

2

Radioactivity

"Physicists always publish their results completely. If our discovery has a commercial future, that is an accident from which we must not profit. And if radium is to be used in the treatment of disease, it seems to me impossible for us to take advantage of that."

—Marie (Skłodowska) Curie

2.1 INTRODUCTION

Knowledge concerning atomic nuclei began as early as 1896 with the discovery of radioactivity.

In 1895, Roentgen discovered the X-rays. The French scientist Becquerel got interested in Roentgen's work. Becquerel was seized of the fact that the production of X-rays was always accompanied by fluorescence from the material of the X-ray tube (glass). He thought that X-rays existed whenever there was fluorescence. To investigate this problem, Becquerel took uranium sulphate, which fluoresces under the action of sunlight. He found that fluorescent uranium sulphate did give out rays, which could affect a photographic plate even when wrapped in thick black paper. Becquerel argued that the fluorescent salt had given rise to X-rays, which had penetrated the black paper and affected the photographic plate.

But he soon saw that he was mistaken. During one such experiment the sky happened to be overcast and the uranium salt was hardly fluorescent. On developing the photographic plate, Becquerel was surprised to see a dark spot on it, as before. He had obviously stumbled on some new kind of rays (1896) which could penetrate the thick wrapper and affect the photographic plate. It was soon established that any salt of uranium emits Becquerel rays. Unlike the X-rays, which appear in an X-ray tube only under special conditions, the Becquerel rays are emitted in a spontaneous manner.

Is uranium the only substance emitting Becquerel rays? Marie Curie found that pitchblende, the ore from which uranium is extracted, emits Becquerel rays with a much stronger intensity

than what its uranium content would. After a long and laborious process of chemical separation, Marie Curie and her husband Pierre Curie discovered two new elements, polonium and radium, which emitted Becquerel rays.

They gave the name 'radioactive' to all substances capable of emitting, Becquerel rays and the phenomenon itself came to be known as 'radioactivity'.

The discovery of radium was a great event, as it was found to be about a million times more radioactive than uranium. This power of radium radiation made it possible to study radioactivity systematically.

■ 2.2 PROPERTIES OF RADIOACTIVE RAYS

- (i) By a calorimetric experiment, Curie estimated that one gm of radium liberates 140 calories in one hour. Though small, this energy is released continuously over a very long period of time.
- (ii) Radioactive rays ionize the surrounding air and affect photographic plates.
- (iii) Radioactive rays act differently on different cells and tissues. Cells that multiply rapidly are most readily destroyed by these rays. This outstanding discovery made radium an invaluable aid to physicians to fight tumours, particularly cancerous growths.
- (iv) Fluorescence is produced in substances like zinc sulphide, barium platinocynide, etc. By adding minute quantities of radium to, say, zinc sulphide, we can get a compound that is continuously luminous in the dark. This can be used to produce luminous watch dials, gun sights and instrument pointers (coating) for those instruments which are required to be read in the dark.
- (v) Rutherford found that a beam of radioactive rays from radium sample splits into three components in a strong magnetic or electric field. See Fig. 2.1.

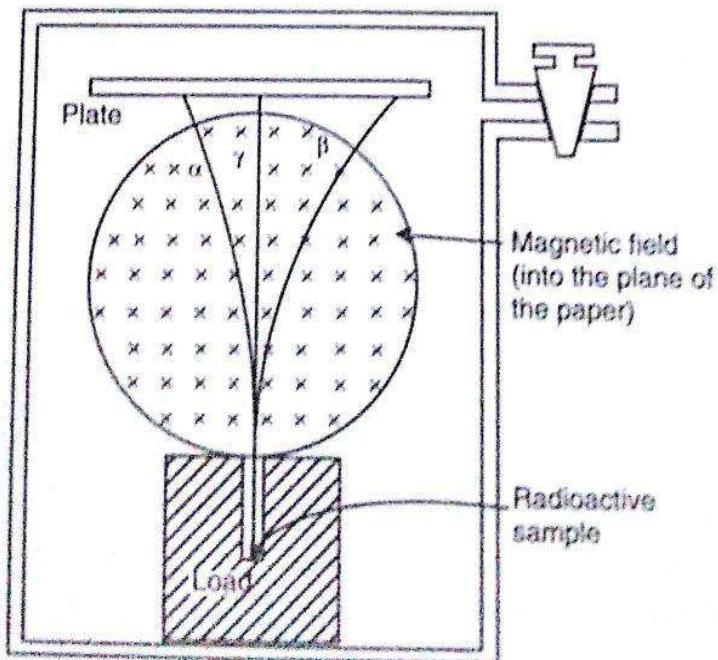


Fig. 2.1. Deflection of α , β , γ rays by a magnetic field.

(a) The α -rays (particles) are the nuclei of helium atoms. This identification was made by Rutherford and Royds in 1909. By the spectroscopic method, they found traces of helium in an originally pure sample of radon gas, which is an alpha emitter. When an electric discharge was passed through a tube containing pure radon gas, initially only characteristic radon lines appeared.

After a day Rutherford found that, Radon lines became somewhat weaker and new lines started appearing. These new lines were identified with helium spectrum. As the days passed, the radon spectrum began weakening while the helium lines grew brighter. Thus, for the first time, people 'saw' decay of an element (radon) and 'birth' of a new element, helium. Such a transformation in which a parent element gives rise to a new element—called the daughter product—by emitting radioactive rays is called a radioactive transformation. For the above example of radon, we can write the radioactive transformation equation as:



Parent element $\rightarrow \alpha$ -particle + daughter product

Notice that during a radioactive transformation, the mass number and total charge is conserved.

Equation 2.1 is an example of nuclear reaction.

Clearly, radioactivity is a nuclear phenomenon. In other words, the radioactive rays come out of the atomic nucleus.

A few examples of alpha decay are,



Alpha rays can be stopped by a thin sheet of paper. On the other hand, they cause intense ionization in air. Most α -particles are emitted with velocities between $\sim 1.5 \times 10^7$ m/s and $\sim 2.2 \times 10^7$ m/s. Any group of α -particles emitted from the same type of nuclei always has a definite velocity and hence a definite energy.

The alpha particles cover a definite distance in a given material, practically without any loss of intensity and then suddenly in a small distance are absorbed completely. The definite distance they travel within a given material is called their range in that material. For example, α -particles

from radium ($\begin{array}{c} 226 \\ 88 \end{array}$ Ra) have a range in air equal to 3.4 cm. at 0°C and 76 cm. of pressure.

We will look at these aspects in chapter 4, where we will see how range is related to energy (See Chapter 1 also) and how one can theoretically account for alpha decay.

(b) The β rays are identical with electrons. A β -particle therefore has a mass ($1/1836$)₁ mass of a proton.

A few examples of β decay are:



Notice that the mass number and charge are conserved and the daughter product moves one place up in the periodic table, as loss of a negative charge by a nucleus would imply gain of a positive charge. Beta rays cause much less ionization in air, but are ~ 100 times more penetrating than α -rays. They can penetrate a sheet of aluminium a few mm thick.

The velocities of β -particles emitted from various nuclei range up to $0.99c$, where $c = 3 \times 10^8$ m/s, is the velocity of light. A particular β -active element emits β -particles with energies varying between zero and a certain maximum. This maximum energy is called the end-point energy. We shall study a number of interesting aspects of β -rays in Chapter IV.

(c) The γ -rays are part of the electromagnetic spectrum. They have wavelengths smaller than those usually associated with the X-rays. Thus, usually γ -ray photons (energy of a photon, $E = hf = hc/\lambda$) are more energetic than the X-ray photons and are even more penetrating than the X-rays. (They are ~ 100 times more penetrating than β -rays.) The wavelength of γ -ray photons ranges between $\sim 1.7 \times 10^{-8}$ cm and $\sim 4 \times 10^{-6}$ cm.

The ionization due to γ -rays is a photoelectric effect. Owing to their large energies, the γ -ray photons can dislodge electrons not only from outer orbits (valence orbits or conduction bands) of the atoms but also from the inner orbits. Besides this photoelectric effect, γ -rays lose energy by (i) Compton scattering, in which the γ -photon collides with an electron and gets scattered with a shift in wavelength [$\Delta\lambda = h/m_0c(1 - \cos\alpha)$], and (ii) pair production in which a γ -photon is converted into a pair consisting of an electron and a positron. For this, the energy of the γ -ray has to be > 1.02 MeV.

We will study γ -rays and γ -decay in some detail in Chapter 4.

2.3 THE LAW OF RADIOACTIVE DECAY

When a nucleus disintegrates by emitting a particle (α or β) or a gamma ray, or by capturing an electron from the atomic shell (K -capture), the process is called radioactive decay. This decay is spontaneous.

By using a Geiger Counter (see Chapter 1), it is possible to study how radioactive decay depends on time.

Let us take a radioactive sample containing N_0 nuclei at time $t = 0$, i.e., at the beginning. We wish to calculate the number N of these nuclei left after time t .

The number of nuclei of a given radioactive sample disintegrating per second is called the activity of that sample.

$$\therefore \frac{dN}{dt} = \text{Rate of decrease of nuclei with time}$$

= Activity at time t .

Experimentally, it is found that the activity at any instant of time t is directly proportional to the number N of parent type nuclei present at that time.

∴

$$-\frac{dN}{dt} \propto N$$

or

$$-\frac{dN}{dt} = \lambda N \quad \dots(2.9)$$

where $\lambda > 0$, is the proportionality constant.

The negative sign indicates that N decreases as t increases.

For unit time, from Eq. 2.9, we get,

$$\lambda = \left(-\frac{dN}{N} \right) \quad \dots(2.10)$$

i.e., λ is fractional change in N per sec. and we see that λ is not a mere proportionality constant, but it gives us the probability of decay per unit interval of time. Hence λ is called the probability constant, the decay constant or the disintegration constant.

Notice that dN is the number of parent nuclei that decay between times t and $t + dt$, and that we have taken N as a continuous variable. We will come back to this in Sec. 2.4.

From Eq. 2.9, by integration we get,

$$\int_{N_0}^N \frac{dN}{N} = - \int_0^t \lambda dt$$

$$N = N_0 e^{-\lambda t} \quad \dots(2.11)$$

or

Here, N_0 = Number of radioactive nuclei at $t = 0$.

