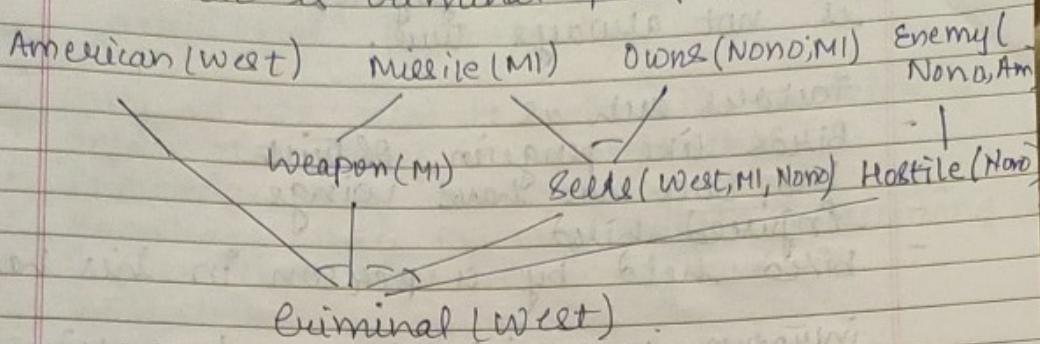


Forward chaining
Col. West is Criminal Proof



- ⇒ FC is data-driven, automatic, unconscious processing
- BC is goal driven, appropriate for problem solving & has lesser complexity.

- ⇒ $\exists x \text{ Dog}(x) \wedge \text{Owns}(\text{Jack}, x)$
 $\forall x (\exists y \text{ Dog}(y) \wedge \text{Owns}(x, y)) \rightarrow \text{AnimalLover}(x)$
 $\forall x \text{ AnimalLover}(x) \rightarrow (\forall y \text{ Animal}(y) \rightarrow \text{Kills}(x, y))$
 $\text{Kills}(\text{Jack}, \text{Tuna}) \vee \text{Kills}(\text{Curiosity}, \text{Tuna})$
 $\text{Cat}(\text{Tuna})$
 $\forall x \text{ Cat}(x) \rightarrow \text{Animal}(x)$

Dealing with uncertainty in AI
9/9/19
 we need to provide a no. of facts (universal) in the knowledge base. We cannot ignore them just because they haven't been explicitly mentioned.

But, there may exist uncertainty in

case base reasoning

Rules.

$\forall x \text{ bird}(x) \rightarrow \text{fly}(x)$
is not always true.

Factors such as:

- Birds like penguin, ostrich has/does not have wings
- Injured bird
- bird held by a person in his hands

introduce uncertainty in the knowledge base
Rules cannot ^(may not) always be true

Thus, we can keep a portion for accommodating uncertainty in every rule.

Ex: $\forall x \text{ bird}(x) \wedge \text{p}(x) \wedge \text{o}(x) \wedge \text{k}(x) \wedge \text{hw}(x) \rightarrow \text{bf}(x) [80\%]$

20% kept aside
for uncertainty, i.e.
this rule yields correct
ans. in 80% cases only

can occur due to lack of knowledge or
lack of info. provided by user or
misrepresentation.

Dealing with uncertainty

- Probability
- Fuzzy logic
- Neural network
- Hidden markup model

Probability:-

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Probability of P₁ event given the occurrence of event P₂ is called conditional probability.

12/9/19

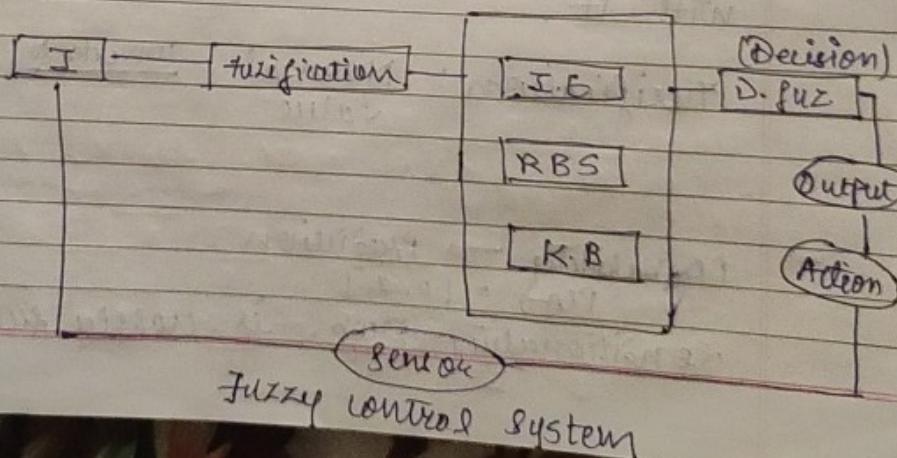
- Why do we need to deal with uncertainty?
- Information may be questionable.
 - Knowledge may be questionable.
 - Knowledge may not be truth preserving.
 - Information may be ambiguous or uncertain.
 - Output is expected in probability.

