

Supporting Information for: Porosity-Permeability Relationships in Mudstone from Pore-Scale Fluid Flow Simulations using the Lattice Boltzmann Method

Harsh Biren Vora¹, Brandon Dugan²

¹Department of Earth, Environment and Planetary Sciences, Rice University, Houston, TX 77005

²Department of Geophysics, Colorado School of Mines, Golden, CO 80401

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Introduction:

This supporting information includes six parts:

- (1) Incorporating heterogenous platelet geometry in mudstone models [Fig. S1]
- (2) Compaction data from smectite, kaolinite and intermediate mudstone models [Table S1]
- (3) Compaction data from natural mudstone models, *NM1* and *NM2* [Table S2]
- (4) Evolution of vertical tortuosity (τ_v) during compaction and fluid injection [Fig. S2]
- (5) Microfracture growth data in compacted intermediate mudstone model [Table S3]
- (6) Macrofracture propagation data in compacted intermediate mudstone model [Table S4]

Figure S1: Scaled-down model of mudstone pore structure *NM2* ($\phi=0.79$), designed after sample 1324B-7H-7. The *NM2* mudstone model consists of 31% smectite, 41% illite and 28% chlorite by volume. (a) Cross sectional view of *NM2* pore structure with bedding layers consisting of smectite, illite and chlorite platelets; (b) Orthogonal view of *NM2*; and (c) Cross sectional view of *NM2* pore structure with platelet rotation ($\theta=10^\circ$) and directions of vertical (q_v) and horizontal flow (q_h).

