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The Effect of Pressure on Porosity and the Transport Properties of Rock

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We reanalyze the flow model proposed by Wyllie and Rose (1950) in which the complicated flow network through the pore phase of rock is replaced by a single representative conduit. Although the model is a very simple representation of the complicated pore phase in rock, we find that it provides an adequate simulation of how the transport properties vary with external pressure. Expressions derived for fluid permeability k and formation factor F are combined to give an expression for the mean hydraulic radius of the pore phase. Using this expression, we show that the exponent r in the empirical relationship $k \propto F^{-r}$ must fall in the range $1 \leq r \leq 3$. Also, we use the expression for hydraulic radius to estimate the crack area per unit volume and the standard deviation of the height of the asperities on the microcrack surfaces for two granites. The values are in reasonable agreement with other estimates.

INTRODUCTION

The porosity of most rocks is interconnected, making the rock permeable to the flow of fluids. The volumetric flow rate q under a pressure gradient $\nabla \cdot p$ is governed by the diffusion equation, as follows:

$$(q/A) = -(k/\mu)\nabla \cdot p \tag{1}$$

where A is the cross-sectional area of the rock, k is rock permeability, and μ is fluid viscosity.

Most rock-forming minerals are insulators, and rock is very highly resistive to the flow of electrical current unless the rock is saturated with an electrolyte. In saturated rock, current flows through the electrolyte filling the interconnected pore spaces. The flow of current, in such cases, also is governed by the diffusion equation

$$J = -c\nabla \cdot v \tag{2}$$

where J is the current flux density, c is rock conductivity, and $\nabla \cdot v$ is the potential gradient.

Note the similarities between the flow of fluid and the flow of electrical current: both processes are described by the diffusion equation and, in both cases, flow characteristics are controlled by the pore phase rather than the rock matrix. Experimental evidence suggests that similarities in flow processes exist even on the microscopic scale. *Brace et al.* [1968] found from measurements of permeability and resistivity at pressures up to 4 kbar that permeability k and conductivity c of one rock, Westerly granite, were linearly related, with a slope of 1.5 ± 0.1, on a log-log plot; i.e., k is proportional to $c^{1.5}$ for Westerly granite. This same relationship was found [*Coyner et al.*, 1979] in experiments on other rocks, although the exponent generally was not 1.5, even in experiments on other samples of Westerly granite. Nevertheless, because of the close correlation of these two properties, we infer from these results that ions flowing through the rock under a potential gradient follow the same path as "particles" of water flowing under a pressure gradient.

Having accepted this description of the flow of fluid and electrical current in saturated rock, we ask the questions, Can the data in these experiments be used to derive new information about the pore phase and the flow process? Can a simple model be devised which describes the transport properties of rock? Are parameters derived from analyzing the model consistent with values measured in independent experiments?

In an attempt to answer these questions, we analyzed the flow of fluid and charged ions in rock, using a model apparently first proposed by *Wyllie and Rose* [1950]. We considered only the changes in flow characteristics caused by increasing the confining pressure acting on the sample. This restriction simplifies the analysis and interpretation considerably, because changes in porosity are entirely due to changes in the cross-sectional area of the flow conduits.

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ANALYSIS

Simple Model

Consider the section of a rock sample shown schematically in Figure 1*a*. The pore spaces, which act as conduits for the flow of fluid or electric current, are assumed to be isotropically distributed; i.e., any plane through the sample exposes the same (in a statistical sense) pore structure. Note that we assume that all porosity is involved in the flow process; that is, dead ends, stagnant regions, and isolated pores are neglected.

One such conduit, the one typical of the sample as a whole, is shown in Figure 1*b*. The average (over the cross section) fluid velocity v_l along a streamline can be written (see, for example, *Brace et al.* [1968])

$$v_l = -\left(\frac{m^2}{b\mu}\right)\left(\frac{dp}{dl}\right) \tag{3}$$

where m, the hydraulic radius, is the ratio of the volume of a conduit to its wetted area; b is a constant which depends on pore shape; μ is fluid viscosity; and (dp/dl) is the pressure gradient along the streamline. The pressure gradient (dp/dl) is related to the pressure gradient (dp/dx) with respect to the speciment axis by

