**Progressive Localization of Microcracks and Acoustic Emissions in Granite and Sandstone: Insights using the Discrete Element Method**

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**Abstract**

1. **Introduction**

The mechanical loading of rocks results in local inelastic processes that produce microcracks [Kranz, 1983], and their cooperative interaction leads to formation of macroscopic fractures [Wong et al., 1997]. The spatial and temporal evolution of microcracks affects several physical properties of rock such as peak strength, stiffness and permeability. Inelastic deformation in the form of microcracking is accompanied by release of fracture energy, producing Acoustic Emissions. Acoustic Emissions (AEs) are broadly defined as high-frequency transient elastic waves generated by a sudden release of stored strain energy within a material [Lockner, 1993]. While correlation between macroscopic strain and microcracking activity, and associated AE, has been established by experimental [Lockner, 1993] and numerical studies [Hazzard et al., 2000], understanding of mechanical controls over microcrack coalescence and associated AE remains elusive.

During biaxial experiments, the growth of fractures occurs through the interaction of shear and tensile microcracks [Lei et al., 2000], representing a complicated process influenced by lithology [Lei et al., 2004] and confining pressure [Amitrano, 2006]. While fracture growth in crystalline rocks such as granite occurs largely through the coalescence of tensile microcracks [Moore and Lockner, 1995], pore collapse and shear microcracking have been shown to be the dominating mechanisms in non-crystalline rocks such as sandstone [Fortin et al., 2009]. Recently, AE’s have been employed to resolve the differences between proposed models of fracture growth [Baud et al., 2004; Stanchits et al., 2006], but accurate prediction of microcrack mode may be ambiguous due conversion of acoustic waves [Modiriasari et al., 2017] and difficulty in inverting AE’s for double-couple mechanisms [Jalali et al., 2018]. Thus, it is difficult to accurately monitor the spatial and temporal evolution of microcracking and associated failure modes during biaxial experiments. Additionally, constraints over splitting of fracture energy between tensile and shear microcracks remain elusive. Numerical methods provide the ability to monitor stresses and displacement of grains on a micro-scale, complementing the macromechanical behavior observed during laboratory experiments. Thus, we seek to constrain the dominant controls over microcracking mode, their relative contribution to fracture energy during growth of shear fractures in crystalline and non-crystalline rocks using numerical simulations of biaxial experiments.

The process of fracture growth is further influenced by the effects of confining pressure on microcracking mode and spatial distribution of microcracks. Analyses of AE accompanying microcracking during biaxial experiments document that increasing confining pressure results in greater AE energy [Zhang et al., 1990] and spatial distribution of events [Hirata et al., 1987]. However, the influence of confining pressure on mode and spatial distribution of microcracks, and how that corresponds to changes in the observed AE energy remains elusive due to our inability to monitor individual events during laboratory experiments. Thus, there is a need for a modeling approach to monitor the source mechanisms of individual microcracks, their patterns of coalescence, and their effect on AE energy.

The deformation during confined biaxial experiments and associated AE exhibit striking spatial and temporal similarity to earthquakes, obeying the Gutenberg-Richter relationship [Scholz, 1968] and Omori’s Law [Reasenberg and Jones, 1989]. The self-similarity in deformation processes from the laboratory to the field scale are particularly expressed by similar values of dimensionless parameters, such as the fractal dimension (D-value) [Hirata et al., 1987], a statistical measure of distribution of events, and the seismic b-value [Lei et al., 2004], slope of frequency-moment relationship expressed by the events. However, a mechanistic understanding of how the dimensionless parameters are affected by local stress changes, microcracking mode and confining pressure remains elusive. Since acoustic emissions accompanying brittle deformation show similarity over eight orders of magnitude of length [Hanks, 1992], understanding the variation in b-values and D-values from a micromechanical analysis can provide further characterize catastrophic processes along faults.

Numerical modeling approaches offer the ability to investigate the mechanisms of microcracking and brittle fracture growth in detail. Discrete numerical methods are attractive to study brittle fracturing because, much like real rocks, the numerical materials are composed of assemblages of grains. In the discrete numerical approach, particle interactions yield emergent behaviors and heterogeneities form and evolve in response to changing stress conditions and material properties [Morgan 2015]. The Discrete Element Method (DEM) has been employed to simulate rock deformation from laboratory scale experiments to large scale geodynamic processes, including formation of deformation bands in sandstones [Wang et al.,2008], analyze changes in porosity and stress during biaxial experiments [Longjohn et al., 2018], evolution of fault gouges [Guo and Morgan, 2007], evolution of slope failure and landslide processes [Amitrano, 2006], and deformation of fold and thrust belts [Dean et al., 2013]. However, there has been little investigation into contribution of microcracking mode to growth and coalescence of fractures and associated Acoustic Emissions. In this study, we use the Discrete Element Method [Cundall and Strack, 1979] to investigate the micromechanics of brittle fracturing through simulations of biaxial experiments in granular rock material.

1. **The Discrete Element Method**

The discrete element method [Cundall and Strack, 1979] is a particle-based numerical technique that employs a time stepping, finite difference approach to solve Newton’s equations of motion for every particle in a system. The method first solves for forces imposed on the surfaces of each particle by neighboring particles or boundaries and then calculates a displacement based on the acceleration caused by sum of the forces. Particle motions are induced by external forces prescribed by stress or strain rate boundary conditions, and by forces resolved at interparticle contacts. The disequilibrium of forces drives particle displacements. The numerical code used is RICEBAL, based on open-source code TRUBAL [Cundall and Strack, 1979]. RICEBAL resembles a numerical sandbox but offers added value by allowing material properties and mechanical states to be monitored throughout simulations and be correlated with deformation behavior and structure.

