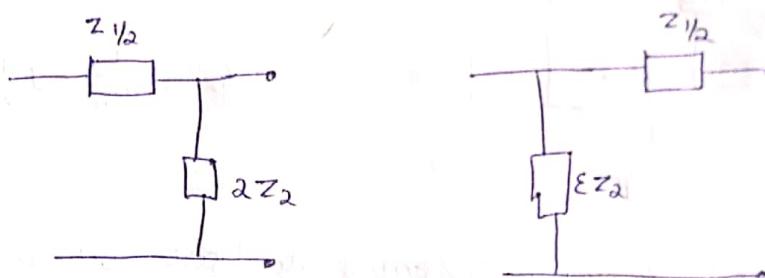
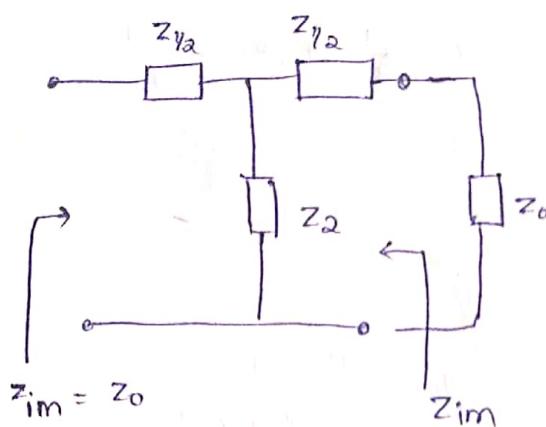


29-10-19

FILTER DESIGN

After Mid - 2



→ On cascading these two, we get a  $\pi$  type circuit.

Now from the initial figure, we can write

$$z_{in} = z_{1/2} + z_2 \parallel [z_0 + z_{1/2}]$$

$$\therefore z_{in} = \frac{z_2 [z_0 + z_{1/2}] + z_{1/2} [z_0 + z_2 + z_{1/2}]}{z_0 + z_2 + z_{1/2}}$$

$$\text{on solving keep } z_{in} = z_0, \text{ we get } z_0 = \sqrt{\frac{z_1^2}{4} + z_1 z_2}$$

$$\text{Also, } z_{in} \Big|_{z_2=\infty} = z_{oc} = \cancel{\frac{z_1}{2}} + z_2$$

$$z_{in} \Big|_{z_2=0} = z_{sc} = z_{1/2} + [z_{1/2} \parallel z_2]$$

$$\begin{aligned} z_{oc} z_{sc} &= z_0^2 \\ &= \frac{z_1^2}{4} + z_1 z_2 \quad \rightarrow ① \end{aligned}$$

Propagation constant (gamma)

$$\gamma = \ln \left[ \frac{I_1}{I_2} \right]$$

here,  $I_1$  = input current

$I_2$  = response current

$\gamma$  = proportional const

on applying KCL, we get

$$\frac{z_1}{2} I_2 + z_0 I_2 + (I_2 - I_1) z_2 = 0$$

$$\underbrace{\frac{I_1}{I_2}}_{e^\gamma} = \frac{z_1}{2 z_2} + \frac{z_0}{z_2} + 1$$

$$\Rightarrow z_2 \cdot e^\gamma = \frac{z_1}{2} + z_2 + z_0$$

$$z_0^2 = [z_2 (e^\gamma - 1) - \frac{z_1}{2}]^2 \rightarrow ②$$

on solving ① and ②, we get

$$0 = z_2 [e^\gamma - 1]^2 - z_1 e^\gamma$$

$$\cosh \gamma = 1 + \frac{z_1}{2 z_2}$$

it can also be written as

$$\sinh \gamma = \sqrt{\cosh^2 \gamma - 1}$$

$$\tan h \gamma = \frac{z_0}{z_2 + z_1/2} = \frac{z_0}{z_{oc}}$$

$$\sinh \left( \frac{\gamma}{2} \right) = \sqrt{\frac{\cosh \gamma - 1}{2}}$$

$$\Rightarrow \sqrt{\frac{z_1}{4 z_2}} = \sinh \left( \frac{\gamma}{2} \right)$$

In the side formula,

→  $z_1$  and  $z_2$  are imaginary

$$\text{if } \frac{z_1}{4z_2} = \text{real}$$

$$z = \alpha + j\beta$$

$$\text{so, } \sqrt{\frac{z_1}{4z_2}} = \sinh\left(\frac{\alpha}{2}\right) \cos\left(\frac{\beta}{2}\right) + j \sin\left(\frac{\beta}{2}\right) \cosh\left(\frac{\alpha}{2}\right)$$

case-A

$$\Rightarrow z_1 = jx_1, \quad z_2 = jx_2$$

here,  $z_1$  and  $z_2$  are same type reactants.

$$\text{so, } \cosh\left(\frac{\alpha}{2}\right) \cdot \sin\left(\frac{\beta}{2}\right) = 0$$

$$\frac{\beta}{2} = 0, n\pi$$

$$\alpha = 2 \sinh^{-1} \sqrt{\frac{z_1}{4z_2}} \quad \left\{ \because \sinh\left(\frac{\beta}{2}\right) = \sqrt{\frac{z_1}{4z_2}} \right\}$$

$$\text{where, } \left| \frac{z_1}{4z_2} \right| > 0$$

case-B

$$\Rightarrow z_1 = jx_1, \quad z_2 = -jx_2$$

ii)  $\alpha = 0$

here,  $z_1$  and  $z_2$  are of diff. type reactants.

$$\text{so, } \sinh\left(\frac{\alpha}{2}\right) \cos\left(\frac{\beta}{2}\right) = 0$$

$$\sin\left(\frac{\beta}{2}\right) = \sqrt{\frac{z_1}{4z_2}}$$

$$\beta = 2 \sin^{-1} \sqrt{\frac{z_1}{4z_2}}$$

$$\text{where } -1 < \frac{z_1}{4z_2} < 0$$

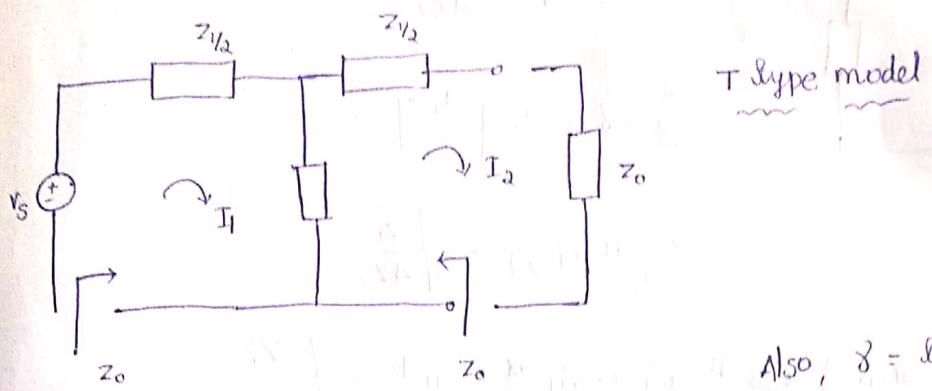
$$\text{iii) } \beta = \pi, \quad \cosh\left(\frac{\alpha}{2}\right) = \sqrt{\frac{z_1}{4z_2}}$$

$$\alpha = 2 \cosh^{-1} \sqrt{\frac{z_1}{4z_2}}$$

$$\text{where } -\infty < \frac{z_1}{4z_2} < -1$$

30-10-19

\* These filter designs are mostly prepared by these reactive elements



$$\text{Also, } \delta = \ln\left(\frac{L_1}{L_2}\right)$$

$$\text{WKT, } z_o = \sqrt{\frac{z_1^2}{4} + z_1 z_2} \Rightarrow \sinh\left(\frac{\delta}{2}\right) = \sqrt{\frac{z_1}{4z_2}}$$

$$= \sqrt{z_1 z_2} \sqrt{1 + \frac{z_1}{4z_2}}$$

$$= \sqrt{z_1 z_2} \left[ 1 + \frac{z_1}{4z_2} \right]$$

$$\text{So, } \sinh\frac{\alpha}{2} \cos\beta + j \cosh\frac{\alpha}{2} \sin\beta = \sqrt{\frac{z_1}{4z_2}}$$

$$z_1 = jx_1, z_2 = jx_2$$

case (A)

$$\frac{z_1}{4z_2} = \frac{x_1}{4x_2} > 0$$

at  $\beta = 0$

$$\sinh\frac{\alpha}{2} = \sqrt{\frac{z_1}{4z_2}}$$

$$\alpha = 2 \sinh^{-1} \sqrt{\frac{z_1}{4z_2}}$$

case (B)

$$z_1 = jx_1, z_2 = -jx_2$$

$$\frac{z_1}{4z_2} = \frac{jx_1}{-4jx_2} = -\frac{x_1}{4x_2}$$

If  $\alpha = 0$

$$\sin \frac{\beta}{2} = \sqrt{\frac{z_1}{4z_2}}$$

$$\beta = 2 \sin^{-1} \sqrt{\frac{z_1}{4z_2}}$$

$$-1 < \frac{z_1}{4z_2} < 0$$

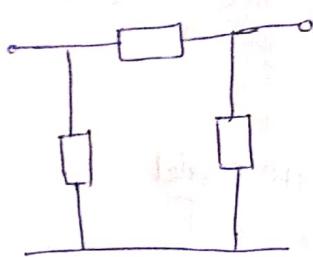
$\beta = \pi$

$$\cosh\frac{\alpha}{2} = \sqrt{\frac{z_1}{4z_2}}$$

$$\alpha = 2 \cosh^{-1} \sqrt{\frac{z_1}{4z_2}}$$

$$-\infty < \frac{z_1}{4z_2} < -1$$

For  $\pi$  type



$$z_0 = \frac{\sqrt{z_1 z_2}}{\sqrt{1 + \frac{z_1}{4z_2}}}$$

$$\gamma = \ln \left[ \frac{I_1}{I_2} \right]$$

$$\Rightarrow \sinh \frac{\gamma}{2} = \sqrt{\frac{z_1}{4z_2}}$$

$$\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} + j \cosh \frac{\alpha}{2} \cdot \sin \frac{\beta}{2} = \sqrt{\frac{z_1}{4z_2}}$$

Remaining all are just same like T type ..

