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CONSOLIDATION TESTING WITH BACK PRESSURE[a]

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SYNOPSIS

The principle of back pressure, which has been applied to develop fully saturated specimens for triaxial testing, has been extended to consolidation testing. The need for reproducing in situ neutral pressures and for having fully saturated specimens for consolidation testing is examined. A new type of consolidometer and loading unit has been constructed for consolidation testing of specimens under back pressure. The apparatus and testing procedure are described. Results obtained from tests an organic silts tested with and without back pressure are presented and compared. Results of back pressure consolidation tests on other soils are mentioned.

INTRODUCTION

The time rate of consolidation of a partially saturated soil under load is significantly different from the time rate of consolidation of a fully saturated

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soil under the same load. There are two basic reasons for the difference: (1) The air bubbles in the partially saturated soil are highly compressible, compared to the relatively incompressible water occupying the corresponding pore space in the saturated soil, and (2) the gas bubbles in the partially saturated soil impede the flow of water in the pores of the soil and thereby reduce the effective permeability of the soil.[4]

If a soil is fully saturated in its in situ condition, the usual condition for soils below the ground-water table, testing in the laboratory should be on

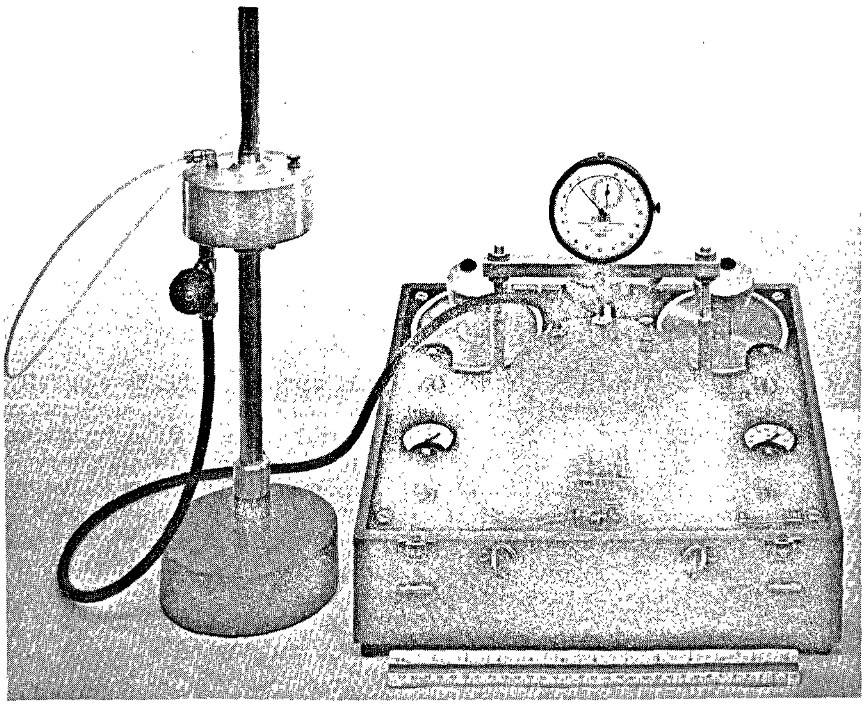


FIG. 1.—BACK PRESSURE CONSOLIDOMETER

fully saturated specimens to properly reproduce field conditions. To assure laboratory duplication of fully saturated field conditions, a special consolidation apparatus has been developed wherein back pressure is maintained on the specimen during testing. The back pressure used may be the in situ neutral pressure prior to loading or a higher pressure, if desired, to facilitate complete solution of all air bubbles in the pore water of the laboratory specimen.

4 Bjerrum, L., and Huder, J., "Measurement of the Permeability of Compacted Clays," Proceedings 4th Internatl. Conf. on Soil Mechanics and Foundations, London, 1957, Vol. I, pp. 6-8.

The apparatus developed for back pressure testing and the testing techniques used are described herein. Also presented are the results of some initial comparative tests on organic silts tested with and without back pressure.

Notation.—The symbols adopted for use in this paper are defined where they first appear and are listed alphabetically in the Appendix.

NEED FOR DUPLICATION OF FIELD CONDITIONS

The importance of duplicating in situ conditions as closely as possible in consolidation tests may readily be noted if the case of a soil subject to high in situ hydrostatic pressure is considered. In this case, the high hydrostatic

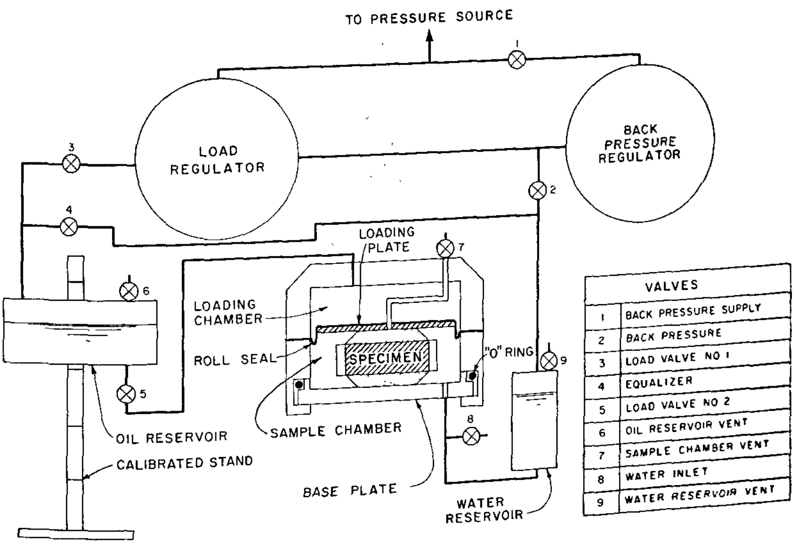


FIG. 2.—BACK PRESSURE CONSOLIDOMETER—SCHEMATIC DIAGRAM

pressure causes gas in the pore space to be dissolved in the pore water. When a sample of the soil is removed from the ground there is an immediate relaxation of applied stresses. The pressure in the pore water reduces to atmospheric pressure or less because of the tendency of the sample to expand; the dissolved gas emerges from solution to form bubbles, most of which become entrapped in the pore water. If this gas is not redissolved in the pore water prior to testing, the soil, which was completely saturated in situ, would be tested in a partially saturated condition. The observed reaction of the specimen to the test load could be significantly different from the reaction of the soil to loading in the field, particularly with respect to the time rate of consolidation.

