PERGAMON

International Journal of Rock Mechanics and Mining Sciences 37 (2000) 867–872

International Journal of
Rock Mechanics
and Mining Sciences

www.elsevier.com/locate/ijrmms

Technical Note

Simulating acoustic emissions in bonded-particle models of rock

J.F. Hazzard*, R.P. Young

Department of Earth Sciences, University of Liverpool, 4 Brownlow Street, Liverpool, L69 7GP, UK

Accepted 9 March 2000

1. Introduction

Recording acoustic emissions (AE) during laboratory testing of rock samples can provide significant additional information on the micro-mechanics of failure processes [1]. Information about the onset and propagation of microcracking and fracture in rock samples subjected to different stress regimes can be determined by recording the time and location of AE during the test. Using AE to map out fault nucleation and propagation may also be useful in the understanding of earthquake mechanisms and may contribute to solving the problem of earthquake prediction [2].

As well as monitoring spatial and temporal patterns of microcracking, recording full-waveform information for AE enables the calculation of source characteristics such as magnitude and mechanism [3,4]. Furthermore, it has been shown that AE amplitudes obey the power law frequency–magnitude relation observed for earthquakes [1,5].

An area of research that is less well-developed is numerical modelling of AEs. Several numerical models have been proposed to simulate cracking and failure in brittle rock specimens [6–8], however, none of these models account for energy released during crack formation (i.e. seismic waves). In this Technical Note, a two-dimensional (2-D) 'bonded-particle' model of rock is proposed where the rock is simulated by an assembly of circular disks with specified stiffnesses connected by bonds of specified strengths. With this model, strain energy can be stored at the particle contacts until the bond strength is overcome, at which time the bond breaks and the stored strain energy is released as kinetic energy in the form of a seismic wave. These 'bonded-particle' models have been used successfully to simulate brittle rock behaviour in previous experiments [9] however, AE information was not recorded. This Technical Note presents a technique for recording AEs in bonded-particle models and applies the technique to a simulated compressive failure test on a model of a granite core sample.

2. Bonded-particle models

2.1. Theory

The Particle Flow Code in two dimensions (PFC^{2D}, [10]) is a discontinuum code that represents a rock mass as an assemblage of circular disks confined by planar walls. A three-dimensional (3-D) version of the code is also available in which the particles are spheres, however, only 2-D models will be discussed here. The distinct element method [11] is used to model the forces and motions of the particles within the assembly. This method is similar to that used in explicit finite-difference analyses and allows information to propagate dynamically through the system. For this reason, PFC^{2D} is a logical choice for modelling AEs and the resulting dynamic output.

The particles move independently of one another and interact only at contacts. They are assumed to be rigid (non-deformable) but overlap can occur at the contacts. Contacts are assumed to exist only at a point and not over some finite surface area as would be the case with fully deformable particles. The particles can be bonded together to simulate a competent rock. The contact bonds can be envisaged as a pair of elastic springs (or a point of glue) with constant normal and

* Corresponding author. Tel.: +44-151-794-5160; fax: +44-151-794-5169.

E-mail address: hazzard@liv.ac.uk (J.F. Hazzard).

PII: S1365-1609(00)00017-4

shear stiffnesses acting at a point. The values assigned to these stiffnesses influence the macro deformation properties of the rock sample (Young's modulus and Poisson's ratio). The contact bonds also have a specified shear and tensile strength. The values assigned to these strengths influence the macro strength of the sample and the nature of cracking and failure that occurs during loading.

The contact bonds allow tension to exist at the contacts until the force at the contact exceeds the strength of the bond, at which time the bond breaks and the tensile force becomes zero. Similarly, the contact can support shear forces until the bond breaks, but in this case the shear force is set to a residual value that depends on the compressive normal force at the contact and the coefficient of friction. The contact behaviour is summarised in Fig. 1.

