CHAPTER 10 Functions

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. The values of $f(x)=3\sin\left(\sqrt{\frac{\pi^2}{16}-x^2}\right)$ lie in the interval .... *(1983 - 1 Mark)*

2. For the function $f(x)=\frac{x}{1+e^{1/x}}, \quad x \neq 0$
$= 0 \quad , \quad x = 0$
the derivative from the right, $f'(0+)=$ ...., and the derivative from the left, $f'(0-)=$ .... *(1983 - 2 Marks)*

3. The domain of the function $f(x)=\sin^{-1}(\log_2 \frac{x^2}{2})$ is given by .... *(1984 - 2 Marks)*

4. Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is .... and out of these .... are onto functions. *(1985 - 2 Marks)*

5. If $f(x)=\sin\ln\left(\frac{\sqrt{4-x^2}}{1-x}\right)$, then domain of $f(x)$ is .... and its range is .... *(1985 - 2 Marks)*

6. There are exactly two distinct linear functions, ..., and .... which map $[-1, 1]$ onto $[0, 2]$. *(1989 - 2 Marks)*

7. If f is an even function defined on the interval (-5, 5), then four real values of x satisfying the equation $f(x)=f\left(\frac{x+1}{x+2}\right)$ are ...., ..., ..., and .... *(1996 - 1 Mark)*

8. If $f(x)=\sin^2 x+\sin^2\left(x+\frac{\pi}{3}\right)+\cos x\cos\left(x+\frac{\pi}{3}\right)$ *and* $g\left(\frac{5}{4}\right)=1$, then $(gof)(x)=$ .... *(1996 - 2 Marks)*

B True / False

1. If $f(x)=(a-x^n)^{1/n}$ where $a>0$ and n is a positive integer, then $f[f(x)]=x$. *(1983 - 1 Mark)*

2. The function $f(x)=\frac{x^2+4x+30}{x^2-8x+18}$ is not one -to -one. *(1983 - 1 Mark)*

3. If $f_1(x)$ and $f_2(x)$ are defined on domains D_1 and D_2 respectively, then $f_1(x)+f_2(x)$ is defined on $D_1 \cup D_2$. *(1988 - 1 Mark)*

C MCQs with One Correct Answer

1. Let R be the set of real numbers. If $f: R \to R$ is a function defined by $f(x)=x^2$, then f is : *(1979)*
   (a) Injective but not surjective
   (b) Surjective but not injective
   (c) Bijective
   (d) None of these.

2. The entire graphs of the equation $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if *(1979)*
   (a) $k<7$ (b) $-5<k<7$
   (c) $k>-5$ (d) None of these.

3. Let $f(x)=|x-1|$. Then *(1983 - 1 Mark)*
   (a) $f(x^2)=(f(x))^2$ (b) $f(x+y)=f(x)+f(y)$
   (c) $f(|x|)=|f(x)|$ (d) None of these

4. If x satisfies $|x-1|+|x-2|+|x-3|\geq 6$, then *(1983 - 1 Mark)*
   (a) $0\leq x\leq 4$ (b) $x\leq -2$ or $x\geq 4$
   (c) $x\leq 0$ or $x\geq 4$ (d) None of these

5. If $f(x)=\cos(\ln x)$, then $f(x)f(y)-\frac{1}{2}\left[f\left(\frac{x}{y}\right)+f(xy)\right]$ has the value *(1983 - 1 Mark)*
   (a) -1 (b) 1/2
   (c) -2 (d) none of these

6. The domain of definition of the function

$y=\dfrac{1}{\log_{10}(1-x)}+\sqrt{x+2}$ is *(1983 - 1 Mark)*

(a) $(-3,-2)$ excluding -2.5 (b) $[0, 1]$ excluding 0.5
(c) $[-2, 1)$ excluding 0 (d) none of these

7. Which of the following functions is periodic? *(1983 - 1 Mark)*
(a) $f(x)=x-[x]$ where [x] denotes the largest integer less than or equal to the real number x
(b) $f(x)=\sin\dfrac{1}{x}$ for $x\neq 0$, $f(0)=0$
(c) $f(x)=x\cos x$
(d) none of these