The simulation of in situ conditions in the consolidometer is accomplished by the following technique.

Prior to the start of the consolidation test, the drainage lines and the water surrounding the specimen are simultaneously subjected to a predetermined pressure called the back pressure. The application of the back pressure causes gas bubbles in the pore space to be redissolved in the pore water. If the back pressure is made equal to the in situ hydrostatic pressure, the stress condition within the liquid phase of the soil in situ has been duplicated. After the back pressure is applied, the consolidation test may proceed in the usual manner.

DESCRIPTION OF APPARATUS

Application of the back pressure principle to one-dimensional consolidation testing required the development of a new type of consolidometer. The consolidometer (patent pending) designed by the staff of the Soils and Foundation Department of Tippetts-Abbett-McCarthy-Stratton, New York, N. Y., in conjunction with Anteus Corp. of Mount Vernon, N. Y., is shown in Fig. 1. A schematic diagram of the apparatus is shown in Fig. 2.

A major feature of the new apparatus is a double chamber assembly. The upper part of the assembly consists of a 5-in. diameter Lucite cylinder that is permanently attached to a steel dome; the lower part is a steel base plate. The upper part is easily attached and sealed to the base plate by a bayonet-type lock and an O-ring seal.

The assembly is divided into a sample chamber and a loading chamber by a roll seal to which is attached a stainless steel plate. This plate provides the means by which load is applied to the porous stone on top of the sample. The roll seal, which is made of a rubber impregnated fabric, provides virtually frictionless operation and is capable of tolerating small amounts of tilting without detrimental effect on the test.

Two specially developed constant bleed regulators, sensitive to a head of 1 in. of water, constitute a second major feature of the apparatus. The first of these regulators is used for applying and controlling the back pressure; the second, a differential regulator, is used for applying and maintaining the load increments that are applied in addition to the back pressure. Pressure for the regulators is supplied from a large volume, high pressure storage tank containing compressed air.

A differential regulator is used to control loading so that if a malfunction should occur in the back pressure regulator, the load would remain constant on the sample. This device also makes possible the application of the same pressure on both sides of the loading plate so that the sample is not loaded during the application of back pressure.

Each regulator is provided with a Lucite disc on the upper surface of which is inscribed a pressure scale. The pressure delivered by the regulator is determined by a pointer that rides in a helical groove machined into the underside of the disc. As the control mechanism of the regulator is turned, the pointer advances and when the needle is directly under an inscribed calibration mark the indicated value of load is made available to the consolidometer. In addition to high sensitivity, the advantages of these regulators over other regulators include low air consumption and practically instantaneous pressure response.

The consolidometer also is equipped with a calibrated stand having an oil reservoir for the application of very low loads ranging from 0.01 tons per sq ft to 0.3 tons per sq ft on a 2-1/2-in. diameter specimen. This stand is calibrated to hundredths of a ton per sq ft. The oil reservoir and the loading section of the chamber are filled with silicone oil. Because the reservoir is of the same cross-sectional area as the roll seal, every inch of movement of the seal causes the same movement in the reservoir oil level. Therefore, the head of oil remains constant even though the specimen consolidates.

The ring normally used in this apparatus is a 2-1/2 in. diameter stainless steel floating ring that is Teflon-lined to decrease wall friction. The apparatus can also accommodate a 4-in. diameter Teflon-lined fixed ring. The Teflon linings can be replaced with little effort should the need develop.

The compression of the sample is measured by an extensometer. The stem of the extensometer is in contact with a needle that passes through a Teflon seal and rests on the upper side of the loading plate.

The maximum capacity of the consolidation apparatus can be expressed as a combination of load and back pressure. For a 2-1/2 in. diameter specimen, the limiting combinations are 64 tons per sq ft and zero back pressure and 32 tons per sq ft and 200 lb per sq in. back pressure. Corresponding values for a 4-in. diameter specimen are 16 tons per sq ft and zero back pressure and 8 tons per sq ft and 200 lb per sq in. back pressure.

TESTING PROCEDURE

Specimen Preparation.—The sample is extruded from a sample liner directly into the Teflon-lined consolidation ring that is fitted with a detachable stainless steel cutting edge. The upper and lower faces of the specimen are trimmed to plane surfaces by using the top and bottom of the ring as guides. A special 1/4-in. spacer disc that is used to push the specimen down from the top of the ring after trimming of the upper face and prior to trimming of the lower face, permits the cutting of a 3/4-in. thick specimen that is centered in the ring prior to insertion of the specimen into the consolidation chamber.

In the chamber the specimen is seated against a saturated, porous stone having the shape of a flat truncated cone and located in the base of the consolidometer; a second similar saturated porous stone is placed on top of the specimen. Once the specimen is in place, the double chamber assembly is locked into position, the compression extensometer frame is attached, the extensometer stem is seated on the consolidation pin, and the sample chamber is filled with water.

