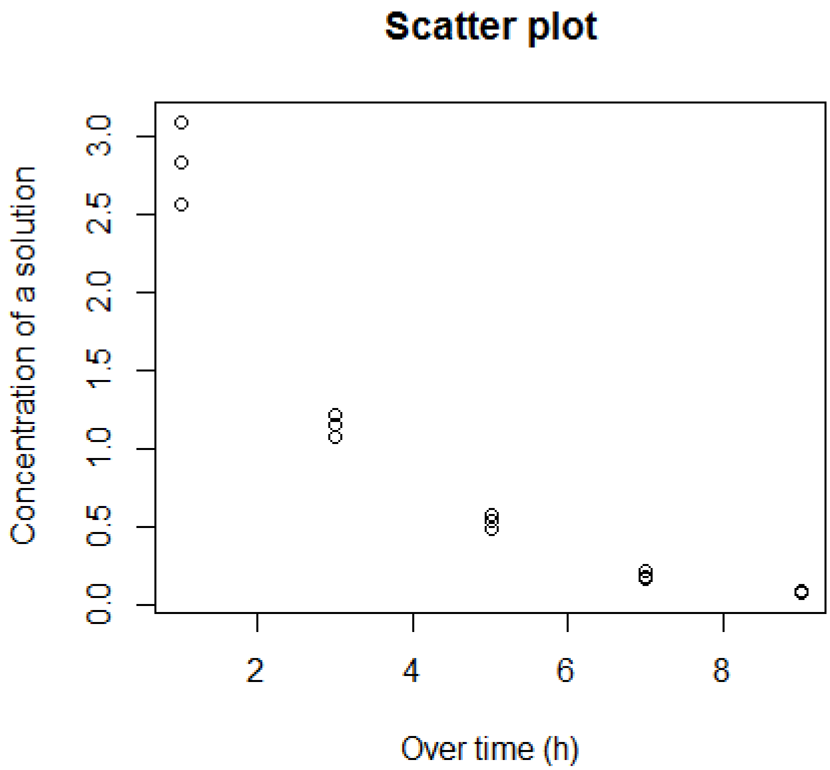
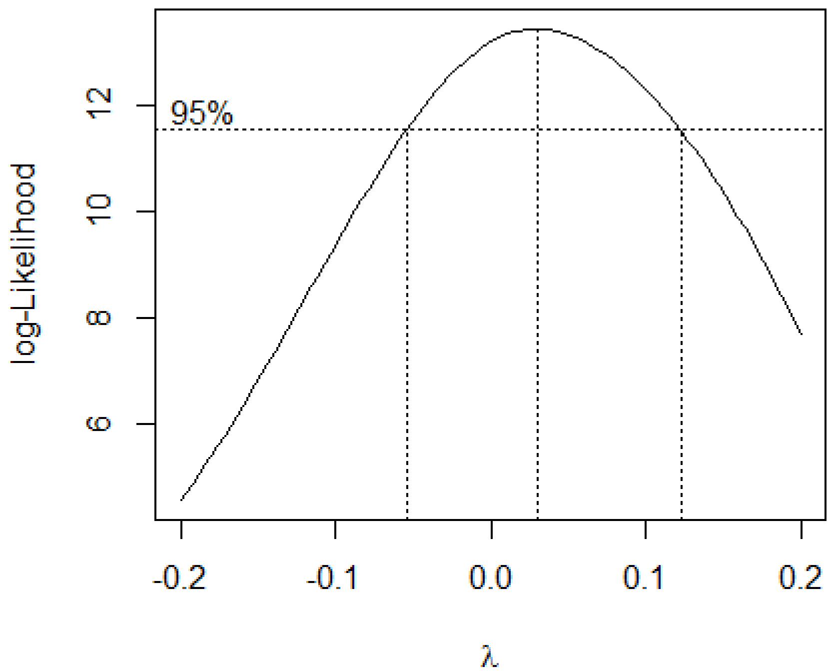
MA 542: Homework 5 Solutions

1. (a) Scatter plot of the data:



Plot shows departure from the linearity of the association. It also show non constant variation for the observations for different time levels. So Y should be transformed. Based on the shape of the graph, the transformation should be $Y' = log_{10}Y$.

