CHAPTER 2

Complex Numbers

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. If the expression *(1987 - 2 Marks)*

$$\frac{\left[\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)+i\tan(x)\right]}{\left[1+2\,i\,\sin\left(\frac{x}{2}\right)\right]}$$

is real, then the set of all possible values of x is ....

2. For any two complex numbers z_1, z_2 and any real number a and b. *(1988 - 2 Marks)*
$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 =$ ....

3. If a, b, c, are the numbers between 0 and 1 such that the points $z_1 = a + i$, $z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle, then $a =$ ....and $b =$ .... *(1989 - 2 Marks)*

4. *ABCD* is a rhombus. Its diagonals *AC* and *BD* intersect at the point *M* and satisfy *BD* = 2*AC*. If the points *D* and *M* represent the complex numbers $1 + i$ and $2 - i$ respectively, then A represents the complex number ....or.......... *(1993 - 2 Marks)*

5. Suppose Z_1, Z_2, Z_3 are the vertices of an equilateral triangle inscribed in the circle $|Z| = 2$. If $Z_1 = 1 + i\sqrt{3}$ then $Z_2 =$ ...., $Z_3 =$ .... *(1994 - 2 Marks)*

6. The value of the expression
$1\cdot(2-\omega)(2-\omega^2)+2\cdot(3-\omega)(3-\omega^2)+....+(n-1).(n-\omega)(n-\omega^2)$,
where ω is an imaginary cube root of unity, is..... *(1996 - 2 Marks)*

B True / False

1. For complex number $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, we write $z_1 \cap z_2$, if $x_1 \le x_2$ and $y_1 \le y_2$. Then for all complex numbers *z* with $1 \cap z$, we have $\frac{1-z}{1+z} \cap \theta$. *(1981 - 2 Marks)*

2. If the complex numbers, Z_1, Z_2 and Z_3 represent the vertices of an equilateral triangle such that
$|Z_1| = |Z_2| = |Z_3|$ then $Z_1 + Z_2 + Z_3 = 0$. *(1984 - 1 Mark)*

3. If three complex numbers are in A.P. then they lie on a circle in the complex plane. *(1985 - 1 Mark)*

4. The cube roots of unity when represented on Argand diagram form the vertices of an equilateral triangle. *(1988 - 1 Mark)*

C MCQs with One Correct Answer

1. If the cube roots of unity are 1, ω, ω^2, then the roots of the equation $(x-1)^3 + 8 = 0$ are *(1979)*
(a) $-1, 1+2\omega, 1+2\omega^2$ (b) $-1, 1-2\omega, 1-2\omega^2$
(c) $-1, -1, -1$ (d) None of these

2. The smallest positive integer n for which *(1980)*
$\left(\frac{1+i}{1-i}\right)^n = 1$ is
(a) $n = 8$ (b) $n = 16$
(c) $n = 12$ (d) none of these

3. The complex numbers $z = x + iy$ which satisfy the equation
$\left|\frac{z-5i}{z+5i}\right| = 1$ lie on *(1981 - 2 Marks)*
(a) the x-axis
(b) the straight line $y = 5$
(c) a circle passing through the origin
(d) none of these

4. If $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$, then *(1982 - 2 Marks)*
(a) $\text{Re}(z) = 0$ (b) $\text{Im}(z) = 0$
(c) $\text{Re}(z) > 0, \text{Im}(z) > 0$ (d) $\text{Re}(z) > 0, \text{Im}(z) < 0$

5. The inequality $|z-4| < |z-2|$ represents the region given by *(1982 - 2 Marks)*
(a) $\text{Re}(z) \ge 0$ (b) $\text{Re}(z) < 0$
(c) $\text{Re}(z) > 0$ (d) none of these

6. If $z = x + iy$ and $\omega = (1-iz)/(z-i)$, then $|\omega| = 1$ implies that, in the complex plane, *(1983 - 1 Mark)*
(a) *z* lies on the imaginary axis
(b) *z* lies on the real axis
(c) *z* lies on the unit circle
(d) None of these

7. The points $z_1, z_2, z_3 z_4$ in the complex plane are the vertices of a parallelogram taken in order if and only if *(1983 - 1 Mark)*
(a) $z_1+z_4=z_2+z_3$ (b) $z_1+z_3=z_2+z_4$
(c) $z_1+z_2=z_3+z_4$ (d) None of these

