

Introduction :

Vedic mathematics & FastMaths

"FastMaths" is a system of reasoning and mathematical working based on ancient Indian teachings called Veda. It is fast , efficient and easy to learn and use.

It is being taught in some of the most prestigious institutions in England and Europe. NASA scientists applied its principles in the area of artificial intelligence.

Vedic mathematics, which simplifies arithmetic and algebraic operations, has increasingly found acceptance the world over. Experts suggest that it could be a handy tool for those who need to solve mathematical problems faster by the day.

In what way FastMaths Methods are different from Conventional Methods?

FastMaths provides answer in one line where as conventional method requires several steps.

What is Vedic Mathematics?

It is an ancient technique, which simplifies multiplication, divisibility, complex numbers, squaring, cubing, square and cube roots. Even recurring decimals and auxiliary fractions can be handled by Vedic mathematics. Vedic Mathematics forms part of Jyotish Shastra which is one of the six parts of Vedangas. The Jyotish Shastra or Astronomy is made up of three parts called Skandas. A Skanda means the big branch of a tree shooting out of the trunk.

Who Brought Vedic Maths to limelight?

The subject was revived largely due to the efforts of Jagadguru Swami Bharathikrishna Tirthaji of Govardhan Peeth, Puri Jaganath (1884-1960). Having researched the subject for years, even his efforts would have gone in vain but for the enterprise of some disciples who took down notes during his last days.

What is the basis of Vedic Mathematics?

The basis of Vedic mathematics, are the 16 sutras, which attribute a set of qualities to a number or a group of numbers. The ancient Hindu scientists (Rishis) of Bharat in 16 Sutras (Phrases) and 120 words laid down simple steps for solving all mathematical problems in

easy to follow 2 or 3 steps.

Vedic Mental or one or two line methods can be used effectively for solving divisions, reciprocals, factorisation, HCF, squares and square roots, cubes and cube roots, algebraic equations, multiple simultaneous equations, quadratic equations, cubic equations, bi-quadratic equations, higher degree equations, differential calculus, Partial fractions, Integrations, Pythagoras theorem, Apollonius Theorem, Analytical Conics and so on.

What is the speciality of Vedic Mathematics?

Vedic scholars did not use figures for big numbers in their numerical notation. Instead, they preferred to use the Sanskrit alphabets, with each alphabet constituting a number. Several mantras, in fact, denote numbers; that includes the famed Gayatri mantra, which adds to 108 when decoded.

How important is Speed?

How fast you can solve a problem is very important. There is a race against time in all the competitions. Only those people having fast calculation ability will be able to win the race. Time saved can be used to solve more problems or used for difficult problems.

Is it useful today?

Given the initial training in modern maths in today's schools, students will be able to comprehend the logic of Vedic mathematics after they have reached the 8th standard. It will be of interest to every one but more so to younger students keen to make their mark in competitive entrance exams.

India's past could well help them make it in today's world.

It is amazing how with the help of 16 Sutras and 16 sub-sutras, the Vedic seers were able to mentally calculate complex mathematical problems.

Introduction :

Learn to calculate 10-15 times faster.

"FastMaths" is a system of reasoning and mathematical working based on ancient Indian teachings called Veda. It is fast , efficient and easy to learn and use.

Example 1 : Finding Square of a number ending with 5

To find the square of 75

Do the following

**Multiply 5 by 5 and put 25 as your right part of answer.
Multiply 7 with the next higher digit ie $(7+1)=8$ gives
56 as the left part of the answer, Answer is 5625**

Example 2 : Calculate 43×47

The answer is 2021 Same theory worked here too.

The above 'rule' works when you multiply 2 numbers with units digits add up to 10 and tenth place same

Example 3 : Find 52×58 ? Answer = 3016 How long this take ?

Example 4: Multiply 52×11

Answer is 572

Write down the number being multiplied and put the total of the digits between 2 digits

52×11 is [5 and $5+2=7$ and 2] , answer is 572

Example 5: Can you find the following within less than a minute?

a) $1001/13$?

b) $1/19$?

Now you can learn Fastmaths techniques with ease at your home in your spare time

Chapter 1 : Numbers

1.1 Numbers

Numbers begins at 1. All other numbers come from one. There are only nine numbers and a zero.

NUMBERS	
0 ? ZERO	5 ? FIVE
1 ? ONE	6 ? SIX
2 ? TWO	7 ? SEVEN
3 ? THREE	8 ? EIGHT
4 ? FOUR	9 ? NINE ?

Starting from number 1 all whole numbers are generated using " By one more than one before".

2 is more than 1; 4 is more than 3; 6 is more than 5 and so on. ?

Whole numbers are also called Natural Numbers
--

Assignments

1. Which Number is 1 more than

- a) 19**
- b) 40**
- c) 189**
- d) 23**
- e) 4589**

2.Which number is 1 less than

- a) 29**

- b) 48**
- c) 2339**
- d) 5**
- e) 65320**

Assignments Answers

1. Which Number is 1 more than

- a) 20**
- b) 41**
- c) 190**
- d) 24**
- e) 4590**

2. Which number is 1 less than

- a) 28**
- b) 47**
- c) 2338**
- d) 4**
- e) 65319**

Chapter 1 : Numbers

1.2 Place Value

Since there are only 9 numbers and a zero we count in groups of 10.