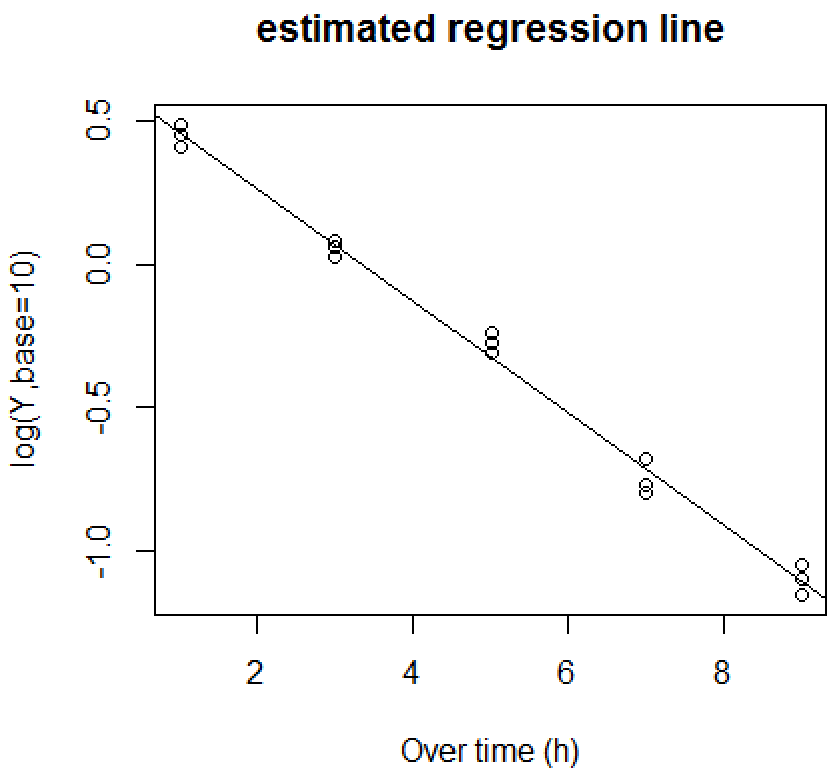
(b) Box-Cox log-likelihood:



So lambda=0 minimizes the SSE. So the transformation Y'=ln Y is suggested.

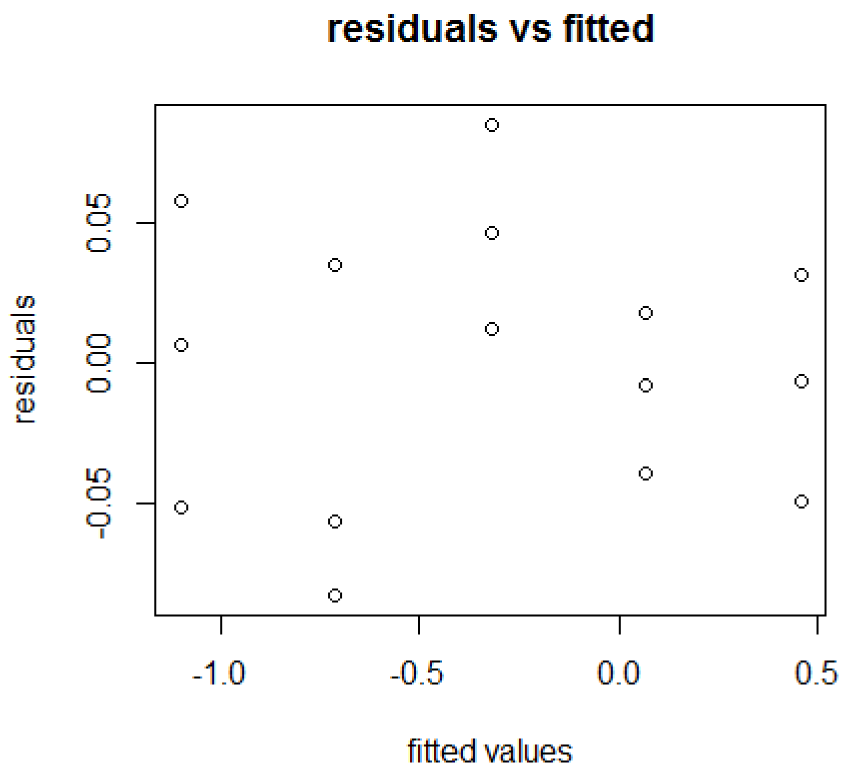
(c) The estimated regression function is $\hat{Y}' = 0.6549 - 0.1954X$, where $\hat{Y}'$ is $\log_{10}(Y)$ and X is the number of hours.

