

1-1.9-24

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Question:

The x -coordinate of a point \mathbf{P} twice its y -coordinate. If \mathbf{P} is equidistant from the points $\mathbf{Q}(2, -5)$ and $\mathbf{R}(-3, 6)$, find the coordinates of \mathbf{P} .

Solution:

Let y -coordinate of \mathbf{P} be a .

Then,

$$\mathbf{P} = \begin{pmatrix} 2a \\ a \end{pmatrix}$$

Now, since \mathbf{P} is equidistant from \mathbf{Q} and \mathbf{R} ,

$$\|\mathbf{P} - \mathbf{Q}\| = \|\mathbf{P} - \mathbf{R}\| \quad (0.1)$$

$$\|\mathbf{P} - \mathbf{Q}\|^2 = \|\mathbf{P} - \mathbf{R}\|^2 \quad (0.2)$$

$$\mathbf{P}^2 + \mathbf{Q}^2 - 2\mathbf{P}\mathbf{Q}^\top = \mathbf{P}^2 + \mathbf{R}^2 - 2\mathbf{P}\mathbf{R}^\top \quad (0.3)$$

$$2\mathbf{P}(\mathbf{R}^\top - \mathbf{Q}^\top) = \mathbf{R}^2 - \mathbf{Q}^2 \quad (0.4)$$

$$\begin{pmatrix} 2a \\ a \end{pmatrix} \begin{pmatrix} -5 & 11 \end{pmatrix} = 8 \quad (0.5)$$

$$a = 8 \quad (0.6)$$

So,

$$\mathbf{P} = \begin{pmatrix} 16 \\ 8 \end{pmatrix}$$