Sources of uncertainty

- * Uncertain data (Incomplete data)
- * Uncertain knowledge
- * Knowledge representation technique
- * Inference technique

Methods to Deal with uncertainty

- * Fuzzy logic * Probabilistic model
- * Hidden markov model * Neural network
- * Uncertainty factor with quantitative F.L



conditional probability:

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

fuzzy logic

Degree of membership

Temperature

10° 15° 20° 25° 30° 35° 40° 45° 50°

→ cold → cool → average

→ warm → hot → very hot

Degree of membership depicts the probability of an item to belong to class A.

If only two classes : full and empty are presented, then probability or degree of membership of each is roughly around 0.5.

Every object has a probabilistic value based on fuzzy value associated with it.

Fixification: Normal value converted to Fuzzy value etc.

(hot/cold)
etc.

⇒ Probability → Prediction.

$$P(A) = [0, 1]$$

Conditionality prob. is closely related to

The dependency between two entities.

$$P(A \cap B) = P\left(\frac{A}{B}\right) * P(B)$$

$$\text{Also, } P(A \cap B) = P\left(\frac{B}{A}\right) * P(A)$$

$$\therefore P\left(\frac{A}{B}\right) * P(B) = P\left(\frac{B}{A}\right) * P(A)$$

$$\Rightarrow P\left(\frac{A}{B}\right) = \frac{P(B/A) * P(A)}{P(B)}$$

When given the prob. of B occurring when A has occurred, we can determine the reverse dependency by using the above formula.

13(a)(i) Representation of fuzzy set.

i) $A = \{ \overset{\text{member}}{(10, 0.1)}, \overset{\text{degree of membership towards A}}{(15, 0.3)}, \dots \}$

$\therefore \mu_B^{(A)} = 0.1$

ii) $A = \{ \overset{\text{DOM}}{10} + \overset{\text{member}}{\frac{0.1}{10}} + \overset{\text{DOM}}{15} + \dots \}$

\Rightarrow Set towards with DOM of every variable is 0 i.e. an empty fuzzy set.

DOM of every variable 1 is universal fuzzy set.

$$\Rightarrow A = \{ \frac{0.1}{10} + \frac{0.2}{20} + \frac{0.3}{30} + \frac{0.8}{70} \}$$

$$B = \left\{ \frac{0.3}{15} + \frac{0.1}{20} + \frac{0.9}{35} + \frac{0.8}{20} \right\}$$

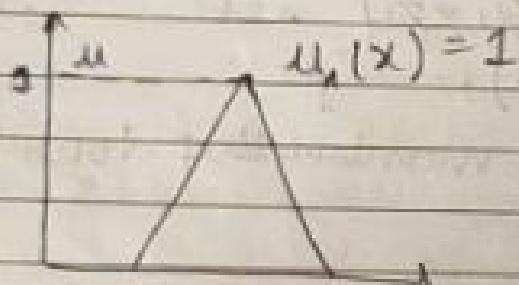
$$\min(A \cap B) = \sum_{x \in X} (\mu_A(x), \mu_B(x))$$

$$\text{Here, } \min(A \cap B) = \left\{ \frac{0}{10} + \frac{0}{15} + \frac{0.1}{20} + \frac{0}{30} + \frac{0}{35} + \frac{0}{70} \right\}$$

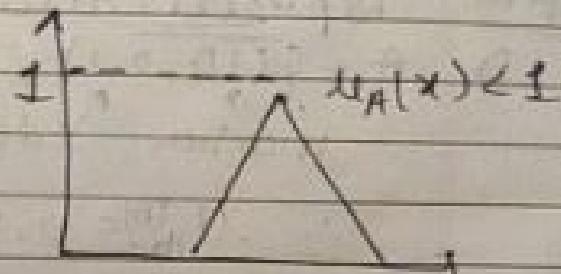
$$= \left\{ \frac{0.1}{20} + \frac{0.8}{70} \right\}$$

Empty fuzzy set: $\mu_A(x) = 0$.

Normal FS: A normal FS is one whose membership func. has at least one element x_i in the universe where its membership func. equals 1.



Normal FS



Non-normal FS

Equality of FS: $\mu_A(x) = \mu_B(x)$

Union of two FS:

$$\mu_{A \cup B}(x) = \max. [\mu_A(x), \mu_B(x)]$$

$$A = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$

$$B = \left\{ \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}$$

$$M_{A \cup B} = \left\{ \frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5} \right\}$$

Intersection :

Fuzzy set - which ele. belongs to both sets?

