Answers and Explanations

1	a	**2**	a	**3**	d	**4**	c	**5**	c	**6**	b	**7**	d	**8**	b	**9**	c	**10**	b
11	c	**12**	c	**13**	b	**14**	b	**15**	d	**16**	d	**17**	c	**18**	b	**19**	a	**20**	c
21	c	**22**	d	**23**	b	**24**	a	**25**	b	**26**	d	**27**	d	**28**	a	**29**	c	**30**	b
31	d	**32**	d	**33**	c	**34**	c	**35**	c	**36**	b	**37**	a	**38**	b	**39**	c	**40**	d
41	b	**42**	b	**43**	a	**44**	d	**45**	d	**46**	b	**47**	c	**48**	c	**49**	b	**50**	b
51	d	**52**	a	**53**	c	**54**	a	**55**	b	**56**	b	**57**	b	**58**	b	**59**	c	**60**	d
61	b	**62**	c	**63**	a	**64**	c	**65**	d	**66**	d	**67**	b	**68**	a	**69**	d	**70**	a
71	d	**72**	a	**73**	b	**74**	a	**75**	c	**76**	d	**77**	d	**78**	d	**79**	c	**80**	c
81	b	**82**	a	**83**	c	**84**	d	**85**	d	**86**	c	**87**	d	**88**	c	**89**	b		

1. a The difference between two integers will be 1, only if one is even and the other one is odd. 4x will always be even, so 17y has to be odd and hence y has to be odd.

Moreover, the number 17y should be such a number that is 1 less than a multiple of 4. In other words, we have to find all such multiples of 17, which are 1 less than a multiple of 4. The first such multiple is 51. Now you will find that as the multiples of 17 goes on increasing, the difference between it and its closest higher multiple of 4 is in the following pattern, 0, 3, 2, 1, e.g. 52 – 51 = 1,

68 – 68 = 0, 88 – 85 = 3, 104 – 102 = 2, 120 – 119 = 1, 136 – 136 = 0

So the multiples of 17 that we are interested in are 3, 7, 11, 15 .

Now since, $x \le 1000,\ 4x \le 4000$. The multiple of 17 closest and less than 4000 is 3995 (17 × 235). And incidentally, 3996 is a multiple of 4, i.e. the difference is 4.

This means that in order to find the answer, we need to find the number of terms in the AP formed by 3, 7, 11, 15 … 235, where a = 3, d = 4.

Since, we know that Tn = a + (n – 1)d,

so 235 = 3 + (n – 1) × 4. Hence, n = 59.

2. a Let x be the fixed cost and y the variable cost

17500 = x + 25y … (i)

30000 = x + 50y … (ii)

Solving the equation (i) and (ii), we get

x = 5000, y = 500

Now if the average expense of 100 boarders be ‘A’. Then

100 × A = 5000 + 500 × 100

$\therefore$ A = 550.

3. b $|r-6| = 11 \Rightarrow r-6 = 11,\ r = 17$

or –(r – 6) = 11, r = –5

$|2q-12| = 8 \Rightarrow 2q - 12 = 8$, q = 10

or 2q – 12 = –8 , q =2

Hence, minimum value of $\frac{q}{r} = \frac{10}{-5} = -2$.

For questions 4 to 6:

Place of worship	Number of flowers before offering	Number of flowers offered	Number of flowers left
1	(15/8)y	y	(7/8)y
2	(7/4) y	y	(3/4)y
3	(3/2) y	y	y/2
4	y	y	0

Starting from the fourth place of worship and moving backwards, we find that number of flowers before entering the first place of worship is $\frac{15}{8}y$.

Hence, number of flowers before doubling = $\frac{15}{16}y$

(but this is equal to 30)

Hence, y = 32

Answer for 4 is (c)

The minimum value of y so that $\frac{15}{16}y$ is a whole number is 16.

Therefore, 16 is the minimum number of flowers that can be offered.

Answer for 5 is (c).

For y = 16, the value of $\frac{15}{16}y = 15$.

Hence, the minimum number of flowers with which Roopa leaves home is 15.

Answer for 6 is (b).

7. d Let m = 1. So, option (a) will give the answer as V_m and option (c) will give the answer as V_1. Both of these cannot be the answers as V_m and V_1 are the amount of volume filled.
Let m = 2. So, option (b) will give the answer as 2 $(1 - V_2)$ and option (d) will give the answer as $2(1 - V_1)$. Now consider option (b).
Actual empty volume > $2(1 - V_2)$. Therefore, for this situation $m(1 - V_1)$ is the only possible answer.

8. b Let m = 1 and n = 1. Option (a) gives the answer as $\frac{1}{4}$ and option (d) gives the answer as 'greatest integer less than or equal to $\frac{1}{2}$'. So, both of these cannot be the answer. Option (b) gives the answer as 'smallest integer greater than or equal to $\frac{1}{2}$' and option (c) gives the answer as 1. But the actual answer can be greater than 1 as the volume of the vessel is 2 l. Hence, (b) is the answer.

9. c The data is not linear. So check (b).
Let the equation be $y = a + bx + cx^2$.
Putting the values of x and y, we get the following result.
$\Rightarrow 4 = a + b + c$, $8 = a + 2b + 4c$ and $14 = a + 3b + 9c$.
Solving these, we get a = 2, b = 1 and c = 1.
So the equation is $y = 2 + x + x^2$.