- Ten Units make a TEN,
- Ten Tens make a HUNDRED .
- Ten Hundreds make a THOUSAND.

PLACE VALUE			
X	X	X ????????X?	
Thousand	Hundred	Ten	Units

The first seven place values are **UNITS, TENS, HUNDREDS, THOUSANDS, TEN-THOUSANDS, HUNDRED-THOUSANDS, and MILLIONS.**

In any number the value of a digit depends upon its position

- The 4 in 41 stands for four Tens
- The two in 42 stands for two Units
- The value of the digit 5 in 452 is five Tens, because it is in the tens column.

The following Number can be written as

$$54321 = 54 \times 1000 + 3 \times 100 + 2 \times 10 + 1 \times 1$$

since

- The 54 in 54321 stands for 54 Thousands
- The 3 in 54321 stands for 3 Hundreds
- The 2 in 54321 stands for 2 Tens
- The 1 in 54321 stands for 1 Units

The number 54,321 says fifty four thousand, three hundred

and twenty one ?

Assignments

1. Find the value of 4 in the following

- a) 430
- b) 947
- c) 14
- d) 125004

2. Write the following numbers in Words

- a) 57
- b) 7002
- c) 405
- d) 9

3. Fill in the blanks

- a) $243 = \underline{\quad} \times 100 + 4 \times \underline{\quad} + \underline{\quad} \times 3$
- b) $45 = 1000 \times \underline{\quad} + 100 \times \underline{\quad} + 10 \times \underline{\quad} + 1 \times \underline{\quad}$
- c) $9 = 100 \times \underline{\quad} + 10 \times \underline{\quad} + 1 \times \underline{\quad}$

4. Write the following numbers in Figures

- a) Two hundred and thirty five
- b) Nine thousand and twenty nine
- c) Four million
- d) Sixty-eight
- e) Twenty four thousand

Assignments Answers

1. Find the value of 4 in the following

- a) HUNDRED
- b) TEN
- c) UNITY
- d) UNITY

2. Write the following numbers in Words

- a) Fifty Seven
- b) Seven thousand two
- c) Four hundred Five
- d) Nine

3. Fill in the blanks

- a) $243 = 2 \times 100 + 4 \times 10 + 1 \times 3$
- b) $45 = 1000 \times 0 + 100 \times 0 + 10 \times 4 + 1 \times 5$
- c) $9 = 100 \times 0 + 10 \times 0 + 1 \times 9$

4. Write the following numbers in Figures

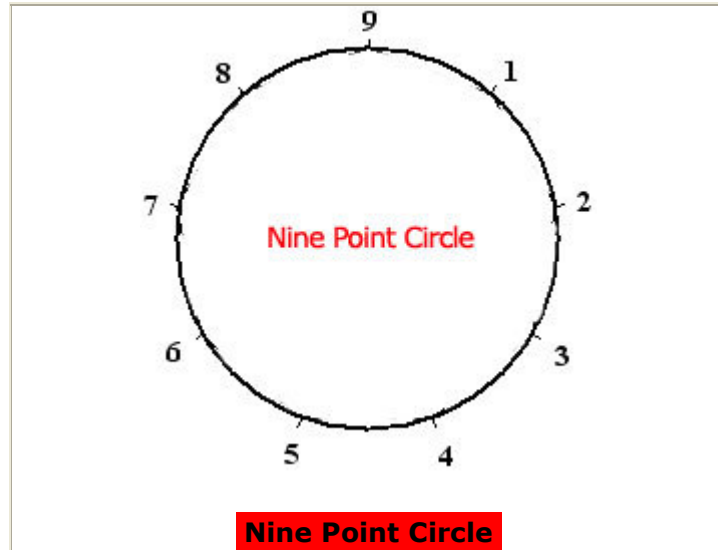
- a) 235
- b) 9029
- c) 4000000
- d) 68
- e) 24000

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Chapter 1 : Numbers

1.3 9-Point Circle

The basic numbers always remain one to nine.



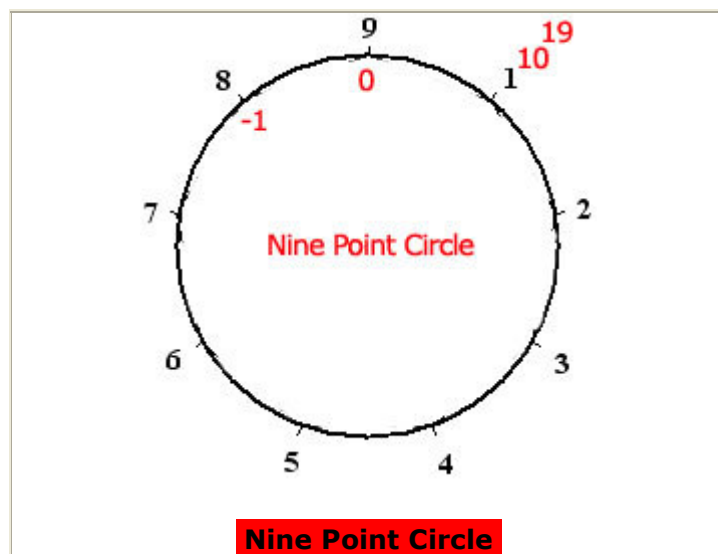
We can represent 9 numbers as shown above. This circle is called a nine-point circle.

The number 1 is the absolute and is inside everything.