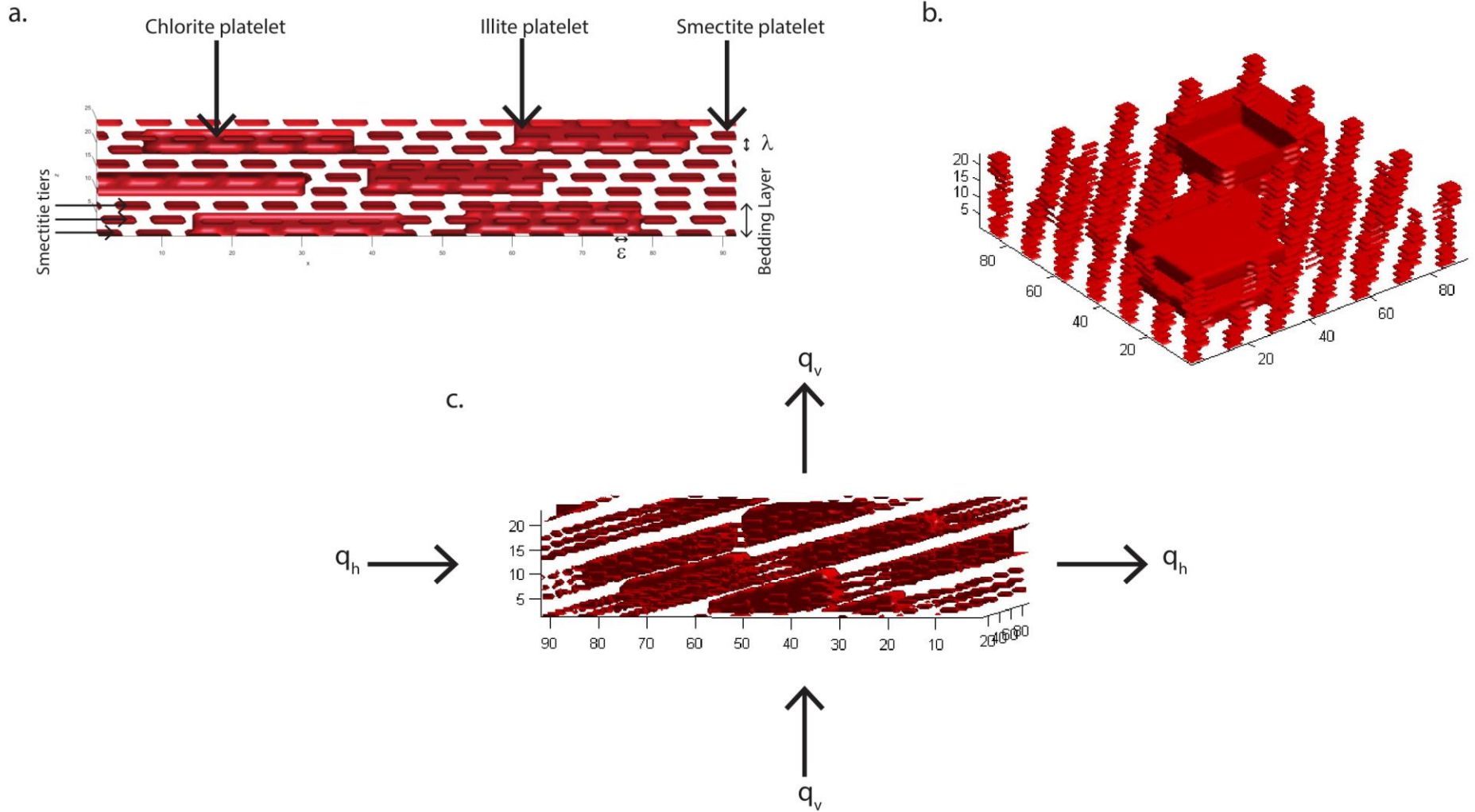


Table S1: Vertical (q_v) and horizontal (q_h) flux during compaction of kaolinite, smectite and intermediate mudstone models, simulated by step-wise decrease in intrabed (ξ), interbed pore throat diameters (λ) and orientation (θ).

Model	Step	Porosity ϕ	Intrabed Pore Throat Width ξ	Interbed Pore Throat Width λ	Platelet Orientation θ	Vertical Flux q_v	Reynolds Number Vertical flow Re_v	Vertical Permeability k_v	Vertical Tortuosity τ_v	Horizontal Flux q_h	Reynolds Number Horizontal flow Re_h	Horizontal Permeability k_h	Horizontal Tortuosity τ_h
			(nm)	(nm)	Degrees	(m/s)		(m ²)		(m/s)		(m ²)	
Kaolinite	1	0.76	3.60×10^2	3.60×10^2	45	7.15×10^{-1}	2.41×10^0	8.31×10^{-15}	3.98	1.12×10^{-1}	1.88×10^{-2}	1.10×10^{-14}	3.98
Kaolinite	2	0.68	2.60×10^2	2.60×10^2	35	3.56×10^{-1}	1.20×10^0	3.42×10^{-15}	5.68	5.16×10^{-2}	8.69×10^{-3}	4.96×10^{-15}	4.36
Kaolinite	3	0.58	1.80×10^2	1.80×10^2	25	9.81×10^{-2}	3.31×10^{-1}	7.84×10^{-16}	8.15	1.77×10^{-2}	2.99×10^{-3}	1.68×10^{-15}	4.53
Kaolinite	4	0.46	1.20×10^2	1.20×10^2	20	2.63×10^{-2}	8.85×10^{-2}	1.78×10^{-16}	11.07	9.27×10^{-3}	1.56×10^{-3}	8.67×10^{-16}	4.97
Kaolinite	5	0.35	8.00×10^1	8.00×10^1	15	5.73×10^{-3}	1.93×10^{-2}	3.42×10^{-17}	14.30	2.33×10^{-3}	3.93×10^{-4}	2.16×10^{-16}	5.00
Kaolinite	6	0.28	6.00×10^1	6.00×10^1	10	1.92×10^{-3}	6.49×10^{-3}	1.07×10^{-17}	16.61	8.69×10^{-4}	1.47×10^{-4}	8.02×10^{-17}	4.29
Kaolinite	7	0.19	4.00×10^1	4.00×10^1	5	3.91×10^{-4}	1.32×10^{-3}	2.02×10^{-18}	19.80	2.60×10^{-4}	4.39×10^{-5}	2.39×10^{-17}	3.31
Kaolinite	8	0.14	4.00×10^1	4.00×10^1	0	1.23×10^{-4}	4.14×10^{-4}	6.33×10^{-19}	21.64	1.56×10^{-4}	2.63×10^{-5}	1.43×10^{-17}	1.74
Smectite	1	0.80	9.00×10^0	9.00×10^0	45	3.56×10^{-1}	4.00×10^{-2}	6.84×10^{-17}	7.08	4.18×10^{-2}	9.40×10^{-5}	1.33×10^{-16}	7.08