10. b $a_1 = 1, a_2 = 7, a_3 = 19, a_4 = 43$.
The difference between successive terms is in series 6, 12, 24, 48, ..., i.e. they are in GP. Hence,

$$a_{100} = a_1 + a\left(\frac{r^n - 1}{r - 1}\right) = 1 + 6\frac{(2^{99} - 1)}{(2 - 1)} = 6 \times 2^{99} - 5$$

11. c $$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{19.21}$$
$$= \frac{1}{2}\left(1 - \frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) + \ldots \frac{1}{2}\left(\frac{1}{19} - \frac{1}{21}\right)$$
$$= \frac{1}{2} - \frac{1}{42} = \frac{(21 - 1)}{42} = \frac{20}{42} = \frac{10}{21}$$

12. c The vehicle travels 19.5 km/L at the rate of 50 km/hr.
So it should travel $\frac{19.5}{1.3}$ km/L at the rate of 70 km/hr = 15 km/L.
The distance covered at 70 km/hr with 10 L = 10 × 15 = 150 km

13. b Use choices. The answer is (b), because $-x < -2$ and $-2 < 2y \Rightarrow -x < 2y$.

14. b Use the choices. If b = 1, then the factors are $(x - a)(x^2 + 1)$. This cannot yield 3 real roots.

15. d $x > 5$, $y < -1$
Use answer choices.
Take x = 6, y = –6. We see none of the statements (1, 2 and 3) is true. Hence the correct option is (d).

16. d Let $y = n^3 - 7n^2 + 11n - 5$
At n = 1, y = 0
$\therefore (n - 1)(n^2 - 6n + 5)$
$= (n - 1)^2 (n - 5)$
Now $(n - 1)^2$ is always positive.
Now for n < 5 the expression gives a negative quantity.
Therefore, the least value of n will be 6.
Hence m = 6.

17. c Let 'x' be the number of males in Mota Hazri.

	Chota Hazri	Mota Hazri
Males	x – 4522	x
Females	2(x – 4522)	x + 4020

$x + 4020 - 2(x - 4522) = 2910 \Rightarrow x = 10154$
$\therefore$ Number of males in Chota Hazri = 10154 – 4522 = 5632

18. b Let the number of students in classes X, Y and Z be a, b and c respectively. Then
Total of X = 83a
Total of Y = 76b
Total of Z = 85c

And $\frac{83a + 76b}{a + b} = 79$, i.e. 4a = 3b

Also $\frac{76b + 85c}{b + c} = 81$, i.e. 4c = 5b

Hence, $b = \frac{4}{3}a$, $c = \frac{5}{4}b = \frac{5}{4} \times \frac{4}{3}a = \frac{5}{3}a$

Average of X, Y and Z = $\frac{83a + 76b + 85c}{a + b + c}$

$$= \frac{83a + 76 \times \frac{4}{3}a + 85 \times \frac{5}{3}a}{a + \frac{4}{3}a + \frac{5}{3}a} = \frac{978}{12} = 81.5$$

19. a Let the cost of 1 burger, 1 shake and 1 fries be x, y and z.
Then
$3x + 7y + z = 120$ ... (i)
$\underline{4x + 10y + z = 164.5}$ ... (ii)
$x + 3y = 44.5$ ... (iii) (ii – i)
Multiplying (iii) by 4 and subtracting (ii) from it, we find
$2y - z = 13.5$ ...(iv)
Subtracting (iv) from (iii), we get x + y + z = 31.

20. c Let the 6th and the 7th terms be x and y.
Then 8th term = x + y
Also $y^2 - x^2 = 517$
$\Rightarrow (y + x)(y - x) = 517 = 47 \times 11$
So y + x = 47
y – x = 11
Taking y = 29 and x = 18, we have 8th term = 47,
9th term = 47 + 29 = 76 and 10th term = 76 + 47 = 123.

21. c $x + y = 1$ and $x > 0$ $y > 0$

Taking $x = y = \frac{1}{2}$, value of

$$\left(x+\frac{1}{x}\right)^2+\left(y+\frac{1}{y}\right)^2=\left(2+\frac{1}{2}\right)^2+\left(2+\frac{1}{2}\right)^2$$

$$=\frac{25}{4}+\frac{25}{4}=\frac{25}{2}$$

It can be easily verified as it is the least value among options.

For questions 22 and 23:

$$BA=\frac{r_1+r_2}{n_1},\ MBA_2=\frac{r_1+r_2}{n_1+n_2}\ \text{and}$$

$$MBA_1=\frac{r_1}{n_1}+\frac{n_2}{n_1}\ \max\left\{0,\ \frac{r_2}{n_2}-\frac{r_1}{n_1}\right\}$$

From BA and MBA_2, we get $BA \geq MBA_2$ because $n_1+n_2 \geq n_1$.

From BA and MBA_1, we get $BA \geq MBA_1$ because

$$\frac{r_1}{n_1}+\frac{r_2}{n_1}\geq\frac{r_1}{n_1}+\frac{r_2}{n_1}\times\frac{n_2}{r_2}\max\left\{0,\frac{r_2}{n_2}-\frac{r_1}{n_1}\right\}.$$

Now from MBA_1 and MBA_2, we get

$$\frac{r_1}{n_1}+\frac{r_2}{n_1}\times\frac{n_2}{r_2}\max\left\{0,\frac{r_2}{n_2}-\frac{r_1}{n_1}\right\}\geq\frac{r_1}{n_1+n_2}+\frac{r_2}{n_1+n_2}.$$

22. d From the above information, $BA \geq MBA_1 \geq MBA_2$

None of these is the right answer.