Fuzzy set - How much of the ele. is in both sets?

$$\Rightarrow A \{ (x_1, 0.8), (x_2, 1), (x_3, 0.2), (x_4, 0.5), (x_5, 0.6), (x_6, 1) \}$$

$$B \{ (x_2, 0), (x_3, 1), (x_5, 1), (x_6, 0.2), (x_7, 0.8) \}$$

$$A \cup B = \{ (x_1, 0.8), (x_2, 1), (x_3, 1), (x_4, 0.5), (x_5, 1), (x_6, 1), (x_7, 0.8) \}$$

$$A \cap B = \{ (x_3, 0.2), (x_5, 0.6), (x_6, 0.2) \}$$

$$\Rightarrow A = \left\{ \frac{0.8}{1} + \frac{0.75}{2} + \frac{0.90}{3} + \frac{0.99}{4} + \frac{1}{5} \right\}$$

$$\bar{A} = 1 - M_A(x)$$

$$\bar{A} = \left\{ \frac{0.2}{1} + \frac{0.25}{2} + \frac{0.1}{3} + \frac{0.01}{4} + \frac{0}{5} \right\}$$

\uparrow complement of a set.

\Rightarrow Product of two FS.

$$M_{A \cdot B}(x) = M_A(x) \cdot M_B(x)$$

16/9/19 1) Strong α -cut : $A_\alpha = \{ x | M_A(x) > \alpha \}$

Weak α -cut : $A_\alpha = \{ x | M_A(x) \geq \alpha \}$

2) Support of fuzzy set

$$\text{Support}(A) = \{x \mid u_A(x) > 0\}$$

3) Core of a fuzzy set

$$\text{core}(A) = \{x \mid u_A(x) = 1\}$$

4) Height, $h(A)$ = is the largest value of $u_A(x)$ for which α -cut is nonempty

[Result of above 4 op. will be a CRISP set]

fuzzy set :

$$A \cap \bar{A} \neq \emptyset$$

$$A \cup \bar{A} \neq 1$$

$$(A \cap B)' = A' \cup B'$$

$$(A \cup B)' = A' \cap B'$$

Ex: $A = \left\{ \frac{0.2}{1} + \frac{0.3}{3} + \frac{0.6}{4} + \frac{0.9}{7} + \frac{1}{8} \right\}$

$$\alpha = 0.5$$

Strong α -cut

$$A_{0.5} = \{4, 7, 8\}$$

$$\alpha = 0.3$$

Weak α -cut

$$A_{0.3} = \{4, 7, 8\}$$

$$A_{\frac{0.3}{2}} = \{2, 4, 7, 8\}$$

// Strong α -cut

Weak α -cut

$$\Rightarrow A = \left\{ \frac{0.1}{2} + \frac{0.4}{3} + \frac{0}{4} + \frac{0.5}{6} + \frac{0}{8} + \frac{1}{9} \right\}$$

$$\text{Supp}(A) = \{2, 3, 6, 9\}$$

Fuzzy singleton is if the support of A has only one variable in the crisp set and its DOM is 1 then it is called fuzzy singleton.

$$\text{supp}(A) = \{x\} \quad \& \quad u_A(x) = 1$$

$$\Rightarrow A = \left\{ \frac{0.2}{x_1} + \frac{0.9}{x_2} + \frac{1}{x_3} + \frac{0.8}{x_4} + \frac{1}{x_5} + \frac{0.9}{x_6} \right\}$$

$$\text{supp}(A) = \{x_3, x_5\}$$

$$\Rightarrow A = \left\{ \frac{0.3}{a} + \frac{0.2}{b} + \frac{0.4}{c} + \frac{0.1}{d} + \frac{0.6}{e} \right\}$$

$$u(A) = 0.6$$

\Rightarrow Fuzzy operations

Cartesian Multiplication

$$C = A \times B$$

$$= [u_c(x) / (a, b) \mid a \in A, b \in B, u_c(x) = \min(u_a(a), u_b(b))]$$

$$A = \frac{0.2}{3} + \frac{1}{5} + \frac{0.5}{7}; \quad B = \frac{0.8}{2} + \frac{0.3}{6}$$

$$A \times B = [\min((0.2)(0.8)) / (3, 2),$$

$$\min((0.2)(0.3)) / (3, 6)$$

$$\min((1)(0.8)) / (5, 2)$$

$$\min((1)(0.3)) / (5, 6)$$

$$\min((0.5)(0.8)) / (7, 2)$$

$$\min((0.5)(0.3)) / (7, 6)]$$

$$= \frac{0.2}{(3,2)} + \frac{0.2}{(3,6)} + \frac{0.8}{(5,2)} + \frac{0.3}{(5,6)} + \frac{0.5}{(7,2)} + \frac{0.3}{(7,6)}$$

