CHAPTER

Inverse Trigonometric **Functions**

Section-A

JEE Advanced/ IIT-JEE

Fill in the Blanks

Let a, b, c be positive real numbers. Let

$$\theta = \tan^{-1} \sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1} \sqrt{\frac{b(a+b+c)}{ca}}$$

$$+\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$$
.

Then $\tan \theta =$ _____

(1981 - 2 Marks)

- The numerical value of $\tan \left\{ 2 \tan^{-1} \left(\frac{1}{5} \right) \frac{\pi}{4} \right\}$ is equal
- The greater of the two angles $A = 2 \tan^{-1} (2\sqrt{2} 1)$ and 3. $B = 3 \sin^{-1}(1/3) + \sin^{-1}(3/5)$ is _____ ___. (1989 - 2 Marks)

MCQs with One Correct Answer

The value of $\tan \left| \cos^{-1} \left(\frac{4}{5} \right) + \tan^{-1} \left(\frac{2}{3} \right) \right|$ is

(1983 - 1 Mark)

- (a) $\frac{6}{17}$ (b) $\frac{7}{16}$ (c) $\frac{16}{7}$

- If we consider only the principle values of the inverse trigonometric functions then the value of

$$\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right)$$
 is (1994)

- (a) $\frac{\sqrt{29}}{3}$ (b) $\frac{29}{3}$ (c) $\frac{\sqrt{3}}{29}$ (d) $\frac{3}{29}$
- The number of real solutions of $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2 + x + 1} = \pi/2$ is

(1999 - 2 Marks)

- (a) zero
- (b) one
- (c) two
- (d) infinite

If $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}-...\right)$

$$+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}-...\right)=\frac{\pi}{2}$$

- for $0 < |x| < \sqrt{2}$, then x equals
- (a) 1/2 (b) 1
- (c) -1/2The value of x for which $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$ is
- (a) 1/2 (c) 0 (b) 1
- If 0 < x < 1, then
 - $\sqrt{1+x^2} \left[\left\{ x \cos \left(\cot^{-1} x \right) + \sin \left(\cot^{-1} x \right) \right\}^2 1 \right]^{1/2} =$
 - (a) $\frac{x}{\sqrt{1+x^2}}$
- (c) $x\sqrt{1+x^2}$
- (d) $\sqrt{1+x^2}$
- The value of cot $\left(\sum_{n=1}^{23} \cot^{-1} \left(1 + \sum_{k=1}^{n} 2k\right)\right)$ is

(JEE Adv. 2013)

(2004S)

(d) -1/2

- (a) $\frac{23}{25}$ (b) $\frac{25}{23}$ (c) $\frac{23}{24}$

MCQs with One or More than One Correct

- The principal value of $\sin^{-1} \left(\sin \frac{2\pi}{3} \right)$ is (1986 2 Marks)
 - (a) $-\frac{2\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$
- If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse

trigonometric functions take only the principal values, then (JEE Adv. 2015) the correct option(s) is (are)

- (a) $\cos \beta > 0$
- (b) $\sin \beta < 0$
- (c) $\cos(\alpha + \beta) > 0$
- (d) $\cos \alpha < 0$

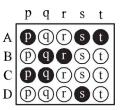
E Subjective Problems

- 2. Find all the solution of $4\cos^2 x \sin x 2\sin^2 x = 3\sin x$ (1983 2 Marks)
- 1. Find the value of : $\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$, where $0 \le \cos^{-1}x \le \pi$ and $-\pi/2 \le \sin^{-1}x \le \pi/2$.

