CHAPTER 17

Indefinite Integrals

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}\,dx = Ax + B\log(9e^{2x} - 4) + C$, then $A = \ldots\ldots, B = \ldots\ldots$ and $C = \ldots\ldots$ *(1990 - 2 Marks)*

C MCQs with One Correct Answer

1. The value of the integral $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x}\,dx$ is *(1995S)*

(a) $\sin x - 6\tan^{-1}(\sin x) + c$
(b) $\sin x - 2(\sin x)^{-1} + c$
(c) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + c$
(d) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + c$

2. If $\int_{\sin x}^{1} t^2 f(t)\,dt = 1 - \sin x$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is *(2005S)*

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) 3 (d) $\sqrt{3}$

3. $\int \frac{x^2 - 1}{x^3\sqrt{2x^4 - 2x^2 + 1}}\,dx =$ *(2006 - 3M, –1)*

(a) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + c$ (b) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + c$
(c) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + c$ (d) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + c$

4. Let $I = \int \frac{e^x}{e^{4x} + e^{2x} + 1}dx$, $J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1}dx$. Then, for an arbitrary constant C, the value of J – I equals *(2008)*

(a) $\frac{1}{2}\log\left(\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1}\right) + C$ (b) $\frac{1}{2}\log\left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1}\right) + C$
(c) $\frac{1}{2}\log\left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1}\right) + C$ (d) $\frac{1}{2}\log\left(\frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1}\right) + C$

5. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{\frac{9}{2}}}\,dx$ equals (for some arbitrary constant K) *(2012)*

(a) $-\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$
(b) $\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$
(c) $-\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$
(d) $\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

E Subjective Problems

1. Evaluate $\int \frac{\sin x}{\sin x - \cos x}dx$ *(1978)*

2. Evaluate $\int \frac{x^2 dx}{(a + bx)^2}$ *(1979)*

3. Evaluate $\int (e^{\log x} + \sin x)\cos x\,dx$. *(1981 - 2 Marks)*

4. Evaluate : $\int \frac{(x-1)e^x}{(x+1)^3}dx$ *(1983 - 2 Marks)*

5. Evaluate the following $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ ***(1984 - 2 Marks)***

6. Evaluate the following $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$ ***(1985 - 2½ Marks)***

7. Evaluate : $\int \left[\frac{(\cos 2x)^{1/2}}{\sin x}\right]dx$ ***(1987 - 6 Marks)***

8. Evaluate $\int (\sqrt{\tan x} + \sqrt{\cot x})dx$ ***(1989 - 3 Marks)***

9. Find the indefinite integral $\int\left(\frac{1}{\sqrt[3]{x}+\sqrt[4]{4}} + \frac{\ln(1+\sqrt[6]{x})}{\sqrt[3]{x}+\sqrt{x}}\right)dx$

(1992 - 4 Marks)

10. Find the indefinite integral $\int \cos 2\theta \ln\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)d\theta$

(1994 - 5 Marks)

11. Evaluate $\int \frac{(x+1)}{x(1+xe^x)^2}dx.$ ***(1996 - 2 Marks)***

12. Integrate $\int \frac{x^3+3x+2}{\left(x^2+1\right)^2(x+1)}dx.$ ***(1999 - 5 Marks)***

13. Evaluate $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx$. ***(2001 - 5 Marks)***

14. For any natural number m, evaluate

$\int (x^{3m}+x^{2m}+x^m)(2x^{2m}+3x^m+6)^{1/m}dx$, $x>0$.

(2002 - 5 Marks)

H Assertion & Reason Type Questions

1. Let $F(x)$ be an indefinite integral of $\sin^2 x$.

STATEMENT-1 : The function $F(x)$ satisfies $F(x+\pi)=F(x)$ for all real x. **because**

STATEMENT-2 : $\sin^2(x+\pi)=\sin^2 x$ for all real x.

(2007 -3 marks)

(a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

Section-B JEE Main / AIEEE

1. If $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log\sin(x-\alpha), +C$, then value of (A, B) is **[2004]**

(a) $(-\cos\alpha, \sin\alpha)$ (b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$ (d) $(\sin\alpha, \cos\alpha)$

2. $\int \frac{dx}{\cos x - \sin x}$ is equal to **[2004]**

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

(b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

3. $\int\left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to **[2005]**

(a) $\frac{\log x}{(\log x)^2+1}+C$ (b) $\frac{x}{x^2+1}+C$

(c) $\frac{xe^x}{1+x^2}+C$ (d) $\frac{x}{(\log x)^2+1}+C$

4. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals **[2007]**

(a) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(c) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(d) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

5. The value of $\sqrt{2}\int \frac{\sin x dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is **[2008]**

(a) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(b) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(c) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(d) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

6. If the $\int \frac{5\tan x}{\tan x-2}dx = x + a\ln|\sin x - 2\cos x| + k$, then a is equal to : **[2012]**

(a) –1 (b) –2

(c) 1 (d) 2

7. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to

(a) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right]+C$ **[JEE M 2013]**

(b) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

(c) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

(d) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right]+C$

8. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to **[JEE M 2014]**

(a) $(x+1)e^{x+\frac{1}{x}}+c$ (b) $-xe^{x+\frac{1}{x}}+c$

(c) $(x-1)e^{x+\frac{1}{x}}+c$ (d) $xe^{x+\frac{1}{x}}+c$

9. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals : **[JEE M 2015]**

(a) $-(x^4+1)^{\frac{1}{4}}+c$

(b) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(d) $(x^4+1)^{\frac{1}{4}}+c$

10. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to : **[JEE M 2016]**

(a) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$

(b) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(c) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$

(d) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

where C is an arbitrary constant.