8. Let $f(x)=\sin x$ and $g(x)=\ln|x|$. If the ranges of the composition functions fog and gof are R_1 and R_2 respectively, then *(1994 - 2 Marks)*
(a) $R_1=\{u:-1\le u<1\}$, $R_2=\{v:-\infty<v<0\}$
(b) $R_1=\{u:-\infty<u<0\}$, $R_2=\{v:-1\le v\le 0\}$
(c) $R_1=\{u:-1<u<1\}$, $R_2=\{v:-\infty<v<0\}$
(d) $R_1=\{u:-1\le u\le 1\}$, $R_2=\{v:-\infty<v\le 0\}$

9. Let $f(x)=(x+1)^2-1,\ x\ge -1$. Then the set $\{x: f(x)=f^{-1}(x)\}$ is *(1995)*
(a) $\left\{0,-1,\dfrac{-3+i\sqrt{3}}{2},\dfrac{-3-i\sqrt{3}}{2}\right\}$
(b) $\{0,1,-1\}$
(c) $\{0,-1\}$
(d) empty

10. The function $f(x)=|px-q|+r|x|$, $x\in(-\infty,\infty)$ where $p>0, q>0, r>0$ assumes its minimum value only on one point if *(1995)*
(a) $p\neq q$ (b) $r\neq q$
(c) $r\neq p$ (d) $p=q=r$

11. Let $f(x)$ be defined for all $x>0$ and be continuous. Let $f(x)$ satisfy $f\left(\dfrac{x}{y}\right)=f(x)-f(y)$ for all x, y and $f(e)=1$. Then *(1995S)*
(a) $f(x)$ is bounded (b) $f\left(\dfrac{1}{x}\right)\to 0$ as $x\to 0$
(c) $xf(x)\to 1$ as $x\to 0$ (d) $f(x)=\ln x$

12. If the function $f: [1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$, then $f^{-1}(x)$ is *(1999 - 2 Marks)*
(a) $\left(\dfrac{1}{2}\right)^{x(x-1)}$ (b) $\dfrac{1}{2}(1+\sqrt{1+4\log_2 x})$
(c) $\dfrac{1}{2}(1-\sqrt{1+4\log_2 x})$ (d) not defined

13. Let $f:R\to R$ be any function. Define $g:R\to R$ by $g(x)=|f(x)|$ for all x. Then g is *(2000S)*
(a) onto if f is onto
(b) one-one if f is one-one
(c) continuous if f is continuous
(d) differentiable if f is differentiable.

14. The domain of definition of the function $f(x)$ given by the equation $2^x+2^y=2$ is *(2000S)*
(a) $0<x\le 1$ (b) $0\le x\le 1$
(c) $-\infty<x\le 0$ (d) $-\infty<x<1$

15. Let $g(x)=1+x-[x]$ and $f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$. Then for all x, $f(g(x))$ is equal to *(2001S)*
(a) x (b) 1 (c) $f(x)$ (d) $g(x)$

16. If $f:[1,\infty)\to[2,\infty)$ is given by $f(x)=x+\dfrac{1}{x}$ then $f^{-1}(x)$ equals *(2001S)*
(a) $(x+\sqrt{x^2-4})/2$ (b) $x/(1+x^2)$
(c) $\left(x-\sqrt{x^2-4}\right)/2$ (d) $1+\sqrt{x^2-4}$

17. The domain of definition of $f(x)=\dfrac{\log_2(x+3)}{x^2+3x+2}$ is
(a) $R\setminus\{-1,-2\}$ (b) $(-2,\infty)$ *(2001S)*
(c) $R\setminus\{-1,-2,-3\}$ (d) $(-3,\infty)\setminus\{-1,-2\}$

18. Let $E=\{1,2,3,4\}$ and $F=\{1,2\}$. Then the number of onto functions from E to F is *(2001S)*
(a) 14 (b) 16 (c) 12 (d) 8

19. Let $f(x)=\dfrac{\alpha x}{x+1}, x\neq -1$. Then, for what value of α is $f(f(x))=x$? *(2001S)*
(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) -1