The number 1 is a factor of every number and every number is a factor to itself. ?

Where do we add 10 on a nine-point Circle?.

Now where do we add 0 ?



Chapter 1 : Numbers

1.3.2 Product:

When two numbers multiplied together the answer is called *product*.

Example

- The product of 3 and 6 is 18??
- The product of 5 and 9 is 45? ?

Multiplying by 1 brings about no change

Any number when multiplied by 0 gives 0

Assignments

Find the Product of

a) 5×4

b) 7×9

c) 6×2

d) 1×0

e) 12×1

Assignments Answers

Find the Product of

a) 20

b) 63

c) 12

d) 0

e) 12

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Chapter 1 : Numbers

1.3.3 Factors:

Numbers, which multiplied together to give a product, are called *factors*.

3 and 8 are factors of 24, because $24 = 3 \times 8$?

A number may also be seen as a factor of itself. Some numbers have more than one pair as factors

All numbers have one and themselves as a factor.

Example 1: Find Factors of 36 ?

Factors of 36

36 can be expressed as $1 \times 36 = 36$, $2 \times 18 = 36$, $3 \times 12 = 36$, $4 \times 9 = 36$, $6 \times 6 = 36$

Factors of 36 are 1,2,3,4,6,9,12,18,36.

The number 1 is a factor of every number

1.3.3.1 Factor pairs

Number 18 has 6 factors; 1,2,3,6, 9,18

18 can be expressed as $1 \times 18 = 18$, $2 \times 9 = 18$, $3 \times 6 =$

18

Arrange Pair factors like (1X18),(2X9), (3X6).?These pair of numbers is called *factor pairs*.

Factor pairs of 18 are (1X18),(2X9), (3X6)

If you know one factor of a number, you can get another using factor pairs.

If you know 44 can be divided by 4, than another factor of 44 must be 11 since $11 \times 4 = 44$

Assignments

List all factors and list factor pairs if any.

- a) 64
- b) 48
- c) 128
- d) 27
- e) 37

Assignments Answers

List all factors and list factor pairs if any.

a) 64

- Factors 1,2,4,8,16,32,64
- Factor Pairs (1,64) (2,32) (4,16) (8,8)

b) 48

- Factors 1,2,3,4,6,8,12,16,24,48
- Factor Pairs (1,48) (2,24) (3,16) (4,12) (6,8)

c) 128

- Factors 1,2,4,8,16,32,64,128
- Factor Pairs (1,128) (2,64) (4,32) (8,16)

d) 27

- Factors 1,3,9,27
- Factor Pairs (1,27) (3,9)

e) 37

- Factors 1,37
- Factor Pairs (1,37)

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Chapter 1 : Numbers

1.3.3.2 Highest common factor (HCF)

Suppose we have 2 numbers 70 and 99

$$70 = 2 \times 5 \times 7$$
$$99 = 3 \times 3 \times 11$$

Looking at the factors, there is no common factor except number 1. There is no factor of one number, which is also a factor of the other number, except for 1. Such pair of numbers is called *relatively prime*; they are prime in relation to each other.

Example 1: Check 18 and 30

$$18 = 2 \times 3 \times 3$$
$$30 = 2 \times 3 \times 5$$

So 18 and 30 are not relatively prime, they have factors in common

**Both numbers can be divided by 2, 3 and $2 \times 3 = 6$
Of these three factor numbers the number 6 is the highest
Common Factor (HCF)**

Example 2: Check 48 and 72

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

So 48 and 72 are not relatively prime, they have factors in common. Of these factor numbers the number $2 \times 2 \times 2 \times 3 = 24$ is the highest Common Factor (HCF)

Example 3: Check 140 and 27

$$140 = 2 \times 2 \times 5 \times 7$$

$$27 = 3 \times 3 \times 3$$

So 140 and 27 are relatively prime. The highest Common Factor (HCF) = 1

When numbers are close together the HCF will also be a factor of the sum and of the difference of the numbers ?

Example 4: Find HCF of 411 and 417?

The above note means the HCF will divide into 411 and 417
also $411 + 417 = 828$

$$417 \div 411 = 6$$

This means that HCF is either 6 or a factor of 6 (6 or 3 or 2 or 1).

Since 6 is not a factor of 411 and 417 , test for 3 or 2
 $HCF(411,417) = 3$

Example 5: Find HCF of 90 and 102

This means the HCF will divide into 102 and 90 also

$$102 + 90 = 192$$

$$102 \div 90 = 12$$

This means that HCF is either 12 or a factor of 12 (12, 6,4,3,2,1)

3 is a common factor of 90 and 102
And 2 also, but not 4 ,Therefore $2 \times 3 = 6$, HCF = 6

$$\text{HCF}(90,102)= 6$$

Assignments

1. Find the following

- a) $\text{HCF}(80,20)=$
- b) $\text{HCF}(68,24)=$
- c) $\text{HCF}(88,38)=$
- d) $\text{HCF}(88,82)=$
- e) $\text{HCF}(63,18)=$

Assignments Answers

1. Find the following

a) $\text{HCF}(80,20)=20$

$$\begin{aligned} 80 &= 2 \times 2 \times 2 \times 2 \times 5 \\ 20 &= 2 \times 2 \times 5 \end{aligned}$$