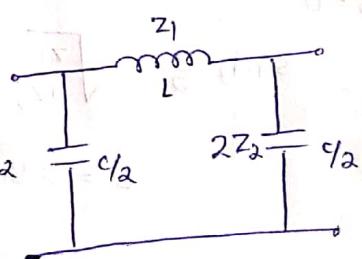
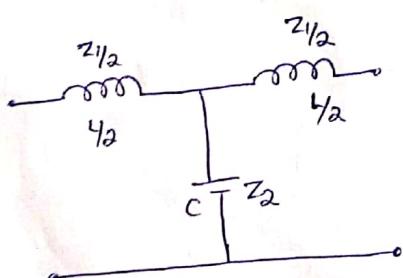
### K-type Filters

\* It is product of impedances

$$z_1 = j\omega L, z_2 = \frac{1}{j\omega C}$$

$$z_1 z_2 = \frac{L}{C} = K^2$$

$$K = \sqrt{\frac{L}{C}}$$



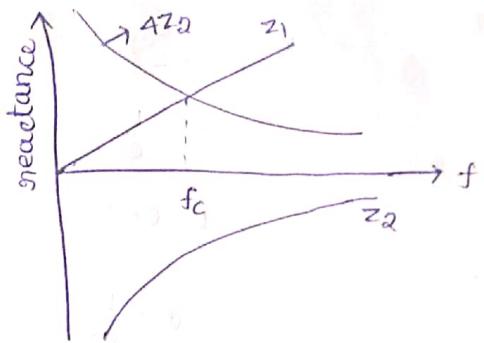
$$\frac{z_1}{4z_2} = 0, \omega = 0$$

$$z_1 = -4z_2$$

$$j\omega C L = \frac{-4}{j\omega L C}$$

$$\Rightarrow \omega^2 LC = 4 \Rightarrow f_c = \frac{1}{\pi \sqrt{LC}}$$

The graph can be drawn as

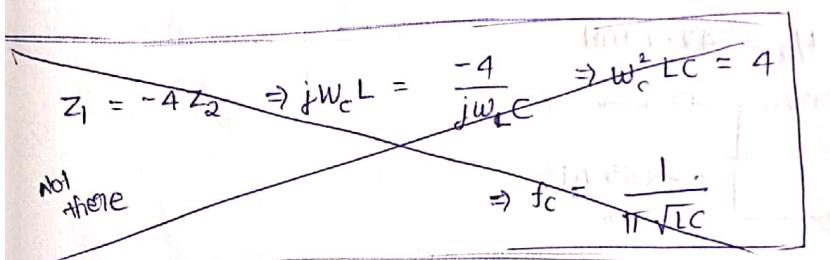


Now,

$$\frac{z_1}{4z_2} = -\frac{\omega^2 LC}{4} = -\frac{4\pi^2 f^2 LC}{4} = -\frac{f^2}{f_c^2}$$

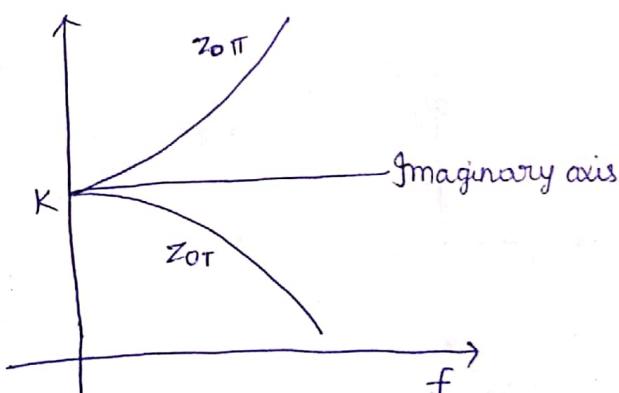
$$\text{So, } -1 \leq -\frac{f^2}{f_c^2} \leq 0 \rightarrow \alpha = 0; \beta = 2 \sin^{-1} \left[ \frac{f}{f_c} \right]$$

$$-\infty < -\frac{f^2}{f_c^2} < -1 \quad \boxed{\frac{f}{f_c} > 1} \rightarrow \beta = \pi; \alpha = 2 \cos^{-1} \left[ \frac{f}{f_c} \right]$$



$$Z_{OT} = \sqrt{z_1 z_2 \left[ 1 + \frac{z_1}{4z_2} \right]} = K \sqrt{\left[ 1 - \left( \frac{f}{f_c} \right)^2 \right]}$$

$$Z_{O\pi} = \frac{\sqrt{z_1 z_2}}{\sqrt{1 + \frac{z_1}{4z_2}}} = \frac{K}{\sqrt{1 - \left( \frac{f}{f_c} \right)^2}}$$



$$f_c^2 = \frac{1}{\pi^2 L C}$$

$$K = \sqrt{\frac{L}{C}}$$

$$f_c^2 = \frac{1}{\pi^2 K^2 C^2}$$

$$K^2 = \frac{L}{C}$$

$$C = \frac{1}{\pi f_c K}$$

$$L = C K^2$$

$$C = \frac{L}{K^2}$$

$$L = \frac{K}{\pi f_c}$$

$$f_c^2 = \frac{K^2}{\pi^2 L^2}$$

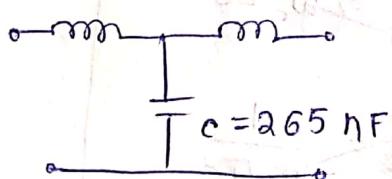
→ Using the given information build a high and low pass filters.

$$f_c = 2 \text{ kHz}$$

$$R_0 = K = 600 \Omega$$

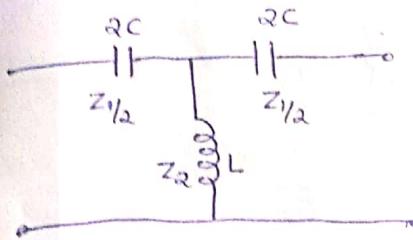
$$\text{on solving } L = 95.4 \text{ mH}$$

$$L_2 = 47.7 \text{ mH}$$



$$C = 265 \text{ nF}$$

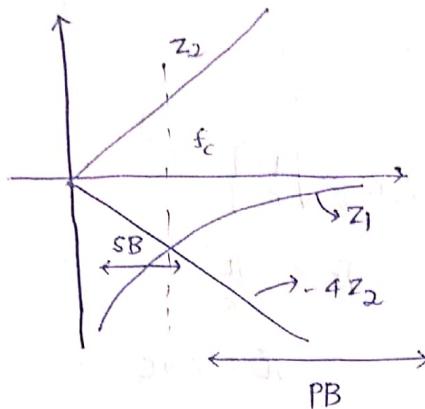
5-11-19



$$-1 < \frac{z_1}{4z_2} < 0$$

$$z_1 = 0 \quad \frac{1}{j\omega C} = 0$$

$$z_1 = -4z_2 \Rightarrow \frac{1}{j\omega C} = -4j\omega L$$



$$\frac{1}{4\omega^2} = \frac{1}{LC} \Rightarrow f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$z_1 = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

$$z_2 = j\omega L$$

WKT,  $z_0 = \sqrt{z_1 z_2 \left[ 1 + \frac{z_1}{4z_2} \right]}$

$$\sinh h(\beta/2) = \sqrt{\frac{z_1}{4z_2}}$$

$$\sinh h(\beta/2) = j \frac{f_c}{f}$$

$$\frac{z_1}{4z_2} = \frac{1}{j\omega C \cdot 4j\omega L} = -\frac{1}{4\omega^2 LC}$$

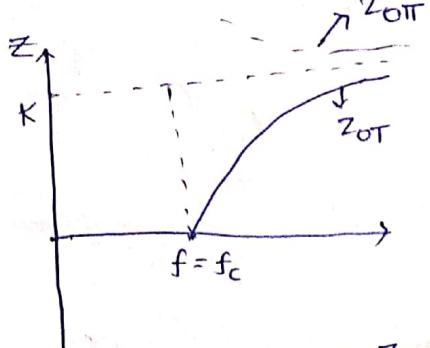
$$\frac{z_1}{4z_2} = -\frac{f_c^2}{f^2}$$

$$SB \Rightarrow \alpha = 2 \cosh^{-1} \left[ \frac{f_c}{f} \right], \beta = \pi$$

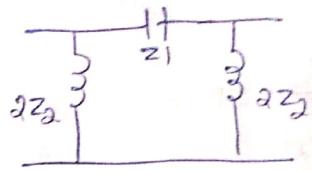
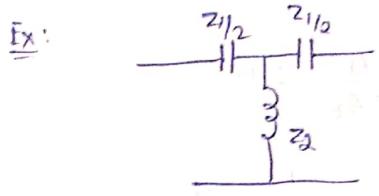
$$PB \Rightarrow \beta = 2 \sin^{-1} \left[ \frac{f_c}{f} \right], \alpha = 0$$

$$Z_{OT} = K \sqrt{1 + \frac{z_1}{4z_2}} = K \sqrt{1 - \left( \frac{f_c}{f} \right)^2}$$

$$Z_{OT} = \frac{K}{\sqrt{1 + \frac{z_1}{4z_2}}} = \frac{K}{\sqrt{1 - \left( \frac{f_c}{f} \right)^2}}$$



Graph betw Z\_{OT} & Z\_{OTT}



$$K = 600 \Omega$$

$$f_c = 1 \text{ kHz}$$

Ans

$$\text{Now, } f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_c = \frac{K}{4\pi L}$$

$$\therefore K = \sqrt{\frac{L}{C}}$$

$$L = \frac{K}{4\pi f_c}$$

$$\sqrt{C} = \frac{\sqrt{L}}{K}$$

$$\sqrt{L} = K \sqrt{C}$$

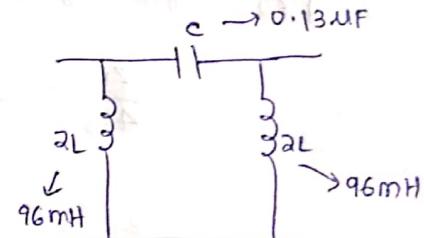
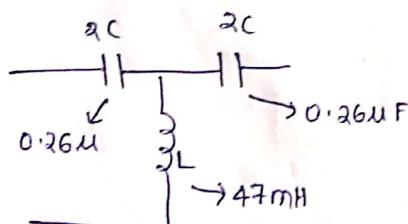
also,  $f_c = \frac{1}{4\pi K f_c}$

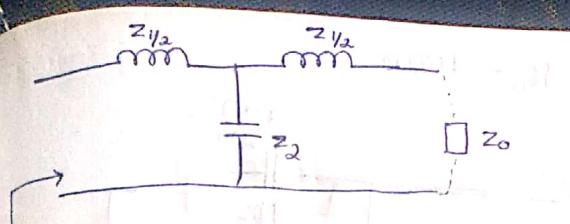
$$C = \frac{1}{4\pi K f_c}$$

on solving we get

$$L = 47 \text{ mH}$$

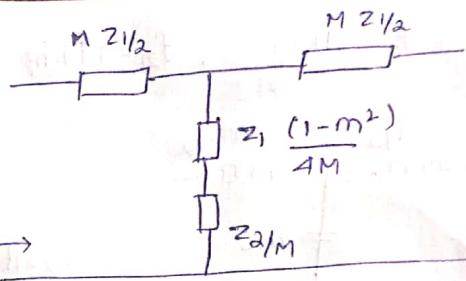
$$C = 0.13 \mu\text{F}$$





$$Z_0 = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$= \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

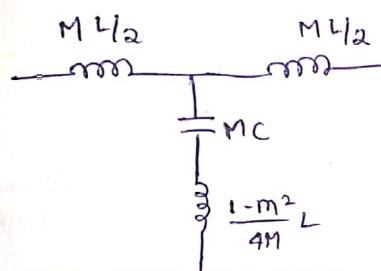


$$Z_0 = \sqrt{\frac{M^2 Z_1^2}{4} + M Z_1 Z_2}$$

$$Z_1^2 + Z_1 Z_2 = \frac{M^2 Z_1^2}{4} + M Z_1 Z_2$$

here  $Z_2' = \frac{Z_1}{4M} (1-M^2) + \frac{Z_2}{M}$

**LOW pass filter**



$$\frac{1}{M w_s C} = \frac{1 - M^2}{4M} w_s L$$

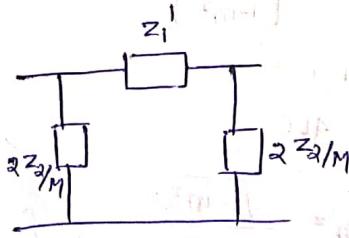
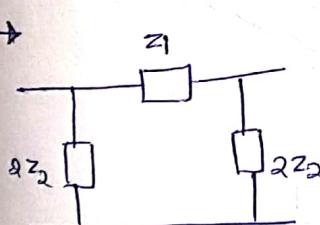
$$w_s^2 = \frac{4}{(1 - M^2) LC}$$

$$f_{\eta} = \frac{f_c}{\sqrt{1 - M^2}}$$

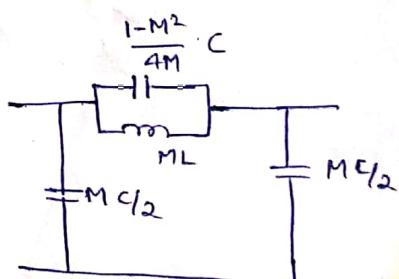
$$f_{\eta} = \frac{1}{\pi \sqrt{LC(1 - M^2)}}$$

