ITM(SLS) BARODA UNIVERSITY, VADODARA

School of Computer Science
Engineering & Technology
(BTECH.SEM-1)
SUBJECT: DISCRETE MATHEMATICS WITH PYTHON

Tutorial-2
Propositional Logics and Predicate Logics

Q1.	Which of these sentences are propositions? What are the truth values of those that are propositions? **a)** Boston is the capital of Massachusetts. **b)** Miami is the capital of Florida. **c)** $2 + 3 = 5$. **d)** $5 + 7 = 10$. **e)** $x + 2 = 11$. **f)** Answer this question.
Q2.	Let p and q be the propositions “The election is decided” and “The votes have been counted,” respectively. Express each of these compound propositions as an English sentence. **a)** $\neg p$ **b)** $p \vee q$ **c)** $\neg p \wedge q$ **d)** $q \rightarrow p$ **e)** $\neg q \rightarrow \neg p$ **f)** $\neg p \rightarrow \neg q$ **g)** $p \leftrightarrow q$ **h)** $\neg q \vee (\neg p \wedge q)$
Q3.	Let p, q, and r be the propositions p: You have the flu. q: You miss the final examination. r: You pass the course. Express each of these propositions as an English sentence. **a)** $p \rightarrow q$ **b)** $\neg q \leftrightarrow r$ **c)** $q \rightarrow \neg r$ **d)** $p \vee q \vee r$ **e)** $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ **f)** $(p \wedge q) \vee (\neg q \wedge r)$
Q4.	Construct a truth table for each of these compound propositions.

	a) $p \to (\neg q \vee r)$ b) $\neg p \to (q \to r)$ c) $(p \to q) \vee (\neg p \to r)$ d) $(p \to q) \wedge (\neg p \to r)$ e) $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$ f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$ g) $(p \oplus q) \wedge (p \oplus \neg q)$
Q5.	Use the truth table to verify commutative, associative, distributive, and De-morgan's law.
Q6.	Check whether the following conditional statements are a tautology or a contradiction by using truth tables. **a)** $(p \wedge q) \to p$ **b)** $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ **c)** $\neg p \to (p \to q)$ **d)** $(p \wedge q) \to (p \to q)$ **e)** $\neg(p \to q) \to p$ **f)** $[(p \to q) \wedge (q \to r)] \to (p \to r)$ ***g)*** $p \wedge (q \wedge \neg p)$
Q7.	Let P (x) denote the statement "$x \leq 4$." What are these truth values? a) P (0) b) P (4) c) P (6)
Q8.	Let Q(x) be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values? a) Q(0) b) Q(−1) c) Q(1) d) $\exists x Q(x)$ e) $\forall x Q(x)$ f) $\exists x \neg Q(x)$ g) $\forall x \neg Q(x)$
Q9.	Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people. a) Someone in your class can speak Hindi. b) Everyone in your class is friendly. c) There is a person in your class who was not born in California. d) A student in your class has been in a movie.

	e) No student in your class has taken a course in logic programming.
Q10.	Express these system specifications using the propositions *p* “The user enters a valid password,” *q* “Access is granted,” and *r* “The user has paid the subscription fee” and logical connectives (including negations). **a)** “The user has paid the subscription fee, but does not enter a valid password.” **b)** “Access is granted whenever the user has paid the subscription fee and enters a valid password.” **c)** “Access is denied if the user has not paid the subscription fee.” **d)** “If the user has not entered a valid password but has paid the subscription fee, then access is granted.”
Q11.	Let $Q(x, y)$ be the statement “x has sent an e-mail message to y,” where the domain for both x and y consists of all students in your class. Express each of these quantifications in English. **a)** $\exists x \exists y Q(x, y)$ **b)** $\exists x \forall y Q(x, y)$ **c)** $\forall x \exists y Q(x, y)$ **d)** $\exists y \forall x Q(x, y)$ **e)** $\forall y \exists x Q(x, y)$ **f)** $\forall x \forall y Q(x, y)$
Q12.	Translate these statements into English, where the domain for each variable consists of all real numbers. Also find truth value a) $\exists x \, \forall y \, (xy = y)$ b) $\forall x \, \forall y(((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$ c) $\forall x \, \forall y \, \exists z(x = y + z)$
Q13.	Use rules of inference to show that the hypothesis “Randy works hard,” “If Randy works hard, then he is a dull boy,” and “If Randy is a dull boy, then he will not get the job” imply the conclusion “Randy will not get the job.”
Q14.	Use direct proof to show that (i) the sum of two odd integers is even. (ii) The sum of two even integers is even.
Q15.	**Define the following terms:** 1. Propositions. 2. Logical connectives with truth table. 3. Tautology, contradiction, and contingency. 4. Converse, contrapositive and inverse. 5. Logically equivalent 6. Predicates 7. Quantifiers, Universal, Existential

	8. Theorem, proofs, lemma, axiom 9. Types of proof.