$$(dp/dl) = (1/\tau)(dp/dx) \tag{4}$$

where τ is the actual path length relative to the apparent path length, i.e.,

$$\tau = dl/dx \tag{5}$$

The total flow q through the conduit is

$$q = v_l A_l \tag{6}$$

where A_l is the cross-sectional area of the conduits normal to

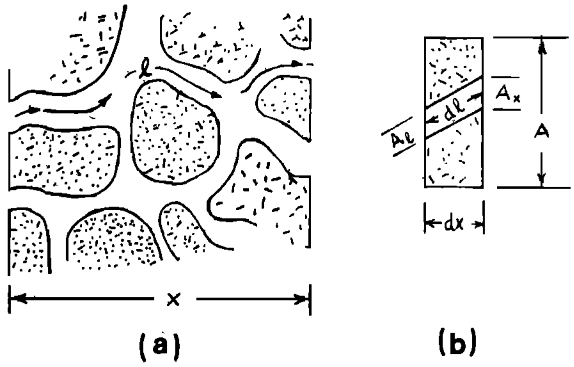

Fig. 1. (*a*) Tortuous path followed by a particle of water or an ion shown by the streamline *l*. (*b*) Representative section of the network chosen such that A_x/A equals the porosity of the rock and dl/dx equals l/x.

the streamline. Permeability k of the sample is defined as

$$k = (q/A)\mu/(dp/dx) \tag{7}$$

where A is the cross-sectional area of the sample. Combining (3), (4), (6), and (7) gives

$$k = (m^2/b)(A_l/A)(1/\tau) \tag{8}$$

Note that the cross-sectional area A_l is less than the conduit area A_x in a plane normal to the specimen axis; the two are related by

$$A_l = A_x/\tau \tag{9}$$

Wyllie and Rose [1950] and *Wyllie and Spangler* [1952] did not differentiate between A_l and A_x, leading to errors in their expressions and in some subsequent analyses [*Wyllie and Gregory*, 1955; *Brace et al.*, 1968]. The errors and resulting confusion are presented in more detail in the discussion section.

The porosity ϕ of a sample with isotropically distributed pores can be found by measuring the cross-sectional area A_x [see *Underwood*, 1970, p. 24]:

$$\phi = A_x/A \tag{10}$$

Combining (8), (9), and (10) gives an expression for the permeability k, as follows:

$$k = (m^2/b)(\phi/\tau^2) \tag{11}$$

The hydraulic radius m is, by definition, the ratio of the pore volume to the wetted area, or

$$m = \phi V/A_s \tag{12}$$

where A_s is the surface area of the pore phase. We see from (11) and (12) that permeability can also be expressed in the form

$$k = (1/b)\phi^3(V/A_s)^2(1/\tau^2) \tag{13}$$

where A_s/V is the surface area per unit volume and b is equal to 2 for circular tubes and equal to 3 for cracks. The parameter τ^2 in (13) was given the name tortuosity by *Rose and Bruce* [1949]. This definition was used by early workers [*Wyllie and Rose*, 1950; *Wyllie and Spangler*, 1952; *Wyllie and Gregory*, 1955], but in recent literature, tortuosity is sometimes defined as τ. Note in (13) that porosity ϕ and pore surface area per unit volume, A_s/V, can be found directly from measurements of the size and shape of the pore phase, whereas tortuosity, although it is a geometrical parameter, cannot be found without measuring flow properties.

The effective electrical conductivity of the rock model shown in Figure 1 is found by following the same procedure used to derive the expression for permeability. The conductance C (current per unit potential difference) of the model is

$$C = c_f(A_l/l) \tag{14}$$

where c_f is the conductivity of the pore fluid. The effective

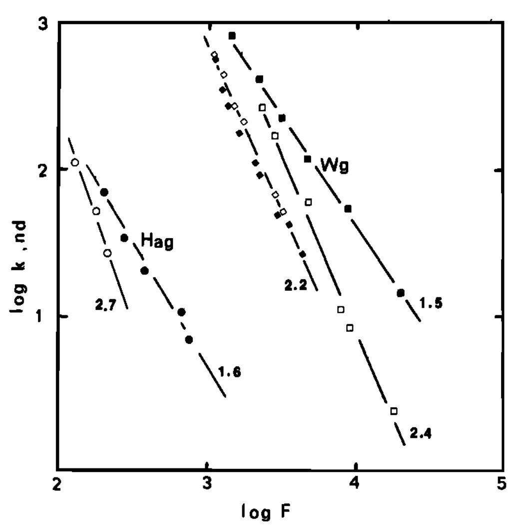

Fig. 2*a*. Log k is found to be proportional to log F in our measurements in three samples of Westerly granite (Wg) and two samples of Henderson augen gneiss (Hag). Note the variation in slope r (given by the numbers adjacent to the lines) from sample to sample. Solid diamonds indicate measurements taken as pressure was increased, and open diamonds indicate measurements taken during unloading.

