MA 542 SPRING - 2018
REGRESSION ANALYSIS

Quiz-2 Name: Key

The following is the **R** output for the **plastic Hardness** example.
(X-elapsed time in hours, Y-hardness in Brinell)

Listing 1: R output

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 168.60000    2.65702   63.45  < 2e-16 ***
X             2.03438    0.09039   22.51 2.16e-12 ***
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05    .    0.1

Residual standard error: 3.234 on 14 degrees of freedom
Multiple R-squared:  0.9731,    Adjusted R-squared:  0.9712
F-statistic: 506.5 on 1 and 14 DF,  p-value: 2.159e-12

> Xbar=mean(X)
> Xbar
[1] 28
> sxi_barX2=sum((X-Xbar)^2)
> sxi_barX2
[1] 1280
```

a) Find an estimate for the variance of estimated change in mean Hardness when the elapsed time is increased by one hour.

$$S^2\{b_1\} = (0.09039)^2 \quad \text{directly from the output}$$

$$= 0.00817$$

b) Find an estimate for the variance of the estimated mean response when the elapsed time is 20 hour. $X_n = 20$

$$S^2\{\hat{Y}_n\} = MSE\left[\frac{1}{n} + \frac{(X_n - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right]$$

$$= (3.234)^2\left[\frac{1}{16} + \frac{(20-28)^2}{1280}\right]$$

$$= 1.1766$$

c) Suppose the elapsed time for a new batch of plastic is 20 hour, find an estimate of the variance of the hardness of the new batch. $X_n = 20$,

$$s^2\{\text{Pred}\} = MSE\left[1 + \frac{1}{n} + \frac{(X_n - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right]$$

$$= (3.234)^2\left[1 + \frac{1}{16} + \frac{64}{1280}\right]$$

$$= 11.6353$$

d) Using part c), find the 95% interval for the hardness of the new batch.

Point Estimate: $\hat{Y}_n = 168.6 + 2.0344(20) = 209.288$

$$t_{1-\alpha/2:14} = t_{0.975:14} = 2.145$$

95% Prediction Interval:

$$\hat{Y}_n \pm t_{1-\alpha/2:n-2} \times s'\{\text{pred}\}$$

$$= 209.288 \pm (2.145)\sqrt{11.6353}$$

$$= (201.971, 216.604)$$