20. Suppose $f(x)=(x+1)^2$ for $x\ge -1$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y=x$, then $g(x)$ equals *(2002S)*
(a) $-\sqrt{x}-1, x\ge 0$ (b) $\dfrac{1}{(x+1)^2}, x>-1$
(c) $\sqrt{x+1}, x\ge -1$ (d) $\sqrt{x}-1, x\ge 0$

21. Let function $f:R\to R$ be defined by $f(x)=2x+\sin x$ for $x\in R$, then f is *(2002S)*
(a) one-to-one and onto
(b) one-to-one but NOT onto
(c) onto but NOT one-to-one
(d) neither one-to-one nor onto

22. If $f:[0,\infty)\longrightarrow[0,\infty)$, and $f(x)=\dfrac{x}{1+x}$ then f is *(2003S)*
(a) one-one and onto
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto

23. Domain of definition of the function

$f(x)=\sqrt{\sin^{-1}(2x)+\frac{\pi}{6}}$ for real valued x, is ***(2003S)***

(a) $\left[-\frac{1}{4},\frac{1}{2}\right]$ (b) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (c) $\left(-\frac{1}{2},\frac{1}{9}\right)$ (d) $\left[-\frac{1}{4},\frac{1}{4}\right]$

24. Range of the function $f(x)=\frac{x^2+x+2}{x^2+x+1}; x\in R$ is ***(2003S)***

(a) $(1,\infty)$ (b) $(1,11/7]$ (c) $(1,7/3]$ (d) $(1,7/5]$

25. If $f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2$ such that $\min f(x)>\max g(x)$, then the relation between b and c, is ***(2003S)***

(a) no real value of b & c (b) $0<c<b\sqrt{2}$

(c) $|c|<|b|\sqrt{2}$ (d) $|c|>|b|\sqrt{2}$

26. If $f(x)=\sin x+\cos x$, $g(x)=x^2-1$, then $g(f(x))$ is invertible in the domain ***(2004S)***

(a) $\left[0,\frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$ (c) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (d) $[0,\pi]$

27. If the functions f(x) and g(x) are defined on $R\to R$ such that

$f(x)=\begin{cases}0, & x\in\text{rational}\\ x, & x\in\text{irrational}\end{cases}$; $g(x)=\begin{cases}0, & x\in\text{irrational}\\ x, & x\in\text{rational}\end{cases}$ then

$(f-g)(x)$ is ***(2005S)***

(a) one-one & onto
(b) neither one-one nor onto
(c) one-one but not onto
(d) onto but not one-one

28. X and Y are two sets and $f: X\to Y$. If $\{f(c)=y;\ c\subset X,\ y\subset Y\}$ and $\{f^{-1}(d)=x;\ d\subset Y,\ x\subset X\}$, then the true statement is ***(2005S)***

(a) $f(f^{-1}(b))=b$ (b) $f^{-1}(f(a))=a$

(c) $f(f^{-1}(b))=b,\ b\subset y$ (d) $f^{-1}(f(a))=a,\ a\subset x$

29. If $F(x)=\left(f\left(\frac{x}{2}\right)\right)^2+\left(g\left(\frac{x}{2}\right)\right)^2$ where $f''(x)=-f(x)$ and $g(x)=f'(x)$ and given that $F(5)=5$, then $F(10)$ is equal to ***(2006 - 3M, –1)***

(a) 5 (b) 10 (c) 0 (d) 15

30. Let $f(x)=\frac{x}{(1+x^n)^{1/n}}$ for $n\ge 2$ and

$g(x)=\underbrace{(fofo...of)}_{f\text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2}g(x)dx$ equals. ***(2007 -3 marks)***

(a) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$ (b) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+K$

(c) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$ (d) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+K$

31. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x)=e^{x^2}+e^{-x^2}$, $g(x)=xe^{x^2}+e^{-x^2}$ and $h(x)=x^2e^{x^2}+e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then ***(2010)***

(a) $a=b$ and $c\ne b$ (b) $a=c$ and $a\ne b$

(c) $a\ne b$ and $c\ne b$ (d) $a=b=c$

32. Let $f(x)=x^2$ and $g(x)=\sin x$ for all $x\in R$. Then the set of all x satisfying (f o g o g o f) (x) = (g o g o f) (x), where (f o g) (x) = $f(g(x))$, is ***(2011)***