 (1981 2 Marks)
- 3. Prove that $\cos \tan^{-1} \sin \cot^{-1} x = \sqrt{\frac{x^2 + 1}{x^2 + 2}}$. (2002 5 Marks)

F Match the Following

DIRECTIONS (Q. 1 & 2): Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:



(2006 - 6M)

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

1. Match the following

Column I

- (A) $\sum_{i=1}^{\infty} \tan^{-1} \left(\frac{1}{2i^2} \right) = t$, then $\tan t =$
- - $\cos \theta_1 = \frac{a}{b+c}, \cos \theta_2 = \frac{b}{a+c}, \cos \theta_3 = \frac{c}{a+b},$

(B) Sides a, b, c of a triangle ABC are in AP and

then
$$\tan^2\left(\frac{\theta_1}{2}\right) + \tan^2\left(\frac{\theta_3}{2}\right) =$$

- (C) A line is perpendicular to x + 2y + 2z = 0 and (r) passes through (0, 1, 0). The perpendicular distance of this line from the origin is
- 2. Let (x, y) be such that $\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}$. (2007)

Match the statements in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I

(A) If
$$a = 1$$
 and $b = 0$, then (x, y)

(B) If
$$a = 1$$
 and $b = 1$, they (x, y)

(C) If
$$a = 1$$
 and $b = 2$, then (x, y)

(D) If
$$a = 2$$
 and $b = 2$, then (x, y)

Column II

Column II

(p)

- (p) lies on the circle $x^2 + y^2 = 1$
- (q) lies on $(x^2-1)(y^2-1)=0$
- (r) lies on y = x
- (s) lies on $(4x^2-1)(y^2-1)=0$

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DIRECTIONS (Q. 3): Following question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

3. Match List I with List II and select the correct answer using the code given below the lists:

(JEE Adv. 2013)

Liet

P. $\left(\frac{1}{y^2} \left(\frac{\cos(\tan^{-1} y) + y \sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)}\right)^2 + y^4\right)^{1/2}$ takes value

 $1. \qquad \frac{1}{2}\sqrt{\frac{5}{3}}$

Q. If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then

2. $\sqrt{2}$

List II

possible value of $\cos \frac{x-y}{2}$ is

R If $\cos\left(\frac{\pi}{4} - x\right) \cos 2x + \sin x \sin 2 \sec x = \cos x \sin 2x \sec x +$

3.

 $\cos\left(\frac{\pi}{4} + x\right)\cos 2x$ then possible value of $\sec x$ is

S. If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right) = \sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right), x \neq 0$,

4. 1

then possible value of x is

Codes:

P Q R S (a) 4 3 1 2 (b) 4 3 2 1 (c) 3 4 2

JEE Main / AIEEE Section-B

- $\cot^{-1}(\sqrt{\cos\alpha}) \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x =$

 - (a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$
- [2002]

[2003]

- (c) tan α
- (d) $\cot \left(\frac{\alpha}{2}\right)$
- The trigonometric equation $\sin^{-1} x = 2 \sin^{-1} a$ 2. has a solution for

 - (a) $|a| \ge \frac{1}{\sqrt{2}}$ (b) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$
 - (c) all real values of a (d) $|a| < \frac{1}{2}$
- If $\cos^{-1} x \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 4xy\cos\alpha + y^2$ is 7. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2}\right)$, equal to
 - (a) $2\sin 2\alpha$
- (b) 4
- (c) $4\sin^2\alpha$
- (d) $-4\sin^2\alpha$
- 4. If $\sin^{-1}\left(\frac{x}{5}\right) + \csc^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the values of x is
- (b) 5
- [2007]

- (c) 1
- (d) 3

- The value of $\cot\left(\cos \operatorname{ec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is
 - (a) $\frac{6}{17}$ (b) $\frac{3}{17}$
 - (c) $\frac{4}{17}$ (d) $\frac{5}{17}$
- If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in [JEE M 2013] (a) x = y = z (b) 2x = 3y = 6z(c) 6x = 3y = 2z (d) $6x = 4v = 2\pi$

where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is: [JEE M 2015]

- (a) $\frac{3x x^3}{1 + 3x^2}$ (b) $\frac{3x + x^3}{1 + 3x^2}$
- (c) $\frac{3x-x^3}{1-3x^2}$ (d) $\frac{3x+x^3}{1-3x^2}$