**2.1. Interparticle Contact Forces and Bonding**

Morgan, 2015 provides a detailed description of the mechanics of RICEBAL, which is summarized here. The force-displacement caused by particle interaction is characterized by normal and shear forces along elastic, frictional contacts, calculated using the following equations respectively [Fig. 1a]:

(Eq. 1)

(Eq. 2)

where *fn* is the repulsive normal force, *kn* is the normal interparticle stiffness, *δn* is the amount of overlap between particles in contact, *fs* is the shear force, *ks* is the shear stiffness, *δs* is the shear offset of particle centers. We implement the non-linear Hertz-Mindlin theory [Johnson, 1985] to calculate *kn* and *ks*, which are related to the elasticity of the particles the contact area of overlapping particles:

(Eq. 3)

(Eq. 4)

Where *Ep* is the Young’s Modulus of the particles, *Gp* is the Shear Modulus of the particles, and *Ra* and *Rb* are the radii of the respective discrete elements. In RICEBAL, *Gp* is an input parameter, which is related to *Ep* through the equation:

(Eq. 5)

Where *νp* is the Poisson’s ratio of particles. Shear forces at interparticle contacts are also limited by friction along particle surfaces (*µp*), as

(Eq. 6)

Where *fsmax* is the threshold force for particles to slide past each other. We implement interparticle bonding to impart cohesion into the material assemblage. The bonds have four key mechanical properties – *Eb*, the Young’s Modulus of the bond, *Gb*, the Shear Modulus of the bond, *Tb*, the tensile strength of the bonds, and *Cb*, the bond cohesion (unconfined shear strength of bonds at *fn*=0). The bonds connect the centers of particles in contact, and bond forces are at zero when particles are not displaced relative to each other (*δn*=*δs*=0). When particles are displaced in tension, the bonds support tensile and shear forces below predefined tensile strength and shear strength. When particles are displaced in compression, the bonds support the shear forces below the predefined shear strength. Normal and shear forces are related through the following equations [Fig.1b]:

In compression (*δn*>0): (Eq. 7)

In tension (*δn*<0): (Eq. 8)

Where *fnmin* is the minimum normal force required to break the bond. Bond-induced interparticle forces are calculated as

(Eq. 9)

(Eq. 10)

where *Ab* is the cross-sectional area of the elastic bond, assumed to be a circle with the radius equal to the smallest particle in contact. When particles are separated in tension (*δn*<0), bond tensile forces are limited by tensile strength and bond area, *Tb*\**Ab*. Shear forces separated during tension (*fn*<0) and compression (*fn*>0) are expressed by

(Eq. 11)

(Eq. 12)

**2.2. Particle Motions**

For each time step, net force and displacement and moment are calculated by summing components of all contact forces acting on a particle. Net force (*Fp*) and net moment (*Mp*) are calculated for each particle by

(Eq. 13)

(Eq. 14)

Where *mp* and *Ip* are mass and moment inertia and *ẍp* and *Ӫp* are the linear and angular accelerations respectively. The particle displacements and roataions are calculated by inverting twice and integrating Eq. 14 and Eq. 15 over each time step for the new particle positions and orientations. At each time step, new particle configurations and contact forces are calculated. At each time step, particle motions are partially damped to dissipate energy in the system, to recreate the inelastic deformation in rocks [Cundall, 1987; Hazzard et al., 2000].

**2.3. Quantification of Stress, Strain and Porosity**

The quantification of bulk parameters during biaxial experiments using RICEBAL are explained in detail by Longjohn et al., 2018. Membrane and platen boundary stresses correspond to the minimum and maximum principal stress (*σ3* and *σ1*). Membrane stresses are maintained constant at a specified confining pressure; the platen stresses are calculated by summing the net forces of the particles at each platen and dividing that sum by the platen area. Stress associated with bond breakage is recorded by dividing fs an fn at failure by the area of the bond, which is equal to the area of the smallest particle in the domain. The displacement of the platens is used to calculate axial strain (*εa*). Bulk porosity (*φ*) is calculated using total volume of the domain and total volume of all the spheres within the sample as:

(Eq. 15)

1. **Biaxial Experiments and Calibration of Materials**

In this study, we develop numerical analogs for Berea Sandstone and Lac du Bonnet Granite, representing two widely studied rock types ranging from weaker non-crystalline rocks to strong crystalline rock. We simulate biaxial experiments for both rock types at ten different confining pressures (0 MPa - 50 MPa), to calibrate bulk behavior of numerical samples to experimental datasets under confined and unconfined conditions.

* 1. **Biaxial Experiments using the Discrete Element Method.**

We simulate samples as granular assemblage of ~60,000 particles with particle radii of 10-40 µm within an initial spatial domain of 0.04 m x 0.03 m [Fig. 2a]. To prepare cohesive samples for biaxial experiments, we preconsolidate our samples to a confining pressure of 10 MPa. The two horizontal confining walls, constructed of rows of particles, are moved inward between rigid vertical walls of particles until preconsolidation stress of 10 MPa is achieved. After consolidation, sample dimensions are 0.019 m x 0.03 m [Fig. 2b]. Porosity of the all samples after preconsolidation is 0.176. Following preconsolidation, the particles along horizontal platens can move independently in the vertical direction, while maintaining their constant confining stress, acting as a membrane to confine the sample during biaxial experiments. At this stage, we introduce interparticle bonds to simulate cohesive rock material. Axial compression is conducted by moving lateral platens inward at a constant velocity. As the vertical platens move inwards, local stresses increase causing failure of interparticle bonds. In this study, we define the failure of each interparticle bond as a microcrack. The induced microcracks coalesce to form a shear fracture, resulting in the failure of the sample [Fig. 2c]. Biaxial experiments are run under confining pressures of 0, 2, 5,10,15,20,25,30,40 and 50 MPa. Macromechanical properties of the sample are collected at increments of 2000 cycles, associated with axial strain increments of 0.001 and relative platen displacement of 0.008 mm. All simulations of biaxial experiments are conducted up to axial strain of 0.103. In our study, micromechanical parameters of discrete particles and bonds are assigned to calibrate bulk properties to two rock types – Berea Sandstone and Lac du Bonnet Granite. This choice of rock types will help us to contrast the fracture growth mechanisms between weak, sedimentary rock (sandstone) and strong, crystalline rock (granite).