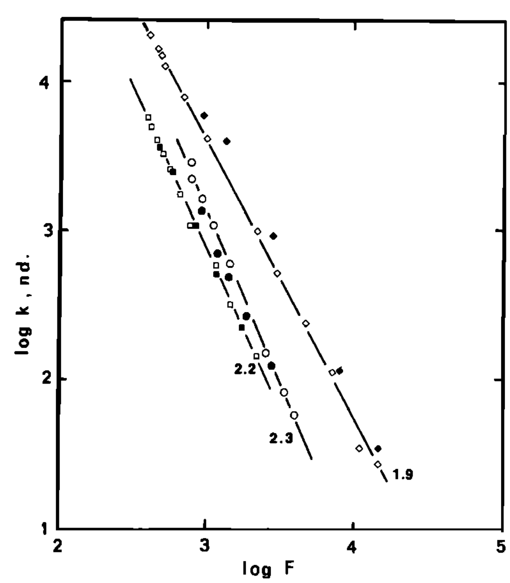

Fig. 2*b*. Log k is found to be proportional to log F in measurements taken in three orthogonal directions for a sample of Chelmsford granite. Squares indicate a path perpendicular to hardway, circles indicate a path perpendicular to rift, and diamonds indicate a path perpendicular to grain. Open symbols indicate increasing pressure, and solid symbols indicate decreasing pressure. Numbers adjoining data are the slopes of the curves.

TABLE 1. Slope r for Various Rocks

Rock	Description	Porosity ϕ, percent	r	Reference
Westerly granite	micr, an_{17}, qtz, mica	1.1	1.8	*Batzle and Simmons* [1983]
Westerly granite			1.5	*Brace et al.* [1968]
Westerly granite			2.4	H. C. Heard (personal communication, 1978)
Westerly granite			2.2	*Coyner et al.* [1979]
Westerly granite (500°)		?	1.9	*Batzle and Simmons* [1983]
Westerly granite (300°)		?	2.0	*Batzle and Simmons* [1983]
Chelmsford granite (perpendicular to hardway)	qtz, micr, an_{22}, mica	1.0	2.2	*Coyner et al.* [1979]
Chelmsford granite (perpendicular to rift)		1.0	1.9	*Coyner et al.* [1979]
Chelmsford granite (perpendicular to grain)		1.0	2.3	*Coyner et al.* [1979]
Henderson augen gneiss	unknown		1.6	W. F. Brace (unpublished data, 1980)
Henderson augen gneiss	unknown		2.7	*Coyner et al.* [1979]
Marysville granite	coarse grained, altered	0.18	2.6	*Batzle* [1978]
Marysville granite	pink, medium grained	2.6	2.8	*Batzle* [1978]
Raft River siltstone* (decreasing load)	friable, argillaceous	~45	2.4	*Batzle* [1978]
Frederick diabase* (700°)	pyr, an_{45}, ox, mica	0.1	6.0 (25–66 bars) 2.1 (66 bars to 2 kbar)	*Batzle and Simmons* [1983]
Pottsville sandstone	qtz, orth, mica, ox	2.9	2.6	W. F. Brace (unpublished data, 1980)
Pigeon Cove granite	orth, qtz, amph	0.6	2.1	*Coyner et al.* [1979]

Micr, microcline; an, plagioclase with anorthosite content; qtz, quartz; pyr, pyroxene; ox, oxides; orth, orthoclase; amph, amphibolite.
*Rocks giving anomalous results. See text.

conductivity c of the sample is

$$c = C(x/A) \tag{15}$$

From (9), (10), (14), and (15) we find

$$c/c_f = \phi/\tau^2$$

or equivalently,

$$F = \tau^2/\phi \tag{16}$$

where F, the formation factor, is c_f/c as defined by *Archie* [1942].

As has been discussed previously, we assume that electrical current and fluid follow the same paths and so tortuosity τ^2 for the two processes is the same. Tortuosity is eliminated by combining (11) and (16), giving an expression for the mean hydraulic radius m:

$$m = (bkF)^{1/2} \tag{17}$$

Effect of Pressure

Increasing the confining pressure acting on jacketed samples causes a decrease in both permeability and conductivity. *Brace et al.* [1968] measured permeability and the formation factor for Westerly granite and found that log k is proportional to log F (Figure 2*a*). Results for other samples of Westerly granite, for Chelmsford granite in three directions, and for Henderson augen gneiss are given in the plots of log F versus log k in Figures 2*a* and 2*b*. There is considerable unexplained variation among the Westerly samples and variation with direction in Chelmsford. Part of the difference between the data by Brace et al. and later data for Westerly granite may reflect the fact that k and F were not measured on the same samples in the Brace et al. early experiments. Nevertheless, the linearity of log k versus log F for all these samples is striking. Rock types other than the crystalline rocks in Figures 2*a* and 2*b* show similar behavior; a listing of the slope r of plots of log k versus log F from some recent measurements is given in Table 1.

