ITM(SLS) BARODA UNIVERSITY, VADODARA

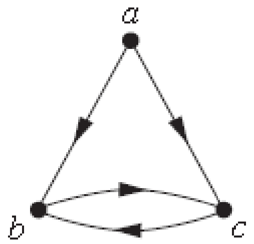
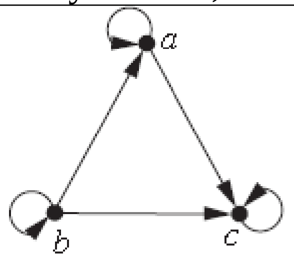
School of Computer Science Engineering & Technology (BTECH.SEM-1)

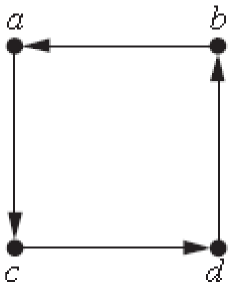
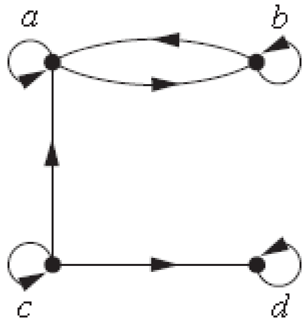
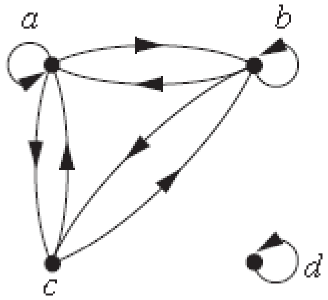
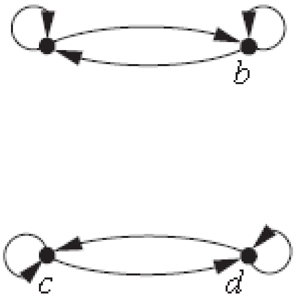
SUBJECT: DISCRETE MATHEMATICS WITH PYTHON

Tutorial-3

Relation, Matrices & Equivalence relation and Partial ordering

Q1.	List all the ordered pairs in the relation $R = \{(a, b) \mid a$ divides $b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$. **a)** Display this relation graphically **b)** Display this relation in tabular form.
Q2.	For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive. **a)** $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$ **b)** $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$ **c)** $\{(2, 4), (4, 2)\}$ **d)** $\{(1, 2), (2, 3), (3, 4)\}$ **e)** $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$ **f)** $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$
Q3.	Determine whether the relation R on the set of all positive integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if **a)** $x \neq y$. **b)** $xy \geq 1$.
Q4.	Determine whether the relation R on the set of all real numbers are reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if **a)** $x + y = 0$ b**)** $x = 2y$.
Q5.	Let $R1 = \{(1, 2), (2, 3), (3, 4)\}$ and $R2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$ be relations from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$. Find **a)** $R1 \cup R2$. **b)** $R1 \cap R2$. **c)** $R1 - R2$. **d)** $R2 - R1$.
Q6.	Let R be the relation R = {(a, b) | a divides b} on the set of positive integers. Find a) R^{-1} b) R' (complement of R)
Q7.	Represent each of these relations on $\{1, 2, 3, 4\}$ with a matrix (with the elements of this set listed in increasing order). a) $\{(1, 2),(1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$ b) $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$

	c) {(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4,3)} d) {(2, 4), (3, 1), (3, 2), (3, 4)}
Q8.	List the ordered pairs in the relations on {1, 2, 3} corresponding to these matrices
	$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
Q9.	Let R1 and R2 be relations on a set A represented by the matrices
	$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $\mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
	Find the matrices that represent a) R1 ∪ R2. b) R1 ∩ R2. c) R2 O R1. d) R1 O R2.
Q10.	Let R be the relation represented by the matrix: $\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Then find matrix represented following:
	a) R^2. **b)** R^3. **c)** R^4.
Q11.	Determine whether the relations represented by the directed graphs shown are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.
	 

	 
	 