* 1. **Micromechanical Parameters and Calibration of Samples**

Experimental laboratory data is used as a reference to adjust micromechanical parameters and calibrate the bulk behavior of our models to replicate Berea Sandstone and Lac du Bonnet granite, materials that we choose to represent porosity and strength end-members in rock mechanics. To capture the geomechanical character of the rocks under unconfined and confined conditions in our models, we calibrate Unconfined Compressive Strength (*UCS*), Young’s Modulus (*E*), and Mohr-Coulomb Cohesion (*C*) and slope (*µ*), to experimental datasets. The pertinent input microparameters are (1) mechanical properties of the particles - Shear Modulus of particles (*Gp*), Poisson’s Ratio of Particles (*νp*), (2) mechanical properties of interparticle bonds - Young’s and Shear Modulus of bonds (*Eb* and *Gb*), Tensile Strength and Cohesion of bonds (*Tb* and *Cb*), and (3) interparticle friction (*f*). The derived microparameters are determined indirectly by assigning values and comparing the response of the model to the geomechanical properties of the physical rock.

*Calibration of Young’s Modulus (E)*: The bulk Young’s Modulus increases with the Young’s Modulus of Particles (*Ep*), which is controlled using input elastic parameters of particles, *Gp* and *νp*, through Eq. 5. We derive a Young’s Modulus of 4.28x109 Pa for Berea Sandstone [Fig. 3] is attained by using values of 2.90x1010 Pa and 0.33 for *Gp* and *νp* respectively [Table 1]. Similarly, we derive a Young’s Modulus of 42.07x109 Pa [Fig. 3] for Lac du Bonnet Granite is attained by using values of 2.00x1012 Pa and 0.26 for *Gp* and *νp* respectively [Table 1].

*Calibration of Unconfined Compressive Strength (UCS)*: *Tb* and *Cb* describe the energy required to break assigned bonds in tensile and shear mechanisms respectively. Scholtès and Donzé, 2013 prescribe that assigned values of *Tb* and *Cb* must have a ratio equal to bulk *UCS*/*TS*, where *TS* is the tensile strength of the rock, to ensure that micromechanical processes are reflective of the bulk geomechanical behavior of the rock. In this study, we maintain a *Tb*/*Cb* ratios of 10 for Berea Sandstone [Bobich, 2005], and 20 for Lac du Bonnet Granite [Martin and Chandler, 1994]. *UCS* shows a direct correlation with *Tb* and *Cb* which are adjusted to match experimental values, while obeying the guidelines of Scholtès and Donzé, 2013. We attain a UCS of 85.05x106 Pa for Berea Sandstone [Fig. 3] by employing values of 9.00x106 Pa and 9.00x107 Pa for *Tb* and *Cb* respectively [Table 1]. Similarly, we attain a UCS of Pa for Lac du Bonnet Granite [Fig. 3] by employing values of 6x108 Pa and 1.2x1010 Pa for *Tb* and *Cb­­* respectively [Table 1].

*Calibration of Mohr-Coulomb Cohesion (C) and slope (µ)*: After attaining the desired values of bulk Young’s Modulus and UCS through unconfined tests, confined compression tests are conducted to attain the desired Mohr-Coulomb behavior (*C* and *µ*) by varying interparticle friction (*f*). Interparticle friction shows a direct correlation with slope of the Mohr-Coulomb envelope, and is adjusted to attain the bulk compressive behavior of Berea Sandstone and Lac du Bonnet Granite. We employ values of 0.3 and 0.7 for Berea Sandstone and Lac du Bonnet Granite respectively [Table 1] to attain desired Mohr-Coulomb behavior, described by the following equations [Fig. 4]:

(Eq. 16)

(Eq. 17)

where *τ* is the shear stress (MPa) and *σn* is the normal stress (MPa) on the sample. Thus, through retroactive adjustment of micromechanical parameters we calibrate the bulk properties of simulated samples of Berea Sandstone and Lac du Bonnet Granite to match experimentally derived geomechanical properties [Table 2].

1. **Characterizing Simulated Microcracks and Acoustic Emissions**

In order to characterize the progression of fracture growth and associated material damage, we document several quantities and derive characteristic relationships that can be compared among experiments. Each bond breakage event is assumed to be a microcrack in the modeled rock samples. As axial stress is applied to the vertical platens during the numerical biaxial experiments, fractures grow by the coalescence of emergent microcracks. During each simulated biaxial experiment, we track the spatial and temporal evolution of microcracks and their mode of failure, and calculate the energy associated with each microcrack. These data yield several dimensionless parameters that describe the evolution of the sample, described below. ~~We calculate the fractal dimension (D-value) using spatial distribution of clustered events, and the evolution of Damage Index using measurements of porosity through an experiment. We implement spatial and temporal clustering of microcrack energy, and calculate the seismic b-value based on frequency-magnitude distribution of the clustered events.~~

**4.1.** **Mode of Microcracks**

Interparticle bonds can fail in either tension or shear, as show in Equations (#)-(#). A tensile microcrack forms when interparticle normal stress exceeds the tensile strength of the bond, resulting in a mode 1 microcrack. Similarly, a shear microcrack results when local shear stress exceeds the shear strength of the bond in compression, resulting in a mode 2 microcrack. A mixed mode microcrack, referred to as tensile-shear, can form when local shear stress exceeds bond shear strength in tension. We document the mode of each microcrack generated, along with the failure stress at the time of bond breakage.

**4.2. Energy of Microcracks and Acoustic Emissions**

As axial stress is applied to the platens, interparticle bonds become distorted prior to failure, accumulating elastic strain energy. Bond failure that accompanies microcrack formation releases this energy instantly, emitting an acoustic signal analogous to an earthquake. The acoustic energy associated with each microcracking event is calculated using the following equation [Tang and Kaiser, 1998]:

(Eq. 18)  
Where *Ef* is the acoustic energy associated with a micro-fracture, *Cf* is the elastic modulus of the bond broken, *σcf* is the peak strength of the failed element and *vf* is the volume of microcrack. RICEBAL provides us with the ability to monitor stress associated with each broken bond *σcf*. Volume of a microcrack (*vf*) is taken as the sum of the areas of the two particles bounding the broken bond. If the microcrack fails in shear, *Cf* takes the value of shear modulus of the bond (*Gb*); if the microcrack fails in tension, *Cf* takes the value of Young’s modulus of the bond (*Eb*). Total fracture energy is calculated as the sum of energy from all microcracking events during a biaxial experiment, up to an axial strain of 0.103.