So 80 and 20 are not relatively prime, they have factors in common. Of these factor numbers the number $2 \times 2 \times 5 = 20$ is the highest Common Factor (HCF)

b) $\text{HCF}(68,24)=4$

$$\begin{aligned} 68 &= 2 \times 2 \times 17 \\ 24 &= 2 \times 2 \times 6 \end{aligned}$$

So 68 and 24 are not relatively prime, they have factors in common. Of these factor numbers the number $2 \times 2 = 4$ is the highest Common Factor (HCF)

c) $\text{HCF}(88,38)= 2$

$$\begin{aligned} 88 &= 2 \times 2 \times 2 \times 11 \\ 38 &= 2 \times 19 \end{aligned}$$

So 88 and 38 are not relatively prime, they have factors in common. Of these factor numbers the number 2 is the highest Common Factor (HCF)

$$d) \text{HCF}(88,82)=2$$

$$\begin{aligned} 88 &= 2 \times 2 \times 2 \times 11 \\ 82 &= 2 \times 41 \end{aligned}$$

So 88 and 82 are not relatively prime, they have factors in common. Of these factor numbers the number 2 is the highest Common Factor (HCF)

$$e) \text{HCF}(63,18)=9$$

$$\begin{aligned} 63 &= 3 \times 3 \times 7 \\ 18 &= 2 \times 3 \times 3 \end{aligned}$$

So 63 and 18 are not relatively prime, they have factors in common. Of these factor numbers the number $3 \times 3 = 9$ is the highest Common Factor (HCF)

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Chapter 1 : Numbers

1.3.4 Divisibility

The number 1 is not a product and cannot be divided. A number, which is a product, is divisible by any one of its factors.

10 is a product of 2 and 5 and so 2 and 5 are factors of 10.

10 can be divided by 2 or 5 without any reminders

$$10 / 5 = 2 \text{ or } 10 / 2 = 5$$

1.3.5 Prime Numbers

Some numbers will have only one pair of factors

$11 = 11 \times 1$ and there are no other numbers which multiply together to give 11.

Such numbers are called *prime numbers*. ?

The first few prime numbers are 1, 3, 5, 7, 11, 13, 17, 19..... ?

Assignments

Find Prime Number from the following
31, 49, 147, 97, 81

Assignments Answer

Find Prime Number from the following
31, 49, 147, 97, 81
Answer : 31, 97.

All other numbers have more than 1 factor. 49 can be written as 7×7 and 81 can be written as 3×27 or 9×9

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Chapter 1 : Numbers

1.3.6 The number 2.

The number two stands for 2 types of beings in the creation, good and evil. So the number two divides the creation into two types of beings. It also divides the number into two sorts, odd and even.

1.3.6.1 Odd and Even Numbers ?

Numbers which have 2 as a factor are called Even Numbers, which do not have 2 as a factor, are called Odd.

The even numbers are 2, 4, 6, 8, 10, 12, 14, and so on.

Any number which ends in a 2,4,6,8, or 0 is even.

The odd numbers are 1,3,5,7,9,11,13, and so on.

Any number, which ends in a 1,3,5,7 or 9, is an odd number. An odd number cannot be divided into two equal parts.

1.3.6.2. Multiples

Multiple means many. If we take number 1 many times, we arrive at 1,2,3,4,5.... Similarly if we take number two many times, we arrive at 2,4,6,8.... These are all multiples of two.

A multiple of a number is that number multiplied by any number ? ?

Assignments

**1. Find the Odd numbers from the following
3, 6, 7, 12, 15, 19, 21, 10,100**

**2. Find the Even numbers from the following
13, 26, 70, 12, 19, 39, 61, 102,150**

Assignments Answers

**1. Find the Odd numbers from the following
3, 7,15, 19, 21**

**2. Find the Even numbers from the following
26, 70, 12, 102,150**

Chapter 1 : Numbers

1.3.7. The Number 9

In our number system number nine is the largest digit

The digital root of a number can be obtained by summing the digits of the number, for example, for 23, digital root is $2 + 3 = 5$. ? We will learn more about digital roots in chapter 3.

The digit sum or Digital root of a number is unchanged if 9 is added to it or subtracted from it.

Table of 9

- **$9 \times 1 = 9$**
- **$9 \times 2 = 18$**
- **$9 \times 3 = 27$**
- **$9 \times 4 = 36$**
- **$9 \times 5 = 45$**
- **$9 \times 6 = 54$**
- **$9 \times 7 = 63$**
- **$9 \times 8 = 72$**
- **$9 \times 9 = 81$**

If you read the answers as two columns the left column goes up by one more than one before (1,2,3,4,5....) and the right column goes down by one less than the one before (9,8,7,6,5...)

1.3.7.1 By Addition and By Subtraction?

When adding or subtracting numbers which end in 9 or 9's use the following method.

