

# CHAPTER

# 5

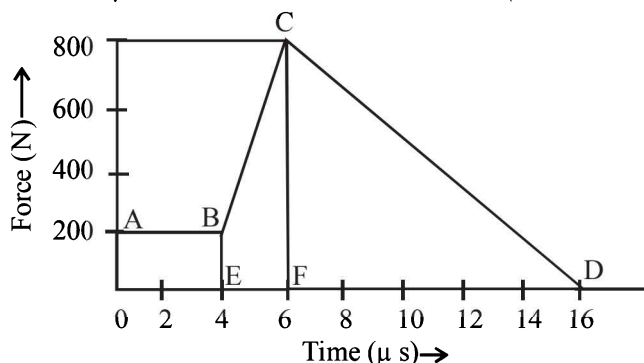
# Momentum and Impulse

## Section-A

## JEE Advanced/IIT-JEE

### A Fill in the Blanks

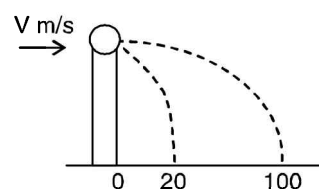
- A particle of mass  $4m$  which is at rest explodes into three fragments. Two of the fragments each of mass  $m$  are found to move with a speed  $v$  each in mutually perpendicular directions. The total energy released in the process of explosion is ..... (1987 - 2 Marks)
- The magnitude of the force (in newtons) acting on a body varies with time  $t$  (in micro seconds) as shown in the fig  $AB$ ,  $BC$  and  $CD$  are straight line segments. The magnitude of the total impulse of the force on the body from  $t = 4 \mu s$  to  $t = 16 \mu s$  is .....Ns. (1994 - 2 Marks)



### C MCQs with One Correct Answer

- Two particles of masses  $m_1$  and  $m_2$  in projectile motion have velocities  $\vec{v}_1$  and  $\vec{v}_2$  respectively at time  $t = 0$ . They collide at time  $t_0$ . Their velocities become  $\vec{v}_1'$  and  $\vec{v}_2'$  at time  $2t_0$  while still moving in air. The value of  $|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)|$  is (2001S)
  - zero
  - $(m_1 + m_2)gt_0$
  - $\frac{1}{2}(m_1 + m_2)gt_0$
  - $2(m_1 + m_2)gt_0$
- Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is (2002S)
  - 30 m/s
  - 20 m/s
  - 10 m/s
  - 5 m/s
- A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity  $V$  m/s in a

horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity  $V$  of the bullet is (2011)



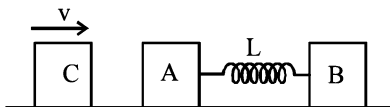
- 250 m/s
  - $250\sqrt{2}$  m/s
  - 400 m/s
  - 500 m/s
- A particle of mass  $m$  is projected from the ground with an initial speed  $u_0$  at an angle  $\alpha$  with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed  $u_0$ . The angle that the composite system makes with the horizontal immediately after the collision is (JEE Adv. 2013)

- $\frac{\pi}{4}$
- $\frac{\pi}{4} + \alpha$
- $\frac{\pi}{2} - \alpha$
- $\frac{\pi}{2}$

### D MCQs with One or More than One Correct

- A ball hits the floor and rebounds after an inelastic collision. In this case (1986 - 2 Marks)
  - the momentum of the ball just after the collision is the same as that just before the collision.
  - the mechanical energy of the ball remains the same in the collision
  - the total momentum of the ball and the earth is conserved
  - the total energy of the ball and the earth is conserved
- A shell is fired from a cannon with a velocity  $v$  (m/sec.) at an angle  $\theta$  with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (in m/sec.) of the other piece immediately after the explosion is (1986 - 2 Marks)
  - $3v \cos \theta$
  - $2v \cos \theta$
  - $\frac{3}{2}v \cos \theta$
  - $\sqrt{\frac{3}{2}}v \cos \theta$

