Numerical Study of Constant Velocity and Oscillatory Lid Driven Cavity flow

S Deekshit Reddy
ME15B060

T Sree Harsha
ME15B070

ABSTRACT

In this report we present a numerical study of constant velocity and oscillatory lid driven cavity flow. The flow was simulated for various Reynolds numbers. Finite volume method is used to discretize the governing equation and SIMPLE algorithm is used for pressure-velocity coupling. Upwind scheme is used to solve convection. The entire code is written in C++ .

Nomenclature

p Pressure
u velocity component of fluid in x-direction
v velocity component of fluid in y-direction
μ dynamic viscosity of the fluid
ρ density of the fluid
ν kinematic viscosity of fluid
Re Reynolds number
St Stokes number

1 Introduction

The lid driven cavity flow has long been used a test or validation case for new codes or new solution methods. The problem geometry is simple and two-dimensional, and the boundary conditions are also simple. The standard case is fluid contained in a square domain with Dirichlet boundary conditions on all sides, with three stationary sides and one moving side (with velocity tangent to the side).

Lid driven cavity flow is one important benchmark for Navier Stokes solvers due to its simple geometry and can also serve as a simplified model for industrial applications like wave-induced flow in sandpits etc. In this report we have done a transient lid driven cavity flow with lid oscillating with a specific frequency.

The Stokes number is defined as the ratio of the characteristic time of a particle (or droplet) to a characteristic time of the flow or of an obstacle. Here Stokes number is defined as $St = \omega^2 A/\nu$ where ω is frequency of oscillations and A is amplitude of oscillations.

2 Governing Equations

The flow can be solved with the help of Navier Stokes equations and the Continuity equation. The flow is assumed to be transient, laminar and incompressible. The governing equations for the flow are as follows :

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

X momentum equation:

$$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

Y momentum equation:

$$\rho\frac{\partial v}{\partial t} + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \mu\frac{\partial^2 v}{\partial x^2} + \mu\frac{\partial^2 v}{\partial y^2} \tag{3}$$

3 Boundary Conditions

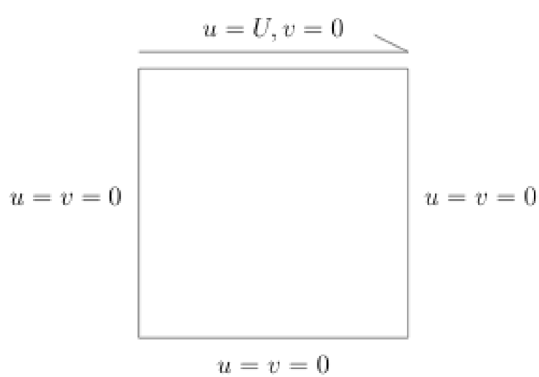

Fig. 1. Lid driven cavity flow

No slip boundary conditions at all faces and no flow boundary conditions are applied at the stationary walls. The boundary conditions for all the boundaries can be seen in Fig. 1. The moving wall's velocity is function of time. Equation 4 gives the velocity of the moving wall as a function of time. Neumann condition is applied for pressure near the walls.

$$U = U_o cos(\omega t) \tag{4}$$

$$Re = \frac{\rho U_o H}{\mu} \tag{5}$$

4 Solution Method

Finite Volume method is used to discretize the governing equations. Upwind scheme is being used for convection term and central differencing scheme for the diffusion term. SIMPLE algorithm is employed for flow field calculation. Fully implicit scheme is used for transient formulation.

5 Grid Convergence

One way of verifying the code is grid convergence test. Steady state solutions are calculated for four different grid sizes:125 x 125, 100 x 100, 75 x 75 and 50 x 50. The velocity of the lid in this test is 0.001 m/s. We can see that from table 1 that the velocity value does not change with increase in grid resolution after a point.

Grid size	$u(l/2,h)$
50 x 50	0.000921
75 x 75	0.000934
100 x 100	0.000934
125 x 125	0.000934

Table 1. Grid Convergence Results

6 Results

6.1 Steady State flow predictions of constant velocity lid driven cavity

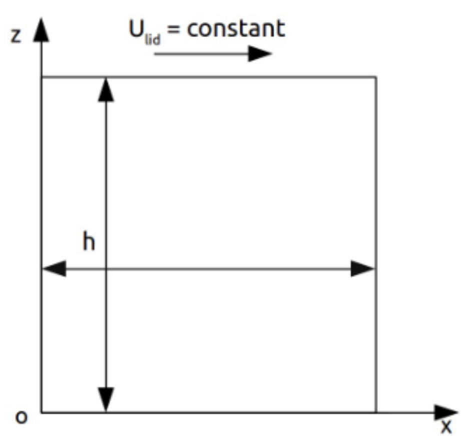

Fig. 2.

