White Noise = ARIMA(0,0,0)

- Fixed, constant mean
- Fixed, constant variance
- No correlation over time

- simulating WN : arima.sim(model=c(0,0,0), n, mean, sd)
- estimating WN model : arima(x, order=c(0,0,0))

- No autocorrelation for any lags

Random Walk (RW) = ARIMA(0,1,0) without a constant

- example of non-stationary process
- no specified mean or variance
- strong dependence over time

- its changes or increments are white noise
    - $\boxed{\mathbf{Y_t = Y_{t-1} + e}}$; e = mean zero white noise
    - has on only one parameter, variance of white noise

- $Y_t - Y_{t-1} = e$, white noise with mean 0
    - i.e diff(Y) = WN
- **RW with Drift:** ARIMA(0,1,0) with a constant
    - $Y_t = C + Y_{t-1} + e$, two parameters C and variance
    - i.e $Y_t - Y_{t-1} = C + e$, WN with mean C and variance

- the RW ACF plot is likely to show large **autocorrelation** for many lags without quick decay to zero

The Autoregressive Model

μ , ϕ , σ_e

$\boxed{\mathbf{(Y_t - \mu) = \phi * (Y_{t-1} - \mu) + e}}$; **e is WN(0, σ_e^2)**

Three parameters : Mean - μ, Slope - ϕ, WN variance σ_e^2

- If phi = 0, Y_t is WN with (μ, σ_e^2)
- If phi != 0, Y_t is auto correlated
- If phi = 1 and μ =0, Its a Random Walk, which is not stationary
- Large values of ϕ, leads to greater autocorrelation

- Negative values of ϕ result in oscillatory time series

> **Persistence** is defined by a high correlation between an observation and its lag, while anti-persistence is defined by a large amount of variation between an observation and its lag

> Example and ACFs : pg 5-6

> Simulating: arima.sim(model = list(ar), n) ; -1 <= ar <= 1

> **AR Model Estimation and Forecasting**

- $(Y_t - \mu) = \phi * (Y_{t-1} - \mu) + e$; e is $WN(0, \sigma_e^2)$

  arima(x, model=c(1,0,0))

  ar1 = $\hat{\phi}$

  Intercept = $\hat{\mu}$

  $\sigma^2 = \widehat{\sigma_e^2}$ of WN

- Forecasting : $\boxed{\widehat{\mathbf{Y_t}} = \hat{\boldsymbol{\mu}} + \hat{\boldsymbol{\phi}} * (\widehat{\mathbf{Y_{t-1}}} - \hat{\boldsymbol{\mu}})}$

  - predict(mode, h)

  - $\hat{e} = Y_t - \widehat{Y}t$

  - Forecast Std.Error calculated ???

  - Interval = $\boxed{\widehat{\mathbf{Y_t}} \pm \mathbf{2} * \boldsymbol{SE}}$ for 95% confidence

- Dissipating **autocorrelation** across several lags
- More Info: https://www.otexts.org/fpp/8/3

The Simple Moving Average

- $\boxed{\mathbf{Y_t = \boldsymbol{\mu} + e_t + \Theta\, e_{t-1}}}$
  - Mean: μ
  - Slope: Θ
  - WN variance: σ_e^2
- If slope = 0, then Y_t is $WN(\mu, \sigma_e^2)$
- If slope !=0, then Y_t is auto-correlated
- Larger value of Θ leads to greater autocorrelation
- Negative value of Θ leads to oscillatory time series
- Simulation

- arima.sim(model=list(ma= Θ), n)

More Info: https://www.otexts.org/fpp/8/4

- Estimation

$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

$$\epsilon_t \sim WhiteNoise(0, \sigma_\epsilon^2)$$

```
> MA_inflation_changes <- arima(inflation_changes, order = c(0, 0, 1))
> print(MA_inflation_changes)

Coefficients:
          ma1  intercept
      -0.7932     0.0010
s.e.   0.0355     0.0281
sigma^2 estimated as 8.882
```