**4.3. Damage Index (*DI*)**

For each biaxial simulation, we use the measured porosity (*φ*) at each increment of axial strain to calculate the Damage Index (*DI*) defined by Renaud et al., 2017 as:

(Eq. 19)

where *φi* is the porosity of the sample before the onset of the biaxial experiment. After preconsolidation to 10 MPa, the initial porosity of all samples in this study is 0.176. A positive value for *DI* at peak stress indicates the growth of a dilatant fracture zone, whereas a negative value of *DI* indicates a compactant and shear dominated fracture zone [Renaud et al., 2017].

**4.4. Characterizing Microcrack Distribution (D-value)**

The distributions of AE source locations have been widely used to study crack redistribution during failure processes in laboratory experiments. In this study, we quantify the distribution of microcrack events using the correlation integral (*C(R)*) [Hirata et. al, 1987] defined as:

, (Eq. 20)

where *NR(r<R)* is the number of AE source pairs separated by a distance *r* shorter than *R*, and *N* is the number of sources analyzed. The source distribution of AE’s has been shown to be fractal in nature, where the correlation integral *C(R)* is proportional to *RD*, where *D* is the fractal dimension of the distribution:

(Eq. 21)

In our study, we calculate representative *D*-value for each simulated biaxial experiment. In two dimensions, *D*=2 indicates completely random distribution of microcracks. Lower values of D indicate more localized microcracks, concentrated in zones of more intense damage. However, since the *D*-value does not carry information regarding the shape of spatial distribution, we supplement fractal analysis with visual inspection of the actual microcracks generated during biaxial experiments.

**4.5. Spatial and Temporal Clustering of Microcrack Energy**

Microfractures in real rocks defines a broad range of lengths and energies, and show strong spatial and temporal correlation [Main et al., 1989]. Due to the narrow range grain size distribution in our models, the energy associated with microcrack formation, which corresponds to individual bond breakage events, also falls into a narrow range. To correct for the limitations of the discrete element approach, we implement a spatial and temporal clustering algorithm developed by Hazzard and Young, 2000 [Fig. 5]. When a bond breaks, the energy of all microcracks in the source regions is summed to define the initial energy of a macro-event (*E0*). We implement a source dimension of three times the radius of the largest particle in our domain (r=0.0012 m), as used by Hazzard and Young, 2000. The even duration is calculated by assuming that each crack is an expanding shear fracture, which can propagate as slowly as 0.5 times the shear wave velocity of the material [Madariaga, 1976]. In our methodology, we assume constant shear wave velocities of 1500 m/s (corresponding to an event duration of 1.6x10-6 s) for Berea Sandstone and 3000 m/s for Lac du Bonnet Granite (corresponding to an event duration of 8x10-7 s) [Mavko et al., 2009]. Since we apply a constant strain rate of 1.2x10-7 m/s in our simulations, the event durations correspond to 1.5% and 2.5% of total applied strain, which we simplify to an average event duration of 2 % of total strain. The energy of all the individual microcracks occurring in the source region and through the duration of the macro-event are summed, and constitute the energy of a macro-event. We calculate the moment magnitude (*M*) associated with the energy of each clustered event (*Ec*) using the relationship defined by Kanamori, 1983 as:

(Eq. 22)

**4.6. Acoustic Emission Frequency-Magnitude Relationships (b-values)**

The AE amplitude distribution during fracturing experiments has been shown to obey the Gutenberg-Richter relationship [Richter, 1958] observed for crustal earthquakes [Mogi, 1962; Scholz, 1968]. According to this relationship, the distribution of AE size is expressed by a power law:

(Eq. 23)

Where *A* is the maximum amplitude of AE, *N* is the number of events with amplitude greater than *A*, and *a* and *b* are constants. The logarithmic form is often used to linearize the relationship:

(Eq. 24)

Where *M*=log(*A*) is the AE magnitude and the exponent *b* is the scaling of the AE magnitude distribution. The evolution of *b* value has been used to study stages of fracture development [Lei et al., 2006] and earthquake forecasting [Mogi, 1967]. In this study, we analyze the effects of lithology and confining pressure on the evolution of b-value during biaxial experiments. We calculate a characteristic b-value for each biaxial experiment by deriving the slope of the frequency-magnitude behavior of clustered AE events.

1. **Results**

**5.1. Progressive Localization of Damage: Berea Sandstone**

As a first demonstration of our simulation results, we examine the growth of a macrofracture in Berea Sandstone under a confining pressure of 15 MPa. We identify four stages in the mechanical behavior [Fig. 6], as observed by the stress-strain behavior of the sample, and prescribed by experimental [Amitrano, 2003] and numerical studies [Longjohn et al., 2018]. Stage 1 (initiation), corresponding to an axial strain range from 0 to 0.023, is characterized by increasing rock strength and a linear stress-strain curve. This initial stage of the biaxial experiment is characterized by low microfracturing activity, 97% of which are generated in tensile mode. Microcracking activity during Stage 1 increases as axial strain on sample increases. Stage 2 (nucleation), corresponding to an axial strain range from 0.024 to 0.038, begins with the introduction of non-linearity in the stress-strain behavior of the sample till peak stress is reached. Stage 2 is characterized by increasing rock strength and a decrease in slope of the stress-strain curve, corresponding to strain hardening behavior of the sample. Stage 2 of the biaxial experiment is characterized by very high microfracturing activity, generated in both shear (25%) and tensile (75%) modes. Microcrack growth increases as we approach peak stress of rock, especially in shear mode. Stage 3, corresponding to an axial strain range from 0.039 to 0.062, defines the post-peak stress-strain behavior of rock until residual strength of rock is attained. Stage 3 (localization) is characterized by decreasing rock strength corresponding to strain softening behavior of the sample. Stage 3 is characterized by high microfracturing activity, generated in both shear (19%) and tensile (81%) modes. Microcrack growth declines as we approach residual strength of rock, especially in shear mode. Stage 4, corresponding to an axial strain range from 0.063 to 0.103, defines the frictional sliding behavior of the rock sample. Stage 4 is characterized by nearly constant rock strength corresponding to the residual strength of rock. Stage 4 is characterized by very low microcracking activity, 94% of which are generated dominantly in tensile mode. Microcracking activity decreases slowly as we approach the end of the experiment at an axial strain of 0.103. A total of 5442 microcracks developed in the sandstone sample at a confining pressure of 15 MPa, with 18% of them occurring in shear mode generated largely during Stage 2 and Stage 3 of the biaxial experiment.