After a bond breaks in PFC, stress is redistributed and this may then cause more cracks to form nearby. If the rock model is suitably stressed, then these bond breakages will localise into an inclined macrofracture — eventually causing sample failure. In this way, deformation and fracture of rocks is modelled *directly* by allowing micromechanical damage to evolve, instead of *indirectly* by using constitutive equations as is the case in most continuum models. It has been shown that PFC can fairly accurately reproduce the fundamental mechanical behaviour of a range of rock types subjected to different stress regimes [9,12,13]. The rest of this Technical Note will now describe how realistic acoustic information can be obtained from the models.

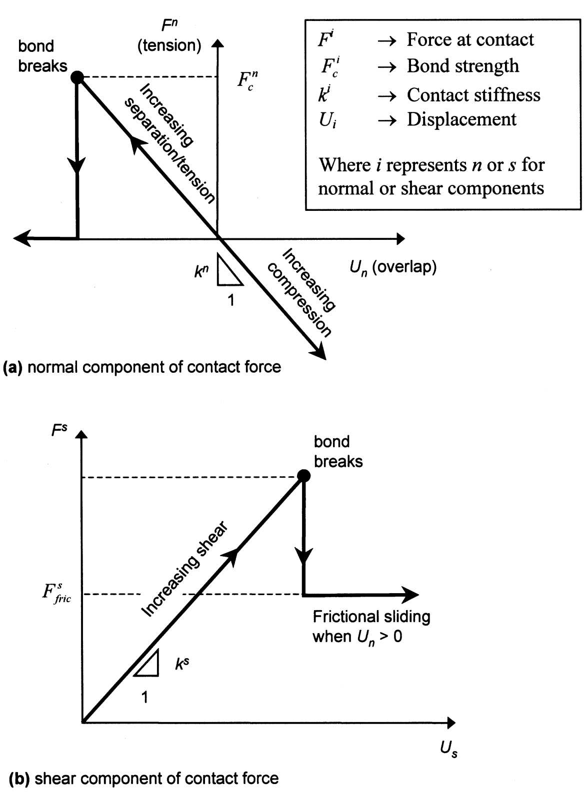

Fig. 1. Constitutive behaviour for a bonded point contact in PFC^{2D} (from Ref. [10]).

2.2. Microcracking and AE

It has been shown that each bond breakage in PFC can be considered a microcrack in the modelled rock [14]. Experimental evidence suggests that microcracks in real rocks generally form at defects such as grain boundaries or voids. Many different micromechanisms have been proposed to explain the formation of microcracks at these defect locations (e.g. sliding on pre-existing cracks, elastic mismatch between grains, etc.). In PFC, each bond breakage (or microcrack) represents parting or sliding at a grain contact or plane of weakness. Although this is obviously a simplification of what occurs in reality, it has been shown that this micromechanical representation of cracking can yield realistic cracking patterns and can reproduce the macromechanical behaviour of a variety of modelled rock types [9,12,13].

It is known that microcracks often generate acoustic energy that propagates through the rock and can be recorded to provide information about the cracks [1,4]. This phenomenon also occurs in PFC. Once a PFC particle assembly has been created and contact bonds are installed, the rock model can be loaded by moving the confining walls. As stress is applied to the model boundaries, local forces build up and strain energy is stored at the particle contacts according to:

$$E_c = \frac{1}{2}\left(\frac{|F^n|^2}{k^n} + \frac{|F^s|^2}{k^s}\right) \tag{1}$$

See Fig. 1 for a description of the parameters. Eventually the local force is sufficient to break the connecting bond at which time the two particles rapidly move apart until stopped by local confinement. If there were no confinement, and no damping then all of the strain energy stored at the contact would be converted into kinetic energy for a tensile bond breakage. However, for most bond breakages in PFC models, only a fraction of the strain energy stored at the contact will contribute to the particle motion because some energy will always be lost to numerical damping, friction at particle contacts and transfer of strain energy to neighbouring particles. In this way energy release can vary between different simulated AE.

The kinetic energy of each particle can be calculated by:

$$E_k = \tfrac{1}{2}(mv_x^2 + mv_y^2 + I\omega^2) \tag{2}$$

where m is the particle mass, v is the velocity in the x-

or y-direction, I is the moment of inertia and ω is the angular velocity. In a closely packed particle assembly, this kinetic energy manifests itself as a seismic wave that propagates out from the location of the bond breakage, similar to an AE in real rocks. Seismic source information can then be calculated for each bond breakage by observing the forces and motions of the source particles or by actually recording waveforms at points far from the source and applying standard waveform inversion techniques [12]. A method for determining seismic information from these simulated AE is described below.

