CHAPTER 3 Quadratic Equation and Inequations (Inequalities)

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. The coefficient of x^{99} in the polynomial $(x-1)(x-2)\,....(x-100)$ is .... *(1982 - 2 Marks)*
2. If $2+i\sqrt{3}$ is a root of the equation $x^2+px+q=0$, where p and q are real, then $(p, q) = ($.................. , ...$)$. *(1982 - 2 Marks)*
3. If the product of the roots of the equation $x^2-3kx+2e^{2\ln k}-1=0$ is 7, then the roots are real for $k =$ .... *(1984 - 2 Marks)*
4. If the quadratic equations $x^2+ax+b=0$ and $x^2+bx+a=0$ $(a \neq b)$ have a common root, then the numerical value of $a+b$ is .... *(1986 - 2 Marks)*
5. The solution of equation $\log_7 \log_5(\sqrt{x+5}+\sqrt{x})=0$ is .... *(1986 - 2 Marks)*
6. If $x<0, y<0, x+y+\dfrac{x}{y}=\dfrac{1}{2}$ and $(x+y)\dfrac{x}{y}=-\dfrac{1}{2}$, then $x =$ .... and $y =$ ...... *(1990 - 2 Marks)*
7. Let n and k be positive such that $n \geq \dfrac{k(k+1)}{2}$. The number of solutions $(x_1, x_2, ...x_k)$, $x_1 \geq 1$, $x_2 \geq 2$, ...,$x_k \geq k$, all integers, satisfying $x_1+x_2+....+x_k=n$, is .... *(1996 - 2 Marks)*
8. The sum of all the real roots of the equation $|x-2|^2+|x-2|-2=0$ is .... *(1997 - 2 Marks)*

B True / False

1. For every integer $n>1$, the inequality $(n!)^{1/n} < \dfrac{n+1}{2}$ holds. *(1981 - 2 Marks)*
2. The equation $2x^2+3x+1=0$ has an irrational root. *(1983 - 1 Mark)*
3. If $a<b<c<d$, then the roots of the equation $(x-a)(x-c)+2(x-b)(x-d)=0$ are real and distinct. *(1984 - 1 Mark)*
4. If $n_1, n_2, ...n_p$ are p positive integers, whose sum is an even number, then the number of odd integers among them is odd. *(1985 - 1 Mark)*
5. If $P(x)=ax^2+bx+c$ and $Q(x)=-ax^2+dx+c$, where $ac \neq 0$, then $P(x)Q(x)=0$ has at least two real roots. *(1985 - 1 Mark)*
6. If x and y are positive real numbers and m, n are any positive integers, then $\dfrac{x^n y^m}{(1+x^{2n})(1+y^{2m})} > \dfrac{1}{4}$ *(1989 - 1 Mark)*

C MCQs with One Correct Answer

1. If ℓ, m, n are real, $\ell \neq m$, then the roots by the equation: $(\ell-m)x^2-5(\ell+m)x-2(\ell-m)=0$ are *(1979)*
   (a) Real and equal (b) Complex
   (c) Real and unequal (d) None of these.
2. The equation $x+2y+2z=1$ and $2x+4y+4z=9$ have *(1979)*
   (a) Only one solution
   (b) Only two solutions
   (c) Infinite number of solutions
   (d) None of these.
3. If x, y and z are real and different and *(1979)* $u=x^2+4y^2+9z^2-6yz-3zx-2xy$, then u is always.
   (a) non negative (b) zero
   (c) non positive (d) none of these
4. Let $a>0$, $b>0$ and $c>0$. Then the roots of the equation $ax^2+bx+c=0$ *(1979)*
   (a) are real and negative (b) have negative real parts
   (c) both (a) and (b) (d) none of these
5. Both the roots of the equation $(x-b)(x-c)+(x-a)(x-c)+(x-a)(x-b)=0$ are always
   (a) positive (b) real *(1980)*
   (c) negative (d) none of these.
6. The least value of the expression $2\log_{10}x-\log_x(0.01)$, for $x>1$, is *(1980)*
   (a) 10 (b) 2
   (c) -0.01 (d) none of these.
7. If (x^2+px+1) is a factor of (ax^3+bx+c), then *(1980)*
   (a) $a^2+c^2=-ab$ (b) $a^2-c^2=-ab$
   (c) $a^2-c^2=ab$ (d) none of these