8. If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that $c=(1-r)a+rb$ and $w=(1-r)u+rv$, where r is a complex number, then the two triangles *(1985 - 2 Marks)*
(a) have the same area (b) are similar
(c) are congruent (d) none of these

9. If $\omega\ (\neq 1)$ is a cube root of unity and $(1+\omega)^7=A+B\omega$ then A and B are respectively *(1995S)*
(a) 0, 1 (b) 1, 1 (c) 1, 0 (d) $-1, 1$

10. Let z and ω be two non zero complex numbers such that $|z|=|\omega|$ and $\operatorname{Arg} z+\operatorname{Arg}\omega=\pi$, then z equals *(1995S)*
(a) ω (b) $-\omega$ (c) $\overline{\omega}$ (d) $-\overline{\omega}$

11. Let z and ω be two complex numbers such that $|z|\le 1$, $|\omega|\le 1$ and $|z+i\omega|=|z-i\overline{\omega}|=2$ then z equals *(1995S)*
(a) 1 or i (b) i or $-i$ (c) 1 or -1 (d) i or -1

12. For positive integers n_1, n_2 the value of the expression $(1+i)^{n_1}+(1+i^3)^{n_1}+(1+i^5)^{n_2}+(1+i^7)^{n_2}$, where $i=\sqrt{-1}$ is a real number if and only if *(1996 - 1 Marks)*
(a) $n_1=n_2+1$ (b) $n_1=n_2-1$
(c) $n_1=n_2$ (d) $n_1>0, n_2>0$

13. If $i=\sqrt{-1}$, then $4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$ is equal to *(1999 - 2 Marks)*
(a) $1-i\sqrt{3}$ (b) $-1+i\sqrt{3}$ (c) $i\sqrt{3}$ (d) $-i\sqrt{3}$

14. If $\arg(z)<0$, then $\arg(-z)-\arg(z)=$ *(2000S)*
(a) π (b) $-\pi$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

15. If z_1, z_2 and z_3 are complex numbers such that *(2000S)*
$|z_1|=|z_2|=|z_3|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$, then $|z_1+z_2+z_3|$ is
(a) equal to 1 (b) less than 1
(c) greater than 3 (d) equal to 3

16. Let z_1 and z_2 be n^{th} roots of unity which subtend a right angle at the origin. Then n must be of the form *(2001S)*
(a) $4k+1$ (b) $4k+2$ (c) $4k+3$ (d) $4k$

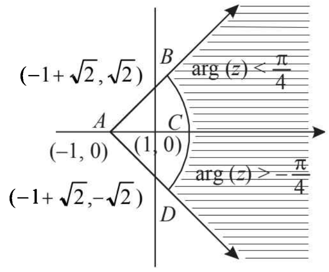
17. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$ are the vertices of a triangle which is *(2001S)*
(a) of area zero
(b) right-angled isosceles
(c) equilateral
(d) obtuse-angled isosceles

18. For all complex numbers z_1, z_2 satisfying $|z_1|=12$ and $|z_2-3-4i|=5$, the minimum value of $|z_1-z_2|$ is *(2002S)*
(a) 0 (b) 2 (c) 7 (d) 17

19. If $|z|=1$ and $\omega=\frac{z-1}{z+1}$ (where $z\neq -1$), then $\operatorname{Re}(\omega)$ is *(2003S)*
(a) 0 (b) $-\frac{1}{|z+1|^2}$
(c) $\left|\frac{z}{z+1}\right|\cdot\frac{1}{|z+1|^2}$ (d) $\frac{\sqrt{2}}{|z+1|^2}$

20. If $\omega\ (\neq 1)$ be a cube root of unity and $(1+\omega^2)^n=(1+\omega^4)^n$, then the least positive value of n is *(2004S)*
(a) 2 (b) 3 (c) 5 (d) 6

21. The locus of z which lies in shaded region (excluding the boundaries) is best represented by *(2005S)*



(a) $z:|z+1|>2$ and $|\arg(z+1)|<\pi/4$
(b) $z:|z-1|>2$ and $|\arg(z-1)|<\pi/4$
(c) $z:|z+1|<2$ and $|\arg(z+1)|<\pi/2$
(d) $z:|z-1|<2$ and $|\arg(z+1)|<\pi/2$

