ITM(SLS)BARODA UNIVERSITY , VADODARA

School of Computer Science Engineering& Technology (BTECH.SEM-1)

SUBJECT: DISCRETE MATHEMATICS WITH PYTHON

TUTORIAL -1 SETS, FUNCTION & COUNTING

SETS

Q1. Define the following giving an example of each:
(a) Set (b) Subset (c) Proper Set (d) Equal Set (e) Finite & Infinite Set
(f) Power Set (g) Null Set (h) Singleton Set (i) Cardinality of a Set
(j) Disjoint Set.

Q2. List the members of these sets:

(a) $A=\{ x|x$ is a real number $\ni x^2=1 \}$

(b) $B=\{ x|x$ is a positive integer less than 100 $\}$

(c) $C=\{ x|x$ is the square of an integer and $x<100 \}$

(d) $D=\{ x|x$ is an integer $\ni x^2=2 \}$

Q3. Determine whether each of these pairs of sets are equal :
(a) $A=\{1,3,3,3,5,5,5,5,5\}$ & $B=\{5,3,1\}$ (b) $A=\{\{1\}\}$ & $B=\{1,\{1\}\}$
(c) $A=\{\phi\}$ & $B=\phi$ (d) $A=\{ x|x$ is an integer and $x^3=1$ & $B=\{1\}$

Q4. For each of the following sets, determine whether 2 is an element of that set:

(a) $A=\{ x\in R|x$ is an integer greater than 1 $\}$

(b) $B=\{ x\in R|x$ is the square of an integer$\}$

(c) $C=\{2,\{2\}\}$ (d) $D=\{\{2\},\{2,\{2\}\}\}$ (e) $E=\{\{\{2\}\}\}$

Q5. Determine whether each of these statements is true or false:

(a) $0\in\phi$ (b) $\phi\in\{0\}$ (c) $\{0\}\in\phi$ (d) $\{\phi\}\in 0$ (e) $x\in\{x\}$

(f) $\{x\}\subseteq\{x\}$ (g) $\{x\}\in\{x\}$ (h) $\{x\}\in\{\{x\}\}$ (i) $\phi\in\{x\}$

(j) $\phi\in\{x\}$.

Q6. Use a Venn Diagram to illustrate the set of all months of the year whose names do not contain the letter R in the set of all months of the year.

Q7. What is the cardinality of each of these sets?
(a) $\{a\}$ (b) $\{a, b\}$ (c) $\{\phi, \{\phi\}\}$

Q8. Let $A=\{a, b, c, d\}$ and $B=\{y, z\}$. Find (a) A X B (b) B X A

Q9. Let A = Set of students who live within one Km of school & B = Set of students who walk to class. Describe in words, the students in each of thesesets.

(a) $A\cap B$ (b) $A\cup B$ (c) A – B (d) B – A

Q10. Let $A=\{0, 2, 4, 6, 8, 10\}$, $B=\{0, 1, 2, 3, 4, 5, 6\}$ &
$C=\{4, 5, 6, 7, 8, 9, 10\}$. Find

(I) (a) $A\cap B\cap C$ (b) $A\cup B\cup C$ (c) $(A\cup B)\cap C$ (d) $(A\cap B)\cup C$

(II) Draw Venn diagrams for each of these combinations of sets A, B & C mentioned in previous question where A', B', C' are complements of sets.
(a) $A\cap(B-C)$ (b) $(A\cap B)\cup(A\cap C)$ (c) $(A\cap B')\cup(A\cap C')$

Q11. What can you say about sets A & B if we know that

(a) $A \cup B = A$ (b) $A \cap B = A$ (c) $A - B = A$

(d) $A \cap B = B \cap A$

(e) $A - B = B - A$

Q12. Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

(I) Express each of these sets with bit strings where the i^{th} bit in the string is 1 if is in the set & 0 otherwise .

(a) $\{3, 4, 5\}$ (b) $\{1, 3, 6, 10\}$ (c) $\{2, 3, 4, 7, 8, 9\}$.

(II) Find set specified by each of these bit strings:

(a) 1111001111 (b) 0101111000 (c) 1000000001

FUNCTIONS

Q13. Determine whether f is a function from R to R:

(a) $f(x) = \frac{1}{x}$ (b) $f(x) = \sqrt{x}$

(c) $f(x) = \pm[1 + x^2]^{1/2}$

Q14. Determine f is function from Z *to* R *if*

(a) $f(n) = \pm n$ (b) $f(n) = [n^2 + 1]^{1/2}$

(c) $f(n) = \frac{1}{n^2 - 4}$

Q16. Determine f is function from the set of all bit strings to the set of the set ofintegers if :
(a) *f(s)* is the position of a 0 bit in S. (b) *f(s)* is the number of a 1 bit in S.