Example : Find $75 + 39$

Add 40 to 75 and take 1 off. $75 + 39 = 75 + 30 - 1 = 114$

Example : Find $122 - 59$

Subtract 60 from 122 and put 1 back. $122 - 60 + 1 = 63$

Assignments

Find the following

a) $132 + 49 =$

b) $34 + 29 =$

c) $63 - 19 =$

d) $56 - 9 =$

e) $79 + 19 =$

Assignments Answers

Find the following

a) $132 + 49 = 181$

- **Add 50 to 132 and take 1 off.**
- **$132 + 49 = 132 + 50 - 1 = 182 - 1 = 181$**

b) $34 + 29 = 63$

- **Add 30 to 34 and take 1 off.**
- **$34 + 29 = 34 + 30 - 1 = 64 - 1 = 63$**

c) $63 - 19 = 44$

- **Subtract 20 to 63 and add 1.**
- **$63 - 19 = 63 - 20 + 1 = 43 + 1 = 44$**

d) $56 - 9 = 47$

- Subtract 10 to 56 and add 1.
- $56 - 9 = 56 - 10 + 1 = 46 + 1 = 47$

e) $79 + 19 = 98$

- Add 20 to 79 and take 1 off.
- $79 + 19 = 79 + 20 - 1 = 99 - 1 = 98$

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Chapter 1 : Numbers

1.3.8. The Number 10

The number ten is 1 with a zero next to it. So multiplying a number by ten the answer is the same but with a zero on the end.

Example 1 Find 84×10

$$84 \times 10 = 840$$

Example 2: Find 77×10

$$77 \times 10 = 770$$

The effect of multiplying a number by ten is to move every digit in that number one place to the left and a zero is added to the end.

When multiplying decimal fraction by 10. Each number is moved into the next column to the left. The effect of this is to move the decimal point one place to the right.

Example 3. Find 0.4761×10

$$0.4761 \times 10 = 4.761$$

Assignments

Find the following

$$a) 44 \times 10 =$$

$$b) 71 \times 10 =$$

$$c) 0.123 \times 10 =$$

$$d) 0.567 \times 10 =$$

$$e) 10.25 \times 10 =$$

Assignments Answers

Find the following

$$a) 44 \times 10 = 440$$

$$b) 71 \times 10 = 710$$

$$c) 0.123 \times 10 = 1.23$$

$$d) 5.67$$

$$e) 102.5$$

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Chapter 1 : Numbers

1.3.9 Square Numbers

			000
		00	000
	0	00	000 ?
	1	2	3
Number of Squares	1	2	3
Number of Counts	1	4	9

The numbers 1,4,9,16...are called Square Numbers because you can arrange the number of counters to form a Square. The 4 Counters are in 2 rows of 2. The 9 counters are in 3 rows and 3 columns.

$$1 \times 1 = 1$$

$$2 \times 2 = 4$$

$$3 \times 3 = 9$$

So if we square a number we multiply it by itself.

3 Squared is $3 \times 3 = 9$;
 4 Squared is $4 \times 4 = 16$; ?

Square numbers always have an odd number of factors. All other numbers have an even number of factors

1.3.10 Triangular Numbers

			0
		0	00
	0	00	000 ?
	1	2	3
Number of Squares	1	2	3
Number of Counts	1	3	6

The numbers 1,3,6....are called Triangular Numbers because you can arrange the number of counters to form a Triangle.

1.3.11 Cube Numbers

	1	2	3
Number of Cube	1	2	3
Number of Counts	1	8	27

Numbers 1, 8, 27 are called Cube numbers because you can arrange that many cubes to form a larger cube.

The length, breadth and height of cubes are always same.

- $1 \times 1 \times 1 = 1$
- $2 \times 2 \times 2 = 8$
- $3 \times 3 \times 3 = 27$

?

If we cube a number we multiply it by itself twice?

Examples:

3 cubed is $3 \times 3 \times 3 = 27$;

4 Cubed is $4 \times 4 \times 4 = 64$; ??

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Chapter 1 : Numbers

1.3.12.1 Doubling and Halving

Multiply by 4

Since $4 = 2 \times 2$, we can multiply a number 4 by doubling it and doubling the answer ?

Find $35 \times 4 = ?$

**Simply double 35 to 70, then double 70 to 140.
 $35 \times 4 = 140$**

Multiply by 8

Since $8 = 2 \times 2 \times 2$, we can multiply a number 8 by doubling it three times ?

Find $26 \times 8 = ?$

**Simply double 26 to 52, doubling 52 to 104, doubling 104 gives 208.
 $26 \times 8 = 208$**

Divide by 4 ?

Similarly if we halved a number and then halved again we would be dividing the number by 4.

Divide 72 by 4

We halve 72 twice; Half of 72 is 36, half of 36 is 18

Divide by 8

Similarly if we halved a number 3 times we would be dividing the number by 8

Divide 104 by 8

We halve 104 three times; Half of 104 is 52, Half of 52 is 26, half of 26 is 13

General

Find 14×18

Halving 14 and 18 gives 7 and 9. Since $7 \times 9 = 63$, we double this twice. We get 126 and 252

So $14 \times 18 = 252$

You will learn more techniques in next chapters.