22. a, b, c are integers, not all simultaneously equal and ω is cube root of unity ($\omega\neq 1$), then minimum value of $|a+b\omega+c\omega^2|$ is *(2005S)*
(a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

23. Let $\omega=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$, then the value of the det.
$\begin{vmatrix}1 & 1 & 1\\ 1 & -1-\omega^2 & \omega^2\\ 1 & \omega^2 & \omega^4\end{vmatrix}$ is *(2002 - 2 Marks)*
(a) 3ω (b) $3\omega(\omega-1)$
(c) $3\omega^2$ (d) $3\omega(1-\omega)$

24. If $\frac{w-\overline{w}z}{1-z}$ is purely real where $w=\alpha+i\beta$, $\beta\neq 0$ and $z\neq 1$, then the set of the values of z is *(2006 - 3M, –1)*
(a) $\{z:|z|=1\}$ (b) $\{z:z=\overline{z}\}$
(c) $\{z:z\neq 1\}$ (d) $\{z:|z|=1, z\neq 1\}$

25. A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is ***(2007 -3 marks)***

(a) $3e^{i\pi/4}+4i$ (b) $(3-4i)e^{i\pi/4}$
(c) $(4+3i)e^{i\pi/4}$ (d) $(3+4i)e^{i\pi/4}$

26. If $|z|=1$ and $z\neq\pm1$, then all the values of $\dfrac{z}{1-z^2}$ lie on

(a) a line not passing through the origin ***(2007 -3 marks)***
(b) $|z|=\sqrt{2}$
(c) the x-axis
(d) the y-axis

27. A particle P starts from the point $z_0=1+2i$, where $i=\sqrt{-1}$. It moves horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i}+\hat{j}$ and then it moves through an angle $\dfrac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by ***(2008)***

(a) $6+7i$ (b) $-7+6i$
(c) $7+6i$ (d) $-6+7i$

28. Let $z=\cos\theta+i\sin\theta$. Then the value of $\sum\limits_{m=1}^{15}\text{Im}(z^{2m-1})$ at $\theta=2°$ is ***(2009)***

(a) $\dfrac{1}{\sin 2°}$ (b) $\dfrac{1}{3\sin 2°}$ (c) $\dfrac{1}{2\sin 2°}$ (d) $\dfrac{1}{4\sin 2°}$

29. Let $z=x+iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation : $z\bar{z}^3+\bar{z}z^3=350$ is ***(2009)***

(a) 48 (b) 32 (c) 40 (d) 80

30. Let z be a complex number such that the imaginary part of z is non-zero and $a=z^2+z+1$ is real. Then a cannot take the value ***(2012)***

(a) -1 (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{3}{4}$

31. Let complex numbers α and $\dfrac{1}{\alpha}$ lie on circles $(x-x_0)^2+(y-y_0)^2=r^2$ and $(x-x_0)^2+(y-y_0)^2=4r^2$. respectively. If $z_0=x_0+iy_0$ satisfies the equation $2|z_0|^2=r^2+2$, then $|\alpha|=$ ***(JEE Adv. 2013)***

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{\sqrt{7}}$ (d) $\dfrac{1}{3}$

D MCQs with One or More than One Correct

1. If $z_1=a+ib$ and $z_2=c+id$ are complex numbers such that $|z_1|=|z_2|=1$ and $\text{Re}(z_1\bar{z}_2)=0$, then the pair of complex numbers $w_1=a+ic$ and $w_2=b+id$ satisfies – ***(1985 - 2 Marks)***

(a) $|w_1|=1$ (b) $|w_2|=1$
(c) $\text{Re}(w_1\overline{w}_2)=0$ (d) none of these

2. Let z_1 and z_2 be complex numbers such that $z_1\neq z_2$ and $|z_1|=|z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\dfrac{z_1+z_2}{z_1-z_2}$ may be ***(1986 - 2 Marks)***

(a) zero (b) real and positive
(c) real and negative (d) purely imaginary
(e) none of these.