Loading.—To prepare for loading, the oil reservoir is lowered to the zero head position and the loading chamber and sample chamber are opened to the atmosphere. This permits the loading plate to move slowly downward under its own weight until it is in contact with the upper porous stone as evidenced by lack of movement of the compression extensometer. After the plate is in contact, the loading and sample chambers are closed to the atmosphere and the load valve on the oil reservoir is opened. The back pressure valve, the equalizer valve (which permits application of back pressure to both sides of the loading plate), and the back pressure supply valve are then opened. The predetermined back pressure then is applied in small increments at intervals of approximately 30 min. Application of the required back pressure normally takes 6 hr. The sample then is permitted to adjust under full back pressure for a period of approximately 70 hr.

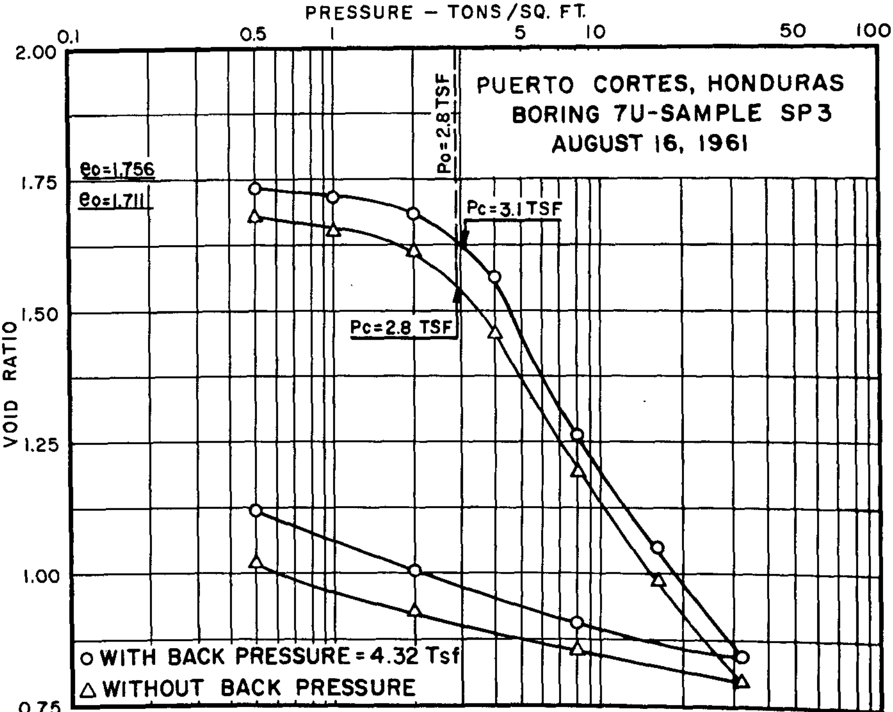
During the application of back pressure, one of three things may occur insofar as the specimen height is concerned. The sample may swell, remain unchanged, or compress. If the sample tends to swell, either the increase in thickness of specimen under zero applied load may be noted, or the pressure required to maintain the specimen at constant thickness (swelling pressure) may be measured. Generally, it is the swelling pressure that is measured. This measurement is accomplished by closing the equalizer valve and raising the oil reservoir to maintain the extensometer at its original reading. If the specimen compresses, the amount and time rate of compression would be observed.

Following the period of adjustment under full back pressure the loading sequence is started. The load valve on the oil reservoir and the equalizer valve are closed. In the case of low loads, the first load is applied by raising the oil reservoir; higher loads are applied by adjustment of the load regulator. The load valve is opened (zero time for the load increment) and time-compression readings are taken at prescribed elapsed time intervals as in the standard test. These readings are usually continued for a period of 24 hr. At the end of this period, the load valve is closed, another setting is made on the load regulator, the load valve is opened to apply the next load, and time-compression readings are taken again. This procedure is repeated until the entire load cycle has been completed. The load on the specimen is then removed in increments, as in the standard test by reducing the pressure delivered by the differential regulator. The unload cycle is continued until the load on the specimen is equivalent to the first load applied in the load cycle. Time-compression readings for the unload cycle are taken for that period of time required for the time-compression curves to become horizontal or for 24 hr, whichever occurs first.

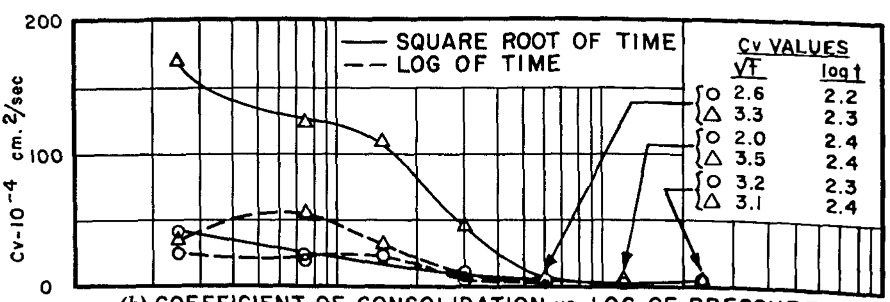
On completion of the unload cycle, the load regulator and the oil head are set at zero and the equalizer valve is opened. The amount of swell is measured until the sample ceases to expand. When the swell readings have been completed, the back pressure regulator is reduced to zero, the water is removed from the chamber, the apparatus is disassembled, and the sample is removed.

COMPARATIVE TESTS

General.—Tests with and without back pressure have been performed on two materials from different locations and with different stress histories. Both materials had a high natural degree of saturation. Consequently, the effect of the back pressure would not be expected to be too striking in these tests. However, several interesting facts are revealed by the test data. The results of the tests on each of these materials are examined individually in the paragraphs that follow. Numerous other tests, both with and without back pressure but not comparative, have been performed with the new consolidometer. The results of these tests tend to confirm the conclusions from the comparative tests regarding instantaneous compression.



