CHAPTER 20 Vector Algebra and Three Dimensional Geometry

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Let $\vec{A}$, $\vec{B}$, $\vec{C}$ be vectors of length 3, 4, 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}, \vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ *to* $\vec{A}+\vec{B}$. Then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is .... ***(1981 - 2 Marks)***
2. The unit vector perpendicular to the plane determined by $P(1,-1,2)$, $Q(2,0,-1)$ and $R(0,2,1)$ is .... ***(1983 - 1 Mark)***
3. The area of the triangle whose vertices are $A(1,-1,2)$, $B(2,1,-1)$, $C(3,-1,2)$ is .... ***(1983 - 1 Mark)***
4. A, B, C and D, are four points in a plane with position vectors ***a, b, c*** and ***d*** respectively such that $(\vec{a}-\vec{d})(\vec{b}-\vec{c})=(\vec{b}-\vec{d})(\vec{c}-\vec{a})=0$ ***(1984 - 2 Marks)***
   The point D, then, is the .... of the triangle ABC.
5. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and the vectors $\vec{A}=(1,a,a^2)$, $\vec{B}=(1,b,b^2)$, $\vec{C}=(1,c,c^2)$, are non-coplanar, then the product $abc=$ .... ***(1985 - 2 Marks)***
6. If $\vec{A}\,\vec{B}\,\vec{C}$ are three non-coplanar vectors, then –
   $$\frac{\vec{A}\cdot\vec{B}\times\vec{C}}{\vec{C}\times\vec{A}\cdot\vec{B}}+\frac{\vec{B}\cdot\vec{A}\times\vec{C}}{\vec{C}\cdot\vec{A}\times\vec{B}}= ....$$ ***(1985 - 2 Marks)***
7. If $\vec{A}=(1,1,1)$, $\vec{C}=(0,1,-1)$ are given vectors, then a vector B satisfying the equations $\vec{A}\times\vec{B}=\vec{C}$ and $\vec{A}\cdot\vec{B}=3$ .... ***(1985 - 2 Marks)***
8. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ $(a\neq b\neq c\neq 1)$ are coplanar, then the value of $\frac{1}{(1-a)}+\frac{1}{(1-b)}+\frac{1}{(1-c)}=$ .... ***(1987 - 2 Marks)***
9. Let $b=4\hat{i}+3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. All vectors in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$, respectively, are given by .... ***(1987 - 2 Marks)***
10. The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are.......and ....respectively. ***(1988 - 2 Marks)***
11. Given that $\vec{a}=(1,1,1)$, $\vec{c}=(0,1,-1)$, $\vec{a}.\vec{b}=3$ and $\vec{a}\times\vec{b}=\vec{c}$, then $\vec{b}=$ .... ***(1991 - 2 Marks)***
12. A unit vector coplanar with $\vec{i}+\vec{j}+2\vec{k}$ and $\vec{i}+2\vec{j}+\vec{k}$ and perpendicular to $\vec{i}+\vec{j}+\vec{k}$ is .... ***(1992 - 2 Marks)***
13. A unit vector perpendicular to the plane determined by the points $P(1,-1,2)$ $Q(2,0,-1)$ and $R(0,2,1)$ is .... ***(1994 - 2 Marks)***
14. A nonzero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}$, $\hat{i}+\hat{k}$. The angle between $\vec{a}$ and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is .... ***(1996 - 2 Marks)***
15. If $\vec{b}$ and $\vec{c}$ are any two non-collinear unit vectors and $\vec{a}$ is any vector, then $(\vec{a}.\vec{b})\vec{b}+(\vec{a}.\vec{c})\vec{c}+\frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}(\vec{b}\times\vec{c})=$ .... ***(1996 - 2 Marks)***
16. Let $OA=a$, $OB=10a+2b$ and $OC=b$ where O, A and C are non-collinear points. Let p denote the area of the quadrilateral $OABC$, and let q denote the area of the parallelogram with OA and OC as adjacent sides. If $p=kq$, then $k=$ .... ***(1997 - 2 Marks)***

B True / False

1. Let $\vec{A}, \vec{B}$ and $\vec{C}$ be unit vectors suppose that $\vec{A}.\vec{B}=\vec{A}.\vec{C}=0$, and that the angle between $\vec{B}$ and $\vec{C}$ is $\pi/6$. Then $\vec{A}=\pm 2(\vec{B}\times\vec{C})$. ***(1981 - 2 Marks)***

2. If $X.A = 0, X.B = 0, X.C = 0$ for some non-zero vector X, then $[A\ B\ C] = 0$ *(1983 - 1 Mark)*
3. The points with position vectors $a + b, a - b$, and $a + kb$ are collinear for all real values of k. *(1984 - 1 Mark)*
4. For any three vectors $\vec{a}, \vec{b}$, and $\vec{c}$,

   $(\vec{a} - \vec{b}) . (\vec{b} - \vec{c}) \times (\vec{c} - \vec{a}) = 2\vec{a} . \vec{b} \times \vec{c}.$ *(1989 - 1 Mark)*

C MCQs with One Correct Answer

1. The scalar $\vec{A}.(\vec{B}+\vec{C}) \times (\vec{A}+\vec{B}+\vec{C})$ equals : *(1981 - 2 Marks)*

   (a) 0 (b) $[\vec{A}\ \vec{B}\ \vec{C}]+[\vec{B}\ \vec{C}\ \vec{A}]$

   (c) $[\vec{A}\ \vec{B}\ \vec{C}]$ (d) None of these

2. For non-zero vectors $\vec{a}, \vec{b}, \vec{c}$, $|(\vec{a} \times \vec{b}).\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$ holds if and only if *(1982 - 2 Marks)*

   (a) $\vec{a}.\vec{b} = 0,\ \vec{b}.\vec{c} = 0$ (b) $\vec{b}.\vec{c} = 0,\ \vec{c}.\vec{a} = 0$

   (c) $\vec{c}.\vec{a} = 0\ \vec{a}.\vec{b} = 0$ (d) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

3. The volume of the parallelopiped whose sides are given by $\overrightarrow{OA} = 2i - 2j$, $\overrightarrow{OB} = i + j - k$, $\overrightarrow{OC} = 3i - k$, is *(1983 - 1 Mark)*

   (a) $\frac{4}{13}$ (b) 4

   (c) $\frac{2}{7}$ (d) none of these

4. The points with position vectors $60i + 3j$, $40i - 8j$, $ai - 52j$ are collinear if *(1983 - 1 Mark)*

   (a) $a = -40$ (b) $a = 40$

   (c) $a = 20$ (d) none of these

5. Let $\vec{a}, \vec{b}, \vec{c}$, be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$, are vectors defined by the relations $\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, $\vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, $\vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ then the value of the expression $(\vec{a} + \vec{b}) . \vec{p} + (\vec{b} + \vec{c}) . \vec{q} + (\vec{c} + \vec{a}) . \vec{r}$ is equal to *(1988 - 2 Marks)*

   (a) 0 (b) 1 (c) 2 (d) 3.

6. Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is *(1993 - 1 Marks)*

   (a) the Arithmetic Mean of a and b
   (b) the Geometric Mean of a and b
   (c) the harmonic Mean of a and b
   (d) equal to zero

7. Let $\vec{p}$ and $\vec{q}$ be the position vectors of P and Q respectively, with respect to O and $|\vec{p}| = p, |\vec{q}| = q$. The points R and S divide PQ internally and externally in the ratio 2 : 3 respectively. If OR and OS are perpendicular then *(1994)*

   (a) $9q^2 = 4q^2$ (b) $4p^2 = 9q^2$

   (c) $9p = 4q$ (d) $4p = 9q$

8. Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}, \beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}, \gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$ *(1994)*

   (a) are collinear
   (b) form an equilateral triangle
   (c) form a scalene triangle
   (d) form a right angled triangle

9. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k}, \vec{c} = \hat{k} - \hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a}.\vec{d} = 0 = [\vec{b}\ \vec{c}\ \vec{d}]$, then $\vec{d}$ equals *(1995S)*

   (a) $\pm\frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$ (b) $\pm\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

   (c) $\pm\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (d) $\pm\hat{k}$

10. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{(\vec{b} + \vec{c})}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is *(1995S)*

    (a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\pi/2$ (d) π

11. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = 0$. If $|\vec{u}| = 3, |\vec{v}| = 4$ and $|\vec{w}| = 5$, then $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$ is *(1995S)*

    (a) 47 (b) -25 (c) 0 (d) 25

12. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non coplanar vectors, then *(1995S)* $(\vec{a} + \vec{b} + \vec{c}). [(\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})]$ equals

    (a) 0 (b) $[\vec{a}\ \vec{b}\ \vec{c}]$

    (c) $2[\vec{a}\ \vec{b}\ \vec{c}]$ (d) $-[\vec{a}\ \vec{b}\ \vec{c}]$

13. Let $\boldsymbol{a} = \boldsymbol{2i} + \boldsymbol{j} - \boldsymbol{2k}$ and $\boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j}$. If c is a vector such that $\boldsymbol{a} . \boldsymbol{c} = |\boldsymbol{c}|, |\boldsymbol{c} - \boldsymbol{a}| = 2\sqrt{2}$ and the angle between $(\boldsymbol{a} \times \boldsymbol{b})$ and $\boldsymbol{c}$ is 30°, then $|(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c}| =$ *(1999 - 2 Marks)*

    (a) 2/3 (b) 3/2 (c) 2 (d) 3

14. Let $a = 2i + j + k, b = i + 2j - k$ and a unit vector c be coplanar. If c is perpendicular to a, then $c =$ *(1999 - 2 Marks)*

    (a) $\frac{1}{\sqrt{2}}(-j + k)$ (b) $\frac{1}{\sqrt{3}}(-i - j - k)$

    (c) $\frac{1}{\sqrt{5}}(i - 2j)$ (d) $\frac{1}{\sqrt{3}}(i - j - k)$

15. If the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ form the sides BC, CA and AB respectively of a triangle ABC, then ***(2000S)***

(a) $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0$ (b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(c) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}$ (d) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

16. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be such that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=0$. Let P_1 and P_2 be planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is ***(2000S)***

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

17. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a}-\vec{b}, 2\vec{b}-\vec{c}, 2\vec{c}-\vec{a}\right]=$ ***(2000S)***

(a) 0 (b) 1 (c) $-\sqrt{3}$ (d) $\sqrt{3}$

18. Let $\vec{a}=\vec{i}-\vec{k}$, $\vec{b}=x\vec{i}+\vec{j}+(1-x)\vec{k}$ and $\vec{c}=y\vec{i}+x\vec{j}+(1+x-y)\vec{k}$. Then $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ depends on ***(2001S)***

(a) only x (b) only y
(c) Neither x Nor y (d) both x and y

19. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then

$\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2$ does NOT exceed ***(2001S)***

(a) 4 (b) 9 (c) 8 (d) 6

20. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is ***(2002S)***

(a) 45° (b) 60°

(c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{2}{7}\right)$

21. Let $\vec{V}=2\vec{i}+\vec{j}-\vec{k}$ and $\vec{W}=\vec{i}+3\vec{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $|\vec{U}\vec{V}\vec{W}|$ is

(a) -1 (b) $\sqrt{10}+\sqrt{6}$ ***(2002S)***

(c) $\sqrt{59}$ (d) $\sqrt{60}$

22. The value of k such that $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=7$, is ***(2003S)***

(a) 7 (b) -7
(c) no real value (d) 4

23. The value of 'a' so that the volume of parallelopiped formed by $\hat{i}+a\hat{j}+\hat{k}$, $\hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum is ***(2003S)***

(a) -3 (b) 3 (c) $1/\sqrt{3}$ (d) $\sqrt{3}$

24. If $\vec{a}=(\hat{i}+\hat{j}+\hat{k}), \vec{a}.\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then $\vec{b}$ is

(a) $\hat{i}-\hat{j}+\hat{k}$ (b) $2\hat{j}-\hat{k}$ ***(2004S)***

(c) $\hat{i}$ (d) $2\hat{i}$

25. If the lines

$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then the value of k is ***(2004S)***

(a) 3/2 (b) 9/2 (c) $-2/9$ (d) $-3/2$

26. The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is ***(2004S)***

(a) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ (b) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$

(c) $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$ (d) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

27. A variable plane at a distance of the one unit from the origin cuts the coordinates axes at A, B and C. If the centroid $D(x, y, z)$ of triangle ABC satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then the value k is ***(2005S)***

(a) 3 (b) 1 (c) $\frac{1}{3}$ (d) 9

28. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, non-coplanar vectors and

$\vec{b}_1=\vec{b}-\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{b}_2=\vec{b}+\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$,

$\vec{c}_1=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}+\frac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_2=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{b}_1\cdot\vec{c}}{|\vec{b}_1|^2}\vec{b}_1$,

$\vec{c}_3=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{c}|^2}\vec{a}+\frac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_4=\vec{c}-\frac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a}=\frac{\vec{b}.\vec{c}}{|\vec{b}|^2}\vec{b}_1$,

then the set of orthogonal vectors is ***(2005S)***

(a) $(\vec{a}, \vec{b}_1, \vec{c}_3)$ (b) $(\vec{a}, \vec{b}_1, \vec{c}_2)$

(c) $(\vec{a}, \vec{b}_1, \vec{c}_1)$ (d) $(\vec{a}, \vec{b}_2, \vec{c}_2)$

29. A plane which is perpendicular to two planes $2x-2y+z=0$ and $x-y+2z=4$, passes through $(1,-2,1)$. The distance of the plane from the point $(1, 2, 2)$ is ***(2006 - 3M, –1)***

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

30. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-\hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is *(2006 - 3M, –1)*

(a) $4\hat{i}-\hat{j}+4\hat{k}$ (b) $3\hat{i}+\hat{j}-3\hat{k}$

(c) $2\hat{i}+\hat{j}-2\hat{k}$ (d) $4\hat{i}+\hat{j}-4\hat{k}$

31. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is *(2007 -3 marks)*

(a) zero (b) one (c) two (d) three

32. Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Which one of the following is correct ? *(2007 -3 marks)*

(a) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$

(b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$

(c) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}\neq\vec{0}$

(d) $\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}$ are mutually perpendicular

33. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a},\hat{b},\hat{c}$ such that $\hat{a}.\hat{b}=\hat{b}.\hat{c}=\hat{c}.\hat{a}=\frac{1}{2}$. Then, the volume of the parallelopiped is *(2008)*

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

34. Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t+\hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then, *(2008)*

(a) $\hat{u}=\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ *and* $M=(1+\hat{a}\cdot\hat{b})^{1/2}$

(b) $\hat{u}=\frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ *and* $M=(1+\hat{a}\cdot\hat{b})^{1/2}$

(c) $\hat{u}=\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ *and* $M=(1+2\hat{a}\cdot\hat{b})^{1/2}$

(d) $\hat{u}=\frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ *and* $M=(1+2\hat{a}\cdot\hat{b})^{1/2}$