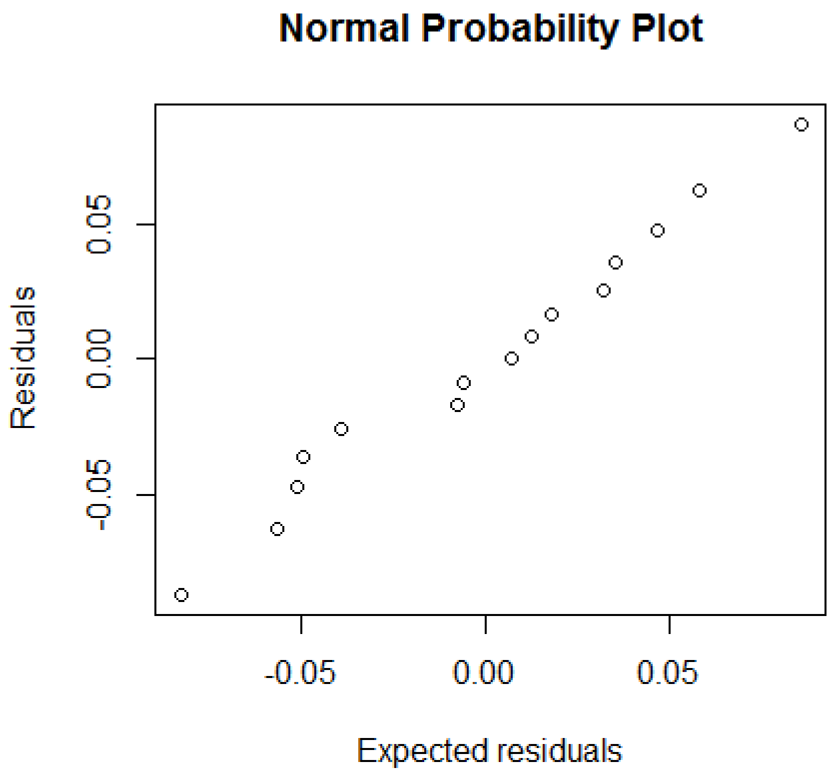
(d) Plot the estimated regression line and the transformed data:



The regression line appears to be a good fit to the transformed data.

(e) Residuals vs fitted values and normal probability plot:

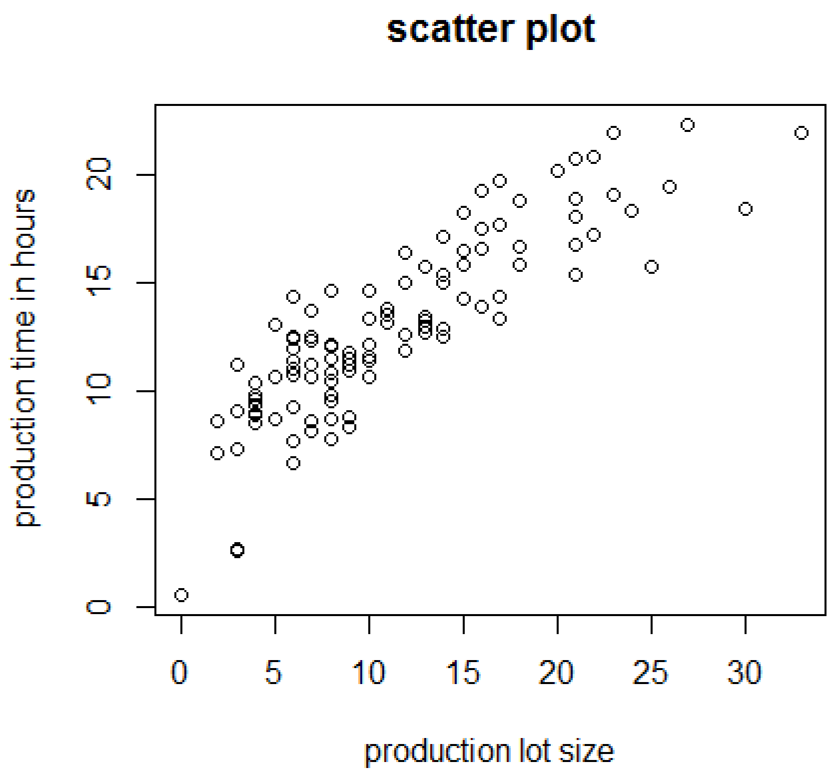




The plot of residuals versus fitted values shows all positive residuals at $X = 5$, but otherwise no pattern. The normal probability plot shows little evidence of nonnormality.