(a) $\pm\sqrt{n\pi}, n\in\{0,1,2,....\}$

(b) $\pm\sqrt{n\pi}, n\in\{1,2,....\}$

(c) $\frac{\pi}{2}+2n\pi, n\in\{...-2,-1,0,1,2....\}$

(d) $2n\pi, n\in\{...-2,-1,0,1,2,....\}$

33. The function $f:[0,3]\to[1,29]$, defined by $f(x)=2x^3-15x^2+36x+1$, is ***(2012)***

(a) one–one and onto (b) onto but not one–one

(c) one–one but not onto (d) neither one–one nor onto

D MCQs with One or More than One Correct

1. If $y=f(x)=\frac{x+2}{x-1}$ then ***(1984 - 3 Marks)***

(a) $x=f(y)$
(b) $f(1)=3$
(c) y increases with x for $x<1$
(d) f is a rational function of x

2. Let $g(x)$ be a function defined on $[-1, 1]$. If the area of the equilateral triangle with two of its vertices at (0,0) and $[x, g(x)]$ is $\frac{\sqrt{3}}{4}$, then the function $g(x)$ is ***(1989 - 2 Marks)***

(a) $g(x)=\pm\sqrt{1-x^2}$ (b) $g(x)=\sqrt{1-x^2}$

(c) $g(x)=-\sqrt{1-x^2}$ (d) $g(x)=\sqrt{1+x^2}$

3. If $f(x)=\cos[\pi^2]x+\cos[-\pi^2]x$, where $[x]$ stands for the greatest integer function, then ***(1991 - 2 Marks)***

(a) $f\left(\frac{\pi}{2}\right)=-1$ (b) $f(\pi)=1$

(c) $f(-\pi)=0$ (d) $f\left(\frac{\pi}{4}\right)=1$

4. If $f(x)=3x-5$, then $f^{-1}(x)$ ***(1998 - 2 Marks)***

(a) is given by $\frac{1}{3x-5}$
(b) is given by $\frac{x+5}{3}$
(c) does not exist because f is not one-one
(d) does not exist because f is not onto.

5. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin \sqrt{x})^2$, then
(a) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$ *(1998 - 2 Marks)*
(b) $f(x) = \sin x$, $g(x) = |x|$
(c) $f(x) = x^2$, $g(x) = \sin \sqrt{x}$
(d) f and g cannot be determined.

6. Let $f: (0, 1) \to \boldsymbol{R}$ be defined by $f(x) = \dfrac{b-x}{1-bx}$, where b is a constant such that $0 < b < 1$. Then
(a) f is not invertible on $(0, 1)$
(b) $f \neq f^{-1}$ on $(0, 1)$ and $f'(b) = \dfrac{1}{f'(0)}$
(c) $f = f^{-1}$ on $(0, 1)$ and $f'(b) = \dfrac{1}{f'(0)}$
(d) f^{-1} is differentiable $(0, 1)$

7. Let $f: (-1, 1) \to IR$ be such that $f(\cos 4\theta) = \dfrac{2}{2-\sec^2\theta}$ for $\theta \in \left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$. Then the value (s) of $f\left(\dfrac{1}{3}\right)$ is (are)
(a) $1-\sqrt{\dfrac{3}{2}}$ (b) $1+\sqrt{\dfrac{3}{2}}$ (c) $1-\sqrt{\dfrac{2}{3}}$ (d) $1+\sqrt{\dfrac{2}{3}}$

8. The function $f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$ has a local minimum or a local maximum at $x =$ *(JEE Adv. 2013)*
(a) -2 (b) $\dfrac{-2}{3}$ (c) 2 (d) $\dfrac{2}{3}$

9. Let $f: \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \to R$ be given by $f(x) = (\log(\sec x + \tan x))^3$. Then *(JEE Adv. 2014)*
(a) $f(x)$ is an odd function
(b) $f(x)$ is one-one function
(c) $f(x)$ is an onto function
(d) $f(x)$ is an even function

10. Let $a \in R$ and let $f: R \to R$ be given by $f(x) = x^5 - 5x + a$. Then *(JEE Adv. 2014)*
(a) $f(x)$ has three real roots if $a > 4$
(b) $f(x)$ has only real root if $a > 4$
(c) $f(x)$ has three real roots if $a < -4$
(d) $f(x)$ has three real roots if $-4 < a < 4$