35. Let $P(3, 2, 6)$ be a point in space and Q be a point on the line

$$\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k})$$

Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x-4y+3z=1$ is *(2009)*

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

36. If $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=1$ and $\vec{a}\cdot\vec{c}=\frac{1}{2}$, then *(2009)*

(a) $\vec{a},\vec{b},\vec{c}$ are non-coplanar

(b) $\vec{b},\vec{c},\vec{d}$ are non-coplanar

(c) $\vec{b},\vec{d}$ are non-parallel

(d) $\vec{a},\vec{d}$ are parallel and $\vec{b},\vec{c}$ are parallel

37. A line with positive direction cosines passes through the point $P(2,-1,2)$ and makes equal angles with the coordinate axes. The line meets the plane

$$2x+y+z=9$$

at point Q. The length of the line segment PQ equals *(2009)*

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

38. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a *(2010)*

(a) parallelogram, which is neither a rhombus nor a rectangle

(b) square

(c) rectangle, but not a square

(d) rhombus, but not a square

39. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is *(2010)*

(a) $x+2y-2z=0$ (b) $3x+2y-2z=0$

(c) $x-2y+z=0$ (d) $5x+2y-4z=0$

40. If the distance of the point $P(1,-2,1)$ from the plane $x+2y-2z=\alpha$, where $\alpha>0$, is 5, then the foot of the perpendicular from P to the plane is *(2010)*

(a) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$ (b) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$

(c) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$ (d) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

41. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD}=\hat{i}+2\hat{j}+2\hat{k}$

The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by *(2010)*

(a) $\frac{8}{9}$ (b) $\frac{\sqrt{17}}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4\sqrt{5}}{9}$

42. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by *(2011)*

(a) $\hat{i}-3\hat{j}+3\hat{k}$ (b) $-3\hat{i}-3\hat{j}-\hat{k}$
(c) $3\hat{i}-\hat{j}+3\hat{k}$ (d) $\hat{i}+3\hat{j}-3\hat{k}$

43. The point P is the intersection of the straight line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is *(2012)*

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

44. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$ is *(2012)*

(a) $5x-11y+z=17$ (b) $\sqrt{2}x+y=3\sqrt{2}-1$
(c) $x+y+z=\sqrt{3}$ (d) $x-\sqrt{2}y=1-\sqrt{2}$

45. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}|=\sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})=(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is *(2012)*

(a) 0 (b) 3 (c) 4 (d) 8

46. Let P be the image of the point (3,1,7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1}=\frac{y}{z}=\frac{z}{1}$ is *(JEE Adv. 2016)*

(a) $x+y-3z=0$ (b) $3x+z=0$
(c) $x-4y+7z=0$ (d) $2x-y=0$

D MCQs with One or More than One Correct

1. Let $\vec{a}=a_1i+a_2j+a_3k$, $\vec{b}=b_1i+b_2j+b_3k$ and $\vec{c}=c_1i+c_2j+c_3k$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$ is equal to *(1986 - 2 Marks)*

(a) 0
(b) 1
(c) $\frac{1}{4}(a_1^2+a_2^2+a_2^3)(b_1^2+b_2^2+b_3^2)$
(d) $\frac{3}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)(c_1^2+c_2^2+c_3^2)$

2. The number of vectors of unit length perpendicular to vectors $\vec{a}=(1, 1, 0)$ and $\vec{b}=(0, 1, 1)$ is *(1987 - 2 Marks)*

(a) one (b) two (c) three (d) infinite
(e) None of these.

3. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}-2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$, whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$, is : *(1993 - 2 Marks)*

(a) $2\hat{i}+3\hat{j}-3\hat{k}$ (b) $2\hat{i}+3\hat{j}+3\hat{k}$
(c) $-2\hat{i}-\hat{j}+5\hat{k}$ (d) $2\hat{i}+\hat{j}+5\hat{k}$

4. The vector $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$ is *(1994)*

(a) a unit vecotr
(b) makes an angle $\frac{\pi}{3}$ with the vector $(2\hat{i}-4\hat{j}+3\hat{k})$
(c) parallel to the vector $\left(-\hat{i}+\hat{j}-\frac{1}{2}\hat{k}\right)$
(d) perpendicular to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

5. If $a=i+j+k, \vec{b}=4i+3j+4k$ and $c=i+\alpha j+\beta k$ are linearly dependent vectors and $|c|=\sqrt{3}$, then *(1998 - 2 Marks)*

(a) $\alpha=1, \beta=-1$ (b) $\alpha=1, \beta=\pm1$
(c) $\alpha=-1, \beta=\pm1$ (d) $\alpha=\pm1, \beta=1$

6. For three vectors u, v, w which of the following expression is not equal to any of the remaining three? *(1998 - 2 Marks)*

(a) $u \bullet (v\times w)$ (b) $(v\times w)\bullet u$
(c) $v \bullet (u\times w)$ (d) $(u\times v)\bullet w$

7. Which of the following expressions are meaningful? *(1998 - 2 Marks)*

(a) $u\,(v\times w)$ (b) $(u\bullet v)\bullet w$
(c) $(u\bullet v)\,w$ (d) $u\times(v\bullet w)$

8. Let a and b be two non-collinear unit vectors. If $u = a-(a\,.\,b)\,b$ and $v = a\times b$, then $|v|$ is *(1999 - 3 Marks)*

(a) $|u|$ (b) $|u|+|u\,.\,a|$
(c) $|u|+|u\,.\,b|$ (d) $|u|+u.(a+b)$

9. Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2. Plane P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and that P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between vector $\vec{A}$ and a given vector $2\hat{i}+\hat{j}-2\hat{k}$ is *(2006 - 5M, –1)*

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{4}$

10. The vector (s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are *(2011)*

(a) $\hat{j}-\hat{k}$ (b) $-\hat{i}+\hat{j}$ (c) $\hat{i}-\hat{j}$ (d) $-\hat{j}+\hat{k}$

11. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane (s) containing these two lines is (are) *(2012)*

(a) $y+2z=-1$ (b) $y+z=-1$
(c) $y-z=-1$ (d) $y-2z=-1$

12. A line l passing through the origin is perpendicular to the lines *(JEE Adv. 2013)*

$$l_1:(3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k}, \quad -\infty<t<\infty$$

$$l_2:(3+2s)\hat{i}+(3+2s)\hat{j}+(2+s)\hat{k}, \quad -\infty<s<\infty$$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is (are)

(a) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ (b) $(-1,-1,0)$
(c) $(1,1,1)$ (d) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

13. Two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2: x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then α can take value(s) *(JEE Adv. 2013)*

(a) 1 (b) 2 (c) 3 (d) 4

14. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then *(JEE Adv. 2014)*

(a) $\vec{b}=\left(\vec{b}.\vec{z}\right)\left(\vec{z}-\vec{x}\right)$

(b) $\vec{a}=\left(\vec{a}.\vec{y}\right)\left(\vec{y}-\vec{z}\right)$

(c) $\vec{a}.\vec{b}=-\left(\vec{a}.\vec{y}\right)\left(\vec{b}.\vec{z}\right)$

(d) $\vec{a}=-\left(\vec{a}.\vec{y}\right)\left(\vec{z}-\vec{y}\right)$

15. From a point $P(\lambda,\lambda,\lambda)$, perpendicular PQ and PR are drawn respectively on the lines $y=x, z=1$ and $y=-x, z=-1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is/(are) *(JEE Adv. 2014)*

(a) $\sqrt{2}$ (b) 1 (c) -1 (d) $-\sqrt{2}$

16. In R^3, consider the planes $P_1: y=0$ and $P_2: x+z=1$. Let P_3 be the plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point $(0, 1, 0)$ from P_3 is 1 and the distance of a point (α,β,γ) from P_3 is 2, then which of the following relations is (are) true? *(JEE Adv. 2015)*