8. The number of real solutions of the equation $|x|^2-3|x|+2=0$ is *(1982 - 2 Marks)*
(a) 4 (b) 1 (c) 3 (d) 2

9. Two towns A and B are 60 km apart. A school is to be built to serve 150 students in town A and 50 students in town B. If the total distance to be travelled by all 200 students is to be as small as possible, then the school should be built at *(1982 - 2 Marks)*
(a) town B (b) 45 km from town A
(c) town A (d) 45 km from town B

10. If p, q, r are any real numbers, then *(1982 - 2 Marks)*
(a) $\max(p,q) < \max(p,q,r)$
(b) $\min(p,q) = \frac{1}{2}(p+q-|p-q|)$
(c) $\max(p,q) < \min(p,q,r)$
(d) none of these

11. The largest interval for which $x^{12}-x^9+x^4-x+1>0$ is *(1982 - 2 Marks)*
(a) $-4<x\le 0$ (b) $0<x<1$
(c) $-100<x<100$ (d) $-\infty<x<\infty$

12. The equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ has *(1984 - 2 Marks)*
(a) no root (b) one root
(c) two equal roots (d) infinitely many roots

13. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ lies in the interval *(1984 - 2 Marks)*
(a) $[\frac{1}{2},2]$ (b) $[-1,2]$ (c) $[-\frac{1}{2},1]$ (d) $[-1,\frac{1}{2}]$

14. If $\log_{0.3}(x-1)<\log_{0.09}(x-1)$, then x lies in the interval – *(1985 - 2 Marks)*
(a) $(2,\infty)$ (b) $(1,2)$
(c) $(-2,-1)$ (d) none of these

15*. If α and β are the roots of $x^2+px+q=0$ and α^4, β^4 are the roots of $x^2-rx+s=0$, then the equation $x^2-4qx+2q^2-r=0$ has always *(1989 - 2 Marks)*
(a) two real roots
(b) two positive roots
(c) two negative roots
(d) one positive and one negative root

* Question has more than one correct option.

16. Let a, b, c be real numbers, $a\ne 0$. If α is a root of $a^2x^2+bx+c=0$. β is the root of $a^2x^2-bx-c=0$ and $0<\alpha<\beta$, then the equation $a^2x^2+2bx+2c=0$ has a root γ that always satisfies *(1989 - 2 Marks)*
(a) $\gamma=\frac{\alpha+\beta}{2}$ (b) $\gamma=\alpha+\frac{\beta}{2}$
(c) $\gamma=\alpha$ (d) $\alpha<\gamma<\beta$.

17. The number of solutions of the equation $\sin(e)^x=5^x+5^{-x}$ is
(a) 0 (b) 1 *(1990 - 2 Marks)*
(c) 2 (d) Infinitely many

18. Let α, β be the roots of the equation $(x-a)(x-b)=c, c\ne 0$. Then the roots of the equation $(x-\alpha)(x-\beta)+c=0$ are *(1992 - 2 Marks)*
(a) a, c (b) b, c
(c) a, b (d) $a+c, b+c$

19. The number of points of intersection of two curves $y=2\sin x$ and $y=5x^2+2x+3$ is *(1994)*
(a) 0 (b) 1 (c) 2 (d) ∞

20. If p, q, r are +ve and are in $A.P.$, the roots of quadratic equation $px^2+qx+r=0$ are all real for *(1994)*
(a) $\left|\frac{r}{p}-7\right|\ge 4\sqrt{3}$ (b) $\left|\frac{p}{r}-7\right|\ge 4\sqrt{3}$
(c) all p and r (d) no p and r

21. Let $p,q\in\{1,2,3,4\}$. The number of equations of the form $px^2+qx+1=0$ having real roots is *(1994)*
(a) 15 (b) 9 (c) 7 (d) 8

22. If the roots of the equation $x^2-2ax+a^2+a-3=0$ are real and less than 3, then *(1999 - 2 Marks)*
(a) $a<2$ (b) $2\le a\le 3$
(c) $3<a\le 4$ (d) $a>4$

23. If α and β $(\alpha<\beta)$ are the roots of the equation $x^2+bx+c=0$, where $c<0<b$, then *(2000S)*
(a) $0<\alpha<\beta$ (b) $\alpha<0<\beta<|\alpha|$
(c) $\alpha<\beta<0$ (d) $\alpha<0<|\alpha|<\beta$