(f) $\hat{Y} = 10^{0.6549 - 0.1954X} = 4.517 \times 6.638^X$

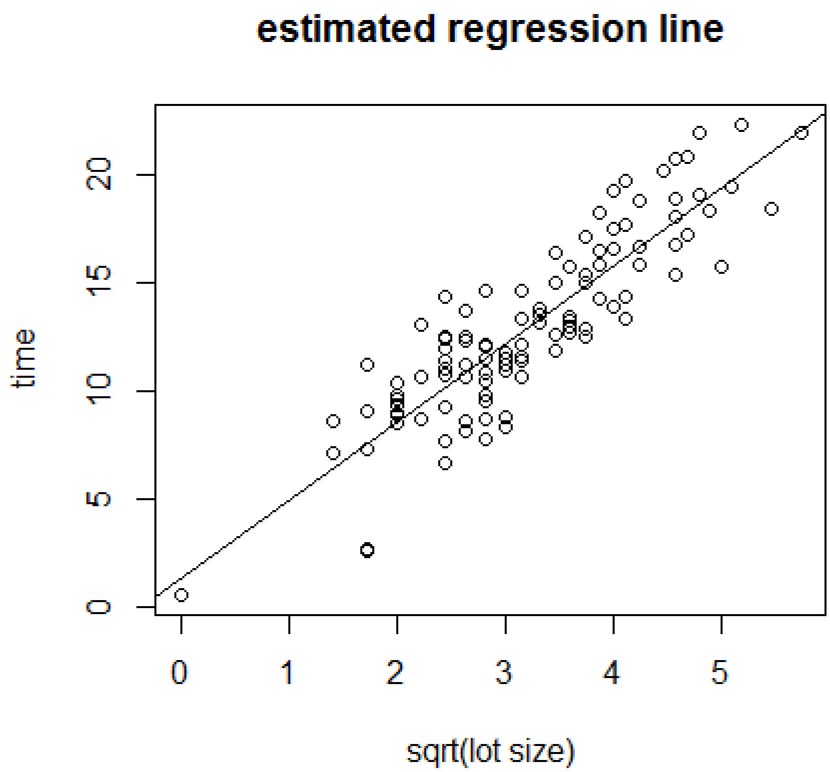
2. (a) Scatter plot of the data:



Scatter plot shows the association is not linear but the variation is approximately constant for each X-level (all the point are in a band with a constant width). So X should be transformed.

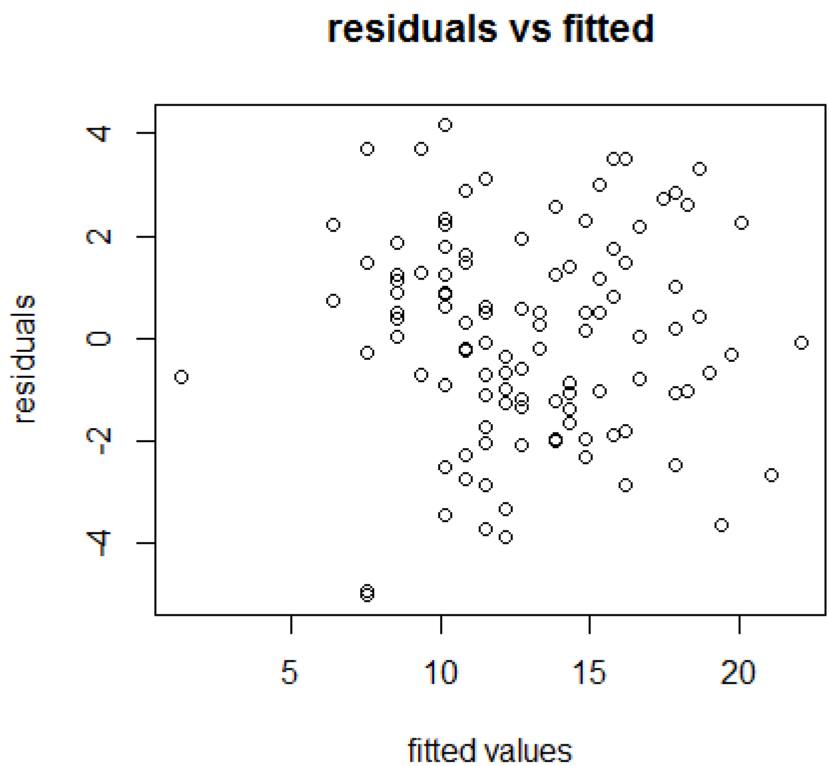
(b) The estimated regression function is $\hat{Y} = 1.2547 - 3.6235X'$, where $X' = \sqrt{X}$.

