CHAPTER 15 Matrices and Determinants

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Let $p\lambda^4+q\lambda^3+r\lambda^2+s\lambda+t=\begin{vmatrix}\lambda^2+3\lambda & \lambda-1 & \lambda+3\\ \lambda+1 & -2\lambda & \lambda-4\\ \lambda-3 & \lambda+4 & 3\lambda\end{vmatrix}$

be an identity in λ, where p, q, r, s and t are constants. Then, the value of t is .... *(1981 - 2 Marks)*

2. The solution set of the equation $\begin{vmatrix}1 & 4 & 20\\ 1 & -2 & 5\\ 1 & 2x & 5x^2\end{vmatrix}=0$ is .... *(1981 - 2 Marks)*

3. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of determinant chosen is positive is .... *(1982 - 2 Marks)*

4. Given that $x=-9$ is a root of $\begin{vmatrix}x & 3 & 7\\ 2 & x & 2\\ 7 & 6 & x\end{vmatrix}=0$ the other two roots are .... and .... *(1983 - 2 Marks)*

5. The system of equations

$\lambda x+y+z=0$

$-x+\lambda y+z=0$

$-x-y+\lambda z=0$

Will have a non-zero solution if real values of λ are given by .... *(1984 - 2 Marks)*

6. The value of the determinant $\begin{vmatrix}1 & a & a^2-bc\\ 1 & b & b^2-ca\\ 1 & c & c^2-ab\end{vmatrix}$ is .... *(1988 - 2 Marks)*

7. For positive numbers x, y and z, the numerical value of the determinant $\begin{vmatrix}1 & \log_x y & \log_x z\\ \log_y x & 1 & \log_y z\\ \log_z x & \log_z y & 1\end{vmatrix}$ is .... *(1993 - 2 Marks)*

B True/ False

1. The determinants *(1983 - 1 Mark)*

$\begin{vmatrix}1 & a & bc\\ 1 & b & ca\\ 1 & c & ab\end{vmatrix}$ and $\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$ are not identically equal.

2. If $\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\begin{vmatrix}a_1 & b_1 & 1\\ a_2 & b_2 & 1\\ a_3 & b_3 & 1\end{vmatrix}$

then the two triangles with vertices (x_1,y_1), (x_2,y_2), (x_3,y_3), and (a_1,b_1), (a_2,b_2), (a_3,b_3) must be congruent. *(1985 - 1 Mark)*

C MCQs with One Correct Answer

1. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of A consisting of all determinants with value -1. Then
   (a) C is empty *(1981 - 2 Marks)*
   (b) B has as many elements as C
   (c) $A=B\cup C$
   (d) B has twice as many elements as elements as C

2. If $\omega(\neq 1)$ is a cube root of unity, then

$$\begin{vmatrix}1 & 1+i+\omega^2 & \omega^2\\ 1-i & -1 & \omega^2-1\\ -i & -i+\omega-1 & -1\end{vmatrix}=$$ *(1995S)*

   (a) 0 (b) 1 (c) i (d) ω

3. Let a, b, c be the real numbers. Then following system of equations in x, y and z *(1995S)*

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1,\ \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,$$

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \text{ has}$$

   (a) no solution (b) unique solution
   (c) infinitely many solutions (d) finitely many solutions

4. If A and B are square matrices of equal degree, then which one is correct among the followings? *(1995S)*
(a) $A+B=B+A$ (b) $A+B=A-B$
(c) $A-B=B-A$ (d) $AB=BA$

5. The parameter, on which the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon is *(1997 - 2 Marks)*
(a) a (b) p (c) d (d) x

6. If $f(x)=\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(100)$ is equal to *(1999 - 2 Marks)*
(a) 0 (b) 1 (c) 100 (d) –100

7. If the system of equations $x-ky-z=0$, $kx-y-z=0$, $x+y-z=0$ has a non-zero solution, then the possible values of k are *(2000S)*
(a) $-1, 2$ (b) $1, 2$ (c) $0, 1$ (d) $-1, 1$