11. Let $f(x) = \sin\left(\dfrac{\pi}{6}\sin\left(\dfrac{\pi}{2}\sin x\right)\right)$ for all $x \in R$ and $g(x) = \dfrac{\pi}{2} \sin x$ for all $x \in R$. Let $(fog)(x)$ denote $f(g(x))$ and $(gof)(x)$ denote $g(f(x))$. Then which of the following is (are) true? *(JEE Adv. 2015)*
(a) Range of f is $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$
(b) Range of fog is $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$
(c) $\displaystyle\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$
(d) There is an $x \in R$ such that $(gof)(x) = 1$

E Subjective Problems

1. Find the domain and range of the function $f(x) = \dfrac{x^2}{1+x^2}$. Is the function one-to-one? *(1978)*
2. Draw the graph of $y = |x|^{1/2}$ for $-1 \le x \le 1$. *(1978)*
3. If $f(x) = x^9 - 6x^8 - 2x^7 + 12x^6 + x^4 - 7x^3 + 6x^2 + x - 3$, find $f(6)$. *(1979)*
4. Consider the following relations in the set of real numbers R.
$R = \{(x, y); x \in R, y \in R, x^2 + y^2 \le 25\}$
$R' = \left\{(x, y) : x \in R, y \in R, y \ge \dfrac{4}{9}x^2\right\}$
Find the domain and range of $R \cap R'$. Is the relation $R \cap R'$ a function? *(1979)*
5. Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Prove that there is a bijective mapping from A to B. *(1981 - 2 Marks)*
6. Let f be a one-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(x) = 1, f(y) \neq 1, f(z) \neq 2$ determine $f^{-1}(1)$. *(1982 - 3 Marks)*
7. Let R be the set of real numbers and $f: R \longrightarrow R$ be such that for all x and y in R $|f(x) - f(y)| \le |x-y|^3$. Prove that $f(x)$ is a constant. *(1988 - 2 Marks)*
8. Find the natural number 'a' for which $\displaystyle\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function '$f$' satisfies the relation $f(x+y) = f(x)f(y)$ for all natural numbers x, y and further $f(1) = 2$. *(1992 - 6 Marks)*
9. Let $\{x\}$ and $[x]$ denotes the fractional and integral part of a real number x respectively. Solve $4\{x\} = x + [x]$. *(1994 - 4 Marks)*
10. A function $f: IR \to IR$, where IR is the set of real numbers, is defined by $f(x) = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$. Find the interval of values of α for which f is onto. Is the function one-to-one for $\alpha = 3$? Justify your answer. *(1996 - 5 Marks)*
11. Let $f(x) = Ax^2 + Bx + C$ where A, B, C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers $2A, A+B$ and C are all integers. Conversely, prove that if the numbers $2A$, $A+B$ and C are all integers then $f(x)$ is an integer whenever x is an integer. *(1998 - 8 Marks)*

F Match the Following

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	○

1. Let the function defined in column 1 have domain $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and range $(-\infty,\infty)$ ***(1992 - 2 Marks)***

Column I	**Column II**
(A) $1+2x$	(p) onto but not one-one
(B) $\tan x$	(q) one- one but not onto
	(r) one- one and onto
	(s) neither one-one nor onto

2. Let $f(x)=\dfrac{x^2-6x+5}{x^2-5x+6}$ ***(2007 -6 marks)***

Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	***Column II***
(A) If $-1<x<1$, then $f(x)$ satisfies	(p) $0<f(x)<1$
(B) If $1<x<2$, then $f(x)$ satisfies	(q) $f(x)<0$
(C) If $3<x<5$, then $f(x)$ satisfies	(r) $f(x)>0$
(D) If $x>5$, then $f(x)$ satisfies	(s) $f(x)<1$

I Integer Value Correct Type

1. Let $f:[0,4\pi]\to[0,\pi]$ be defined by $f(x)=\cos^{-1}(\cos x)$. The number of points $x\in[0,4\pi]$ satisfying the equation

$f(x)=\dfrac{10-x}{10}$ is ***(JEE Adv. 2014)***

Section-B JEE Main / AIEEE

1. The domain of $\sin^{-1}[\log_3(x/3)]$ is **[2002]**

(a) $[1,9]$ (b) $[-1,9]$ (c) $[-9,1]$ (d) $[-9,-1]$

2. The function $f(x)=\log\left(x+\sqrt{x^2+1}\right)$, is **[2003]**

(a) neither an even nor an odd function
(b) an even function
(c) an odd function
(d) a periodic function.

