R Codes for Chapter-5
Matrix Approach to Simple Linear Regression

1. Getting Started

A vector can be defined by using "c( )", but this is not the type of entities we want in the matrix approach.

```
> A = c(4, 7, 10) # vector
> A
[1]  4  7 10
```

To transform a Colum vector to a "n by 1" matrix, "as.matrix(vector name)" can be used.

```
> AA=as.matrix(A) #Transform avector to a matrix
> AA
     [,1]
[1,]    4
[2,]    7
[3,]   10
```

Here note that AA is a 3 by 1 matrix. most of the following commands work only for a matrix (not for a vector). We also can use the following ways to define matrices.

```
> B=matrix(c(1,2,3,4,5,6,7,8,9,10),ncol=2,nrow=5)
                                              #matrix with 2colms and 5 rows.
> B
     [,1] [,2]
[1,]    1    6
[2,]    2    7
[3,]    3    8
[4,]    4    9
[5,]    5   10


> C=matrix(c(rep(1,5),rep(2,5)),ncol=2,nrow=5) #matrix with 2colms and 5 rows
.
> C
     [,1] [,2]
[1,]    1    2
[2,]    1    2
[3,]    1    2
[4,]    1    2
[5,]    1    2

> D=matrix(c(rep(1,3),rep(2,3),rep(3,3)),,ncol=3)
> D
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    1    2    3
[3,]    1    2    3
```

2. Matrix Operations

a) Transpose of a matrix

```
> TransA=t(A)
> TransA
     [,1] [,2] [,3]
[1,]    4    7   10
> TransB=t(B)
> TransB
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    2    3    4    5
[2,]    6    7    8    9   10
```

b) Addition

```
> sumBC=B+C
> sumBC
     [,1] [,2]
[1,]    2    8
[2,]    3    9
[3,]    4   10
[4,]    5   11
[5,]    6   12
```

c) Subtraction

```
> BminC=B-C
> BminC
     [,1] [,2]
[1,]    0    4
[2,]    1    5
[3,]    2    6
[4,]    3    7
[5,]    4    8
```

d) Constant times a Matrix

```
> FTB=4*B
> FTB
     [,1] [,2]
[1,]    4   24
[2,]    8   28
[3,]   12   32
[4,]   16   36
[5,]   20   40
```

e) Matrix Multiplication

```
> BTtC=B%*%t(C) #dimentions shoud match
> BTtC
     [,1] [,2] [,3] [,4] [,5]
[1,]   13   13   13   13   13
[2,]   16   16   16   16   16
[3,]   19   19   19   19   19
[4,]   22   22   22   22   22
[5,]   25   25   25   25   25
```

f) Inverse

The R package "MASS" should be uploaded first to use the command "ginv".

```
> library(MASS) #loading MASS
> D=matrix(c(rep(1,3),rep(2,3),rep(3,3)),,ncol=3)
> InvD=ginv(D)  #Should be a square matrix.
> InvD
           [,1]       [,2]       [,3]
[1,] 0.02380952 0.02380952 0.02380952
[2,] 0.04761905 0.04761905 0.04761905
[3,] 0.07142857 0.07142857 0.07142857
```

3. Linear Regression

Here we use Plastic hardness Example.

```
> X=cbind(rep(1,16),c(16,16,16,16,24,24,24,24,32,32,32,32,40,40,40,40))
                                                       #Creating the matrix X
> Y=as.matrix(Hardness)
```

a) $(\boldsymbol{X}'\boldsymbol{X})^{-1}$

```
> A=ginv(t(X)%*%X)
> A
          [,1]        [,2]
[1,]  0.675000 -0.02187500
[2,] -0.021875  0.00078125
```

b) b

```
> b=ginv(t(X)%*%X)%*%t(X)%*%Y
> b
           [,1]
[1,] 168.600000
[2,]   2.034375
```

So $b_0 = 168.6$ and $b_1 = 2.034375$.