24. If a, b, c, d are positive real numbers such that $a+b+c+d=2$, then $M=(a+b)(c+d)$ satisfies the relation
(a) $0\le M\le 1$ (b) $1\le M\le 2$ *(2000S)*
(c) $2\le M\le 3$ (d) $3\le M\le 4$

25. If $b>a$, then the equation $(x-a)(x-b)-1=0$ has *(2000S)*
(a) both roots in (a,b)
(b) both roots in $(-\infty, a)$
(c) both roots in $(b,+\infty)$
(d) one root in $(-\infty,a)$ and the other in $(b,+\infty)$

26. For the equation $3x^2+px+3=0, p>0$, if one of the root is square of the other, then p is equal to *(2000S)*
(a) 1/3 (b) 1 (c) 3 (d) 2/3

27. If $a_1, a_2, \ldots, a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1+a_2+\ldots\ldots+a_{n-1}+2a_n$ is *(2002S)*
(a) $n(2c)^{1/n}$ (b) $(n+1)c^{1/n}$
(c) $2nc^{1/n}$ (d) $(n+1)(2c)^{1/n}$

28. The set of all real numbers x for which $x^2-|x+2|+x>0$, is *(2002S)*
(a) $(-\infty,-2)\cup(2,\infty)$ (b) $(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$
(c) $(-\infty,-1)\cup(1,\infty)$ (d) $(\sqrt{2},\infty)$

29. If $\alpha\in\left(0,\frac{\pi}{2}\right)$ then $\sqrt{x^2+x}+\frac{\tan^2\alpha}{\sqrt{x^2+x}}$ is always greater than or equal to *(2003S)*
(a) $2\tan\alpha$ (b) 1 (c) 2 (d) $\sec^2\alpha$

30. For all 'x', $x^2 + 2ax + 10 - 3a > 0$, then the interval in which 'a' lies is *(2004S)*
(a) $a < -5$ (b) $-5 < a < 2$ (c) $a > 5$ (d) $2 < a < 5$

31. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is *(2004S)*
(a) $p^3 - q(3p - 1) + q^2 = 0$ (b) $p^3 - q(3p + 1) + q^2 = 0$
(c) $p^3 + q(3p - 1) + q^2 = 0$ (d) $p^3 + q(3p+1) + q^2 = 0$

32. Let a, b, c be the sides of a triangle where $a \neq b \neq c$ and $\lambda \in R$. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then *(2006 - 3M, –1)*
(a) $\lambda < \frac{4}{3}$ (b) $\lambda > \frac{5}{3}$
(c) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$ (d) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

33. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is *(2007 -3 marks)*
(a) $\frac{2}{9}(p-q)(2q-p)$ (b) $\frac{2}{9}(q-p)(2p-q)$
(c) $\frac{2}{9}(q-2p)(2q-p)$ (d) $\frac{2}{9}(2p-q)(2q-p)$

34. Let p and q be real numbers such that $p \neq 0, p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is *(2010)*
(a) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
(b) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(c) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
(d) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

35. Let (x_0, y_0) be the solution of the following equations
$(2x)^{\ell n 2} = (3y)^{\ell n 3}$
$3^{\ell n x} = 2^{\ell n y}$
Then x_0 is *(2011)*
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 6

36. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is *(2011)*
(a) 1 (b) 2 (c) 3 (d) 4

37. A value of b for which the equations
$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$
have one root in common is *(2011)*
(a) $-\sqrt{2}$ (b) $-i\sqrt{3}$ (c) $i\sqrt{5}$ (d) $\sqrt{2}$

38. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has *(JEE Adv. 2014)*
(a) one purely imaginary root
(b) all real roots
(c) two real and two purely imaginary roots
(d) neither real nor purely imaginary roots

39. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and B_1 are the roots of the equation $x^2 - 2x \sec \alpha + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan \theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals *(JEE Adv. 2016)*
(a) $2(\sec \theta - \tan \theta)$ (b) $2 \sec \theta$
(c) $-2 \tan\theta$ (d) 0

D MCQs with One or More than One Correct

1. For real x, the function $\frac{(x-a)(x-b)}{x-c}$ will assume all real values provided *(1984 - 3 Marks)*
(a) $a > b > c$ (b) $a < b < c$
(c) $a > c > b$ (d) $a < c < b$