Results from the two experiments marked with an asterisk in Table 1 are anomalous. Data from the Raft River siltstone are plotted in Figure 3. Note that permeability decreases with virtually no change in conductivity during the loading cycle, but normal behavior, with a slope of 2.4, is experienced during unloading. Raft River is a very porous weak rock from a geothermal area, and, apparently, the pore spaces were crushed by the confining pressure during the loading cycle. Crushing causes a decrease in both porosity and hydraulic radius. Consequently, the decrease in permeability, which depends on both hydraulic radius and porosity (see equation (11)), was disproportionately large in comparison with the decrease in conductivity, which depends only on porosity (see equation (16)). No crushing occurred during the unloading cycle, and behavior was normal.

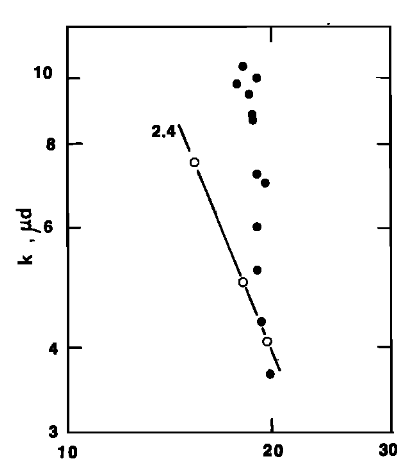

Fig. 3. Log k versus log F for Raft River siltstone [*Batzle*, 1978]. Crushing of the sample as the pressure was increased caused permeability to decrease with very little change in formation factor (solid circles). Normal behavior (open circles) was observed as pressure was decreased. The slope r during unloading is 2.4.

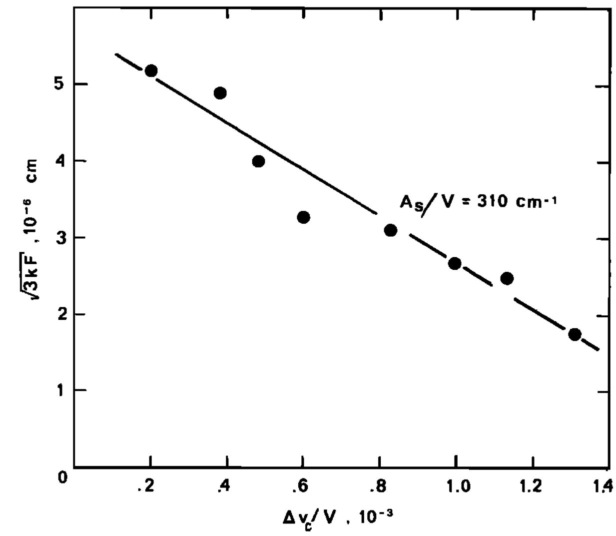

Fig. 4*a*. Westerly granite [*Coyner et al.*, 1979]. Change in aperture $(3kF)^{1/2}$ is proportional to the change $\Delta v_c/V$ in crack porosity as pressure is increased. The slope gives crack area per unit volume, A_s/V.

The other anomalous result was for diabase; the value of the slope r was approximately 6.0 at confining pressures up to 68 bars and then changed abruptly to 2.1 for pressures up to 2 kbar. Only three data points are available to define the abnormally high slope at low pressure, but one data point taken as the pressure was lowered also fell on the curve. The high slope, if it is real, may be a result of the heat treatment which the sample was subjected to before the experiment. Frederick diabase has virtually no porosity, and the temperature of this sample was increased to 700°C to produce microcracks. This severe treatment may have introduced a pore phase which does not conform to the model here and which is not found in the other samples in Table 1, where all or nearly all of the pore phase was formed by natural processes. We conclude that log k is linearly related to log c (or, equivalently, log F) for virtually all rocks loaded under confining pressure in the elastic range.