Thus we see that the law of radioactive decay is exponential in character. Figure 2.2 depicts a typical experimental decay curve for radon.

Notice that only half the amount of radon present initially remains after 3.83 days; only one-fourth of it remains after 7.66 days and only one-eighth remains after 15.32 days.

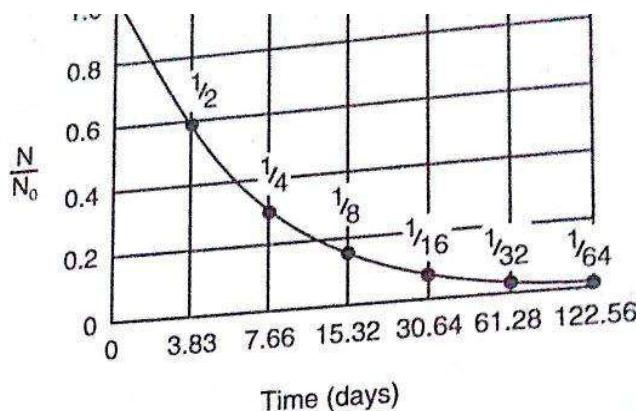


Fig. 2.2. Decay curve for radon, $\left({}^{222}_{88}\text{Rn}\right)$.

The plot shows that in a fixed time interval (say, 3.83 days for radon) a fixed fraction (1/2 for radon) of the amount of radioactive substance at the beginning of the interval decays. This fraction is independent of the amount of radioactive substance or whether you have a freshly prepared sample or not and instead depends only on the interval of time. This is a characteristic of the exponential nature of the law of radioactive decay.

The decay constant λ is a characteristic of radioactive substance and it depends in no way on the amount of the substance present.

Half Life (T): It is convenient to define a time interval during which half of a given sample of radioactive substance decays. This interval is called the half life or half value period of that substance, denoted by T .

$$\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda T}$$

$$e^{\lambda T} = 2$$

or

$$\lambda T = \log_e 2 = 0.693$$

$$T = \frac{0.693}{\lambda}$$

...(2.12)

Mean Life (τ): Individual radioactive atoms may have life spans between zero and infinity. Hence it is meaningful to talk about the average or mean life τ , defined as,

$$\tau = \frac{\text{Total life time of all nuclei in a given sample}}{\text{Total number of nuclei in that sample}} = \frac{1}{\lambda} \quad \dots(2.13)$$

To evaluate Eq. 2.13, see Fig. 2.3.

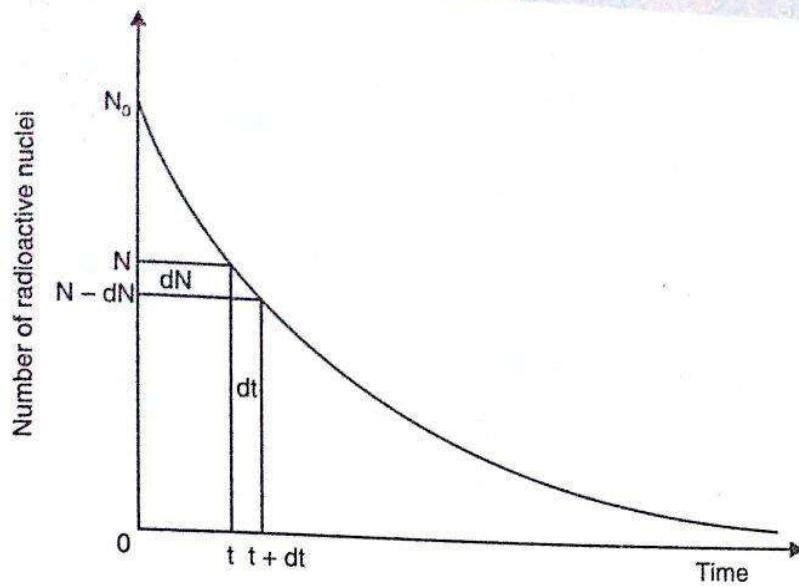


Fig. 2.3. Curve shows how dN number of nuclei decay in time dt .

From the curve, one can see that each of the dN number of radioactive nuclei has lived a life of t seconds, i.e., the total life span of dN nuclei is $(dN \cdot t)$ seconds.

∴ We can put Eq. 2.13 as,

$$\begin{aligned} \tau &= \frac{\int_0^0 t dN}{\int_0^{N_0} dN} \\ &= \frac{-N_0 \lambda \int_0^\infty t e^{-\lambda t} dt}{-N_0}, \quad \therefore dN = -\lambda N dt = -\lambda N_0 e^{-\lambda t} dt \\ &= \lambda \int_0^\infty t e^{-\lambda t} dt, \end{aligned}$$

which on integration by parts becomes,

$$\tau = \lambda \cdot \frac{1}{\lambda^2} = \frac{1}{\lambda} \quad \dots(2.14)$$

From Fig. 2.2,

T of radon = 3.83 days

i.e. in n half lives, $1/2^n$ of the original sample survives.

Also, ; $T = \frac{0.693}{\lambda}$

∴ λ of radon = $\lambda_{Rn} = 2.1 \times 10^{-6} \text{ sec}^{-1}$.

ILLUSTRATIVE EXAMPLE

Example 2.1: According to measurements by Rutherford and Geiger, one gram of radium emits in one second 3.7×10^{10} alpha particles. Estimate the half life of radium.

Solution: The decay constant of radium is,

$$\lambda_{Ra} = \frac{3.7 \times 10^{10}}{2.7 \times 10^{21}} \text{ sec}^{-1} \quad \text{see Eq. 2.10}$$

where 2.7×10^{21} = Number of radium atoms in one gram of radium.

(\because There are 6.02×10^{23} atoms [Avogadro's number] in one gm-atom of radium-226)

$$\therefore \lambda_{Ra} = 1.37 \times 10^{-11} \text{ sec}^{-1}$$

$$\therefore \text{Half life, } T_{Ra} = \frac{0.693}{\lambda_{Ra}} \equiv 5 \times 10^{10} \text{ sec} \equiv 1600 \text{ yrs.}$$

$$\text{Unit of Activity} = \frac{dN}{dt} = \lambda N$$

The most commonly used unit is the curie. It was originally based on the rate of decay of a gram of radium. Experiments have yielded the result that there are about 3.7×10^{10} disintegrations per second per gram of radium. This number is taken as a standard and is called the curie. Thus by definition,

$$\text{One curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \frac{\text{disintegrations}}{\text{sec}} \quad \dots(2.15)$$

This is applicable to all types of nuclear disintegrations.

A Curie of activity is a very strong source of radiation.

Thus, one has

$$1 \text{ millicurie} = 1 \text{ m Ci} = 10^{-3} \text{ Ci}$$

and

$$1 \text{ microcurie} = 1 \mu \text{ Ci} = 10^{-6} \text{ Ci.}$$

Sometimes one uses another unit for activity, called the rutherford.

$$1 \text{ rutherford} = 1 \text{ rd} = 10^6 \frac{\text{disintegrations}}{\text{sec.}} \quad \dots(2.16)$$

$$1 \text{ mrд} = 10^{-3} \text{ rd}$$

$$1 \mu\text{rd} = 10^{-6} \text{ rd}$$

From Eq. 2.8,

$$\text{Activity} = \left| \frac{dN}{dt} \right| = \lambda N = \frac{0.693}{T} N$$

Notice that a very short-lived substance gives rise to large activity, even if it is present in minute quantities.

ILLUSTRATIVE EXAMPLE

Example 2.2: Calculate the activity of (i) One gram of radium $^{226}_{88}\text{Ra}$, whose half life is 1622 years and (ii) 3×10^{-9} kg of active gold, $^{200}_{79}\text{Au}$, whose half life is 48 mins.

Solution:

$$(i) \quad N = 1 \text{ gm} \left(\frac{1 \text{ gm-mole}}{226 \text{ gm}} \right) \left(6.03 \times 10^{23} \frac{\text{atoms}}{\text{gm-mole}} \right)$$

$$= 2.66 \times 10^{21} \text{ atoms}$$

$$\text{and } \lambda = \frac{0.693}{T}$$

$$= \frac{0.693}{1622 \times 365 \times 24 \times 60 \times 60} \text{ sec}^{-1} = 1.355 \times 10^{-11} \text{ sec}^{-1}$$

$$\therefore \text{Activity} = \lambda N = 3.604 \times 10^{10} \frac{\text{disintegrations}}{\text{sec}}$$

$$= 0.974 \text{ Ci}$$

$$\sim 1 \text{ Ci}$$

$$(ii) \quad N = \left(3 \times 10^{-6} \text{ gm} \right) \left(\frac{1 \text{ gm-mole}}{200 \text{ gm}} \right) \left(6.03 \times 10^{23} \frac{\text{atoms}}{\text{gm-mole}} \right)$$

$$= 9.04 \times 10^{15} \text{ atoms.}$$

$$\text{and } \lambda = \frac{0.693}{T}$$

$$= \frac{0.693}{48 \times 60} \text{ sec}^{-1}$$

$$= 2.406 \times 10^{-4} \text{ sec}^{-1}$$

$$\text{Activity} = \lambda N = 2.18 \times 10^{12} \frac{\text{disintegrations}}{\text{sec}}$$

$$\approx 58.9 \text{ Ci}$$

Notice that in (ii) the quantity of active substance is only about a millionth compared to that in (i), though the activity in (ii) is ~ 60 times that in (i).

2.4 STATISTICAL NATURE OF RADIOACTIVITY

In the previous section we have treated N , the number of radioactive nuclei present at time t , as a continuous variable. We are justified in doing this as long as N is very large compared to dN , the number of nuclei decaying between times t and $t + dt$. This is usually the case, as even in a minute quantity of radioactive material we have a very large number of nuclei, e.g., in one microgram of radium, there are still 2.7×10^{15} radium nuclei.

Actually N varies discontinuously and the smallest value of dN is one, corresponding to decay of a single nucleus. Thus we see that, the law of radioactive decay is valid only for a sample of radioactive material which is sufficiently large to enable us to treat dN as a differential. The concepts of half life and mean life become meaningless when you have a sample containing only a few nuclei. For example, if you are given say, 100 active nuclei on a sample holder, and the half life of the material is one hour, then it does not mean that you will be left with 50 nuclei after one hour! As the number of nuclei in the sample is increased—though we cannot say which nuclei will decay at a given instant—we can say that about half of them will decay after one half life.