$$\Rightarrow x = \frac{0.6}{3} + \frac{0.7}{8} + \frac{0.9}{9} + \frac{1}{10}$$

$$y = \frac{0.5}{2} + \frac{0.4}{3}$$

$$z = x \times y$$

$$= \left[\frac{0.5}{(3,2)} + \frac{0.4}{(3,3)} + \frac{0.5}{(8,2)} + \frac{0.4}{(8,3)} + \frac{0.5}{(9,2)} + \right. \\ \left. \frac{0.4}{(9,3)} + \frac{0.5}{(10,2)} + \frac{0.4}{(10,3)} \right]$$

$$\Rightarrow A \cdot B = u_A(x) \cdot u_B(x) / x \quad | \quad x \in A, x \in B$$

$$\text{Ex: } A = \frac{0.2}{3} + \frac{1}{5} + \frac{0.5}{7}$$

$$B = \frac{0.1}{3} + \frac{0.3}{7} + \frac{0.2}{8}$$

$$A \cdot B = \frac{(0.2)(0.1)}{3} + \frac{(1)(0)}{5} + \frac{(0.5)(0.3)}{7} + \\ \left. \frac{(0.2)(0)}{8} \right]$$

$$= \frac{0.02}{3} + \frac{0.15}{7}$$

$$\Rightarrow \text{Exponent of } A \\ A^x = \{ u_A(x)^x / x \mid x \in A \}$$

$$\text{Ex: } A = \frac{0.5}{4} + \frac{1}{5} + \frac{0.4}{6}$$

$$A^2 = \frac{(0.5)^2}{4} + \frac{(1)^2}{5} + \frac{(0.4)^2}{6}$$

$$= \frac{0.25}{4} + \frac{1}{5} + \frac{0.16}{6}$$

Algebraic Sum

$$A + B = \mu_A(x) + \mu_B(x) - (\mu_A(x) \cdot \mu_B(x))$$

$$\text{Ex: } A = \frac{0.2}{1} + \frac{1}{2} + \frac{0.4}{3}$$

$$B = \frac{0.3}{1} + \frac{0.4}{2} + \frac{0.2}{7}$$

μ_{A+B} \leftarrow event

$P(E|H)$ \curvearrowright hypothesis

Bounded sum ($A \oplus B$)

$$\mu_{A \oplus B}(x) = \min(1, \mu_A(x) + \mu_B(x))$$

$$\mu_{A \ominus B}(x) = \min(1, \mu_A(x) - \mu_B(x))$$

→ Machines generally use CRISP dataset while, humans & rules tend to use fuzzy values.

Fuzzy control: combine use of fuzzy linguistic variables with fuzzy logic.

→ Fuzzy logic deals with imprecision and vagueness, but not uncertainty.

A A	D D
OD DD	AA AA

Box B,
 $(0.6 - \text{prob. of choosing } B_1)$

B_2
 (0.4)

Joint Prob. Dist. (JPD):

Box	Fruit	
	F = D	F = A
B = B ₁	0.4	0.2
B = B ₂	0.24	0.16

If we want to get the overall prob. of choosing an orange, we simply add the column ($F = A$). This is marginal probability.

→ If $P(A/B) = P(A)$, then events A & B are independent

$$\begin{aligned} \text{Ex: } P(B | J, TM, E, A) \\ = \frac{P(B \wedge J \wedge TM \wedge E \wedge A)}{P(J \wedge TM \wedge E \wedge A)} \end{aligned}$$

$$\begin{aligned} &= P(J | TM \wedge E \wedge A) * P(TM \wedge E \wedge A) \end{aligned}$$

$$\begin{aligned} \Rightarrow P(C, A, B) &= P(C | A, B) * P(A, B) \\ &= P(C | A, B) * P(A | B) * P(B) \end{aligned}$$

$\neg B, \neg E, \exists A, J, M$

$$\Rightarrow P(\neg B | \neg E, A, J, M)$$

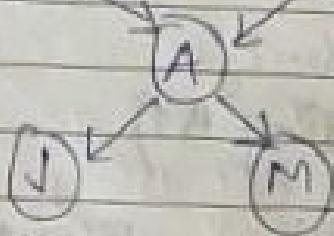
independent of all other ctry., so $P(B)$

$$P(\neg B) * P(\neg E) * P(A | \neg B, \neg E) * P(\neg J | A) * P(\neg M | A)$$

$$P(B) = 0.001$$

(B)

$$(E) P(E) = 0.002$$



9/19

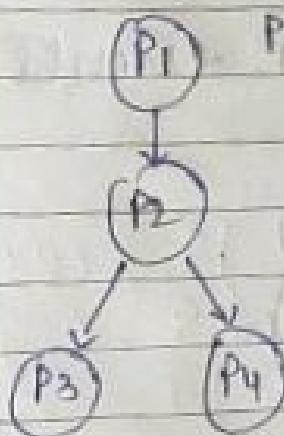
B E

B	E	P(A)
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	P(J)
T	0.90
F	0.05

A	P(M)
T	0.70
F	0.01

conditional Probability
table)