3. If z_1 and z_2 are two nonzero complex numbers such that $|z_1+z_2|=|z_1|+|z_2|$, then Arg z_1 – Arg z_2 is equal to ***(1987 - 2 Marks)***

(a) $-\pi$ (b) $-\dfrac{\pi}{2}$ (c) 0 (d) $\dfrac{\pi}{2}$
(e) π

4. The value of $\sum\limits_{k=1}^{6}\left(\sin\dfrac{2\pi k}{7}-i\cos\dfrac{2\pi k}{7}\right)$ is ***(1987 - 2 Marks)***

(a) -1 (b) 0 (c) $-i$ (d) i
(e) None

5. If ω is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ equals ***(1998 - 2 Marks)***

(a) 128ω (b) -128ω (c) $128\omega^2$ (d) $-128\omega^2$

6. The value of the sum $\sum\limits_{n=1}^{13}(i^n+i^{n+1})$, where $i=\sqrt{-1}$, equals ***(1998 - 2 Marks)***

(a) i (b) $i-1$ (c) $-i$ (d) 0

7. If $\begin{vmatrix}6i & -3i & 1\\ 4 & 3i & -1\\ 20 & 3 & i\end{vmatrix}=x+iy$, then ***(1998 - 2 Marks)***

(a) $x=3, y=2$ (b) $x=1, y=3$
(c) $x=0, y=3$ (d) $x=0, y=0$

8. Let z_1 and z_2 be two distinct complex numbers and let $z=(1-t)z_1+tz_2$ for some real number t with $0<t<1$. If Arg (w) denotes the principal argument of a non-zero complex number w, then ***(2010)***

(a) $|z-z_1|+|z-z_2|=|z_1-z_2|$
(b) $\text{Arg}(z-z_1)=\text{Arg}(z-z_2)$
(c) $\begin{vmatrix}z-z_1 & \bar{z}-\bar{z}_1\\ z_2-z_1 & \bar{z}_2-\bar{z}_1\end{vmatrix}$
(d) $\text{Arg}(z-z_1)=\text{Arg}(z_2-z_1)$

9. Let $w = \frac{\sqrt{3}+i}{2}$ and P = $\{w^n : n = 1, 2, 3, ...\}$. Further $H_1 = \left\{z \in \mathbb{C} : \text{Re}\, z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in \mathbb{C} : \text{Re}\, z < \frac{-1}{2}\right\}$, where c is the set of all complex numbers. If $z_1 \in P \cap H_1, z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 =$ *(JEE Adv. 2013)*

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

10. Let a, b $\in \mathbb{R}$ and $a^2 + b^2 \neq 0$.

Suppose $S = \left\{Z \in C : Z = \frac{1}{a+ibt}, + \in \mathbb{R}, t \neq 0\right\}$, where $i = \sqrt{-1}$. If z = x+ iy and z $\in$ S, then (x, y) lies on *(JEE Adv. 2016)*

(a) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for a > 0, b $\neq$ 0

(b) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for a < 0, b $\neq$ 0

(c) the x-axis for a $\neq$ 0, b = 0

(d) the y-axis for a = 0, b $\neq$ 0

E Subjective Problems

1. Express $\frac{1}{1-\cos\theta + 2i\sin\theta}$ in the form $x + iy$. *(1978)*

2. If $x = a + b, y = a\gamma + b\beta$ and $z = a\beta + b\gamma$ where γ and β are the complex cube roots of unity, show that $xyz = a^3 + b^3$. *(1978)*

3. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$. *(1979)*

4. Find the real values of x and y for which the following equation is satisfied $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$ *(1980)*

5. Let the complex number z_1, z_2 and z_3 be the vertices of an equilateral triangle. Let z_0 be the circumcentre of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$. *(1981 - 4 Marks)*

6. Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if $z_1^2 + z_2^2 - z_1 z_2 = 0$. *(1983 - 3 Marks)*

7. If $1, a_1, a_2 ......, a_{n-1}$ are the n roots of unity, then show that $(1-a_1)(1-a_2)(1-a_3) ....(1-a_{n-1}) = n$ *(1984 - 2 Marks)*

8. Show that the area of the triangle on the Argand diagram formed by the complex numbers z, iz and $z + iz$ is $\frac{1}{2}|z|^2$. *(1986 - 2½ Marks)*