8. Let $\omega=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is *(2002S)*
(a) 3ω (b) $3\omega(\omega-1)$ (c) $3\omega^2$ (d) $3\omega(1-\omega)$

9. The number of values of k for which the system of equations $(k+1)x+8y=4k$; $kx+(k+3)y=3k-1$ has infinitely many solutions is *(2002S)*
(a) 0 (b) 1 (c) 2 (d) infinite

10. If $A=\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2=B$, is *(2003S)*
(a) 1 (b) –1
(c) 4 (d) no real values

11. If the system of equations $x+ay=0$, $az+y=0$ and $ax+z=0$ has infinite solutions, then the value of a is *(2003S)*
(a) –1 (b) 1
(c) 0 (d) no real values

12. Given $2x-y+2z=2$, $x-2y+z=-4$, $x+y+\lambda z=4$ then the value of λ such that the given system of equation has NO solution, is *(2004S)*
(a) 3 (b) 1 (c) 0 (d) -3

13. If $A=\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3|=125$ then the value of α is *(2004S)*
(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 5

14. $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1}=\left[\frac{1}{6}(A^2+cA+dI)\right]$, then the value of c and d are *(2005S)*
(a) $(-6,-11)$ (b) $(6,11)$ (c) $(-6,11)$ (d) $(6,-11)$

15. If $P=\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ and $A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q=PAP^T$ and $x=P^TQ^{2005}P$ then x is equal to *(2005S)*
(a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$
(c) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$
(d) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

16. Consider three points $P=(-\sin(\beta-\alpha),-\cos\beta)$, $Q=(\cos(\beta-\alpha),\sin\beta)$ and $R=(\cos(\beta-\alpha+\theta),\sin(\beta-\theta))$, where $0<\alpha,\beta,\theta<\frac{\pi}{4}$. Then, *(2008)*
(a) P lies on the line segment RQ
(b) Q lies on the line segment PR
(c) R lies on the line segment QP
(d) P, Q, R are non-collinear

17. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is *(2010)*
(a) 0 (b) 2^9-1 (c) 168 (d) 2

18. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$ where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is *(2011)*

(a) 2 (b) 6 (c) 4 (d) 8

19. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is *(2012)*

(a) 2^{10} (b) 2^{11} (c) 2^{12} (d) 2^{13}

20. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that *(2012)*

(a) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (b) $PX = X$

(c) $PX = 2X$ (d) $PX = -X$

21. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\dfrac{q_{31} + q_{32}}{q_{21}}$ equals *(JEE Adv. 2016)*

(a) 52 (b) 103

(c) 201 (d) 205

D MCQs with One or More than One Correct

1. The determinant $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$ is equal to zero, if *(1986 - 2 Marks)*

(a) a, b, c are in A. P.

(b) a, b, c are in G. P.

(c) a, b, c are in H. P.

(d) α is a root of the equation $ax^2 + bx + c = 0$

(e) $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$.

2. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then *(1998 - 2 Marks)*

(a) $x = 3, y = 1$ (b) $x = 1, y = 3$

(c) $x = 0, y = 3$ (d) $x = 0, y = 0$

3. Let M and N be two 3×3 non-singular skew- symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2 (M^TN)^{-1} (MN^{-1})^T$ is equal to *(2011)*

(a) M^2 (b) $-N^2$ (c) $-M^2$ (d) MN

4. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are) *(2012)*

(a) -2 (b) -1 (c) 1 (d) 2

5. For 3×3 matrices M and N, which of the following statement(s) is (are) **NOT** correct? *(JEE Adv. 2013)*

(a) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(b) MN – NM is skew symmetric for all symmetric matrices M and N

