CHAPTER 6 Sequences and Series

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. The sum of integers from 1 to 100 that are divisible by 2 or 5 is .... *(1984 - 2 Marks)*
2. The solution of the equation $\log_7 \log_5(\sqrt{x+5}+\sqrt{x}) = 0$ is .... *(1986 - 2 Marks)*
3. The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $n\,(n+1)^2/2$, when n is even. When n is odd, the sum is .... *(1988 - 2 Marks)*
4. Let the harmonic mean and geometric mean of two positive numbers be the ratio 4 : 5. Then the two number are in the ratio .... *(1992 - 2 Marks)*
5. For any odd integer n $\geq$ 1, n^3-$(n$-$1)^3$+...+$(-1)^{n\text{-}1}$ 1^3= .... *(1996 - 1 Mark)*
6. Let p and q be roots of the equation $x^2 - 2x + A = 0$ and let r and s be the roots of the equation $x^2 - 18x + B = 0$. If $p < q < r < s$ are in arithmetic progression, then A = ... and B = ... *(1997 - 2 Marks)*

C MCQs with One Correct Answer

1. If x, y and z are pth, qth and rth terms respectively of an A.P. and also of a G.P., then $x^{y-z}y^{z-x}z^{x-y}$ is equal to :
   (a) xyz (b) 0 (c) 1 (d) None of these
2. The third term of a geometric progression is 4. The product of the first five terms is *(1982 - 2 Marks)*
   (a) 4^3 (b) 4^5 (c) 4^4 (d) none of these
3. The rational number, which equals the number $2.\overline{357}$ with recurring decimal is *(1983 - 1 Mark)*
   (a) $\frac{2355}{1001}$ (b) $\frac{2379}{997}$ (c) $\frac{2355}{999}$ (d) none of these
4. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in — *(1985 - 2 Marks)*
   (a) A.P. (b) G.P. (c) H.P. (d) none of these
5. Sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+$.............. is equal to *(1988 - 2 Marks)*
   (a) $2^n - n - 1$ (b) $1 - 2^{-n}$
   (c) $n + 2^{-n} - 1$ (d) $2^n + 1$.
6. The number $\log_2 7$ is *(1990 - 2 Marks)*
   (a) an integer (b) a rational number
   (c) an irrational number (d) a prime number
7. If $ln(a+c)$, $ln\,(a-c)$, $ln\,(a-2b+c)$ are in A.P., then *(1994)*
   (a) a, b, c are in A.P. (b) a^2, b^2, c^2 are in A.P.
   (c) a, b, c are in G.P. (d) a, b, c are in H.P.
8. Let $a_1, a_2, ...\, a_{10}$ be in A, P, and $h_1, h_2, ...h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then a_4h_7 is *(1999 - 2 Marks)*
   (a) 2 (b) 3 (c) 5 (d) 6
9. The harmonic mean of the roots of the equation $\left(5+\sqrt{2}\right)x^2 - \left(4+\sqrt{5}\right)x + 8 + 2\sqrt{5} = 0$ is *(1999 - 2 Marks)*
   (a) 2 (b) 4 (c) 6 (d) 8
10. Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is 3/4, then *(2000S)*
    (a) $a = \frac{4}{7}, r = \frac{3}{7}$ (b) $a = 2, r = \frac{3}{8}$
    (c) $a = \frac{3}{2}, r = \frac{1}{2}$ (d) $a = 3, r = \frac{1}{4}$
11. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are *(2001S)*
    (a) –2, –32 (b) –2, 3 (c) –6, 3 (d) –6, –32
12. Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are *(2001S)*
    (a) NOT in A.P./G.P./H.P. (b) in A.P.
    (c) in G.P. (d) in H.P.
13. If the sum of the first $2n$ terms of the A.P. 2, 5, 8, …, is equal to the sum of the first n terms of the A.P. 57, 59, 61, …, then n equals *(2001S)*
    (a) 10 (b) 12 (c) 11 (d) 13
14. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. if $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is *(2002S)*
    (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$ (c) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (d) $\frac{1}{2} - \frac{1}{\sqrt{2}}$
15. An infinite G.P. has first term 'x' and sum '5', then x belongs to *(2004S)*
    (a) $x < -10$ (b) $-10 < x < 0$
    (c) $0 < x < 10$ (d) $x > 10$
16. In the quadratic equation $ax^2 + bx + c = 0$, $\Delta = b^2 - 4ac$ and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$, are in G.P. where α, β are the root of $ax^2 + bx + c = 0$, then *(2005S)*
    (a) $\Delta \neq 0$ (b) $b\Delta = 0$ (c) $c\Delta = 0$ (d) $\Delta = 0$