23. b BA = 50 where there is no incomplete innings means

$r_2 = n_2 = 0 \Rightarrow \frac{r_1}{n_1}=50$

$$MBA_1=\frac{r_1}{n_1}+\frac{n_2}{n_1}\max\left[0,\left(\frac{r_2}{n_2}-\frac{r_1}{n_1}\right)\right]$$

$$=50+\frac{1}{n_1}\max\left[0,\left(\frac{45}{1}-50\right)\right]$$

$= 50 + 0 = 50$

$$BA=\frac{r_1+r_2}{n_1}=\frac{50n_1+45}{n_1}=50+\frac{45}{n_1}>50$$

$$MBA_2=\frac{r_1+r_2}{n_1+n_2}=\frac{50n_1+45}{n_1+1}=50-\frac{5}{n+1}$$

Hence, BA will increase, MBA_2 will decrease.

24. a Equation of quadratic equation is

$ax^2 + bx + c = 0$

$x^2 + bx + c = 0$

First roots = (4, 3)

Sum of the roots $=\frac{-b}{a}=-7\Rightarrow b=7$.

Product of the roots $=\frac{c}{a}=12\Rightarrow c=12$.

$\therefore$ Equation formed $x^2 - 7b + 12 = 0$ ... (i)

Another boy gets the wrong roots (2, 3).

$\therefore$ Sum of the roots $=\frac{-b}{a}=-5\Rightarrow b=5$.

Product of the roots $=\frac{c}{a}=6\Rightarrow c=6$.

Equation formed $x^2 - 5b + 6 = 0$ ... (ii)

$x^2 + b'x + c_1 = 0$

$b' = 2 + 3$

$\therefore c = 6$

Hence, $x^2 - 7x + 6 = 0$

$\Rightarrow x^2 - 6x - x + 6 = 0$

$\Rightarrow x(x - 6) - 1(x - 6) = 0$

$\Rightarrow (x - 6)(x - 1) = 0$

$\therefore x = 6, 1$

Hence, the actual roots = (6, 1).

Alternate method:

Since constant = 6[3 × 2] and coefficient of

x = [–4x – 3x] = –7

Since quadratic equation is

x^2 – (Sum of roots)x + Product of roots = 0 or

$x^2 - 7x + 6 = 0$

Solving the equation $(x - 6)(x - 1) = 0$ or x = (6, 1).

25. b $f(x) + f(y) = \log\left(\frac{1+x}{1-x}\right)+\log\left(\frac{1+y}{1-y}\right)$

$$=\log\left(\frac{(1+x)\ (1+y)}{(1-x)(1-y)}\right)$$

$$=\log\left(\frac{1+x+y+xy}{1+xy-x-y}\right)$$

$$=\log\left(\frac{1+xy+x+y}{1+xy-(x+y)}\right)$$

$$=\log\left(\frac{1+\left(\frac{x+y}{1+xy}\right)}{1-\left(\frac{x+y}{1+xy}\right)}\right)$$

$$=f\left(\frac{x+y}{1+xy}\right)$$

26. d $x_0 = x$
$x_1 = -x$
$x_2 = -x$
$x_3 = x$
$x_4 = x$
$x_5 = -x$
$x_6 = -x$
………..
$\Rightarrow$ Choices (a), (b), (c) are incorrect.

27. d $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + zy + zx)$
$\Rightarrow x^2 + y^2 + z^2 = 19$
$\Rightarrow$ y + z cannot be simultaneously = 0
else $xy + zy + zx = 0 \Rightarrow x^2 < 19 \Rightarrow x < \sqrt{19} \simeq 4.4$

28. a Coefficient of $x^n = \frac{1}{2}(n+1)(n+4)$

$S = 2 + 5x + 9x^2 + 14x^3 + ...$
$xS = 2x + 5x^2 + ...$
$S(1-x) = 2 + 3x + 4x^2 + 5x^3 + ...$

Let $S_1 = S(1-x) \Rightarrow S_1 = 2 + 3x + 4x^2 + ...$

$xS_1 = 2x + 3x^2 + ...$
$S_1(1-x) = 2 + x + x^2 + ...$

$S_1(1-x) = 2 + \frac{x}{1-x}$

$S(1-x^2) = 2 + \frac{x}{1-x} \Rightarrow S = \frac{2-x}{(1-x)^3}$

29. c $x^2 + 5y^2 + z^2 = 4yx + 2yz$

$(x^2 + 4y^2 - 4yx) + z^2 + y^2 - 2yz = 0$

$(x - 2y)^2 + (z - y)^2 = 0$
It can be true only if x = 2y and z = y

30. b Arithmetic mean is more by 1.8 means sum is more by 18. So ba – ab = 18
b > a because sum has gone up, e.g. 31 – 13 = 18
Hence, b – a = 2

31. d $x - 1 \le [x] \le x$
$2x + 2y - 3 \le L(x,y) \le 2x + 2y \Rightarrow a - 3 \le L \le a$
$2x + 2y - 2 \le R(x,y) \le 2x + 2y \Rightarrow a - 2 \le R \le a$
Therefore, $L \le R$
Note: Choice (b) is wrong, otherwise choice (a) and choice (c) are also not correct. Choose the numbers to check.

32. d $\frac{A^2}{x} + \frac{B^2}{x-1} = 1 \Rightarrow A^2(x-1) + B^2x = x^2 - x$
This is a quadratic equation.
Hence, number of roots = 2 or 1 (1 in the case when both roots are equal).