(c) MN is symmetric for all symmetric matrices M and N

(d) (adj M) (adj N) = adj (MN) for all invertible matrices M and N

6. Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $p^2 \neq 0$, when n = *(JEE Adv. 2013)*

(a) 57 (b) 55 (c) 58 (d) 56

7. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if *(JEE Adv. 2014)*

(a) The first column of M is the transpose of the second row of M

(b) The second row of M is the transpose of the first column of M

(c) M is a diagonal matrix with non-zero entries in the main diagonal

(d) The product of entries in the main diagonal of M is not the square of an integer

8. Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then *(JEE Adv. 2014)*

(a) determinant of $(M^2 + MN^2)$ is 0

(b) there is 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(c) determinant of $(M^2 + MN^2) \geq 1$

(d) for a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

9. Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha ?$$

(JEE Adv. 2015)

(a) –4 (b) 9 (c) –9 (d) 4

10. Let X and Y be two arbitrary, 3 × 3, non-zero, skew-symmetric matrices and Z be an arbitrary 3 × 3, non zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric? *(JEE Adv. 2015)*

(a) $Y^3Z^4 - Z^4Y^3$ (b) $X^{44} + Y^{44}$
(c) $X^4Z^3 - Z^3X^4$ (d) $X^{23} + Y^{23}$

11. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = [q_{ij}]$ is a matrix such that PQ = kI, where $k \in \mathbb{R}$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then *(JEE Adv. 2016)*

(a) $a = 0, k = 8$ (b) $4a - k + 8 = 0$
(c) $\det(P\,\text{adj}\,(Q)) = 2^9$ (d) $\det(Q\,\text{adj}\,(P)) = 2^{13}$

12. Let $a, \lambda, \mu, \in \mathbb{R}$. Consider the system of linear equations

$ax + 2y = \lambda$
$3x - 2y = \mu$

Which of the following statement(s) is (are) correct? *(JEE Adv. 2016)*

(a) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ.
(b) If $a \neq -3$, then the system has a unique solution for all values of λ and μ.
(c) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$.
(d) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$.

E Subjective Problems

1. For what value of k do the following system of equations possess a non trivial (i.e., not all zero) solution over the set of rationals Q?

$x + ky + 3z = 0$
$3x + ky - 2z = 0$
$2x + 3y - 4z = 0$

For that value of k, find all the solutions for the system. *(1979)*

2. Let a, b, c be positive and not all equal. Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.

(1981 - 4 Marks)

3. Without expanding a determinant at any stage, show that

$$\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = xA + B,$$ where A and B are determinants of order 3 not involving x. *(1982 - 5 Marks)*

4. Show that

$$\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$$

(1985 - 2 Marks)

5. Consider the system of linear equations in x, y, z:

$(\sin 3\theta)\ x - y + z = 0$
$(\cos 2\theta)\ x + 4y + 3z = 0$
$2x + 7y + 7z = 0$

Find the values of θ for which this system has nontrivial solutions. *(1986 - 5 Marks)*

6. Let $\Delta a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$.

Show that $\sum_{a=1}^{n} \Delta a = c$, a constant. *(1989 - 5 Marks)*

7. Let the three digit numbers A 28, 3B9, and 62 C, where A, B, and C are integers between 0 and 9, be divisible by a fixed integer k. Show that the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is divisible by k. *(1990 - 4 Marks)*

8. If $a \neq p$, $b \neq q$, $c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$. Then find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ *(1991 - 4 Marks)*

9. For a fixed positive integer n, if *(1992 - 4 Marks)*

$$D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$$

then show that $\left[\frac{D}{(n!)^3} - 4\right]$ is divisible by n.

10. Let λ and α be real. Find the set of all values of λ for which the system of linear equations

$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$, $x + (\cos\alpha)y + (\sin\alpha)z = 0$,

$-x + (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution. For $\lambda = 1$, find all values of α.