Basically therefore, the radioactivity phenomenon is statistical in nature. We cannot say which radioactive nucleus will decay at a given instant of time, though we can predict what portion of the given sample will decay in an interval of time, provided the given sample is sufficiently large. This is characteristic of all quantum mechanical phenomena, e.g., emission of light when atoms de-excite.

It is possible to obtain the law of radioactive decay purely on the basis of statistical arguments, as follows.

Assume that disintegration of a nucleus depends only on the law of chance and the probability P for a nucleus to decay is the same for all nuclei of its kind. Further, P is independent of the age or past history of a nucleus.

$\therefore P$ depends only on time interval and for short intervals, it is proportional to the interval, i.e.,

$$P = \lambda \Delta t \quad \dots(2.17)$$

The probability Q_1 , that a nucleus will not decay during the time interval Δt , is:

$$Q_1 = 1 - P = 1 - \lambda \Delta t$$

The probability Q_2 , that a nucleus will not decay during time $2\Delta t$, is:

$$Q_2 = (1 - P)(1 - P) = (1 - \lambda \Delta t)^2$$

In general, the probability Q_n , that a nucleus survives n such intervals, is:

$$Q_n = (1 - \lambda \Delta t)^n \quad \dots(2.18)$$

Consider a finite time interval t , made up of n intervals of Δt , i.e.,

$$\Delta t = t/n$$

∴

Notice that in (ii) the quantity of active substance is only about a million times greater than in (i), though the activity in (ii) is ~ 60 times that in (i).

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In general, the probability Q_n , that a nucleus survives n such intervals, is:

$$Q_n = (1 - \lambda \Delta t)^n$$

Consider a finite time interval t , made up of n intervals of Δt , i.e.

$$\Delta t = t/n$$

$$Q_n = (1 - \lambda t/n)^n \quad \dots(2.18)$$

But the limit of this quantity Q_n , as $n \rightarrow \infty$ is N/N_0 , the surviving fraction after time t .

$$\begin{aligned} \text{Now } Q_n &= 1 - n \frac{\lambda t}{n} + \frac{n(n-1)}{2!} \frac{\lambda^2 t^2}{n^2} - \frac{n(n-1)(n-2)}{3!} \frac{\lambda^3 t^3}{n^3} + \dots \\ &= 1 - \lambda t + \left(1 - \frac{1}{n}\right) \frac{\lambda^2 t^2}{2!} - \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \frac{\lambda^3 t^3}{3!} + \dots \\ \therefore \lim_{n \rightarrow \infty} Q_n &= \frac{N}{N_0} = 1 - \lambda t + \frac{\lambda^2 t^2}{2!} - \frac{\lambda^3 t^3}{3!} + \dots \\ &= e^{-\lambda t} \end{aligned}$$

giving the law of radioactive decay,

$$N = N_0 e^{-\lambda t}.$$

2.5 THE STATISTICAL ERRORS OF NUCLEAR PHYSICS

Using a detector like the Geiger Counter (see Chapter 1), it is possible to count the radioactive particles. From the preceding section, we know that the time t necessary for the observation of any finite number of counts N is subject to statistical fluctuations, giving rise to an error in the observed counting rate, $n = N/t$.

This is fundamental to the phenomenon of radioactivity and not a property of the instrument. We shall show below that the number of counts observed in a given time obeys a Poisson distribution.

Let the probability that N particles are observed in time t be P_N . Suppose that the interval t is divided into n equal intervals so small that the probability of emission of two particles within an interval is negligible.

\therefore The probability of the emission of one particle in a given interval is, N/n , where N is the average number.

The purpose of this section is to treat the fluctuations or differences between the actual number of decaying nuclei N and the average number \bar{N} , sometimes denoted by $\langle N \rangle$.

The probability of emission of N particles in the first N intervals and none in the remaining $(n - N)$ intervals is:

$$\left(\frac{\bar{N}}{n}\right)^N \left(1 - \frac{\bar{N}}{n}\right)^{n-N} \quad \dots (2.19)$$

This is only one possible way of obtaining N particles in the total time t . The first particle could have been in any one of the n intervals, the second one in any one of the remaining $(n - 1)$ and so on, so that finally N^{th} in any of the remaining $(n + 1 - N)$. The N^{th} particle can choose any one of the intervals between the one it occupies and all the $n - N$ unoccupied

intervals. Thus the number of ways of distributing the N particles in the n intervals is:

$$n(n-1)(n-2) \dots (n-N+1)$$

However, all of these ways are not independent, since the particles may be interchanged without influencing the result. Therefore, the number of essentially different ways

$$= \frac{n(n-1)(n-2) \dots (n-N+1)}{\text{Number of ways of interchanging particles}}$$

Any one of the N particles can be chosen as the first, any one of the remaining $(N-1)$ as the second and so on.

$$\therefore \text{Number of ways of interchanging particles} = N!$$

Hence, probability of obtaining N counts is,

$$P_N = \frac{n(n-1)(n-2) \dots (n-N+1)}{N!} \left(\frac{\bar{N}}{n} \right)^N \left(1 - \frac{\bar{N}}{n} \right)^{n-N} \quad \dots (2.20)$$

This is the binomial distribution law.

When $n \rightarrow \infty$, Eq. (2.20) becomes

$$\begin{aligned} P_N &= \frac{n^N}{N!} \left(\frac{\bar{N}}{n} \right)^N e^{-\bar{N}} \\ &= \frac{\bar{N}^N e^{-\bar{N}}}{N!} \end{aligned} \quad \dots (2.21)$$

This is the famous Poisson distribution formula. It is illustrated in Fig. 2.4.

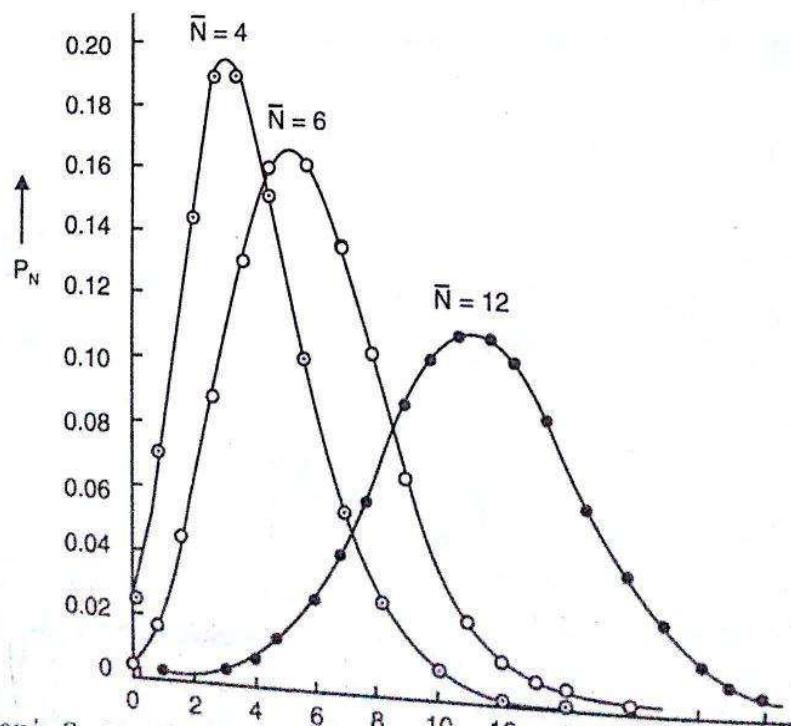


Fig. 2.4. Poisson's Series. Notice that as N becomes larger, the curve becomes more symmetrical about the maximum.

Strictly speaking, Eq. 2.21 is valid for only integral values of N . However, a continuous curve of P_N against N plotted through the points given by this formula yields a curve with an asymmetrical maximum for small values of N . As N becomes large the curve becomes symmetrical about the maximum and it can be shown analytically that the Poisson distribution approaches a Gauss error curve.

Then,

$$P_N = \frac{1}{\sqrt{2\pi\bar{N}}} e^{-\left(\frac{(N-\bar{N})^2}{2\bar{N}}\right)} \quad \dots(2.22)$$

which is the Gauss law. Proof of this is left to the reader (see Prob. 2.18).

\therefore If on an average \bar{N} counts are observed in given time t , the standard deviation in the number of counts,

$$S = \sqrt{\bar{N}} \quad \dots(2.23)$$

$$\left(\because \text{the precision index } h = \frac{1}{\sqrt{2\bar{N}}} \text{ and } S = \frac{\sqrt{2}}{h} \right)$$

Usually a single reading N is taken, which is assumed to be close enough to \bar{N} . Thus standard deviation,

$$S = \sqrt{N}$$

To appreciate this, let us take an example from laboratory data.

ILLUSTRATIVE EXAMPLE

Example 2.3: A cesium ($^{137}_{55}\text{Cs}$) sample was counted using a Geiger counter and the number of beta particles emitted in 1 min. was recorded in 10 intervals, each of 1 min. duration. The data is shown in the table below. The first column shows the observed counts, the second one shows the difference between average number of counts and particular 1 min. count.

Show that the standard deviation of a single measurement is in satisfactory agreement with that given by

$$S = \sqrt{\bar{N}}$$

N	$N - \bar{N}$	$(N - \bar{N})^2$
28792	13	169
28640	-139	19321
28510	-269	72361

Contd...

70		-99	9801
28680		61	3721
28840		44	1936
28823		1	1
28780		196	38416
28975		231	53361
29010		-39	1521
28740			
$\bar{N} = 28779$			$\Sigma 190807$

Solution: Average counts for one min. interval = 28779. Since there are 9 intervals, standard deviation of a single measurement

$$= \left(\frac{190807}{9} \right)^{\frac{1}{2}} \\ \cong 150$$

Now, we know that,

$$S = \sqrt{\bar{N}} = (28779)^{\frac{1}{2}} \cong 169, \text{ in good agreement.}$$

■ 2.6 RADIOACTIVE GROWTH AND DECAY

Let us consider decay of the type,



Our aim is to find out the abundance of substance *B* if *A* decays to *B* and *B* decays to *C*.

