Physics -1 Lab SM203P
Experiment - 1

Experiment 1: Calculation of the acceleration due to gravity using a bouncing ball & finding the coefficient of restitution

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1 Aim

To determine the acceleration due to gravity and coefficient of restitution of energy of the tennis ball used to measure the same.

2 Apparatus used

1. A tennis ball
2. PhyPhox application on a mobile device
3. A meter scale/measuring tape

3 Theory

To calculate g, we first make the following assumptions:

1. We will ignore air drag (which is almost valid if the ball is dropped from a low height).
2. The time of contact between ball and floor is very short compared to the free-fall time.

Under the following assumptions, we can use the relation,

$$s = \frac{1}{2}gt^2$$

to calculate g. But calculating the time taken for a single bounce will introduce intolerable uncertainty. So we consider the time taken by the ball for n bounces. The height reached by the ball after every bounce will be lower and lower, so we define the coefficient of restitution of energy in the following way:

$$\begin{aligned} e &= \frac{mgh_1}{mgh_o} \\ &= \frac{h_1}{h_o} \end{aligned} \tag{1}$$

Taking t_1 as the time between the 1^{st} and 2^{nd} bounces,

$$g = \frac{2 \cdot h_1}{(t_1/2)^2} \quad = 8h_1/t_1^2 \tag{2}$$

Using 1,

$$g = 8h_o e/t_1^2 \tag{3}$$

We define t_n^* as the time between the 1^{st} and n^{th} bounce,

$$\begin{aligned} t_n^* &= t_1 + t_2 + \ldots + t_n \\ &= t_1 + 2\sqrt{2h_2/g} + \ldots + 2\sqrt{2h_n/g} \\ &= t_1 + 2(t_1/2)(\sqrt{e}) + \ldots + 2(t_1/2)(\sqrt{e})^{n-1} \\ &= t_1[1 + \sqrt{e} + \ldots + (\sqrt{e})^{n-1}] \\ &= t_1 \frac{(\sqrt{e})^n - 1}{\sqrt{e} - 1} \end{aligned} \tag{4}$$

From 4,

$$t_1 = t_n^* \left[\frac{\sqrt{e} - 1}{(\sqrt{e})^n - 1} \right] \tag{5}$$

Replacing into 3 we obtain,

$$g = \frac{8 \cdot h_o \cdot e}{(t_n^*)^2} \cdot \left[\frac{(\sqrt{e})^n - 1}{\sqrt{e} - 1} \right]^2 \tag{6}$$

4 Error Propagation

$$Relative\ Error\ (\Delta error) = \frac{absolute\ error}{measurement} * 100\%$$

$$\Delta h_0 = \frac{error\ in\ h_0}{measured\ h_0} * 100\% \ , \quad \Delta h_1 = \frac{error\ in\ h_1}{measured\ h_1} * 100\%$$

Dividing or multiplying two quantities results in a relative error equal to the sum of their respective relative errors,

$$\frac{\Delta e}{e} = \frac{\Delta h_0}{h_0} + \frac{\Delta h_1}{h_1} \tag{7}$$

To calculate the error propagation in g, we take log and differentiate equation 6 to obtain:

$$\frac{\Delta g}{g} = \frac{\Delta h_0}{h_0} + 2\frac{\Delta t_n^*}{t_n^*} + \frac{\Delta e}{e} + \frac{\Delta e}{\sqrt{e}(\sqrt{e} - 1)} + \frac{n(e^{n/2-1})\Delta e}{e^{n/2} - 1} \tag{8}$$

5 Procedure

1. Using a meter scale, or measuring tape, measure a distance of upto 0.5 meters or more and affix it using a pen or anything else convenient. (For example, we have used 1 meter, 1.5 meters)

2. Hold your tennis ball at that position.

3. Open the PhyPhox application on your phone, and open (in)elastic collisions.

4. Release the ball at the same instant of clicking on the start button of your phone.

5. Then the application will tell you height of ball released, time between each successive occurrences of the ball touching the ground.

6. Discard any readings in which ball is released irregularly or if the ball bounces of an irregularly shaped object. As a practice, always do the experiment on a smooth, even surface. Also discard readings where the initial Height of release of the ball from the phone is more than 5% of the height chosen. (For example, if height chosen is 1m, keep readings only where the initial height is recorded between 95-105 cm).

7. Repeat the process 5 to 10 times.

8. Fix a value of n, where n is the numbers of bounces recorded by PhyPhox minus one.

9. Use H1, Ho, T(n) to calculate acceleration due to gravity(g), and coefficient of restitution(e) and record all of them in a table with each readings range of error values.