The spatial distribution of microcracks generated during the four stages of the biaxial experiment show significant variation. Stage 1 is characterized by distributed tensile microcracking through the rock sample, indicating onset of dilatancy in the sample [Fig. 7a]. Stage 2 is characterized by growth of shear and tensile microcracks, predominantly around asperities created in Stage 1 [Fig. 7b]. Shear microcracks generated in Stage 2 of the biaxial experiment frequently occur as clusters, enveloped by tensile microcracks. During this nucleation stage, the shear and tensile microcracks coalesce to form small fractures through the sample. Stage 3 is characterized by the coalescence of pre-existing microcracks and smaller fractures into a through-going shear rupture, assisted by newly generated shear and tensile microcracks [Fig. 7c]. Microcracking activity during Stage 3 is very localized, with new asperities largely confined to the process zone of the developed shear fracture. Shear and tensile microcracks also coalesce to form smaller conjugate fractures, originating from the primary shear rupture. Stage 4 is characterized by shearing along the macrofracture surface [Fig. 7d]. Microcracks are generated by the rupture of surface asperities and gouge fracturing, caused by plucking of grains along the shear fracture and associated conjugate fractures from sliding of fractured blocks of the rock. Thus, the nucleation and localization of fracture in sandstone is controlled by the cooperative interaction of tensile and shear microcracks. This indicates interplay between competing processes – dilatancy and compactant/shear behavior. As a result, we observe a net decrease in volume of the sample was observed prior to failure [Fig. 8]. At *σ/σfailure*=1, we calculate a negative value of the Damage Index (*DI*), indicating the formation of a compactant fracture zone. Using the location of microcrack distribution through the sample, we calculate a fractal dimension (D-value) of 1.57 associated with biaxial deformation of Berea sandstone under a confining pressure of 15 MPa [Fig. 9].

We calculate a total of 3.55 J of fracture energy released as AE during the biaxial experiment on Berea Sandstone under a confining pressure of 15 MPa, exhibiting an exponential trend [Fig. 10]. AE energy release from microcracking activity during Stage 1 is small, contributing to 5% of the AE energy release. Stage 2 of the experiment, corresponding to the highest microcracking activity, is also associated with the highest release in fracture energy, with 49% of the total energy released as AE. Although, only 25% of the total microcracking from Stage 2 occur in shear mode, they contribute to 76% of the fracture energy released. Stage 3 of the biaxial experiment, corresponding to the coalescence of the shear fracture, is associated with release of 38% of the total fracture energy. Similar to Stage 2, only 19% of all the total microcracking in Stage 3 occurs in shear mode, but contribute to 75% of fracture energy released. Frictional sliding in Stage 4 results in a small amount of fracture energy release as AE. Stage 4 is associated with 7% a release of total fracture energy, with 39% released by microcracks occurring in shear mode. We calculate range of -5.4 to -3.8 of moment magnitudes after clustering of microcrack energy. The slope of the frequency-magnitude distribution yields a b-value of 1.40 for biaxial deformation of Berea sandstone under a confining pressure of 15 MPa [Fig. 11].

Thus, the growth of shear fracture in Berea sandstone occurs through the cooperative coalescence of tensile and shear microcracks. While the abundance of shear microcracks is less than tensile, shear microcracking is localized in the fracture zone, and is the dominant mode of fracture energy release as AE. The nature of the fracture zone is controlled by the competing processes of dilatant and compressive forces.

**3.2. Progressive Localization of Damage: Lac du Bonnet Granite**

A comparison simulation to that described above is carried out on the numerical analog to the Lac du Bonnet Granite. The biaxial experiment, also conducted at 15 MPa confining pressure, results in some similar deformation trends as Berea Sandstone, but also some distinct differences, which are highlighted here. The growth of macrofractures in Lac du Bonnet Granite is dominated by tensile microcracking in four distinct stages [Fig. 12a]: Stage 1, corresponding to an axial strain from 0 to 0.007, is characterized by a linear stress-strain curve and rapid increase in rock strength attributed to the high Youngs Modulus of granite. Stage 2, corresponding to an axial strain range from 0.008 to 0.015, is the strain hardening phase of the experiment and is characterized by very high tensile microcracking activity as we attain peak strength of rock. Stage 3, corresponding to an axial strain range from 0.016 to 0.036, defines the post-peak strain-weakening behavior of rock, and is characterized by high tensile microcracking activity. Microcrack growth declines as we approach residual strength of rock. Stage 4, corresponding to an axial strain range from 0.037 to 0.103, defines the frictional sliding behavior of the granite sample and is characterized by residual strength of rock and low microcracking activity. A total of 2598 microcracks developed in the granite sample at a confining pressure of 15 MPa, with ~98% of them occurring in tensile mode generated largely during Stage 2 and Stage 3 of the biaxial experiment.