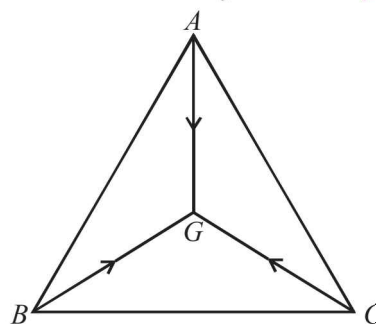
3. Two blocks  $A$  and  $B$ , each of mass  $m$ , are connected by a massless spring of natural length  $L$  and spring constant  $K$ . The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in fig. A third identical block  $C$ , also of mass  $m$ , moves on the floor with a speed  $v$  along the line joining  $A$  and  $B$ , and collides elastically with  $A$ . Then (1993-2 Marks)



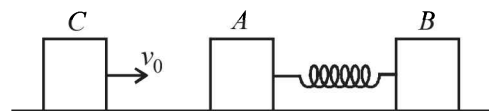
- the kinetic energy of the  $A$ - $B$  system, at maximum compression of the spring, is zero.
  - the kinetic energy of the  $A$ - $B$  system, at maximum compression of the spring, is  $mv^2/4$ .
  - the maximum compression of the spring is  $v\sqrt{(m/K)}$
  - the maximum compression of the spring is  $v\sqrt{(m/2K)}$
4. The balls, having linear momenta  $\vec{p}_1 = \vec{p}_i$  and  $\vec{p}_2 = -\vec{p}_i$ , undergo a collision in free space. There is no external force acting on the balls. Let  $\vec{p}'_1$  and  $\vec{p}'_2$  be their final momenta. The following option (s) is (are) NOT ALLOWED for any non-zero value of  $p, a_1, a_2, b_1, b_2, c_1$  and  $c_2$ . (2008)
- $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$   
 $\vec{p}'_2 = a_2\hat{i} + b_2\hat{j}$
  - $\vec{p}'_1 = c_1\hat{k}$   
 $\vec{p}'_2 = c_2\hat{k}$
  - $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$   
 $\vec{p}'_2 = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$
  - $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j}$   
 $\vec{p}'_2 = a_2\hat{i} + b_1\hat{j}$
5. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of  $2 \text{ ms}^{-1}$ . Which of the following statement(s) is (are) correct for the system of these two masses? (2010)
- Total momentum of the system is  $3 \text{ kg ms}^{-1}$
  - Momentum of 5 kg mass after collision is  $4 \text{ kg ms}^{-1}$
  - Kinetic energy of the centre of mass is  $0.75 \text{ J}$
  - Total kinetic energy of the system is  $4 \text{ J}$
6. A particle of mass  $m$  is attached to one end of a mass-less spring of force constant  $k$ , lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time  $t = 0$  with an initial velocity  $u_0$ . When the speed of the particle is  $0.5 u_0$ , it collides elastically with a rigid wall. After this collision (JEE Adv. 2013)
- The speed of the particle when it returns to its equilibrium position is  $u_0$
  - The time at which the particle passes through the equilibrium position for the first time is  $t = \pi\sqrt{\frac{m}{k}}$
  - The time at which the maximum compression of the spring occurs is  $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$
  - The time at which the particle passes through the equilibrium position for the second time is  $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

## E Subjective Problems

- A body of mass  $m$  moving with velocity  $V$  in the  $X$ -direction collides with another body of mass  $M$  moving in  $Y$ -direction with velocity  $v$ . They coalesce into one body during collision. Calculate : (1978)
  - the direction and magnitude of the momentum of the final body.
  - the fraction of initial kinetic energy transformed into heat during the collision in terms of the two masses.
- Three particles  $A, B$  and  $C$  of equal mass move with equal speed  $V$  along the medians of an equilateral triangle as shown in figure. They collide at the centroid  $G$  of the triangle. After the collision,  $A$  comes to rest,  $B$  retraces its path with the speed  $V$ . What is the velocity of  $C$ ? (1982 - 2 Marks)



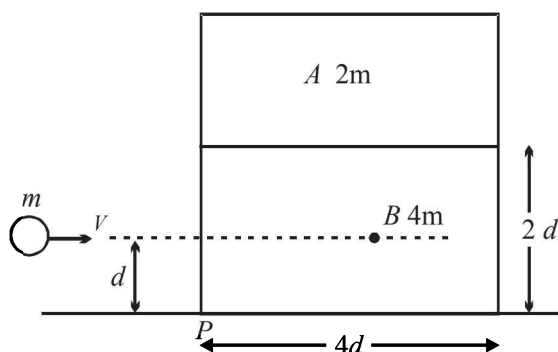
- Two bodies  $A$  and  $B$  of masses  $m$  and  $2m$  respectively are placed on a smooth floor. They are connected by a spring. A third body  $C$  of mass  $m$  moves with velocity  $v_0$  along the line joining  $A$  and  $B$  and collides elastically with  $A$  as shown in Fig.



