CHAPTER 12 Differentiation

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, then $\frac{dy}{dx} =$ .... *(1982 - 2 Marks)*

2. If $f_r(x)$, $g_r(x)$, $h_r(x)$, $r = 1, 2, 3$ are polynomials in x such that $f_r(a) = g_r(a) = h_r(a)$, $r = 1, 2, 3$

and $F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$ then $F'(x)$ at $x = a$ is .... *(1985 - 2 Marks)*

3. If $f(x) = \log_x (\ln x)$, then $f'(x)$ at $x = e$ is .... *(1985 - 2 Marks)*

4. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is .... *(1986 - 2 Marks)*

5. If $f(x) = |x - 2|$ and $g(x) = f[f(x)]$, then $g'(x) =$ .... for $x > 20$ *(1990 - 2 Marks)*

6. If $xe^{xy} = y + \sin^2 x$, then at $x = 0$, $\frac{dy}{dx} =$ .... *(1996 - 1 Mark)*

B True/ False

1. The derivative of an even function is always an odd function. *(1983 - 1 Mark)*

C MCQs with One Correct Answer

1. If $y^2 = P(x)$, a polynomial of degree 3, then $2\frac{d}{dx}\left(y^3 \frac{d^2y}{dx^2}\right)$ equals *(1988 - 2 Marks)*
   (a) $P'''(x) + P'(x)$ (b) $P''(x)P'''(x)$
   (c) $P(x)P'''(x)$ (d) a constant

2. Let $f(x)$ be a quadratic expression which is positive for all the real values of x. If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x, *(1990 - 2 Marks)*
   (a) $g(x) < 0$ (b) $g(x) > 0$
   (c) $g(x) = 0$ (d) $g(x) \geq 0$

3. If $y = (\sin x)^{\tan x}$, then $\frac{dy}{dx}$ is equal to *(1994)*
   (a) $(\sin x)^{\tan x}(1 + \sec^2 x \log \sin x)$
   (b) $\tan x (\sin x)^{\tan x - 1}.\cos x$
   (c) $(\sin x)^{\tan x} \sec^2 x \log \sin x$
   (d) $\tan x (\sin x)^{\tan x - 1}$

4. If $x^2 + y^2 = 1$ then *(2000)*
   (a) $yy'' - 2(y')^2 + 1 = 0$ (b) $yy'' + (y')^2 + 1 = 0$
   (c) $yy'' + (y')^2 - 1 = 0$ (d) $yy'' + 2(y')^2 + 1 = 0$

5. Let $f: (0, \infty) \to R$ and $F(x) = \int_0^x f(t)dt$. If $F(x^2) = x^2(1 + x)$, then $f(4)$ equals *(2001S)*
   (a) 5/4 (b) 7 (c) 4 (d) 2

6. If y is a function of x and $\log(x + y) - 2xy = 0$, then the value of $y'(0)$ is equal to *(2004S)*
   (a) 1 (b) -1 (c) 2 (d) 0

7. If $f(x)$ is a twice differentiable function and given that $f(1) = 1; f(2) = 4, f(3) = 9$, then *(2005S)*
   (a) $f''(x) = 2$ for $\forall\ x \in (1, 3)$
   (b) $f''(x) = f'(x) = 5$ for some $x \in (2, 3)$
   (c) $f''(x) = 3$ for $\forall\ x \in (2, 3)$
   (d) $f''(x) = 2$ for some $x \in (1, 3)$

8. $\frac{d^2x}{dy^2}$ equals *(2007 -3 marks)*
   (a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (b) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$
   (c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

9. Let $g(x) = \log f(x)$ where $f(x)$ is twice differentible positive function on $(0, \infty)$ such that $f(x+1) = x f(x)$. Then, for $N = 1, 2, 3, \ldots\ldots\ldots$ *(2008)*

$$g''\left(N+\frac{1}{2}\right)-g''\left(\frac{1}{2}\right)=$$

(a) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N-1)^2}\right\}$

(b) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N-1)^2}\right\}$

(c) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N+1)^2}\right\}$

(d) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N+1)^2}\right\}$

10. Let $f : [0, 2] \to R$ be a function which is continuous on $[0, 2]$ and is differentiable on $(0, 2)$ with $f(0) = 1$. Let $F(x) = \int_0^{x^2} f\left(\sqrt{t}\right)dt$ for $x \in [0,2]$. If $F'(x) = f'(x)$ for all $x \in (0,2)$, then $F(2)$ equals *(JEE Adv. 2014)*

(a) $e^2 - 1$ (b) $e^4 - 1$

(c) $e - 1$ (d) e^4

D MCQs with One or More than One Correct

1. Let $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$. Then *(JEE Adv. 2016)*

(a) $g'(2) = \frac{1}{15}$ (b) $h'(1) = 666$

(c) $h(0) = 16$ (d) $h(g(3)) = 36$

E Subjective Problems

1. Find the derivative of $\sin(x^2 + 1)$ with respect to x from first principle. *(1978)*

2. Find the derivative of

$$f(x) = \begin{cases} \dfrac{x-1}{2x^2-7x+5} & \text{when } x \neq 1 \\ -\dfrac{1}{3} & \text{when } x = 1 \end{cases}$$

at $x = 1$ *(1979)*

3. Given $y = \frac{5x}{3\sqrt{(1-x)^2}} + \cos^2(2x+1)$; Find $\frac{dy}{dx}$. *(1980)*

4. Let $y = e^{x \sin x^3} + (\tan x)^x$. Find $\frac{dy}{dx}$ *(1981 - 2 Marks)*

5. Let f be a twice differentiable function such that

$$f''(x) = -f(x), \text{and } f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2$$