33. c Let the largest piece = 3x
Middle = x
Shortest = 3x – 23
or 3x + x + (3x – 23) = 40
or x = 9
or the shortest piece = 3(9) – 23 = 4
Check choices:
The shortest piece has to be < 20 cm.
27 is wrong choice.
The largest piece is a multiple of 3.
Or (23 + Shortest) should be a multiple of 3.
Answer = 4 cm (Among other choices)

34. c If p = q = r = 1, then expression = 1
Check the choice only, one choice gives the value of expression = 1.

35. c $2^x - x - 1 = 0$
$\Rightarrow 2^x - 1 = x$
If we put x = 0, then this is satisfied and if we put x = 1, then also this is satisfied.
Now we put x = 2, then this is not valid.

36. b For the curves to intersect, $\log_{10} x = x^{-1}$
Thus, $\log_{10} x = \frac{1}{x}$ or $x^x = 10$
This is possible for only one value of x (2 < x < 3).

37. a It is given that $p + q + r \ne 0$, if we consider the first option, and multiply the first equation by 5, second by –2 and third by –1, we see that the coefficients of x, y and z all add up-to zero.
Thus, 5p – 2q – r = 0
No other option satisfies this.

38. a Let 'x' be the number of standard bags and 'y' be the number of deluxe bags.
Thus, $4x + 5y \le 700$ and $6x + 10y \le 1250$
Among the choices, (c) and (d) do not satisfy the second equation.
Choice (b) is eliminated as, in order to maximize profits the number of deluxe bags should be higher than the number of standard bags because the profit margin is higher in a deluxe bag.

39. c Let the 1st term be ‘a’ and common difference be ‘d’ then we have 3^{rd} term = a + 2d
15^{th} term = a + 14d
6^{th} term = a + 5d
11^{th} term = a + 10d
13^{th} term = a + 12d
Since sum of 3^{rd} and 15^{th} term = sum of 6^{th}, 11^{th} and 13^{th} term, therefore we have
2a + 16d = 3a + 27d
$\Rightarrow$ a + 11d = 0
Which is the 12^{th} term.

40. d We can see that x + 2 is an increasing function and 5 – x is a decreasing function. This system of equation will have smallest value at the point of intersection of the two. i.e. 5 – x = x + 2 or x = 1.5.
Thus smallest value of g(x) = 3.5

41. b **Case 1:** If x < 2, then y = 2 – x + 2.5 – x + 3.6 – x = 8.1 – 3x.
This will be least if x is highest i.e. just less than 2.
In this case y will be just more than 2.1

Case 2: If $2 \le x < 2.5$, then y = x – 2 + 2.5 – x + 3.6 – x = 4.1 – x
Again, this will be least if x is the highest case y will be just more than 1.6.

Case 3: If $2.5 \le x < 3.6$, then y = x – 2 + x – 2.5 + 3.6 – x = x – 0.9
This will be least if x is least i.e. X = 2.5.

Case 4: If $x \ge 3.6$, then
y = x – 2 + x – 2.5 + x – 3.6 = 3x – 8.1
The minimum value of this will be at x = 3.6 and y = 2.7
Hence the minimum value of y is attained at x = 2.5

42. b Solution can be found using Statement A as we know both the roots for the equation (viz. $\frac{1}{2}$ and $-\frac{1}{2}$).
Also statement B is sufficient.
Since ratio of c and b = 1, c = b.
Thus the equation = $4x^2 + bx + b = 0$. Since $x = -\frac{1}{2}$ is one of the roots, substituting we get $1 - \frac{b}{2} + b = 0$ or b = –2. Thus c = –2.

43. a Both the series are infinitely diminishing series.
For the first series: First term = $\frac{1}{a^2}$ and r = $\frac{1}{a^2}$
For the second series: First term = $\frac{1}{a}$ and r = $\frac{1}{a^2}$

The sum of the first series = $\dfrac{\frac{1}{a^2}}{1-\frac{1}{a^2}} = \dfrac{1}{a^2-1}$

The sum of the second series = $\dfrac{\frac{1}{a}}{1-\frac{1}{a^2}} = \dfrac{a}{a^2-1}$

Now, from the first statement, the relation can be anything (depending on whether a is positive or negative).
But the second statement tells us, $4a^2 - 4a + 1 = 0$ or $a = \frac{1}{2}$. For this value of a, the sum of second series will always be greater than that of the first.

44. d The number of terms of the series forms the sum of first n natural numbers i.e.
$\frac{n(n+1)}{2}$.
Thus the first 23 letters will account for the first $\frac{23 \times 24}{2}$ = 276 terms of the series.
The 288th term will be the 24th letter which is x.

45. d $p + q = \alpha - 2$ and $pq = -\alpha - 1$
$(p + q)^2 = p^2 + q^2 + 2pq$,
Thus $(\alpha - 2)^2 = p^2 + q^2 + 2(-\alpha - 1)$
$p^2 + q^2 = \alpha^2 - 4\alpha + 4 + 2\alpha + 2$
$p^2 + q^2 = \alpha^2 - 2\alpha + 6$
$p^2 + q^2 = \alpha^2 - 2\alpha + 1 + 5$
$p^2 + q^2 = (\alpha - 1)^2 + 5$
Thus, minimum value of $p^2 + q^2$ is 5.