Assignments

Find the following

a) $128 / 8 =$

b) $28 \times 4 =$

c) $7 \times 8 =$

d) $64 / 4 =$

Assignments Answers

Find the following

a) $128 / 8 = 16$

b) $28 \times 4 = 112$

c) $7 \times 8 = 56$

d) $64 / 4 = 16$

Chapter 1 : Numbers

Assignment ? 1

1. Which Number is 1 more than

1) 199

2) 401

2. Which number is 1 less than

1) 20

2) 309

3. Find the value of 4 in the following

1) 430

2) 947

3) 14

4) 125004

4. Write the following numbers in Words

1) 57

2) 7002

3) 405

4) 9

5. Fill in the blanks

1) $243 = \underline{\hspace{1cm}} \times 100 + 4 \times \underline{\hspace{1cm}} + \underline{\hspace{1cm}} \times 3$

2) $7002 = 1000 \times \underline{\hspace{1cm}} + 100 \times \underline{\hspace{1cm}} + 10 \times \underline{\hspace{1cm}} + 1 \times \underline{\hspace{1cm}}$

3) $45 = 1000 \times \underline{\hspace{1cm}} + 100 \times \underline{\hspace{1cm}} + 10 \times \underline{\hspace{1cm}} + 1 \times \underline{\hspace{1cm}}$

4) $9 = 100 \times \underline{\hspace{1cm}} + 10 \times \underline{\hspace{1cm}} + 1 \times \underline{\hspace{1cm}}$

6. Write the following numbers in Figures

1) Two hundred and thirty five

2) Nine thousand and twenty nine

3) Four million

4) Sixty-eight

5) Twenty four thousand

7. Find the next member of the series

1) 2,4,6,8, ??

2) 8,16,24,32 , ?..

3) 27, 25, 23, 21,?..

4) 36,45,54,63,??..

5) 103, 110, 117, 124,?.

8. Addition and Subtraction

1) $6 ? 3 + 2 =$

2) $67 ? 23 =$

3) $24 + 5 ? 2 =$

4) $346 ? 34 + 23 =$

5) $3 + 4 + 5 + 6 =$

9. List all factors and list factor pairs if any.

1) 64

2) 48

3) 128

4) 27

5) 37

10. Find Prime Number from the following
31, 49, 147, 97, 81

11. Find the following

1) $\text{HCF}(80, 20) =$

2) $\text{HCF}(68, 24) =$

3) $\text{HCF}(88, 38) =$

4) $\text{HCF}(88, 82) =$

5) $\text{HCF}(63, 18) =$

6) $\text{HCF}(66, 64) =$

7) $\text{HCF}(57, 33) =$

8) $\text{HCF}(40, 4) =$

9) $\text{HCF}(60, 26) =$

10) $\text{HCF}(74, 52) =$

12 Find the following

1) $128 \div 8$

2) 28×4

3) 7×8

4) $64 \div 4$

13. Write the following numbers in Ascending and Descending orders

1) 97, 63, 37, 39, 30

2) 11, 50, 5, 6, 0

3) 10, 57, 7, 38, 4

4) 60, 4, 66, 4, 23

5) 65, 37, 37, 22, 25

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Chapter 1 : Numbers

Answers

Assignment ? 1

1. Which Number is 1 more than

- **Ans 200**
- **Ans 402**

2. Which number is 1 less than

- **20 Ans 19**
- **309 Ans 308**

3. Find the value of 4 in the following

- **430 Ans Hundred**
- **947 Ans Ten**
- **14 Ans Unity**
- **124002 Ans Thousand**

4. Write the following numbers in Words

- **57 Fifty Seven**
- **7002 Seven Thousand and two**
- **405 Four Hundred Five**
- **09 Nine**

5. Fill in the blanks

- **$243 = 2 \times 100 + 4 \times 10 + 1 \times 3$**
- **$7002 = 1000 \times 7 + 100 \times 0 + 10 \times 0 + 1 \times 2$**
- **$45 = 1000 \times 0 + 100 \times 0 + 10 \times 4 + 1 \times 5$**
- **$9 = 100 \times 0 + 10 \times 0 + 1 \times 9$**

6. Write the following numbers in Figures

- **Two hundred and thirty five = 235**
- **Nine thousand and twenty nine = 925**

- Four million = 4000000
- Sixty-eight = 68
- Twenty four thousand = 24000

7. Find the next member of the series

- 2,4,6,8, ?? 10
- 8,16,24,32, ?.. 40
- 27, 25, 23, 21,?.. 19
- 36,45,54,63,??.. 72
- 103, 110, 117, 124,?.. 131

8. Addition and Subtraction

- $6 \times 3 + 2 = 5$
- $67 \times 23 = 44$
- $24 + 5 \times 2 = 27$
- $346 \times 34 + 23 = 335$
- $3 + 4 + 5 + 6 = 18$

9. List all factors and list factor pairs if any.

- 64 Factors 1,2,4,8,16,32,64
Factor Pairs (1,64),(2,32),(4,16)(8,8)
- 48 Factors 1,2,3,4,6,8,12,16,24,48
Factor Pairs (1,48),(2,24),(3,16),(4,12),(6,8)
- 128 Factors 1,2,4,8,16,32,64,128
Factor Pairs (1,128),(2,64),(4,32),(8,16)
- 27 Factors 1,3,9,27
Factor Pairs (1,27),(3,9)
- 37 Factors 1,37
Factor Pair (1,37)

10. Find Prime Number from the following

- 31 Prime Number
- 49 Not a Prime Number
- 147 Not a Prime Number
- 97 Prime Number
- 81 Not a Prime Number

11. Find the following

- $\text{HCF}(80,20) = 20$
- $\text{HCF}(68,24) = 4$
- $\text{HCF}(88,38) = 2$
- $\text{HCF}(88,82) = 2$
- $\text{HCF}(63,18) = 9$
- $\text{HCF}(66,64) = 2$
- $\text{HCF}(57,33) = 3$
- $\text{HCF}(40,4) = 4$
- $\text{HCF}(60,26) = 2$
- $\text{HCF}(74,52) = 2$