Let N_A be the number of nuclei of *A* type at any instant. Assume that originally only *A* type was present and the initial number of nuclei of *A* type be N_0 .

Let N_B be the number of nuclei of *B* type at the same instant *t*. At *t* = 0, the initial number is zero.

B is formed as a result of decay of *A*.

Therefore, the number of nuclei entering the *B* category is,

$$-\frac{dN_A}{dt} = \lambda_A N_A$$

where λ_A is decay constant of *A* type.

The number of nuclei leaving the *B* category is,

$$\approx \lambda_B N_B$$

where λ_B is decay constant of *B* type.

\therefore The net change in number of nuclei (per sec) of the *B* category is,

$$\lambda_A N_A - \lambda_B N_B = \frac{dN_B}{dt} \quad \dots(2.24)$$

Now,

$$N_A = N_0 e^{-\lambda_A t}$$

$$\therefore \frac{dN_B}{dt} = \lambda_A N_0 e^{-\lambda_A t} - \lambda_B N_B$$

Multiplying by the integrating factor, $e^{\lambda_B t} dt$

$$e^{\lambda_B t} dN_B = \lambda_A N_0 e^{(\lambda_B - \lambda_A)t} dt - \lambda_B N_B e^{\lambda_B t} dt \quad \dots(2.25)$$

L.H.S. can be now integrated by parts.

$$\int e^{\lambda_B t} dN_B = e^{\lambda_B t} N_B - \int \lambda_B N_B e^{\lambda_B t} dt \quad \dots(2.26)$$

Integrating Eq. 2.25, and substituting Eq. 2.26, we get,

$$e^{\lambda_B t} N_B = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0 e^{(\lambda_B - \lambda_A)t} + C$$

$$\therefore N_B = 0, t = 0,$$

$$C = -\left(\frac{\lambda_A N_0}{\lambda_B - \lambda_A} \right)$$

$$\therefore N_B = \frac{N_0 \lambda_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \quad \dots(2.27)$$

Which is the desired result, giving the number of nuclei of *B* type present at any time *t*.

The decay of *A* type and growth and decay of *B* type is shown in Fig. 2.5.

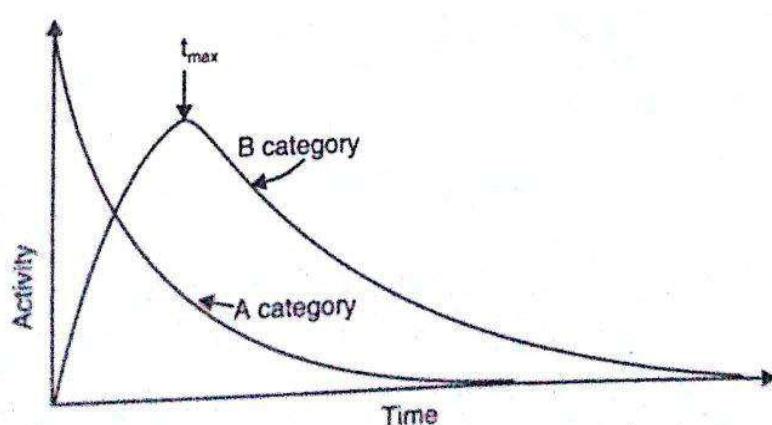


Fig. 2.5. Decay and growth of radioactivity.

ILLUSTRATIVE EXAMPLE

Example 2.4: Show that if there are initially N_0 radioactive nuclei of the parent present, the time at which the number of radioactive daughter nuclei is maximum is:

$$t_{\max} = \frac{\log(\lambda_B/\lambda_A)}{\lambda_B - \lambda_A}$$



Solution: Figure 2.5 indicates how the daughter reaches the maximum activity.

We know that,

$$N_B = \frac{N_0 \lambda_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t})$$

For N_B to be maximum,

$$\frac{dN_B}{dt} = 0 = \frac{N_0 \lambda_A}{\lambda_B - \lambda_A} [-\lambda_A e^{-\lambda_A t_{\max}} + \lambda_B e^{-\lambda_B t_{\max}}]$$

$$\therefore t_{\max} = \frac{\log(\lambda_B/\lambda_A)}{\lambda_B - \lambda_A}$$

2.7 IDEAL EQUILIBRIUM

From Eq. 2.24, at t_{\max} we have

$$\lambda_A N_A = \lambda_B N_B \quad \dots(2.28)$$

since

$$\frac{dN_B}{dt} = 0$$

Thus at time t_{\max} , (and only at t_{\max}), the activity of parent and activity of accumulated daughter are equal.

At t_{\max} (as considered in the illustrative example above), we have:

$$\begin{aligned} \lambda_A N_A &= \lambda_A N_0 e^{-\lambda_A t_{\max}} \\ &= \lambda_A N_0 e^{-\lambda_A \frac{\log(\lambda_B/\lambda_A)}{\lambda_B - \lambda_A}} \\ &= \lambda_A N_0 \left(\frac{\lambda_A}{\lambda_B} \right) e^{\frac{\lambda_A}{\lambda_B - \lambda_A}} \\ &= \lambda_A N_0 \left(\frac{T_B}{T_A} \right) e^{\frac{T_B}{T_A - T_B}} \end{aligned} \quad \dots(2.29)$$

When the activities of parent and daughter are, equal, the situation is called *ideal equilibrium*. Note that this situation exists only at the moment when, time = t_{\max} .

Look at Fig. 2.5 where dN_B/dt is positive between times, $t = 0$ and $t = t_{\max}$. This implies that parent activity in this time range always exceeds daughter activity. Conversely between times $t = t_{\max}$ and $t = \infty$, dN_B/dt is negative, implying that the daughter activity exceeds the activity of its parent.

■ 2.8 TRANSIENT EQUILIBRIUM AND SECULAR EQUILIBRIUM

Consider, the daughter shorter-lived than the parent.

i.e

$$T_A > T_B$$

From Eq. 2.27 the activity of B type is,

$$\lambda_B N_B = N_0 \lambda_A \frac{\lambda_B}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t})$$

Since

$$\lambda_A N_A = \lambda_A N_0 e^{-\lambda_A t},$$

$$\lambda_B N_B = (\lambda_A N_A) \frac{\lambda_B}{\lambda_B - \lambda_A} (1 - e^{-(\lambda_B - \lambda_A)t}) \quad \dots(2.30)$$

or

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{T_A}{T_A - T_B} (1 - e^{-(T_A - T_B)/T_A} \lambda_B t) \quad \dots(2.31)$$

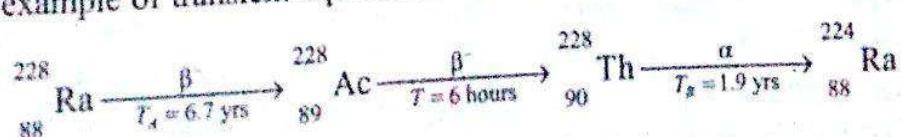
For large t , Eq. 2.31 becomes,

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{T_A}{T_A - T_B} \quad \dots(2.32)$$

When the ratio, $\frac{T_A}{T_A - T_B}$ is greater than one, from Eq. 2.31, it is clear that for large t ,

Equation 2.32 should hold. Since we have $T_A > T_B$ such is the case. When the Eq. 2.32 holds, we say that *transient equilibrium* exists between parent and the daughter. Note that according to Eq. 2.32 the ratio of daughter and parent activities is constant.

A prominent example of transient equilibrium is the following decay:



${}^{228}_{89} \text{Ac}$ has half life of ~ 6 hours and promptly decays into ${}^{228}_{90} \text{Th}$.

Thus we can ignore the presence of ^{228}Ac as an intermediate product. From Eq. 2.32, the activity ratio corresponding to equilibrium is,

$$\begin{aligned}
 \frac{\lambda_B N_B}{\lambda_A N_A} &= \frac{\text{Activity of } ^{228}\text{Th}}{\text{Activity of } ^{228}\text{Ra}} \\
 &= \frac{T_A}{T_A - T_B} \\
 &= \frac{6.7}{6.7 - 1.9} \\
 &= 1.39 \quad \dots(2.33)
 \end{aligned}$$

i.e., it is > 1 .

Thus it is an example of transient equilibrium. Figure 2.6 shows decay of ^{228}Ra and growth of ^{228}Th .

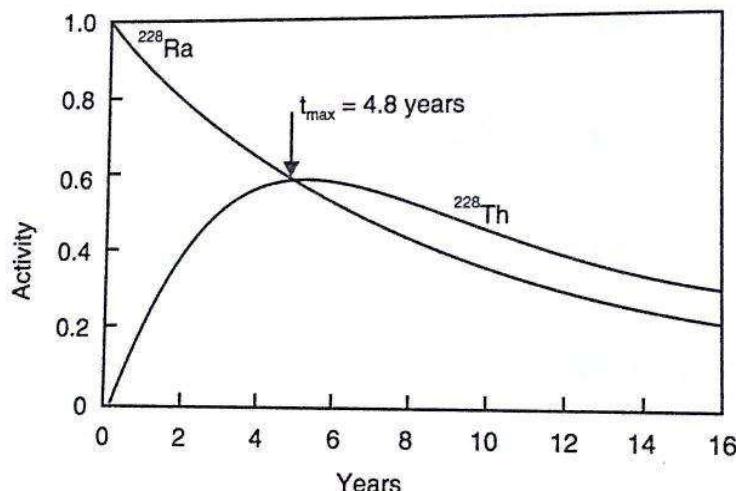


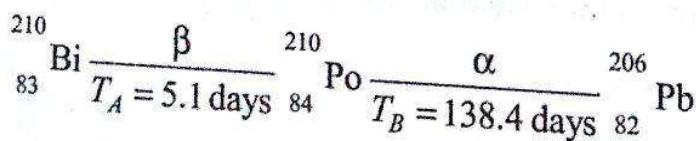
Fig. 2.6. Transient equilibrium between ^{228}Ra parent and ^{228}Th daughter. Initially, the source is pure ^{228}Ra .

Verify that $t_{\max} = 4.8$ yrs (see Example 2.4).

Notice from the Fig. 2.6 that at larger values of time, the activity of the daughter approaches its transient equilibrium value of 1.39 times the remaining parent activity (as given by Eq. 2.33).