$$\therefore M = \sqrt{1 - \left(\frac{f_c}{f_{\eta}}\right)^2}$$

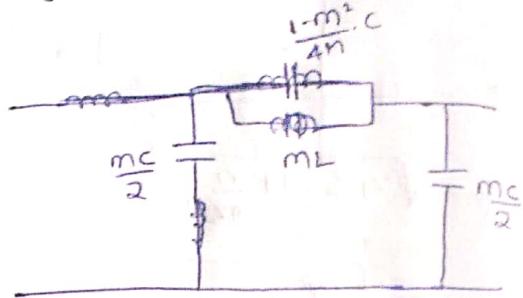
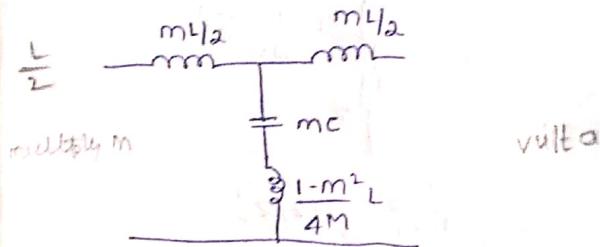
here  $f_{\eta}$  = attenuation



$$Z_1' = \frac{M Z_1 Z_2 \left[ \frac{4M}{1 - M^2} \right]}{M Z_1 + Z_2 \left[ \frac{4M}{1 - M^2} \right]}$$



$$Ex: f_c = 1 \text{ kHz}, f_n = 1.1 \text{ kHz}, R_o = 400 \Omega$$



We can use the same formulas to find L and C

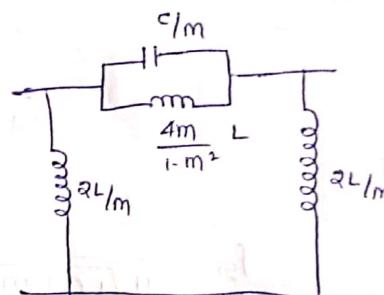
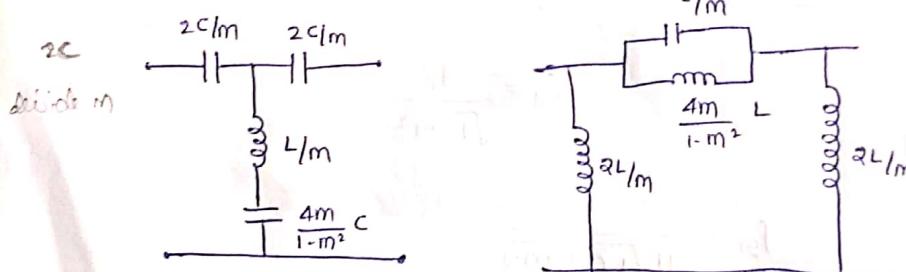
$$L = \frac{K}{2\pi f_c} \quad , \quad C = \frac{1}{2\pi f_c K} \quad \{ \text{take } K = R_o \}$$

$$\text{We get } L = 127 \text{ mH}$$

$$C = 0.79 \mu F$$

6-11-19

### High pass filter



$$\omega_n \frac{L}{M} = \frac{1}{\omega_n C \left[ \frac{4m}{1-m^2} \right]}$$

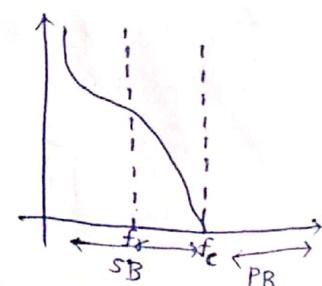
$$\omega_n^2 = \frac{1-m^2}{4LC}$$

$$\Rightarrow f_n = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}, \quad f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_n = f_c \sqrt{1-m^2}$$

$$\therefore M = \sqrt{1 - \left( \frac{f_n}{f_c} \right)^2}$$

$$f_n < f_c$$



Ex: Given  $f_c = 10 \text{ KHz}$

$$R_o = 5 \Omega$$

$$M = 0.4$$

AM

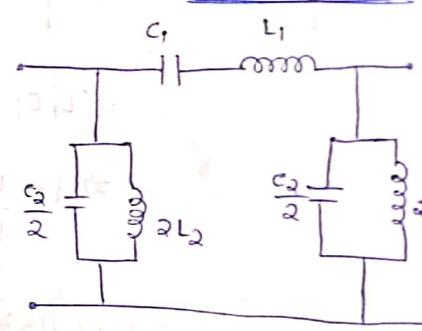
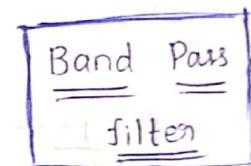
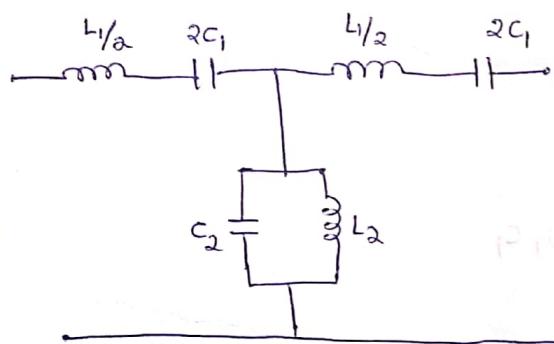
$$L = \frac{K}{4\pi f_c}, \quad C = \frac{1}{4\pi f_c K}$$

$$= \frac{5}{4\pi (10K)}$$

$$L = 39.8 \mu\text{H} \quad \text{and} \quad C = 1.59 \mu\text{F}$$

Now substitute these values in  $\frac{2C}{m}$ ,  $\frac{L}{m}$  and  $\frac{4m}{1-m^2} \cdot C$

Resonance of Series = Resonance of parallel



$$Z_1 = \left[ j\omega L_1 - \frac{j}{\omega C_1} \right] = j \left[ \frac{\omega^2 L_1 C_1 - 1}{\omega C_1} \right]$$

$$Z_2 = \frac{j\omega L_2 \cdot \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}} = \frac{j\omega L_2}{1 - \omega^2 L_2 C_2}$$

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = K^2$$

$$\omega_0 \frac{L_1}{2} = \frac{1}{2\omega_0 C_1}$$

$$\omega_0^2 = \frac{1}{L_1 C_1}$$

$$\omega_0^2 = \frac{1}{L_2 C_2}$$

$$L_1 C_1 = L_2 C_2 \Rightarrow \frac{L_1}{C_2} = \frac{L_2}{C_1}$$

$$-1 < \frac{z_1}{4z_2} < 0$$

$$z_1 = 0 \rightarrow \omega^2 = \frac{1}{L_1 L_2}$$

$$z_1 = \pm j\omega_0$$

$$\hookrightarrow z_1^2 = 4z_1 z_2 = -4K^2$$

$$z_1 w_1 = -z_1 w_2$$

$$\Rightarrow \omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\Rightarrow f_0 = \sqrt{f_1 f_2}$$

$$\left[ \frac{1}{j\omega_1 C_1} + j\omega_1 L_1 \right] = -2jK$$

$$1 - \omega_1^2 L_1 C_1 = 2K\omega_1 C_1$$

$$\Rightarrow 1 - \frac{\omega_1^2}{\omega_0^2} = 2K\omega_1 C_1$$

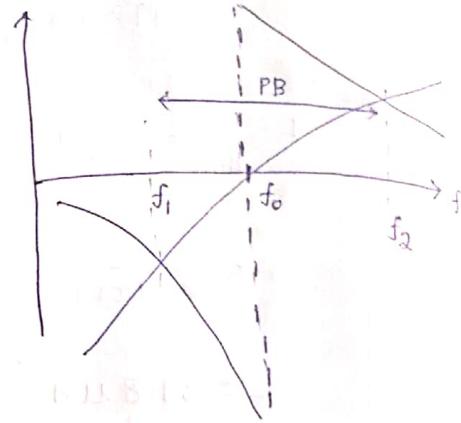
$$1 - \left( \frac{f_1}{f_0} \right)^2 = 2K\omega_1 C_1$$

$$C_1 = \frac{f_2 - f_1}{4\pi f_1 f_2}$$

$$\begin{aligned} L_1 &= \frac{1}{\omega_0^2 C_1} \\ &= \frac{4\pi K f_1 f_2}{\omega_0^2 (f_2 - f_1)} \Rightarrow L_1 = \frac{K}{\pi (f_2 - f_1)} \end{aligned}$$

$$L_2 = \frac{1}{\omega_0^2 C_2} = \frac{(f_2 - f_1) K}{4\pi f_1 f_2}$$

$$C_2 = \frac{L_1}{K^2} = \frac{1}{\pi (f_2 - f_1) K}$$



$$\text{Ex: } K = 500 \text{ v/V}$$

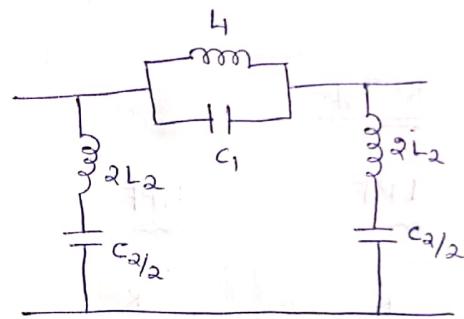
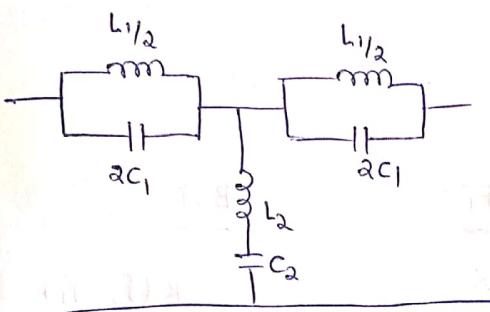
$$f_1 = 1 \text{ KHz}, f_2 = 10 \text{ KHz}$$

on solving we get

$$C_1 = 143 \text{ nF} \quad C_2 = 69.6 \text{ nF}$$

$$L_1 = 17.4 \text{ mH} \quad L_2 = 35.7 \text{ mH}$$

### Band Stop filter



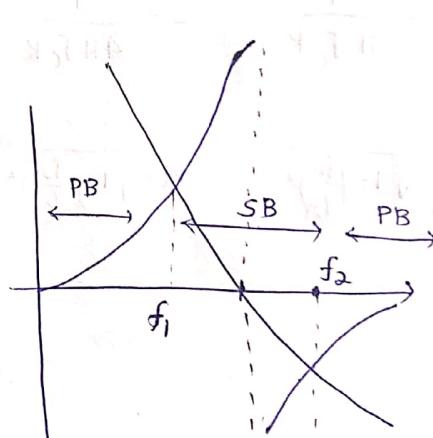
$$-1 < \frac{z_1}{4z_2} < 0$$

$$z_1 z_2 = K^2$$

$$L_1 C_1 = L_2 C_2$$

$$z_1 z_2 = \frac{L_1}{C_1} = \frac{L_2}{C_2} = K^2$$

$$z_2 - j \left[ \frac{1}{\omega_1 C_2} - \omega_1 L_2 \right] = j \frac{K}{2}$$



$$z_2 z_1 = -4z_2^2 = K^2$$

$$z_2 = \pm j \frac{K}{2}$$

$\left. \begin{array}{l} \text{Since series is more easier than parallel} \\ \text{to solve. So we considered} \\ z_2 \text{ other than } z_1 \end{array} \right\}$