17. In the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is *(2009)*

(a) $\frac{n(4n^2-1)c^2}{6}$ (b) $\frac{n(4n^2+1)c^2}{3}$

(c) $\frac{n(4n^2-1)c^2}{3}$ (d) $\frac{n(4n^2+1)c^2}{6}$

18. Let $a_1, a_2, a_3, ...$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is *(2012)*

(a) 22 (b) 23 (c) 24 (d) 25

19. Let $b_i > 1$ for $i = 1, 2, ..., 101$. Suppose $\log_e b_1, \log_e b_2, ..., \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, ..., a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + ... + b_{51}$ and $s = a_1 + a_2 + ... + a5_3$, then *(JEE Adv. 2016)*

(a) $s > t$ and $a_{101} > b_{101}$ (b) $s > t$ and $a_{101} < b_{101}$
(c) $s < t$ and $a_{101} > b_{101}$ (d) $s < t$ and $a_{101} < b_{101}$

D MCQs with One or More than One Correct

1. If the first and the $(2n-1)$st terms of an A.P., a G.P. and an H.P. are equal and their n-th terms are a, b and c respectively, then *(1988 - 2 Marks)*

(a) $a = b = c$ (b) $a \geq b \geq c$

(c) $a + c = b$ (d) $ac - b^2 = 0$.

2. For $0 < \phi < \pi/2$, if

$$x = \sum_{n=0}^{\infty} \cos^{2n}\phi, \; y = \sum_{n=0}^{\infty} \sin^{2n}\phi, \; z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$$

then: *(1993 - 2 Marks)*

(a) $xyz = xz + y$ (b) $xyz = xy + z$

(c) $xyz = x + y + z$ (d) $xyz = yz + x$

3. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then *(1998 - 2 Marks)*

(a) $b_0 = 1, b_1 = 3$ (b) $b_0 = 0, b_1 = n$
(c) $b_0 = -1, b_1 = n$ (d) $b_0 = 0, b_1 = n^2 - 3n + 3$

4. Let T_r be the r^{th} term of an A.P., for $r = 1, 2, 3, ...$ If for some positive integers m, n we have

$T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals *(1998 - 2 Marks)*

(a) $\frac{1}{mn}$ (b) $\frac{1}{m} + \frac{1}{n}$ (c) 1 (d) 0

5. If $x > 1, y > 1, z > 1$ are in G.P., then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in *(1998 - 2 Marks)*

(a) A.P. (b) H.P. (c) GP (d) None of these

6. For a positive integer n, let

$a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... \frac{1}{(2^n) - 1}$. Then *(1999 - 3 Marks)*

(a) $a(100) \leq 100$ (b) $a(100) > 100$
(c) $a(200) \leq 100$ (d) $a(200) > 100$

7. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then *(2008)*

(a) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ (b) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$

(c) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$ (d) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

8. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$ for $n = 1, 2, 3, ...$ Then, *(2008)*

(a) $S_n < \frac{\pi}{3\sqrt{3}}$ (b) $S_n > \frac{\pi}{3\sqrt{3}}$

(c) $T_n < \frac{\pi}{3\sqrt{3}}$ (d) $T_n > \frac{\pi}{3\sqrt{3}}$

9. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s) *(JEE Adv. 2013)*

(a) 1056 (b) 1088 (c) 1120 (d) 1332

E Subjective Problems

1. The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation. $2A + G^2 = 27$
Find the two numbers. *(1979)*

2. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120°, and the common difference is 5°, Find the number of sides of the polygon. *(1980)*

3. Does there exist a geometric progression containing 27, 8 and 12 as three of its terms ? If it exits, how many such progressions are possible ? *(1982 - 3 Marks)*

4. Find three numbers a, b, c, between 2 and 18 such that
(i) their sum is 25
(ii) the numbers 2, a, b sare consecutive terms of an A.P. and
(iii) the numbers b, c, 18 are consecutive terms of a G.P. *(1983 - 2 Marks)*