(a) $2\alpha+\beta+2\gamma+2=0$ (b) $2\alpha-\beta+2\gamma+4=0$
(c) $2\alpha+\beta-2\gamma-10=0$ (d) $2\alpha-\beta+2\gamma-8=0$

17. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z+1=0$ and $P_2: 2x-y+z-1=0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie (s) on M? *(JEE Adv. 2015)*

(a) $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$ (b) $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$
(c) $\left(-\frac{5}{6},0,\frac{1}{6}\right)$ (d) $\left(-\frac{1}{3},0,\frac{2}{3}\right)$

18. Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}$, $\vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12$, $|\vec{b}|=4\sqrt{3}$, $\vec{b}.\vec{c}=24$, then which of the following is (are) true? *(JEE Adv. 2015)*

(a) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=12$ (b) $\frac{|\vec{c}|^2}{2}+|\vec{a}|=30$
(c) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$ (d) $\vec{a}.\vec{b}=-72$

19. Consider a pyramid OPQRS located in the first octant ($x\geq 0, y\geq 0, z\geq 0$) with O as origin, and OP and OR along the x–axis and the y–axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point, T of diagonal OQ such that TS = 3. Then *(JEE Adv. 2016)*

(a) the acute angle between OQ and OS is $\frac{\pi}{3}$
(b) the equation of the plane containing the triangle OQS is $x-y=0$
(c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$
(d) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

20. Let $\hat{u}=u_1\hat{i}+u_2\hat{j}+u_3\hat{k}$ be a unit vector in R^3 and $\hat{w}=\frac{1}{\sqrt{6}}\left(\hat{i}+\hat{j}+2\hat{k}\right)$. Given that there exists a vector $\vec{v}$ in R^3 such that $\left|\hat{u}\times\vec{v}\right|=1$ and $\hat{w}\cdot\left(\hat{u}\times\vec{v}\right)=1$. Which of the following statement(s) is (are) correct? *(JEE Adv. 2016)*

(a) There is exactly one choice for such $\vec{v}$

(b) There are infinitely many choices for such $\vec{v}$

(c) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$

(d) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$

E Subjective Problems

1. From a point O inside a triangle ABC, perpendiculars OD, OE, OF are drawn to the sides BC, CA, AB respectively. Prove that the perpendiculars from A, B, C to the sides EF, FD, DE are concurrent. ***(1978)***

2. $A_1, A_2, \ldots\ldots\ldots\ldots A_n$ are the vertices of a regular plane polygon with n sides and O is its centre. Show that $\sum_{i=1}^{n-1}(\overrightarrow{OA_i} \times \overrightarrow{OA_{i+1}}) = (1-n)(\overrightarrow{OA_2} \times \overrightarrow{OA_1})$ ***(1982 - 2 Marks)***

3. Find all values of λ such that $x, y, z, \neq (0,0,0)$ and $(\vec{i}+\vec{j}+3\vec{k})x+(3\vec{i}-3\vec{j}+\vec{k})y+(-4\vec{i}+5\vec{j})z = \lambda(x\vec{i}\times\vec{j}\,y+\vec{k}\,z)$ where $\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors along the coordinate axes. ***(1982 - 3 Marks)***

4. A vector $\vec{A}$ has components A_1, A_2, A_3 in a right -handed rectangular Cartesian coordinate system $oxyz$. The coordinate system is rotated about the x-axis through an angle $\frac{\pi}{2}$. Find the components of A in the new coordinate system, in terms of A_1, A_2, A_3. ***(1983 - 2 Marks)***

5. The position vectors of the points A, B, C and D are $3\hat{i}-2\hat{j}-\hat{k}, 2\hat{i}+3\hat{j}-4\hat{k}, -\hat{i}+\hat{j}+2\hat{k}$ and $4\hat{i}+5\hat{j}+\lambda\hat{k}$, respectively. If the points A, B, C and D lie on a plane, find the value of λ. ***(1986 - 2½ Marks)***

6. If A, B, C, D are any four points in space, prove that – ***(1987 - 2 Marks)***
$$\left|\overrightarrow{AB}\times\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}\right| = 4 \text{ (area of triangle } ABC)$$

7. Let $OA\,CB$ be a parallelogram with O at the origin and OC a diagonal. Let D be the midpoint of OA. Using vector methods prove that BD and CO intersect in the same ratio. Determine this ratio. ***(1988 - 3 Marks)***

8. If vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, show that
$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = \vec{0}$$ ***(1989 - 2 Marks)***

9. In a triangle OAB, E is the midpoint of BO and D is a point on AB such that $AD : DB = 2 : 1$. If OD and AE intersect at P, determine the ratio $OP : PD$ using vector methods. ***(1989 - 4 Marks)***

10. Let $\vec{A} = 2\vec{i}+\vec{k}$, $\vec{B} = \vec{i}+\vec{j}+\vec{k}$, and $\vec{C} = 4\vec{i}-3\vec{j}+7\vec{k}$. Determine a vector $\vec{R}$. Satisfying $\vec{R}\times\vec{B} = \vec{C}\times\vec{B}$ and $\vec{R}\cdot\vec{A} = 0$ ***(1990 - 3 Marks)***

11. Determine the value of 'c' so that for all real x, the vector $cx\hat{i}-6\hat{j}-3\hat{k}$ and $x\hat{i}+2\hat{j}+2cx\hat{k}$ make an obtuse angle with each other. ***(1991 - 4 Marks)***

12. In a triangle ABC, D and E are points on BC and AC respectively, such that $BD = 2\,DC$ and $AE = 3EC$. Let P be the point of intersection of AD and BE. Find BP/PE using vector methods. ***(1993 - 5 Marks)***

13. If the vectors $\vec{b}, \vec{c}, \vec{d}$, are not coplanar, then prove that the vector $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})+(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})+(\vec{a}\times\vec{d})\times(\vec{b}\times\vec{c})$ is parallel to $\vec{a}$. ***(1994 - 4 Marks)***

14. The position vectors of the vertices A, B and C of a tetrahedron $ABCD$ are $\hat{i}+\hat{j}+\hat{k}$, $\hat{i}$ and $3\hat{i}$, respectively. The altitude from vertex D to the opposite face ABC meets the median line through A of the triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$, find the position vector of the point E for all its possible positions. ***(1996 - 5 Marks)***

15. If A, B and C are vectors such that $|B| = |C|$. Prove that $[(A+B)\times(A+C)]\times(B\times C)(B+C) = 0$. ***(1997 - 5 Marks)***

16. Prove, by vector methods or otherwise, that the point of intersection of the diagonals of a trapezium lies on the line passing through the mid-points of the parallel sides. (You may assume that the trapezium is not a parallelogram.) ***(1998 - 8 Marks)***

17. For any two vectors u and v, prove that ***(1998 - 8 Marks)***
(a) $(u\cdot v)^2+|u\times v|^2 = |u|^2|v|^2$ and
(b) $(1+|u|^2)(1+|v|^2) = (1-u\cdot v)^2+|u+v+(u\times v)|^2$.