(1993 - 5 Marks)

11. For all values of A, B, C and P, Q, R show that

(1994 - 4 Marks)

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

12. Let $a > 0,\ d > 0$. Find the value of the determinant

(1996 - 5 Marks)

$$\begin{vmatrix} \dfrac{1}{a} & \dfrac{1}{a(a+d)} & \dfrac{1}{(a+d)(a+2d)} \\ \dfrac{1}{(a+d)} & \dfrac{1}{(a+d)(a+2d)} & \dfrac{1}{(a+2d)(a+3d)} \\ \dfrac{1}{(a+2d)} & \dfrac{1}{(a+2d)(a+3d)} & \dfrac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

13. Prove that for all values of θ,

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \dfrac{2\pi}{3}\right) & \cos\left(\theta + \dfrac{2\pi}{3}\right) & \sin\left(2\theta + \dfrac{4\pi}{3}\right) \\ \sin\left(\theta - \dfrac{2\pi}{3}\right) & \cos\left(\theta - \dfrac{2\pi}{3}\right) & \sin\left(2\theta - \dfrac{4\pi}{3}\right) \end{vmatrix} = 0$$

(2000 - 3 Marks)

14. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$. ***(2003 - 2 Marks)***

15. If M is a 3×3 matrix, where $\det M = 1$ and $MM^T = I$, where 'I' is an identity matrix, prove that $\det(M - I) = 0$.

(2004 - 2 Marks)

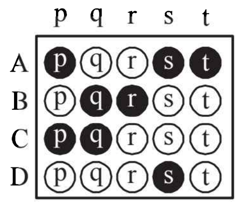
16. If $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $AX = U$ has infinitely many solutions, prove that $BX = V$ has no unique solution. Also show that if $afd \neq 0$, then $BX = V$ has no solution. ***(2004 - 4 Marks)***

F Match the Following

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. Consider the lines given by

$L_1 : x + 3y - 5 = 0;\ L_2 : 3x - ky - 1 = 0;\ L_3 : 5x + 2y - 12 = 0$

Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS. ***(2008)***

Column I	*Column II*
(A) L_1, L_2, L_3 are concurrent, if	(p) $k = -9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q) $k = -\dfrac{6}{5}$
(C) L_1, L_2, L_3 from a triangle, if	(r) $k = \dfrac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(s) $k = 5$

2. Match the Statements/Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS. ***(2008)***

Column I	***Column II***
(A) The minimum value of $\dfrac{x^2+2x+4}{x+2}$ is	(p) 0
(B) Let A and B be 3×3 matrices of real numbers, where A is symmetric, B is skew-symmetric, and $(A+B)(A-B)=(A-B)(A+B)$. If $(AB)^t=(-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q) 1
(C) Let $a=\log_3 \log_3 2$. An integer k satisfying $1<2^{(-k+3^{-a})}<2$, must be less than	(r) 2
(D) If $\sin\theta=\cos\phi$, then the possible values of $\frac{1}{\pi}\left(\theta\pm\phi-\frac{\pi}{2}\right)$ are	(s) 3

G Comprehension Based Questions

PASSAGE - 1

Let $A=\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}$, and U_1, U_2 and U_3 are columns of a 3×3 matrix U. If column matrices U_1, U_2 and U_3 satisfying $AU_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $AU_2=\begin{bmatrix}2\\3\\0\end{bmatrix}$, $AU_3=\begin{bmatrix}2\\3\\1\end{bmatrix}$ evaluate as directed in the following questions.