Smectite	2	0.76	7.00×10^0	7.00×10^0	35	2.10×10^{-1}	2.36×10^{-2}	3.61×10^{-17}	9.31	5.08×10^{-2}	1.14×10^{-4}	1.60×10^{-16}	6.88
Smectite	3	0.72	5.00×10^0	5.00×10^0	25	1.08×10^{-1}	1.21×10^{-2}	1.63×10^{-17}	12.03	4.11×10^{-2}	9.24×10^{-5}	1.29×10^{-16}	6.27
Smectite	4	0.66	4.00×10^0	4.00×10^0	15	3.50×10^{-2}	3.93×10^{-3}	4.60×10^{-18}	15.44	1.66×10^{-2}	3.72×10^{-5}	5.16×10^{-17}	5.06
Smectite	5	0.58	3.00×10^0	3.00×10^0	10	5.02×10^{-3}	5.64×10^{-4}	5.59×10^{-19}	20.10	3.18×10^{-3}	7.15×10^{-6}	9.85×10^{-18}	4.63
Smectite	6	0.45	2.00×10^0	2.00×10^0	5	3.04×10^{-4}	3.42×10^{-5}	2.77×10^{-20}	27.95	2.79×10^{-4}	6.28×10^{-7}	8.59×10^{-19}	3.74
Smectite	7	0.16	1.00×10^0	1.00×10^0	0	1.84×10^{-7}	2.06×10^{-8}	1.30×10^{-23}	50.65	3.06×10^{-7}	6.88×10^{-10}	9.35×10^{-22}	1.71
Intermediate	1	0.73	13.71×10^1	13.71×10^1	45	1.39×10^{-1}	3.12×10^{-1}	6.10×10^{-16}	6.72	1.75×10^{-2}	1.12×10^{-3}	1.11×10^{-15}	6.72
Intermediate	2	0.66	10.28×10^1	10.28×10^1	35	3.73×10^{-2}	8.37×10^{-2}	1.38×10^{-16}	9.60	7.07×10^{-3}	4.54×10^{-4}	4.44×10^{-16}	7.11
Intermediate	3	0.60	80.00×10^0	80.00×10^0	25	1.73×10^{-2}	3.88×10^{-2}	5.59×10^{-17}	12.79	5.78×10^{-3}	3.71×10^{-4}	3.60×10^{-16}	6.68
Intermediate	4	0.50	57.14×10^0	57.14×10^0	20	6.95×10^{-3}	1.56×10^{-2}	1.93×10^{-17}	16.88	3.00×10^{-3}	1.93×10^{-4}	1.86×10^{-16}	7.06
Intermediate	5	0.44	45.71×10^0	45.71×10^0	15	2.98×10^{-3}	6.70×10^{-3}	7.58×10^{-18}	20.03	1.45×10^{-3}	9.31×10^{-5}	8.93×10^{-17}	6.46
Intermediate	6	0.35	34.28×10^0	34.28×10^0	10	1.06×10^{-3}	2.38×10^{-3}	2.45×10^{-8}	24.45	7.22×10^{-4}	4.64×10^{-5}	4.43×10^{-17}	5.60
Intermediate	7	0.25	22.85×10^0	22.85×10^0	5	2.48×10^{-4}	5.57×10^{-4}	5.16×10^{-19}	30.30	2.33×10^{-4}	1.49×10^{-5}	1.42×10^{-17}	4.16
Intermediate	8	0.07	11.42×10^0	11.42×10^0	0	5.54×10^{-6}	1.24×10^{-5}	1.02×10^{-20}	41.61	7.68×10^{-6}	4.93×10^{-7}	4.68×10^{-19}	1.83