  - $\texttt{ma1} = \hat{\theta}$, $\texttt{intercept} = \hat{\mu}$, $\texttt{sigma^2} = \hat{\sigma}_\epsilon^2$

- Forecasting : $\boxed{\widehat{\mathbf{Y_t}} = \hat{\boldsymbol{\mu}} + \hat{\theta} * \widehat{\mathbf{e_{t-1}}}}$

  - predict(mode, h)

  - $\hat{e} = Y_t - \hat{Y}t$

  - Interval = $\boxed{\widehat{\mathbf{Y_t}} \pm \mathbf{2} * \boldsymbol{SE}}$ for 95% confidence

- **Autocorrelation** for the first lag only

MA and AR Models

- MA model:

$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

- AR model:

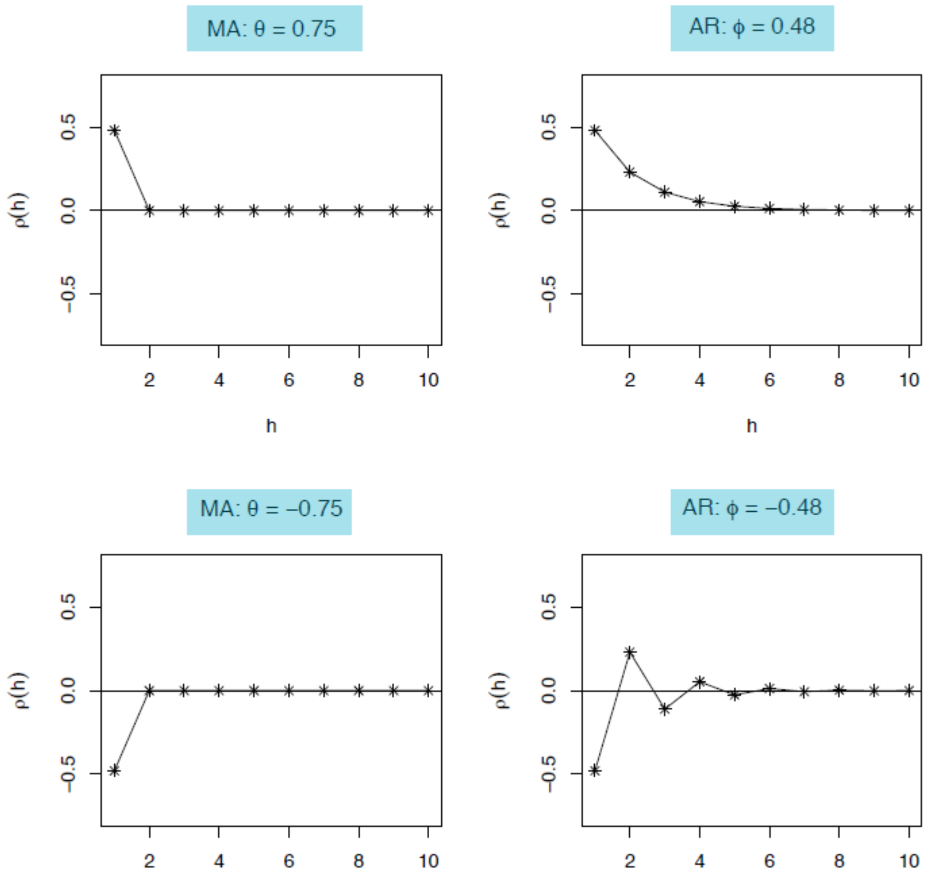
$$(Today - Mean) = Slope * (Yesterday - Mean) + Noise$$

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

- Where:

$$\epsilon_t \sim WhiteNoise(0, \sigma_\epsilon^2)$$

MA models have autocorrelation only at lag-1 where AR models can have it at other lags



Model fitness measured using AIC and BIC, lower the better