5. If $a > 0, b > 0$ and $c > 0$, prove that

$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$ *(1984 - 2 Marks)*

6. If n is a natural number such that

$n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} .... p_k^{\alpha_k}$ and $p_1, p_2, ..., p_k$ are distinct primes, then show that $\ln n \geq k \ln 2$ *(1984 - 2 Marks)*

7. Find the sum of the series :

$\sum_{r=0}^{n} (-1)^r \, {}^nC_r [\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} ......$ up to m terms] *(1985 - 5 Marks)*

8. Solve for x the following equation : *(1987 - 3 Marks)*

$$\log_{(2x+3)}(6x^2+23x+21) = 4 - \log_{(3x+7)}(4x^2+12x+9)$$

9. If $\log_3 2$, $\log_3(2^x - 5)$, and $\log_3\left(2^x - \frac{7}{2}\right)$ are in arithmetic progression, determine the value of x. *(1990 - 4 Marks)*

10. Let p be the first of the n arithmetic means between two numbers and q the first of n harmonic means between the same numbers. Show that q does not lie between p and $\left(\frac{n+1}{n-1}\right)^2 p$. *(1991 - 4 Marks)*

11. If $S_1, S_2, S_3, \ldots\ldots\ldots, S_n$ are the sums of infinite geometric series whose first terms are 1, 2, 3, ..., n and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots\ldots\ldots \frac{1}{n+1}$ respectively, then find the values of $S_1^2 + S_2^2 + S_3^2 + \ldots\ldots\ldots + S_{2n-1}^2$ *(1991 - 4 Marks)*

12. The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in AP. Find the intervals in which β and γ lie. *(1996 - 3 Marks)*

13. Let a, b, c, d be real numbers in G.P. If u, v, w, satisfy the system of equations *(1999 - 10 Marks)*

$$u + 2v + 3w = 6$$
$$4u + 5v + 6w = 12$$
$$6u + 9v = 4$$

then show that the roots of the equation

$$\left(\frac{1}{u}+\frac{1}{v}+\frac{1}{w}\right)x^2 + [(b-c)^2+(c-a)^2+(d-b)^2]\,x + u + v + w = 0$$

and $20x^2 + 10(a-d)^2 x - 9 = 0$ are reciprocals of each other.

14. The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. *(2000 - 4 Marks)*

15. Let $a_1, a_2, \ldots, a_n$ be positive real numbers in geometric progression. For each n, let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean, and harmonic mean of $a_1, a_2, \ldots, a_n$. Find an expression for the geometric mean of $G_1, G_2, \ldots, G_n$ in terms of $A_1, A_2, \ldots, A_n, H_1, H_2, \ldots, H_n$. *(2001 - 5 Marks)*

16. Let a, b be positive real numbers. If a, A_1, A_2, b are in arithmetic progression, a, G_1, G_2, b are in geometric progression and a, H_1, H_2, b are in harmonic progression, show that $\frac{G_1G_2}{H_1H_2} = \frac{A_1+A_2}{H_1+H_2} = \frac{(2a+b)(a+2b)}{9ab}$. *(2002 - 5 Marks)*

17. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P., then prove that either $a = b = c$ or $a, b, -\frac{c}{2}$ form a G.P. *(2003 - 4 Marks)*

18. If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \ldots..(-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$, then find the least natural number n_0 such that $b_n > a_n \ \forall\, n \geq n_0$. *(2006 - 6M)*

G Comprehension Based Questions

PASSAGE - 1

Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r-1)$. Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \ldots$

1. The sum $V_1 + V_2 + \ldots + V_n$ is *(2007 -4 marks)*
   (a) $\frac{1}{12}n(n+1)(3n^2-n+1)$ (b) $\frac{1}{12}n(n+1)(3n^2+n+2)$
   (c) $\frac{1}{2}n(2n^2-n+1)$ (d) $\frac{1}{3}(2n^3-2n+3)$

2. T_r is always *(2007 -4 marks)*
   (a) an odd number (b) an even number
   (c) a prime number (d) a composite number

3. Which one of the following is a correct statement ? *(2007 -4 marks)*
   (a) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 5
   (b) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 6
   (c) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 11
   (d) $Q_1 = Q_2 = Q_3 = \ldots.$

PASSAGE -2

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, Let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