Q17. Find the domain & range of these functions. Note that in each case, to find thedomain, determine the set of elements assigned values by the function

(a) the function that assigns to each bit string the number of ones minus thenumber of zeros.

(b) the function that assigns to each bit string twice the number of zeros in that string.

(c) the function that assigns to each positive integer the largest perfectsquare not exceeding this integer.

(d) the function that assigns to each pair of positive integers the maximum ofthese two integers.

Q18. Determine whether each of the function {a,b,c,d} to itself is one-to-one & onto

(a) $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$

(b) $f(a) = b$, $f(b) = b$, $f(c) = d$, $f(d) = c$

(c) $f(a) = d$, $f(b) = b$, $f(c) = c$, $f(d) = d$

Q19. Determine whether each of the functions from Z to Z is one – one & onto :(a) $f(n) = n - 1$ (b) $f(n) = n^2 + 1$ (c) $f(n) = n^3$

Q20. Determine whether each of the functions from $f : Z \times Z \rightarrow Z$ is onto if:

(a) $f(m, n) = m + n$ (b) $f(m, n) = m^2 + n^2$ (c) $f(m, n) = m$

(d) $f(m, n) = |n|$ (e) $f(m, n) = m - n$

Q21. Determine whether each of these functions is a bijection from R to R :

(a) $f(x) = 2x + 1$ (b) $f(x) = -3x^2 + 7$ (c) $f(x) = \frac{x+1}{x+2}$ (d) $f(x) = x^5 + 1$

Q23. Define $g : Z \to B$ by $g(x) = |x| + 1$. Determine with reasons whether g isone-to-one & whether it is onto in each of the following cases:

(i) $B = Z$ (ii) $B = N$

Q24. The addition & multiplication of real numbers are functions :
add,mult : R *X* R → R where *add (x,y) = x+y* & *mult(x,y)= xy*.Check both for one-one & onto.

Q25. For the following functions check whether f^{-1} exist or not, if exist find The formula for the same:

(i) $f: Q \to Q$, where Q is set of rational numbers given by $f(x) = 3x + 5$.

(ii) $f: A \to B$, where $A = R - \left\{-\frac{1}{2}\right\}$ *and* $B = R - \left\{\frac{1}{2}\right\}$, given by

$$f(x) = \frac{x-3}{2x+1}$$

Q26. If mapping $f: R \to R$ and $g: R \to R$ then find $(fog)(x)$ and $(gof)(x)$ also Check $gof = fog$.

(i) $f(x) = x^3$ and $g(x) = x^3$

(ii) $f(x) = x^2$ and $g(x) = sinx$

(iii) $f(x) = x^3 - 4x$ and $g(x) = x^4$

(iv) $f(x) = x^2 - 3$ and $g(x) = 3x - 4$

(v) $f(x) = x^3$ and $g(x) = x^2$

COUNTING

Q25. List all the permutations of {a,b,c} & how many are there?

Q26. How many permutations of {a,b,c,d,e,f,g} end with *a*?

Q27. Find the value of each of these quantities using formula:

(a) $P(6, 3)$ (b) $P(6, 5)$ (c) $P(8, 1)$ (d) $P(8, 5)$ (e) $P(8, 8)$ (f) $P(10, 9)$

Q28. Find the number of 5-Permutations of the set with nine elements.

Q29. A coin is flipped 10 times where each flip comes up either heads or tails. Howmany possible outcomes

(a) are there in total? (b) contain exactly three heads?

(c) contain at least three heads?

(d) contain the same number of heads & tails?

Q30. Find the expansion using Binomial theorem & simplify:(a) $(x+y)^4$

(b) $(2x+3y)^6$

Q31. Find the coefficient of x^{16} in the expansion of $(2x^2 - \frac{x}{2})^{12}$

Q32. Consider the Binomial expansion of $(x+y)^{20}$. What is the seventh term andthe fifteenth term?

Q33. Find the coefficient of :

(a) $x^5 y^8$ in $(x+y)^{13}$ (b) x^9 in $(2-x)^{19}$ (c) $x^{101} y^{99}$ in $(2x-3y)^{200}$

Q34. What is the row of Pascal's triangle containing the binomial coefficients C(9,k),k=0 to 9?

Q35. How many strings of six letters are there?

Q36. How many ways are there to assign three jobs to five employees if eachemployee can be given more than one job?

Q37. A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels, and plain bagels.
How many ways are there to choose
(a) six bagels? (b) a dozen bagels? (c) two dozen bagels?

Q38. A book publisher has 3000 copies of a discrete mathematics book. How many ways are there to store these books in 3 warehouses if the copies of the book are indistinguishable?

Q39. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$ where x_i, $i = 1,2,3,4,5$ is a non-negative integer such that (a) $x_1 \geq 1$? (b) $x_i \geq 2$ for $i = 1,2,3,4,5$? (c) $0 \leqslant x_1 \leqslant 1$