46. b $(a + b + c + d)^2 = (4m + 1)^2$
Thus, $a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)$ $= 16m^2 + 8m + 1$
$a^2 + b^2 + c^2 + d^2$ will have the minimum value if (ab + ac + ad + bc + bd + cd) is the maximum.
This is possible if a = b = c = d = (m + 0.25) … since a + b + c + d = 4m + 1
In that case 2((ab + ac + ad + bc + bd + cd)
$= 12(m + 0.25)^2 = 12m^2 + 6m + 0.75$
Thus, the minimum value of $a^2 + b^2 + c^2 + d^2$
$= (16m^2 + 8m + 1) - 2(ab + ac + ad + bc + bd + cd)$
$= (16m^2 + 8m + 1) - (12m^2 + 6m + 0.75)$
$= 4m^2 + 2m + 0.25$
Since it is an integer, the actual minimum value
$= 4m^2 + 2m + 1$

47. c Assume the number of horizontal layers in the pile be n.

So $\sum \frac{n(n+1)}{2} = 8436$

$\Rightarrow \frac{1}{2} [\sum n^2 + \sum n] = 8436$

$\Rightarrow \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4} = 8436$

$\Rightarrow n(n+1) \left[\frac{2n+4}{12}\right] = 8436$

$\Rightarrow \frac{n(n+1)(n+2)}{6} = 8436$

$\Rightarrow n(n+1)(n+2) = 36 \times 37 \times 38$
So n = 36

48. c Using $\log a - \log b = \log\left(\frac{a}{b}\right)$, $\frac{2}{y-5} = \frac{y-5}{y-3.5}$, where $y = 2^x$
Solving we get y = 4 or 8 i.e. x = 2 or 3. It cannot be 2 as log of negative number is not defined (see the second expression).

49. b u is always negative. Hence, for us to have a minimum value of $\frac{vz}{u}$, vz should be positive. Also for the least value, the numerator has to be the maximum positive value and the denominator has to be the smallest negative value. In other words, vz has to be 2 and u has to be –0.5.

Hence the minimum value of $\frac{vz}{u} = \frac{2}{-0.5} = -4$.

For us to get the maximum value, vz has to be the smallest negative value and u has to be the highest negative value. Thus, vz has to be –2 and u has to be –0.5.

Hence the maximum value of $\frac{vz}{u} = \frac{-2}{-0.5} = 4$.

50. b GRRRRR, RGRRRR, RRGRRR, RRRGRR, RRRRGR, RRRRRG
GGRRRR, RGGRRR, RRGGRR, RRRGGR, RRRRGG
GGGRRR, RGGGRR, RRGGGR, RRRGGG
GGGGRR, RGGGGR, RRGGGG
GGGGGR, RGGGGG
GGGGGG
Hence 21 ways.

51. d When we substitute two values of x in the above curves, at x = –2 we get
$y = -8 + 4 + 5 = 1$
$y = 4 - 2 + 5 = 7$
Hence at x = –2 the curves do not intersect.
At x = 2, $y_1 = 17$ and $y_2 = 11$
At x = –1, $y_1 = 5$ and 2 and $y_2 = 5$
When x = 0, $y_1 = 5$ and $y_2 = 5$
And at x = 1, $y_1 = 7$ and $y_2 = 7$
Therefore, the two curves meet thrice when x = –1, 0 and 1.

52. a Let us say there are only 3 questions. Thus there are $2^{3-1} = 4$ students who have done 1 or more questions wrongly, $2^{3-2} = 2$ students who have done 2 or more questions wrongly and $2^{3-3} = 1$ student who must have done all 3 wrongly. Thus total number of wrong answers = $4 + 2 + 1 = 7 = 2^3 - 1 = 2^n - 1$.
In our question, the total number of wrong answers = $4095 = 2^{12} - 1$. Thus n = 12.

53. c Here x, y, z are distinct positive real number

So $\frac{x^2(y+z)+y^2(x+2)+z^2(x+y)}{xyz}$

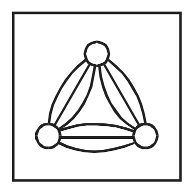
$$= \frac{x}{y} + \frac{x}{z} + \frac{y}{x} + \frac{y}{z} + \frac{z}{x} + \frac{z}{y}$$

$$= \left(\frac{x}{y} + \frac{y}{x}\right) + \left(\frac{y}{z} + \frac{z}{y}\right) + \left(\frac{z}{x} + \frac{x}{z}\right)$$ [We know that

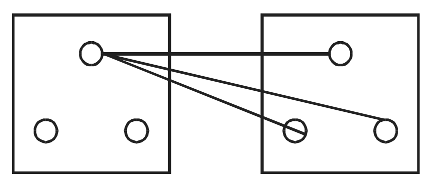
$\frac{a}{b} + \frac{b}{a} > 2$ if a and b are distinct numbers
> 2 + 2 + 2
> 6

54. a The least number of edges will be when one point is connected to each of the other 11 points, giving a total of 11 lines. One can move from any point to any other point via the common point. The maximum edges will be when a line exists between any two points. Two points can be selected from 12 points in $^{12}C_2$ i.e. 66 lines.

55. b Consider first zone. The number of telephone lines can be shown a follows.


= 9 lines

Therefore, total number of lines required for internal connections in each zone = 9 × 4 = 36 lines.
Now consider the connection between any two zones.



Each town in first zone can be connected to three towns in the second zone.
Therefore, the lines required = 3 × 3 = 9
Therefore, total number of lines required for connecting towns of different zones = $^4C_2 \times 9 = 6 \times 9 = 54$
Therefore, total number of lines in all = 54 + 36 = 90

56. b $ax^2 + bx + 1 = 0$
For real roots

$b^2 - 4ac \ge 0$

$\therefore b^2 - 4a(1) \ge 0$

$\therefore b^2 \ge 4a$

For a = 1, 4a = 4, $\therefore$ b = 2, 3, 4
a = 2, 4a = 8, $\therefore$ b = 3, 4
a = 3, 4a = 12, $\therefore$ b = 4
a = 4, 4a = 16, $\therefore$ b = 4
$\therefore$ Number of equations possible = 7.