3. AE monitoring in PFC

If PFC models are run dynamically with low numerical damping, then each time a bond breaks, energy is released and seismic source information can be calculated. A simple technique for calculating seismic source information from the motions of the two particles involved in each bond breakage is described in Ref. [13]. The problem at present is that each bond breakage in PFC releases approximately the same amount of energy because the particles are all approximately the same size. This is not the case for real earthquakes and AE where magnitudes generally exhibit a large range that follow a power-law frequency–magnitude distribution [1,5]. It is therefore postulated that in PFC, microcracks (bond breakages) occurring close together in space and time may be part of the same macro rupturing event. This is probably a realistic assumption as it is known that most seismic events in the field are made up of many smaller scale ruptures and shearing of asperities [15] and that shear fractures generally grow at some finite velocity [16]. For the rest of this discussion therefore, a microcrack will refer to a single bond breakage and an AE will refer to a cluster of one or more bond breakages assumed to be part of the same seismic event.

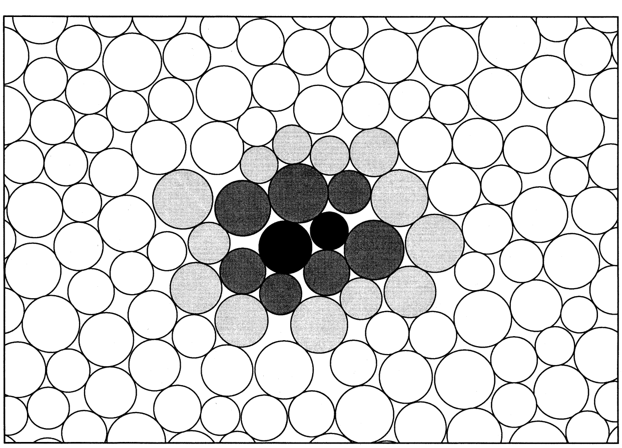

Fig. 2. Particles making up an AE source in PFC^{2D}. Black particles are the parent particles formerly connected by a bond. Dark grey particles are all those in contact with the parent particles. Light grey particles are all particles in contact with the dark grey particles.

The new AE algorithm works as follows:

- When a bond breaks, the kinetic energy of all of the particles in the source region is stored as the initial kinetic energy E_k^0. The size of the source region can be chosen by the user to be either (i) the two parent particles on either side of the crack, (ii) the two parent particles and all particles in contact with

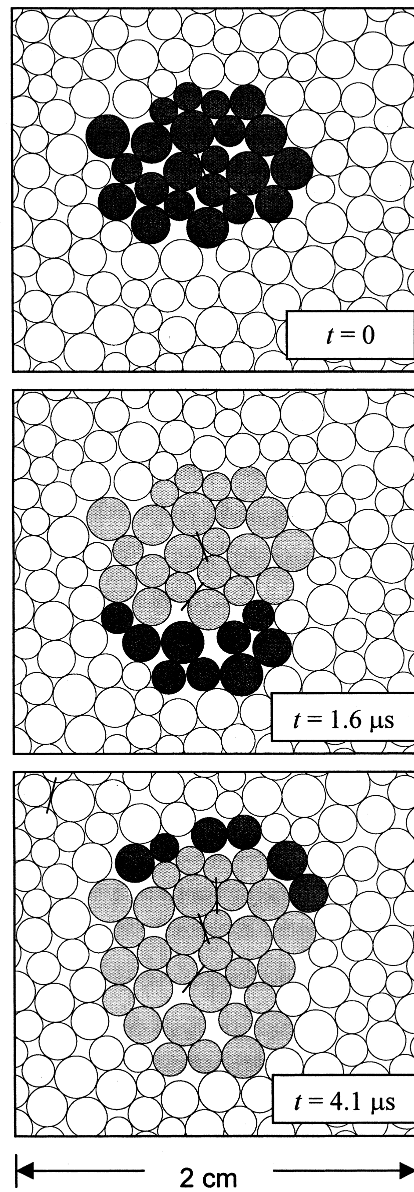

Fig. 3. Three stages of an AE composed of three cracks. Cracks (bond breakages) are shown as black lines. Time from the first bond breakage is shown in each frame. Particles involved in the energy calculations are coloured grey. New particles added to the source area after formation of a new crack are coloured dark grey.