Stage 1 is characterized by distributed tensile microcracking through the rock sample, indicating onset of dilatancy in the sample [Fig. 12b1]. Stage 2 is characterized by growth of shear and tensile microcracks, predominantly around asperities created in Stage 1 [Fig. 12b2]. Shear microcracking during Stage 2 dominantly occurs in the process zone, and local fractures are emergent. Stage 3 is characterized by the coalescence of pre-existing and newly generated microcracks and smaller fractures into a through-going shear rupture [Fig. 12b3]. Conjugate fractures, originating from the primary shear rupture are emergent at this stage, originating from coalescence of tensile microcracks in the gouge of the primary shear fracture. Stage 4 is characterized by shearing along the macrofracture surface [Fig. 12b4] and new microcracks are generated by gouge fracturing. Thus, the nucleation and localization of fractures in granite occurs through the coalescence of microcracks, predominantly generated in tensile mode. The abundance of tensile microcracking in the fracture zone indicates dilatancy as primary force in fracture generation. As a result, we observe a net very little compression in the sample was prior to failure [Fig. 9]. At *σ/σfailure*=1, we calculate a positive value of the Damage Index (*DI*), indicating the formation of a dilatant fracture zone. Using the location of microcrack distribution through the sample, we calculate a fractal dimension (D-value) of 1.27 associated with biaxial deformation of Lac du Bonnet granite under a confining pressure of 15 MPa [Fig. 9].

We calculate a total of 21.91 J of fracture energy released as AE during the biaxial experiment on Lac du Bonnet granite under a confining pressure of 15 MPa, exhibiting an exponential trend [Fig. 12c]. The release of fracture energy in the four stages of the biaxial experiment are 15%, 43%, 31% and 11% respectively. Through the entirety of the experiment, tensile microcracking is the dominant mode of energy release, contributing to 93% of the total AE energy. However, the small number of microcracks generated in shear mode (2% of total microcracks), correspond to events of high energy, contributing to 7% of total energy released. We calculate range of -4.4 to -3.4 of moment magnitudes after clustering of microcrack energy. The slope of the frequency-magnitude distribution yields a b-value of 1.51 for biaxial deformation of Lac du Bonnet granite under a confining pressure of 15 MPa [Fig. 11].

Thus, the growth of shear fracture in Lac du Bonnet granite occurs through the cooperative coalescence of microcracks predominantly generated in tensile mode. Fracture energy is largely released by microcracking in tensile mode. Since tensile microcracking is the primary mode of deformation, the developed fracture zone is dilatant in nature.

**3.2. Effect of Confining Pressure**

To examine how the behavior of each of these materials changes as a function of confining pressure, we carry out identical experiments on both the Berea Sandstone and Lac du Bonnet Granite analogs. As we increase confining pressure from 0 MPa to 50 MPa on samples of Berea Sandstone, we observe an increase in peak strength of rock from 85.05 MPa to 238.82 MPa [Fig. 13a]. As confining pressure increases, rock strength increases, providing greater resistance to the formation of a shear fracture during biaxial deformation experiments. This increased resistance to fracture formation with confining pressure results in an increase in shear microcracks and a decline in tensile microcracks [Fig. 13b]. While the total number of microcracks created during biaxial deformation of sandstone does not vary significantly with confining pressure, microcracking in shear increases from 4% of total events at 0 MPa to 45% of total events at 50 MPa. The increase in shear microcracking and simultaneous decline in tensile microcracking indicates transition from dilatant fracture zones at low confining pressures to shearing fracture zones at high confining pressure. This observation is supported by the variation in calculated Damage Index at *σ*/*σfailure* =1 with confining pressure [Fig. 13c]. At low confining pressures (0-10 MPa), we calculate positive values of the Damage Index at peak strength of rock, indicating formation of a dilatant fracture zone. At higher confining pressures (15-50 MPa), we calculate negative values of the Damage Index at peak strength of rock, indicating formation of a fracture zone dominated by compaction and shear. Total fracture energy released as AE increases from 1.73 J at confining pressure of 0 MPa to 7.97 J at confining pressure of 50 MPa, with the contribution of shear microcracking increasing from 31% to 92% [Fig. 13d]. The increase in fracture energy with confining pressure is due to increase in fraction of shear microcracking, which are associated with larger stress release during their formation. Visual representation of the microcrack locations show an increase in concentration of shear microcracks in the zones of the primary shear fracture and conjugate fractures [Fig 13e]. In general, an increase in confining pressure also correlates with increase in conjugate fractures and spatial distribution of microcracks. As a result of increase in spatial distribution of microcracks, we calculate an increase in fractal dimension (*D*) from 1.43 at confining pressure of 0 MPa to 1.74 at confining pressure of 50 MPa [Fig. 14a]. The increase in confining pressure on sandstone results in increase in number of smaller fractures that eventually merge into the primary shear fracture or form conjugate fractures. The increase in confining pressure results in slower fracture coalescence and increase in events of intermediate moment magnitudes, resulting in a decline in slope of the AE frequency-magnitude distribution. Thus, we calculate a decline in b-values from 1.30 at confining pressure of 0 MPa to 0.98 at a confining pressure of 50 MPa [Fig. 14b].

As we increase confining pressure from 0 MPa to 50 MPa on samples of Lac du Bonnet granite, we observe an increase in peak strength of rock from 233.79 MPa to 433.63 MPa [Fig. 15a], indicating greater resistance to the formation of shear fracture during biaxial experiments. The formation of a fracture under increasing confining pressure is facilitated by an increase in total number of microcracks created during biaxial tests increases from 2110 at a confining pressure of 0 MPa to 3204 at a confining pressure of 50 MPa [Fig. 15b]. While number of shear microcracks increases slightly, the growth of the fracture is facilitated by a significant increase in tensile microcracking with confining pressure, indicating fracture zones in granite are dilatant in nature. However, microcracking in shear mode increases from 2% of total microcracks at a confining pressure of 0 MPa to 4% of total microcracks at a confining pressure of 50 MPa, indicating a decline in dilatant nature with increasing confining pressure. This observation is supported by the variation in calculated Damage Index at *σ*/*σfailure* =1 with confining pressure [Fig. 15c]. At peak strength of rock during each biaxial experiment, we calculate positive values of Damage Index at all confining pressures indicating dilatant fracture zones. However, values of Damage Index decline with increasing confining pressure, indicating increase in compressive nature of fracture zones with confining pressure. Total fracture energy released as AE increases from 17.5 J at confining pressure of 0 MPa to 29.02 J at confining pressure of 50 MPa, with the contribution of shear microcracking increasing from 6% to 12% [Fig. 15d]. Similar to sandstone, the increase in confining pressure results in an increase in spatial distribution of microcracks [Fig. 15e], resulting in an increase in calculated fractal dimension (*D*) from 1.28 at confining pressure of 0 MPa to 1.63 at confining pressure of 50 MPa [Fig 14a]. We calculate a decline in b-values from 1.50 at 0 MPa to 1.04 at confining pressure of 50 MPa for granite due to increase in events of intermediate magnitude [Fig. 14b].