P1	P(P2)	P2	P(P3)
T	0.8	T	0.2
F	0.5	F	0.3

P2	P(P4)
T	0.8
F	0.5

Finding joint probability = $P(B, E, A, J, M)$

$$= P(B) * P(E) * P(A | B, E) * P(J | A) * P(M | A)$$

$$\text{Also, } P(J) = P(A) * P(J | A) + P(J | \neg A) * P(\neg A)$$

$$= P(J \cap A) + P(J \cap \neg A)$$

$$= 0.9 * P(A) + 0.05 * P(\neg A)$$

$$\begin{aligned}
 P(A) &= P(A|B, E) * P(B) * P(E) + \\
 &\quad P(A|\bar{B}, E) * P(\bar{B}) * P(E) + \\
 &\quad P(A|\bar{B}, \bar{E}) * P(\bar{B}) * P(\bar{E}) + \\
 &\quad P(A|\bar{B}, \bar{E}) = P(\bar{B}) * P(\bar{E}) \\
 &= (0.95 \times 0.001 \times 0.002) + \\
 &\quad (0.95 \times 0.001 \times 0.998) + \\
 &\quad (0.29 \times 0.999 \times 0.002) + \\
 &\quad (0.001 \times 0.999 \times 0.998) \\
 &= 0.00252
 \end{aligned}$$

$$\begin{aligned}
 \therefore P(J) &= (0.9 \times 0.0025) + (0.05 \times 0.9975) \\
 &= 0.0521
 \end{aligned}$$