4. Which one of the following statements is correct ? *(2007 -4 marks)*
   (a) $G_1 > G_2 > G_3 > \ldots$
   (b) $G_1 < G_2 < G_3 < \ldots$
   (c) $G_1 = G_2 = G_3 = \ldots$
   (d) $G_1 < G_3 < G_5 < \ldots$ and $G_2 > G_4 > G_6 > \ldots.$

5. Which one of the following statements is correct ? *(2007 -4 marks)*
   (a) $A_1 > A_2 > A_3 > \ldots$
   (b) $A_1 < A_2 < A_3 < \ldots$
   (c) $A_1 > A_3 > A_5 > \ldots$ and $A_2 < A_4 < A_6 < \ldots$
   (d) $A_1 < A_3 < A_5 < \ldots$ and $A_2 > A_4 > A_6 > \ldots$

6. Which one of the following statements is correct ? *(2007 -4 marks)*
   (a) $H_1 > H_2 > H_3 > \ldots$
   (b) $H_1 < H_2 < H_3 < \ldots$
   (c) $H_1 > H_3 > H_5 > \ldots$ and $H_2 < H_4 < H_6 < \ldots$
   (d) $H_1 < H_3 < H_6 < \ldots$ and $H_2 > H_4 > H_6 > \ldots$

H Assertion & Reason Type Questions

1. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1, b_2 = b_1 + a_2, b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.
STATEMENT - 1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.
and
STATEMENT - 2 : The numbers b_1, b_2, b_3, b_4 are in H.P. *(2008)*
(a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
(b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explanation for STATEMENT - 1
(c) STATEMENT - 1 is True, STATEMENT - 2 is False
(d) STATEMENT - 1 is False, STATEMENT - 2 is True

I Integer Value Correct Type

1. Let S_k, $k = 1, 2, \ldots, 100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k + 1)S_k|$ is *(2010)*

2. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$.
if $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to *(2010)*

3. Let $a_1, a_2, a_3 \ldots a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is *(2011)*

4. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k - 20 =$ *(JEE Adv. 2013)*

5. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b + 2$, then the value of $\frac{a^2 + a - 14}{a + 1}$ is *(JEE Adv. 2014)*

6. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is *(JEE Adv. 2015)*

7. The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3) \ldots (1 + x^{100})$ is *(JEE Adv. 2015)*

Section-B JEE Main / AIEEE

1. If 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x-1)$ are in A.P. then x equals **[2002]**

(a) $\log_3 4$ (b) $1-\log_3 4$
(c) $1-\log_4 3$ (d) $\log_4 3$

2. l, m, n are the p^{th}, q^{th} and r^{th} term of a G. P. all positive,

then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals **[2002]**

(a) -1 (b) 2 (c) 1 (d) 0

3. The value of $2^{1/4}.\,4^{1/8}.\,8^{1/16}\ \ldots\ \infty$ is **[2002]**

(a) 1 (b) 2 (c) 3/2 (d) 4

4. Fifth term of a GP is 2, then the product of its 9 terms is **[2002]**

(a) 256 (b) 512
(c) 1024 (d) none of these

5. Sum of infinite number of terms of GP is 20 and sum of their square is 100. The common ratio of GP is **[2002]**

(a) 5 (b) 3/5 (c) 8/5 (d) 1/5

6. $1^3-2^3+3^3-4^3+\ldots+9^3=$ **[2002]**

(a) 425 (b) -425 (c) 475 (d) -475

7. The sum of the series **[2003]**

$\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}$........... up to ∞ is equal to

(a) $\log_e\left(\frac{4}{e}\right)$ (b) $2\log_e 2$

(c) $\log_e 2-1$ (d) $\log_e 2$

8. If $S_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to

(a) $\frac{2n-1}{2}$ (b) $\frac{1}{2}n-1$ **[2004]**

(c) $n-1$ (d) $\frac{1}{2}n$

9. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then $a-d$ equals **[2004]**

(a) $\frac{1}{m}+\frac{1}{n}$ (b) 1 (c) $\frac{1}{mn}$ (d) 0

10. The sum of the first n terms of the series

$1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\ldots$

is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\frac{n^2(n+1)}{2}$ **[2004]**

(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{3n(n+1)}{2}$

11. The sum of series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+$..... is **[2004]**

(a) $\frac{(e^2-2)}{e}$ (b) $\frac{(e-1)^2}{2e}$

(c) $\frac{(e^2-1)}{2e}$ (d) $\frac{(e^2-1)}{2}$

12. If the coefficients of rth, $(r+1)$th, and $(r+2)$th terms in the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation **[2005]**