them, or (iii) all particles in (ii) and all particles in contact with them (see Fig. 2). For the rest of this discussion it will be assumed that the largest source area (iii) is being used.

- The kinetic energy of all of the particles in the source region (E_k) is monitored for the duration of the event and the *change* in kinetic energy is calculated each step by:

$$\Delta E_k = E_k - E_k^0 \tag{3}$$

- The event duration is calculated by assuming that each cluster is an expanding shear fracture. It is known that a shear fracture can propagate as slowly as 0.5 times the shear wave velocity of the material [16]. Therefore, the duration is calculated by dividing the radius of the source area by 0.5 times the shear wave speed of the assembly,
- If no more microcracks form within the source area over the duration of the event, then the kinetic energy associated with the event is the maximum value of ΔE_k attained during the event. The magnitude of the AE can then be calculated by [17]

$$M_e = \tfrac{2}{3}(\log\ \Delta E_k^{\max} - 4.8) \tag{4}$$

where the energy is in Joules.

- If another crack occurs within the source area before the end of the monitoring period then it is incorporated into the event. When this happens, the time is set back to zero and the source area is expanded to include the particles surrounding the new crack (see Fig. 3).
- The kinetic energy of all of the particles in the expanded source region is then monitored until the duration of the event has elapsed and no more cracks form within the source area. The event magnitude is then calculated from Eq. (4).

An example event is shown in Fig. 3 and the kinetic energy associated with the event is shown in Fig. 4. It is clear from this figure how the energies associated with different microcracks can add together constructively to contribute to a larger magnitude energy peak. It is conceivable that if acoustic information was recorded at some distance from this source then the high frequency information would be attenuated and this cluster of microcracks would be interpreted as a single event with a maximum kinetic energy as shown.

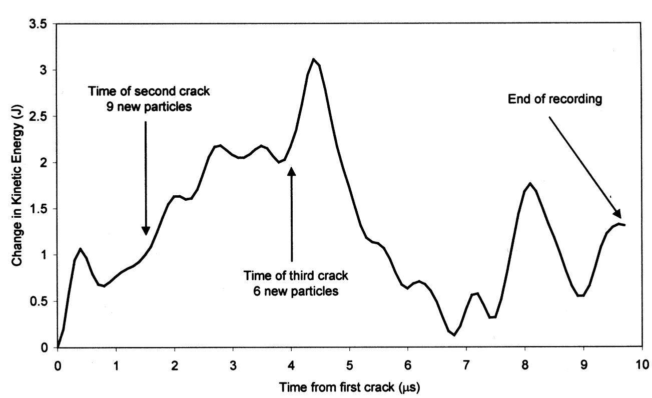

Fig. 4. The kinetic energy of the source particles for an AE composed of three cracks (see Fig. 3).

4. Granite model

To demonstrate the AE modelling technique, a PFC^{2D} model of a core sample of Lac du Bonnet granite was created (Fig. 5). The model is 31.5 × 12.6 cm and is made up of approximately 6000 randomly placed particles ranging in size from 2 to 3 mm. Lac du Bonnet granite is modelled because the mechanical behaviour of this rock type has been studied extensively as part of a feasibility study into the concept of underground nuclear waste disposal [18]. In addition to laboratory and field tests on Lac du Bonnet granite, extensive modelling of this rock type using PFC^{2D} has already been performed [9].