12 Find the following

- $128 / 8 =$ First $128 / 2$ gives 64. again divide by 2 gives 32 , again divide by 2 gives 16 since $8 = 2 \times 2 \times 2$
- $24 \times 4 =$ First 24×2 gives 48 and again 48×2 gives 96 since $4 = 2 \times 2$
- $7 \times 8 =$ First 7×2 gives 14 and again 14×2 gives 28 , again 28×2 gives 56. since $8 = 2 \times 2 \times 2$
- $64 / 4 =$ 64 by 2 gives 32 and again 32 by 2 gives 16

13. Write the following numbers in Ascending and Descending orders

Ascending Order

- 30, 37, 39, 63, 97
- 0, 5, 6, 11, 50
- 4, 7, 10, 38, 57
- 4, 4, 23, 60, 66
- 22, 25, 37, 37, 65

Descending Order

- 97 , 63 , 39 , 37 , 30
- 50 , 11 , 6 , 5 , 0
- 57 , 38 , 10 , 7 , 4
- 66 , 60 , 23 , 4 , 4
- 65 , 37 , 37 , 25 , 22

Chapter 3 : Digital roots or Digital Sum of Numbers

3.1 Digital Roots or Digit Sums

The word **Digit** means the single figure numbers; the numbers from 1 to 9 and zero

Digital Root or Digital Sum of a number : is the remainder when the number is divided by 9.

So for 23, the remainder is 5 because $23 \div 9 = 2$ remainder 5. The digital root is also 5.

The digital root can also be obtained by summing the digits of the number.

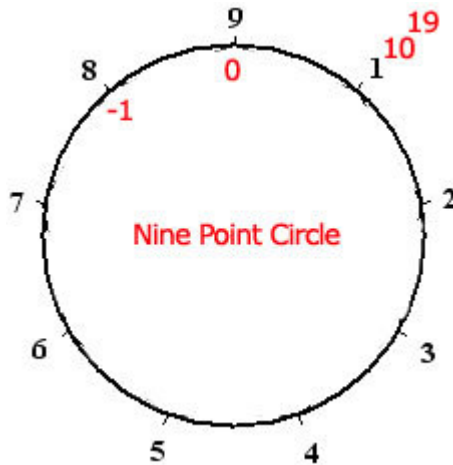
For example,

- Digital sum of 23 is $2 + 3 = 5$.
- Digital sum of 17 is $1 + 7 = 8$
- Digital sum of 763 is $7 + 6 + 3 = 16$. And 16 is a 2-digit number and we add the figures in 16 to get $1 + 6 = 7$. So digital root of 763 is 7

When the sum of digits is greater than 9, you keep adding.
So for 2856, the digital root is $2 + 8 + 5 + 6 = 21$, $2 + 1 = 3$.

For example, with 18, $1 + 8 = 9$, but $18 \div 9 = 2$ remainder 0. Therefore we take a remainder of 0 as being identical with a digital root of 9.

Look at the 9-Point Circle below.



Adding 9 to a number does not affect its digit sum. So 1,10, 19, 28 all have a digit sum of 1

Digital sum of 39409 is $3 + 4 + 0 = 7$, ignore all 9's

Looking again at the 9 point circle, if we count backwards round the circle we see that since 0 comes before 1 and it is logical to put zero at the same place as 9.

In terms of digit sums 9 and 0 are equivalent

Any group of digits in a number that add up to a 9 can also be removed.

Digit sum of 24701 is 5

We see that 2 and 7 which adds up to 9. We can remove 2 and 7 and add up only other digits $4 + 0 + 1 = 5$

Assignments

Q1 Find the digit sum of 16, 27, 203 and 30103

Q2 The digit sum of a 2 digit number is 8 and the digits are same, What is the number?

Q3 The digit sum of a 2 digit number is 9 and the first digit is twice the second . What is the number?

Q4 Find the digit sum of 6437 , 3542 and 673982471

Assignments Answers

Q1 Find the digit sum of 16, 27, 203 and 30103

Ans : Digit Sum of 16 is $1+6 = 7$

Digit Sum of 27 is $2+7 = 9$

Digit Sum of 203 is $2+0+3 = 5$

Digit Sum of 30103 is $3+0+1+0+3 = 7$

Q2 The digit sum of a 2 digit number is 8 and the digits are same, What is the number?

Ans : 44

Q3 The digit sum of a 2 digit number is 9 and the first digit is twice the second . What is the number?

Ans : 36

Q4 Find the digit sum of 6437 , 3542 and 673982471

Ans : Digit Sum of 6437 is 2

Digit Sum of 3542 is 5

Digit Sum of 673982471 is 2

Chapter 3 : Digital roots or Digital Sum of Numbers

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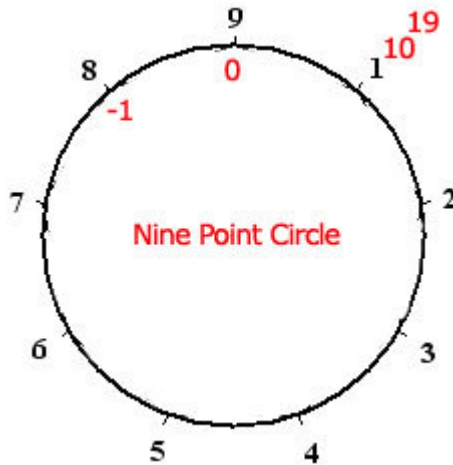
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Digit sum of 24701 is 5

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Q4 Find the digit sum of 6437 , 3542 and 673982471

Assignments Answers

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Ans : Digit Sum of 16 is $1+6=7$

Digit Sum of 27 is $2+7=9$

Digit Sum of 203 is $2+0+3=5$

Digit Sum of 30103 is $3+0+1+0+3=7$

Q2 The digit sum of a 2 digit number is 8 and the digits are same, What is the number?

