CHAPTER
4
Permutations and Combinations

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. In a certain test, a_i students gave wrong answers to atleast i questions, where $i = 1, 2, \ldots, k$. No student gave more than k wrong answers. The total number of wrong answers given is .... *(1982 - 2 Marks)*

2. The side AB, BC and CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices is .... *(1984 - 2 Marks)*

3. Total number of ways in which six '+' and four '−' signs can be arranged in a line such that no two '−' signs occur together is .... *(1988 - 2 Marks)*

4. There are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is _____ .

B True / False

1. The product of any r consecutive natural numbers is always divisible by $r!$. *(1985 - 1 Mark)*

C MCQs with One Correct Answer

1. $^nC_{r-1} = 36$, $^nC_r = 84$ and $^nC_{r+1} = 126$, then r is : *(1979)*
   (a) 1
   (b) 2
   (c) 3
   (d) None of these.

2. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated are *(1982 - 2 Marks)*
   (a) 69760
   (b) 30240
   (c) 99748
   (d) none of these

3. The value of the expression $^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to *(1982 - 2 Marks)*
   (a) $^{47}C_5$
   (b) $^{52}C_5$
   (c) $^{52}C_4$
   (d) none of these

4. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4 ; and then the men select the chairs from amongst the remaining. The number of possible arrangements is *(1982 - 2 Marks)*

   (a) $^6C_3 \times {}^4C_2$
   (b) $^4P_2 \times {}^4P_3$
   (c) $^4C_2 + {}^4P_3$
   (d) none of these

5. A five-digit numbers divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done is *(1989 - 2 Marks)*
   (a) 216
   (b) 240
   (c) 600
   (d) 3125

6. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ? *(2000S)*
   (a) 16
   (b) 36
   (c) 60
   (d) 180

7. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals *(2001S)*
   (a) 5
   (b) 7
   (c) 6
   (d) 4

8. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is *(2002S)*
   (a) 40
   (b) 60
   (c) 80
   (d) 100

9. A rectangle with sides of length $(2m - 1)$ and $(2n - 1)$ units is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is *(2005S)*

   (a) $(m + n - 1)^2$
   (b) 4^{m+n-1}
   (c) $m^2 n^2$
   (d) $m(m + 1)n(n + 1)$
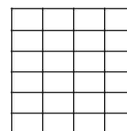
10. If the LCM of p, q is $r^2 t^4 s^2$, where r, s, t are prime numbers and p, q are the positive integers then the number of ordered pair (p, q) is *(2006 - 3M, −1)*
    (a) 252
    (b) 254
    (c) 225
    (d) 224

11. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is *(2007 -3 marks)*
    (a) 360
    (b) 192
    (c) 96
    (d) 48

12. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is *(2009)*
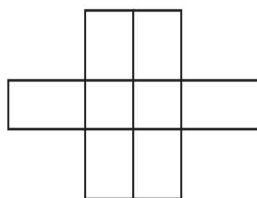   (a) 55
    (b) 66
    (c) 77
    (d) 88

13. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is *(2012)*
    (a) 75 (b) 150 (c) 210 (d) 243

14. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is *(JEE Adv. 2014)*
    (a) 264 (b) 265 (c) 53 (d) 67

15. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 memoers) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is *(JEE Adv. 2016)*
    (a) 380 (b) 320 (c) 260 (d) 95

D MCQs with One or More than One Correct

1. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n- digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is *(1998 - 2 Marks)*
    (a) 6 (b) 7 (c) 8 (d) 9

E Subjective Problems

1. Six X's have to be placed in the squares of figure below in such a way that each row contains at least one X. In how many different ways can this be done. *(1978)*

F Match the Following

2. Five balls of different colours are to be placed in there boxes of different size. Each box can hold all five. In how many different ways can we place the balls so that no box remains empty ? *(1981 - 4 Marks)*

3. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then show that the number of ways in which they can be seated is $\dfrac{m!(m+1)!}{(m-n+1)!}$ *(1983 - 2 Marks)*

4. 7 relatives of a man comprises 4 ladies and 3 gentlemen; his wife has also 7 relatives; 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of the wife's relatives? *(1985 - 5 Marks)*

5. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw? *(1986 - 2½ Marks)*

6. Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side. Determine the number of ways in which the sitting arrangements can be made. *(1991 - 4 Marks)*

7. A committee of 12 is to be formed from 9 women and 8 men. In how many ways this can be done if at least five women have to be included in a committee? In how many of these committees *(1994 - 4 Marks)*
    (a) The women are in majority?
    (b) The men are in majority?

8. Prove by permutation or otherwise $\dfrac{(n^2)!}{(n!)^n}$ is an integer $(n \in I^+)$. *(2004 - 2 Marks)*

9. If total number of runs scored in n matches is $\left(\dfrac{n+1}{4}\right)$ $(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the kth match are given by k. 2^{n+1-k}, where $1 \le k \le n$. Find n. *(2005 - 2 Marks)*

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

1. Consider all possible permutations of the letters of the word ENDEANOEL. Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS. *(2008)*

Column I	Column II
(A) The number of permutations containing the word ENDEA is	(p) 5!