If the daughter is *longer-lived* than the parent, i.e., if $T_A < T_B$, from Eq. 2.31 it can be seen that, the ratio $\frac{\lambda_B N_B}{\lambda_A N_A}$ continuously increases as time t increases.

In other words, after sufficient time, the activity of the daughter becomes independent of the residual activity of the parent, and there can be no equilibrium between them. This is indicated in Fig. 2.7 for the case of



Let us now consider the important case of the daughter being much shorter-lived than parent. Here,

and Eq. 2.30 becomes

$$\lambda_A \ll \lambda_B \quad (T_A \gg T_B)$$

$$\lambda_B N_B = \lambda_A N_A (1 - e^{-\lambda_B t}) \quad \dots(2.34)$$

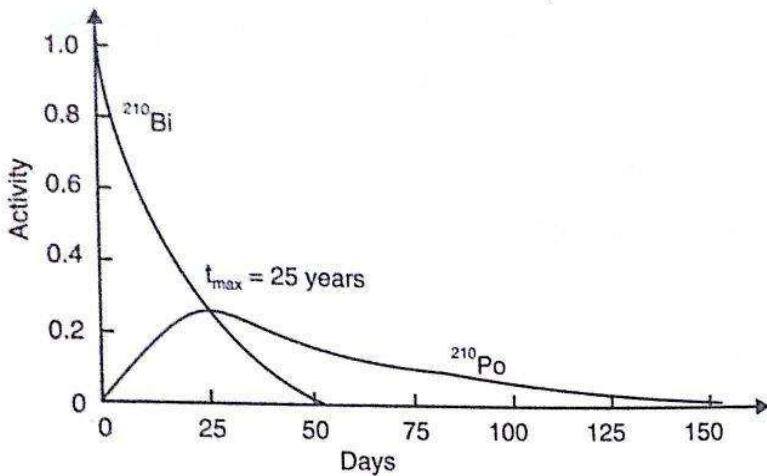


Fig. 2.7. There is no equilibrium between ^{210}Bi (5.1 d) and ^{210}Po (138.4 d).

that is the daughter activity is controlled only by its own decay constant.

It can be at once seen that for,

$$t \gg T_B,$$

the exponential,

$$e^{-\lambda_B t} = e^{-\frac{0.693}{T_B} t}$$

$$\approx 0$$

$$\dots(2.35)$$

and so

$$\lambda_B N_B = \lambda_A N_A$$

It should be kept in mind that Eq. 2.35 is valid only if $T_A \gg T_B$ and $t \gg T_B$.

Thus in these cases, the daughter activity equals the parent activity and the equilibrium is called secular equilibrium.

For a very long-lived parent, on physical grounds, the result given by Eq. 2.35 is expected. In this case, the activity of the parent can be taken as almost constant and so the rate of production of the daughter too is constant. As the daughter quantity grows (accumulates), its rate of decay increases and finally catches up, after sufficient time, with the rate of its production. When the two rates become equal, we say that the daughter is in secular equilibrium with the parent, and,

$$\lambda_B N_B = \lambda_A N_A$$

$$\frac{\lambda_B}{\lambda_A} = \frac{T_A}{T_B} = \frac{N_A}{N_B} \quad \dots(2.36)$$

giving,

Therefore in the case of secular equilibrium, the ratio of the number of parent nuclei and daughter nuclei is constant, and is equal to the ratio of their half lives.

This explains why the percentage of radium contained in uranium was always experimentally found to be the same—an average of one gram of radium per 3.2 tons of pure uranium.

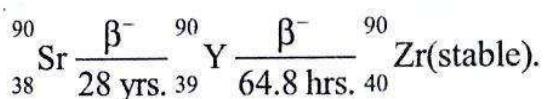
From Eq. 2.36

$$\frac{N_{Ra}}{N_U} = \frac{T_{Ra}}{T_U} = \frac{1620 \text{ yrs.}}{4.5 \times 10^9 \text{ yrs.}} (T_U \gg T_{Ra})$$

$$= 3.6 \times 10^{-7}$$

giving the experimentally observed ratio of 1 gm to 3.2 tons. Actually, radium is not the immediate daughter of uranium and we have to consider a radioactive series. (We will see this in the next section.) However, uranium has the longest half life in the series and hence all other products which are successively formed (except the end-product) are in secular equilibrium with uranium and their per cent content in a given sample of uranium is constant, equal to the ratio of their half lives as given by Eq. 2.36.

Another example of secular equilibrium, (which is not a part of a naturally occurring radioactive series) is,



If we start with a freshly prepared sample of pure ${}^{90}\text{Sr}$, the number of ${}^{90}\text{Y}$ nuclei at first increases as per Eq. 2.34, then almost reaches the saturation value, and from then on slowly decreases with the half life of ${}^{90}\text{Sr}$.

2.9 RADIOACTIVE SERIES

Each series is formed by successive daughter products, all ultimately derived from a single parent. Radioactive nuclei found in nature are said to exhibit natural radioactivity. [There are now a few thousand radioactive isotopes which have been produced in the laboratory, mostly by neutron bombardment.] These are said to exhibit artificial radioactivity. Compared to this, the number of naturally occurring radioactive nuclides is quite small, about 70.

[When naturally occurring radioactive isotopes were studied, scientists found that they could be divided into four series (isotopes between $Z = 81$ and $Z = 92$).

The reason that there are exactly four series is a consequence of the fact that alpha decay reduces the mass number of nucleus by 4.]

Therefore, the nuclei whose mass numbers are all given by

$$A = 4n. \quad \dots(2.37)$$

where n is an integer, can decay in descending order of mass number.

Radioactive nuclides whose mass numbers obey Eq. 2.37 are members of the $4n$ series.

The four series of naturally occurring nuclides are:

Mass Number	Series	Parent	<i>T</i> of parent
$A = 4n$	Thorium Series	$^{232}_{90}\text{Th}$	1.39×10^{10} yrs.
$A = 4n + 1$	Neptunium Series	$^{237}_{93}\text{Np}$	2.25×10^6 yrs.
$A = 4n + 2$	Uranium Series	$^{238}_{92}\text{U}$	4.51×10^9 yrs.
$A = 4n + 3$	Actinium Series	$^{235}_{92}\text{Ac}$	7.07×10^8 yrs.

The members of each of these series can decay into one another (in the same series) in descending order of mass number.

The table gives the half lives of parent nuclide in each of the four series, which is very large, except for the parent of the neptunium series. For $^{237}_{93}\text{Np}$, half life is, $T = 2.25 \times 10^6$ yrs., which is quite short compared to the estimated age of earth which is about $\sim 10^{10}$ yrs. (see the following section). Thus, the members of this series are not found in nature today. However, they have been produced in the laboratory by neutron bombardment of other heavy nuclei. Note that the origin of this series can be traced back to Americium and Plutonium.

This process of chain of alpha and beta decays that leads from parent to stable end-point product in each series is shown in Figs. 2.8, 2.9, 2.10 and 2.11.

→ **Branching:** [A given type of nuclei will normally decay by one particular mode, say by emission of β -particles. But many cases have been found in which a smaller percentage of nuclei will decay by a different mode such as α emission. This phenomenon is known as branching.] For example, 99.96 per cent of nuclei of $^{214}_{83}\text{Bi}$ undergo β decay (see Fig. 2.10) to form ^{214}Po ; whereas 0.04 per cent of nuclei undergo alpha decay to form ^{210}Tl (Thulium).

Let us denote the probability of α emission by one nucleus, in time dt , $\lambda_\alpha dt$ and that of β emission by $\lambda_\beta dt$.

Then the total probability of decay of a nucleus in time dt by either α or β emission is:

$$(\lambda_\alpha + \lambda_\beta)dt$$

Hence the activity is:

$$\frac{dN}{dt} = -(\lambda_\alpha + \lambda_\beta)N. \quad \dots(2.38)$$

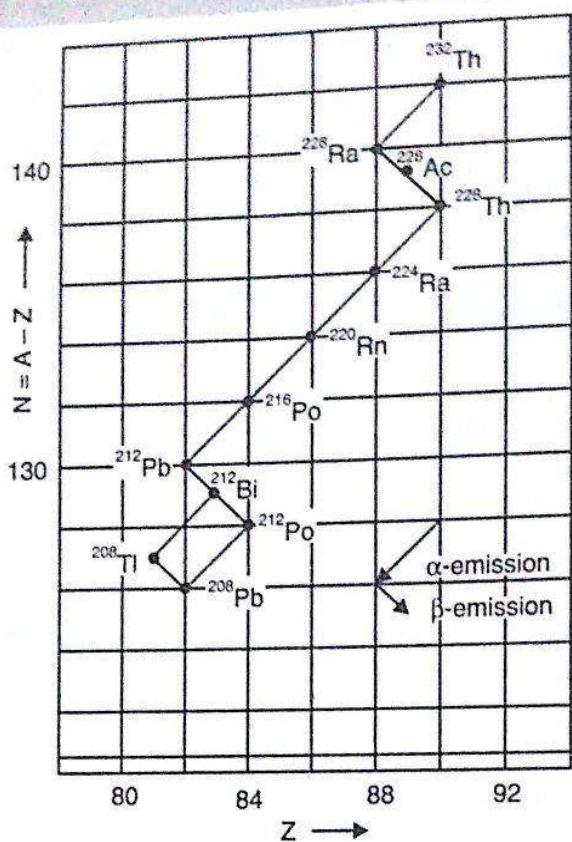


Fig. 2.8. $A = 4n$, thorium series. End product (stable) is ^{208}Pb . Notice the branching where ^{212}Bi may decay by α and then β emission or in the opposite order.

