Expt.3: Nonlinear systems: logistic map & chaos

Due Date: 4th October, 2021 11:59pm.

This exercise is intended as an introduction to you on how very simple systems can ultimately lead to chaotic behaviour.
Three pdf files have been uploaded.

First read the uploaded excerpt from James Gleick's book "Chaos". It is essentially some scientific history of chaos with anecdotes, & of course, an introduction to the subject. Gleick's book is a classic & can easily be read for light & enjoyable reading, as well as learning something new at the same time.

Then look at the 2 pdf files on the logistic map, x_{n+1} = r x_n (1 - x_n), where r is the control parameter that can have values upto 4.
In general, in a nonlinear system, one value of the control parameter can lead to several distinct values of the variable being studied.

The same is true for the logistic map, first envisaged as a simple model for population growth. First, for values 1<=r<3, the system has only one value. Increase r a little more, the system oscillates between 2 stable values. Increase it some more, the "period" now doubles from 2 to 4 -- there are 4 stable values. This is known as "period-doubling", a signature of a nonlinear system that eventually becomes chaotic.

WHAT is chaotic behaviour? Chaos implies a sensitivity to initial conditions -- an infinitesimal change in values can completely change the dynamics of the system.

Your assignment is to write a code (in any language you choose) to do the following:

1. Reproduce the time-series plots (x_n vs n) shown in the logistic_map_info.pdf file -- i.e. generate the data for different r values, & then plot x_n vs n for those r values.

2. Plot the time-series plot for a value of r given by r = 3.853XXX where XXX is your roll number. For example, if your roll number is IMT2020008, r = 3.853008. If your roll number is IMT2020508, r = 3.853508.
Compare the time-series plots you get with slight changes in r. See how a small change in the control parameter can completely change the evolution of the system.

3a.

Now plot what is known as the "orbit diagram" or "bifurcation diagram" for the logistic map -- x vs r -- i.e. all possible x values that can be reached for each r value, r running from 1 to 4. Refer to the reading material to understand what plot you have to get.

3b. For a value of $r > r_\infty$ you will come across a window with a stable period-3 cycle. Show a detailed view of this window – what do you see? Can you see self-similarity there?

4. Calculate the value of the Feigenbaum constants (see page 4 of the logistic_map_info.pdf file, for example).

This can easily be done in a couple of days. You have upto a week for this.
Each group has to summarize what it has done **in a single document / report** -- submit ONE report, **the contributions of each member of the group – code, data, plots & calculations of each individual member will be different and have to be separately included**. This is essential.
You have a week to submit the report.