2. If S is the set of all real x such that $\frac{2x-1}{2x^3 + 3x^2 + x}$ is positive, then S contains *(1986 - 2 Marks)*
(a) $\left(-\infty, -\frac{3}{2}\right)$ (b) $\left(-\frac{3}{2}, -\frac{1}{4}\right)$
(c) $\left(-\frac{1}{4}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, 3\right)$
(e) none of these

3. If a, b and c are distinct positive numbers, then the expression $(b + c - a)(c + a - b)(a + b - c) - abc$ is *(1986 - 2 Marks)*
(a) positive (b) negative
(c) non-positive (d) non-negative
(e) none of these

4. If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$ then a, b, c, d *(1987 - 2 Marks)*
(a) are in A. P. (b) are in G. P.
(c) are in H. P. (d) satisfy $ab = cd$
(e) satisfy none of these

5. The equation $x^{3/4(\log_2 x)^2 + \log_2 x - 5/4} = \sqrt{2}$ has *(1989 - 2 Marks)*
(a) at least one real solution
(b) exactly three solutions
(c) exactly one irrational solution
(d) complex roots.

6. The product of n positive numbers is unity Then their sum is *(1991 - 2 Marks)*
(a) a positive integer (b) divisible by n
(c) equal to $n+\frac{1}{n}$ (d) never less than n

7. Number of divisor of the form $4n+2\,(n\geq 0)$ of the integer 240 is *(1998 - 2 Marks)*
(a) 4 (b) 8 (c) 10 (d) 3

8. If $3^x = 4^{x-1}$, then $x=$ *(JEE Adv. 2013)*
(a) $\frac{2\log_3 2}{2\log_3 2-1}$ (b) $\frac{2}{2-\log_2 3}$
(c) $\frac{1}{1-\log_4 3}$ (d) $\frac{2\log_2 3}{2\log_2 3-1}$

9. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) *a* subset(s) of S? *(JEE Adv. 2015)*
(a) $\left(-\frac{1}{2},-\frac{1}{\sqrt{5}}\right)$ (b) $\left(-\frac{1}{\sqrt{5}},0\right)$
(c) $\left(0,\frac{1}{\sqrt{5}}\right)$ (d) $\left(\frac{1}{\sqrt{5}},\frac{1}{2}\right)$

E Subjective Problems

1. Solve for x : $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$ *(1978)*

2. If $(m\,,n) = \frac{(1-x^m)(1-x^{m-1})\ldots\ldots\ldots(1-x^{m-n+1})}{(1-x)(1-x^2)\ldots\ldots\ldots\ldots(1-x^n)}$ *(1978)*
where m and n are positive integers ($n \leq m$), show that $(m, n+1) = (m-1, n+1) + x^{m-n-1}(m-1, n)$.

3. Solve for x : $\sqrt{x+1} - \sqrt{x-1} = 1$. *(1978)*

4. Solve the following equation for x : *(1978)*
$2\log_x a + \log_{ax} a + 3\log_{a^2x} a = 0\,,\, a>0$

5. Show that the square of $\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$ is a rational number. *(1978)*

6. Sketch the solution set of the following system of inequalities:
$x^2 + y^2 - 2x \geq 0;\ 3x - y - 12 \leq 0;\ y - x \leq 0;\ y \geq 0.$ *(1978)*

7. Find all integers x for which *(1978)*
$(5x-1) < (x+1)^2 < (7x-3)$.

8. If α, β are the roots of $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + rx + s = 0$, evaluate $(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)$ in terms of p, q, r and s.
Deduce the condition that the equations have a common root. *(1979)*

9. Given $n^4 < 10^n$ for a fixed positive integer $n \geq 2$, prove that $(n+1)^4 < 10^{n+1}$. *(1980)*

10. Let $y = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$ *(1980)*
Find all the real values of x for which y takes real values.