c) $\widehat{\boldsymbol{Y}}$

```
> Yhat=X%*%b
> Yhat
         [,1]
 [1,] 201.150
 [2,] 201.150
 [3,] 201.150
 [4,] 201.150
 [5,] 217.425
 [6,] 217.425
 [7,] 217.425
 [8,] 217.425
 [9,] 233.700
[10,] 233.700
[11,] 233.700
```

```
[12,] 233.700
[13,] 249.975
[14,] 249.975
[15,] 249.975
[16,] 249.975
```

d) H

```
> H=X%*%ginv(t(X)%*%X)%*%t(X)
> H
        [,1]   [,2]   [,3]   [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10] [,11] [,12]  [,13]  [,14]  [,15]  [,16]
 [1,]  0.175  0.175  0.175  0.175 0.100 0.100 0.100 0.100 0.025 0.025 0.025 0.025 -0.050 -0.050 -0.050 -0.050
 [2,]  0.175  0.175  0.175  0.175 0.100 0.100 0.100 0.100 0.025 0.025 0.025 0.025 -0.050 -0.050 -0.050 -0.050
 [3,]  0.175  0.175  0.175  0.175 0.100 0.100 0.100 0.100 0.025 0.025 0.025 0.025 -0.050 -0.050 -0.050 -0.050
 [4,]  0.175  0.175  0.175  0.175 0.100 0.100 0.100 0.100 0.025 0.025 0.025 0.025 -0.050 -0.050 -0.050 -0.050
 [5,]  0.100  0.100  0.100  0.100 0.075 0.075 0.075 0.075 0.050 0.050 0.050 0.050  0.025  0.025  0.025  0.025
 [6,]  0.100  0.100  0.100  0.100 0.075 0.075 0.075 0.075 0.050 0.050 0.050 0.050  0.025  0.025  0.025  0.025
 [7,]  0.100  0.100  0.100  0.100 0.075 0.075 0.075 0.075 0.050 0.050 0.050 0.050  0.025  0.025  0.025  0.025
 [8,]  0.100  0.100  0.100  0.100 0.075 0.075 0.075 0.075 0.050 0.050 0.050 0.050  0.025  0.025  0.025  0.025
 [9,]  0.025  0.025  0.025  0.025 0.050 0.050 0.050 0.050 0.075 0.075 0.075 0.075  0.100  0.100  0.100  0.100
[10,]  0.025  0.025  0.025  0.025 0.050 0.050 0.050 0.050 0.075 0.075 0.075 0.075  0.100  0.100  0.100  0.100
[11,]  0.025  0.025  0.025  0.025 0.050 0.050 0.050 0.050 0.075 0.075 0.075 0.075  0.100  0.100  0.100  0.100
[12,]  0.025  0.025  0.025  0.025 0.050 0.050 0.050 0.050 0.075 0.075 0.075 0.075  0.100  0.100  0.100  0.100
[13,] -0.050 -0.050 -0.050 -0.050 0.025 0.025 0.025 0.025 0.100 0.100 0.100 0.100  0.175  0.175  0.175  0.175
[14,] -0.050 -0.050 -0.050 -0.050 0.025 0.025 0.025 0.025 0.100 0.100 0.100 0.100  0.175  0.175  0.175  0.175
[15,] -0.050 -0.050 -0.050 -0.050 0.025 0.025 0.025 0.025 0.100 0.100 0.100 0.100  0.175  0.175  0.175  0.175
[16,] -0.050 -0.050 -0.050 -0.050 0.025 0.025 0.025 0.025 0.100 0.100 0.100 0.100  0.175  0.175  0.175  0.175
```

e) SSE

```
> SSE=t(Y)%*%Y-t(b)%*%t(X)%*%Y
> SSE
        [,1]
[1,] 146.425
```

f) MSE

```
> n=length(X)
> MSE=SSE/(n-2)
> MSE
         [,1]
[1,] 4.880833
```

g) $\widehat{Y}_h$

```
> Xh=c(1,30)
> Yhat_h=t(Xh)%*%b
> Yhat_h
         [,1]
[1,] 229.6313
```

h) $\boldsymbol{S^2\{b\}}$

Here note that, MSE should be transformed to a vector first.

```
> S2b=as.vector(MSE)*ginv(t(X)%*%X)
> S2b
           [,1]         [,2]
[1,]  3.2945625 -0.106768229
[2,] -0.1067682  0.003813151
```

i) $\boldsymbol{S^2\{Pred\}}$

```
> Xh=c(1,30)
> S2pred=as.vector(MSE)*(1+t(Xh)%*%ginv(t(X)%*%X)%*%Xh)
> S2pred
         [,1]
[1,] 5.201138
```