1. The value $|U|$ is ***(2006 - 5M, –2)***

(a) 3 (b) –3 (c) $\frac{3}{2}$ (d) 2

2. The sum of the elements of the matrix U^{-1} is ***(2006 - 5M, –2)***

(a) –1 (b) 0 (c) 1 (d) 3

3. The value of $[3\ 2\ 0]\,U\begin{bmatrix}3\\2\\0\end{bmatrix}$ is ***(2006 - 5M, –2)***

(a) 5 (b) $\frac{5}{2}$ (c) 4 (d) $\frac{3}{2}$

PASSAGE - 2

Let $\mathscr{A}$ be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

4. The number of matrices in $\mathscr{A}$ is ***(2009)***

(a) 12 (b) 6 (c) 9 (d) 3

5. The number of matrices A in $\mathscr{A}$ for which the system of linear equations

$$A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

has a unique solution, is ***(2009)***

(a) less than 4
(b) at least 4 but less than 7
(c) at least 7 but less than 10
(d) at least 10

6. The number of matrices A in $\mathscr{A}$ for which the system of linear equations

$$A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

is inconsistent, is ***(2009)***

(a) 0 (b) more than 2
(c) 2 (d) 1

PASSAGE - 3

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_P=\left\{A=\begin{bmatrix}a&b\\c&a\end{bmatrix}:a,b,c\in\{0,1,2,\ldots,p-1\}\right\}$$ ***(2010)***

7. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is

(a) $(p-1)^2$ (b) $2(p-1)$
(c) $(p-1)^2+1$ (d) $2p-1$

8. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is
[**Note:** The trace of a matrix is the sum of its diagonal entries.]

(a) $(p-1)(p^2-p+1)$ (b) $p^3-(p-1)^2$
(c) $(p-1)^2$ (d) $(p-1)(p^2-2)$

9. The number of A in T_p such that det (A) is not divisible by p is

(a) $2p^2$ (b) p^3-5p
(c) p^3-3p (d) p^3-p^2

PASSAGE - 4

Let a, b and c be three real numbers satisfying *(2011)*

$$[a\,b\,c]\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix}=[0\,0\,0] \qquad \text{...(E)}$$

10. If the point $P(a, b, c)$, with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is
(a) 0 (b) 12 (c) 7 (d) 6

11. Let ω be a solution of $x^3 - 1 = 0$ with $\text{Im}(\omega) > 0$, if $a = 2$ with b and c satisfying (E), then the value of $\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}$ is equal to
(a) -2 (b) 2 (c) 3 (d) -3

12. Let $b = 6$, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$ is
(a) 6 (b) 7 (c) $\frac{6}{7}$ (d) ∞

H Assertion & Reason Type Questions

1. Consider the system of equations
$x - 2y + 3z = -1$
$-x + y - 2z = k$
$x - 3y + 4z = 1$

STATEMENT - 1 : The system of equations has no solution for $k \neq 3$ and

STATEMENT-2 : The determinant $\begin{vmatrix}1 & 3 & -1\\ -1 & -2 & k\\ 1 & 4 & 1\end{vmatrix} \neq 0$, for $k \neq 3$. *(2008)*

(a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
(b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explaination for STATEMENT - 1
(c) STATEMENT - 1 is True, STATEMENT - 2 is False
(d) STATEMENT - 1 is False, STATEMENT - 2 is True

I Integer Value Correct Type

1. Let ω be the complex number $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying $\begin{vmatrix}z+1 & \omega & \omega^2\\ \omega & z+\omega^2 & 1\\ \omega^2 & 1 & z+\omega\end{vmatrix}=0$ is equal to *(2010)*

2. Let k be a positive real number and let

$$A=\begin{bmatrix}2k-1 & 2\sqrt{k} & 2\sqrt{k}\\ 2\sqrt{k} & 1 & -2k\\ -2\sqrt{k} & 2k & -1\end{bmatrix} \text{ and } B=\begin{bmatrix}0 & 2k-1 & \sqrt{k}\\ 1-2k & 0 & 2\sqrt{k}\\ -\sqrt{k} & -2\sqrt{k} & 0\end{bmatrix}$$

If det (adj A) + det (adj B) = 10^6. then [k] is equal to
[**Note** : adj M denotes the adjoint of square matrix M and $[k]$ denotes the largest integer less than or equal k. *(2010)*