Table S2: Vertical (q_v) and horizontal (q_h) flux during compaction of *NM1* (designed after sample 1324C-1H-1) and *NM2* (designed after sample 1324B-7H-7) mudstone models, simulated by step-wise decrease in intrabed (ξ), interbed pore throat diameters (λ) and orientation (θ).

Model	Step	Porosity ϕ	Intrabed Pore Throat Width ξ	Interbed Pore Throat Width λ	Platelet Orientation Θ	Vertical Flux q_v	Reynolds Number Vertical flow Re_v	Vertical Permeability k_v	Vertical Tortuosity τ_v	Horizontal Flux q_h	Reynolds Number Horizontal flow Re_h	Horizontal Permeability k_h	Horizontal Tortuosity τ_h
			(nm)	(nm)	Degrees	(m/s)		(m ²)		(m/s)		(m ²)	
NM1	1	0.72	5.05×10^1	5.05×10^1	15	5.03×10^{-2}	2.71×10^{-2}	1.54×10^{-16}	11.30	5.08×10^{-2}	7.42×10^{-4}	1.39×10^{-15}	3.91
NM1	2	0.67	3.85×10^1	3.85×10^1	10	1.90×10^{-2}	1.03×10^{-2}	5.19×10^{-17}	13.67	2.43×10^{-2}	3.55×10^{-4}	6.53×10^{-16}	3.43
NM1	3	0.60	2.65×10^1	2.65×10^1	6	3.94×10^{-3}	2.12×10^{-3}	9.39×10^{-18}	17.01	6.22×10^{-3}	9.08×10^{-5}	1.64×10^{-16}	2.94
NM1	4	0.49	1.45×10^1	1.45×10^1	3	3.36×10^{-4}	1.81×10^{-4}	6.86×10^{-19}	22.45	6.70×10^{-4}	9.78×10^{-6}	1.73×10^{-17}	2.47
NM1	5	0.32	2.5×10^0	2.5×10^0	0	4.17×10^{-6}	2.25×10^{-6}	7.09×10^{-21}	32.95	1.05×10^{-5}	1.53×10^{-7}	2.66×10^{-19}	1.52
NM2	1	0.58	6.25×10^1	6.25×10^1	12	1.41×10^{-2}	7.62×10^{-3}	3.11×10^{-17}	12.46	8.16×10^{-3}	1.19×10^{-4}	2.27×10^{-16}	3.69
NM2	2	0.50	3.85×10^1	3.85×10^1	9	3.13×10^{-3}	1.69×10^{-3}	6.65×10^{-18}	15.34	2.63×10^{-3}	3.85×10^{-5}	7.07×10^{-17}	3.60
NM2	3	0.45	2.65×10^1	2.65×10^1	6	1.31×10^{-3}	7.09×10^{-4}	2.74×10^{-18}	17.51	1.50×10^{-3}	2.19×10^{-5}	3.96×10^{-17}	3.12
NM2	4	0.37	1.45×10^1	1.45×10^1	3	3.16×10^{-4}	1.71×10^{-4}	6.46×10^{-19}	20.76	5.06×10^{-4}	7.39×10^{-6}	1.30×10^{-17}	2.50
NM2	5	0.25	2.5×10^0	2.5×10^0	0	2.39×10^{-5}	1.29×10^{-5}	4.79×10^{-20}	26.54	5.96×10^{-5}	8.71×10^{-7}	1.51×10^{-18}	1.60

Fig. S2: Vertical tortuosity increases as porosity declines during compaction (τ_v) (a) in kaolinite, smectite and intermediate mudstone models (b) in heterogenous mudstone models, *NM1* (designed after sample 1324C-1H-1) and *NM2* (designed after sample 1324B-7H-7).