The model was loaded by applying a constant strain rate to the top and bottom walls (platens) while the side walls were servo-controlled to maintain a confining stress of 20 MPa. The stress–strain behaviour of the model compared to an actual laboratory test [18] is shown in Fig. 6. It can be seen that the stiffness and strength of the actual sample is well reproduced by the model (due to our choice of contact stiffnesses and strengths) and that the rock sample fails without the need to incorporate any constitutive macro failure criteria. The differences between the model and laboratory behaviour can be explained by noting that round

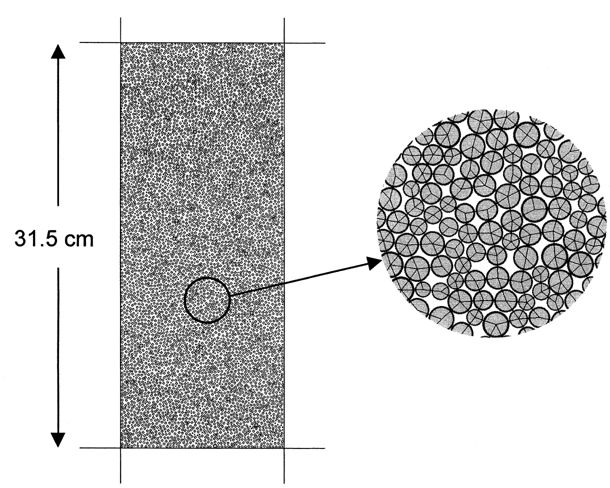

Fig. 5. PFC^{2D} model of Lac du Bonnet granite core sample. Black lines show the locations of bonds connecting the particles.

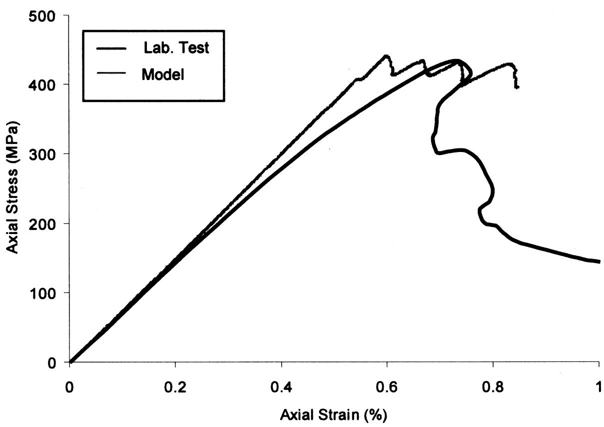

Fig. 6. Stress/strain behaviour of laboratory test [18] and model during compression of Lac du Bonnet granite core sample with 20 MPa confining pressure.

particles are being used to simulate a crystalline rock. For example, in the actual rock the stiffness apparently decreases with increasing strain due to opening microcracks, however, in the model, the microcracks have a very low aperture and therefore their volume contribution is smaller. Despite these differences, the PFC model of granite has been shown to reproduce many of the mechanical behavioural characteristics of this rock subjected to different stress conditions [9,13]. The suitability of using PFC to model crystalline rocks is discussed elsewhere [14] and therefore the remainder of this discussion will focus on the acoustic output from the model.

AE were monitored throughout the loading experiment using the algorithm described above. These AE are plotted in Fig. 7. The size of the circle/ellipse in Fig. 7 corresponds to the extent of the event. 'Best-fit' ellipses were drawn for all AE consisting of more than two cracks to more accurately show the extent (linearity) of the events. Ellipse geometries were determined by obtaining the covariance of the set of crack locations and using the eigenvalues and eigenvectors of the covariance matrix to specify the length and orientation of the ellipse axes. An example is shown inset in Fig. 7.

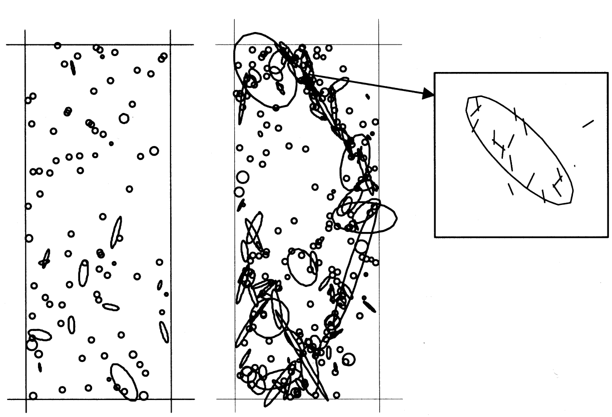

Fig. 7. AE occurring before (left) and after (right) the peak stress. AE consisting of three or more cracks are plotted as best-fit ellipses. One ellipse is shown with the crack locations to show the quality of the fit.