(a) VOID RATIO vs. LOG OF PRESSURE



(b) COEFFICIENT OF CONSOLIDATION vs. LOG OF PRESSURE

FIG. 3.—COMPARATIVE TEST RESULTS—MATERIAL NO. 1

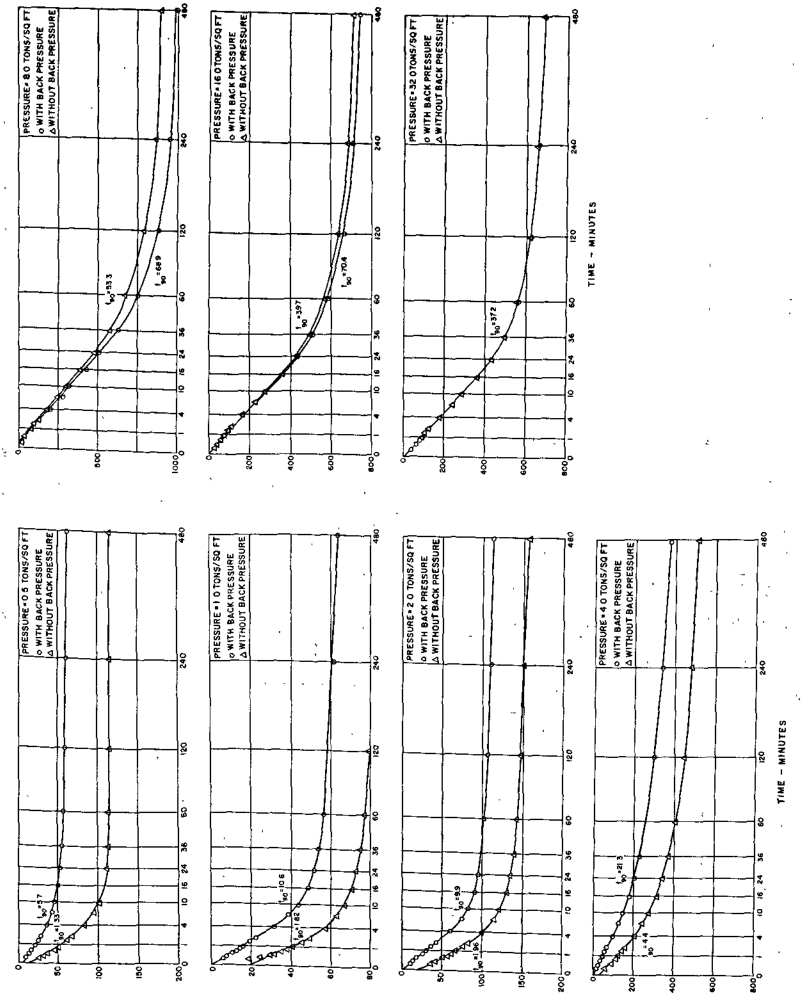


FIG. 4.—COMPRESSION VS. SQUARE ROOT OF TIME CURVES—MATERIAL NO. 1

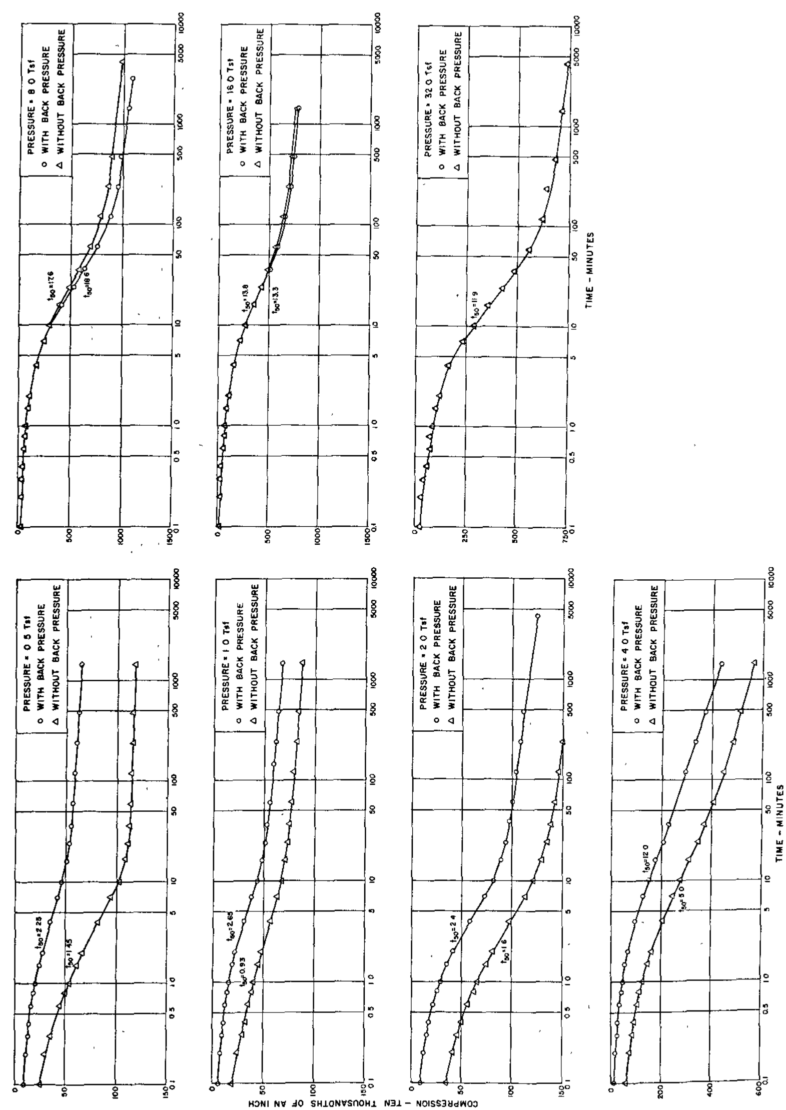
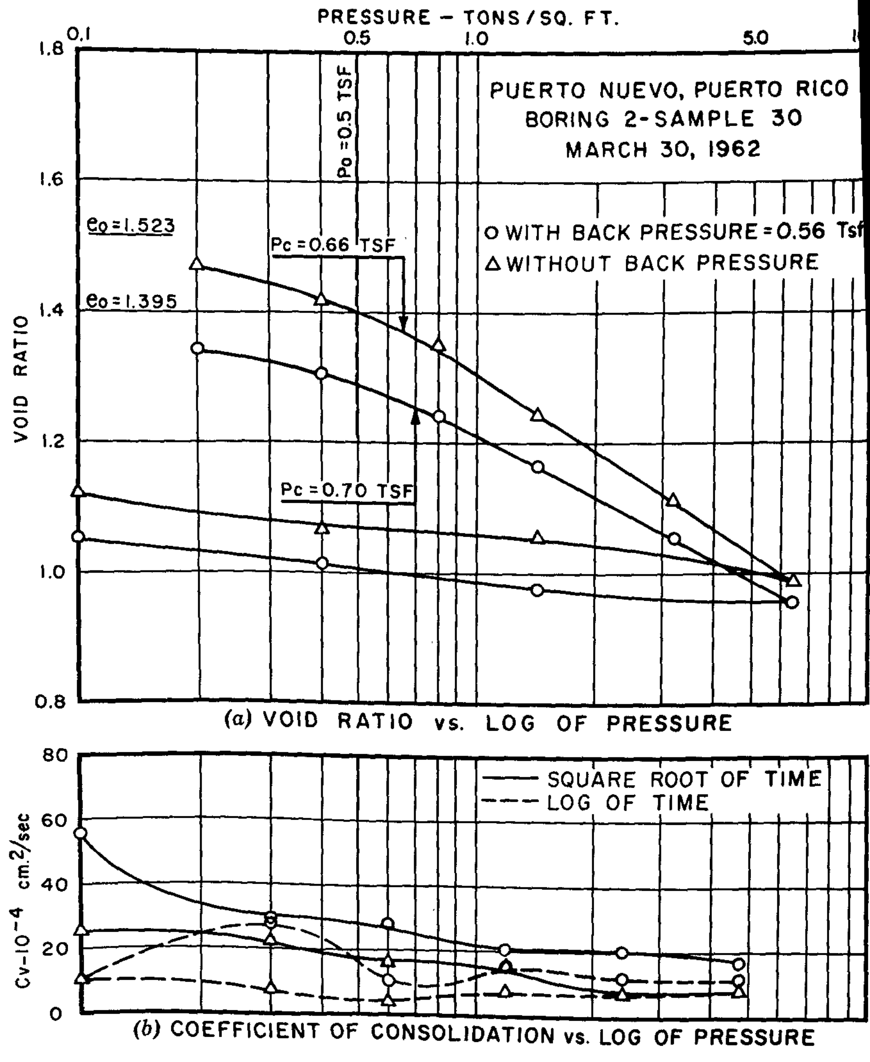


FIG. 5.—COMPRESSION VS. LOGARITHM OF TIME CURVES—MATERIAL NO. 1



(a) VOID RATIO vs. LOG OF PRESSURE

(b) COEFFICIENT OF CONSOLIDATION vs. LOG OF PRESSURE

FIG. 6.—COMPARATIVE TEST RESULTS—MATERIAL NO. 2

It should be noted that in all tests mentioned herein the coefficient of consolidation has been computed by a method devised to minimize the effect of secondary consolidation on the values obtained. It is common knowledge that secondary consolidation has a considerable influence on the latter parts of time rate of compression curves. Therefore, instead of determining the c_v values by using an average height of sample, 2 H, equal to the average of the height at the beginning of the load cycle and the height at the end of a 24-hr period as is common practice, the following were used:

The square root of time

$$2\,H_{avg} = Z_i - \frac{d_s + d_{90}}{2} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

and the logarithm of time

$$2\,H_{avg} = Z_i - \frac{d_s + d_{50}}{2} \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