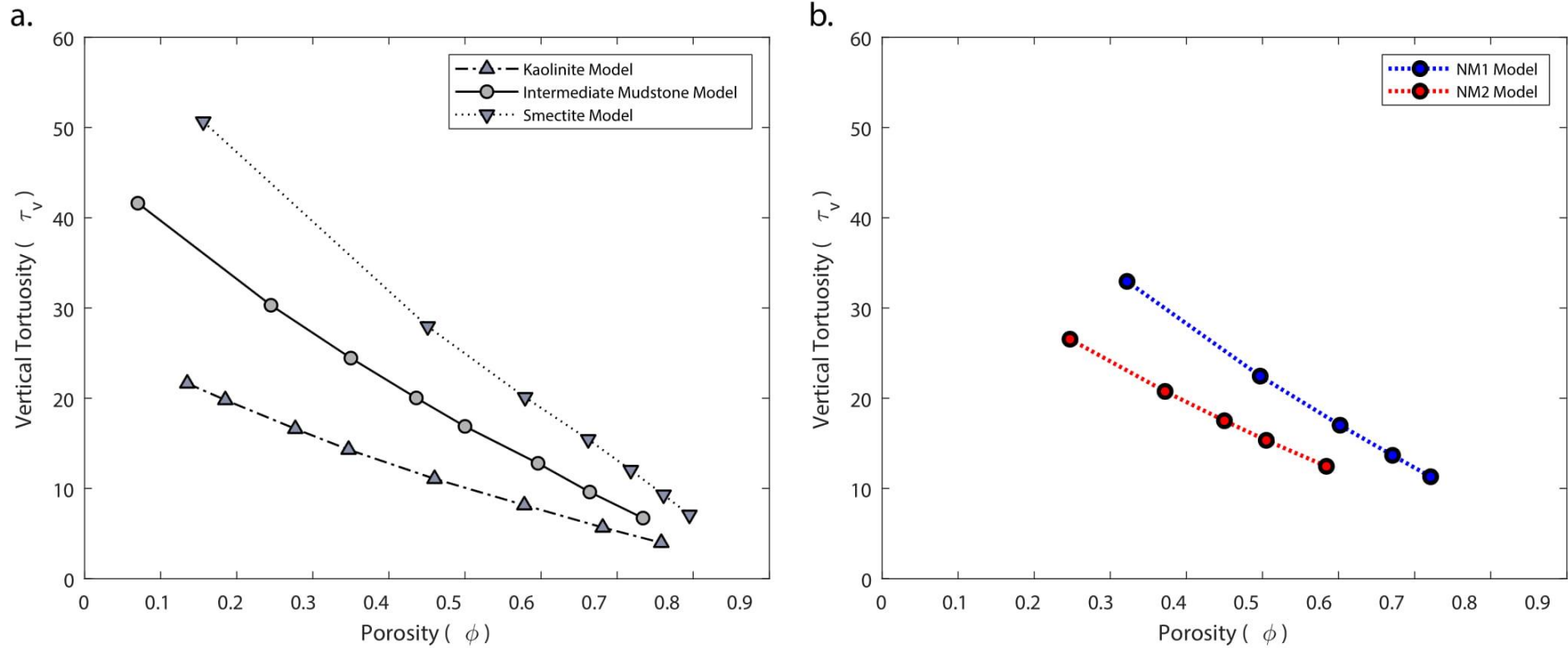


Table S3: Vertical flux (q_v^{mf}) during growth of microfractures through compacted intermediate mudstone, simulated by step-wise increase in microfracture width (ξ^{mf}).

Step	Porosity ϕ	Micro-fracture Width ξ^{mf}	Effective Fracture Width ε_{eff}^{mf}	Interbed Pore Throat Width λ	Vertical Flux q_v^{mf}	Reynolds Number Vertical flow Re_v	Vertical Permeability k_v^{mf}
		(nm)	(nm)	(nm)	(m/s)		(m ²)
1	0.07	11.42 x10 ⁰	0.00 x10 ⁰	11.42 x10 ⁰	5.54 x 10 ⁻⁶	1.24 x 10 ⁻⁵	1.02 x 10 ⁻²⁰
2	0.10	57.14 x10 ⁰	1.37 x 10 ²	11.42 x10 ⁰	1.56 x 10 ⁻⁴	3.52 x 10 ⁻⁴	2.89 x 10 ⁻¹⁹
3	0.13	10.20 x10 ¹	2.74 x 10 ²	11.42 x10 ⁰	1.05 x 10 ⁻³	2.37 x 10 ⁻³	1.95 x 10 ⁻¹⁸
4	0.18	18.28 x10 ¹	5.14 x 10 ²	11.42 x10 ⁰	7.99 x 10 ⁻³	1.79 x 10 ⁻²	1.48 x 10 ⁻¹⁷
5	0.25	29.71 x10 ¹	8.57 x 10 ²	11.42 x10 ⁰	4.68 x 10 ⁻²	1.05 x 10 ⁻¹	8.66 x 10 ⁻¹⁷
6	0.29	37.70 x10 ¹	1.10 x 10 ³	11.42 x10 ⁰	1.12 x 10 ⁻⁴	2.52 x 10 ⁻¹	2.07 x 10 ⁻¹⁶

Table S4: Vertical flux (q_v^{frac}) during propagation of macrofracture through compacted intermediate mudstone, simulated by step-wise increase in fracture width (ξ^{frac}).

Step	Porosity ϕ	Macro-fracture Width ξ^{frac}	Effective Fracture Width ε_{eff}^{frac}	Interbed Pore Throat Width λ	Vertical Flux q_v^{frac}	Reynolds Number Vertical flow Re_v	Vertical Permeability k_v^{frac}
		(nm)	(nm)	(nm)	(m/s)		(m ²)
1	0.07	11.42 x10 ⁰	0.00 x10 ⁰	11.42 x10 ⁰	5.54 x 10 ⁻⁶	1.24 x 10 ⁻⁵	1.02 x 10 ⁻²⁰
2	0.12	37.71 x 10 ¹	3.66 x 10 ²	11.42 x10 ⁰	9.41 x 10 ⁻⁶	2.11 x 10 ⁻⁵	1.74 x 10 ⁻²⁰
3	0.17	70.85 x 10 ¹	6.97 x 10 ²	11.42 x10 ⁰	1.17 x 10 ⁻⁵	2.64 x 10 ⁻⁵	2.17 x 10 ⁻²⁰
4	0.21	11.09 x 10 ²	1.10 x 10 ³	11.42 x10 ⁰	3.83 x 10 ⁻⁴	8.60 x 10 ⁻⁴	7.07 x 10 ⁻¹⁹
5	0.25	14.74 x 10 ²	1.46 x 10 ³	11.42 x10 ⁰	5.81 x 10 ⁻³	1.31 x 10 ⁻²	1.07 x 10 ⁻¹⁷
6	0.32	22.06 x 10 ²	2.19 x 10 ³	11.42 x10 ⁰	6.64 x 10 ⁻²	1.49 x 10 ⁻¹	1.22 x 10 ⁻¹⁶