9. Let $Z_1 = 10 + 6i$ and $Z_2 = 4 + 6i$. If Z is any complex number such that the argument of $\frac{(Z-Z_1)}{(Z-Z_2)}$ is $\frac{\pi}{4}$, then prove that $|Z - 7 - 9i| = 3\sqrt{2}$. *(1990 - 4 Marks)*

10. If $iz^3 + z^2 - z + i = 0$, then show that $|z| = 1$. *(1995 - 5 Marks)*

11. If $|Z| \leq 1, |W| \leq 1$, show that

$$|Z - W|^2 \leq (|Z| - |W|)^2 + (Arg\, Z - Arg\, W)^2$$

(1995 - 5 Marks)

12. Find all non-zero complex numbers Z satisfying $\overline{Z} = iZ^2$. *(1996 - 2 Marks)*

13. Let z_1 and z_2 be roots of the equation $z^2+pz+q = 0$, where the coefficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane. If $\angle AOB = \alpha \neq 0$ and $OA = OB$, where O is the origin, prove that

$p^2 = 4q \cos^2\left(\frac{\alpha}{2}\right)$. *(1997 - 5 Marks)*

14. For complex numbers z and w, prove that $|z|^2 w - |w|^2 z = z - w$ if and only if $z = w$ or $z\overline{w} = 1$. *(1999 - 10 Marks)*

15. Let a complex number α, $\alpha \neq 1$, be a root of the equation $z^{p+q} - z^p - z^q + 1 = 0$, where p, q are distinct primes. Show that either $1 + \alpha + \alpha^2 + ... + \alpha^{p-1} = 0$ or $1 + \alpha + \alpha^2 + ... + \alpha^{q-1} = 0$, but not both together. *(2002 - 5 Marks)*

16. If z_1 and z_2 are two complex numbers such taht $|z_1| < 1 < |z_2|$ then prove that $\left|\frac{1 - z_1\overline{z}_2}{z_1 - z_2}\right| < 1$. *(2003 - 2 Marks)*

17. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$ where $|a_r| < 2$. *(2003 - 2 Marks)*

18. Find the centre and radius of circle given by

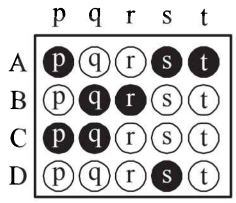
$$\left|\frac{z-\alpha}{z-\beta}\right| = k, k \neq 1$$

where, $z = x + iy$, $\alpha = \alpha_1 + i\alpha_2$, $\beta = \beta_1 + i\beta_2$ *(2004 - 2 Marks)*

19. If one the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}\, i$. Find the other vertices of the square. *(2005 - 4 Marks)*

F Match the Following

DIRECTIONS (Q. 1 and 2) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. $z \neq 0$ is a complex number **(1992 - 2 Marks)**

Column I	**Column II**
(A) $\operatorname{Re} z = 0$	(p) $\operatorname{Re} z^2 = 0$
(B) $\operatorname{Arg} z = \frac{\pi}{4}$	(q) $\operatorname{Im} z^2 = 0$
	(r) $\operatorname{Re} z^2 = \operatorname{Im} z^2$

2. Match the statements in **Column I** with those in **Column II**. ***(2010)***
[Note : Here z takes values in the complex plane and Im z and Re z denote , respectively, the imaginary part and the real part of z.]

Column I	**Column II**
(A) The set of points z satisfying $\lvert z - i\lvert z\rvert\rvert = \lvert z + i\lvert z\rvert\rvert$ is contained in or equal to	(p) an ellipse with eccentricity $\frac{4}{5}$
	(q) the set of points z satisfying $\operatorname{Im} z = 0$
(B) The set of points z satisfying $\lvert z+4\rvert + \lvert z-4\rvert = 10$ is contained in or equal to	(r) the set of points z satisfying $\lvert\operatorname{Im} z\rvert \leq 1$
(C) If $\lvert w\rvert = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to	(s) the set of points z satisfying $\lvert\operatorname{Re} z\rvert < 2$
(D) If $\lvert w\rvert = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to.	(t) the set of points z satisfying $\lvert z\rvert \leq 3$