3. Let M be a 3 × 3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix},\ M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}, \text{ and } M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}.$$

Then the sum of the diagonal entries of M is *(2011)*

4. The total number of distinct $x \in \mathbb{R}$ for which

$$\begin{vmatrix}x & x^2 & 1+x^3\\ 2x & 4x^2 & 1+8x^3\\ 3x & 9x^2 & 1+27x^3\end{vmatrix}=10 \text{ is}$$ *(JEE Adv. 2016)*

5. Let $z=\frac{-1+\sqrt{3}i}{2}$, where $i=\sqrt{-1}$, and r, s $\in$ {1, 2, 3}. Let $P=\begin{bmatrix}(-z)^r & z^{2s}\\ z^{2s} & z^r\end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is *(JEE Adv. 2016)*

Section-B JEE Main / AIEEE

1. If $a > 0$ and discriminant of $ax^2+2bx+c$ is –ve, then

$$\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix} \text{ is equal to}$$ **[2002]**

(a) +ve (b) $(ac-b^2)(ax^2+2bx+c)$
(c) –ve (d) 0

2. If the system of linear equations **[2003]**
$x+2ay+az=0$; $x+3by+bz=0$; $x+4cy+cz=0$;
has a non - zero solution, then a, b, c.
(a) satisfy $a+2b+3c=0$ (b) are in A.P
(c) are in G.P (d) are in H.P.

3. If $1, \omega, \omega^2$ are the cube roots of unity, then

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix} \text{ is equal to}$$ **[2003[**

(a) ω^2 (b) 0 (c) 1 (d) ω

4. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then **[2003[**

(a) $\alpha = 2ab, \beta = a^2+b^2$
(b) $\alpha = a^2+b^2,\ \beta = ab$
(c) $\alpha = a^2+b^2,\ \beta = 2ab$
(d) $\alpha = a^2+b^2,\ \beta = a^2-b^2$.

5. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is **[2004]**
(a) $A^2 = I$
(b) $A = (-1)\,I$, where I is a unit matrix
(c) A^{-1} does not exist
(d) A is a zero matrix

6. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$. and $10\,B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverseof matrix A, then α is **[2004]**
(a) 5 (b) –1 (c) 2 (d) –2

7. If $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots.$ are in G.P., then the value of the determinant **[2004]**

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}, \text{ is}$$

(a) –2 (b) 1 (c) 2 (d) 0

8. If $A^2 - A + I = 0$, then the inverse of A is **[2005]**
(a) $A+I$ (b) A (c) $A-I$ (d) $I-A$

9. The system of equations
$\alpha x + y + z = \alpha - 1$
$x + \alpha y + z = \alpha - 1$
$x + y + \alpha z = \alpha - 1$
has infinite solutions, if α is **[2005]**
(a) -2 (b) either -2 or 1
(c) not -2 (d) 1

10. If $a^2+b^2+c^2 = -2$ and **[2005]**

$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix},$$

then f(x) is a polynomial of degree
(a) 1 (b) 0 (c) 3 (d) 2

11. If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_n, \ldots\ldots$ are in G.P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

is equal to **[2005]**
(a) 1 (b) 0
(c) 4 (d) 2

12. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A-B)(A+B)$, then which of the following will be always true? **[2006]**
(a) $A = B$
(b) $AB = BA$
(c) either of A or B is a zero matrix
(d) either of A or B is identity matrix

13. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then **[2006]**
(a) there cannot exist any B such that AB = BA
(b) there exist more then one but finite number of B′s such that AB = BA

(c) there exists exactly one B such that AB = BA
(d) there exist infinitely many B's such that AB = BA

14. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is **[2007]**
(a) divisible by x but not y
(b) divisible by y but not x
(c) divisible by neither x nor y
(d) divisible by both x and y