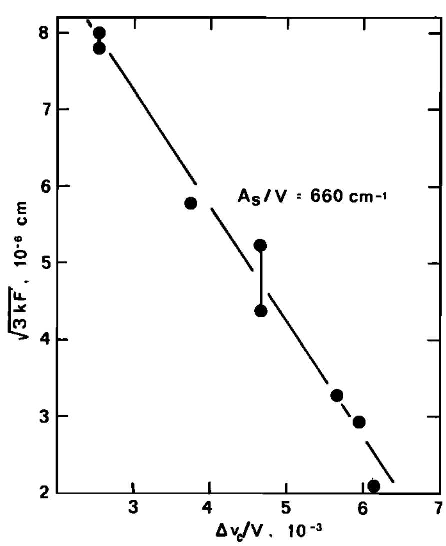

Fig. 4*b*. Westerly granite preheated to 500° [*Batzle and Simmons*, 1983]. Change in aperture $(3kF)^{1/2}$ is proportional to change $\Delta v_c/V$ in crack porosity as pressure is increased. Note the increase in crack area per unit volume, A_s/V, caused by preheating the sample. Data joined by lines were taken at the same conditions, giving an estimate of reproducibility.

Flow Through Fractures

The increased resistance to the flow of fluid and electrical current caused by increased confining pressure is primarily due to the closure of porosity in the form of cracks. The elastic deformation of pores other than cracks decreases cross-sectional flow area by less than a few percent for ordinary pressure changes; clearly, such small area changes are insufficient to account for the large changes in transport properties that are observed. Closing cracks, however, has a large effect because small changes in pressure cause relatively large changes in the cross-sectional areas of flow passages.

We can check whether the simple model under discussion simulates the effect of pressure on transport properties in rocks where cracks are important elements in the network. First, note that the change Δa in half aperture is related to the change Δm in hydraulic radius by

$$\Delta a = \Delta m \tag{18}$$

In Table 1 we see that the slope r of plots of log k versus log c falls in the range $1.5 \le r \le 2.8$. An expression for r for the simple flow model under discussion can be derived by combining (11), (16), and (18). In the elastic range, A_s/V is nearly independent of pressure, and changes in porosity are almost entirely due to changes in crack aperture. We find for deformation in this range that r is given by the expression

$$r = \frac{d \ln k}{dp}\left(\frac{d \ln c}{dp}\right)^{-1}$$
$$= \frac{3d(\ln a)/dp - d(\ln \tau^2)/dp}{d(\ln a)/dp - d(\ln \tau^2)/dp} \tag{19}$$

Solving (19) for constant r gives

$$\tau^2/\tau_0{}^2 = (a/a_0)^{-[(3-r)/(r-1)]} \tag{20}$$

where subscript zero refers to some arbitrary datum. We see in (20) that the slope r is a measure of the sensitivity of the tortuosity to changes in crack aperture a: as r approaches 3,

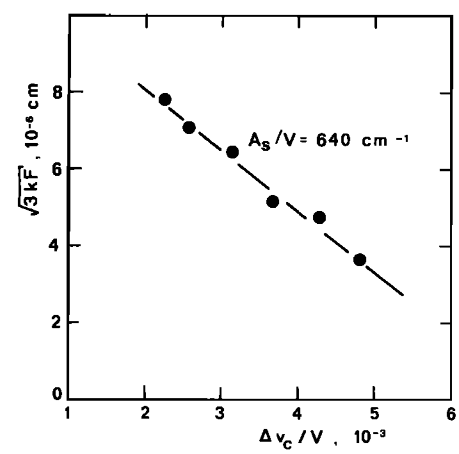

Fig. 4*c*. Chelmsford granite, averaged over three orthogonal directions [*Coyner et al.*, 1979]. Change in aperture $(3kF)^{1/2}$ is proportional to change in $\Delta v_c/V$ in crack porosity as pressure is increased. Crack area per unit volume, A_s/V, for Chelmsford granite is approximately twice that for Westerly granite in Figure 4*a*.

tortuosity is nearly independent of aperture, whereas small changes in aperture result in very large changes in tortuosity for samples where r is near unity. Apparently, the range $1 \leq r \leq 3$ represents the complete range of possible behavior, at least for this model where A_s/V is independent of pressure.

We also can use the model to compare crack aperture derived from transport properties with the value found from measurements of elastic deformation. The change in half aperture is found from measurements of permeability and conductivity using (11) and (16) (where $b = 3$ for cracks) with the result

$$\Delta a = \Delta(3kF)^{1/2} \tag{21}$$

The change in aperture can also be expressed in terms of the change Δv_c in pore volume:

$$\Delta a = (\Delta v_c/V)(V/A_s) \tag{22}$$

where A_s is the wetted area. The parameter $\Delta v_c/V$ in (22) is the change in crack porosity. *Walsh* [1965], using a simple geometric construction, analyzed crack porosity in rocks and showed that it can be calculated from measurements of volumetric strain for jacketed samples under confining pressure (see, for example, *Brace* [1965] and *Feves et al.* [1977]).