Find $h(10)$ if $h(5) = 11$ *(1982 - 3 Marks)*

6. If α be a repeated root of a quadratic equation $f(x) = 0$ and $A(x)$, $B(x)$ and $C(x)$ be polynomials of degree 3, 4 and 5 respectively, then show that $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is divisible by $f(x)$, where prime denotes the derivatives. *(1984 - 4 Marks)*

7. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$ *(1989 - 2 Marks)*

8. Find $\frac{dy}{dx}$ at $x = -1$, when

$$(\sin y)^{\sin\left(\frac{\pi}{2}x\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x \tan(\ln(x+2)) = 0$$

(1991 - 4 Marks)

9. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$,

prove that $\frac{y'}{y} = \frac{1}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$.

(1998 - 8 Marks)

H Assertion & Reason Type Questions

1. Let $f(x) = 2 + \cos x$ for all real x.

STATEMENT - 1 : For each real t, there exists a point c in $[t, t+\pi]$ such that $f'(c) = 0$ because

STATEMENT - 2 : $f(t) = f(t + 2\pi)$ for each real t.

(2007 -3 marks)

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

2. Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is continuous, $g(0) \neq 0$. $g'(0) = 0$, $g''(0) \neq 0$, and $f(x) = g(x) \sin x$

STATEMENT - 1 : $\lim_{x \to 0} [g(x) \cot x - g(0) \operatorname{cosec} x] = f''(0)$

and

STATEMENT - 2 : $f'(0) = g(0)$ *(2008)*

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is NOT** a correct explaination for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

I Integer Value Correct Type

1. If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is *(2009)*

2. Let $f(\theta) = \sin\left(\tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$.

Then the value of $\dfrac{d}{d(\tan\theta)}(f(\theta))$ is *(2011)*

Section-B JEE Main / AIEEE

1. If $y = (x + \sqrt{1 + x^2})^n$, then $(1 + x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is **[2002]**

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$

2. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and

$F(t) = \int_0^t f(t-y)g(y)\,dy$, then **[2003]**

(a) $F(t) = te^{-t}$ (b) $F(t) = 1 - te^{-t}(1+t)$

(c) $F(t) = e^t - (1+t)$ (d) $F(t) = te^t$.

3. If $f(x) = x^n$, then the value of **[2003[**

$f(1) - \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} - \dfrac{f'''(1)}{3!} + \ldots\ldots\ldots \dfrac{(-1)^n f^n(1)}{n!}$ is

(a) 1 (b) 2^n (c) $2^n - 1$ (d) 0

4. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A. P, then $f'(a), f'(b), f'(c)$ are in **[2003]**

(a) Arithmetic -Geometric Progression

(b) A.P

(c) G.P

(d) H.P.

5. If $x = e^{y + e^{y + e^{y + \ldots\infty}}}$, $x > 0$, then $\dfrac{dy}{dx}$ is **[2004]**

(a) $\dfrac{1+x}{x}$ (b) $\dfrac{1}{x}$ (c) $\dfrac{1-x}{x}$ (d) $\dfrac{x}{1+x}$

6. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is **[2005]**

(a) 1 (b) 0 (c) 3 (d) 2

7. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals **[2005]**

(a) -2 (b) 3 (c) 2 (d) 1

8. Let $f: R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\dfrac{1}{48}\right)$. Then $\lim_{x \to 2} \int_6^{f(x)} \dfrac{4t^3}{x-2} dt$ equals **[2005]**

(a) 24 (b) 36 (c) 12 (d) 18

9. The set of points where $f(x) = \dfrac{x}{1+|x|}$ is differentiable is

(a) $(-\infty, 0) \cup (0, \infty)$ (b) $(-\infty, -1) \cup (-1, \infty)$

(c) $(-\infty, \infty)$ (d) $(0, \infty)$ **[2006]**

10. If $x^m . y^n = (x+y)^{m+n}$, then $\dfrac{dy}{dx}$ is **[2006]**

(a) $\dfrac{y}{x}$ (b) $\dfrac{x+y}{xy}$ (c) xy (d) $\dfrac{x}{y}$

11. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals **[2009]**

(a) 1 (b) $\log 2$ (c) $-\log 2$ (d) -1

12. Let $f: (-1, 1) \to \mathbf{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$ **[2010]**

(a) -4 (b) 0 (c) -2 (d) 4

13. $\dfrac{d^2x}{dy^2}$ equals : **[2011]**

(a) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$ (b) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$

(c) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$ (d) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$

14. If $y = \sec(\tan^{-1}x)$, then $\dfrac{dy}{dx}$ at $x = 1$ is equal to : **[JEE M 2013]**

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{2}$ (c) 1 (d) $\sqrt{2}$

15. If g is the inverse of a function f and $f'(x) = \dfrac{1}{1+x^5}$, then $g'(x)$ is equal to: **[JEE M 2014]**

(a) $\dfrac{1}{1+\{g(x)\}^5}$ (b) $1+\{g(x)\}^5$

(c) $1+x^5$ (d) $5x^4$

16. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha\log|x| + \beta x^2 + x$ then **[JEE M 2014]**

(a) $\alpha = 2, \beta = -\dfrac{1}{2}$ (b) $\alpha = 2, \beta = \dfrac{1}{2}$

(c) $\alpha = -6, \beta = \dfrac{1}{2}$ (d) $\alpha = -6, \beta = -\dfrac{1}{2}$