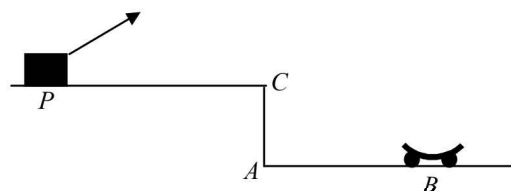
At a certain instant of time  $t_0$  after collision, it is found that the instantaneous velocities of  $A$  and  $B$  are the same. Further at this instant the compression of the spring is found to be  $x_0$ . Determine (i) the common velocity of  $A$  and  $B$  at time  $t_0$ ; and (ii) the spring constant. (1984- 6 Marks)

- A ball of mass  $100 \text{ gm}$  is projected vertically upwards from the ground with a velocity of  $49 \text{ m/sec}$ . At the same time another identical ball is dropped from a height of  $98 \text{ m}$  to fall freely along the same path as that followed by the first ball. After some time the two balls collide and stick together and finally fall to the ground. Find the time of flight of the masses. (1985 - 8 Marks)
- A bullet of mass  $M$  is fired with a velocity  $50 \text{ m/s}$  at an angle with the horizontal. At the highest point of its trajectory, it collides head-on with a bob of mass  $3M$  suspended by a massless string of length  $10/3 \text{ metres}$  and gets embedded in the bob. After the collision, the string moves through an angle of  $120^\circ$ . Find
  - the angle  $\theta$ ;
  - the vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet. Take  $g = 10 \text{ m/s}^2$

6. A block 'A' of mass  $2m$  is placed on another block 'B' of mass  $4m$  which in turn is placed on a fixed table. The two blocks have a same length  $4d$  and they are placed as shown in fig. The coefficient of friction (both static and kinetic) between the block 'B' and table is  $\mu$ . There is no friction between the two blocks. A small object of mass  $m$  moving horizontally along a line passing through the centre of mass (cm.) of the block B and perpendicular to its face with a speed  $v$  collides elastically with the block B at a height  $d$  above the table. (1991 - 4 + 4 Marks)

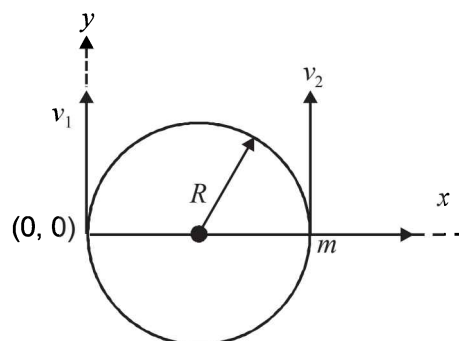


- (a) What is the minimum value of  $v$  (call it  $v_0$ ) required to make the block A topple?
- (b) If  $v = 2v_0$ , find the distance (from the point P in the figure) at which the mass  $m$  falls on the table after collision. (Ignore the role of friction during the collision).
7. A cart is moving along  $+x$  direction with a velocity of  $4 \text{ m/s}$ . A person on the cart throws a stone with a velocity of  $6 \text{ m/s}$  relative to himself. In the frame of reference of the cart the stone is thrown in  $y$ - $z$  plane making an angle of  $30^\circ$  with vertical  $z$ -axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from the branch of a tree by means of a string of length  $L$ . A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine : (1997 - 5 Marks)
- (i) The speed of the combined mass immediately after the collision with respect to an observer on the ground,
- (ii) The length  $L$  of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.
8. A car P is moving with a uniform speed of  $5\sqrt{3} \text{ m/s}$  towards a carriage of mass  $9 \text{ kg}$  at rest kept on the rails at a point B as shown in figure. The height AC is  $120 \text{ m}$ . Cannon balls of  $1 \text{ kg}$  are fired from the car with an initial velocity  $100 \text{ m/s}$  at an angle  $30^\circ$  with the horizontal. The first cannon ball hits the stationary carriage after a time  $t_0$  and sticks to it. Determine  $t_0$ . (2001 - 10 Marks)



At  $t_0$ , the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage, what will be the horizontal velocity of the carriage just after the second impact?