1. **Discussions**

**6.1. Comparison of Fracture growth in Berea Sandstone and Lac du Bonnet Granite**

The formation of shear fractures in granite has similarities and differences when compared to the deformation in Berea Sandstone. Stage 1 (initiation), is characterized by distributed tensile microfracturing and low AE energy release in both rock types. During Stage 2 (nucleation), we observe very little microcracking in shear microcracking in granite, whereas shear mode microcracks are abundant in sandstone. Stage 2 is associated with highest AE energy release for both rock types, however, the dominant contribution to release of fracture energy is tensile microcracking for granite as opposed to shear microcracking in sandstone. While shear microcracks generated in Stage 2 are localized to the zone of shear in granite, their distribution through the sample is much greater than sandstone extending beyond the process zone. Stage 3 (localization) is characterized confined microcracking in the fracture zone in both granite and sandstone, but shows significant differences in mode of microcracks generated. While the growth of shear fracture is facilitated by the coalescence of tensile microcracks in, both shear and tensile microcracks are abundant in this phase of the experiment in sandstone. While release of fracture energy as AE remains high during stage 3, it is facilitated dominantly by tensile microcracks in granite as opposed to shear microcracks in sandstone. Stage 4 (frictional sliding) is characterized by tensile microcracking and gouge rupturing in both rock types, with low AE energy release.

The mechanism of fracture evolution is different in sandstone and granite [Fig. 16]. In weaker rocks such as sandstone, the growth of the fracture occurs through the progressive coalescence of shear and tensile microcracks. The damage in the fracture zone is characterized by the interplay of dilatant and compactant forces. At low confining pressures, tensile microcracks dominate the fracture zone, resulting in the formation of a dilatant fracture zone. At high confining pressures, shear microcracking replaces tensile microcracking, resulting in the formation of a fracture zone dominated by compaction and shear. AE energy associated with fracture formation increases with confining pressure due to increase in shear microcracking. Shear microcracks are associated with a larger stress drop as they overcome both tensile and compressive forces during formation. Thus, the increase in shear microcracking with confining pressure results in an increase in AE energy in sandstone. In strong rocks such as granite, the growth of fracture occurs predominantly through coalescence of tensile microcracks, resulting in formation of dilatant fracture zones [Fig. 16b]. As confining pressure increases, the increased resistance to fracture coalescence is countered by an increase in microcracking activity. Energy of AE is dominated by tensile mode microcracks. AE energy increases with confining pressure on rock as number of tensile microcracks increases.

As we increase confining pressure on rock from 0 MPa to 50 MPa, *D*-values increase from 1.43 to 1.74 in sandstone and from 1.28 to 1.63 in granite [Fig. 14a]. We calculate higher values of *D* in sandstone when compared to granite due to greater distribution of microcrack events. Since sandstone is mechanically weaker than granite, the stress required to generate asperities is lower. As a result, we calculate higher number of events and greater distribution of microcracks during biaxial deformation of sandstone in comparison with granite. As we increase confining pressure on rock from 0 MPa to 50 MPa, *b*-values decline from 1.29 to 0.98 in sandstone and from 1.50 to 1.04 in granite [Fig. 14b]. The difference in b-values can be attributed to greater number of events of intermediate magnitude during biaxial deformation of sandstone. The growth of a shear fracture in sandstone is a slower process, involving the coalescence of greater number of microcracks, resulting in events of intermediate magnitude. In comparison, the localization of a shear fracture in granite occurs through rapid coalescence of small events, culminating into few events of large magnitude. As a result, we derive lower values of *b* for sandstone when compared to granite.

**6.2. Validation of Models and Comparison with Experimental Datasets**

We test the validity of our model results by comparing our results to experimental datasets and highlight how our study can add to the current understanding of fracture formation. Our simulation results show that the growth of a macrofracture in Berea Sandstone occurs through the coalescence of microcracks occurring in shear and tensile modes, with the fraction of shear microcracks increasing with confining pressure [Fig. 13]. The growth of shear fractures in sandstones has been shown to occur through coalescence of tensile and shear microcracks using AE source mechanism [Fortin et al., 2009] and microstructural analyses [Menendez et al., 1996]. Our simulation results show that the growth of shear microcracks occurs primarily in Stage 2 and 3 of a biaxial experiment, during the genesis and coalescence of the shear macrofracture as rock approaches its peak strength [Fig. 6]. The growth of microcracks in shear mode has also been shown to occur dominantly near the peak stress of the rock [Lei et al, 2004], with fraction of microcracking occurring in shear increasing with confining pressure [Baud et al., 2004]. However, there is little knowledge over how fracture energy is split between shear and tensile microcracks. Our simulation results add this body of work by showing that shear microcracks are the dominant mode of fracture energy release in weaker rocks such as sandstone, due to large stress drops associated with them during the nucleation and localization of shear fractures [Fig. 10]. The increase in confining pressure results in switch in mechanism, from dilatant to shear dominated [Fig. 13]. Thus, the increase in shear microcracking with confining pressure results in the greater AE energy observed in laboratory studies. Experimental deformation experiments on Berea Sandstone show a transition from dilating to compacting fracture zones [Menendez et al., 1996; Besuelle, 2001]. Our simulation results show that the nature of the fracture zone is an interplay between the abundance of shear and tensile microcracks in the process zone. At low confining pressures, we observe dilatant fracture zones as tensile microcracks dominate the fracture and gouge region. At high confining pressures, the dominant damage mechanism is compactant, resulting in abundant shear microcracks in the fracture and gouge region. As a result, we observe compactant fracture zones in sandstones under high confining pressure conditions [Fig. 13c]. The switch in microcracking processes has been attributed to the decrease in internal friction with increasing confining pressure, as shear cracks are less pressure sensitive than tensile microcracks [Amitrano, 2003].