in which d_s is the corrected compression dial reading at 0% primary compression; d_{50} equals the compression dial reading at 50% primary compression by log fitting method; d_{90} represents the compression dial reading at 90% primary compression by square root fitting method; 2 H_{avg} denotes the average height of sample for a given load increment; and Z_i indicates the thickness of the sample at the start of load increment.

TABLE 1.—TEST DATA—MATERIAL NO. 1

Classification; Gray Organic Silt (OH)		
LL = 81	PI = 37	G_s = 2.75
	With back pressure	**Without back pressure**
Dry Unit Weight, in pounds per cubic foot	62.3	63.3
Initial Water Content, in percentage	64.3	62.0
Final Water Content, in percentage	41.9	39.8
Initial Void Ratio	1.756	1.711
Final Void Ratio	1.120	1.020
Initial Degree of Saturation, in percentage	100	99.7
Final Degree of Saturation, in percentage	100	100

Material No. 1.—The results of a standard consolidation test and a consolidation test with a back pressure of 4.32 tons per sq ft on specimens from a single sample are shown in Figs. 3 through 5. Both tests were performed in the new consolidometer. The sample is a gray organic silt from a boring in the port area at Puerto Cortes, Honduras. It was obtained at a depth of

143 ft below mean sea level by means of a Shelby tube sampler. The silt ha a liquid limit of 81%, a plasticity index of 37%, and a specific gravity of 2.7 Other pertinent data regarding the specimens are shown in Table 1.