We can now compare changes in aperture inferred from permeability and conductivity with more direct measurements. Equations (21) and (22) indicate that $(3kF)^{1/2}$ should be directly proportional to Δa derived from measurements of bulk volume, the constant of proportionality equaling A_s/V. We calculated these parameters from data reported for a sample of Chelmsford granite and two samples of Westerly granite, one of which had been heat treated to increase crack density. The results are plotted in Figures 4*a*, 4*b*, and 4*c*. We see that the relationship between the two parameters is approximately linear, as predicted by the simple model. The surface area of cracks per unit volume, A_s/V, calculated from the slope of the curves is shown on the figures. Note the surprisingly large variation in A_s/V between the two types of granite and the large increase in crack area caused by the heat treatment.

Rough Fractures

The effect of pressure on transport properties can be examined further if we assume that the microcracks are rough sur-

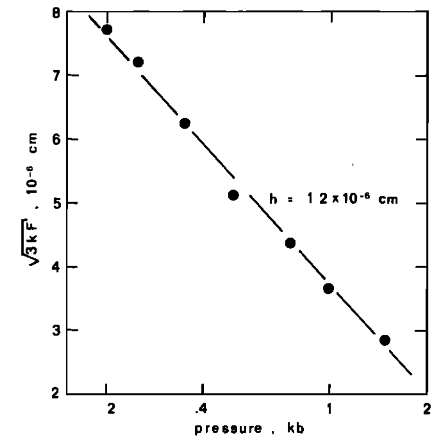

Fig. 5*b*. Chelmsford granite, averaged over three orthogonal directions. Aperture $(3kF)^{1/2}$ is proportional to log pressure. The slope h is the standard deviation of the asperity height.

faces with known topography. Here we follow the model proposed by *Greenwood and Williamson* [1966] and further developed by *Whitehouse and Archard* [1970] where the topography was assumed to be random. This model seems to be appropriate here: Greenwood and Williamson demonstrated by direct measurement that the surfaces of a variety of metal samples have approximately random topography, and the model was found to be adequate in several studies of rock properties [*Walsh and Grosenbaugh*, 1979; *Walsh*, 1981; *Scholz and Hickman*, 1983] where the fracture surfaces were microcracks, tensile fractures, and saw cuts.

Here, using an approximation suggested by *Greenwood and Williamson* [1966], we assume that the distribution of asperity heights can be approximated by an exponential; one can show for this case that the change da in aperture resulting from a change dp in pressure is given by the expression

$$da = \sqrt{2}h(dp/p) \tag{23}$$

where h is the standard deviation of the height distribution. The expression for da in (23) differs from that given by *Walsh and Grosenbaugh* [1979], because a rederivation [*Brown and Scholz*, 1983] of the stiffness of two random surfaces in contact shows that it is not simply half that for a random surface on a rigid flat surface, as assumed by Walsh and Grosenbaugh. Substituting the expression for aperture given by (17) and (18) into (23) and rearranging gives

$$d(3kF)^{1/2} = \sqrt{2}h(dp/p) \tag{24}$$

we see from (24) that the slope of a plot of $(3kF)^{1/2}$ versus ln p should be constant, that is,

$$d(3kF)^{1/2}/d(\ln p) = \sqrt{2}h \tag{25}$$

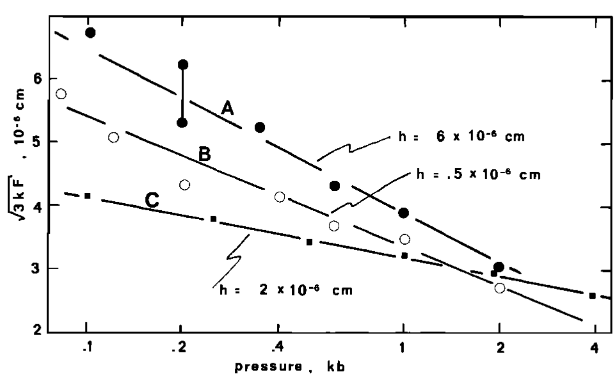

Fig. 5*a*. Westerly granite. Aperture $(3kF)^{1/2}$ varies linearly with log pressure according to theory developed by *Greenwood and Williamson* [1966]. The slope indicated by the numbers adjacent to the curves gives the standard deviation h of the height of the asperities on the crack surfaces. Curve A, Westerly granite preheated to 500° [*Batzle and Simmons*, 1983]; curve B, Westerly granite [*Batzle and Simmons*, 1983]; curve C, Westerly granite [*Brace et al.*, 1968].