$$C_2 = \frac{1}{K\pi} \left[ \frac{f_2 - f_1}{f_1 f_2} \right]$$

$$L_2 = \frac{1}{\omega_0^2 C_2} = \frac{K}{4\pi(f_2 - f_1)}$$

$$L_1 = C_2 K^2$$

$$= \frac{K}{\pi} \left[ \frac{f_2 - f_1}{f_1 f_2} \right]$$

$$C_1 = \frac{L_2}{K^2}$$

$$= \frac{1}{4\pi K(f_2 - f_1)}$$

$$\underline{\text{Ex:}} \quad K = 600\Omega$$

$$f_1 = 2\text{kHz}, f_2 = 6\text{kHz}$$

on calculating we get  $C_2 = 0.17\mu\text{F}$

$$L_2 = 11.93\text{ mH}$$

$$L_1 = 63.66\text{ mF}$$

$$C_1 = \cancel{0.33\text{nF}}$$

$$= 33\text{nF}$$

### K-Type

LPF



$$L = \frac{K}{\pi f_c}$$

HPF



$$L = \frac{K}{4\pi f_c}$$

BPF



$$L_1 = \frac{K}{\pi(f_2 - f_1)}$$

BSF



$$L_1 = \frac{K(f_2 - f_1)}{\pi f_1 f_2}$$

$$C = \frac{1}{\pi f_c K}$$

$$C = \frac{1}{4\pi f_c K}$$

$$C_2 = \frac{1}{\pi K(f_2 - f_1)}$$

$$C_1 = \frac{1}{4\pi K(f_2 - f_1)}$$

$$M = \sqrt{1 - \left(\frac{f_c}{f_2}\right)^2}$$

$$M = \sqrt{1 - \left(\frac{f_1}{f_c}\right)^2}$$

$$L_2 = \frac{K(f_2 - f_1)}{4\pi f_1 f_2}$$

$$L_2 = \frac{K}{4\pi(f_2 - f_1)}$$

$$C_1 = \frac{(f_2 - f_1)}{4\pi K f_1 f_2}$$

$$C_2 = \frac{1}{\pi K} \frac{(f_2 - f_1)}{f_1 f_2}$$

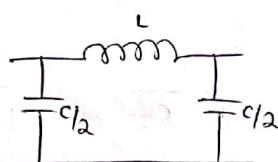
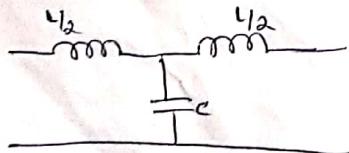
Ex: For an LPF, having  $f_c = 2\text{kHz}$ ,  $R_o = K = 600\Omega$  Find  $L, \alpha$

$$L = \frac{K}{\pi f_c} = 95.49\text{ mH}$$

$$\alpha = 2 \cos^{-1} \left( \frac{f}{f_c} \right)$$

$$= 3.143$$

Ex:



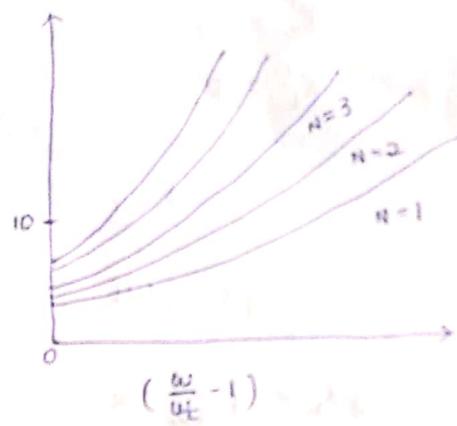
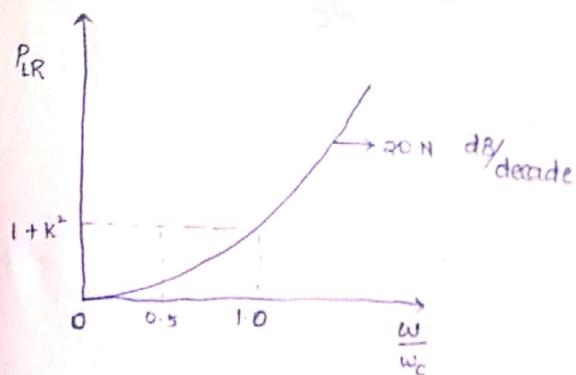
For an HPF,  $f_c = 10\text{kHz}$ ,  $K = 600\Omega$

find  $\alpha$  at  $f = 8\text{kHz}$

$$\begin{aligned} L &= 4.7\text{mH} \\ C &= 13.2\text{nF} \\ \alpha &= 1.38 \end{aligned}$$

12-11-19

$$\text{Power Loss Ratio, } P_{LR} = 1 + K^2 \left( \frac{\omega}{\omega_c} \right)^{2N}$$



Equal Ripple [Chebyshev]

$$P_{LR} = 1 + K^2 T_N^{2N} \left( \frac{\omega}{\omega_c} \right)^{2N}$$

$$-1 \leq T_N(x) \leq 1$$

$$P_{LR} = \frac{K^2}{4} \left( \frac{2\omega}{\omega_c} \right)^{2N}$$

$$T_N(x) = \frac{1}{2} (2x)^N \quad \omega > \omega_c$$

Ex: Take  $f_c = 2 \text{ GHz}$

$$Z_0 = 50 \Omega$$

15 dB @ 3 GHz

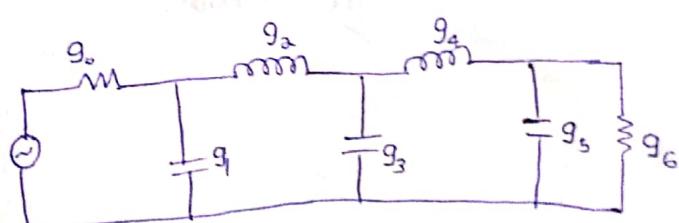
$$\rightarrow N = 5$$

$$\text{Also, } g_1 = 0.618 \quad g_4 = 1.618$$

$$g_2 = 1.618 \quad g_5 = 0.618$$

$$g_3 = 2.000$$

from the information given



Important substitutions are

$$L' = R_o L \Rightarrow R_o = 50 \Omega$$

$$C' = \frac{C}{R_o}$$

$$R_s' = R_o$$

$$R_L' = R_o R_L$$

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

f<sub>req</sub>    Scaling

$$jx = j\omega L$$

$$jx_k = j \frac{\omega}{\omega_c} L_k$$

$$= j\omega$$

$$jB_x = j \frac{\omega}{\omega_c} C_k = j\omega C'_k \quad \text{where } C'_k = \frac{C_k}{\omega_c}$$

$$L'_k = \frac{R_o L_k}{\omega_c}$$

$$C'_k = \frac{C_k}{R_o \omega_c}$$

$$\text{Now, } g_1 = c_1 \Rightarrow C'_1 = \frac{c_1}{2\pi(50)(2 \times 10^9)} = \frac{0.618}{50 \times 2 \times 10^9}$$

$$g_2 = L_2 \Rightarrow L'_2 = \frac{50 \times 1.618}{2\pi(2 \times 10^9)}$$

$$g_3 = c_3 \Rightarrow C'_3 = \frac{c_3}{50(2\pi)(2 \times 10^9)}$$

$$g_4 = L_4 \Rightarrow L'_4 = \frac{50 \times 1.618}{2\pi(2 \times 10^9)}$$

$$g_5 = c_5 \Rightarrow C'_5 = \frac{0.618}{2\pi(50)(2 \times 10^9)}$$

$$\text{Now, } \omega \leftarrow \frac{\omega_0}{\omega} - \frac{\omega_c}{\omega}$$

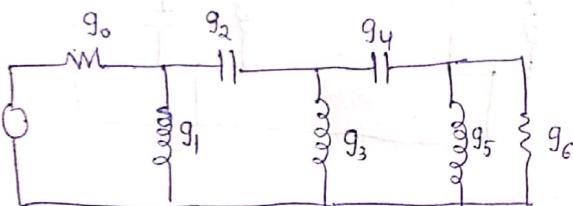
$$jX = j\omega L = -j \frac{\omega_c}{\omega} L = \frac{1}{j\omega c'}$$

$$c' = \frac{1}{\omega_c L R_o}$$

$$jB = j\omega C = -j \frac{\omega_c}{\omega} C = \frac{1}{j\omega L'}$$

$$L' = \frac{R_o}{\omega_c C}$$

The circuit can be drawn as



$$L' = \frac{50}{2\pi(2 \times 10^9)(0.618)}$$

$$C'_2 = \frac{1}{50(0.618)(2\pi \times 10^9)}$$

$$L'_3 = \frac{50}{2\pi \times 2 \times 10^9 \times 2}$$

$$C'_4 =$$

$$L'_5 =$$

$$\left[ \frac{1}{\omega_1} - \frac{1}{\omega_2} \right] \Delta$$

$$\Delta = \frac{\omega_a - \omega_1}{\omega_0}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\omega \leftarrow \frac{\omega_0}{(\omega_a - \omega_1)} \left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right] = \frac{1}{\Delta} \left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]$$

$$jX = j\omega L = j \frac{1}{\Delta} \left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right] L_K = j \frac{\omega L_K}{\Delta \omega_0} - j \frac{\omega_0 L_K}{\Delta \omega} \\ = j\omega L'_K + \frac{1}{j\omega c'_K}$$

$$L'_K = \frac{4kR_o}{\Delta \omega_0}$$

$$c'_K = \frac{\Delta}{\omega_0 L'_K}$$

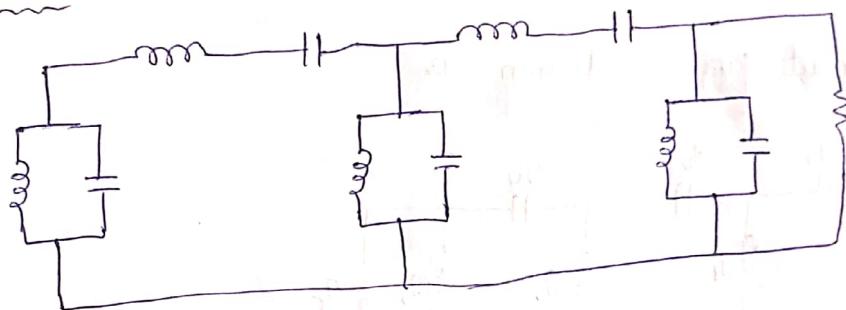
$$jB = j\omega c = j \frac{1}{\Delta} \left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right] c = \frac{j\omega c_K}{\Delta \omega_0} - j \frac{\omega_0 c_K}{\Delta \omega}$$

$$= j\omega c_K' - j \frac{1}{\omega L_K'}$$

$$\Rightarrow C_K' = \frac{c_K}{\omega_0 R_0 \Delta}$$

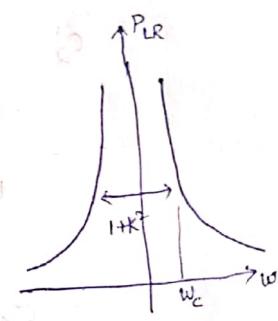
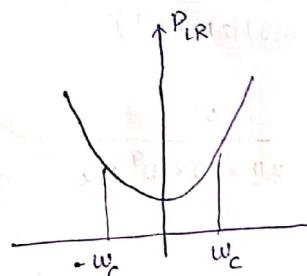
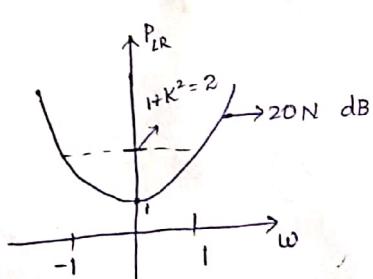
$$L_K' = \frac{\Delta R_0}{\omega_0 c_K}$$

circuit

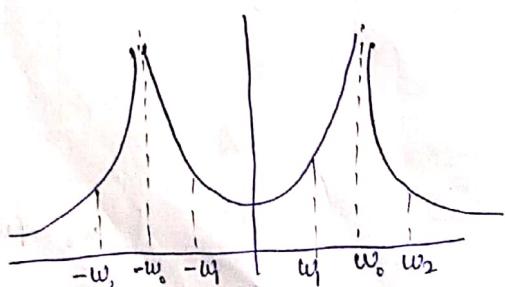


13-11-19

$$P_{LR} = 1 + K^2 \left[ \frac{\omega}{\omega_c} \right]^{2N} \rightarrow IL = 10 \cdot \log P_{LR}$$



$$\omega \leftarrow \Delta \left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]^{-1}$$



BSF:

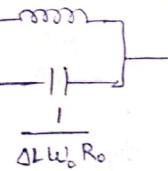
$$jX = j\omega L = j\Delta L$$

$$\left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]$$

$$\omega \leftarrow \Delta \left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]^{-1}$$

$$L = \frac{\Delta L R_0}{\omega_0}$$

$$\Rightarrow jB = j\omega \left[ \frac{1}{\Delta L \omega_0} \right] + \frac{1}{j\omega \left( \frac{\Delta L}{\omega_0} \right)}$$



$$jB = j\omega C = \frac{j\Delta C}{\left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]} \Rightarrow jX = \frac{j\omega}{\omega_0 \Delta C} + \frac{1}{j\omega \left( \frac{\Delta C}{\omega_0} \right)}$$

where  $L = \frac{R_0}{\omega_0 \Delta C}$

$$C = \frac{\Delta C}{\omega_0 R_0}$$

LPF  $\sim$

$$\frac{L}{\omega_c}$$

HPF  $\sim$

$$\frac{1}{T} \frac{1}{\omega_c L}$$

BPF  $\sim$

$$\frac{1}{T} \frac{L}{\Delta \omega_0}$$

BSF  $\sim$

$$\frac{1}{T} \frac{L \Delta}{\omega_0} \quad \frac{1}{\omega_0 L \Delta}$$

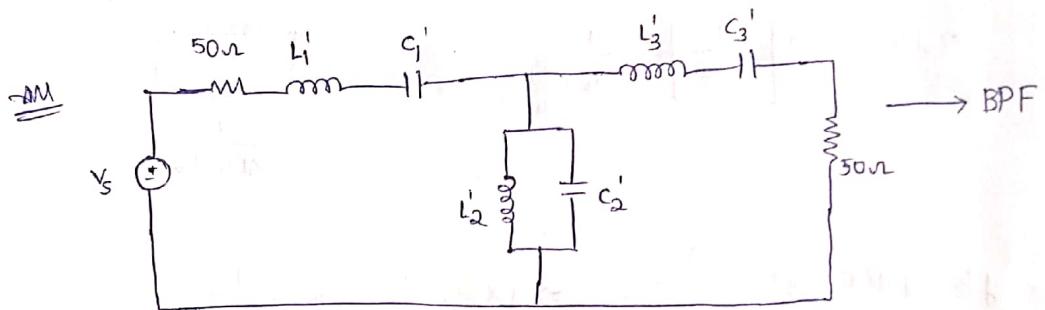
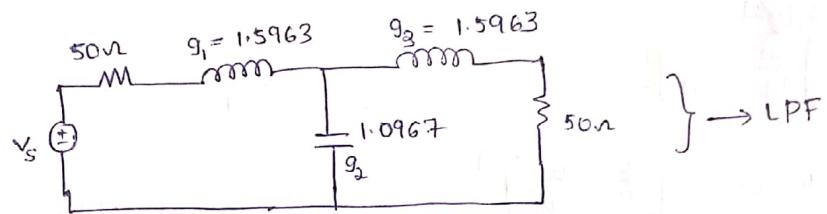
$$\frac{1}{T} \frac{C}{\omega_c}$$

$$\frac{1}{\omega_c C}$$

$$\frac{\Delta}{\omega_0 C} \quad \frac{1}{T} \frac{C}{\Delta \omega_0}$$

$$\frac{1}{\Delta \omega_0 C} \quad \frac{1}{T} \frac{\Delta C}{\omega_0}$$

Ex:  $R_o = 50\Omega$        $f_{GHz} = 1 GHz$        $\Delta = 10\%$       }  $Giga = 10^9$



Here  $\omega_o = 2\pi f_o$   
 $= 2\pi (1 GHz)$

This is BPF. So  $L' = \frac{LR_o}{\Delta\omega_o}$

$$= 0.127 \mu H$$

$$= L_3'$$

$$C_1' = 0.2 \text{ PF} = C_3' = \frac{\Delta}{\omega_o R_o}$$

} for inductor  $R_o$  will be at up  
and for capacitor  $R_o$  will be at down

$$L_2' = \frac{\Delta R_o}{\omega_o C}$$

$$= 0.725 nH$$

$$C_2' = 34 \text{ PF} = \frac{C}{\Delta\omega_o R_o}$$

Now for BSF

$$L_1' = \frac{L \Delta R_o}{\omega_o} = \frac{1.5963 (0.1) 50}{2\pi (10^9)} = 1.27 nH = L_3'$$

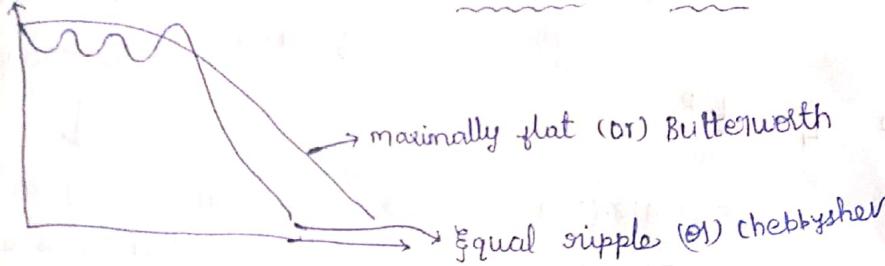
$$C_1' = \frac{1}{\Delta\omega_o L R_o} = 19.9 \text{ PF} = C_3'$$

$$L_2' = 72.5 nH = \frac{R_o}{\Delta\omega_o C}$$

$$C_2' = 3.45 \text{ PF} = \frac{C \Delta}{\omega_o R_o}$$

14-11-19

### Re-discussion of class



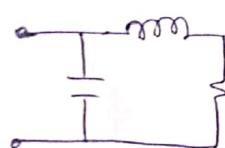
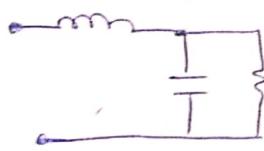
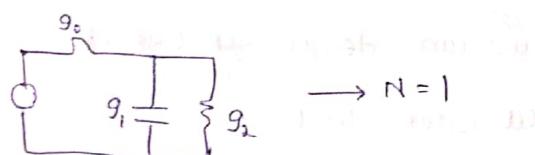
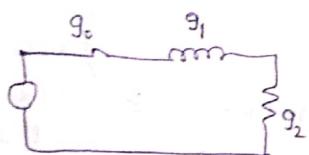
If  $N=1$

then we will have  $N+1$  elements

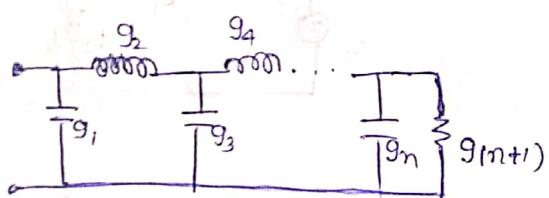
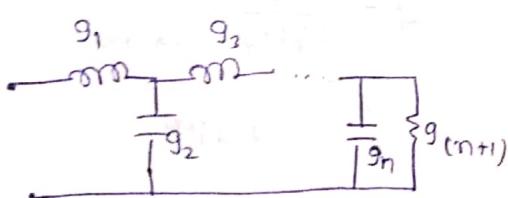
And the  $(N+1)$ th element will be load resistance.

If  $N=1$  and we have  $g_1$  and  $g_2$

then  $g_2$  is the  $(N+1)$ th element



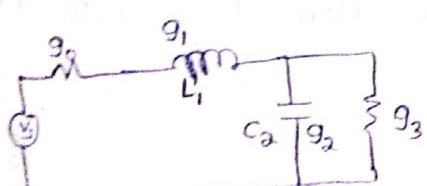
$\rightarrow N=2$



If  $N=n$

here  $n$  may be odd or even

→ let's take an example of  $N=2$



Important

$$L' = L R_o = g_1 R_o$$

$$C' = \frac{C}{R_o} = \frac{g_2}{R_o}$$

$$R'_o = g_1 R_o$$

$$R'_L = g_3 R_o$$

Now for the side example lets take

$$g_1 = 1.414$$

$$g_2 = 1.414$$

$$g_3 = 1.0$$

$$\left. \begin{array}{l} \\ \end{array} \right\} N=2$$

$$f_C = 1 \text{ GHz}$$

$$R_o = 50 \Omega$$

LPF

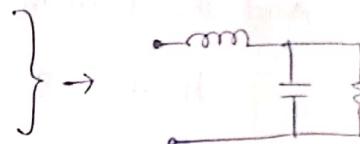
$$\text{so, now } L_1^1 = \frac{L_1 R_o}{w_c}$$

$$= \frac{1.414(50)}{2\pi \times 10^9}$$

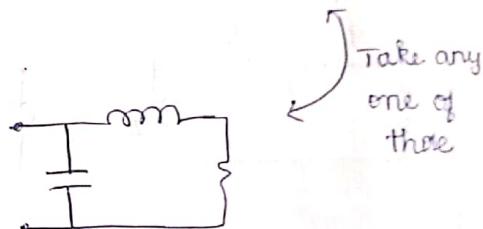
$$= 11.25 \text{ nH}$$

$$C_2^1 = \frac{g_2}{R_o w_c} = \frac{1.414}{50(2\pi \times 10^9)}$$

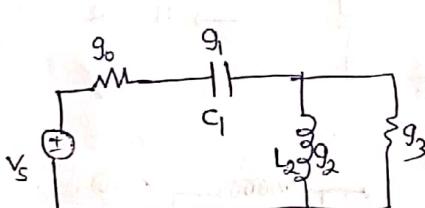
$$= 4.4 \text{ pF}$$



Now we can design for case of capacitor first



HPF



$$C_1^1 = \frac{1}{w_c L R_o} = \frac{1}{(2\pi \times 10^9)(1.414)(50)}$$

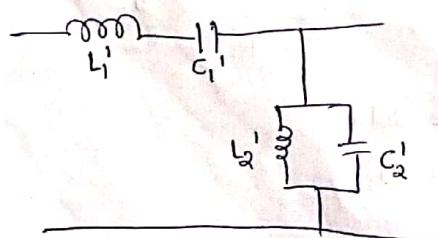
$$= 2.2 \text{ pF}$$

$$L_2^1 = \frac{R_o}{w_c C} = \frac{50}{2\pi \times 10^9 (1.414)} = 5.63 \times 10^{-9}$$

$$= 5.63 \text{ nH}$$

BPF

here we will show a single element as both inductor and capacitor.



$$L_2' = \frac{\Delta R_0}{w_0 c_2}$$

$$= \frac{0.1(50)}{2\pi \times 10^9 \times 1.414} = 563 \text{ PH}$$

$$C_2 = \frac{C_0}{\Delta w_0 R_0} = \frac{1.414}{(0.11727 \times 10^9) \times 50} = 45 \text{ PF}$$

BSF

Same like BPF we do it but by replacing some of the elements.

19-11-19

$$P(S) = \{ s^4 + 6s^3 + 2s^2 + s + 1 \} \text{ Divide this into odd and even powers.}$$

$$M(s) = s^4 + 2s^2 + 1$$

$$N(s) = 6s^3 + s$$

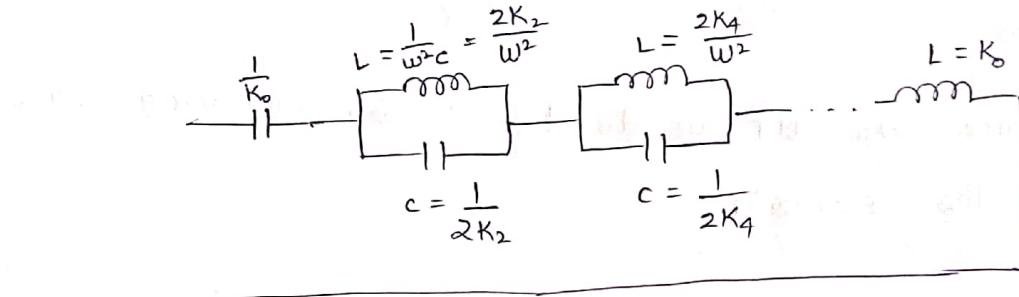
$$\begin{aligned}
 & \text{Now, } 6s^3 + s) \left( s^4 + 2s^2 + 1 \right) \left( \frac{s}{6} \right. \\
 & \quad \left. \overline{s^4 + \frac{s^2}{6}} \right) \\
 & \quad \left( \frac{11}{6}s^2 + 1 \right) 6s^3 + s \left( \frac{36}{11}s \right. \\
 & \quad \left. \overline{6s^3 + \frac{36s}{11}} \right) \\
 & \quad \left( -\frac{25}{11}s \right) \frac{11}{6}s^2 + 1 \left( -\frac{121}{150}s \right. \\
 & \quad \left. \overline{\frac{11}{6}s^2} \right) \\
 & \quad \left( -\frac{25}{11}s \right) \left( -\frac{25}{11}s \right)
 \end{aligned}$$

Since on obtaining 0 as remainder we got  $-\frac{25}{11}$ s as a quotient which is -ve.