The void ratio versus log of pressure curves given in Fig. 3 indicate slightly greater amounts of compression for a given load in the standard tes than in the test with back pressure. This variation, however, is within th normal range of variation for test specimens from the same sample. The tw void ratio versus log of pressure curves are substantially parallel. Thus, th coefficients of compressibility of the two specimens are substantially th same and this is the critical item for proper comparison of the time rate of compression.

The main difference between the test with back pressure and the standar test is shown by the time rate of compression curves of Fig. 4. Presented i this figure are comparative square root of time curves for pressures of 0.5 1.0, 2.0, 4.0, 8.0, 16.0, and 32.0 tons per sq ft. The time rate curve for 1. ton per sq ft is typical of the time rate curves for loads below the preconsolidation load of approximately 3.0 tons per sq ft, and the time rate curve fo 32.0 tons per sq ft is typical for loads above the preconsolidation load. Th time rate curve for the 4.0 tons per sq ft load is intermediate between th two. For the 1.0 ton per sq ft pressure and other pressures below the preconsolidation pressure, a marked difference occurs between the t_{90} tim value determined in the standard test compared with that determined in th back pressure test. Large differences in the coefficient of consolidation, c_v, result therefrom. These differences are shown in Fig. 3 in which the coefficients of consolidation for the standard test are 4 times to 5 times those for the back pressure consolidation test. Differences that are similar although not as large, are observed in the t_{50} values derived therefrom and shown in Fig. 3. For loads of 0.5, 1.0, 2.0, and 4.0 tons per sq ft, the square root of time versus compression curves of Fig. 4 shows that data from the back pressure test resulted in much better straight line relationships for the initial part of the curves did the data from the standard test. This is an indication that the back pressure test yields results that are more nearly correct, because the theory of consolidation indicates that the initial parts of the compression versus square root of time curves should be straight lines. Because of the better initial straight line part of the back pressure test curves, the c_v values from the square root of time curves appear to agree more closely with the c_v values obtained from the logarithm of time curves than do the corresponding values from the standard test.

It is interesting to plot the average of the c_v values obtained from the square root of time and logarithm of time curves for tests with and without back pressure. The resultant curves are much smoother than the c_v curves from either the square root of time or logarithm of time fitting methods. These average curves may give the best approximation of c_v because the square root method fits the theoretical curve to the initial part of the laboratory curve and the logarithm method fits it to the middle of the curve.

For Material No. 1 the average c_v from the back pressure test is much less than the average c_v for the standard test in the lower load range. On the basis that the permeability of a soil is less when air bubbles are present in the pore water than when the water is free of air bubbles, the rate of consolidation in the standard test where air bubbles are present would be less than in the back pressure consolidation test where no air bubbles should be