The steady flow predictions with R_e = 100 and R_e=500 at aspect ratios (L/H) of 1,2 and 4 are presented in this section.The lid is assumed to move with constant velocity.

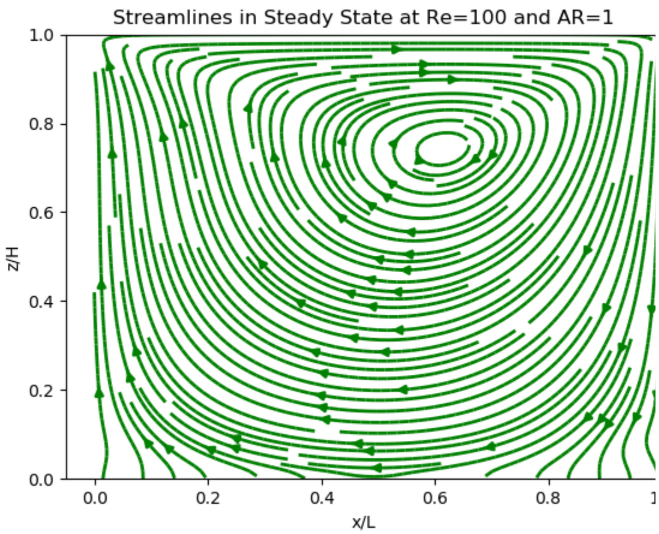

Fig. 3. R_e=100 Aspect Ratio=1

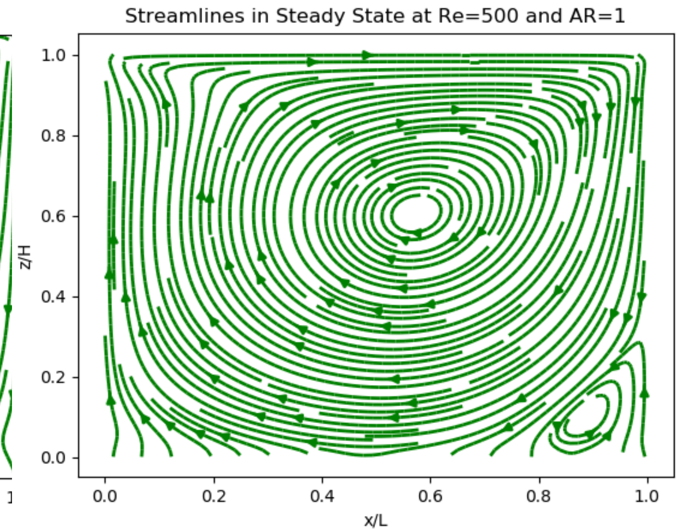

Fig. 4. R_e=500 Aspect Ratio=1

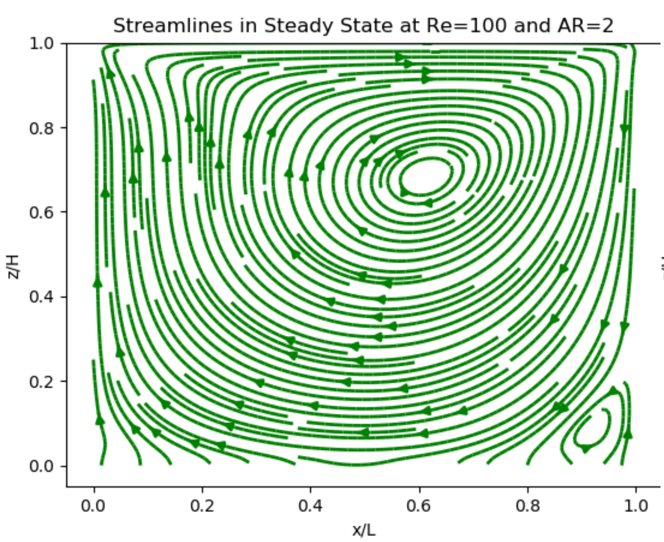