18. Let u and v be unit vectors. If w is a vector such that $w+(w\times u) = v$, then prove that $|(u\times v)\cdot w| \le 1/2$ and that the equality holds if and only if u is perpendicular to v. ***(1999 - 10 Marks)***

19. Show, by vector methods, that the angular bisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices. ***(2001 - 5 Marks)***

20. Find 3–dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying
$\vec{v}_1.\vec{v}_1 = 4, \vec{v}_1.\vec{v}_2 = -2, \vec{v}_1.\vec{v}_3 = 6, \vec{v}_2.\vec{v}_2 = 2, \vec{v}_2.\vec{v}_3 = -5, \vec{v}_3.\vec{v}_3 = 29$ ***(2001 - 5 Marks)***

21. Let $\vec{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$ and

$\vec{B}(t) = g_1(t)\hat{i} + g_2(t)\hat{j}, t \in [0, 1]$,

where f_1, f_2, g_1, g_2 are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are nonzero vectors for all t and $\vec{A}(0) = 2\hat{i} + 3\hat{j}$, $\vec{A}(1) = 6\hat{i} + 2\hat{j}$, $\vec{B}(0) = 3\hat{i} + 2\hat{j}$ and $\vec{B}(1) = 2\hat{i} + 6\hat{j}$. Then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t. ***(2001 - 5 Marks)***

22. Let V be the volume of the parallelopiped formed by the vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. If a_r, b_r, c_r, where $r = 1, 2, 3$, are non-negative real numbers and $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$, show that $V \le L^3$. ***(2002 - 5 Marks)***

23. (i) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).
(ii) If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it. ***(2003 - 4 Marks)***

24. If $\vec{u}, \vec{v}, \vec{w}$, are three non-coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$ respectively and $\vec{x}, \vec{y}, \vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. Prove that $[\vec{x}\times\vec{y} \;\; \vec{y}\times\vec{z} \;\; \vec{z}\times\vec{x}] = \frac{1}{16}[\vec{u} \; \vec{v} \; \vec{w}]^2 \sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$.
(2003 - 4 Marks)

25. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are distinct vectors such that $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$. Prove that $(\vec{a}-\vec{d}).(\vec{b}-\vec{c}) \ne 0$ i.e. $\vec{a}.\vec{b} + \vec{d}.\vec{c} \ne \vec{d}.\vec{b} + \vec{a}.\vec{c}$
(2004 - 2 Marks)

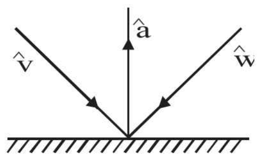
26. Find the equation of plane passing through (1, 1, 1) & parallel to the lines L_1, L_2 having direction ratios (1,0,–1), (1,–1,0). Find the volume of tetrahedron formed by origin and the points where these planes intersect the coordinate axes.
(2004 - 2 Marks)

27. A parallelopiped 'S' has base points A, B, C and D and upper face points A', B', C' and D'. This parallelopiped is compressed by upper face $A'B'C'D'$ to form a new parallelopiped 'T' having upper face points A'', B'', C'' and D''. Volume of parallelopiped T is 90 percent of the volume of parallelopiped S. Prove that the locus of 'A''', is a plane.
(2004 - 2 Marks)

28. P_1 and P_2 are planes passing through origin. L_1 and L_2 are two line on P_1 and P_2 respectively such that their intersection is origin. Show that there exists points A, B, C, whose permutation A', B', C' can be chosen such that (i) A is on L_1, B on P_1 but not on L_1 and C not on P_1 (ii) A' is on L_2, B' on P_2 but not on L_2 and C' not on P_2
(2004 - 4 Marks)

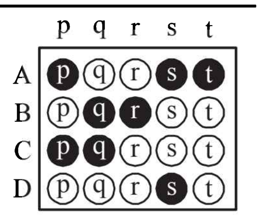
29. Find the equation of the plane containing the line $2x - y + z - 3 = 0$, $3x + y + z = 5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, –1). ***(2005 - 2 Marks)***

30. If the incident ray on a surface is along the unit vector $\hat{v}$, the reflected ray is along the unit vector $\hat{w}$ and the normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$. ***(2005 - 4 Marks)***



F Match the Following

DIRECTIONS (Q. 1-6) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. Match the following : ***(2006 - 6M)***

(A) Two rays $x + y = |a|$ and $ax - y = 1$ intersects each other in the first quadrant in the interval $a \in (a_0, \infty)$, the value of a_0 is — (p) 2

(B) Point (α, β, γ) lies on the plane $x + y + z = 2$. — (q) $\frac{4}{3}$

Let $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\hat{k}\times(\hat{k}\times\vec{a}) = 0$, then $\gamma =$

(C) $\left|\int_0^1 (1-y^2)\,dy\right| + \left|\int_1^0 (y^2-1)\,dy\right|$ — (r) $\left|\int_0^1 \sqrt{1-x}\,dx\right| + \left|\int_{-1}^0 \sqrt{1+x}\,dx\right|$

(D) If $\sin A \sin B \sin C + \cos A \cos B = 1$, then the value of $\sin C =$ — (s) 1

2. Consider the following linear equations

$ax + by + cz = 0; \quad bx + cy + az = 0; \quad cx + ay + bz = 0$

Match the conditions/expressions in **Column I** with statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the *ORS*. *(2007)*

Column I	**Column II**
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p) the equations represent planes meeting only at asingle point
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q) the equations represent the line $x = y = z$.
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r) the equations represent identical planes.
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s) the equations represent the whole of the three dimensional space.

3. Match the statements / expressions given in **Column-I** with the values given in **Column-II**. *(2009)*

Column-I	**Column-II**
(A) Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(p) $\frac{\pi}{6}$
(B) Points of discontinuity of the unction $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, f where $[y]$ denotes the largest integer less than or equal to y	(q) $\frac{\pi}{4}$
(C) Volume of the parallelopiped with its edges represented by the vectors $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}$ and $\hat{i} + \hat{j} + \pi\hat{k}$	(r) $\frac{\pi}{3}$
(D) Angle between vector $\vec{a}$ and $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a} + \vec{b} + \sqrt{3}\,\vec{c} = \vec{0}$	(s) $\frac{\pi}{2}$
	(t) π

4. Match the statements/expressions given in **Column-I** with the values given in **Column-II**. *(2009)*

Column-I	**Column-II**
(A) The number of solutions of the equation $x\,e^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p) 1
(B) Value(s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q) 2
(C) Value(s) of k for which $\|x-1\| + \|x-2\| + \|x+1\| \|+\|x+2\| = 4k$ has integer solution(s)	(r) 3
(D) If $y' = y + 1$ and $y(0) = 1$, then value(s) of y (1n 2)	(s) 4
	(t) 5

5. Match the statement in **Column-1** with the values in **Column -II** *(2010)*

Column – I	**Column – II**
(A) A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x - \frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If length $PQ = d$, then d^2 is	(p) -4
(B) The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0
(C) Non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b} = 0$. $(\vec{b} - \vec{a}).(\vec{b} + \vec{c}) = 0$ and $2\|\vec{b} + \vec{c}\| = \|\vec{b} - \vec{a}\|$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are	(r) 4

(D) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$

and $f(x) = \sin\left(\frac{9x}{2}\right) \Big/ \sin\left(\frac{x}{2}\right)$ *for* $x \neq 0$ (s) 5

The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is

(t) 6

6. Match the statements given in Column-I with the values given in Column-II. *(2011)*

Column-I **Column-II**

(A) If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}, \vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is (p) $\frac{\pi}{6}$

(B) If $\int_a^b (f(x) - 3x)\, dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is (q) $\frac{2\pi}{3}$

(C) The value of $\frac{\pi^2}{\ell n 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is (r) $\frac{\pi}{3}$

(D) The maximum value of $\left|Arg\left(\frac{1}{1-z}\right)\right|$ for $|z| = 1, z \neq 1$ is given by (s) π

(t) $\frac{\pi}{2}$

DIRECTIONS (Q. 7-9) : *Each question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

7. Match List I with List II and select the correct answer using the code given below the lists : *(JEE Adv. 2013)*

List I **List II**

P. Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c})$ and $2(\vec{c} \times \vec{a})$ is 1. 100

Q. Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a} + \vec{b}), 3(\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is 2. 30

R. Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$ is 3. 24

S. Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is 4. 60

Codes:

	P	Q	R	S
(a)	4	2	3	1
(b)	2	3	1	4
(c)	3	4	1	2
(d)	1	4	3	2

8. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}, L_2 \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 = 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2.