57. b $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$

$$\log_{10}\left[\frac{x}{\sqrt{n}}\right] = \log_x 100$$

$$\therefore \log_{10}\sqrt{x} = \frac{\log_{10}100}{\log_{10}x}$$

$$\therefore \frac{1}{2}\log_{10}x = \frac{2}{\log_{10}x}$$

$$\therefore (\log_{10}x)^2 = 4$$

$$\therefore \log_{10}x = \pm 2$$

$$\therefore \log_{10}x = 2 \quad \text{or} \quad \log_{10}x = -2$$

$$\therefore 10^2 = x \quad \text{or} \quad 10^{-2} = x$$

$$\therefore x = 100 \quad \text{or} \quad x = \frac{1}{100}$$

58. b $\frac{1}{3}\log_3 M + 3\log_3 N = 1 + \log_{0.008} 5$

$$\frac{1}{3}\log_3(M^{1/3}N) = 1 + \frac{(\log 10 - \log 2)}{\log 8 - \log 1000}$$

$$\frac{1}{3}\log_3(M^{1/3}N) = 1 - \frac{(1-\log 2)}{3(1-\log 2)}$$

$$\Rightarrow \frac{1}{3}\log_3(M^{1/3}N) = 1 - \frac{1}{3} = \frac{2}{3}$$

or $M^{1/3}N = 3^{2/3}$
or $MN^9 = 3^2$
or $N^9 = 9/M$

59. c $5x + 19y = 64$
We see that if y =1, we get an integer solution for x = 9, now if y changes (increases or decreases) by 5x will change (decrease or increase) by 19.
Looking at the options, if x = 256, we get y = 64.
Using these values we see options (a), (b) and (d) are eliminated and also that these exists a solution for $250 < x \le 300$.

60. d Sum of $\log m + \log\left(\frac{m^2}{n}\right) + \log\left(\frac{m^3}{n^2}\right) + \cdots$ n terms such problem must be solved by taking the value of number of terms. Let's say 2 and check the given option. If we look at the sum of 2 terms of the given series it comes out to be $\log m + \log\frac{m^2}{n} \Rightarrow \log\frac{m \times m^2}{n} = \log\left(\frac{m^3}{n}\right)$

Now look at the option and put number of terms as 2, only option (d) validates the above mentioned answer.

As $\log\left[\frac{m^{(n+1)}}{n^{(n-1)}}\right]^{\frac{n}{2}} \Rightarrow \log\left[\frac{m^3}{n}\right]^1 \Rightarrow \log\left(\frac{m^3}{n}\right)$

61. b xyz = 4
y – x = z – y
2y = x + z
y is the AM of x, y, z.

Also $\sqrt[3]{xyz} = 4^{\frac{2}{3}} \Rightarrow \sqrt[3]{xyz} = 2^{\frac{1}{3}}$

AM ≥ GM

$y \ge 2^{\frac{2}{3}}$

Therefore, the minimum value of y is $2^{\frac{2}{3}}$.

62. c Let $S = 1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4}$ … (i)

$$\therefore \frac{1}{7}S = \frac{1}{7} + \frac{4}{7^2} + \frac{9}{7^3} + \frac{16}{7^4} \quad \ldots \text{(ii)}$$

(i) – (ii) gives,

$$S\left(1-\frac{1}{7}\right) = 1 + \frac{3}{7} + \frac{5}{7^2} + \frac{7}{7^3} + \frac{9}{7^4} \quad \ldots \text{(iii)}$$

$$\frac{1}{7}\times S\left(1-\frac{1}{7}\right) = \frac{1}{7} + \frac{3}{7^2} + \frac{5}{7^3} + \frac{7}{7^4} \quad \ldots \text{(iv)}$$

(iii) – (iv) gives,

$$S\left(1-\frac{1}{7}\right) - \frac{1}{7}S\left(1-\frac{1}{7}\right) = 1 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \frac{2}{7^4}\cdots$$

$$\therefore S\left(1-\frac{1}{7}\right)\left(1-\frac{1}{7}\right) = 1 + \frac{2}{7}\left[1 + \frac{1}{7} + \frac{1}{7^2} + \cdots\infty\right]$$

$$\therefore S\left(1-\frac{1}{7}\right)^2 = 1 + \frac{2}{7}\times\frac{1}{1-\frac{1}{7}}$$

$$\therefore S\left(\frac{6}{7}\right)^2 = 1 + \frac{2}{7}\times\frac{7}{6}$$

$$\therefore S\times\frac{36}{49} = 1 + \frac{1}{3}$$

$$\therefore S = \frac{49}{36}\times\frac{4}{3}$$

$$S = \frac{49}{27}$$

63\. a Let α is the common root.

$\therefore \alpha^3 + 3\alpha^2 + 4\alpha + 5 = 0$

$\alpha^3 + 2\alpha^3 + 7\alpha + 3 = 0$

$\alpha^2 - 3\alpha + 2 = 0$

$\alpha = 2,\ \alpha = 1$

But the above values of α do not satisfy any of the equations. Thus, no root is common.