Fig. 5. R_e=100 Aspect Ratio=2

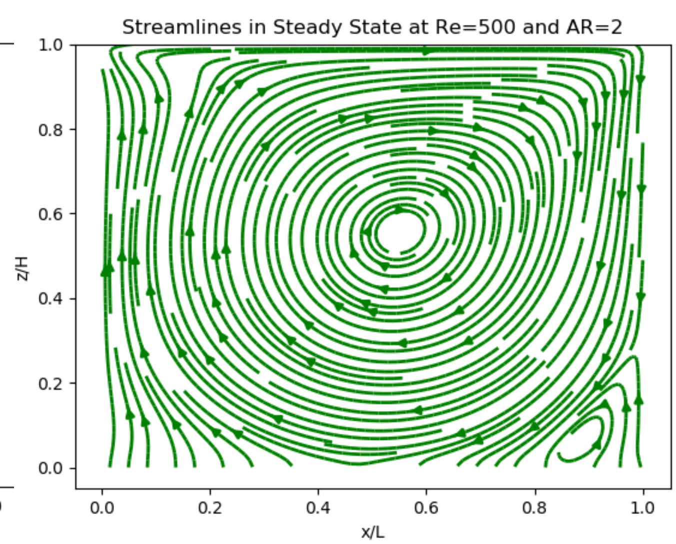

Fig. 6. R_e=500 Aspect Ratio=2

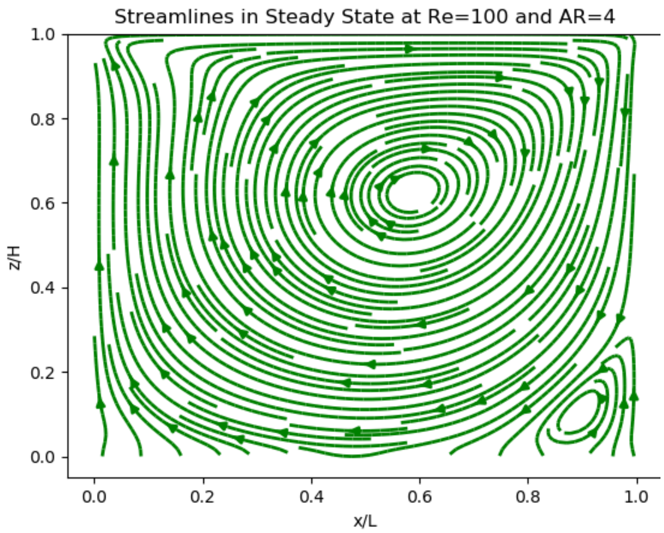

Fig. 7. R_e=100 Aspect Ratio=4

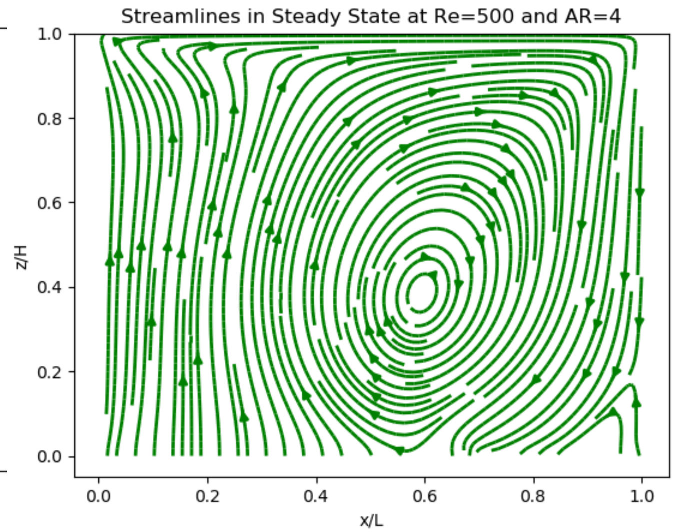

Fig. 8. R_e=500 Aspect Ratio=4

6.2 Oscillatory Lid driven cavity flow

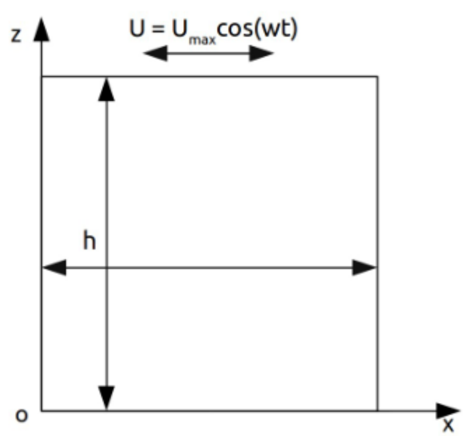

Fig. 9.