6 Observation tables, Error Analysis

n	3	3	3	3	3
h_0(m)	1.5089 ± 0.0001	1.5355 ± 0.0001	1.5174 ± 0.0001	1.4793 ± 1.4402	1.4402 ± 0.0001
h_1(m)	0.9254 ± 0.0001	0.9323 ± 0.0001	0.9288 ± 0.0001	0.9249 ± 0.0001	0.9055 ± 0.0001
e = h_1 / h_0	0.6133 ± 0.0001	0.6072 ± 0.0001	0.6121 ± 0.0001	0.6252 ± 0.0001	0.6287 ± 0.0001
$t_n{}^*$(s)	2.097 ± 0.001	2.099 ± 0.001	2.095 ± 0.001	2.098 ± 0.001	2.086 ± 0.001
g(m/s^2)	9.6683 ± 0.0008	9.6404 ± 0.0011	9.7065 ± 0.0009	9.8117 ± 0.0002	9.7628 ± 0.0002

IMT2020065 Shridhar Sharma

$$\text{Average coefficient of restitution} = \bar{e} = \frac{\sum_{i=1}^{n} e_i}{n} = 0.6173$$

$$\text{Average least count of g} = \bar{\Delta g} = 0.0006$$

$$\text{Average acceleration due to gravity} = \bar{g} = \frac{\sum_{i=1}^{n} g_i}{n} = 9.7179 \pm 0.0006\ m/s^2$$

$$\text{Acceleration due to gravity as measured by PhyPhox} = g(correct) = 9.75\ m/s^2$$

n	3	3	3	3	3
h_0(m)	1.005 ± 0.0001	0.981 ± 0.0001	1.025 ± 0.0001	1.01 ± 0.0001	1.014 ± 0.0001
h_1(m)	0.592 ± 0.0001	0.583 ± 0.0001	0.595 ± 0.0001	0.535 ± 0.0001	0.592 ± 0.0001
e = h_1 / h_0	0.588 ± 0.0001	0.594 ± 0.0001	0.581 ± 0.0001	0.581 ± 0.0001	0.584 ± 0.0001
t_n^*(s)	1.64 ± 0.001	1.63 ± 0.001	1.64 ± 0.001	1.63 ± 0.001	1.63 ± 0.001
g(m/s^2)	9.71 ± 0.017	9.72 ± 0.017	9.70 ± 0.017	9.88 ± 0.017	9.77 ± 0.017

IMT2020553 Abhinav Mahajan

$$\text{Average coefficient of restitution} = \bar{e} = \frac{\sum_{i=1}^{n} e_i}{n} = 0.586$$

$$\text{Average least count of g} = \bar{\Delta g} = 0.0017$$

$$\text{Average acceleration due to gravity} = \bar{g} = \frac{\sum_{i=1}^{n} g_i}{n} = 9.75 \pm 0.0017\ m/s^2$$

$$\text{Acceleration due to gravity as measured by PhyPhox} = g(correct) = 9.80\ m/s^2$$

n	3	3	3	3	3
h_0(m)	1.466 ± 0.0001	1.482 ± 0.0001	1.488 ± 0.0001	1.527 ± 0.0001	1.485 ± 0.0001
h_1(m)	0.842 ± 0.0001	0.856 ± 0.0001	0.852 ± 0.0001	0.874 ± 0.0001	0.850 ± 0.0001
e = h_1 / h_0	0.574 ± 0.0001	0.574 ± 0.0001	0.572 ± 0.0001	0.572 ± 0.0001	0.572 ± 0.0001
t_n^*(s)	1.94 ± 0.001	1.96 ± 0.001	1.94 ± 0.001	1.97 ± 0.001	1.950 ± 0.001
g(m/s^2)	9.71 ± 0.014	9.73 ± 0.014	9.74 ± 0.014	9.71 ± 0.014	9.71 ± 0.014

IMT2020539 Shaurya Agrawal

$$\text{Average coefficient of restitution} = \bar{e} = \frac{\sum_{i=1}^{n} e_i}{n} = 0.57$$

$$\text{Average least count of g} = \bar{\Delta g} = 0.0014$$

$$\text{Average acceleration due to gravity} = \bar{g} = \frac{\sum_{i=1}^{n} g_i}{n} = 9.72 \pm 0.0014\ m/s^2$$

$$\text{Acceleration due to gravity as measured by PhyPhox} = g(correct) = 9.77\ m/s^2$$

n	3	3	3	3	3
h_0(m)	1.444 ± 0.0001	1.573 ± 0.0001	1.448 ± 0.0001	1.470 ± 0.0001	1.450 ± 0.0001
h_1(m)	0.919 ± 0.0001	0.936 ± 0.0001	0.931 ± 0.0001	0.943 ± 0.0001	0.937 ± 0.0001
e = h_1 / h_0	0.6363 ± 0.0113	0.5953 ± 0.0110	0.6436 ± 0.0114	0.6412 ± 0.0116	0.6416 ± 0.0112
t_n^*(s)	2.128 ± 0.001	2.111 ± 0.001	2.157 ± 0.001	2.150 ± 0.001	2.127 ± 0.001
g(m/s^2)	9.61 ± 0.894	9.41 ± 0.884	9.58 ± 0.879	9.73 ± 0.895	9.69 ± 0.893