3. Domain of definition of the function $f(x)=\dfrac{3}{4-x^2}$ $+\log_{10}(x^3-x)$, is **[2003]**

(a) $(-1,0)\cup(1,2)\cup(2,\infty)$ (b) $(a,2)$
(c) $(-1,0)\cup(a,2)$ (d) $(1,2)\cup(2,\infty)$.

4. If $f:R\to R$ satisfies $f(x+y)=f(x)+f(y)$, for all x, $y\in R$ and $f(1)=7$, then $\sum\limits_{r=1}^{n} f(r)$ is **[2003]**

(a) $\dfrac{7n(n+1)}{2}$ (b) $\dfrac{7n}{2}$
(c) $\dfrac{7(n+1)}{2}$ (d) $7n+(n+1)$.

5. A function f from the set of natural numbers to integers defined by **[2003]**

$$f(n)=\begin{cases}\dfrac{n-1}{2}, \text{when n is odd}\\[2mm] -\dfrac{n}{2}, \text{when n is even}\end{cases}$$ is

(a) neither one -one nor onto
(b) one-one but not onto
(c) onto but not one-one
(d) one-one and onto both.

6. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is [2004]

(a) $\{1,2,3,4,5\}$ (b) $\{1,2,3,4,5,6\}$
(c) $\{1,2,3,4,\}$ (d) $\{1,2,3,\}$

7. If $f : R \to S$, defined by

$f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is [2004]

(a) $[-1,3]$ (b) $[-1,1]$ (c) $[0,1]$ (d) $[0,3]$

8. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then [2004]

(a) $f(x) = -f(-x)$ (b) $f(2+x) = f(2-x)$
(c) $f(x) = f(-x)$ (d) $f(x+2) = f(x-2)$

9. The domain of the function $f(x) = \dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is [2004]

(a) $[1,2]$ (b) $[2,3)$ (c) $[1,2]$ (d) $[2,3]$

10. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\dfrac{2x}{1-x^2}$, then f is both one - one and onto when B is the interval [2005]

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right)$ (c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

11. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched? [2005]

	Interval	Function
(a)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(b)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(c)	$\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$
(d)	$(-\infty, -4)$	$x^3 + 6x^2 + 6$

12. A real valued function $f(x)$ satisfies the functional equation
$$f(x-y) = f(x)f(y) - f(a-x)f(a+y)$$
where a is a given constant and $f(0) = 1$, $f(2a-x)$ is equal to [2005]

(a) $-f(x)$ (b) $f(x)$
(c) $f(a) + f(a-x)$ (d) $f(-x)$

13. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function,

$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$, is defined, is [2007]

(a) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right)$
(c) $[0, \pi]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

14. Let f: $N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$.
Show that f is invertible and its inverse is [2008]

(a) $g(y) = \dfrac{3y+4}{3}$ (b) $g(y) = 4 + \dfrac{y+3}{4}$
(c) $g(y) = \dfrac{y+3}{4}$ (d) $g(y) = \dfrac{y-3}{4}$

15. Let $f(x) = (x+1)^2 - 1$, $x \geq -1$

Statement -1 : The set $\{x : f(x) = f^{-1}(x) = \{0, -1\}$
Statement-2 : f is a bijection. [2009]

(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

16. For real x, let $f(x) = x^3 + 5x + 1$, then [2009]

(a) f is onto R but not one-one
(b) f is one-one and onto R
(c) f is neither one-one nor onto R
(d) f is one-one but not onto R

17. The domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is [2011]

(a) $(0, \infty)$ (b) $(-\infty, 0)$
(c) $(-\infty, \infty) - \{0\}$ (d) $(-\infty, \infty)$