(c) Plot the estimated regression line and the transformed data:

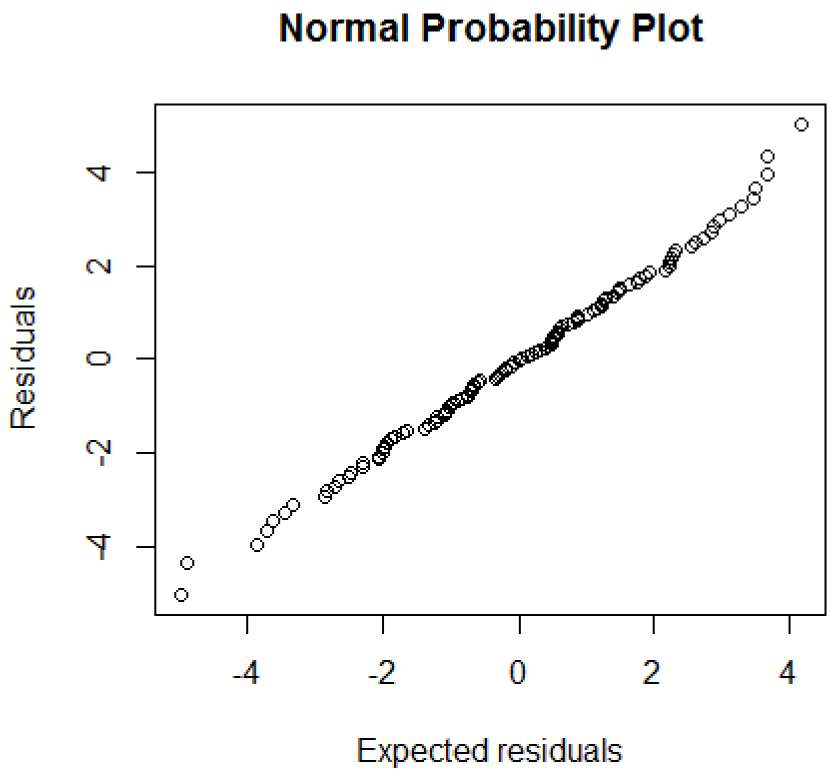


The regression model appears to be a good fit for the transformed data.

(d) Residuals vs fitted values and normal probability plot:



All the residuals are in a horizontal band and so the regression function is linear. Variance of residuals looks approximately same for each X-level and so error term is constant. Also no specific pattern in the plot and that gives the evidence for independent error terms.



Here since the plot is nearly linear, error terms are normal.

(e) $\hat{Y} = 1.2547 + 3.6235\sqrt{X}$.

3. (a) 90% Bonferroni joint confidence intervals for β_0 and β_1 are (162.9013, 174.2987) and (1.8405, 2.2282). Interpretation: In at least 90% of all experiments with these same X values, the confidence intervals so computed will respectively contain the true β_0 and β_1 simultaneously.

   (b) Using formula (4.5) in book, b_0 and b_1 are negatively correlated. The CIs in part (a) above do not consider this.

   (c) The 90% joint confidence interval means that both will be in the interval at least 90% of the time.

4. (a) The Bonferroni intervals are given below.

Time	Estimate	Interval	
20	209.288	206.727	211.848
30	229.631	227.676	231.587
40	249.975	246.782	253.168

Interpretation of 90% simultaneous confidence: In 90% of all possible samples, all three intervals will contain the true mean hardnesses at their respective X values.

(b) The Bonferroni multiplier is $B = t(0.9833, 14) = 2.358773$. A Working-Hotelling multiplier is $W = \sqrt{2F(0.9, 2, 14)} = 2.335152$, which is more efficient.