$$P(M) = P(M|A) * P(A) + P(M|\bar{A}) * P(\bar{A})$$

$$\Rightarrow \text{Rule: } P(A, B) = P(A|B) * P(B)$$

Q2. (i) $P(\bar{P}_3) = ?$

$$= 1 - P(P_3)$$

$$P(P_3) = P(P_3|P_2) * P(P_2) + P(P_3|\bar{P}_2) * P(\bar{P}_2)$$

$$P(P_2) = P(P_2|P_1) * P(P_1) + P(P_2|\bar{P}_1) * P(\bar{P}_1)$$

$$= (0.8 \times 0.4) + (0.5 \times 0.6)$$

$$= 0.32 + 0.3 = 0.62$$

$$P(P_3) = (0.2 \times 0.62) + (0.3 \times 0.38) = 0.238$$

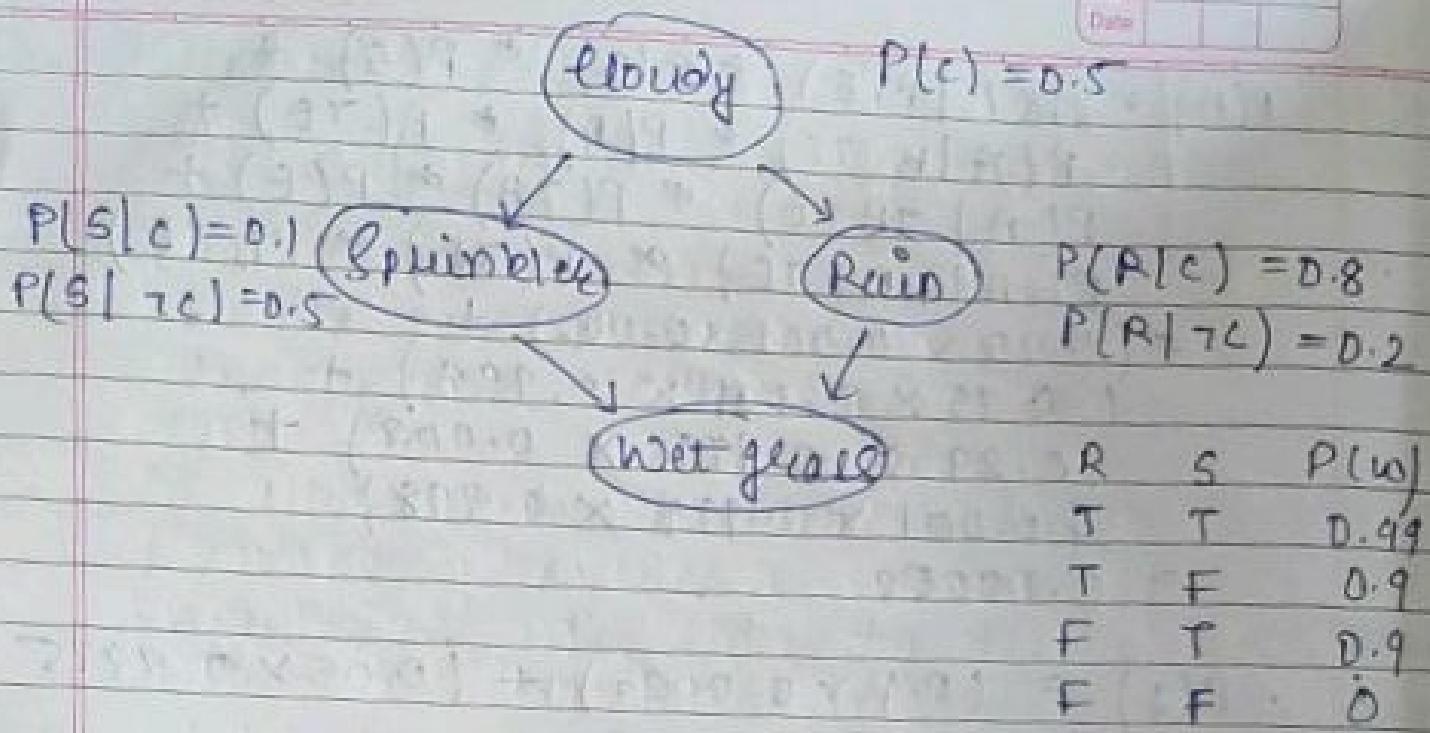
$$\therefore P(\bar{P}_3) = 0.762$$

$$(ii) P(P_2 | \bar{P}_3) \quad (iii) P(P_1 | P_2, \bar{P}_3) \quad (iv) P(P_1 | P_3, P_4)$$

Q3. $P(C, R, S, W) = ?$

$$P(R) = ?$$

$$P(W) = ?$$



Q4

$$\begin{aligned}
 & A \rightarrow B \rightarrow C \\
 P(c|B,A) &= \frac{P(A,B,c)}{P(A,B)} \\
 &= \frac{P(A) * P(B|A) * P(c|B)}{P(A) * P(B|A)} \\
 &= P(c|B)
 \end{aligned}$$

Q2

$$\begin{aligned}
 \text{(ii)} \quad P(P_2 | \neg P_3) &= P(P_2) = 0.62 \\
 \text{(iii)} \quad P(P_1 | P_2, \neg P_3) &= P(P_1) = 0.4 \\
 \text{(iv)} \quad P(P_1 | P_3, P_4) &= P(P_1) = 0.4
 \end{aligned}$$

$$\begin{aligned}
 \underline{\text{Q3}} \quad P(C, R, S, W) &= P(C) * P(R|C) * P(S|C) * \\
 &\quad P(W|R, S)
 \end{aligned}$$

$$P(R|C) = 0.8 \quad P(S|C) = 0.1$$

$$P(W|R, S) = P(W|R) * P(W|S) = 0.99$$

$$\begin{aligned}
 \therefore P(C, R, S, W) &= 0.5 * 0.8 * 0.1 * 0.99 \\
 &= 0.0396
 \end{aligned}$$

$$\begin{aligned}
 P(R) &= P(R|C) * P(C) + P(R|\neg C) * P(\neg C) \\
 &= (0.8 \times 0.5) + (0.2 \times 0.5) \\
 &= 0.4 + 0.1 = 0.5
 \end{aligned}$$

$$\begin{aligned}
 P(W) &= P(W|R, S) * P(R) * P(S) + \\
 &\quad P(W|R, \neg S) * P(R) * P(\neg S) + \\
 &\quad P(W|\neg R, S) * P(\neg R) * P(S) + \\
 &\quad P(W|\neg R, \neg S) * P(\neg R) * P(\neg S)
 \end{aligned}$$

$$\begin{aligned}
 P(S) &= P(S|C) * P(C) + P(S|\neg C) * P(\neg C) \\
 &= (0.1 \times 0.5) + (0.5 \times 0.5) \\
 &= 0.05 + 0.25 = 0.3
 \end{aligned}$$

$$\begin{aligned}
 P(W) &= (0.99 \times 0.5 \times 0.3) + \\
 &\quad (0.9 \times 0.5 \times 0.7) + \\
 &\quad (0.9 \times 0.5 \times 0.3) + \\
 &\quad (0 \times 0.5 \times 0.7) = 0.5985
 \end{aligned}$$

Non-monotonic reasoning

When we add some additional knowledge inside a rule ~~which~~ the rule may not hold true for all cases, it is called non-monotonic reasoning or logic.

$$\text{Ex: } \forall x \text{ bird}(x) \rightarrow \text{fly}(x). \quad (\text{holds true})$$

$$\forall x \text{ bird}(x) \wedge \text{penguin}(x) \rightarrow \text{fly}(x) \quad (\text{untrue})$$

Monotonic logic

If a rule is true, it will hold true for all cases even if we add some knowledge to it.

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Page No.

Date

Frame-Based Expert Systems

- Type of Knowledge Representation Technique

Frame is a data structure with typical knowledge about a particular object or concept. (collection of slots)

Each frame has its own name and set of attributes associated with it.

Frames provide a means of organizing knowledge in slots to describe various attributes.

Slots are used to store values. It may contain a default value or pointer to another frame.