In oscillatory Lid-driven cavity flow, the lid moves in simple harmonic motion with Amplitude U_{max}, $frequency$ω and time period T.Transient Flow predictions at constant frequency=π/6 at R_e=6000, S_t=209.43 and R_e=12000, S_t=837.73 are presented in this section.

The u and w velocity profiles at x/L=0.5 and varying z at different times in a time period at R_e=6000 and R_e=12000.

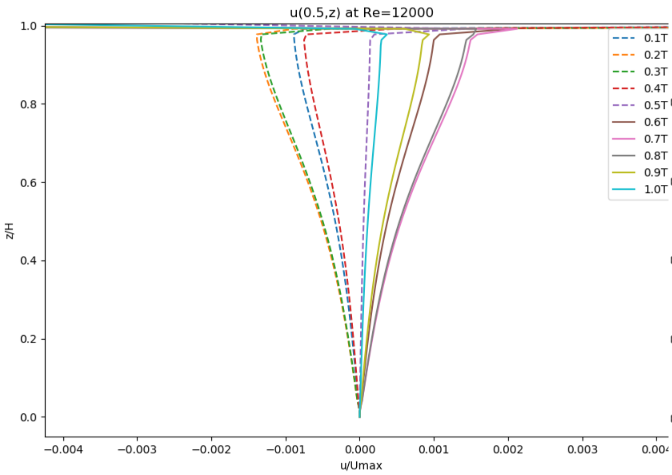

Fig. 10. u(0.5,z/H) R_e=12000 S_t=837.73

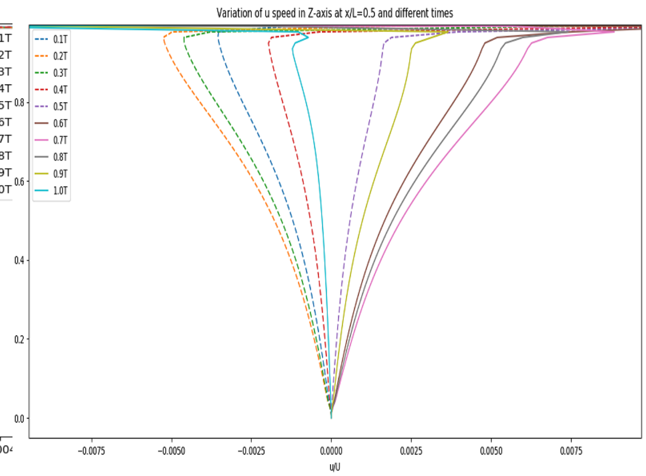

Fig. 11. u(0.5,z/H) R_e=6000 S_t=209.43

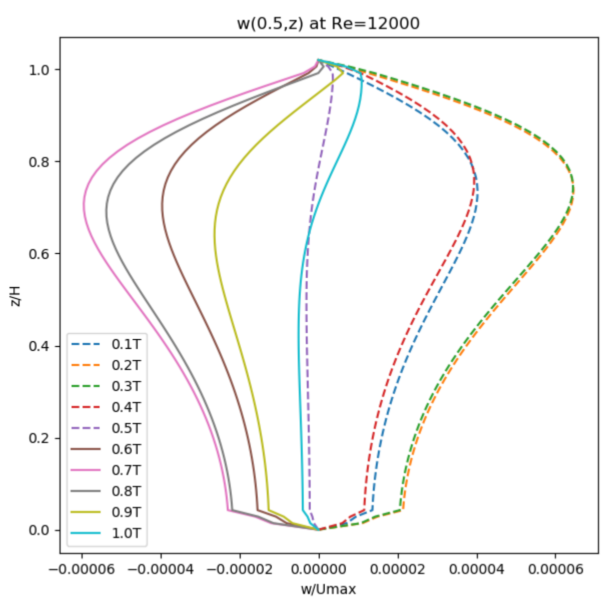

Fig. 12. w(0.5,z/H) R_e=12000 S_t=837.73

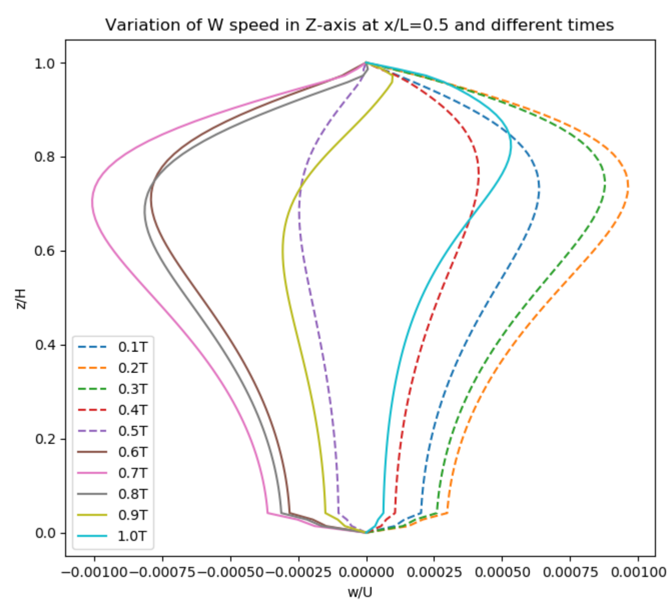

Fig. 13. w(0.5,z/H) R_e=6000 S_t=209.43

The u and w velocity profiles at z/H=0.5 and varying x at different times in a time period at R_e=6000 and R_e=12000.