(c) The Bonferroni multiplier is $B = t(0.95, 14) = 1.76131$. In this case, the Scheffe multiplier is the same as the Working-Hotelling multiplier in (b), so Bonferroni is more efficient.

5. The normal equation (4.13) in book shows it.

YOU ALSO CAN SEE THE FOLLOWING FOR DETAIL ANSWERS FOR THE LAST THREE QUESTIONS.

HW-6 Answer key

4.5

Q1) From previous HWs, we know that

a) $b_0 = 168.6$, $b_1 = 2.034$, $S^2\{b_0\} = 2.657^2$, $S^2\{b_1\} = 0.09039^2$

family confidence coefficient $= 1-\alpha = 0.9$

$\therefore \alpha = 0.1$

$\alpha/2 = 0.05 \quad \alpha/2 = 0.05$

$\therefore$ Bonferroni confidence coefficients are $1-\alpha/2$.

$\therefore$ 90% Bonferroni confidence intervals for β_0 and β_1 are

$b_0 \pm S\{b_0\} t_{0.975;14}$ and $b_1 \pm S\{b_1\} t_{0.975;14}$

$168.6 \pm (2.657)(2.145)$ and $2.034 \pm (0.09039)(2.145)$

$(162.9012, 174.299)$ and $(1.840, 2.228)$ //

meaning:

Both parameters β_0 and β_1 are covered by their confidence intervals with at least 90% probability, for the same sample.

b) (did not grade)

$$cov(b_0, b_1) = \frac{-\bar{X}\sigma^2}{\Sigma(X_i-\bar{X})},$$

Since $\bar{X} > 0$, $cov(b_0, b_1) < 0$.

ie b_0 and b_1 are negetively correlated.

c) Family Confidence Coefficent means that both Prediction intervals ~~one~~ for β_0 and β_1 are correct with at least 90% probability.

4.9

Q2

a) confidence coefficient for each parameter = $1 - \alpha/3$ = [illegible] %

X_h	$\hat{Y}_h$	$s\{\hat{Y}_h\}$
20	$168.6 + 2.034(20) = 209.286$	$\sqrt{(3.234)^2\left[\frac{1}{16} + \frac{(20-28)^2}{1280}\right]} = 1.085$
30	$168.6 + 2.034(30) = 229.631$	$\sqrt{(3.234)^2\left[\frac{1}{16} + \frac{(30-28)^2}{1280}\right]} = 0.828$
40	$168.6 + 2.034(40) = 249.975$	$\sqrt{(3.234)^2\left[\frac{1}{16} + \frac{(40-28)^2}{1280}\right]} = 1.353$

∴ 90% joint confidence intervals for the mean response ~~at~~ respectively at X-levels 20, 30 and 40 are:

20 ⟶ $209.286 \pm 1.085 \cdot t_{1-0.1/6;14} = (206.728, 211.847)$

30 ⟶ $229.631 \pm 0.828 \cdot t_{1-0.1/6;14} = (227.676, 231.586)$

40 ⟶ $249.975 \pm 1.353\, t_{1-\frac{0.1}{6};14} = (246.782, 253.168)$

b) Here we have to compare Bonferroni intervals with Working-Hotelling intervals. We can easily compare "B" values with "W" values.

$$B = t_{1-\frac{0.1}{6};14} = 2.360$$

$$W = \sqrt{2F(1-0.1, 2, 14)} = \sqrt{2(2.726)} = 2.335$$

~~Since W > B,~~

Since $B > W$, Bonferroni intervals are wider than Working-Hotelling intervals.

So Bonferroni Procedure is not the most efficient one.

c)

For $X_n = 30$, $s\{pred\} = \sqrt{(3.234)^2\left[1 + \frac{1}{16} + \frac{(30-28)^2}{1280}\right]} = 3.34$

For $X_n = 40$, $s\{pred\} = \sqrt{(3.234)^2\left[1 + \frac{1}{16} + \frac{(40-28)^2}{1280}\right]} = 3.51$

Here we have to compare Scheffe procedure and Bonferroni Procedure to ~~the~~ find the most efficient one. We can use "S" and "B" values for that.

$$S = \sqrt{gF(1-\alpha; g; n-2)} = \sqrt{2F(1-0.9; 2; 14)} = 2.335$$

$$B = t(1-\alpha/2g, n-2) = t_{0.975;14} = 2.145$$

Since $B < S$, Bonferroni Procedure is the most efficient one.