Info. in slot:

- Frame name
- Relationship of frame to other frames
- Slot value
- Default slot value
- Range of slot value
- Procedural info.

Script Representation

Used to represent events.

Its components include:

- i) Entry conditions. must be satisfied before events in the script can occur.
- ii) Results
- iii) People
- iv) Roles

v) Traces

vi) Scenes

Actions: ATRANS, PTRANS, PROPEL, MOVE, GRASP,
INVEST, EXPEL, MTRANS, MBUILD,
CONC, SPEAK, ATTEND

[↑] conceptualize or think
about an idea

Ex: Restaurant Script

Script: Restaurant

Roles: S = Customer C = Cook M = Cashier
W = Waiter D = Owner F = Food.

Track: Coffee Shop Menu

Props: Tables Money
Check Menu

Scenes: Entering, Ordering, Exiting

Advantages:

- capable of predicting implicit events
- single coherent interpretation may be built up from a collection of observations

20/19

Search optimization

Random search

Informed

uninformed

Fuzzy Computing

Genetic
Algo.

AC

Instead of going for every case a case is picked up at random and it is evaluated. These are then compared to arrive at an optimal solution. This is the goal of random search.

Hence, in genetic algo:

generate population (randomly)

(Utility func.)

(Fitness func.)

Regenerate solution using population

Regeneration:



so Solutions
(population)

g
(upper limit)
(good)

b
(lower limit)
(bad)

Pick pairs: 1 g — 1 b → generate new soln.
 1 g - 1 g
 1 b 1 b

∴ population increases.

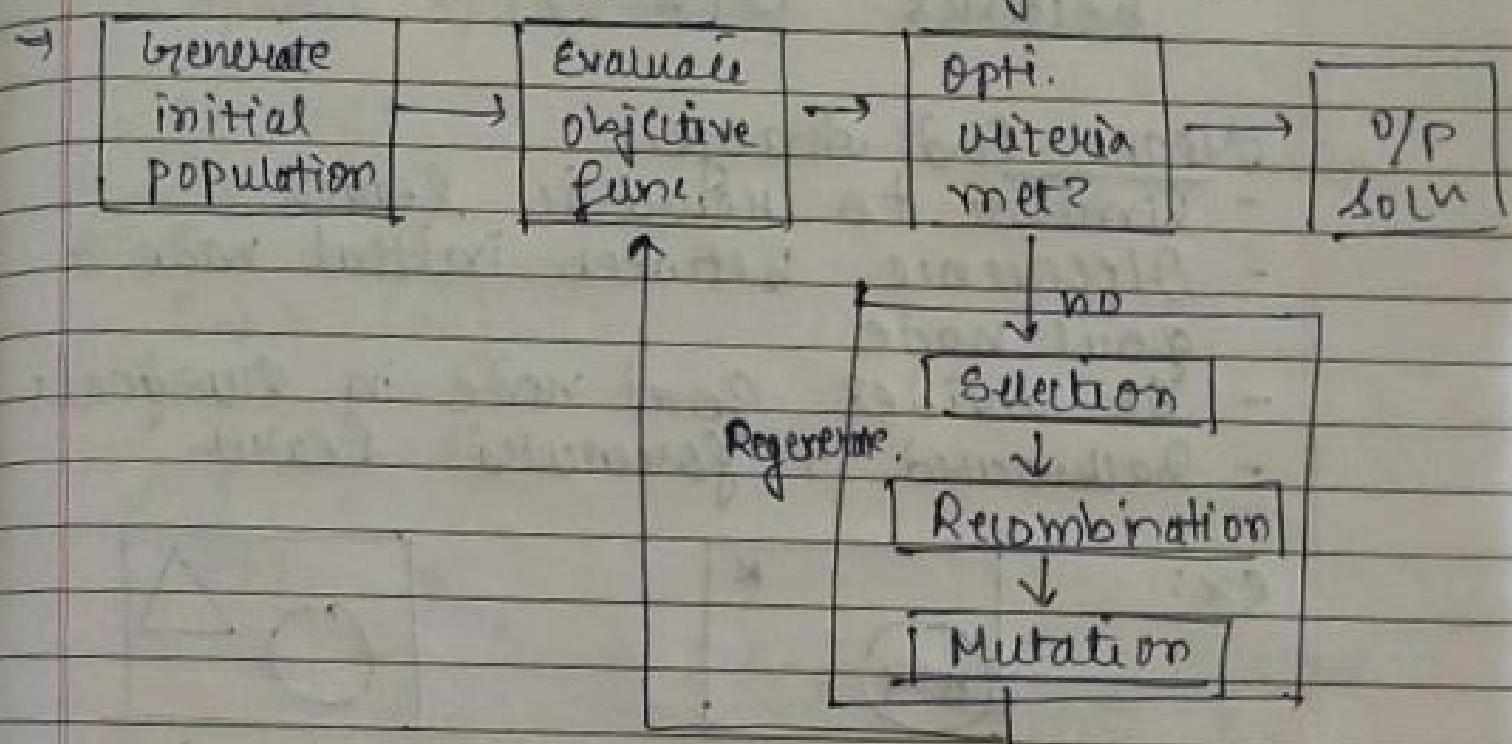
further it is increased by changing the sequence of soln.