9. A particle of mass  $m$ , moving in a circular path of radius  $R$  with a constant speed  $v_2$  is located at point  $(2R, 0)$  at time  $t = 0$  and a man starts moving with a velocity  $v_1$  along the  $+ve$   $y$ -axis from origin at time  $t = 0$ . Calculate the linear momentum of the particle w.r.t. the man as a function of time. (2003 - 2 Marks)



## H Assertion & Reason Type Questions

1. **STATEMENT-1** : In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision. (2007)
- STATEMENT-2** : In an elastic collision, the linear momentum of the system is conserved.
- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
- (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
- (c) Statement-1 is True, Statement-2 is False.
- (d) Statement-1 is False, Statement-2 is True.

## I Integer Value Correct Type

1. A bob of mass  $m$ , suspended by a string of length  $l_1$ , is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass  $m$  suspended by a string of length  $l_2$ , which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the

vertical plane, the ratio  $\frac{l_1}{l_2}$  is (JEE Adv. 2013)

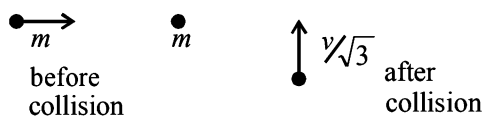
## Section-B

## JEE Main / AIEEE

1. A machine gun fires a bullet of mass 40 g with a velocity  $1200 \text{ ms}^{-1}$ . The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most? [2004]

(a) Two (b) Four  
(c) One (d) Three

2. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass. After collision the 1<sup>st</sup> mass moves with velocity  $\frac{v}{\sqrt{3}}$  in a direction perpendicular to the initial direction of motion. Find the speed of the 2<sup>nd</sup> mass after collision. [2005]



(a)  $\sqrt{3}v$  (b)  $v$   
(c)  $\frac{v}{\sqrt{3}}$  (d)  $\frac{2}{\sqrt{3}}v$

3. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is  $4 \text{ ms}^{-1}$ . The kinetic energy of the other mass is

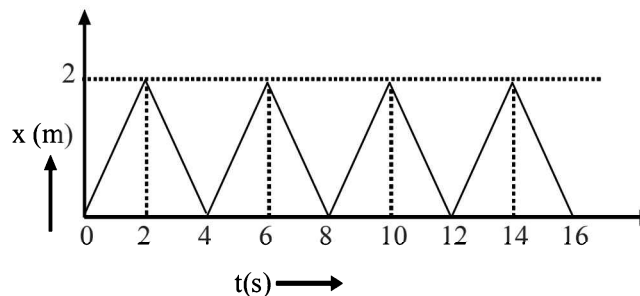
(a) 144 J (b) 288 J [2006]  
(c) 192 J (d) 96 J

4. **Statement -1:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

**Statement -2:** Principle of conservation of momentum holds true for all kinds of collisions. [2010]

(a) Statement -1 is true, Statement -2 is true; Statement -2 is the correct explanation of Statement -1.  
(b) Statement -1 is true, Statement -2 is true; Statement -2 is **not** the correct explanation of Statement -1  
(c) Statement -1 is false, Statement -2 is true.  
(d) Statement -1 is true, Statement -2 is false.

5. The figure shows the position–time ( $x - t$ ) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is [2010]



(a) 0.4 Ns (b) 0.8 Ns  
(c) 1.6 Ns (d) 0.2 Ns

6. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements. [JEE Main 2013]

**Statement - I:** A point particle of mass  $m$  moving with speed  $v$  collides with stationary point particle of mass  $M$ . If the

maximum energy loss possible is given as  $f\left(\frac{1}{2}mv^2\right)$  then

$$f = \left( \frac{m}{M + m} \right).$$

**Statement - II:** Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(a) Statement - I is true, Statement - II is true, Statement - II is the correct explanation of Statement - I.  
(b) Statement - I is true, Statement - II is true, Statement - II is not the correct explanation of Statement - II.  
(c) Statement - I is true, Statement - II is false.  
(d) Statement - I is false, Statement - II is true.

7. A particle of mass  $m$  moving in the  $x$  direction with speed  $2v$  is hit by another particle of mass  $2m$  moving in the  $y$  direction with speed  $v$ . If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to :

[JEE Main 2015]

(a) 56% (b) 62%  
(c) 44% (d) 50%