64\. c $1 - \frac{1}{n} < x \le 3 + \frac{1}{n}$

Put n = 1

$\therefore 0 < x \le 4$

65\. d $36 \le n \le 72$

$$x = \frac{n^2 + 2\sqrt{n}(n+4) + 16}{n + 4\sqrt{n} + 4}$$

Put x = 36.

$$\therefore x = \frac{(36)^2 + 2\times 6\times 40 + 16}{36 + 24 + 4}$$

Which is least value of ‘n’ = 28

66\. d 13x + 1 < 2z and z + 3 = $5y^2$

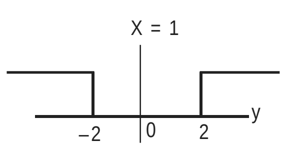
13x + 1 < 2 $(5y^2 - 3)$

$13x + 1 < 10y^2 - 6$

$13x + 7 < 10y^2$ put x = 1

$20 < 10y^2$ $y^2 > 2$

$y^2 > \sqrt{2}$ $(y^2 - 2) > 0$



67\. b x = –|a| b

Now a – xb = a – (– |a| b) b

= a + |a| b^2

$\therefore$ a – xb = a + ab^2 … $a \ge 0$ OR a – xb

= a – ab^2 …a < 0

= a(1 + b^2) = a(1 – b^2)

Consider first case:

As $a \ge 0$ and $|b| \ge 1$, therefore (1 + b^2) is positive.

$\therefore$ a (1 + b^2) ≥ 0

$\therefore$ a – xb ≥ 0

Consider second case.

As a < 0 and $|b| \ge 1$, therefore (1 – b^2) ≤ 0

$\therefore$ a (1 – b^2) ≥ 0 (Since –ve × -ve = +ve and 1 – b^2 can be zero also), i.e. a – xb ≥ 0

Therefore, in both cases a – xb ≥ 0.

68\. a $g^2 = g * g = h$

$g^3 = g^2 * g = h * g = f$

$g^4 = g^3 * g = f * g = e$

$\therefore n = 4$

69\. d $f \oplus [f * \{f \oplus (f * f)\}]$

$= f \oplus [f * \{f \oplus h\}]]$

$= f \oplus [f * e\}]$

$= f \oplus [f]]$

= h

70\. a $e^8 = e^2 * e^2 * e^2$

$= e * e * e$

= e

If we observe a $*$ anything = a

$\therefore$ a^{10} = a

$\therefore \{\underline{a^{10} * (f^{10} \oplus g^9)}\} \oplus e^8$

$= a \oplus e$

= e

71\. d It will go by elimination.

9 – 7 = 2 is even, therefore option (a) not possible.

2 × 9 = 18 is even, therefore option (b) not possible.

$\frac{3+9}{3} = \frac{12}{3} = 4$ is even, therefore option (c) is not possible.

$\therefore$ The correct option is (d).

72\. a Given

$t_1 + t_2 + ... + t_{11} = t_1 + t_2 + ... + t_{19}$ (for an A.P.)

$$\Rightarrow \frac{11}{2}[2a + (11-1)d] = \frac{19}{2}[2a + (19-1)d]$$

22 a + 110d = 28 a + 342 d

16 a + 232 d = 0

2a + 29 d = 0

$$\Rightarrow \frac{30}{2}[2a + (30-1)d] = 0$$

$\Rightarrow S_{30\text{terms}} = 0$

73\. b We have

$f(0) = 0^3 - 4(0) + p = p$

$f(1) = 1^3 - 4(1) + p = p - 3$

If P and P – 3 are of opp. signs then p(p – 3) < 0

Hence 0 < p < 3.

74. a We have

(1) $10^{10} < n < 10^{11}$

(2) Sum of the digits for 'n' = 2

Clearly-

(n)min = 10000000001 (1 followed by 9 zeros and finally 1)

Obviously, we can form 10 such numbers by shifting '1' by one place from right to left again and again.

Again, there is another possibility for 'n'

n = 20000000000

So finally : No. of different values for n = 10 + 1 = 11 ans.

75. c If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = r$

then there are only two possibilities.

(i)

If $a+b+c \neq 0$, then

$$\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = \frac{a+b+c}{(b+c)+(c+a)+(a+b)}$$

$$= \frac{a+b+c}{2(a+b+c)} = \frac{1}{2}$$

(ii)

If a+ b + c = 0, then

b + c = – a

c + a = – b

a + b = – c

Hence $\frac{a}{b+c} = \frac{a}{(-a)} = -1$

Similarly, $\frac{b}{c+a} = \frac{c}{a+b} = -1$

Therefore option (3) is the correct one 1/2 or –1

76. d $$y = \frac{1}{2 + \frac{1}{3+y}}$$

$$\Rightarrow y = \frac{3+y}{7+2y}$$

$$\Rightarrow 2y^2 + 6y - 3 = 0$$

$$\Rightarrow y = \frac{-6 \pm \sqrt{36+24}}{4}$$

$$= \frac{-6 \pm \sqrt{60}}{4} = \frac{-3 \pm \sqrt{15}}{2}$$

Since 'y' is a +ve number, therefore:

$y = \frac{\sqrt{15} - 3}{2}$ ans.

77. d When a > 0, b < 0,

ax^2 and –b |x| are non negative for all x,

i.e. $ax^2 - b|x| \geq 0$

$\therefore$ $ax^2 - b|x|$ is minimum at x = 0 when a > 0, b < 0.

78. d

Family	Adults	Children
1	0, 1, 2	3, 4, 5, ……….
II	0, 1, 2	3, 4, 5, ……….
III	0, 1, 2	3, 4, 5, ……….