Data from the three experiments on Chelmsford granite and Westerly granite used in Figure 4 and data from a different experiment on Westerly granite are plotted as a function of pressure in Figures 5*a* and 5*b*. The slope of the curves is constant in agreement with (25). The numbers next to each curve are the values of the standard deviation h of asperity height calculated from the slope using (25). We see that the values of h for these granites are of the order of 10^{-6} cm, with h for the Westerly granite samples being several times greater than that for Chelmsford granite.

TABLE 2. Comparison of Predicted and Measured Values of A_s/V

Rock	Predicted Value, cm^{-1}	Measured Value, cm^{-1}
Westerly granite	310	170 [*Hadley*, 1976] 130 [*Tapponier and Brace*, 1976; *Wong*, 1982] 46 [*Simmons et al.*, 1975]
Chelmsford granite	640	640 [*Simmons et al.*, 1975]

DISCUSSION

In the previous section we derived (11) and (16) expressing permeability and electrical conductivity in terms of porosity and the hydraulic radius and tortuosity of the path followed by the fluid and the electric current. The model is that proposed by *Wyllie and Rose* [1950], where the pore network is replaced by a single representative tube having a constant cross-sectional area and an effective length greater than that of the sample. This model has been used in a number of analyses of permeability and conductivity since it was proposed. *Wyllie and Rose* [1950] and *Wyllie and Spangler* [1952] in their derivations of the formation factor and permeability introduced two errors which were repeated in some of the subsequent studies. In particular, *Wyllie and Spangler* [1952] used an incorrect expression for fluid velocity and did not distinguish between the conduit cross-sectional area A_l normal to the streamline and the area A_x normal to the sample axis (as related here by (9)). The two errors canceled one another in Wyllie and Spangler's derivation of fluid flow properties, and so their expression for permeability is equivalent to (11) or (13). However, confusing A_x and A_l led *Wyllie and Rose* [1950] to an incorrect expression for the formation factor. *Cornell and Katz* [1953] corrected this mistake and arrived at an expression equivalent to (16). *Wyllie and Gregory* [1955] and *Brace et al.* [1968] followed the orginal derivations, leading to a correct expression for permeability and an incorrect expression for the formation factor. D. R. Stephens and W. Lin (unpublished manuscript, 1978) recognized the difference between A_x and A_l but used Wyllie and Spangler's expression for fluid velocity; therefore they found a correct expression for the formation factor and an incorrect expression for permeability. A good review of all but the recent work on this topic and reference to other analyses not mentioned here is given by *Bear* [1972].

In Table 1 we collected data from experiments where permeability and conductivity were measured as a function of pressure for a variety of rocks which showed, almost without exception, that permeability is related to conductivity by a power law; i.e., $k \sim c^r$. We showed, using our expressions for permeability and conductivity, that $1 \leq r \leq 3$, in agreement with the values gathered in Table 1. One exception is the Raft River siltstone, where the very high slope during the loading cycle is very likely due to crushing of the sample. The slope ($r = 2.4$) during the unloading cycle, where no crushing occurs, is normal. Another exception is a sample of Frederick diabase previously thermally cracked at 700°C. The slope r is constant, with a value of 2.1, at pressure in the range from 68 bars to 2 kbar, but the slope is approximately 6.0 at pressures less than 68 bars. We have no explanation for this anomalous behavior at low pressures other than that the severe heat treatment the sample was subjected to before the experiment may have introduced a pore phase which differed from that in the model and that in the other rocks in Table 1.

Next we investigated the effect of pressure on transport properties for this model when the representative flow conduit has the form of a crack. We derived an expression (equation (21)) for crack aperture based on measurements of permeability and conductivity as a function of pressure and compared values calculated from this equation with values calculated from measurements of crack porosity. The relationship (Figures 4*a*, 4*b*, and 4*c*) between the two values was linear, as predicted.