Match List I with List II and select the correct answer using the code given below the lists : *(JEE Adv. 2013)*

	List I		List II
P.	$a =$	1.	13
Q.	$b =$	2.	-3
R.	$c =$	3.	1
S.	$d =$	4.	-2

Codes:

	P	Q	R	S
(a)	3	2	4	1
(b)	1	3	4	2
(c)	3	2	1	4
(d)	2	4	1	3

9. Match List I with List II and select the correct answer using the code given below the lists : *(JEE Adv. 2014)*

	List - I		List - II
P.	Let $y(x) = \cos\left(3\cos^{-1}x\right)$, $x \in [-1,1]$, $x \neq \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{\left(x^2-1\right)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equals	1.	1
Q.	Let $A_1, A_2, ..., A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point A_k, $k = 1, 2, ..., n$. If $\left\lvert\sum_{k=1}^{n-1}\left(\vec{a}_k \times \vec{a}_{k+1}\right)\right\rvert = \left\lvert\sum_{k=1}^{n-1}\left(\vec{a}_k \cdot \vec{a}_{k+1}\right)\right\rvert$, then the minimum value of n is	2.	2
R.	If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is	3.	8
S.	Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is	4.	9

	P	Q	R	S
(a)	4	3	2	1
(b)	2	4	3	1
(c)	4	3	1	2
(d)	2	4	1	3

DIRECTIONS (Q. 10 & 11) : *Refer to Directions (1-6).*

10. Match the following : *(JEE Adv. 2015)*

Column I	Column II
(A) In R^2, if the magnitude of the projection vector of the vector $\alpha\hat{i}+\beta\hat{j}$ on $\sqrt{3}\hat{i}+\hat{j}$ is $\sqrt{3}$ and if $\alpha = 2+\sqrt{3}\beta$, then possible value of $\lvert\alpha\rvert$ is/are	(p) 1
(B) Let a and b be real numbers such that the function $f(x)=\begin{cases}-3ax^2-2, & x<1\\ bx+a^2, & x\ge 1\end{cases}$ if differentiable for all $x \in R$. Then possible value of a is (are)	(q) 2
(C) Let $\omega \neq 1$ be a complex cube root of unity. If $(3-3\omega+2\omega^2)^{4n+3}+(2+3\omega-3\omega^2)^{4n+3}+(-3+2\omega+3\omega^2)^{4n+3}=0$, then possible value (s) of n is (are)	(r) 3
(D) Let the harmonic mean of two positive real numbers a and b be 4. If q is a positive real number such that a, 5, q, b is an arithmetic progression, then the value(s) of $\lvert q-a \rvert$ is (are)	(s) 4
	(t) 5

11. Match the following : *(JEE Adv. 2015)*

Column I	Column II
(A) In a triangle ΔXYZ, let a, b, and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $2(a^2-b^2)=c^2$ and $\lambda=\dfrac{\sin(X-Y)}{\sin Z}$, then possible values of n for which $\cos(n\pi\lambda)=0$ is (are)	(p) 1
(B) In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y, and Z respectively. If $1+\cos 2X-2\cos 2Y = 2\sin X \sin Y$, then possible value (s) of $\dfrac{a}{b}$ is (are)	(q) 2
(C) In R^2, let $\sqrt{3}i+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ be the position vectors of X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\dfrac{3}{\sqrt{2}}$, then possible value(s) of $\lvert\beta\rvert$ is (are)	(r) 3
(D) Suppose that $F(\alpha)$ denotes the area of the region bounded by $x = 0$, $x=2, y^2=4x$ and $y=\lvert\alpha x-1\rvert+\lvert\alpha x-2\rvert+\alpha x$, where $\alpha \in \{0, 1\}$. Then the value(s) of $F(\alpha)+\dfrac{8}{3}\sqrt{2}$, when $\alpha=0$ and $\alpha=1$, is (are)	(s) 5
	(t) 6

G Comprehension Based Questions

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2} \quad L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

1. The unit vector perpendicular to both L_1 and L_2 is ***(2008)***

(a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

2. The shortest distance between L_1 and L_2 is ***(2008)***

(a) 0 (b) $\frac{17}{\sqrt{3}}$

(c) $\frac{41}{5\sqrt{3}}$ (d) $\frac{17}{5\sqrt{3}}$

3. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is ***(2008)***

(a) $\frac{2}{\sqrt{75}}$ (b) $\frac{7}{\sqrt{75}}$

(c) $\frac{13}{\sqrt{75}}$ (d) $\frac{23}{\sqrt{75}}$

H Assertion & Reason Type Questions

1. Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.
STATEMENT-1 : The parametric equations of the line of intersection of the given planes are $x = 3 + 14t, y = 1 + 2t, z = 15t$. **because**

STATEMENT-2 : The vector $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of given planes. ***(2007 -3 marks)***

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

2. Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

STATEMENT-1 : $\overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq \overrightarrow{0}$. **because**

STATEMENT-2 : $\overrightarrow{PQ} \times \overrightarrow{RS} = \overrightarrow{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \overrightarrow{0}$.

(2007 -3 marks)

(a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

3. Consider three planes

$P_1 : x - y + z = 1$ $P_2 : x + y - z = 1$
$P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.
STATEMENT - 1Z : At least two of the lines L_1, L_2 and L_3 are non-parallel and
STATEMENT - 2 : The three planes doe not have a common point. ***(2008)***

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is NOT** a correct explanation for STATEMENT - 1
(C) STATEMENT - 1 is True, STATEMENT - 2 is False
(D) STATEMENT - 1 is False, STATEMENT - 2 is True

I Integer Value Correct Type

1. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then find the value of $\left(2\vec{a}+\vec{b}\right)$. $\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$. ***(2010)***

2. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then find $|d|$. ***(2010)***

3. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is ***(2011)***

4. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying ***(2012)***
$|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

5. Consider the set of eight vectors
$V = \left\{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\right\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is ***(JEE Adv. 2013)***

6. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k-20=$ *(JEE Adv. 2013)*

7. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c} = p\vec{a}+q\vec{b}+r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is *(JEE Adv. 2014)*

8. Suppose that $\vec{p},\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p},\vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r}),(\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x, y and z, respectively, then the value of $2x+y+z$ is *(JEE Adv. 2015)*

Section-B JEE Main / AIEEE

1. A plane which passes through the point (3, 2, 0) and the line $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$ is **[2002]**
   (a) $x-y+z=1$ (b) $x+y+z=5$
   (c) $x+2y-z=1$ (d) $2x-y+z=5$

2. If $|\vec{a}|=4, |\vec{b}|=2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$ then $(\vec{a}\times\vec{b})^2$ is equal to **[2002]**
   (a) 48 (b) 16
   (c) $\vec{a}$ (d) none of these

3. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are vectors such that $[\vec{a}\ \vec{b}\ \vec{c}]=4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=$ **[2002]**
   (a) 16 (b) 64 (c) 4 (d) 8

4. If $\vec{a},\vec{b},\vec{c}$ are vectors show that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is **[2002]**
   (a) 60° (b) 30° (c) 45° (d) 90°