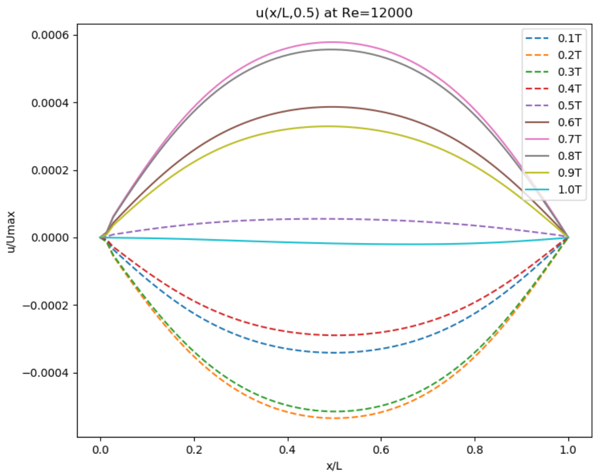

Fig. 14. u(x/L,0.5) R_e=12000 S_t=837.73

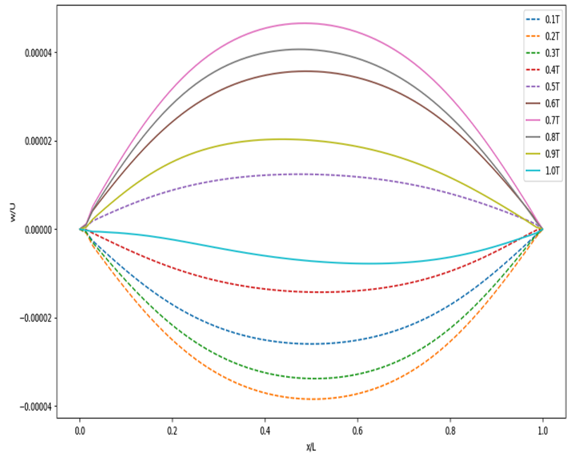

Fig. 15. u(x/L,0.5) R_e=6000 S_t=209.43

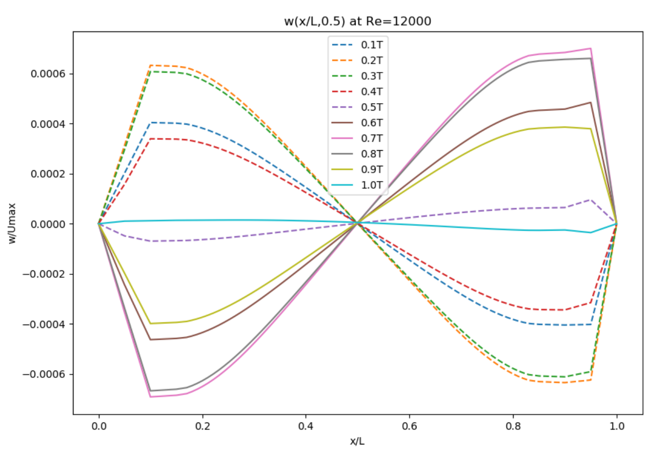

Fig. 16. w(x/L,0.5) R_e=12000 S_t=837.73

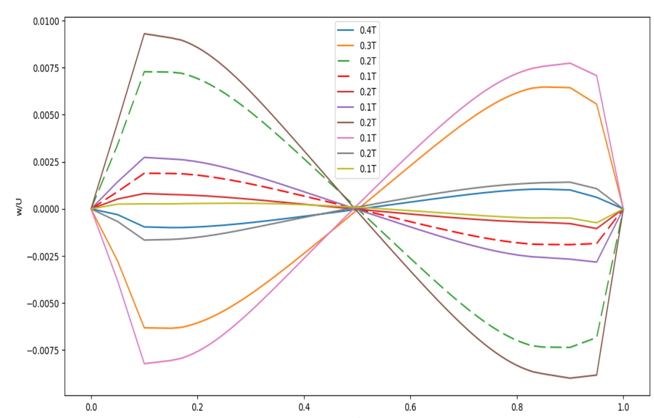

Fig. 17. w(x/L,0.5) R_e=6000 S_t=209.43

The streamlines at R_e=6000 at different time instances are shown below

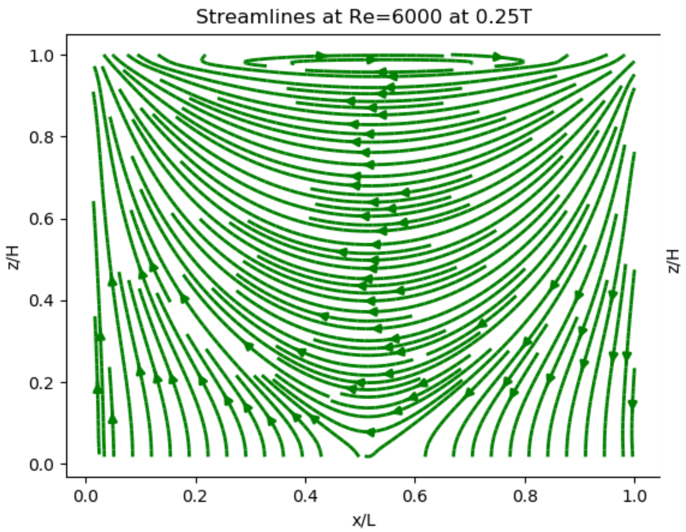

Fig. 18. R_e=6000 S_t=209.43 at t=T/4

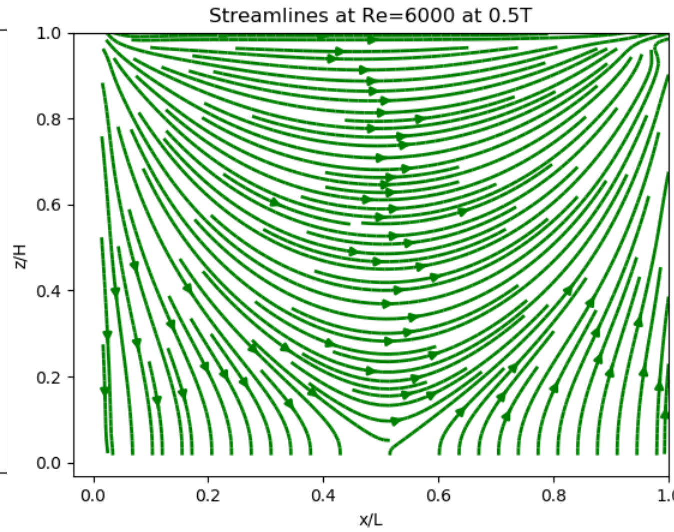

Fig. 19. R_e=6000 S_t=209.43 at t=T/2

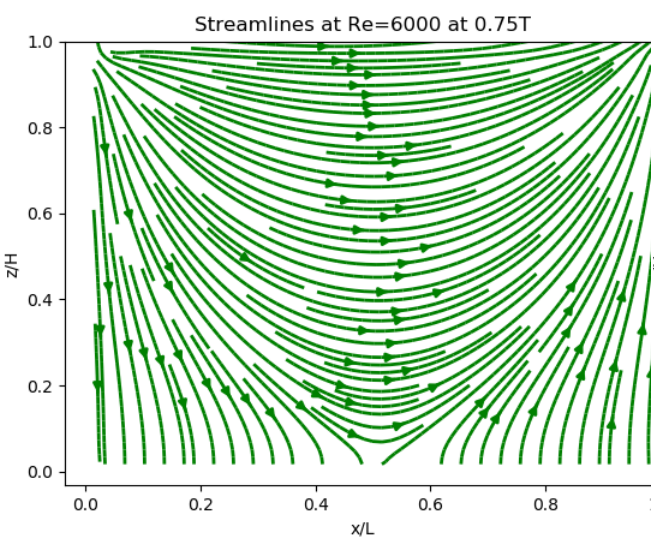