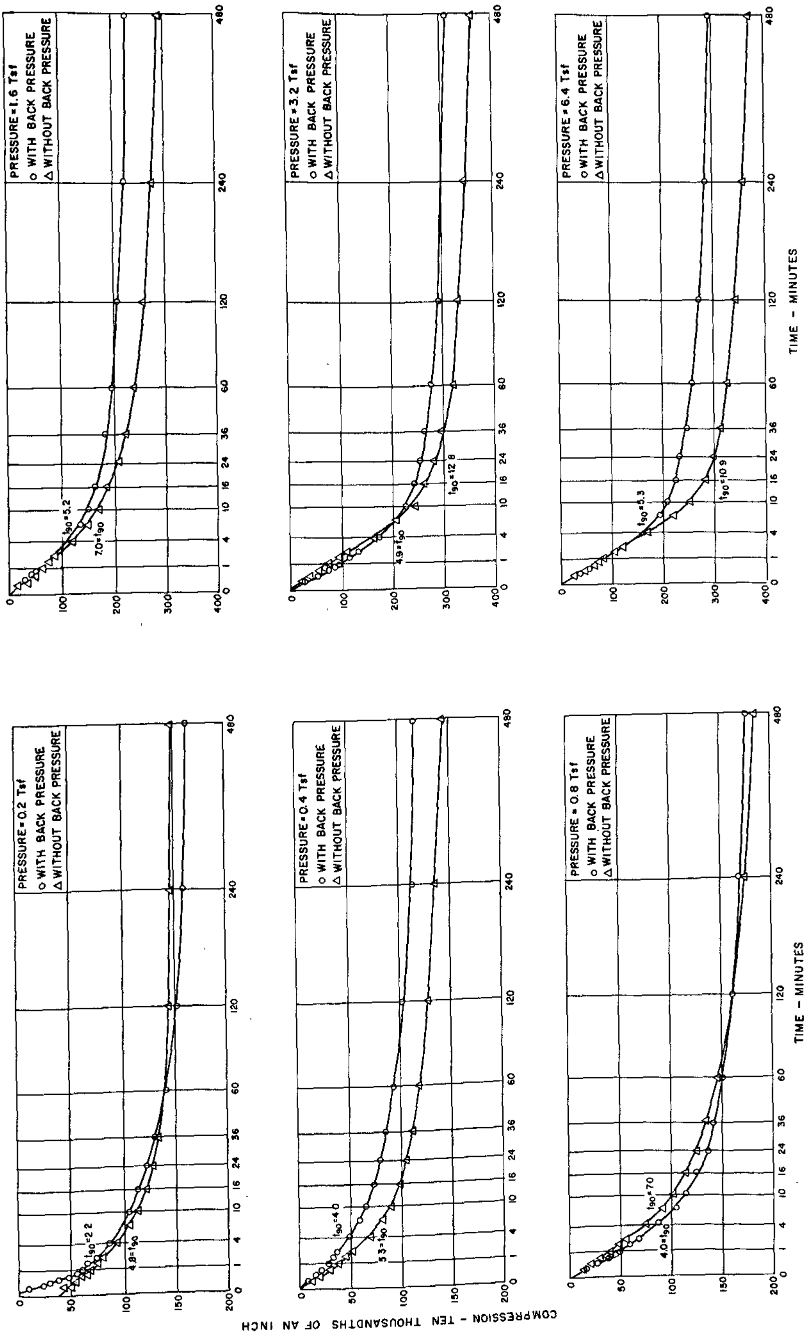


FIG. 7.—COMPRESSION VS. SQUARE ROOT OF TIME CURVES—MATERIAL NO. 2

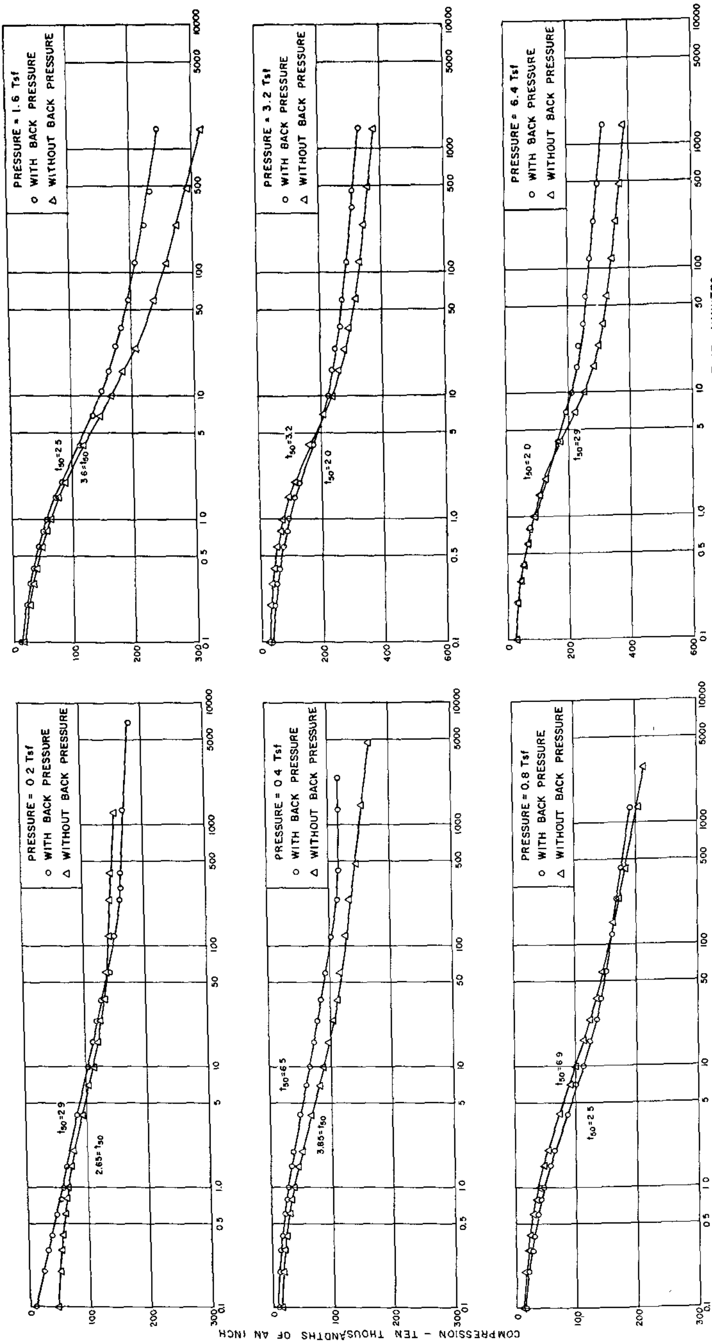


FIG. 8.—COMPRESSION VS. LOGARITHM OF TIME CURVES—MATERIAL NO. 2

present. The reason for the reverse being observed is not immediately clear. A possible explanation is that besides affecting the permeability of the soil, the air bubbles also affect the rate of dissipation of pore pressure. At the beginning of the test, the bubbles permit a more or less instantaneous compression to take place. As the test proceeds, they gradually expand as the hydrostatic excess pressure in the pore water decreases. The same amount of pore fluid is expelled from the specimen with air bubbles as the one without air bubbles. Consequently, the instantaneous compression that occurred at the beginning of the test without expulsion of pore fluid is compensated for in the later part of the test when somewhat more pore fluid has to be expelled than in the test on the saturated specimen resulting from the expansion of the air bubbles back to atmospheric pressure. Because of the expulsion of pore fluid at the latter part of the test when the hydrostatic excess pressures are less, the shape of the time rate of consolidation curve for the soil with air bubbles should be different from the shape of the curve for a soil without air bubbles. More data comparing specimens with and without air bubbles are required to establish the effect of bubbles on the rate of consolidation.

TABLE 2.—TEST DATA—MATERIAL NO. 2

Classification: Gray Organic Silt (OH)		
LL = 58.5	PI = 23.0	G_s = 2.77
	With back pressure	Without back pressure
Dry Unit Weight, in pounds per cubic foot	72.1	68.5
Initial Water Content, in percentage	49.7	54.1
Final Water Content, in percentage	37.4	39.7
Initial Void Ratio	1.395	1.523
Final Void Ratio	1.056	1.124
Initial Degree of Saturation, in percentage	98.6	98.4
Final Degree of Saturation, in percentage	98.1	97.8

Material No. 2.—The results of a standard consolidation test and a consolidation test with a back pressure of 0.56 ton per sq ft on specimens from a single sample of another gray organic silt are shown in Figs. 6 through 8. This material is from a boring in the vicinity of the Puerto Nuevo port area at San Juan, Puerto Rico. It was obtained at a depth of 22 ft below ground surface by a stationary piston sampler. The silt has a liquid limit of 58%, a plasticity index of 23%, and a specific gravity of 2.77. Additional pertinent data regarding the specimens are shown in Table 2.