The slope of the line relating the values of aperture calculated from the two types of measurements was used to calculate the wetted area of the cracks per unit volume of sample. We have no independent measurements of crack area for the Westerly granite samples used in this study, but some measurements have been made on other samples of Westerly granite. *Hadley* [1976], using scanning electron microscope (SEM) photomicrographs, measured a total crack length per unit area of approximately 68 cm^{-1}, and *Wong* [1982], using other SEM measurements made by *Tapponier and Brace* [1976], reported a value of 51 cm^{-1}. *Simmons et al.* [1975] used an optical microscope to measure the length of cracks in a sample of Westerly granite heated to 108°C; they found a value of 18 cm^{-1}. These values can be converted to wetted area A_s per unit volume V using the relationship [*Underwood*, 1970, p. 24]

$$A_s/V = (8/\pi)L_A \tag{26}$$

where L_A is crack length per unit area. We find that the wetted area of cracks per unit volume is approximately 170 cm^{-1} for Hadley's sample of Westerly granite, 130 cm^{-1} for Tapponier's, and 46 cm^{-1} for the measurements made by Simmons et al. Simmons et al. also studied Chelmsford granite under an optical microscope. They found that crack length per unit area was distributed anisotropically, with values varying with direction by a factor of 2. Taking an average of the values over all directions, we find that crack length per unit area is approximately 250 cm^{-1}, equivalent to a crack area A_s per unit volume of 640 cm^{-1}.

A comparison of these directly measured values of crack area A_s per unit volume with values estimated from permeability, formation factor, and elastic volume charge in Figure 4 is presented in Table 2. We see that the values for Chelmsford granite are the same, whereas the largest value (Hadley's) for Westerly granite from SEM measurements is approximately half the value estimated from transport and elastic properties. Could anisotropy be the cause of the discrepancy between measured and estimated values for Westerly granite? Although the very close agreement between the values for Chelmsford must be coincidental, both parameters used in Figure 4 and the microscopic measurements were calculated from values averaged over direction for this rock. Westerly granite also is anisotropic, but the effect could not be included here, because the influence of orientation is known for some, but not all, of the measurements needed for the comparison. Other questions about Table 2 arise, for example, the disparity between the measurements using the SEM and optical microscopes. These questions must remain unanswered until a more thorough study is available where all measurements are made on the same sample.

In the final step of our study we assumed that the microcracks in rocks have rough surfaces. Crack aperture from our calculations using measurements of permeability and the formation factor was analyzed using *Greenwood and Williamson*'s [1966] theory for deformation of rough surfaces in contact. Our results for three samples of Westerly granite and one sample of Chelmsford granite are presented in Figures 5*a* and 5*b*. The linearity of these plots shows that Greenwood and Williamson's theory, with the assumption that the frequency distribution of asperity heights is approximately exponential, describes the deformation of these rocks adequately. The values of h (where the exponential function is $e^{-a/h}$) are listed next to individual plots in Figures 5*a* and 5*b*. Note that h for the as-found sample of Westerly granite is nearly the same as the value for the sample heat treated at 500°C, suggesting that the surfaces of the new cracks are similar to those of the preexisting cracks. The value of h from the *Brace et al.* [1968] experiment appears to be considerably smaller, but the values may be spurious because k and F were measured on two separate samples. Taking a value of 0.5×10^{-6} cm for h and our estimate of crack area per unit volume of 310 cm^{-1} (Table 2), we calculate that the dimensionless parameter $A_s h/V$ has a value of 1.5×10^{-4}; *Walsh and Grosenbaugh* [1979], using published static and dynamic measurements of elastic compressibility, estimated values for $A_s h/V$ of 5×10^{-4} and 2×10^{-4}, in reasonable agreement with the study here.

Jones [1975] and *Jones and Owens* [1979], on empirical grounds, and *Walsh* [1981], on the basis of Greenwood and Williamson's theory, proposed that $k^{1/3}$ should vary linearly with log p. The linearity of the plots in Figures 5*a* and 5*b* suggests that k^n, where n is not necessarily equal to one third, should be proportional to log p. Applying the relationship that k is proportional to F^{-r} obtained from Figures 2*a* and 2*b*, we see that $(3kF)^{1/2}$ is proportional to k^n, where $n = (r - 1)/2r$. As is discussed above, our results indicate that $1 \le r \le 3$, leading to values for n in the range $0 \le n \le \frac{1}{3}$. The proposed value for n of $\frac{1}{3}$ apparently is an upper limit. However, n, whatever its value, is relatively small, and so precise measurements and careful sample preparation will be required to establish its value experimentally.

Note added in proof. M. S. Paterson in an independent analysis also rederived the expression for permeability and for the formation factor for the simple model discussed here. His derivation leads to the same expressions as ours given by (11) and (16). His derivation, together with a thorough discussion of previous work and comparison between theory and data, was recently published [*Paterson*, 1983].

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