It can be seen that before the peak stress is reached, cracking in the model is sparse and randomly located. Most of the AE are quite small. However, after the peak stress, the AE localise along conjugate inclined failure planes — just as in laboratory tests of this type. Some of the AE after the peak stress are very large, showing the ability of the AE algorithm to cluster individual cracks into larger events. It is also interesting to observe that the ellipses representing the AE clusters generally have their major axes oriented along the inclined faults.

To examine the magnitudes of the AE, it was assumed that the AE followed a Gutenberg–Richter type relationship:

$$\log\ N = a - bM \tag{5}$$

where N is the number of events with magnitudes greater than M, and a and b are constants.

The cumulative number of events is plotted against magnitude in Fig. 8 and the b-value is shown as the slope of the straight part of the curve. For comparison, a plot is shown where magnitudes were calculated assuming each crack (bond breakage) is a single event. It is clear that the new AE modelling technique produces events with a much larger range in magnitudes and therefore yields a much more realistic b-value. In laboratory tests on Lac du Bonnet granite, b-values for recorded AE ranged from 1.3 to 2.3 [19] and in exper-

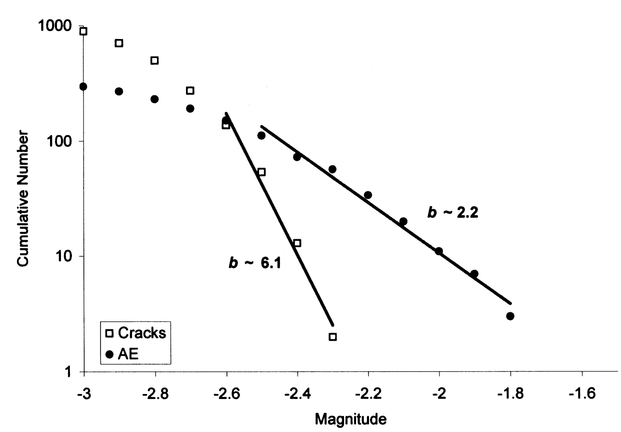

Fig. 8. Frequency-magnitude plots for two different methods of calculating event magnitude in PFC. 'Crack' magnitudes are calculated assuming each bond breakage is an event and the source area comprises two particles. 'AE' magnitudes assume crack clusters and larger source areas as described in the text. A line is fit to the straight part of the curve in each case and the b-value is equal to the slope of this curve.

iments conducted on Westerly granite [2] b-values were found to range from 1.1 to 2.4. The value of 2.2 obtained from the PFC granite model is within these ranges, giving confidence in the ability of the AE algorithm in PFC^{2D} to yield realistic acoustic information.

5. Conclusions

The increasing power of computers is making the use of discontinuum models to simulate rock behaviour more and more feasible. One attraction of using these types of models is that they possess the ability to fracture and break apart under stress. In this way the micromechanics of rock failure can be examined in detail. An exciting extension of these studies is the possibility of recording seismic information each time a microcrack occurs. In this Technical Note an algorithm was presented that records location, extent and magnitude of AEs generated by these models.

A distinct-element code was used because this calculation technique enables information to propagate dynamically through the model. Therefore, each time a crack forms, kinetic energy in the form of seismic waves propagates through the particle assembly. This kinetic energy is used to calculate AE magnitudes. In order to obtain realistic magnitude distributions from the model, clustering of individual microcracks into larger seismic events needed to be performed. The algorithm assumes that seismic events represent a rupture that grows to some finite extent over a finite amount of time. Each AE may therefore consist of several smaller microcracks that represent shearing of asperities or opening along grain-scale planes of weakness with each microcrack contributing to the total kinetic energy of the AE. It was shown that if this algorithm is applied during a loading test on a modelled sample or rock, then realistic magnitude distributions could be obtained.