→ The main problem with the traditional

Search techniques is that they may end up with local optima rather than a global one.

Non-Traditional: a) Genetic Algo.
b) Simulated Annealing
c) Ant Colony optimisation

Genetic Algo: Based on natural evolution of human genes.



Guided random search mechanism

because we have a utility func to know whether a soln is good/bad

10/19

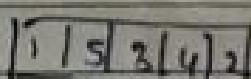
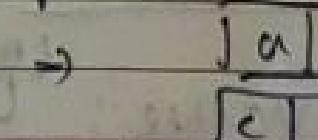
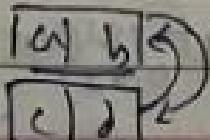
(19)

(49)

mutation:



crossover point



Problem Solving techniques

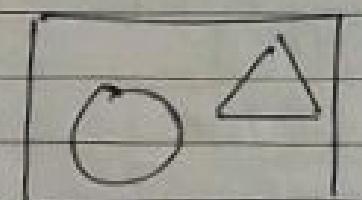
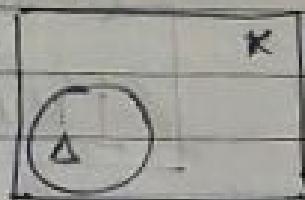
Bidirectional search

- Blind search technique
- 2 Agents / algorithms
- Start from start node and simultaneously the end node
- When both coincide or arrive at a common soln, the process ends
- Depth = $\frac{d}{2}$ ∴ Time complexity becomes $O(b^{\frac{d}{2}})$

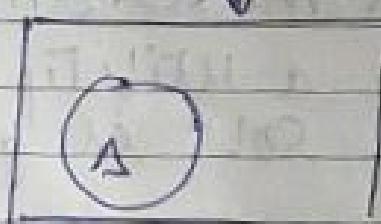
Means-End analysis

- Similar to heuristic search
- Difference between initial node & goal node
- It divides goal node in sub-goals
- Backward + forward search

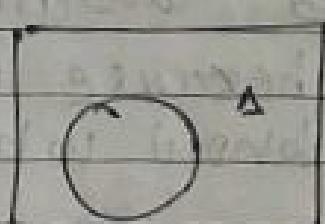
Ex:



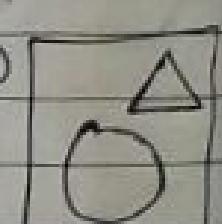
(remove) ↓ Initial



(move) →



(expand) →

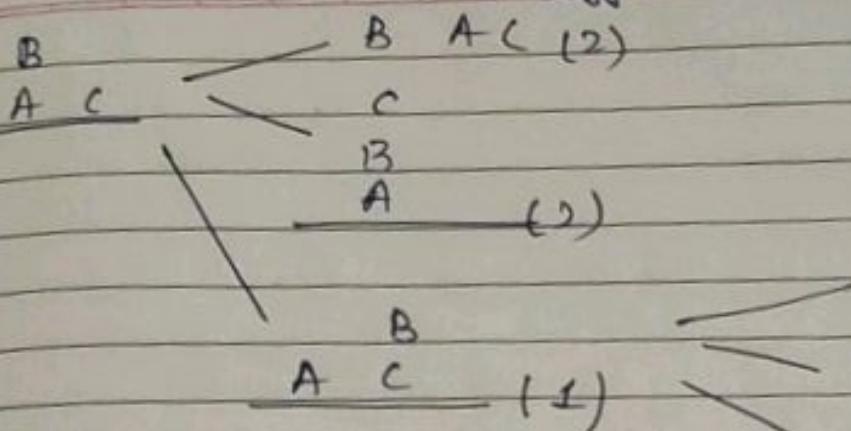


Ex: Initial : B
 A C

Goal : C
 B
 A

Difference value : $0^{(\text{lat. (right (const))})} + 1^{(\text{const})} + 1^{(\text{some})} = 2$

diff



Simulated Annealing

- Global optimization achieved.

$$c = \text{init}$$

$$E_c = E(c)$$

$$; T = T_{\max} \rightarrow T_{\min}$$

$$N = \text{Next}$$

$$E_N = E(N)$$

$$\Delta E = E_N - E_c$$

$$\text{if } (\Delta E > 0)$$

$$e = N$$

else

$$e^{\frac{-\Delta E}{T}}$$

$$\geq \text{rand}(0, 1)$$

~~less~~

$$c = N$$