Our simulation results show that the growth and coalescence of a macrofracture in Lac du Bonnet granite occurs dominantly through the coalescence of tensile microcracks [Fig. 12a]. Similarly, the growth of fractures during biaxial experiments in granites has been shown to occur through coalescence of tensile microcracks using AE source mechanism [Lockner et al., 1991] and microstructural analyses [Peng and Johnson, 1972]. The increase in microcrack and AE energy with confining pressure has been experimentally observed granitic rocks [Stanchits et al., 2006]. Our model results show that the number of microcracks generated during biaxial experiments in granite increases with confining pressure in response to the increased resistance provided by confining pressure [Fig. 15b]. The increase in number of microcracks generated with confining pressure results explains the increase in AE energy release observed by experimental and numerical studies. Our results from deformation of granite samples also shows a small increase in shear microcracking with confining pressure [Fig. 14b], similar to behavior defined by granite in laboratory experiments [Jaeger et al., 2009]. Escartin et al., 1997 show a decline in dilatant nature of fractures in granite with increase in confining pressure. Our results reproduce this behavior [Fig. 15c], exhibiting a declining trend bulk dilation trend with confining pressure. The decline in dilatant nature of fractures is due to the pressure sensitivity of tensile microcracks, which close preferentially under high confining pressure [Wong et al, 2001], reducing the fracture porosity.

Calculations of two-dimensional fractal D-value from experimentally observed AE yield values ranging from 1 to 1.8 in sandstones and granite, with D-values increasing with confining pressure [Amitrano, 2003]. In our models, we calculate values of 1.57-1.83 for Berea Sandstone and 1.27-1.63 for Lac du Bonnet granite, with D-values increasing with confining pressure from 0 MPa to 50 MPa. This indicates an increase in distribution of microcracking with confining pressure in weak and strong rocks. The increase in confining pressure results in greater distribution of applied axial stress through the rock sample. As a result, spatial distribution of microcracking increases with confining pressure, often considered a precursor to onset of grain crushing and ductile deformation in rocks [Wong et al., 1997; Zhang et al, 1990]. Calculations of seismic b-value from experimentally observed AE yield values ranging from 2.2 to 1.2 in sandstones [Amitrano, 2003] and from 2.2 to 0.8 in granites [Rao and Lakshmi, 2005], with b-values declining with confining pressure. In our models, we calculate values between 0.98-1.30 for Berea Sandstone and 1.04-1.51 for Lac du Bonnet granite, with b-values decreasing with confining pressure from 0 MPa to 50 MPa. Increase in confining pressure on rock results in slower coalescence of the fracture. As a result, there is a rise in the number of events of intermediate magnitude, resulting in a decline in the slope of the frequency-magnitude distribution.

**4.3. Scale independence of Deformation Patterns**

The deformation patterns during formation of shear fractures during biaxial experiments on samples are analogous to slip along faults [Lockner, 1993; Main and Meredith, 1989], particularly for shear faulting, which is supposed to be the main mechanism for earthquakes [Scholz, 1990]. Two widely acknowledged models of fracture propagation include the beam buckling model, proposed by Peng and Johnson, 1972, and the wing-crack model, proposed by Pollard and Segall, 1987. Peng and Johnson, 1972, through their beam buckling model suggest that faults evolve from an array of extensional microcracks. This mechanism is similar to the process of shear fracture growth recorded in our granite models. Pollard and Segall, 1987 suggest that both tensile and shear microcracks coalesce to form a fault of uneven surface. Field evidence and theory have shown that tensile fractures, referred to as wing-cracks, form around the tips of pre-existing asperities when loaded in shear, and the faulting occurs through the interaction of the extensional wing-cracks [Blenkinsop, 2008; Horii and Nemat-Nasser, 1985]. This deformation mechanism is similar to the process of shear fracture growth recorded in our sandstone models, which occurs through the progressive localization of shear and tensile microcracks. The differences occurring in deformation patterns of granite and sandstone models suggest that the process of fracture coalescence may be highly influenced by the mechanical properties of the lithology.

AE associated with microcracking also shows remarkable statistical similarity to the seismic characteristics of earthquakes, obeying the Gutenberg-Richter relationship [Scholz, 1968] and Omori’s Law [Reasenberg and Jones, 1989]. Calculated seismic b-values of earthquake sequences follow a unimodal distribution ranging from 0.5 to 2.5, with the mean value of approximately 1 [Shi and Bolt, 1982]. Calculated D-values of earthquake hypocenters typically lie within a range of 1.3 and 1.8, with a mean value of approximately 1.6 [Hirata, 1989]. In our study, we record b-values ranging from 0.98 to 1.66 and D-values ranging from 1.37 to 1.58, lying within range of values calculated during earthquakes. The relationship between b-value and D-value has been used to predict location of faulting from observed seismicity [Hirata, 1989] and understand the relationship between spatial heterogeneities and stress in the earth’s crust [Weimer and Wyss, 1997]. While some studies propose a direct relationship between *b* and *D* [Goebel et al., 2017; Aki, 1981], while others propose an inverse relationship between the two parameters [Wyss et al., 2004; Amitrano, 2003; Hirata, 1989]. In this study, we calculate values of *b* and *D* from biaxial deformation experiments on numerical analogs of sandstone and granite, exhibiting an inverse relationship (R2=0.92) [Fig. 17]:

(Eq. 25)

As we increase confining pressure on rock, *b*-values decline and *D*-values increase during the biaxial experiments, consistent with the experimental and numerical evidence presented by Amitrano, 2003. However, studies analyzing on the temporal variation in *b* and *D* during biaxial experiments show a direct relationship between the two parameters [Goebel et al., 2017; Lei et al., 2006]. This indicates a difference between the bulk behavior exhibited during deformation and the temporal variation of these dimensionless parameters, warranting further examination of temporal variation in micromechanical behavior during biaxial experiments. In the next analysis, we will analyze the temporal variation in dimensionless parameters such as b and D during deformation experiments, and decouple the effects of confining pressure from emergent variation during deformation.