Definite Integrals and Applications of Integrals

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Section-B — JEE Main / AIEEE

1) $\int_0^{10\pi} |\sin(x)|\, dx$ is
   a) 20
   b) 8
   c) 10
   d) 18

(2002)

2) $I_n = \int_0^{\frac{\pi}{4}} \tan^n(x)\, dx$ then $\lim_{n\to\infty} n(I_n + I_{n+2})$ equals
   a) $\frac{1}{2}$
   b) 1
   c) ∞
   d) zero

(2002)

3) $\int_0^2 \left(x^2\right) dx$ is
   a) $2 - \sqrt{2}$
   b) $2 + \sqrt{2}$
   c) $\sqrt{2} - 1$
   d) $-\sqrt{2} - \sqrt{3} + 5$

(2002)

4) $\int_{-\pi}^{\pi} \frac{2x(1+\sin(x))}{1+\cos^2(x)}\, dx$ is
   a) $\frac{\pi^2}{4}$
   b) π^2
   c) zero
   d) $\frac{\pi}{2}$

(2002)

5) If $y = f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of $\frac{3}{4}$ square unit with the axes then $\int_0^2 x f'(x)\, dx$ is
   a) $\frac{3}{2}$

b) 1

c) $\frac{5}{4}$

d) $-\frac{3}{4}$

(2002)

6) The area bounded by the curves $y = \ln(x)$, $y = \ln(|x|)$, $y = |\ln(x)|$ and $y = |\ln(|x|)|$

a) 4 *sq. units*

b) 6 *sq. units*

c) 10 *sq. units*

d) none of these

(2002)

7) The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is

a) 6 *sq. units*

b) 2 *sq. units*

c) 3 *sq. units*

d) 4 *sq. units*

(2003)

8) If $f(a + b - x) = f(x)$ then $\int_a^b xf(x)\,dx$ is equal to

a) $\frac{a+b}{2}\int_a^b f(a + b + x)\,dx$

b) $\frac{a+b}{2}\int_a^b f(b - x)\,dx$

c) $\frac{a+b}{2}\int_a^b f(x)\,dx$

d) $\frac{b-a}{2}\int_a^b f(x)\,dx$

(2003)

9) Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)\,g(x)\,dx$, is

a) $e + \frac{e^2}{2} + \frac{5}{2}$

b) $e - \frac{e^2}{2} - \frac{5}{2}$

c) $e + \frac{e^2}{2} - \frac{3}{2}$

d) $e - \frac{e^2}{2} - \frac{3}{2}$

(2003)

10) The value of the integral $I = \int_0^1 (x)\,(1 - x)^n\,dx$ is

a) $\frac{1}{n+1} + \frac{1}{n+2}$

b) $\frac{1}{n+1}$

c) $\frac{1}{n+2}$

d) $\frac{1}{n+1} - \frac{1}{n+2}$

(2003)

11) $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is

a) $e + 1$

b) $e - 1$

c) $1 - e$

d) e

(2004)

12) The value of $\int_{-2}^{3} \left|1 - x^2\right| dx$ is

a) $\frac{1}{3}$

b) $\frac{14}{3}$

c) $\frac{7}{3}$

d) $\frac{28}{3}$

(2004)

13) The value of $I = \int_{0}^{\frac{\pi}{2}} \frac{(\sin(x)+\cos(x))^2}{\sqrt{1+\sin(2x)}} dx$ is

a) 3

b) 1

c) 2

d) 0

(2004)

14) If $\int_{0}^{\pi} xf(\sin(x))\,dx = A\int_{0}^{\frac{\pi}{2}} f(\sin(x))\,dx$, then A is

a) 2π

b) π

c) $\frac{\pi}{4}$

d) 0

(2004)

15) If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} (x)\, g(x(1-x))\,dx$ and $I_2 = \int_{f(-a)}^{f(a)} g(x(1-x))\,dx$, then the value of $\frac{I_2}{I_1}$ is

a) 1

b) -3

c) -1

d) 2

(2004)