Matrices and Determinants

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Section-A — JEE Advanced/ IIT-JEE

I. Fill in the Blanks:

1) How many 3×3 matrices M with entries from $(0,1,2)$ are there, for which the sum of the diagonal entries of M^TM is 5?
   (a) 126
   (b) 198
   (c) 162
   (d) 135

(JEE Adv. 2017)

2) Let $M=\begin{vmatrix}\sin^4\theta & -1-\sin^2\theta\\ 1+\cos^2\theta & \cos^4\theta\end{vmatrix}=\alpha I+\beta M^{-1}$ Where $\alpha=\alpha(\theta)$ and $\beta=\beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If a^* is the minimum of the set $(\alpha(\theta):\theta\in[0,2\pi))$ and b^* is the minimum of the set $(\beta(\theta):\theta\in[0,2\pi))$. Then the value of a^*+b^* is
   (a) $-\frac{31}{16}$
   (b) $-\frac{17}{16}$
   (c) $-\frac{37}{16}$
   (d) $-\frac{29}{16}$

(JEE Adv. 2019)

II. MCQs with More than One Correct

1) The determinant $\begin{vmatrix}a & b & a\alpha+b\\ b & c & b\alpha+c\\ a\alpha+b & b\alpha+c & 0\end{vmatrix}$ is equal to zero, if
   (a) a,b,c are in A.P.
   (b) a,b,c are in G.P.
   (c) a,b,c are in H.P.
   (d) α is a root of the equation $ax^2+bx+c=0$
   (e) $(x-\alpha)$ is a factor of ax^2+bx+c

(1986-2 Marks)

2) If $\begin{vmatrix}6i & -3i & 1\\ 4 & 3i & -1\\ 20 & 3 & i\end{vmatrix}=x+iy$, then
   (a) $x=3,y=1$
   (b) $x=1,y=3$
   (c) $x=0,y=3$
   (d) $x=0,y=0$

(1998-2 Marks)

3) Let M and N be two 3×3 non-singulr skew-symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2\left(M^TN^{-1}\right)^{-1}\left(MN^{-1}\right)^T$ is equal to
   (a) M^2
   (b) $-N^2$
   (c) $-M^2$
   (d) MN

(2011)

4) If the adjoint of a 3×3 matrix P is $\begin{vmatrix}1 & 4 & 4\\ 2 & 1 & 7\\ 1 & 1 & 3\end{vmatrix}$, then the possible value(s) of the determinant of P is (are)
   (a) -2
   (b) -1
   (c) 1
   (d) 2

(2012)

5) For 3×3 matrices M and N, which of the following statement(s) is (are) NOT correct?
   (a) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric
   (b) MN-NM is skew symmetric for all matrices M and N.
   (c) MN is symmetric for all symmetric matrices M and N.
   (d) (adjM)(adjN) = adj(MN) for all invertible matrices M and N.

(JEE Adv. 2013)

6) Let ω be a complex cube root of unity with $\omega\neq1$ and $P=p_{ij}$ be a $n\times n$ matrix with

$p_{ij} = \omega^{i+j}$. Then $p^2 \neq 0$, when $n =$

(a) 57
(b) 55
(c) 58
(d) 56

(JEE Adv. 2013)

7) Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if

(a) The first column of M is the transpose of the second row of M
(b) The second row of M is the transpose of the first column of M
(c) M is a diagonal matrix with non-zero entries in the main diagonal
(d) The product of entries in the main:wq diagonal of M is not the square of an integer

(JEE Adv. 2014)

8) Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then

(a) determinant of $(M^2 + N^2)$ is 0
(b) there is 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix
(c) determinant of $(M^2 + MN^2) \geq 1$
(d) determinant of $(M^2+MN^2)U$ equals the zero matrix then U is the zero matrix

(JEE Adv. 2014)

9) Which of the following values of α satisfy the equation $\begin{vmatrix}(1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2\\(2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2\\(3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2\end{vmatrix} = -648\alpha$?

(a) -4
(b) 9
(c) -9
(d) 4

(JEE Adv. 2015)

10) Let X and Y be two arbitrary, 3×3, non-zero, skew-symmetric matrices and Z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

(a) $Y^3Z^4 - Z^4Y^3$
(b) $X^{44} + Y^{44}$
(c) $X^4Z^3 - Z^3X^4$
(d) $X^{23} + Y^{23}$

(JEE Adv. 2015)

11) Let P = $\begin{vmatrix}3 & -1 & -2\\2 & 0 & \alpha\\3 & -5 & 0\end{vmatrix}$, where $\alpha \in \mathbb{R}$. Suppose Q= $\left[q_{ij}\right]$ is a matrix such that PQ=kI, where k $\in \setminus$, k $\neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and det(Q)= $\frac{k^2}{2}$, then

(a) $a = 0, k = 8$
(b) $4a - k + 8 = 0$
(c) det (Padj (Q)) = 2^9
(d) det (Qadj (P)) = 2^{13}

(JEE Adv. 2016)

12) Let $a, \lambda, \mu, \in \mathbb{R}$. Consider the system of linear equations

$$ax + 2y = \lambda$$

$$3x - 2y = \mu$$

Which of the following statement(s) is (are) correct?

(a) If $a = -3$, then the system has infinitely many solutions for all value of λ and μ.
(b) If $a \neq -3$, then the system has unique solution for all values of λ and μ.
(c) If $\lambda + \mu = 0$, then the system has infinitely many solutions for a = -3.
(d) If $\lambda+\mu \neq 0$, then the system has no solution for a = -3

(JEE Adv. 2016)

13) Which of the following is (are) not the square of a 3×3 matrix with real entries?

(a) $\begin{vmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{vmatrix}$
(b) $\begin{vmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{vmatrix}$
(c) $\begin{vmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{vmatrix}$
(d) $\begin{vmatrix}-1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{vmatrix}$

(JEE Adv. 2017)