DIRECTIONS (Q. 3) : *Following question has matching lists. The codes for the list have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

3. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, ..., 9.$ ***(JEE Adv. 2014)***

List-I	**List-II**
P. For each z_k there exists as z_j such that $z_k . z_j = 1$	**1.** True
Q. There exists a $k \in \{1, 2, ..., 9\}$ such that $z_1 . z = z_k$ has no solution z in the set of complex numbers	**2.** False
R. $\frac{\lvert 1-z_1\rvert\lvert 1-z_2\rvert....\lvert 1-z_9\rvert}{10}$ equals	**3.** 1
S. $1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals	**4.** 2

	P	Q	R	S
(a)	1	2	4	3
(b)	2	1	3	4
(c)	1	2	3	4
(d)	2	1	4	3

G Comprehension Based Questions

PASSAGE-1

Let A, B, C be three sets of complex numbers as defined below

$A = \{z : \text{Im } z \geq 1\}$

$B = \{z : |z - 2 - i| = 3\}$

$C = \{z : \text{Re}((1-i)z) = \sqrt{2}\}$

1. The number of elements in the set $A \cap B \cap C$ is *(2008)*
   (a) 0 (b) 1 (c) 2 (d) ∞
2. Let z be any point in $A \cap B \cap C$.
   Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between *(2008)*
   (a) 25 and 29 (b) 30 and 34
   (c) 35 and 39 (d) 40 and 44
3. Let z be any point $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between
   (a) –6 and 3 (b) –3 and 6 *(2008)*
   (c) –6 and 6 (d) –3 and 9

PASSAGE-2

Let $S = S_1 \cap S_2 \cap S_3$, where

$S_1 = \{z \in \mathbb{C} : |z| < 4\}$, $S_2 = \left\{z \in \mathbb{C} : \text{Im}\left[\dfrac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$

and $S_3 = \{z \in \mathbb{C} : \text{Re}\, z > 0\}$.

4. Area of $S =$ *(JEE Adv. 2013)*
   (a) $\dfrac{10\pi}{3}$ (b) $\dfrac{20\pi}{3}$ (c) $\dfrac{16\pi}{3}$ (d) $\dfrac{32\pi}{3}$
5. $\min\limits_{z \in S} |1 - 3i - z| =$ *(JEE Adv. 2013)*
   (a) $\dfrac{2-\sqrt{3}}{2}$ (b) $\dfrac{2+\sqrt{3}}{2}$
   (c) $\dfrac{3-\sqrt{3}}{2}$ (d) $\dfrac{3+\sqrt{3}}{2}$

I Integer Value Correct Type

1. If z is any complex number satisfying $|z - 3 - 2i| \leq 2$, then the minimum value of $|2z - 6 + 5i|$ is *(2011)*
2. Let $\omega = e^{\frac{i\pi}{3}}$, and a, b, c, x, y, z be non-zero complex numbers such that *(2011)*
   $a + b + c = x$
   $a + b\omega + c\omega^2 = y$
   $a + b\omega^2 + c\omega = z$
   Then the value of $\dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is
3. For any integer k, let $\alpha_k = \cos\left(\dfrac{k\pi}{7}\right) + i \sin\left(\dfrac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression $\dfrac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|}$ is *(JEE Adv. 2015)*

Section-B JEE Main / AIEEE

1. z and w are two nonzero complex numbers such that $|z| = |w|$ and Arg z + Arg w = π then z equals **[2002]**
   (a) $\overline{\omega}$ (b) $-\overline{\omega}$ (c) ω (d) $-\omega$
2. If $|z - 4| < |z - 2|$, its solution is given by **[2002]**
   (a) Re(z) > 0 (b) Re(z) < 0
   (c) Re(z) > 3 (d) Re(z) > 2
3. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be **[2002]**
   (a) an ellipse (b) a hyperbola
   (c) a circle (d) none of these
4. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$ and $Arg(z) - Arg(\omega) = \dfrac{\pi}{2}$, then $\bar{z}\omega$ is equal to **[2003]**
   (a) $-i$ (b) 1 (c) -1 (d) i
5. Let Z_1 and Z_2 be two roots of the equation $Z^2 + aZ + b = 0$, Z being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then **[2003]**
   (a) $a^2 = 4b$ (b) $a^2 = b$ (c) $a^2 = 2b$ (d) $a^2 = 3b$
6. If $\left(\dfrac{1+i}{1-i}\right)^x = 1$ then **[2003]**
   (a) $x = 2n + 1$, where n is any positive integer
   (b) $x = 4n$, where n is any positive integer
   (c) $x = 2n$, where n is any positive integer
   (d) $x = 4n + 1$, where n is any positive integer.
7. Let z and w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg $zw = \pi$. Then arg z equals **[2004]**
   (a) $\dfrac{5\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{3\pi}{4}$ (d) $\dfrac{\pi}{4}$