IMT2020085 Harshadeep Donapati

$$\text{Average coefficient of restitution} = \bar{e} = \frac{\sum_{i=1}^{n} e_i}{n} = 0.6316$$
$$\text{Average least count of g} = \bar{\Delta g} = 0.883$$
$$\text{Average acceleration due to gravity} = \bar{g} = \frac{\sum_{i=1}^{n} g_i}{n} = 9.604 \pm 0.883\ m/s^2$$
$$\text{Acceleration due to gravity as measured by PhyPhox} = g(correct) = 9.67\ m/s^2$$

n	3	3	3	3	3
h_0(m)	1 ± 0.0001	1 ± 0.0001	1 ± 0.0001	1.5 ± 0.0001	1.5 ± 0.0001
h_1(m)	0.558 ± 0.0001	0.562 ± 0.0001	0.560 ± 0.0001	0.541 ± 0.0001	0.526 ± 0.0001
e = h_1 / h_0	0.558 ± 0.00016	0.562 ± 0.00015	0.56 ± 0.00016	0.5413 ± 0.0001	0.526 ± 0.0001
t_n^*(s)	1.563 ± 0.003	1.607 ± 0.003	1.597 ± 0.003	1.901 ± 0.003	1.93 ± 0.003
g(m/s^2)	9.71 ± 0.027	9.30 ± 0.024	9.36 ± 0.025	9.32 ± 0.023	8.85 ± 0.023

IMT2020126 Ayushmaan Singh

$$\text{Average coefficient of restitution} = \bar{e} = \frac{\sum_{i=1}^{n} e_i}{n} = 0.55$$
$$\text{Average least count of g} = \bar{\Delta g} = 0.024$$
$$\text{Average acceleration due to gravity} = \bar{g} = \frac{\sum_{i=1}^{n} g_i}{n} = 9.22 \pm 0.024\ m/s^2$$
$$\text{Acceleration due to gravity as measured by PhyPhox} = g(correct) = 9.34\ m/s^2$$

n	3	3	3	3	3
h_0(m)	1.1768 ± 0.0001	1.1338 ± 0.0001	1.1625 ± 0.0001	1.1458 ± 0.0001	1.1307 ± 0.0001
h_1(m)	0.6868 ± 0.0001	0.6262 ± 0.0001	0.6538 ± 0.0001	0.6441 ± 0.0001	0.6171 ± 0.0001
e = h_1 / h_0	0.584 ± 0.0001	0.552 ± 0.0001	0.570 ± 0.0001	0.562 ± 0.0001	0.563 ± 0.0001
t_n^*(s)	1.731 ± 0.001	1.606 ± 0.001	1.721 ± 0.001	1.635 ± 0.001	1.625 ± 0.001
g(m/s^2)	9.76 ± 0.0002	9.86 ± 0.0002	9.74 ± 0.0002	9.82 ± 0.0002	9.76 ± 0.0002

IMT2020057 Vishnutha Sheela

$$\text{Average coefficient of restitution} = \bar{e} = \frac{\sum_{i=1}^{n} e_i}{n} = 0.5629$$
$$\text{Average least count of g} = \bar{\Delta g} = 0.0002$$
$$\text{Average acceleration due to gravity} = \bar{g} = \frac{\sum_{i=1}^{n} g_i}{n} = 9.7886 \pm 0.0002\ m/s^2$$
$$\text{Acceleration due to gravity as measured by PhyPhox} = g(correct) = 9.71\ m/s^2$$

7 Sources of error

7.1 Instrumental errors

This occurs when the instruments used have faulty scaling and the measurements taken get distorted by the equipment. In this experiment, care has been taken to avoid these errors and measurements have been cross checked with multiple instruments. The measuring scale and tape enforce each other's accuracy, and it has been made sure that they don't provide conflicting measurements.

7.2 Environmental errors

Inconsistencies in the surroundings may affect the workings of the experiment, resulting in errors in the readings. We have performed the procedures in a setup where the effects of such external causes is minimised. Effects of windy atmosphere, temperature fluctuations and inconsistent bouncing of the ball, all have been avoided to the best of our abilities. The resulting outcomes can be considered fairly independent of such changes in the environment.

7.3 Procedural errors

These occur when different procedures are used to obtain the same final result. Since neither procedure can be perfectly accurate, slightly different answers are obtained by following and comparing both. This error has been avoided completely by all members sticking to a single procedure.

7.4 Human errors

These are caused by the limitations of the human ability, and are of two types.

7.4.1 Transcriptional errors

They occur when data is incorrectly recorded. The prevention of such errors relies solely on the attention of the person carrying out the procedures and recording data. All members have put down the data attentively to the best of their ability, and all calculations done in computerised systems have been carried out without mistakes. Parallax error has been avoided by taking all scale measurements as close to the target as possible, and reading the scale from a perpendicular angle.

7.4.2 Estimation errors

These occur in reading recorded values, like on a scale. Efforts have been made to keep the measurements consistent between all recordings.

Most of the avoidable sources of errors have been considered to reduce the final amount of error in the calculations. Least count errors stemming from the resolution of the scale and stopwatch have been accounted for in the calculations too.