$\therefore$  The given  $P(s)$  is not a Horwitz polynomial.

## Foster's 1<sup>st</sup> method

$$Z(s) = \frac{K_0}{s} + \frac{2K_2 s}{s^2 + \omega_2^2} + \frac{2K_4 s}{s^2 + \omega_4^2} \dots + \frac{K_\infty s}{s^2 + \omega_\infty^2}$$

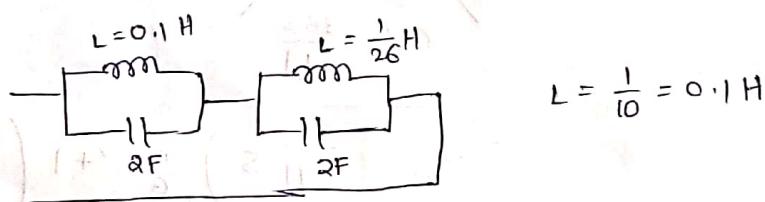


$$\text{Now, } Z(s) = \frac{s(s^2 + 9)}{(s^2 + 5)(s^2 + 13)}$$

$$= \frac{2K_2 s}{(s^2 + 5)} + \frac{2K_4 s}{(s^2 + 13)}$$

$$2K_2 s = [(s^2 + 5) Z(s)] \Big|_{s^2 = -5}$$

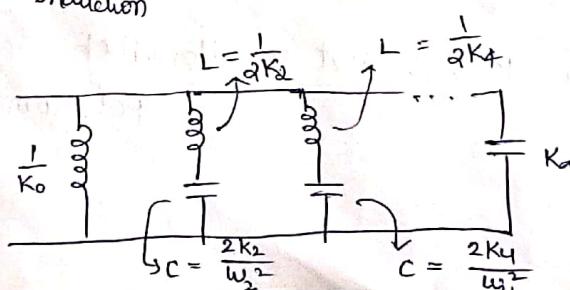
$$2K_2 s = \frac{s(s^2 + 9)}{s^2 + 13} = \frac{4}{8} = \frac{1}{2}$$



## Foster's 2<sup>nd</sup> method

$$Y(s) = \frac{K_0}{s} + \frac{2K_2 s}{s^2 + \omega_2^2} + \dots + \frac{2K_4 s}{s^2 + \omega_4^2} + \dots + \frac{K_\infty s}{s^2 + \omega_\infty^2}$$

Induction



$$\text{Now, } y(s) = \frac{(s^2+5)(s^2+13)}{s(s^2+9)}$$

here we want to do one division as the numerator power is greater than denominator.

$$\begin{array}{r} s^3 + 9s \\ \hline s^4 + 18s^2 + 65 \end{array}$$

$$\begin{array}{r} s^4 + 9s^2 \\ \hline 9s^2 + 65 \end{array}$$

$$\text{So, } y(s) = s + \frac{9s^2 + 65}{s^3 + 9s}$$

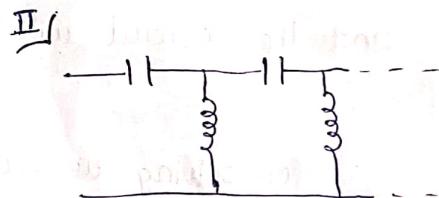
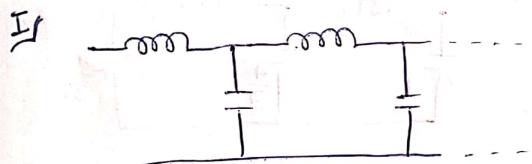
$$= s + \frac{k_0}{s} + \frac{2k_2 s}{s^2 + 9} \Rightarrow 2k_2 = \frac{16}{9}$$

$$k_0 = \frac{65}{9}$$

20-11-19

### Cauer's form

There are 2 forms in it.



Ex

$$Z(s) = \frac{s(s^2+4)(s^2+6)}{(s^2+3)(s^2+5)}$$

I form

$$= \frac{s^5 + 10s^3 + 24s}{s^4 + 8s^2 + 15}$$

here numerator power is bigger than denominator.

$$\text{So, } s^4 + 8s^2 + 15 \) s^5 + 10s^3 + 24s \rightarrow Z_1$$

$$\frac{s^5 + 8s^3 + 15s}{2s^3 + 9s}$$

$$\frac{2s^3 + 9s}{s^4 + 8s^2 + 15} \) s^2 \rightarrow Y_1$$

$$\frac{s^4 + \frac{9}{2}s^2}{\frac{7}{2}s^2 + 15}$$

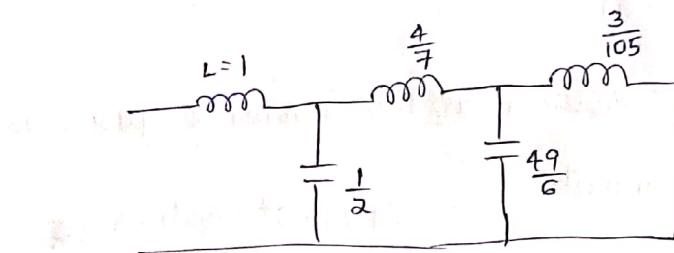
$$\frac{\frac{7}{2}s^2 + 15}{2s^3 + 9s} \) \frac{4}{7}s \rightarrow Z_2$$

$$\frac{2s^3 + \frac{60}{7}s}{\frac{3}{7}s}$$

$$\frac{\frac{3}{7}s}{\frac{7}{2}s^2 + 15} \) \frac{49}{6}s \rightarrow Y_2$$

$$\frac{\frac{7}{2}s^2}{15} \) \frac{3}{7}s \) (\frac{3}{165}s \rightarrow Z_3$$

Now the circuit will be based on those quotients



as the question is  $Z(s)$   
1st quotient is  $Z_1$ , then  
so 1st inductor than  
capacitor

(Ex.)

$$Z(s) = \frac{(s^2+3)(s^2+5)}{s(s^2+4)(s^2+6)}$$

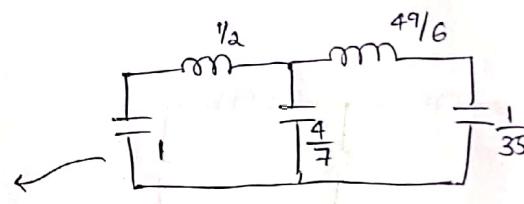
here denominator has more power than numerator

so reverse it so,  $\gamma(s) = \frac{s(s^2+4)(s^2+6)}{(s^2+3)(s^2+5)}$

Now do the same process as earlier

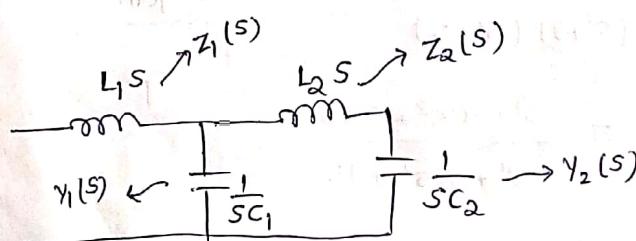
And the 1st quotient is considered as  $Y_1$  than  $Z_1$

and the circuit will be



on solving we can  
get those values.

Formula:



we can get the real eqn. by

$$Z(s) = Z_1(s) + \frac{1}{\frac{1}{Y_1(s)} + \frac{1}{Z_2(s) + \frac{1}{Y_2(s)}}}$$

power of num < denominator

$$Y(s) = \frac{(s^2 + 3)(s^2 + 5)}{s(s^2 + 4)(s^2 + 6)}$$

II form  
~~asym~~

$$= \frac{s^4 + 8s^2 + 15}{s^5 + 10s^3 + 24s} = \frac{15 + 8s^2 + s^4}{24s + 10s^3 + s^5}$$

$$24s + 10s^3 + s^5) \left( 15 + 8s^2 + s^4 \right) \left( \frac{15}{24s} \right) \longrightarrow y_1$$

$$\frac{15}{24} + \frac{150}{24} S^4$$

$$\left( \frac{7}{4}S^2 + \frac{3}{8}S^4 \right) 24S + 10S^3 + S^5 \quad \left( \frac{576}{42S} \right) \rightsquigarrow z_1$$

$$\frac{24S + \frac{36}{7}S^3}{7}$$

$$\frac{34}{7} s^3 + s^5 \quad ) \frac{7}{4} s^2 + \frac{3}{8} s^4 \quad \overline{\underline{48965}}$$

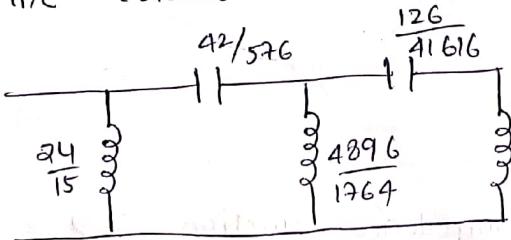
$$\frac{\frac{3}{4}S^2 + \frac{49}{13G}S^4}{\frac{1}{68}S^4} \left( \frac{34}{7} S^3 + S^5 \right) \left( \frac{41616}{12G5} \right)$$

on solving we get

$$Y_2 = \frac{1764}{48965}, \quad \nearrow \frac{49}{1365}$$

$$y_3 = \frac{72}{48965} \quad | \quad \begin{array}{r} 5^5 \\ \hline 68 \\ \hline 0 \end{array}$$

So the circuit is

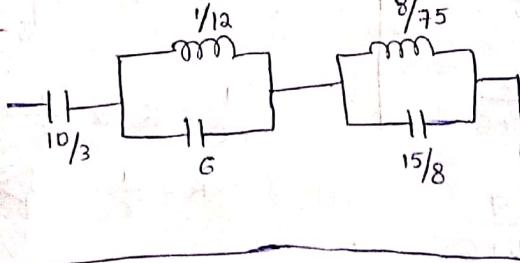


### Example

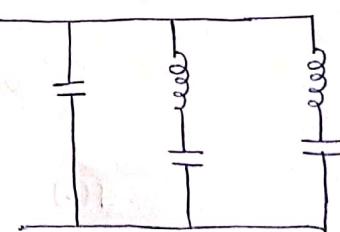
$$Z(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)(s^2+5)}$$

write all the foster forms and Cauer forms.