As per the question, we need to satisfy three conditions namely:

1. Adults (A) > Boys (B)
2. Boys (B) > Girls (G)
3. Girls (G) > Families (F)

Clearly, if the number of families is 2, maximum number of adults can only be 4. Now, for the second condition to be satisfied, every family should have atleast two boys and one girl each. This will result in non-compliance with the first condition because adults will be equal to boys. If we consider the same conditions for 3 families, then all three conditions will be satisfied.

79. c Given equation is x + y = xy

$\Rightarrow xy - x - y + 1 = 1$

$\Rightarrow (x - 1)(y - 1) = 1$

$x - 1 = 1 \; \& \; y - 1 = 1$ or $x - 1 = -1 \; \& \; y - 1 = -1$

Clearly (0, 0) and (2, 2) are the only pairs that will satisfy the equation.

80. c Given $a_1 = 81.33$; $a_2 = -19$

Also:

$a_j = a_{j-1} - a_{j-2}$, for $j \geq 3$

$\Rightarrow a_3 = a_2 - a_1 = -100.33$

$a_4 = a_3 - a_2 = -81.33$

$a_5 = a_4 - a_3 = 19$

$a_6 = a_5 - a_4 = +100.33$

$a_7 = a_6 - a_5 = +81.33$

$a_8 = a_7 - a_6 = -19$

Clearly onwards there is a cycle of 6 and the sum of terms in every such cycle = 0. Therefore, when we add a_1, a_2, a_3... upto a_{6002}, we will eventually be left with $a_1 + a_2$ only i.e. 81.33 – 19 = 62. 33.

81. b $u = (\log_2 x)^2 - 6\log_2 x + 12$

$x^u = 256$

Let $\log_2 x = y \Rightarrow x = 2^y$

$$x^u = 2^8 \Rightarrow uy = 8 \Rightarrow u = \frac{8}{y}$$

$$\frac{8}{y} = y^2 - 6y + 12 \Rightarrow y^3 - 6y^2 + 12y - 8 = 0$$

$$\Rightarrow (y-2)^3 = 0 \Rightarrow y = 2$$

$$\Rightarrow x = 4, \quad u = 4$$

82. a Since Group (B) contains 23 questions, the marks associated with this group are 46.
Now check for option (1). If Group (C) has one question, then marks associated with this group will be 3. This means that the cumulative marks for these two groups taken together will be 49. Since total number of questions are 100, Group (A) will have 76 questions, the corresponding weightage being 76 marks. This satisfies all conditions and hence is the correct option. It can be easily observed that no other option will fit the bill.

83. c Since Group (C) contains 8 questions, the corresponding weightage will be 24 marks. This figure should be less than or equal to 20% of the total marks. Check from the options . Option (3) provides 13 or 14 questions in Group (B), with a corresponding weightage of 26 or 28 marks. This means that number of questions in Group (A) will either be 79 or 78 and will satisfy the desired requirement.

84. d $\dfrac{30^{65}-29^{65}}{30^{64}+29^{64}} > 1$

as $30^{65}-29^{65} > 30^{64}+29^{64}$

$30^{64}(30-1) > 29^{64}(29+1)$

$30^{64} \times 29 > 29^{64} \times 30$

$30^{63} > 29^{63}$

Hence option (d)

85. d If p = 1! = 1
Then p + 2 = 3 when divided by 2! remainder will be 1.
If p = 1! + 2 × 2! = 5
Then p + 2 = 7 when divided by 3! remainder is still 1.
Hence p = 1! + (2 × 2!) + (3 × 3!) + … + (10 × 10!) when divided by 11! leaves remainder 1

Alternative method:
P = 1 + 2.2! + 3.3!+ ….10.10!
= (2 –1)1! + (3 – 1)2! + (4 – 1)3! + ….(11 – 1)10!
=2! – 1! + 3! – 2! + ….. 11! –10!
= 1 + 11!
Hence the remainder is 1.

86. c $a_1 = 1$, $a_{n+1} - 3a_n + 2 = 4n$
$a_{n+1} = 3a_n + 4n - 2$
when n = 2 then $a_2 = 3 + 4 - 2 = 5$
when n = 3 then $a_3 = 3 \times 5 + 4 \times 2 - 2 = 21$
So, it is satisfying $3^n - 2 \times n$
Hence $a_{100} = 3^{100} - 2 \times 100$

87. d $P = \log_x\left(\dfrac{x}{y}\right) + \log_y\left(\dfrac{y}{x}\right)$

$= \log_x x - \log_x y + \log_y y - \log_y x$

$= 2 - \log_x y - \log_y x$

$t = \log_x y$

$\Rightarrow p = 2 - \dfrac{1}{t} - t = -\left(\sqrt{t} + \dfrac{1}{\sqrt{t}}\right)^2$

Which can never be 1

88. c $x = \sqrt{4+\sqrt{4-x}} \Rightarrow x^2 = 4 + \sqrt{4-x}$

$\left(x^2 - 4\right) = \sqrt{4-x}$

Now put the values from options.
Only 3rd option satisfies the condition.

89. d There are two equations to be formed 40 m + 50 f = 1000
250 m + 300 f + 40 × 15 m + 50 × 10 × f = A
850 m + 8000 f = A
m and f are the number of males and females A is amount paid by the employer.
Then the possible values of f = 8, 9, 10, 11, 12
If f = 8
M = 15
If f = 9, 10, 11 then m will not be an integer while f = 12 then m will be 10.
By putting f = 8 and m = 15, A = 18800. When f = 12 and m = 10 then A = 18100
Therefore the number of males will be 10.