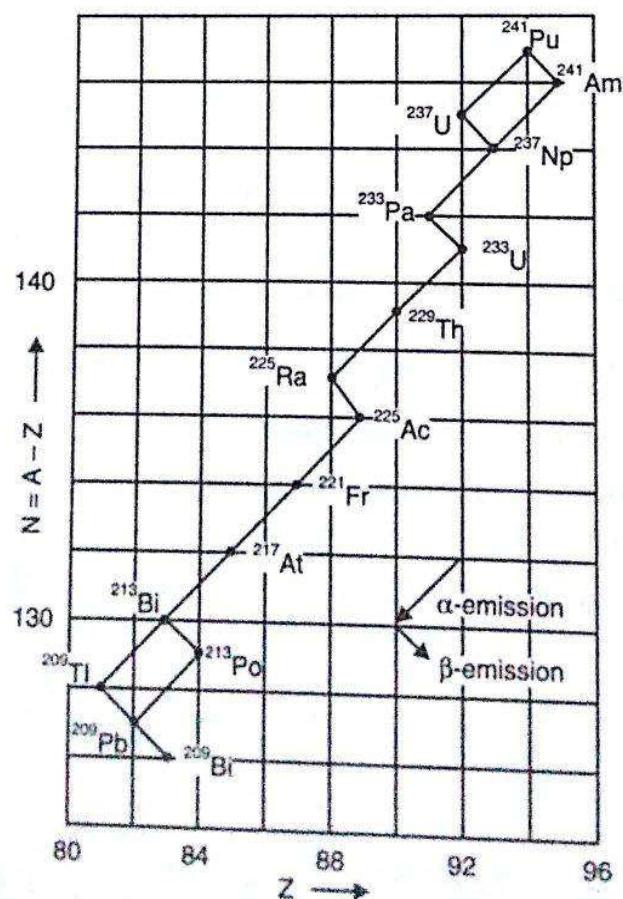


Fig. 2.9. $A = 4n + 1$, neptunium series. End product (stable) is ^{209}Bi . ^{213}Bi may decay by α emission followed by β emission or in the reverse order.

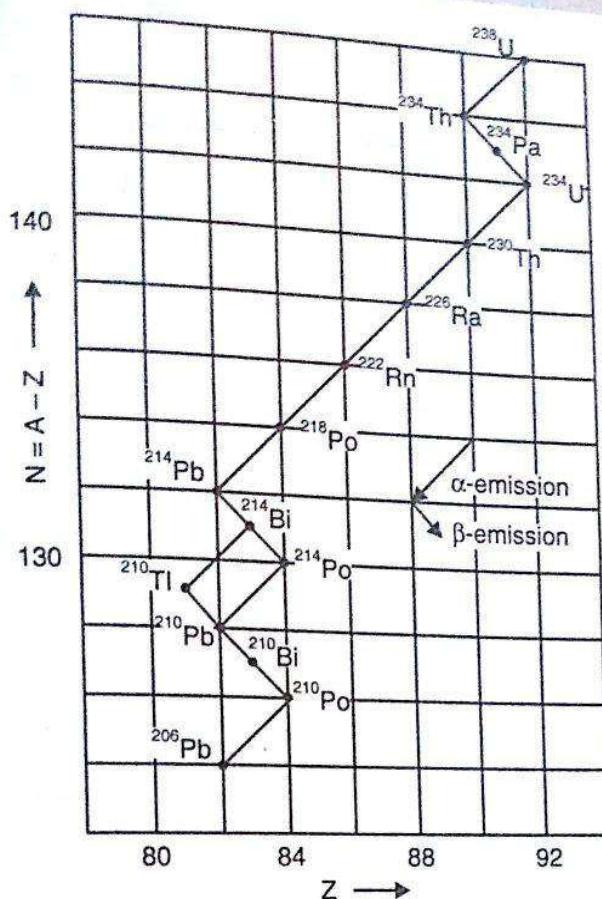


Fig. 2.10. $A = 4n + 2$, uranium series. Stable end product is ^{206}Pb . Again notice branching.

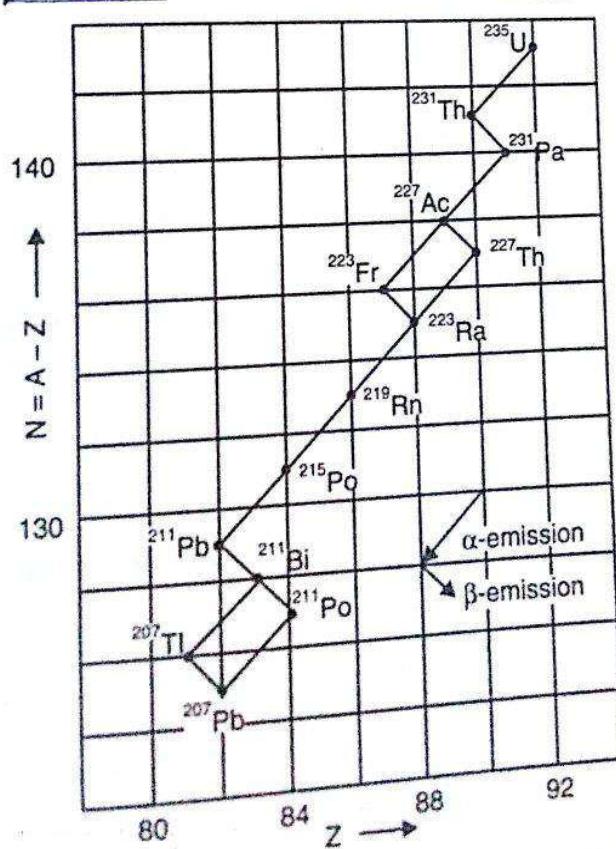


Fig. 2.11. $A = 4n + 3$, actinium series. Stable, end product is ^{207}Pb . Note the branching at ^{227}Ac and ^{211}Bi .

giving mean life,

$$\tau = \frac{1}{\lambda_a + \lambda_\beta}$$

The ratio

$$\frac{\lambda_a}{\lambda_\beta} = \text{branching ratio.}$$

From Eq. 2.39 it follows that,

$$\frac{1}{\tau} = \frac{1}{\tau_a} + \frac{1}{\tau_\beta} \quad \dots(2.40)$$

ILLUSTRATIVE EXAMPLE

Example 2.5: One is often required to deal with a situation where a radioactive substance, initially absent, is formed at a constant rate. For example, using particle accelerators (see Chapter 1), it is possible to artificially produce radioactive nuclei. Assume that a particular type of nucleus with decay constant λ is produced at a steady rate of P nuclei per second. Find the number of nuclei $N(t)$ present t seconds after the production starts.

Solution: Rate of formation of nuclei = P

Rate of decay of nuclei = λN at time t .

$$\therefore \frac{dN}{dt} = P - \lambda N.$$

We know that at time $t = 0$, $N = 0$

\therefore Multiply by integrating factor $e^{\lambda t} dt$,

$$e^{\lambda t} dN = Pe^{\lambda t} dt - \lambda N e^{\lambda t} dt.$$

Integrating $e^{\lambda t} dN$ by parts and R.H.S. in a straightforward way

$$Ne^{\lambda t} - \int N \lambda e^{\lambda t} dt = \frac{P}{\lambda} e^{\lambda t} - \int N \lambda e^{\lambda t} dt + C$$

$$C = -\frac{P}{\lambda}, \quad \because \text{at } t = 0, N = 0$$

$$Ne^{\lambda t} = \frac{P}{\lambda} (e^{\lambda t} - 1)$$

or

$$N = \frac{P}{\lambda} (1 - e^{-\lambda t})$$

which is the required result.

2.10 RADIOACTIVE ISOTOPES OF LIGHTER ELEMENTS

Though the lowest atomic number among the naturally occurring radioactive series is found to be 81, radioactive isotopes of lower atomic numbers have also been found in nature. Most of these isotopes have very long half lives and very low abundances, so that the activity of samples is extremely small. Continuous and extensive testing of nuclear weapons has added new radioactive isotopes to the atmosphere. ^{90}Sr is an example of this nuclear pollution. It is a serious hazard as half life of ^{90}Sr is 27.7 years. Some of the prominent radioactive isotopes, mainly of the lighter elements, are given in Table 2.1.

TABLE 2.1

Atomic number Z	Element	Mass number A	Type of radioactivity	Half life T in years
1	Hydrogen	3	Beta	12.26
6	Carbon	14	Beta	5730
19	Potassium	40	Beta	1.3×10^9
38	Strontium	90	Beta	27.7
57	Lanthanum	138	Beta	1.1×10^{11}
60	Neodymium	144	Alpha	2.4×10^{15}
62	Samarium	147	Alpha	1.06×10^{11}
72	Hafnium	174	Alpha	2×10^{15}
78	Platinum	190	Alpha	6×10^{11}
82	Lead	204	Alpha	1.4×10^{17}

The nuclei of ^{14}C are being constantly produced by cosmic ray neutron bombardment of ^{14}N . Hence its activity is continuously replenished.

2.11 ARTIFICIAL RADIOACTIVITY

Rutherford was the first man to accomplish, in 1919, the artificial transmutation of one element into another. His apparatus is schematically represented by Fig. 2.12.

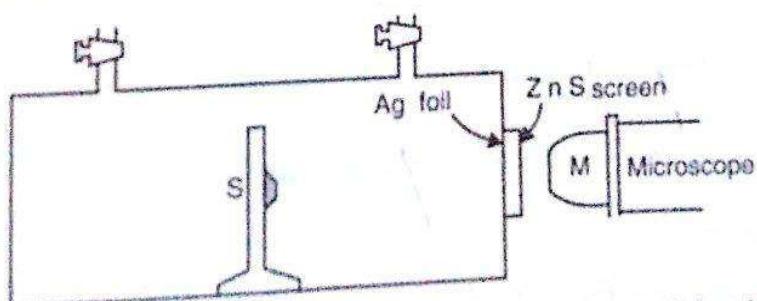
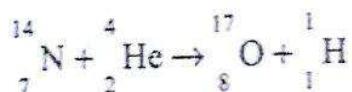


Fig. 2.12. Apparatus used by Rutherford to study the first successful artificial disintegration of nuclei.

The chamber was filled with nitrogen and alpha particles from ^{212}Po source (S in Fig. 2.12) were made to bombard the nuclei of nitrogen. A sheet of silver foil thick enough to absorb

α -particles was placed over an opening in the chamber. The screen made from ZnS was placed outside this opening and a microscope M could observe the scintillations produced on this screen (Zinc sulphide is fluorescent under radioactive rays.) Scintillations were observed when the chamber was filled with nitrogen. Rutherford concluded that the scintillations were produced by particles which were ejected from nitrogen nuclei as a result of α -bombardment. Later these particles were identified as protons (by the magnetic deflection method) having a range of ~ 50 cm in air.