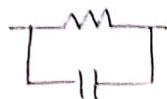
F-X



F-II



RC circuit



→ Impedance

RL circuit

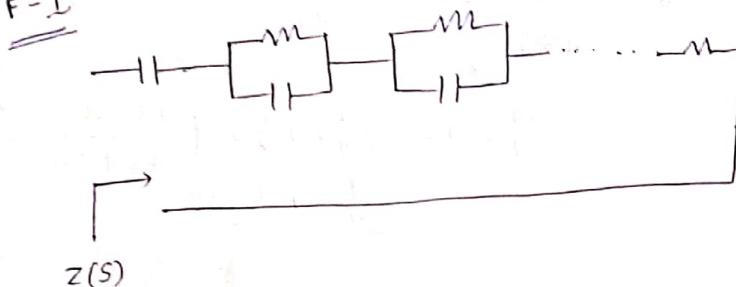


→ Admittance

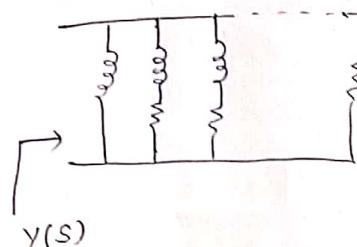
$$\{RC\} \quad Z(s) = \frac{Z_1(s) \cdot Z_2(s)}{Z_1(s) + Z_2(s)} = \frac{\frac{R}{sC}}{R + \frac{1}{sC}} = \frac{R}{sRC + 1} = \frac{\frac{1}{C}}{s + \frac{1}{RC}}$$

$$\{RL\} \quad Y(s) = \frac{1}{sL + R} = \frac{\frac{1}{L}}{s + \frac{R}{L}} \quad \text{↔ } \frac{a}{s+b}$$

F-I



F-II



\* If  $|z(0)| > |z(\infty)|$ , then we have to follow F-I also  
the power of num  $\leq$  power of denom in faster terms

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$$\text{Ex: } F(s) = \frac{6(s+3)(s+9)}{s(s+6)}$$

consider it as a impedance function

Also it is satisfying the above 2 conditions. (\*)

partial fraction

$$\frac{s^2+6s}{s^2+6s} \quad \frac{6s^2+72s+162}{6s^2+36s}$$

$$\therefore F(s) = 6 + \frac{36s+162}{s^2+6s}$$

$$= 6 + \frac{A}{s} + \frac{B}{s+6}$$

$$\therefore F(s) = 6 + \frac{27}{s} + \frac{9}{s+6} \quad \frac{a}{s+b} = \frac{\frac{1}{C}}{s + \frac{1}{RC}}$$

$$A = s \left( \frac{36s+162}{s(s+6)} \right) \Big|_{s=0} = 27$$

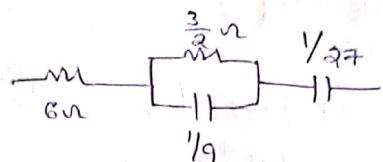
$$B = (s+6) \left( \frac{36s+162}{s(s+6)} \right) \Big|_{s=-6} = 9$$

$$\frac{1}{RC} = 6 \quad \& \quad \frac{1}{C} = 9$$

$$R = \frac{1}{6C}$$

$$R = \frac{9}{6} = \frac{3}{2}$$

$\therefore$  Foster I form is

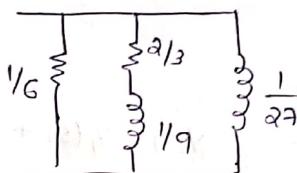


NOW consider the example as Admittance function

$$\text{So, } Y(s) = 6 + \frac{27}{s} + \frac{9}{s+6}$$

$\therefore$  Foster II form is

circuit is

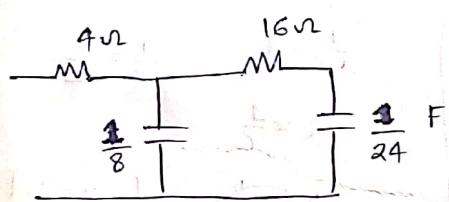


Ex:  $F(s) = \frac{4(s+1)(s+3)}{s(s+2)}$

first we have to remove pole at infinity

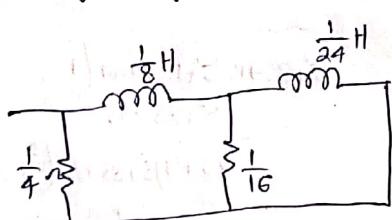
$$F(s) = \frac{4s^2 + 16s + 12}{s^2 + 2s}$$

Circuit is Caueray I form



$$\begin{aligned}
 & s^2 + 2s \quad 4s^2 + 16s + 12 \quad z_1(s) \\
 & \frac{4s^2 + 8s + 0}{8s + 12} \quad s^2 + 8s + 0 \quad z_2(s) \\
 & \frac{8s + 12}{s^2 + 12s} \quad 8s + 12 \quad z_3(s) \\
 & \frac{4s}{8s} \quad 4s \quad z_4(s)
 \end{aligned}$$

another Caueray II form



here the capacitor values wont be changed.

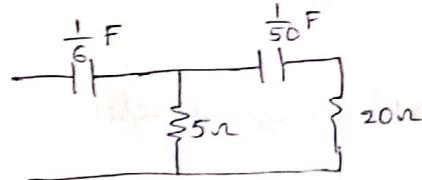
Caueray form II

We write the function in ascending order

$$\text{i.e. } F(s) = \frac{12 + 16s + 4s^2}{2s + s^2}$$

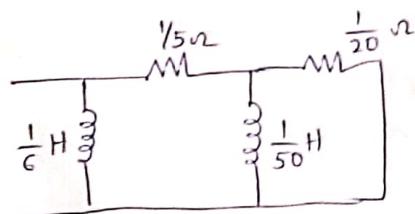
And the circuit is

as Impedance func



If we consider it as a Admittance func

the circuit is



$$\frac{2s+5^2}{12+16s} \left( \frac{6}{5} \rightarrow Z_1(s) \right)$$

$$\frac{2s+5^2}{10s+45^2} \left( \frac{1}{5} \rightarrow Y_1(s) \right)$$

$$2s + \frac{45^2}{5}$$

$$\frac{1/5 s^2}{1/5 s^2} \left( \frac{50}{5} \rightarrow Z_2(s) \right)$$

$$10s$$

$$4s^2 \left( \frac{1}{5} s^2 \left( \frac{1}{20} \right) \right)$$

$$\frac{s^2}{5}$$

$$Y_2(s)$$

$$0$$

$$Y(s) = Y_1(s) + \frac{1}{Z_1(s) + \frac{1}{Y_2(s) + \frac{1}{Z_2(s)}}}$$

Ex:

$$Z(s) = \frac{(s+4)(s+6)}{(s+3)(s+5)}$$

find Foster I and Caley I

$$Z(s) = \frac{s^2 + 10s + 24}{s^2 + 8s + 15}$$

$$= 1 + \frac{2s+9}{s^2 + 8s + 15}$$

circuit is

$$= 1 + \frac{A}{s+3} + \frac{B}{s+5}$$

$$= 1 + \frac{3/2}{s+3} + \frac{1/2}{s+5}$$

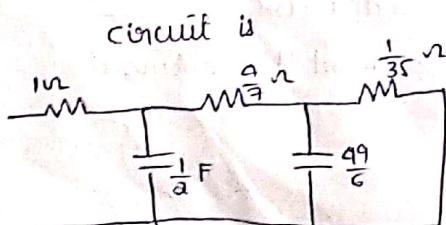
$$= \frac{(s+4)(s+6)}{s^2 + 8s + 15} \left( \frac{1}{s^2 + 8s + 15} \right)$$

$$\frac{1}{RC} = 3 \quad \frac{1}{C} = \frac{3}{2}$$

$$R = \frac{3}{C}$$

$$\left\{ \therefore B = \frac{1}{2} \right\}$$

Now lets do caley forms



$$\frac{s^2 + 8s + 15}{s^2 + 10s + 24} \left( \frac{1}{s^2 + 10s + 24} \right)$$

$$\frac{2s+9}{2s+9} \left( \frac{9}{2} \right)$$

$$\frac{s^2 + 9s}{\frac{7}{2}s + 15} \left( \frac{4}{7} \right)$$

$$\frac{2s + 60}{2s + 60} \left( \frac{40}{7} \right)$$

$$\frac{\frac{3}{7}s}{\frac{3}{7}s} \left( \frac{40}{7} \right)$$

$$\frac{\frac{7}{2}s}{15} \left( \frac{40}{7} \right)$$

$$\frac{\frac{3}{2}}{15} \left( \frac{40}{7} \right)$$

$$0$$

$$Z(s) = \frac{R}{s+R} + \frac{R}{s+R} \cdot \frac{\alpha s}{s+R}$$

$$\therefore Y(s) = \frac{1}{s+R} + \frac{\alpha s}{s+R} = \frac{s+R}{s+R} = 1$$

$$Y(s) = \frac{1}{s+R} + \frac{\alpha s}{s+R} = \frac{s+R}{s+R} = 1$$

$$Z(s) = \frac{R}{s+R}$$

The condition is  $Z(\infty) > Z(0)$

$$F(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

Foster form I

$$F(s) = \frac{s^2 + 4s + 3}{s^2 + 6s + 8}$$

on expanding this like before we will get negative terms so we should do

$$\frac{F(s)}{s} = \frac{(s+1)(s+3)}{s(s+2)(s+4)}$$

Now we will get +ve terms

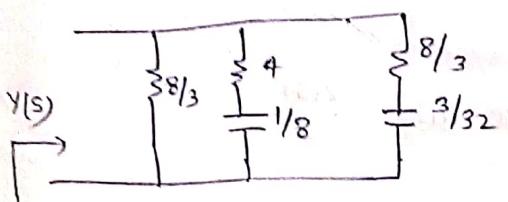
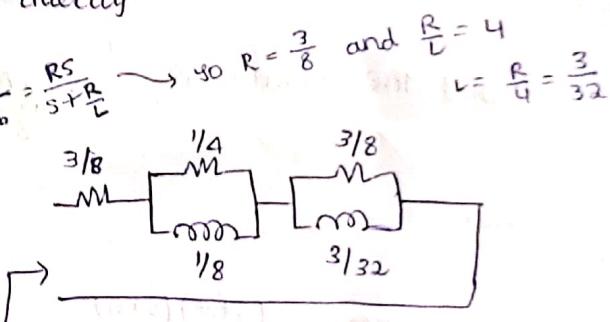
$$\frac{F(s)}{s} = \frac{s^2 + 4s + 3}{s^3 + 6s^2 + 8s}$$

apply partial fraction directly.

$$\frac{s^3 + 6s^2 + 8s}{s^2 + 6s + 8} = s^2 + 4s + 3 \left( \frac{1}{s} \right)$$

on solving, we get

$$F(s) = \frac{3}{8} + \frac{(1/4)s}{s+2} + \frac{(3/8)s}{s+4}$$

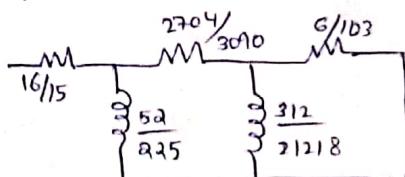


Cauchy form I

$$Z_{RC}(s) = \frac{2(s+2)(s+4)}{(s+3)(s+5)} = \frac{2s^2 + 12s + 16}{s^2 + 8s + 15}$$

Here we will get coefficients as -ve. So we use II form

$$\therefore Z(s) = \frac{16 + 12s + 2s^2}{15 + 8s + s^2}$$



$$\text{Ex: } F(s) = \frac{3(s+3)(s+5)}{(s+4)(s+6)} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ write faster from I and II}$$

We have to see which is suitable whether  $R_L$  or  $R_C$ .

$$\text{If } z(0) > z(\infty) \rightarrow R_C$$

$$z(\infty) > z(0) \rightarrow R_L$$

$$\text{Here we got } z(0) = \frac{15}{8} \text{ and } z(\infty) = \frac{3}{4}$$

so we consider  $R_L$  circuit.

i.e.   $\Rightarrow \frac{z(s)}{s} = \frac{R}{s + \frac{R}{L}}$

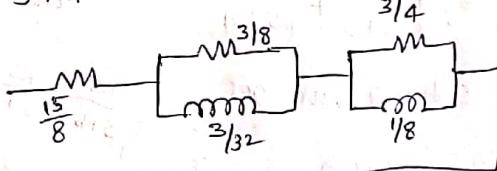
We get -ve terms. So we applied  $\frac{z(s)}{s}$

$$\Rightarrow \frac{z(s)}{s} = \frac{3(s+3)(s+5)}{s(s+4)(s+6)}$$

$$\frac{z(s)}{s} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+6} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ direct partial fraction}$$

$$\therefore z(s) = \frac{15}{8} + \frac{\left(\frac{3}{8}\right)s}{s+4} + \frac{\left(\frac{3}{4}\right)s}{s+6}$$

and the circuit is



$$\text{Ex: } F(s) = \frac{(s+4)(s+6)}{(s+5)(s+7)}$$

we will get -ve values. so take  $\frac{z(s)}{s}$

$$\frac{z(s)}{s} = \frac{A}{s} + \frac{B}{s+5} + \frac{C}{s+7}$$

$$\therefore z(s) = \frac{24}{35} + \frac{\left(\frac{110}{35}\right)s}{s+5} + \frac{\left(\frac{3}{14}\right)s}{s+7}$$