15. Let $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$. If $\left|A^2\right| = 25$, then $|\alpha|$ equals **[2007]**
(a) 1/5 (b) 5 (c) 5^2 (d) 1

16. Let A be a 2 × 2 matrix with real entries. Let I be the 2 × 2 identity matrix. Denote by tr(A), the sum of diagonal entries of a. Assume that $A^2 = I$. **[2008]**
Statement-1 : If $A \neq I$ and $A \neq -I$, then det(A) = –1
Statement-2 : If $A \neq I$ and $A \neq -I$, then tr (A) ≠ 0.
(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

17. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz, y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to **[2008]**
(a) 2 (b) –1 (c) 0 (d) 1

18. Let A be a square matrix all of whose entries are integers. Then which one of the following is true? **[2008]**
(a) If det $A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(b) If det $A \neq \pm 1$, then A^{-1} exists and all its entries are non integers
(c) If det $A = \pm 1$, then A^{-1} exists but all its entries are integers
(d) If det $A = \pm 1$, then A^{-1} need not exists

19. Let A be a 2 × 2 matrix
Statement -1 : adj (adj A) = A
Statement -2 : |adj A| = |A| **[2009]**
(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement -1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement -2 is true. Statement-2 is a correct explanation for Statement-1.

20. Let a, b, c be such that $b(a + c) \neq 0$ if **[2009]**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$$

then the value of n is :
(a) any even integer (b) any odd integer
(c) any integer (d) zero

21. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is **[2010]**
(a) 5 (b) 6
(c) at least 7 (d) less than 4

22. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define
Tr(A) = sum of diagonal elements of A and
$|A|$ = determinant of matrix A.
Statement - 1 : Tr(A) = 0.
Statement -2 : $|A| = 1$. **[2010]**
(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

23. Consider the system of linear equations ; **[2010]**
$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$
The system has
(a) exactly 3 solutions (b) a unique solution
(c) no solution (d) infinite number of solutions

24. The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution is **[2011]**
(a) 2 (b) 1 (c) zero (d) 3

25. Let A and B be two symmetric matrices of order 3.
Statement-1: $A(BA)$ and $(AB)A$ are symmetric matrices.
Statement-2: AB is symmetric matrix if matrix multiplication of A with B is commutative. **[2011]**
(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

26. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to : **[2012]**

(a) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

27. Let P and Q be 3 × 3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$ then determinant of $(P^2 + Q^2)$ is equal to : **[2012]**
(a) –2 (b) 1 (c) 0 (d) –1

28. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to : **[JEE M 2013]**

(a) 4 (b) 11 (c) 5 (d) 0

29. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to: **[JEE M 2014]**

(a) 1 (b) −1 (c) $\alpha\beta$ (d) $\frac{1}{\alpha\beta}$

30. If A is a 3 × 3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals: **[JEE M 2014]**

(a) B^{-1} (b) $(B^{-1})'$ (c) $I + B$ (d) I

31. The set of all values of λ for which the system of linear equations :

$2x_1 - 2x_2 + x_3 = \lambda x_1$
$2x_1 - 3x_2 + 2x_3 = \lambda x_2$
$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution **[JEE M 2015]**

(a) contains two elements
(b) contains more than two elements
(c) is an empty set
(d) is a singleton

32. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3 × 3 identity matrix, then the ordered pair (a, b) is equal to: **[JEE M 2015]**

(a) (2, 1) (b) (–2, – 1)
(c) (2, – 1) (d) (–2, 1)

33. The system of linear equations

$x + \lambda y - z = 0$
$\lambda x - y - z = 0$
$x + y - \lambda z = 0$

has a non-trivial solution for: **[JEE M 2016]**

(a) exactly two values of λ.
(b) exactly three values of λ.
(c) infinitely many values of λ.
(d) exactly one value of λ.

34. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = A A^T, then 5a + b is equal to : **[JEE M 2016]**

(a) 4 (b) 13
(c) −1 (d) 5