11. For what values of m, does the system of equations
$3x + my = m$
$2x - 5y = 20$
has solution satisfying the conditions $x > 0, y > 0$. *(1980)*

12. Find the solution set of the system *(1980)*
$x + 2y + z = 1;$
$2x - 3y - w = 2;$
$x \geq 0;\ y \geq 0;\ z \geq 0;\ w \geq 0.$

13. Show that the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution. *(1982 - 2 Marks)*

14. mn squares of euqal size are arranged to from a rectangle of dimension m by n, where m and n are natural numbers. Two squares will be called 'neighbours' if they have exactly one common side. A natural number is written in each square such that the number written in any square is the arithmetic mean of the numbers written in its neighbouring squares. Show that this is possible only if all the numbers used are equal. *(1982 - 5 Marks)*

15. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n-th power of the other, then show that
$(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$ *(1983 - 2 Marks)*

16. Find all real values of x which satisfy $x^2 - 3x + 2 > 0$ and $x^2 - 2x - 4 \leq 0$ *(1983 - 2 Marks)*

17. Solve for x; $(5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10$ *(1985 - 5 Marks)*

18. For $a \leq 0$, determine all real roots of the equation $x^2 - 2a|x-a| - 3a^2 = 0$ *(1986 - 5 Marks)*

19. Find the set of all x for which $\frac{2x}{(2x^2+5x+2)} > \frac{1}{(x+1)}$ *(1987 - 3 Marks)*

20. Solve $|x^2 + 4x + 3| + 2x + 5 = 0$ *(1988 - 5 Marks)*

21. Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$, then show that $1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$. *(1995 - 5 Marks)*

22. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c, and d denote the lengths of the sides of the quadrilateral, prove that $2 \leq a^2+b^2+c^2+d^2 \leq 4$. *(1997 - 5 Marks)*

23. If α, β are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha+\delta, \beta+\delta$ are the roots of $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant δ, then prove that $\dfrac{b^2-4ac}{a^2} = \dfrac{B^2-4AC}{A^2}$. *(2000 - 4 Marks)*

24. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. *(2001 - 4 Marks)*

25. If $x^2 + (a-b)x + (1-a-b) = 0$ where $a, b \in R$ then find the values of a for which equation has unequal real roots for all values of b. *(2003 - 4 Marks)*

26. If a, b, c are positive real numbers. Then prove that $(a+1)^7(b+1)^7(c+1)^7 > 7^7 a^4 b^4 c^4$ *(2004 - 4 Marks)*

27. Let a and b be the roots of the equation $x^2 - 10cx - 11d = 0$ and those of $x^2 - 10ax - 11b = 0$ are c, d then the value of $a+b+c+d$, when $a \neq b \neq c \neq d$, is. *(2006 - 6M)*

H Assertion & Reason Type Questions

1. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \dfrac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$

STATEMENT - 1 : $(p^2-q)(b^2-ac) \geq 0$

and

STATEMENT - 2 : $b \neq pa$ or $c \neq qa$ *(2008)*

(a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

(b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explanation for STATEMENT - 1

(c) STATEMENT - 1 is True, STATEMENT - 2 is False

(d) STATEMENT - 1 is False, STATEMENT - 2 is True

I Integer Value Correct Type

1. Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations :

$$3x - y - z = 0$$
$$-3x + z = 0$$
$$-3x + 2y + z = 0$$

Then the number of such points for which $x^2 + y^2 + z^2 \leq 100$ is *(2009)*

2. The smallest value of k, for which both the roots of the equation

$$x^2 - 8kx + 16(k^2 - k + 1) = 0$$

are real, distinct and have values at least 4, is *(2009)*

3. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} where $a > 0$ is *(2011)*

4. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is *(2011)*

Section-B JEE Main / AIEEE

1. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is **[2002]**

(a) $3x^2 - 19x + 3 = 0$ (b) $3x^2 + 19x - 3 = 0$
(c) $3x^2 - 19x - 3 = 0$ (d) $x^2 - 5x + 3 = 0$.

2. Difference between the corresponding roots of $x^2+ax+b=0$ and $x^2+bx+a=0$ is same and $a \neq b$, then **[2002]**

(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$
(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

3. Product of real roots of the equation $t^2x^2+|x|+9=0$ **[2002]**

(a) is always positive (b) is always negative
(c) does not exist (d) none of these

4. If p and q are the roots of the equation $x^2+px+q=0$, then **[2002]**

(a) $p = 1, q = -2$ (b) $p=0, q=1$
(c) $p = -2, q = 0$ (d) $p = -2, q = 1$

5. If a, b, c are distinct +ve real numbers and $a^2+b^2+c^2=1$ then $ab + bc + ca$ is **[2002]**

(a) less than 1 (b) equal to 1
(c) greater than 1 (d) any real no.

6. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\dfrac{a}{c}, \dfrac{b}{a}$ and $\dfrac{c}{b}$ are in **[2003]**

(a) Arithmetic - Geometric Progression
(b) Arithmetic Progression
(c) Geometric Progression
(d) Harmonic Progression.

7. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is **[2003]**

(a) $-\dfrac{1}{3}$ (b) $\dfrac{2}{3}$ (c) $-\dfrac{2}{3}$ (d) $\dfrac{1}{3}$

8. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is

(a) 3 (b) 2 (c) 4 (d) 1

9. The real number x when added to its inverse gives the minimum value of the sum at x equal to **[2003]**

(a) –2 (b) 2 (c) 1 (d) –1

10. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation **[2004]**

(a) $x^2 - 18x - 16 = 0$ (b) $x^2 - 18x + 16 = 0$
(c) $x^2 + 18x - 16 = 0$ (d) $x^2 + 18x + 16 = 0$

11. If $(1-p)$ is a root of quadratic equation $x^2 + px + (1-p) = 0$ then its root are **[2004]**

(a) $-1, 2$ (b) $-1, 1$ (c) $0, -1$ (d) $0, 1$

12. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots , then the value of 'q' is **[2004]**
(a) 4 (b) 12 (c) 3 (d) $\frac{49}{4}$

13. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $-\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then **[2005]**
(a) $a = b + c$ (b) $c = a + b$
(c) $b = c$ (d) $b = a + c$

14. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval **[2005]**
(a) $(5, 6]$ (b) $(6, \infty)$ (c) $(-\infty, 4)$ (d) $[4, 5]$

15. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$, respectively, then the value of $2 + q - p$ is **[2006]**
(a) 2 (b) 3 (c) 0 (d) 1

16. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less then 4, lie in the interval **[2006]**
(a) $-2 < m < 0$ (b) $m > 3$
(c) $-1 < m < 3$ (d) $1 < m < 4$

17. If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is **[2006]**
(a) $\frac{1}{4}$ (b) 41 (c) 1 (d) $\frac{17}{7}$

18. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is **[2007]**
(a) $(3, \infty)$ (b) $(-\infty, -3)$ (c) $(-3, 3)$ (d) $(-3, \infty)$.

19. **Statement-1 :** For every natural number $n \geq 2$,
$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots\dots\dots + \frac{1}{\sqrt{n}} > \sqrt{n}.$$
Statement-2 : For every natural number $n \geq 2$,
$\sqrt{n(n+1)} < n + 1.$ **[2008]**
(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

20. The quadritic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[2009]**
(a) 1 (b) 4 (c) 3 (d) 2

21. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is : **[2009]**
(a) less than $4ab$ (b) greater than $-4ab$
(c) less than $-4ab$ (d) greater than $4ab$

22. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|Z|$ is equal to : **[2009]**
(a) $\sqrt{5} + 1$ (b) 2 (c) $2 + \sqrt{2}$ (d) $\sqrt{3} + 1$

23. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[2010]**
(a) -1 (b) 1 (c) 2 (d) -2

24. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : **[2012]**
(a) infinite number of real roots
(b) no real roots
(c) exactly one real root
(d) exactly four real roots

25. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in $[0, 1]$ **[JEE M 2013]**
(a) lies between 1 and 2 (b) lies between 2 and 3
(c) lies between -1 and 0 (d) does not exist.

26. The number of values of k, for which the system of equations : **[JEE M 2013]**
$(k + 1)x + 8y = 4k$
$kx + (k + 3)y = 3k - 1$
has no solution, is
(a) infinite (b) 1 (c) 2 (d) 3

27. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in$ R, have a common root, then $a : b : c$ is **[JEE M 2013]**
(a) 1 : 2 : 3 (b) 3 : 2 : 1 (c) 1 : 3 : 2 (d) 3 : 1 : 2

28. If $a \in$ R and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval: **[JEE M 2014]**
(a) $(-2, -1)$ (b) $(-\infty, -2) \cup (2, \infty)$
(c) $(-1, 0) \cup (0, 1)$ (d) $(1, 2)$

29. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is: **[JEE M 2014]**
(a) $\frac{\sqrt{34}}{9}$ (b) $\frac{2\sqrt{13}}{9}$ (c) $\frac{\sqrt{61}}{9}$ (d) $\frac{2\sqrt{17}}{9}$

30. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to : **[JEE M 2015]**
(a) 3 (b) -3 (c) 6 (d) -6

31. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is : **[JEE M 2016]**
(a) 6 (b) 5
(c) 3 (d) -4