(a) $m^2-m(4r-1)+4r^2-2=0$

(b) $m^2-m(4r+1)+4r^2+2=0$

(c) $m^2-m(4r+1)+4r^2-2=0$

(d) $m^2-m(4r-1)+4r^2+2=0$

13. If $x=\sum_{n=0}^{\infty}a^n$, $y=\sum_{n=0}^{\infty}b^n$, $z=\sum_{n=0}^{\infty}c^n$ where a, b, c are in A.P and $|a|<1, |b|<1, |c|<1$ then x, y, z are in **[2005]**

(a) G.P.
(b) A.P.
(c) Arithmetic - Geometric Progression
(d) H.P.

14. The sum of the series **[2005]**

$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+$ ....ad inf. is

(a) $\frac{e-1}{\sqrt{e}}$ (b) $\frac{e+1}{\sqrt{e}}$ (c) $\frac{e-1}{2\sqrt{e}}$ (d) $\frac{e+1}{2\sqrt{e}}$

15. Let a_1, a_2, a_3........... be terms on A.P. If

$\frac{a_1+a_2+\ldots\ldots\ldots a_p}{a_1+a_2+\ldots\ldots\ldots+a_q}=\frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals **[2006]**

(a) $\frac{41}{11}$ (b) $\frac{7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{11}{41}$

16. If $a_1, a_2, ..., a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to **[2006]**

(a) $n(a_1 - a_n)$ (b) $(n-1)(a_1 - a_n)$

(c) na_1a_n (d) $(n-1)a_1a_n$

17. The sum of series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} -$ .... upto infinity is **[2007]**

(a) $e^{-\frac{1}{2}}$ (b) $e^{+\frac{1}{2}}$ (c) e^{-2} (d) e^{-1}

18. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is equals **[2007]**

(a) $\sqrt{5}$ (b) $\frac{1}{2}\left(\sqrt{5}-1\right)$

(c) $\frac{1}{2}\left(1-\sqrt{5}\right)$ (d) $\frac{1}{2}\sqrt{5}$.

19. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is **[2008]**

(a) –4 (b) –12 (c) 12 (d) 4

20. The sum to infinite term of the series

$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+...$ is **[2009]**

(a) 3 (b) 4 (c) 6 (d) 2

21. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference –2, then the time taken by him to count all notes is **[2010]**

(a) 34 minutes (b) 125 minutes

(c) 135 minutes (d) 24 minutes

22. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after **[2011]**

(a) 19 months (b) 20 months

(c) 21 months (d) 18 months

23. **Statement-1:** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement-2: $\sum_{k=1}^{n}\left(k^3-(k-1)^3\right)=n^3$, for any natural number n. **[2012]**

(a) Statement-1 is false, Statement-2 is true.

(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(d) Statement-1 is true, statement-2 is false.

24. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is **[JEE M 2013]**

(a) $\frac{7}{81}(179-10^{-20})$ (b) $\frac{7}{9}(99-10^{-20})$

(c) $\frac{7}{81}(179+10^{-20})$ (d) $\frac{7}{9}(99+10^{-20})$

25. If $(10)^9 + 2(11)^1(10^8) + 3(11)^2(10)^7 + ...$ $+10(11)^9 = k(10)^9$, then k is equal to: **[JEE M 2014]**

(a) 100 (b) 110 (c) $\frac{121}{10}$ (d) $\frac{441}{100}$

26. Three positive numbers form an increasing G. P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ratio of the G.P. is: **[JEE M 2014]**

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$ (c) $\sqrt{2}+\sqrt{3}$ (d) $3+\sqrt{2}$

27. The sum of first 9 terms of the series.

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+....$ **[JEE M 2015]**

(a) 142 (b) 192 (c) 71 (d) 96

28. If m is the A.M. of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals : **[JEE M 2015]**

(a) $4\,lmn^2$ (b) $4\,l^2m^2n^2$

(c) $4\,l^2mn$ (d) $4\,lm^2n$

29. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is : **[JEE M 2016]**

(a) 1 (b) $\frac{7}{4}$

(c) $\frac{8}{5}$ (d) $\frac{4}{3}$

30. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+......,$ is $\frac{16}{5}$m, then m is equal to : **[JEE M 2016]**

(a) 100 (b) 99

(c) 102 (d) 101