_0^{10\pi} |\sin x| dx$ is
 (a) 20
 (b) 8
 (c) 10
 (d) 18
 (2002)
2. $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$ then $\lim_{n \rightarrow \infty} n[I_n + I_{n+2}]$ equals
 (a) $\frac{1}{2}$
 (b) 1
 (c) ∞
 (d) zero
 (2002)
3. $\int_0^2 [x^2] dx$ is
 (a) $2 - \sqrt{2}$
 (b) $2 + \sqrt{2}$
 (c) $\sqrt{2} - 1$
 (d) $-\sqrt{2} - \sqrt{3} + 5$
 (2002)
4. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is
 (a) $\frac{\pi^2}{4}$
 (b) π^2
 (c) zero
 (d) $\frac{\pi}{2}$
 (2002)
5. If $y = f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of $\frac{3}{4}$ square unit with the axes then $\int_0^2 xf'(x)dx$ is
 (a) $\frac{3}{2}$
 (b) 1
 (c) $\frac{5}{4}$
 (d) $-\frac{3}{4}$
 (2002)
6. The area bounded by the curves $y = \ln x$, $y = \ln|x|$, $y = |\ln x|$ and $y = |\ln|x||$
 (a) 4 sq. units
 (b) 6 sq. units
 (c) 10 sq. units
 (d) none of these
 (2002)
7. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is
 (a) 6 sq. units
 (b) 2 sq. units
 (c) 3 sq. units
 (d) 4 sq. units
 (2003)
8. If $f(a + b - x) = f(x)$ then $\int_a^b xf(x)dx$ is equal to
 (a) $\frac{a+b}{2} \int_a^b f(a + b + x)dx$
 (b) $\frac{a+b}{2} \int_a^b f(b - x)dx$
 (c) $\frac{a+b}{2} \int_a^b f(x)dx$
 (d) $\frac{b-a}{2} \int_a^b f(x)dx$
 (2003)
9. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$, is
 (a) $e + \frac{e^2}{2} + \frac{5}{2}$
 (b) $e - \frac{e^2}{2} - \frac{5}{2}$
 (c) $e + \frac{e^2}{2} - \frac{5}{2}$
 (d) $e - \frac{e^2}{2} - \frac{5}{2}$
 (2003)
10. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is
 (a) $\frac{1}{n+1} + \frac{1}{n+2}$
 (b) $\frac{1}{n+1}$
 (c) $\frac{1}{n+2}$

d) $\frac{1}{n+1} - \frac{1}{n+2}$ (2003)

11. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{\frac{r}{n}}$ is

- a) $e + 1$
- b) $e - 1$
- c) $1 - e$
- d) e

(2004)

12. The value of $\int_{-2}^3 |1 - x^2| dx$ is

- a) $\frac{1}{3}$
- b) $\frac{14}{3}$
- c) $\frac{7}{3}$
- d) $\frac{28}{3}$

(2004)

13. The value of $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$ is

- a) 3
- b) 1
- c) 2
- d) 0

(2004)

14. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is

- a) 2π
- b) π
- c) $\frac{\pi}{4}$
- d) 0

(2004)

15. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} x g x (1-x) dx$ and $I_2 =$

$\int_{f(-a)}^{f(a)} g x (1-x) dx$, then the value of $\frac{I_2}{I_1}$ is

- a) 1
- b) -3
- c) -1
- d) 2

(2004)