Fig. 20. R_e=6000 S_t=209.43 at t=3T/4

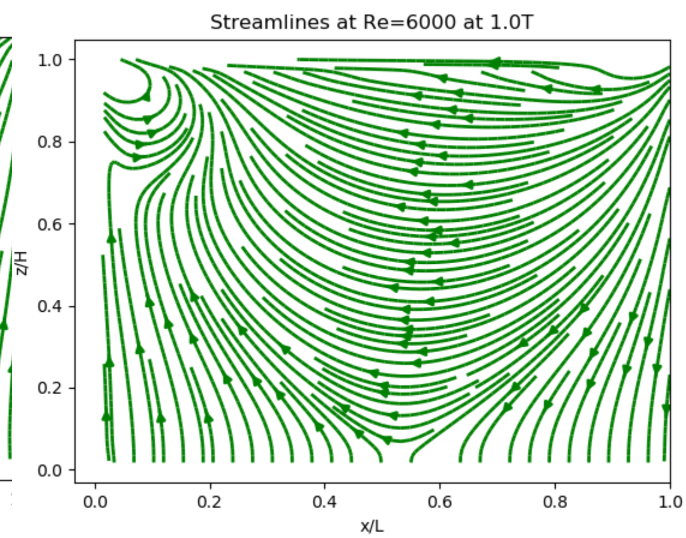

Fig. 21. R_e=6000 S_t=209.43 at t=T

The streamlines at Re=12000 at different time instances are shown below

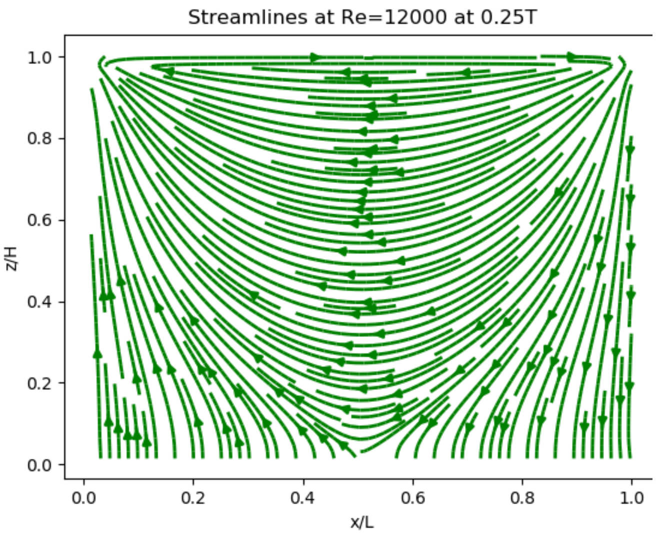

Fig. 22. R_e=12000 S_t=837.73 at t=T/4

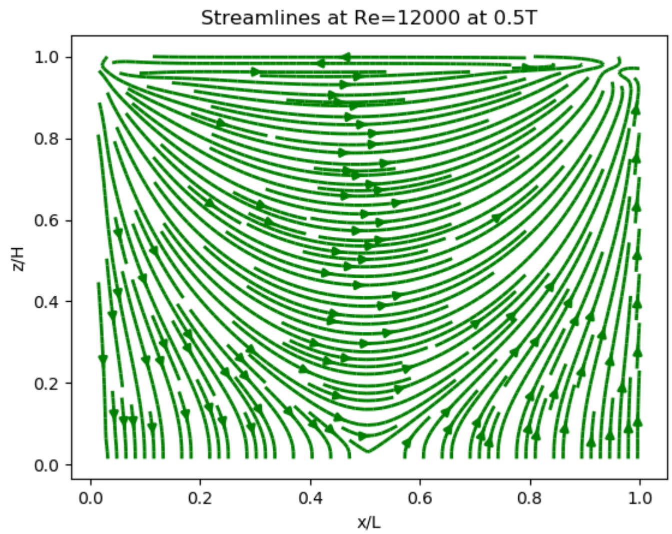

Fig. 23. R_e=12000 S_t=837.73 at t=T/2

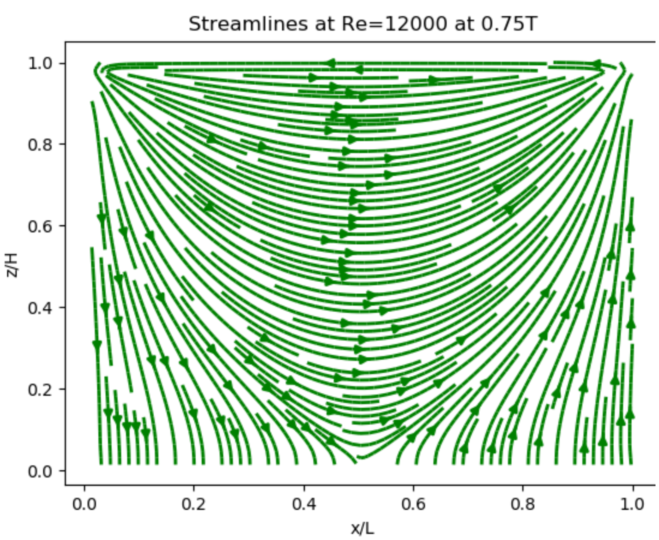

Fig. 24. R_e=12000 S_t=837.73 at t=3T/4

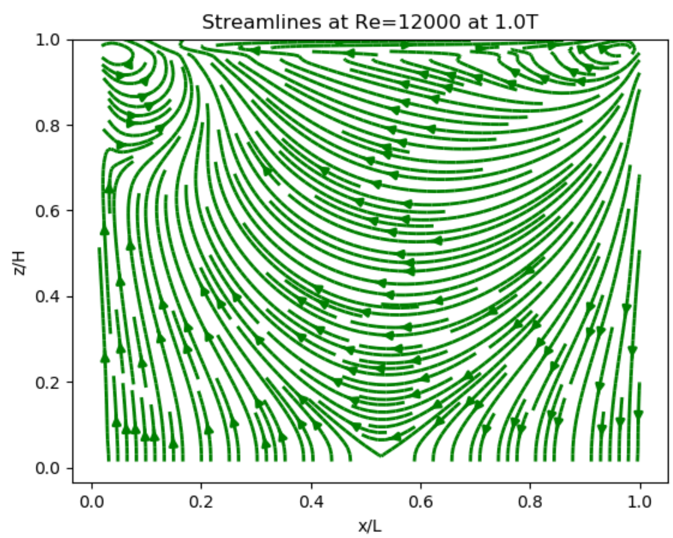

Fig. 25. R_e=12000 S_t=837.73 at t=T

7 Observations and Conclusion