The reaction can be written as:



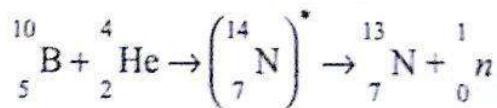
After the discovery of the neutron in 1932 by Chadwick, it became possible to see that many of the induced nuclear transformations are radioactive. Today we have a few thousand radioactive species that have been made in the laboratory, by using accelerators.

The decay theories are the same for these artificially produced radioactive substances as the same laws that govern natural radioactivity hold.

In 1934, M. and Mme. Curie-Joliot were bombarding light element by α -particles and these studies led to important observations. They found that the bombarded substances continued to emit radiation even after the source of α -particles had been removed. Magnetic deflection experiments showed that the radiations consisted of positrons (positron is the anti-particle of the electron, having a +ve charge of the same magnitude as that of the electron).

The half life T of the positron emitters was measured. Curie-Joliot explained this phenomenon by suggesting the following nuclear process:

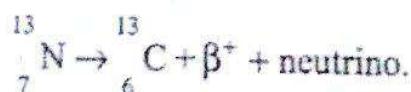
Step I: Bombardment of ^{10}B by α -particles giving a neutron and a residual nucleus rich in protons.



Compound
nucleus

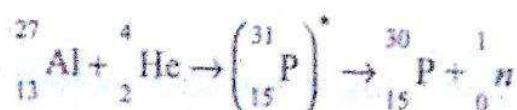
(See Chapter 4)

Step II: Proton rich residual nucleus decays by positron emission.

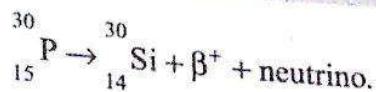


Half life $T = 9.96$ min.

Another reaction studied by Curie-Joliot:



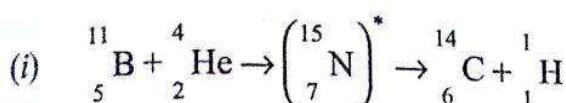
and then,



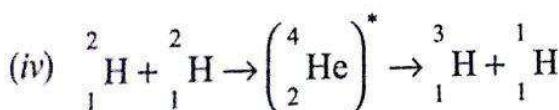
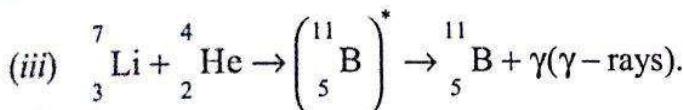
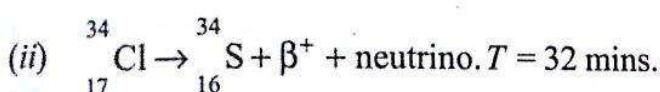
Half life $T = 2.5$ mins.

After the pioneering work by Curie-Joliot in the study of artificial radioactivity, a massive programme to produce artificially radioactive isotopes was launched by various laboratories all over the world using particle accelerators, nuclear reactors (as neutron source) to bombard elements by different projectiles. A study of these reactions and the artificial radioactivity produced has enabled us to experimentally understand the structure of over a thousand isotopes of different elements. In this sense, in nuclear physics today, we know much more about nuclear energy levels and their properties experimentally than what we know of nuclear matter theoretically.

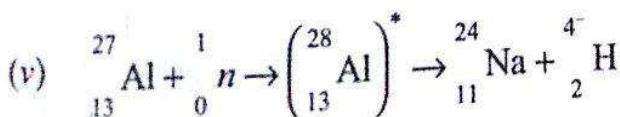
Some more examples of artificial radioactivity are given below:



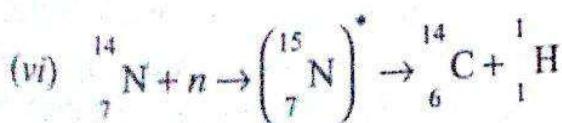
followed by ${}_{6}^{14}\text{C} \rightarrow {}_{7}^{14}\text{N} + \beta^- + \text{anti-neutrino. } T = 5730 \text{ yrs.}$



followed by, ${}_{1}^3\text{H} \rightarrow {}_{2}^3\text{He} + \beta^- + \text{anti-neutrino. } T = 12.26 \text{ hrs.}$



followed by, ${}_{11}^{24}\text{Na} \rightarrow {}_{12}^{24}\text{Mg} + \beta^- + \text{anti-neutrino. } T = 15 \text{ hrs.}$



followed by, ${}_{6}^{14}\text{C} \rightarrow {}_{7}^{14}\text{N} + \beta^- + \text{anti-neutrino.}$

A systematic study of energetics of nuclear reactions is undertaken in Chapter 3. As can be noticed, most of the artificially produced radioactive isotopes have small half lives. This can be a valuable aid in the identification of an element by chemical analysis, e.g., Curie-Joliot, in their boron reaction made a target of

BN (Boron nitride)

This they irradiated with α -particles for several minutes and then heated it with NaOH (caustic soda).

The chemical reaction was:

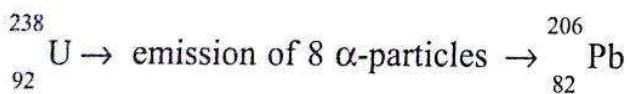


They found that the gaseous NH_3 (ammonia) was the only radioactive chemical product.

This clearly indicated that the nitrogen $\left({}_{7}^{13}\text{N}\right)$ was the radioactive element produced in this experiment. The short half lives also enable a quick identification, as half lives are characteristic of the emitting nuclei.

■ 2.12 DETERMINATION OF THE AGE OF THE EARTH

Radioactivity is the best clock we can employ to estimate the absolute age of the earth, as it is totally unaffected by environmental change or natural upheavals like earthquakes, storms etc. The determination of geological ages is done very often by the so-called lead methods. It involves the following nuclear processes:



To illustrate how an estimate of time can be made, consider the ${}_{92}^{238}\text{U}$ series. The stable end-product is ${}_{82}^{206}\text{Pb}$, and so we can take $\lambda_{\text{Pb}} = 0$. The half life of ${}^{238}\text{U}$ is 4.5×10^9 years.

Hence after sufficient time, (say, a billion years) the only elements present in any appreciable amount will be uranium and lead. This is because, all elements in the uranium series will be in secular equilibrium with the parent ${}^{238}\text{U}$ and only ${}^{206}\text{Pb}$ will not be in equilibrium, and hence quantity of Pb will continuously go on increasing while uranium will continuously get depleted. This is indicated in Fig. 2.13 which depicts the flow of water from a reservoir into a collector, through a chain of other tanks in between. The diameters of the connecting pipes determine

the 'probability' of decay (rate of flow). The situation is analogous with decay of a radioactive series, finally ending in a stable product.

Therefore it is possible for us to apply Eq. 2.27 not only to the first and second elements ($A \rightarrow B$) but also to the first and last, (^{238}U and ^{206}Pb in the case referred).

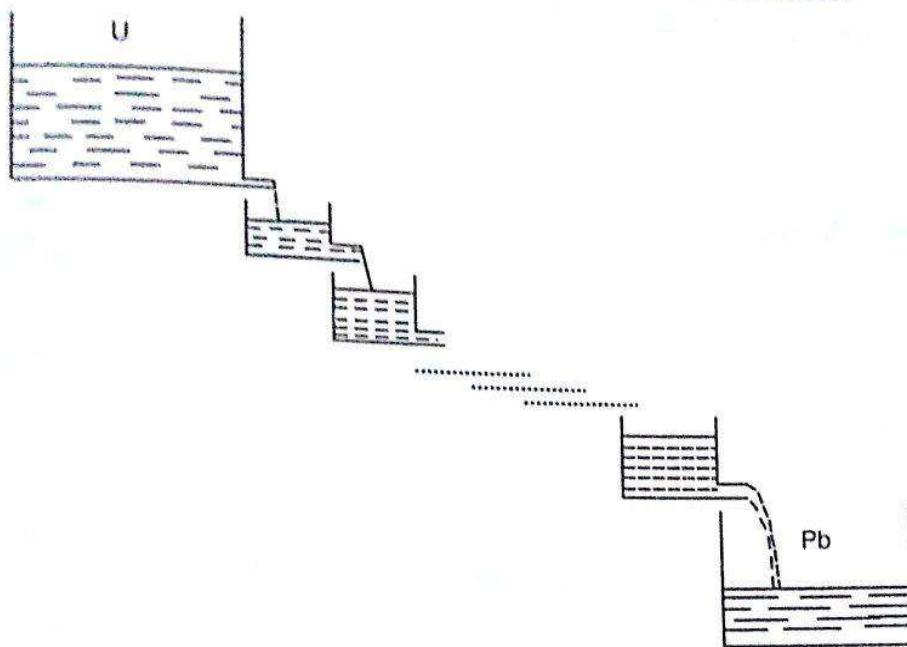


Fig. 2.13. Water flow analogy to illustrate depletion of U and accumulation of Pb. Note that Pb cannot be in equilibrium with U. All other members of the series are, and so they are present in minute and fixed amounts.

Equation 2.27 is:

$$N_B = \frac{N_0 \lambda_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t})$$

For our case,

$$\lambda_A = \lambda_U$$

$$\lambda_B = \lambda_{\text{Pb}} = 0 \quad \text{as Pb is stable.}$$

$$N_s = N_{\text{Pb}}$$

$$N_0 = N_v$$

$$N_{\text{Pb}} = -N_v (e^{-\lambda_U t} - 1)$$

$$= N_v (1 - e^{-\lambda_U t}) \quad \dots(2.41)$$

^{238}U ore always contains ^{206}Pb , which can be assumed to be of radioactive origin (^{206}Pb is the end-product).

- i.e., Present number of Pb atoms + Present number of U atoms
 = Number of U atoms originally present.

i.e.,

$$N_{\text{Pb}} + N_{\text{U}} = N_r$$

...(2.41)

Eqs. 2.41 and 2.42 can be solved simultaneously to give:

$$t = \frac{1}{\lambda_{\text{U}}} \log \left(\frac{N_{\text{Pb}} + N_{\text{U}}}{N_{\text{U}}} \right) \quad \dots(2.42)$$

Thus by spectro-chemically analysing a sample and knowing its uranium and lead content, it is possible to estimate the age of that sample. Of course, the most important factor is to ensure that no helium or uranium has escaped the rock sample during its lifetime.