The back pressure used in the test on material no. 2, although equal to the in situ neutral pressure, is much lower than the back pressure used in the test on material no. 1. Thus, it will be noted that the effect of the back pressure is less pronounced in the test on material no. 2.

Unlike the curves for material no. 1, the void ratio versus logarithm of pressure curves for material no. 2 which are presented in Fig. 6 indicate a somewhat larger amount of compression in the test without back pressure than in the test with back pressure. This may be attributed to the larger initial void ratio of the specimen tested without back pressure and to normal variations in the reaction to load of specimens taken from the same sample. This difference affects but does not invalidate the comparison of the time rates of compression.

As in the case of material no. 1, there is a difference between the time rate of compression as determined from the standard test and the test with back pressure. The square root of time curves presented in Fig. 7 show that for all loadings the t_{90} values are smaller in the back pressure test. The logarithm of time curves presented in Fig. 8 show similar results with reference to the t_{50} values, except that for the two lowest loads t_{50} was less for the standard test than for the back pressure test. However, in neither set of curves is there observed the marked difference in t_{90} or t_{50} values above and below the preconsolidation load as found in the curves for material no. 1. Fig. 6 shows that, for all cases, the coefficient of consolidation for the back pressure test is higher than for the standard test. Thus plots of the average c_v, as described in the paragraph on material no. 1, also would show the average c_v for the back pressure test to be somewhat higher than the average c_v for the test without back pressure. This would be expected on the basis of decreased permeability caused by the presence of air bubbles in the standard test. In general, it also appears that the c_v values from the square root of time curves agree more closely with the values from the logarithm of time curves in the case of the back pressure test. However, this is not as obvious as it is in the case of material no. 1. The close agreement, as noted in the evaluation of the tests on material no. 1, is probably the result of better straight line relationships in the initial parts of the square root curves from the back pressure test, as compared to the curves from standard test.

In both series of comparative tests reported herein, instantaneous compression occurred in virtually all loadings below the preconsolidation load in the standard test. Instantaneous compressions did not occur in the standard test at loadings above the precompression loading. However, no instantaneous compressions occurred at all in the back pressure consolidation test. Instantaneous compressions generally occur in standard-type consolidation tests. This has been found to be true, in general, for numerous other tests that have been performed. D. W. Taylor[5] attributes these initial compressions at least in part to immediate compression of a small amount of gas in the pores. Instantaneous compression did not occur in the back pressure consolidation test because all bubbles were dissolved by the back pressure. Also, instantaneous compression was not noticeable at high loadings above the preconsolidation loading in the standard test, because any compression of air bubbles were relatively small compared to the total compression occurring under these loadings.

In some instances, the time rate curves for back pressure consolidation tests when plotted to the square root of time scale have a straight line part that intersects the ordinate scale above the origin. The actual points plot in

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[5] Taylor, D. W., "Fundamentals of Soil Mechanics," John Wiley and Sons, Inc., New York, N.Y., 1948, pp. 241-242.

a curve from the origin to the straight line part. The reason for this may be necessity to overcome some structural bond in the soil.

CONCLUSIONS

A new type of consolidometer combining ease of operation and the capacity for varying pore pressures and consolidation pressures has been developed to permit the performance of consolidation tests using back pressure. The results of tests on two organic silts indicate that the consolidation characteristics of organic silts as determined by tests with and without back pressure applied to the pore water differ significantly. Tests on one material show the compression, as determined from the standard test, to be substantially the same as that determined from the back pressure test. The coefficients of consolidation for this material, as determined from both types of tests, are significantly different below the preconsolidation pressure where compression of air bubbles in the standard test is a significant part of the total compression. No air bubbles are present in the back pressure consolidation test wherein the use of a high back pressure causes free gas in the pore water to be dissolved in the water and results in a closer duplication of field conditions. Comparative tests on the second material reveal the compression to be somewhat greater in the standard test than in the back pressure test, but this is considered attributable to causes other than the effect of back pressure. For this material, the coefficients of consolidation as determined from the back pressure test were consistently higher than those from the standard test. This difference is believed to be the result of a reduction in permeability caused by the presence of air bubbles in the pore water in the standard test.

Tests performed with back pressure virtually eliminate the initial compression commonly found in the time-compression curves from standard tests. Consolidation characteristics of soils that are saturated <u>in situ</u> should be determined by tests having back pressures at least equal to the <u>in situ</u> neutral pressures.

Additional data, preferably from tests performed on samples with a higher percentage of air voids, are required to substantiate these findings. In particular, it will be necessary during comparative consolidation tests to obtain direct determinations of the permeability of the samples under various pressures for comparison with the permeability as determined from the consolidation test data. On the basis of results obtained, it is expected that the use of consolidation testing with back pressure will permit more accurate prediction of time-rates of settlement for structures founded on saturated soils.

ACKNOWLEDGMENTS

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Vernon, N. Y., who developed the regulators and mechanical details, constructed the back pressure consolidometer, and supplied the photograph and schematic diagram used herein.

APPENDIX.—NOTATION

The following symbols have been adopted for use in this paper:

d_s = corrected compression dial reading at 0% primary compression;

d_{50} = compression dial reading at 50% primary compression by log fitting method;

d_{90} = compression dial reading at 90% primary compression by square root fitting method;

$2\ H_{avg}$ = average height of sample for a given load increment; and

Z_i = thickness of sample at start of load increment.