1. In constant velocity lid-driven cavity at steady state, it can be observed that the region of uniform vorticity grows with increasing Reynolds number.
2. As the Aspect Ratio increases, the corner vortices become stronger and bigger
3. The boundary layer thickness also decreases with increase in Reynolds number
4. From the predictions on Oscillatory lid driven cavity, it can be seen that the velocity profiles at time t and t+T/2 are almost mirror images of each other
5. The number of vortices increase with increase in Reynolds number as well as time
6. The oscillatory lid driven cavity exhibits much more complex dynamic behaviour compared to constant velocity lid driven cavity

References

[1] $https://www.cfd-online.com/Wiki/Lid-driven-cavity-problem$

[2] Numerical Implementations for 2D Lid-Driven Cavity Flow in Stream Function Formulation $https://www.hindawi.com/journals/isrn/2012/871538/$
[3] J Zhu, L E Holmedal, D Myrhaug and H Wang, Fluid flow in steady and oscillatory lid- square cavities, IOP Conf. Series: Materials Science and Engineering 276 (2017) 012015
[4] Time dependent simulation of the Driven Lid Cavity at High Reynolds Number $https://arxiv.org/pdf/0809.3098.pdf$
[5] Lid driven Square Cavity flow $http://www.scielo.br/pdf/jbsmse/v31n3/a04v31n3.pdf$