Chapter-6 : Multiple Regression

Statistical results are done in matrix form.

* First Order model with two predictor variables.

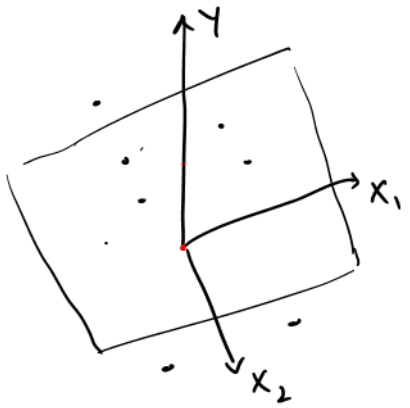
$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i , \longrightarrow (*)$$

Here,

* X_1, X_2 – Predictor variables.
* $\beta_0, \beta_1, \beta_2$ – model parameters
* Assuming $E[\varepsilon_i] = 0$, the regression function for model (*) is

$$E[Y] = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

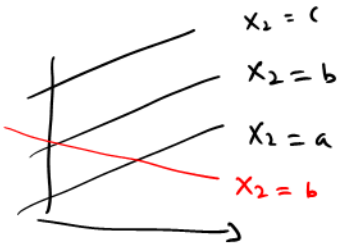
Here note that the regression function is a regression plane. (not a line as in SLR).



* Meaning of the regression coefficients.
* β_0 – the Y- intercept of the regression plane
  - mean of the response Y when $X_1 = 0$ and $X_2 = 0$ (No meaning when 0 is not in the range of X_1 or X_2).
* β_1 – is the change in mean response per unit increase in X_1 when X_2 is held constant.

* β_2 is the change in mean response per unit increase in X_2 when X_1 is held constant.

ie $\frac{\partial(E(Y))}{\partial X_1} = \beta_1$ and $\frac{\partial(E(Y))}{\partial X_2} = \beta_2$.



* Additive effect (not to interact)

When the effect of X_1 on the mean response does not depend on the level of X_2 and correspondingly effect of X_2 does not depend on levels of X_1, the predictors are said to have additive effect.

The regression model (*) is designed such that X_1 and X_2 have additive effects.

First order model with more than two predictor variables

The regression model with the predictors $X_1, X_2, \dots X_{p-1}$ ($p-1$ predictors, p – model parameters) is

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots + \beta_{p-1} X_{ip-1} + \xi_i$$

$$= \beta_0 + \sum_{k=1}^{p-1} \beta_k X_{ik} + \xi_i$$

$$= \sum_{k=0}^{p-1} \beta_k X_{ik} + \xi_i \quad , \quad \text{where } X_{i0} = 1 \quad , \; i = 1, 2, \dots n.$$

Assuming $E[\xi_i] = 0$, the response function is

$$E[Y_i] = \beta_0 + \beta_1 X_1 + \dots + \beta_{p-1} X_{p-1}.$$

This response function is a hyperplane and it is not longer possible to picture this.

* The parameter β_k $(k=1,2,\dots p-1)$, indicates the change in the mean response $E[Y]$ with a unit increase in X_k when all the other predictor variables are held constant.