(B) The number of permutations in which the letter E occurs in the first and the last positions is

(q) $2 \times 5!$

(C) The number of permutations in which none of the letters D, L, N occurs in the last five positions is

(r) $7 \times 5!$

(D) The number of permutations in which the letters A, E, O occur only in odd positions is

(s) $21 \times 5!$

G | Comprehension Based Questions

PASSAGE - 1

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n–digit integers ending with digit 0. **(2012)**

1. The value of b_6 is
   - (a) 7
   - (b) 8
   - (c) 9
   - (d) 11

2. Which of the following is correct?
   - (a) $a_{17} = a_{16} + a_{15}$
   - (b) $c_{17} \neq c_{16} + c_{15}$
   - (c) $b_{17} \neq b_{16} + c_{16}$
   - (d) $a_{17} = c_{17} + b_{16}$

I | Integer Value Correct Type

1. Consider the set of eight vectors
   $V = \left\{ a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\} \right\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is
   (JEE Adv. 2013)

2. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is **(JEE Adv. 2014)**

3. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is **(JEE Adv. 2014)**

4. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\dfrac{m}{n}$ is **(JEE Adv. 2015)**

Section-B | JEE Main / AIEEE

1. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are **[2002]**
   - (a) 216
   - (b) 375
   - (c) 400
   - (d) 720

2. Number greater than 1000 but less than 4000 is formed using the digits 0, 1, 2, 3, 4 (repetition allowed). Their number is **[2002]**
   - (a) 125
   - (b) 105
   - (c) 375
   - (d) 625

3. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4 and 5 without repetition. Total number of such numbers are **[2002]**
   - (a) 312
   - (b) 3125
   - (c) 120
   - (d) 216

4. The sum of integers from 1 to 100 that are divisible by 2 or 5 is **[2002]**
   - (a) 3000
   - (b) 3050
   - (c) 3600
   - (d) 3250

5. If ^nC_r denotes the number of combination of n things taken r at a time, then the expression $^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals **[2003]**
   - (a) $^{n+1}C_{r+1}$
   - (b) $^{n+2}C_r$
   - (c) $^{n+2}C_{r+1}$
   - (d) $^{n+1}C_r$.

6. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is **[2003]**
   - (a) 346
   - (b) 140
   - (c) 196
   - (d) 280

7. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by **[2003]**
   - (a) $7! \times 5!$
   - (b) $6! \times 5!$
   - (c) $30!$
   - (d) $5! \times 4!$

8. How many ways are there to arrange the letters in the word GARDEN with vowels in alphabetical order **[2004]**
   - (a) 480
   - (b) 240
   - (c) 360
   - (d) 120

9. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is **[2004]**
   - (a) $^8C_3$
   - (b) 21
   - (c) 3^8
   - (d) 5

10. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number **[2005]**
   - (a) 601
   - (b) 600
   - (c) 603
   - (d) 602

11. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are of be selected, if a voter votes for at least one candidate, then the number of ways in which he can vote is **[2006]**
    (a) 5040   (b) 6210   (c) 385   (d) 1110

12. The set $S = \{1, 2, 3, ..., 12\}$ is to be partitioned into three sets A, B, C of equal size. Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is **[2007]**

    (a) $\dfrac{12!}{(4!)^3}$ (b) $\dfrac{12!}{(4!)^4}$ (c) $\dfrac{12!}{3!(4!)^3}$ (d) $\dfrac{12!}{3!(4!)^4}$

13. In a shop there are five types of ice-creams available. A child buys six ice-creams.
    Statement-1 : The number of different ways the child can buy the six ice-creams is $^{10}C_5$.
    Statement -2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row. **[2008]**
    (a) Statement -1 is false, Statement-2 is true
    (b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
    (c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
    (d) Statement -1 is true, Statement-2 is false

14. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
    (a) $8. \, ^6C_4. \, ^7C_4$ (b) $6.7. \, ^8C_4$
    (c) $6. 8. \, ^7C_4.$ (d) $7. \, ^6C_4. \, ^8C_4$

15. From 6 different novels and 3 different dictionaries,4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangement is: **[2009]**
    (a) at least 500 but less than 750
    (b) at least 750 but less than 1000
    (c) at least 1000
    (d) less than 500

16. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is **[2010]**
    (a) 36   (b) 66   (c) 108   (d) 3

17. **Statement-1:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $^9C_3$.
    Statement-2: The number of ways of choosing any 3 places from 9 different places is $^9C_3$. **[2011]**
    (a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
    (b) Statement-1 is true, Statement-2 is false.
    (c) Statement-1 is false, Statement-2 is true.
    (d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

18. These are 10 points in a plane, out of these 6 are collinear, if N is the number of triangles formed by joining these points. then: **[2012]**
    (a) $n \le 100$ (b) $100 < n \le 140$
    (c) $140 < n \le 190$ (d) $n > 190$

19. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is : **[2012]**
    (a) 880   (b) 629   (c) 630   (d) 879

20. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is : **[JEE M 2013]**
    (a) 7   (b) 5   (c) 10   (d) 8

21. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is : **[JEE M 2015]**

    (a) 120   (b) 72   (c) 216   (d) 192

22. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is : **[JEE M 2016]**
    (a) 52nd (b) 58th
    (c) 46th (d) 59th