8. If $z = x - i\,y$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2)$ is equal to **[2004]**

(a) -2 (b) -1 (c) 2 (d) 1

9. If $|z^2 - 1| = |z|^2 + 1$, then z lies on **[2004]**

(a) an ellipse (b) the imaginary axis
(c) a circle (d) the real axis

10. If the cube roots of unity are $1, \omega, \omega^2$ then the roots of the equation $(x-1)^3 + 8 = 0$, are **[2005]**

(a) $-1, -1 + 2\omega, -1 - 2\omega^2$
(b) $-1, -1, -1$
(c) $-1, 1 - 2\omega, 1 - 2\omega^2$
(d) $-1, 1 + 2\omega, 1 + 2\omega^2$

11. If z_1 and z_2 are two non- zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg z_1 $-$ arg z_2 is equal to **[2005]**

(a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 0 (d) $\frac{-\pi}{2}$

12. If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on **[2005]**

(a) an ellipse (b) a circle
(c) a straight line (d) a parabola

13. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is **[2006]**

(a) i (b) 1 (c) -1 (d) $-i$

14. If $z^2 + z + 1 = 0$, where z is complex number, then the value of

$$\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \ldots\ldots\ldots + \left(z^6+\frac{1}{z^6}\right)^2$$ is

[2006]

(a) 18 (b) 54
(c) 6 (d) 12

15. If $|z + 4| \le 3$, then the maximum value of $|z+1|$ is **[2007]**

(a) 6 (b) 0 (c) 4 (d) 10

16. The conjugate of a complex number is $\frac{1}{i-1}$ then that complex number is **[2008]**

(a) $\frac{-1}{i-1}$ (b) $\frac{1}{i+1}$ (c) $\frac{-1}{i+1}$ (d) $\frac{1}{i-1}$

17. Let R be the real line. Consider the following subsets of the plane $R \times R$:
$S = \{(x, y): y = x + 1 \text{ and } 0 < x < 2\}$
$T = \{(x, y): x - y \text{ is an integer}\}$,
Which one of the following is true? **[2008]**

(a) Neither S nor T is an equivalence relation on R
(b) Both S and T are equivalence relation on R
(c) S is an equivalence relation on R but T is not
(d) T is an equivalence relation on R but S is not

18. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals **[2010]**

(a) 1 (b) 2 (c) ∞ (d) 0

19. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that : **[2011]**

(a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$
(c) $\beta \in (1, \infty)$ (d) $\beta \in (0, 1)$

20. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals **[2011]**

(a) $(1, 1)$ (b) $(1, 0)$
(c) $(-1, 1)$ (d) $(0, 1)$

21. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies : **[2012]**

(a) either on the real axis or on a circle passing through the origin.
(b) on a circle with centre at the origin
(c) either on the real axis or on a circle not passing through the origin.
(d) on the imaginary axis.

22. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals: **[JEE M 2013]**

(a) $-\theta$ (b) $\frac{\pi}{2} - \theta$ (c) θ (d) $\pi - \theta$

23. If z is a complex number such that $|z| \ge 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$: **[JEE M 2014]**

(a) is strictly greater than $\frac{5}{2}$
(b) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$
(c) is equal to $\frac{5}{2}$
(d) lie in the interval $(1, 2)$

24. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\overline{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a: **[JEE M 2015]**

(a) circle of radius 2.

(b) circle of radius $\sqrt{2}$.

(c) straight line parallel to x-axis

(d) straight line parallel to y-axis.

25. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is: **[JEE M 2016]**

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$