Although only 2-D simulations were shown here, the algorithm also works in three dimensions. Future work on this code now needs to concentrate on determining a mechanism or moment tensor associated with each AE — perhaps by monitoring the movements of all of the particles involved in an event and fitting a seismic source mechanism to these particle motions.

Acknowledgements

The authors wish to thank David Potyondy and Peter Cundall at Itasca Consulting Group for their valuable assistance and advice.

References

[1] Lockner DA. The role of acoustic emission in the study of rock fracture. Int J Rock Mech Min Sci and Geomech Abstr 1993;30:883–99.

[2] Lockner DA, Byerlee JD, Kuksenko V, Ponomarev A, Sidorin A. Quasi-static fault growth and shear fracture energy in granite. Nature 1991;350(7):39–42.

[3] Falls SD, Young RP, Carlson SR, Chow T. Ultrasonic tomography and acoustic emission in hydraulically fractured Lac du Bonnet granite. J Geophys Res 1992;97:6867–84.

[4] Pettitt WS, Young RP, Marsden JR. Investigating the mechanics of microcrack damage induced under true-triaxial unloading, SPE 47319. In: Proc. SPE/ISRM Rock Mechanics in Petroleum Engineering. Trondheim: Society of Petroleum Engineers, 1998. p. 509–18.

[5] Main IG, Meredith PG. Classification of earthquake precursors from a fracture mechanics model. Tectonophysics 1989;167:273–83.

[6] Lockner DA, Madden TR. A multiple-crack model of brittle fracture 1. Non-time-dependent simulations. J Geophys Res 1991;96:19,623–42.

[7] Malan DF, Napier JAL. Computer modelling of granular material microfracturing. Tectonophysics 1995;248:21–37.

[8] Blair SC, Cook NGW. Analysis of compressive fracture in rock using statistical techniques: Part I, a non-linear rule-based model. Int J Rock Mech Min Sci and Geomech Abstr 1998;35:837–48.

[9] Potyondy DO, Cundall PA, Lee C. Modeling of rock using bonded assemblies of circular particles. In: Aubertin M editor. Proc. Second North American Rock Mechanics Symposium — NARMS '96, Montreal. Rotterdam: Balkema, 1996. pp. 1934–44.

[10] Itasca Consulting Group Inc. PFC^{2D} (Particle Flow Code in 2 Dimensions). Version 2.0, ICG, Minneapolis, Minnesota, 1999.

[11] Cundall PA, Strack O. A discrete element model for granular assemblies. Geotechnique 1979;29:47–65.

[12] Hazzard JF, Maxwell SC, Young RP. Micromechanical modelling of acoustic emissions, SPE47320. In: Eurorock 98, Proc. SPE/ISRM Rock Mechanics in Petroleum Engineering, Trondheim, 1998. p. 519–26.

[13] Potyondy D, Cundall PA, Lee C. Modeling of notch formation in the URL mine-by tunnel: Phase I — micromechanical experiments. Report to Atomic Energy of Canada Ltd, Itasca Consulting Group, Minneapolis, 1995.

[14] Hazzard JF, Young RP, Maxwell SC. Micromechanical modelling of cracking and failure in brittle rocks. J Geophys Res 2000(in press).

[15] Scholz CH. The mechanics of earthquakes and faulting. Cambridge: Cambridge University Press, 1990.

[16] Madariaga R. Dynamics of a circular expanding fault. Bull Seism Soc Am 1976;66:639–66.

[17] Hanks TC, Kanamori H. A moment magnitude scale. J Geophys Res 1979;84:2348–50.

[18] Martin CD. Strength of massive Lac du Bonnet granite around underground openings. Ph.D. thesis, University of Manitoba, Winnipeg, Manitoba, Canada, 1993.

[19] Falls SD. Ultrasonic imaging and acoustic emission studies of microcrack development in Lac du Bonnet granite. Ph.D. thesis, Queen's University, Kingston, Ontario, Canada, 1993.