Q A parallel RLC network with

$$R = 5 \Omega, L = 100 \text{ mH}, C = 1 \text{ mF}$$

a) Compute Quality factor (Q)

b) Determine at which freq the impedance magnitude drops to 90% of its max value

② A parallel RLC circuit with  $L = 5 \text{ mH}$

$$Q_0 = 6.5, \omega_0 = 1000 \text{ rad/sec}$$

Determine approximate values of input impedance at

a) 500 b) 750 c) 900 rad/sec

1-ans Quality factor (Q) =  $R \sqrt{\frac{C}{L}}$  {for parallel}

$$\text{a) } Q = 5 \sqrt{\frac{1 \times 10^{-3}}{100 \times 10^{-3}}} = 0.5$$

$$= \frac{5}{10} \Rightarrow 0.5$$

b)  $Z_{\max} = (0.9) R$   
 $= 4.5 \Omega$

$$Y(w) = \frac{1}{Z_{\max}} = \frac{1}{4.5}$$

$$|Y(w)| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega e^{-\frac{1}{Rw}}\right)^2}$$

$$\left|\frac{1}{4.5}\right|^2 = \frac{1}{20.25} + \left[\omega 10^3 \cdot \frac{1}{\omega + 10^3}\right]^2$$

on solving, we get

$$\omega_1 = 62.7 \text{ rad/sec}$$

$$\omega_2 = 159.5 \text{ rad/sec}$$

$$\text{Ans} \quad |Y| = \sqrt{\frac{1}{R^2} + \left(\omega_C - \frac{1}{\omega_L}\right)^2}$$

$$= \sqrt{\frac{1}{R^2} + \left(\frac{\omega Q_0}{R w_0} - \frac{w_0 Q_0}{\omega R}\right)^2}$$

$$= \sqrt{\frac{1}{R^2} + \frac{Q_0^2}{R^2} \left[\frac{\omega}{w_0} - \frac{w_0}{\omega}\right]^2}$$

$$Y(w) = \frac{1}{R} \sqrt{1 + Q_0^2 \left(\frac{\omega}{w_0} - \frac{w_0}{\omega}\right)^2}$$

$$R = Q_0 w_0 L$$

$$R = 32.5 \Omega$$

$$Y(500) = \frac{1}{32.5} \sqrt{1 + (6.5)^2 \left[ \frac{500}{1000} - \frac{1000}{500} \right]}$$

$$= 0.3016, \quad Z_{(500)} = 3.3156 \Omega$$

$$Y(750) = 0.0551, \quad Z_{(750)} = \cancel{18.15} \Omega$$

$$Y(900) = 0.0349, \quad Z_{(900)} = 28.65 \Omega$$

③ RLC circuit is connected in series with

$$R = 5 \Omega, L = 20 \text{ mH}, C = 1 \text{ mF}$$

Calculate  $Q_0$ , bandwidth, mag of impedance at  $0.95 w_0$

④ If the freq is 40, 80 rad/sec find parallel equiv of series combination of an resistor and

a) 100 mF capacitor

b) 3 mH inductor

$$3-\underline{\text{ans}} \quad Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}} \quad \{ \text{for series} \}$$

↳ Just inverse of parallel

$$\therefore Q_0 = 0.894$$

$$L = \frac{QR}{\omega_0}, \quad C = \frac{1}{Q\omega_0 R}$$

Here we have L and C. So,  $\omega_0 = \frac{1}{\sqrt{LC}}$

= 223.713

$$\text{Also, } Z = \sqrt{R^2 + \left( \frac{QR\omega}{\omega_0} - \frac{Q\omega_0 R}{\omega} \right)^2} \\ = R \sqrt{1 + Q^2 \left( 0.95 - \frac{1}{0.95} \right)^2}$$

$$|Z| = 5\Omega$$

$$\beta = \frac{\omega_0}{Q_0}$$

$$= \frac{R}{L} \Rightarrow 250$$

↓  
bandwidth

7-11-19

Tutorial

① BPF

Given,  $f_1 = 2.5 \text{ K}$

also,  $K = 650 \mu$

$$f_2 = 5.5 \text{ K}$$

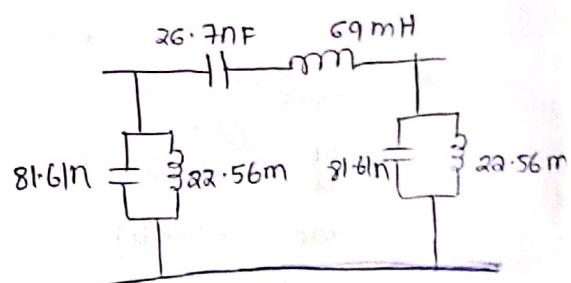
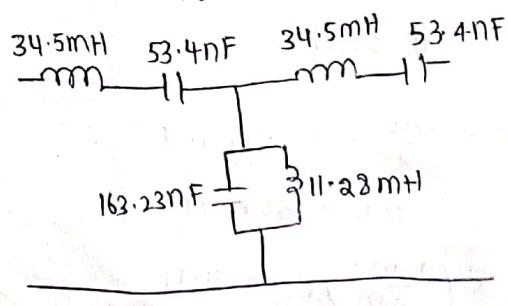
$$\text{Now, } L_1 = \frac{K}{\pi(f_2 - f_1)} = \frac{650}{\pi(3\text{K})} \Rightarrow 69 \text{ mH}$$

$$C_2 = \frac{1}{\pi K(f_2 - f_1)} = \frac{1}{\pi 650 (3\text{K})} \Rightarrow 163.23 \text{nF}$$

$$L_2 = \frac{K(f_2 - f_1)}{4\pi f_1 f_2} = \frac{650 (3\text{K})}{4\pi (13.75 \text{K}^2)} \Rightarrow 11.28 \text{ mH}$$

$$C_1 = \frac{(f_2 - f_1)}{4\pi K f_1 f_2} = \frac{3\text{K}}{4\pi (650) (13.75 \text{K}^2)} \Rightarrow 26.7 \text{nF}$$

Designing circuit



② BSF

Given  $f_1 = 2.7 \text{ K}$  also  $K = 650 \mu\Omega$   
 $f_2 = 5.7 \text{ K}$

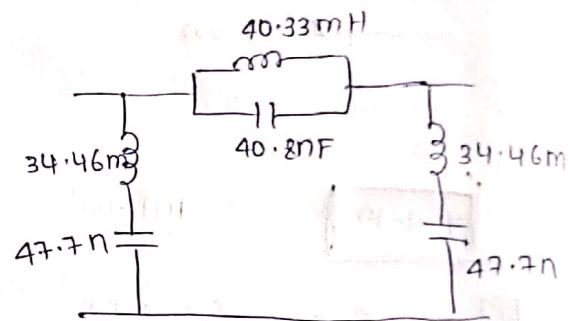
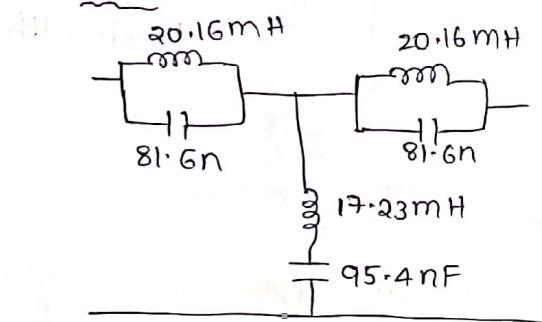
$$\text{Now, } L_1 = \frac{K(f_2 - f_1)}{\pi f_1 f_2} = \frac{650(3K)}{\pi(15.39K^2)} = 40.33 \text{ mH}$$

$$C_1 = \frac{1}{4\pi K(f_2 - f_1)} = \frac{1}{4\pi(650)(3K)} = 40.8 \text{ nF}$$

$$L_2 = C_1 K^2 = (40.8n)(422.5K) = 17.23 \text{ mH}$$

$$C_2 = \frac{L_1}{K^2} = \frac{40.33 \text{ m}}{422.5 \text{ K}} = 95.4 \text{ nF}$$

Circuit



③

LPF

Given,  $R_o = 550 \mu\Omega$ ,  $f_c = 1.7 \text{ K Hz}$

and  $f_{r2} = 2.2 \text{ K}$   $f_2 \rightarrow$  attenuation

$$\text{Now, } L = \frac{K}{\pi f_c} = \frac{550}{\pi(1.7K)} = 102.9 \text{ mH}$$

$$C = \frac{1}{\pi f_c K} = \frac{1}{\pi(1.7K)(550)} = 0.34 \mu\text{H}$$

$$M = \sqrt{1 - \left(\frac{f_c}{f_r}\right)^2} = \sqrt{1 - \left(\frac{2.2K}{1.7K}\right)^2} = 0.638$$

T-section

$$\frac{mL}{2} = 32.68 \text{ mH}$$

$$mc = 0.21 \mu\text{F}$$

$$\left[\frac{1-m^2}{4m}\right]L = 24.6 \text{ mH}$$

π-section

$$\frac{mc}{2} = 0.107 \mu\text{F}$$

$$mL = 65 \text{ mH}$$

$$\left(\frac{1-m^2}{4m}\right)c = 79 \text{ nF}$$

④ HPF

Given  $R_o = 650 \Omega$

$$f_c = 5 \text{ kHz} \quad f_{\eta} = 3.8 \text{ kHz}$$

$$\text{Now, } L = \frac{K}{4\pi f_c} = \frac{650}{4\pi(5k)} = 10.3 \text{ mH}$$

$$C = \frac{1}{4\pi f_c K} = \frac{1}{4\pi(5k)(650)} = 24 \text{ nF}$$

$$M = \sqrt{1 - \left(\frac{f_{\eta}}{f_c}\right)^2} = \sqrt{1 - \left(\frac{3.8k}{5k}\right)^2} = 0.649$$

T-section

$$\frac{2C}{m} = 73.95 \text{ nF}$$

$$\frac{L}{m} = 15.87 \text{ mH}$$

$$\left(\frac{4m}{1-m^2}\right) C = 107.3 \text{ mH}$$

π-section

$$\frac{2L}{M} = 31.74 \text{ mH}$$

$$\frac{C}{M} = 36.97 \text{ nF}$$

$$\left(\frac{4m}{1-m^2}\right) L = 46.19 \text{ mH}$$

12-11-19

Given  $N = 4$   
 $f_c = 3 \text{ kHz}$  and  $R_o = 75 \Omega$

$$\begin{aligned} g_1 &= 0.7654 & g_3 &= 1.8478 \\ g_2 &= 1.8478 & g_4 &= 0.7654 \end{aligned}$$

Design LPF and HPF

LPF

$$L' = \frac{LR_o}{w_c} = \frac{1.8478(75)}{2\pi \times 3 \times 10^3} = \frac{138.585 \times 10^{-9}}{18.849} = 7.35 \text{ nH}$$

$$C' = \frac{C}{R_o w_c} = \frac{0.7654}{75(2\pi \times 10^3)} = 1.6 \text{ PF}$$

HPF

$$C' = \frac{1}{w_c LR_o} = \frac{1}{(2\pi \times 10^3)(75)(0.7654)} = 2.7 \text{ PF}$$

$$L' = \frac{R_o}{w_c C} = \frac{75}{(2\pi \times 10^3)(1.6)} = 6.45 \text{ nH}$$