Q 12	Let R be the relation on the set {1, 2, 3, 4} containing the ordered pairs {(1, 3), (2, 4),(3, 1),(3, 4),(4, 3),(4, 1), (1, 4)}. Find R^2, R^3, R^4, R^*
Q 13	Use Warshall's algorithm to find the transitive closures of these relations on {1, 2, 3, 4}. **a)** {(1, 2), (2,1), (2,3), (3,4), (4,1)} **b)** {(2, 1), (2,3), (3,1), (3,4), (4,1), (4, 3)} **c)** {(1, 2), (1,3), (1,4), (2,3), (2,4), (3, 4)} **d)** {(1, 1), (1,4), (2,1), (2,3), (3,1), (3, 2), (3,4), (4, 2)}
Q 14	Define following: (i) Equivalence relation. (ii) Equivalence class. (iii) Partition of the set. (iv) Congruence Modulo m
Q 15	Let R be the relation on the set of real numbers such that aRb if and only if a − b is an integer. Is R an equivalence relation?
Q 16	Let *R* be an equivalence relation on a set *A*. These statements for elements *a* and *b* of *A* are equivalent: (*i*) *aRb* (*ii*) [*a*] = [*b*] (*iii*) [*a*] ∩ [*b*] _= ∅
Q 17	What are the sets in the partition of the integers arising from congruence modulo 4?
Q 18	Let *R* be the relation on the set of ordered pairs of positive integers such that *((a, b), (c, d))* ∈ *R* if and only if *ad* = *bc*. Show that *R* is an equivalence relation.
Q 19	What is the congruence class [*n*]5 (that is, the equivalence class of *n* with respect to congruence modulo 5) when *n* is **a)** 2? **b)** 3? **c)** 6? **d)** −3?
Q 20	Which of these collections of subsets are partitions of {1, 2, 3, 4, 5, 6}? **a)** {1, 2}, {2, 3, 4}, {4, 5, 6} **b)** {1}, {2, 3, 6}, {4}, {5} **c)** {2, 4, 6}, {1, 3, 5} **d)** {1, 4, 5}, {2, 6}

Q 21	List the ordered pairs in the equivalence relations produced by these partitions of {0, 1, 2, 3, 4, 5}. **a)** {0}, {1, 2}, {3, 4, 5} **b)** {0, 1}, {2, 3}, {4, 5} **c)** {0, 1, 2}, {3, 4, 5} **d)** {0}, {1}, {2}, {3}, {4}, {5}
Q 22	Define following: (i) Partial order relation and set. (ii) Comparable and incomparable elements (iii) Poset and Total order set (iv) Hasse digram with examples (v) GLB and LUB (vi) Lattice
Q 23	*Which of these relations on {0, 1, 2, 3} are partial orderings? Determine the properties of a partial ordering that the others lack.* ***a)*** *{(0, 0), (2, 2), (3, 3)}* ***b)*** *{(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 3)}* ***c)*** *{(0, 0), (1, 1), (1, 2), (2, 2), (3, 1), (3, 3)}* ***d)*** *{(0, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (2, 3), (3, 0), (3, 3)}* ***e)*** *{(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3, 3)}*
Q 24	Find the greatest lower bound and the least upper bound of the sets {3, 9, 12} and {1, 2, 4, 5, 10}, if they exist, in the poset $(\mathbf{Z}^+, \mid)$.
Q 25	Determine whether $(P(S), \subseteq)$ is a lattice where S is a set.
Q 26	Answer these questions for the poset $(\{3, 5, 9, 15, 24, 45\}, \mid)$. **a)** Find the maximal elements. **b)** Find the minimal elements. **c)** Is there a greatest element? **d)** Is there a least element? **e)** Find all upper bounds of {3, 5}. **f)** Find the least upper bound of {3, 5}, if it exists. **g)** Find all lower bounds of {15, 45}. **h)** Find the greatest lower bound of {15, 45}, if it exists.
Q 27	List all ordered pairs in the partial ordering with the accompanying Hasse diagram.

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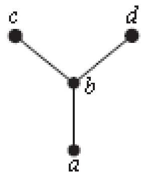
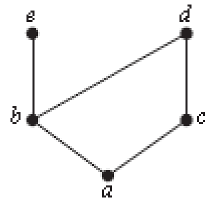
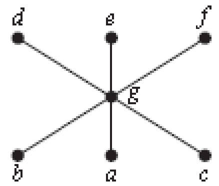
<tr><td></td><td>
</td><td>
</td><td>
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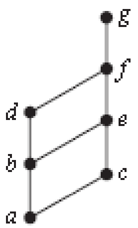
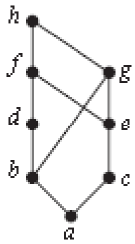
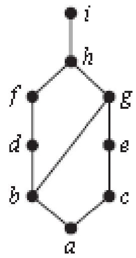
<tr><td>Q 28</td><td colspan="3">Draw the Hasse diagram for divisibility on the set
a) {1, 2, 3, 4, 5, 6}. b) {3, 5, 7, 11, 13, 16, 17}. c) {2, 3, 5, 10, 11, 15, 25}.
d) {1, 3, 9, 27, 81, 243}. e) {1, 2, 3, 4, 5, 6, 7, 8}. f) {1, 2, 3, 5, 7, 11, 13}.
g) {1, 2, 3, 6, 12, 24, 36, 48}. h) {1, 2, 4, 8, 16, 32, 64}</td></tr>
<tr><td>Q29.</td><td colspan="3">Determine whether these posets are lattices.
a) $(\{1, 3, 6, 9, 12\}, \mid)$ b) $(\{1, 5, 25, 125\}, \mid)$
c) $(\mathbf{Z}, \geq)$
d) $(P(S), \supseteq)$, where $P(S)$ is the power set of a set S</td></tr>
<tr><td>Q30.</td><td colspan="3">Determine whether the posets with these Hasse diagrams are lattices.</td></tr>
<tr><td></td><td>
</td><td>
</td><td>
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