The oldest surface rocks have been found to have an age of about 3×10^9 years.

When the same method is applied to determine the age of meteorites, it is found that oldest of these are about 4.5×10^9 years. This corresponds to the age of the earth and is different from the segregation of surface rocks.

Another method may be mentioned. It consists of using the $^{206}\text{Pb}/^{207}\text{Pb}$ isotopic ratio of radiogenic lead.

Using Eq. 2.41 it is easy to see that

$$\frac{^{206}\text{Pb}}{^{207}\text{Pb}} = \frac{^{238}\text{U}(1 - e^{-\lambda_U t})}{^{235}\text{U}(1 - e^{-\lambda'_U t})} \quad \dots(2.44)$$

where

$$\lambda_U = \lambda \text{ of } ^{238}\text{U}$$

and

$$\lambda'_U = \lambda \text{ of } ^{235}\text{U}.$$

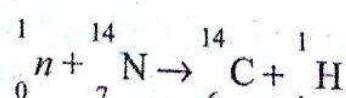
From Eq. 2.44, t can be evaluated.

The main disadvantage of this method is that radon which forms in the decay of ^{238}U has a half life of 3.82 days and being gaseous, might escape from the system.

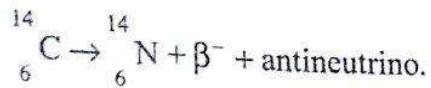
Thus for this method or the first method, the rock samples have to be taken from massive pitchblende, deeply buried. In such a case, the gas would get trapped in rock cavities and radon ($T = 3.82$ days) cannot be expected to diffuse very far from the dense rocks.

2.13 CARBON DATING—ARCHAEOLOGICAL TIME SCALE

Cosmic rays continuously form $^{14}_6\text{C}$ in atmosphere. The nuclear reaction is



$^{14}_6\text{C}$ is a neutron-rich (6 protons and 8 neutrons) isotope of carbon, which is β^- active.



Half life of $^{14}_6\text{C}$ is 5730 years.

Thus $^{14}_6\text{C}$ would not have been present in the atmosphere, had it not been continuously replenished. ^{14}C gets combined with hydrogen and oxygen and eventually finds its way into all organic matter. Since ^{14}C has a half life of 5730 years it is an ideal radioactive isotope for studying the age of civilisations and is extensively used in archaeology and anthropology.

When an animal or plant dies, its intake of carbon stops and from that moment decay of ^{14}C is the only process that continues. Coal and petroleum are organic in nature, but they are so old that there is no trace of ^{14}C in them. This has been established by experiments. In a young animal or tree, ^{14}C activity is same as atmospheric carbon which is about 15 disintegrations per gm per min.

By carefully measuring the ^{14}C activity of a fossil or dead tree, it is possible to estimate its age. This is shown in the problem given below.

ILLUSTRATIVE EXAMPLE

Example 2.6: In an archaeological expedition, charcoal from an ancient fire-pit was excavated. This sample showed a ^{14}C activity of 11.3 counts per gm per min. The absolute activity of ^{14}C in a living tree is independent of species and it is ~ 15.3 counts per gm per min. Estimate the age of the charcoal sample.

Solution: We have,

$$11.3 = 15.3 e^{-\lambda t} \quad \frac{dN}{dt} = \lambda e^{-\lambda t}$$

$$\lambda = \frac{0.693}{T} = \frac{0.693}{5730 \text{ years}} \quad ? \quad \text{or what?}$$

where

Didn't get

$$1.354 = e^{\lambda t} = e^{0.000121t} \quad \text{where } t \text{ is in year}$$

t = age of charcoal sample in years

$$= \frac{\log_e 1.354}{0.000121} \text{ years}$$

$$= 2504.65 \text{ years.}$$

Thus

EXERCISES

1. Calculate the weight 1Ci of,

$$(i) {}_{92}^{238}\text{U}, T = 4.5 \times 10^9 \text{ yrs.}$$

$$(ii) {}_{92}^{234}\text{U}, T = 2.48 \times 10^5 \text{ yrs.}$$

$$(iii) {}_{82}^{212}\text{Pb}, T = 10.6 \text{ hrs.}$$

$$(iv) {}_{84}^{212}\text{Po}, T = 3.04 \times 10^{-7} \text{ sec.}$$

2. Calculate the volume of 1 Ci of radon $\left({}_{86}^{222}\text{Rn}\right)$ at N.T.P. (at 0°C and 760 mm of Hg pressure). Half life of radon = 3.82 days.

3. Sixty hours after a sample of beta emitter ${}_{11}^{24}\text{Na}$ has been prepared, only 6.25% of it remains undecayed. What is the half life of this isotope?

4. The mass of one millicurie (mCi) of ${}_{82}^{214}\text{Pb}$ is $3 \times 10^{-14} \text{ kg}$. Find the half life of this isotope. Also calculate the mean life.

5. Radium has a half life of ~ 1600 yrs. The earth's age is $\sim 4 \times 10^9$ yrs. Thus one expects the amount of radium present to be negligibly small. But still Mme Curie was able to extract radium from uranium ore. Explain.

6. The nucleus of ${}_{92}^{238}\text{U}$ decays into a stable isotope of some element through successive emissions of 8 alpha particles and 6 beta particles. Identify the stable element, and give its Z and A values.

7. If a radioactive element disintegrates over a time equal to its average life τ , what fraction of the original amount remains?

8. The activity of a sample of ${}_{24}^{55}\text{Cr}$ is measured at the end of 5 min. intervals. The following activities are measured in millicuries: 19.2, 7.13, 2.65, 0.99 and 0.37. Plot $\log(\text{activity})$ against time and determine the half life from the graph.

9. A mass of 4.6 gm of uranium oxide (U_3O_8) has a certain activity. What is the mass of $\text{U}(\text{NO}_3)_6$ that gives the same activity? Take uranium to be ${}_{92}^{238}\text{U}$.

10. Give examples to illustrate the generality of the law of exponential decay and growth.

11. Consider the decay series:



Equation 2.27 gives the amount of B at time t . B decays into stable substance C. Determine how the amount of C varies with time.

12. (i) In a sample of 1 kg of ^{232}Th , how many decays occur per day?
(ii) If all the alpha particles could escape from the given thorium and the sample were electrically insulated, how much negative charge would accumulate on the sample? (T of $^{232}\text{Th} = 1.41 \times 10^{10}$ yrs.)
13. When the situation corresponds to transient equilibrium, daughter activity always exceeds the activity of the parent substance. Describe how you visualize such a situation, apart from the way it is analytically given by Eq. 2.32.
14. Express t_{\max} obtained in solved Example 2.4, in terms of mean life τ and half life T of parent and daughter, A and B respectively. Is t_{\max} physically acceptable for both, $T_A > T_B$ and $T_B > T_A$? Explain.
15. There are no physical cases known for which $T_A = T_B$, where as before, A is parent and B is daughter. But if you consider $T_A = T_B$, show that,

$$t_{\max} = \sqrt{\tau_A \tau_B}$$

(Hint: Put $T_A = T_B (1 + \delta)$ where $\delta \ll 1$).

16. ^{212}Bi decays to ^{208}Tl by α emission in 35 per cent of the disintegration and to ^{212}Po by β emission in 65 per cent of the disintegration. If the total half life is 60.6 min., find the decay constants for α and β modes of decay and also the branching ratio.
17. Radioactive isotope ^{32}P is produced by neutron bombardment of ^{31}P target in a nuclear reactor at a steady rate of 5×10^{10} nuclei per sec. If the half life of ^{32}P is 14.5 days, calculate the time required to give maximum activity of ^{32}P . (The maximum activity is sometimes referred to as saturation activity.)
18. Prove that for a Poisson distribution,

$$\sum_{n=0}^{\infty} P_n = 1$$

What does this verify?

19. In order to measure a counting rate with an accuracy of 1 per cent, we must count at least 10,000 decays; or for an accuracy of 1 in 10^3 , a measurement of 10^6 decay events is required. Discuss and explain.
20. A certain sample gives a counting rate of 140 counts per min. on a Geiger counter. The observation is continued for 21 mins. The background of the counter is 40 counts per min., which is based on an observation of 38 min. duration. Find the expected net counting rate of the sample and its probable error.
21. Isotopes of lead are produced in a uranium mineral (originally lead-free) by a radioactive decay of the isotopes ^{235}U and ^{238}U . Suppose the mineral is 6.5×10^8 years old. Calculate the isotopic ratio, $^{206}\text{Pb}/^{207}\text{Pb}$.
- Half lives are, $^{238}\text{U} = 4.51 \times 10^9$ years
 $^{235}\text{U} = 7.07 \times 10^8$ years.
22. An archaeologist finds a piece of wood in an excavated house. When this is examined by a nuclear geologist, it is found to weigh 100 gm and shows a ^{14}C activity of 650 disintegrations per min. Explain how the geologist must have obtained the age of the wood sample and estimate it yourself. Assume that (i) living trees show a ^{14}C activity of 15 disintegrations per gm per min. and (ii) T of ^{14}C is 5,730 years.
23. ^{140}La samples resulting from decay $^{140}\text{Ba} \xrightarrow[300\text{hr}]{\beta} {}^{140}\text{La}$ are to be used in a series of experiments.

Whenever the La activity reaches the maximum, La is separated from its parent, chemically, to form a sample for study. Ba continues to produce La and when the new La growth reaches a maximum value,

0

the second sample is separated. This procedure is continued as long as samples so obtained have an activity greater than 1 mCi at separation.

(i) If the initial amount of pure ^{140}Ba is 5 mCi, how many useful samples can be obtained?

(ii) What will be the activity of the last sample of ^{140}La at the time of separation?

24. What activity of ^{131}I is accumulated (during a 2 hr. bombardment of ^{130}Te (d, p) \rightarrow $^{131}\text{Te} \rightarrow$ ^{131}I , if the yield of ^{131}Te is 0.92 mCi per hour)?

(T of ^{131}I = 8 days.)

25. In a radio-active transformation, the parent element has a half life T_p which is very much greater than the half-life T_D of the daughter element. Find out the time required, in terms of T_D , to have secular equilibrium between the parent and the daughter, within, (i) 90%, (ii) 99% and (iii) 99.9%.