5. If $|\vec{a}|=5$, $|\vec{b}|=4$, $|\vec{c}|=3$ thus what will be the value of $|\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}|$, given that $\vec{a}+\vec{b}+\vec{c}=0$ **[2002]**
   (a) 25 (b) 50 (c) –25 (d) –50

6. If the vectors $\vec{c}$, $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\hat{b}=\hat{j}$ are such that $\vec{a}$, $\vec{c}$ and $\vec{b}$ form a right handed system then $\vec{c}$ is : **[2002]**
   (a) $z\hat{i}-x\hat{k}$ (b) $\vec{0}$
   (c) $y\hat{j}$ (d) $-z\hat{i}+x\hat{k}$

7. $\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{b}$ then $|\vec{a}|:|\vec{b}|:|\vec{c}|$
   (a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ (b) $\sqrt{34}:\sqrt{45}:39$ **[2002]**
   (c) $34:39:45$ (d) $39:35:34$

8. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ then $\vec{a}+\vec{b}+\vec{c}=$ **[2002]**
   (a) abc (b) –1 (c) 0 (d) 2

9. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\pi/4$ with plane $x+y=3$ are **[2002]**
   (a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$
   (c) $1, 1, 2$ (d) $\sqrt{2}, 1, 1$

10. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ *and* $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n}=0$ and $\vec{v}.\hat{n}=0$, then $|\vec{w}.\hat{n}|$ is equal to **[2003]**
    (a) 3 (b) 0 (c) 1 (d) 2.

11. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}-3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The total work done by the forces is
    (a) 50 units (b) 20 units **[2003]**
    (c) 30 units (d) 40 units.

12. The vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ & $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is **[2003]**
    (a) $\sqrt{288}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d) $\sqrt{33}$

13. The shortest distance from the plane $12x+4y+3z=327$ to the sphere $x^2+y^2+z^2+4x-2y-6z=155$ is

(a) 39 (b) 26 (c) $11\frac{4}{13}$ (d) 13

14. The two lines $x=ay+b, z=cy+d$ and $x=a'y+b', z=c'y+d'$ will be perpendicular, if and only if **[2003]**

(a) $aa'+cc'+1=0$
(b) $aa'+bb'+cc'+1=0$
(c) $aa'+bb'+cc'=0$
(d) $(a+a')(b+b')+(c+c')=0$.

15. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{1}=\frac{z-5}{1}$ are coplanar if **[2003]**

(a) $k=3$ or -2 (b) $k=0$ or -1
(c) $k=1$ or -1 (d) $k=0$ or -3

16. $\vec{a},\vec{b},\vec{c}$ are 3 vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=1$ $|\vec{b}|=2, |\vec{c}|=3$, , then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to **[2003]**

(a) 1 (b) 0 (c) –7 (d) 7

17. The radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ is **[2003]**

(a) 4 (b) 1 (c) 2 (d) 3

18. A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1) B(2, 1, 3) and C(–1, 1, 2). Then the angle between the faces OAB and ABC will be **[2003]**

(a) 90^o (b) $\cos^{-1}\left(\frac{19}{35}\right)$

(c) $\cos^{-1}\left(\frac{17}{31}\right)$ (d) 30^o

19. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and vectors $(1,a,a^2)$, $(1,b,b^2)$ and $(1,c,c^2)$ are non- coplanar, then the product abc equals **[2003]**

(a) 0 (b) 2 (c) –1 (d) 1

20. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}, \hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a **[2003[**

(a) parallelogram but not a rhombus
(b) square
(c) rhombus
(d) rectangle.

21. If $\vec{u},\vec{v}$ *and* $\vec{w}$ are three non- coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals **[2003]**

(a) $3\vec{u}.\vec{v}\times\vec{w}$ (b) 0
(c) $\vec{u}.\vec{v}\times\vec{w}$ (d) $\vec{u}.\vec{w}\times\vec{v}$

22. Two system of rectangular axes have the same origin. If a plane cuts them at distances a,b,c and a',b',c' from the origin then **[2003]**

(a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(b) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

(c) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(d) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$.

23. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is **[2004]**

(a) $\frac{9}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{3}{2}$

24. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x=y+a=z$ and $x+a=2y=2z$. The co-ordinates of each of the points of intersection are given by **[2004]**

(a) $(2a,3a,3a),(2a,a,a)$ (b) $(3a,2a,3a),(a,a,a)$
(c) $(3a,2a,3a),(a,a,2a)$ (d) $(3a,3a,3a),(a,a,a)$

25. If the straight lines **[2004]**

$x=1+s, y=-3-\lambda s, z=1+\lambda s$ and $x=\frac{t}{2}, y=1+t, z=2-t$,

with parameters s and t respectively, are co-planar, then λ equals.

(a) 0 (b) –1
(c) $-\frac{1}{2}$ (d) –2

26. The intersection of the spheres

$x^2+y^2+z^2+7x-2y-z=13$ and

$x^2+y^2+z^2-3x+3y+4z=8$

is the same as the intersection of one of the sphere and the plane **[2004]**

(a) $2x-y-z=1$ (b) $x-2y-z=1$
(c) $x-y-2z=1$ (d) $x-y-z=1$

27. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals **[2004]**

(a) 0 (b) $\lambda\vec{b}$ (c) $\lambda\vec{c}$ (d) $\lambda\vec{a}$

28. A particles is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by **[2004]**

(a) 15 (b) 30 (c) 25 (d) 40

29. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non coplanar for **[2004]**

(a) no value of λ
(b) all except one value of λ
(c) all except two values of λ
(d) all values of λ

30. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}$, $\vec{w}$ are perpendicular to each other then $|\vec{u}-\vec{v}+\vec{w}|$ equals **[2004]**

(a) 14 (b) $\sqrt{7}$ (c) $\sqrt{14}$ (d) 2

31. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals **[2004]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

32. If C is the mid point of AB and P is any point outside AB, then **[2005]**

(a) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$ (b) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$
(c) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=\vec{0}$ (d) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{0}$

33. If the angel θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}\,z+4=0$ is such that $\sin\theta=\frac{1}{3}$ then the value of λ is **[2005]**

(a) $\frac{5}{3}$ (b) $\frac{-3}{5}$
(c) $\frac{3}{4}$ (d) $\frac{-4}{3}$

34. The angle between the lines $2x=3y=-z$ and $6x=-y=-4z$ is **[2005]**

(a) 0° (b) 90°
(c) 45° (d) 30°

35. If the plane $2ax-3ay+4az+6=0$ passes through the midpoint of the line joining the centres of the spheres $x^2+y^2+z^2+6x-8y-2z=13$ and $x^2+y^2+z^2-10x+4y-2z=8$ then a equals **[2005]**

(a) -1 (b) 1
(c) -2 (d) 2

36. The distance between the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(i-j+4k)$ and the plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$ is **[2005]**

(a) $\frac{10}{9}$ (b) $\frac{10}{3\sqrt{3}}$
(c) $\frac{3}{10}$ (d) $\frac{10}{3}$

37. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to **[2005]**

(a) $3\vec{a}^2$ (b) $\vec{a}^2$
(c) $2\vec{a}^2$ (d) $4\vec{a}^2$

38. If non zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is **[2005]**

(a) $(-1,2)$ (b) $(-1,-2)$
(c) $(1,-2)$ (d) $\left(1,-\frac{1}{2}\right)$

39. Let a, b and c be distinct non- negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is **[2005]**

(a) the Geometric Mean of a and b
(b) the Arithmetic Mean of a and b
(c) equal to zero
(d) the Harmonic Mean of a and b

40. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number then $[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}] = [\vec{a}\quad \vec{b}+\vec{c}\quad \vec{b}]$ for **[2005]**

(a) exactly one value of λ

(b) no value of λ

(c) exactly three values of λ

(d) exactly two values of λ

41. Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on **[2005]**

(a) only y (b) only x

(c) both x and y (d) neither x nor y

42. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius **[2005]**

(a) 3 (b) 1 (c) 2 (d) $\sqrt{2}$

43. If $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{a} \times (\bar{b} \times \bar{c})$ where $\bar{a}, \bar{b}$ and $\bar{c}$ are any three vectors such that $\bar{a}.\bar{b} \neq 0$, $\bar{b}.\bar{c} \neq 0$ then $\bar{a}$ and $\bar{c}$ are **[2006]**

(a) inclined at an angle of $\frac{\pi}{3}$ between them

(b) inclined at an angle of $\frac{\pi}{6}$ between them

(c) perpendicular

(d) parallel

44. The values of a, for which points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right angled triangle with $C = \frac{\pi}{2}$ are

(a) 2 and 1 (b) – 2 and – 1 **[2006]**

(c) – 2 and 1 (d) 2 and – 1

45. The two lines $x = ay + b$, $z = cy + d$; and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other if **[2006]**

(a) $aa' + cc' = -1$ (b) $aa' + cc' = 1$

(c) $\frac{a}{a'} + \frac{c}{c'} = -1$ (d) $\frac{a}{a'} + \frac{c}{c'} = 1$

46. The image of the point (–1, 3, 4) in the plane $x - 2y = 0$ is

(a) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (b) (15,11,4) **[2006[**

(c) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (d) None of these

47. If a line makes an angle of $\pi/4$ with the positive directions of each of x- axis and y- axis, then the angle that the line makes with the positive direction of the z-axis is **[2007]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

48. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u} \times 3\hat{v}$ is a unit vector for **[2007]**

(a) no value of θ

(b) exactly one value of θ

(c) exactly two values of θ

(d) more than two values of θ

49. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the cooordinates of the other end of the diameter are **[2007]**

(a) (4, 3, 5) (b) (4, 3, – 3)

(c) (4, 9, – 3) (d) (4,–3, 3).

50. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vectors $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals **[2007]**

(a) – 4 (b) – 2

(c) 0 (d) 1.

51. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then cos α equals **[2007]**

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

52. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β? **[2008]**

(a) $\alpha = 2, \beta = 2$ (b) $\alpha = 1, \beta = 2$

(c) $\alpha = 2, \beta = 1$ (d) $\alpha = 1, \beta = 1$

53. The non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is **[2008]**

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

54. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

(a) $a=2, b=8$ (b) $a=4, b=6$
(c) $a=6, b=4$ (d) $a=8, b=2$

55. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to **[2008]**

(a) –5 (b) 5
(c) 2 (d) –2

56. Let the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$. Then (α, β) equals **[2009]**

(a) $(-6, 7)$ (b) $(5, -15)$
(c) $(-5, 5)$ (d) $(6, -17)$

57. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are : **[2009]**

(a) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (b) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

(c) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (d) $6, -3, 2$

58. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u}\ p\vec{v}\ p\vec{\omega}]-[p\vec{v}\ \vec{\omega}\ q\vec{u}]-[2\vec{\omega}\ q\vec{v}\ q\vec{u}]=0$ holds for : **[2009]**

(a) exactly two values of (p, q)
(b) more than two but not all values of (p, q)
(c) all values of (p, q)
(d) exactly one value of (p, q)

59. Let $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c}=\vec{0}$ and $\vec{a}.\vec{b}=3$ **[2010]**

(a) $2\hat{i}-\hat{j}+2\hat{k}$ (b) $\hat{i}-\hat{j}-2\hat{k}$

(c) $\hat{i}+\hat{j}-2\hat{k}$ (d) $-\hat{i}+\hat{j}-2\hat{k}$

60. If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu)=$ **[2010]**

(a) $(2, -3)$ (b) $(-2, 3)$
(c) $(3, -2)$ (d) $(-3, 2)$

61. **Statement -1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x-y+z=5$.
Statement -2: The plane $x-y+z=5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4). **[2010]**

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

62. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals **[2010]**

(a) 45° (b) 60° (c) 75° (d) 30°

63. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals **[2011]**

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$

(c) $\frac{5}{3}$ (d) $\frac{2}{3}$

64. If $\vec{a}=\frac{1}{\sqrt{10}}(3\hat{i}+\hat{k})$ and $\vec{b}=\frac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})$, then the value of $(2\vec{a}-\vec{b})\left[(\vec{a}\times\vec{b})\times(\vec{a}+2\vec{b})\right]$ is **[2011]**

(a) –3 (b) 5 (c) 3 (d) –5

65. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}.\vec{d}=0$. Then the vector $\vec{d}$ is equal to **[2011]**

(a) $\vec{c}+\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (b) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

(c) $\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (d) $\vec{b}-\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

66. **Statement-1:** The point $A(1, 0, 7))$ is the mirror image of the point $B(1, 6, 3)$ in the line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ **[2011]**

Statement-2: The line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

67. Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is : **[2012]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

68. A equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is : **[2012]**

(a) $x - 2y + 2z - 3 = 0$ (b) $x - 2y + 2z + 1 = 0$
(c) $x - 2y + 2z - 1 = 0$ (d) $x - 2y + 2z + 5 = 0$

69. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to: **[2012]**

(a) -1 (b) $\frac{2}{9}$
(c) $\frac{9}{2}$ (d) 0

70. Let $ABCD$ be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincide with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[2012]**

(a) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (b) $\vec{r} = -\vec{q} + \frac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

(c) $\vec{r} = \vec{q} - \frac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (d) $\vec{r} = -3\vec{q} - \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

71. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is **[JEE M 2013]**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{9}{2}$

72. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have **[JEE M 2013]**

(a) any value (b) exactly one value
(c) exactly two values (d) exactly three values

73. If the vectors $\overline{AB} = 3\hat{i} + 4\hat{k}$ and $\overline{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is **[JEE M 2013]**

(a) $\sqrt{18}$ (b) $\sqrt{72}$
(c) $\sqrt{33}$ (d) $\sqrt{45}$

74. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line: **[JEE M 2014]**

(a) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

(b) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

(c) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

(d) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

75. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is **[JEE M 2014]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

76. If $\left[\vec{a}\times\vec{b} \;\; \vec{b}\times\vec{c} \;\; \vec{c}\times\vec{a}\right] = \lambda\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$ then λ is equal to **[JEE M 2014]**

(a) 0 (b) 1 (c) 2 (d) 3

77. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is : **[JEE M 2015]**

(a) $\frac{2}{3}$ (b) $\frac{-2\sqrt{3}}{3}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{-\sqrt{2}}{3}$

78. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is : **[JEE M 2015]**

(a) $x + 3y + 6z = 7$ (b) $2x + 6y + 12z = -13$
(c) $2x + 6y + 12z = 13$ (d) $x + 3y + 6z = -7$

79. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is **[JEE M 2015]**

(a) $3\sqrt{21}$ (b) 13 (c) $2\sqrt{14}$ (d) 8

80. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to : **[JEE M 2016]**

(a) 5 (b) 2 (c) 26 (d) 18

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81. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\sqrt{3}}{2}\left(\vec{b} + \vec{c}\right)$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is: **[JEE M 2016]**

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{2}$

82. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is : **[JEE M 2016]**

(a) $\frac{10}{\sqrt{3}}$ (b) $\frac{20}{3}$

(c) $3\sqrt{10}$ (d) $10\sqrt{3}$