Permutation and Combination-2

Geometrical Arrangement

I. 4 boys and 4 girls have to be seated around a circular table such that no two girls are adjacent to each other. In how many ways can they be seated?

II. 8 people have to be seated on a rectangular table with 4 each on the longer sides. In how many ways can they be seated?

Arranging Objects when few are identical-

I. In how many ways can the letters of the word "MISSISSIPPI" be arranged?

II. In how many ways can the letters of the word "OPPOSITION" be arranged so that the three O's and the S are together?

III. Find the number of natural number solutions to $a + b + c + d = 20$

IV. In how many different ways can I purchase a total of 10 fruits if the fruit seller has mangoes, apples and oranges? Assume the fruit-seller has more than 10 fruits of each variety.

Exercise:

1. In how many ways can 10 people be seated across a circular table if

i. There are 11 identical chairs placed equally apart around the table

A. 12! B. 11! C. 10! D. 9!

ii. If there are 11 distinctly coloured chairs placed equally apart around the table.

A. 12! B. 11! C. 10! D. 9!

iii. If there are 10 identically coloured chair and 1 chair is distinctly coloured.

A. 12! B. 11! C. 10! D. 9!

2. 4 men and 4 women have to be seated in a circle such that all the men are together and all the women are also together. In how many ways can they be seated?

A. 576 B. 24 C. 48 D. 1152

3. 4 managers, 2 vice-presidents and 1 president have to be seated in a circle for a meeting such that the two vice-presidents sit on either side of the president. In how many ways can they be seated?

A. 120 B. 240 C. 360 D. 48

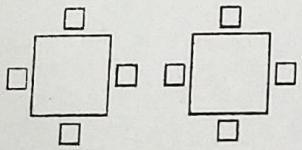
4. In how many ways can 6 Indians and 8 Americans sit across a circular table with 14 equispaced chairs such that no two Indians are sitting next to each other?

A. 7! × 5! B. 8! × 6! C. 7! × 7! D. 7! × 6!

5. In how many ways can 6 couples be seated around a circular table such that each couple is sitting together?

A. 11! B. 6! × 26 C. 5! × 25 D. 5! × 26

6. In how many ways can eight people be seated across two identical square tables, with one on each side, as shown in the following picture



A. 8! B. 2 × 7! C. 4 × 7! D. 7!

7. Find the number of natural number solutions to $a + b + c + d = 20$ such that ...

i. ... each of a, b, c and d are even.

ii. ... each of a, b, c and d are odd.

A. ${}^{13}C_3$, ${}^{11}C_3$ B. ${}^{9}C_3$, ${}^{7}C_3$ C. ${}^{9}C_3$, ${}^{11}C_3$ D. ${}^{13}C_3$, ${}^{7}C_3$

8. There are 15 intermediate stations on a railway line from one terminus to another. In how many ways can 4 of these stations be chosen as halts for the train such that between any two of *these* 4 halts there are atleast 2 stations where the train does not halt?

A. ${}^{11}C_4$ B. ${}^{10}C_4$ C. ${}^{9}C_4$ D. ${}^{8}C_4$

9. Find the number of natural number solutions to the equation $a + b + c \leq 12$.

A. ${}^{11}C_2$ B. ${}^{12}C_3$ C. ${}^{13}C_3$ D. ${}^{13}C_2$

10. In how many ways can the letters of the word "WEDNESDAY" be arranged such that the three vowels are at the two extremes and the middle of the arrangment?

A. $\frac{3!}{2!} \times \frac{6!}{2!}$ B. $\frac{9!}{3! \times 6!}$ C. $\frac{9!}{2! \times 2!}$ D. $\frac{3! \times 6!}{4!}$

11. In how many ways can the letters of the word "ENTERTAINER" be arranged such that no two consonants are together?

A. $2 \times 5! \times 6!$ B. $2 \times \frac{5!}{3!} \times \frac{6!}{(2!)^3}$ C. $\frac{5!}{3!} \times \frac{6!}{(2!)^3}$ D. $\frac{11!}{3! \times (2!)^2} - \frac{6!}{(2!)^3}$

12. How many numbers can be formed using each of the digits 1, 1, 2, 2, 2, 3 and 4?

A. 4! B. $\frac{7!}{2! \times 3!}$ C. $7! - 2! \times 3!$ D. $7! - 2! \times 3! \times 2!$ E. $8 \times 6!$