Ans : 44

Q3 The digit sum of a 2 digit number is 9 and the first digit is twice the second . What is the number?

Ans : 36

Q4 Find the digit sum of 6437 , 3542 and 673982471

Ans : Digit Sum of 6437 is 2

Digit Sum of 3542 is 5

Digit Sum of 673982471 is 2

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Chapter 3 : Digital roots or Digital Sum of Numbers

3.2 Divisibility rules for 9 and 3

An easy test for 9 is to look at the sum of the digits.

Take any number like 243 and add the digits. If the sum is 9 then the number is divisible by 9.

Patterns within the 9? table shown below.

Table of 9

9 X 1 = 9 Digit Sum is 9

9 X 2 = 18 Digit Sum is 9

9 X 3 = 27 Digit Sum is 9

9 X 4 = 36 Digit Sum is 9

9 X 5 = 4 5 Digit Sum is 9
 9 X 6 = 5 4 Digit Sum is 9
 9 X 7 = 6 3 Digit Sum is 9
 9 X 8 = 7 2 Digit Sum is 9
 9 X 9 = 8 1 Digit Sum is 9
 9 X 10 = 9 0 Digit Sum is 9

When a number is divisible by 9 the digit sum is also 9

When a number is divisible by 3 the digit sum is 3,6 or 9

Assignments

Check the following numbers divisible by 3

Q1. 12

Q2. 15

Q3. 20

Q4. 36

Q5. 50

Check the following numbers divisible by 9

Q1. 18

Q2. 45

Q3. 30

Q4. 12825

Q5. 66273

Assignments Answers

Check the following numbers divisible by 3

Q1. 12 YES

Q2. 15 YES

Q3. 20 NO

Q4. 36 YES

Q5. 50 NO

Check the following numbers divisible by 9

Q1. 18 YES Q2. 45 YES
 Q3. 30 NO Q4. 12825 YES
 Q5. 66273 NO

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Chapter 4 : Digital roots or Digital Sum of Numbers

3.3 Digital roots applied to sequences

Various symmetries can be discovered within sequences by plotting the digital roots on a circle of nine points.

Answers to the multiplication tables provide some easy examples as shown below.

2X table and Digital Roots

2	4	6	8	10	12	14	16	18	20	22	24
2	4	6	8	1	3	5	7	9	2	4	6

3X table and Digital Roots

3	6	9	12	15	18	21	24	27	30	33	36
3	6	9	3	6	9	3	6	9	3	6	9

4X table and Digital Roots

4	8	12	16	20	24	28	32	36	40	44	48
4	8	3	7	2	6	1	5	9	4	8	3

5X table and Digital Roots

5	10	15	20	25	30	35	40	45	50	55	60
5	1	6	2	7	3	8	4	9	5	1	6

6X table and Digital Roots

6	12	18	24	30	36	42	48	54	60	66	72
6	3	9	6	3	9	6	3	9	6	3	9

7X table and Digital Roots

7	14	21	28	35	42	49	56	63	70	77	84
7	5	3	1	8	6	4	2	9	7	5	3

8X table and Digital Roots

8	16	24	32	40	48	56	64	72	80	88	96
8	7	6	5	4	3	2	1	9	8	7	6

9X table and Digital Roots

9	18	27	36	45	54	63	72	81	90	99	108
9	9	9	9	9	9	9	9	9	9	9	9

10X table and Digital Roots

10	20	30	40	50	60	70	80	90	100	110	120
1	2	3	4	5	6	7	8	9	1	2	3

11X table and Digital Roots

11	22	33	44	55	66	77	88	99	101	112	123
2	4	6	8	1	3	5	7	9	2	4	6

12X table and Digital Roots

12	24	36	48	60	72	84	96	108	120	132	144
3	6	9	3	6	9	3	6	9	3	6	9

The pattern for a number is the same as the pattern of its complement from 9.

For example:

The pattern for 4 is the same as the pattern for 5 [from 9, complement of 4 is 5] except one is the reverse of the other.

Digital root patterns for two-digit multiplication tables are the same as those of the digital roots of those two-digit numbers.

For example:

The pattern for 12 is the same as that for $1+2 = 3$.

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Chapter 3 : Digital roots or Digital Sum of Numbers

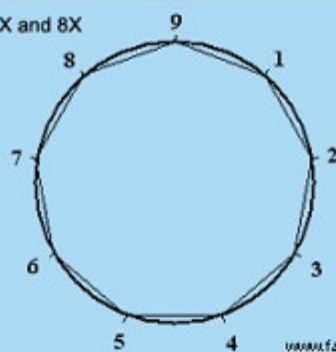
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Various symmetries can be discovered within sequences by plotting the digital roots on a circle of nine points.

Answers to the multiplication tables provide some easy examples.

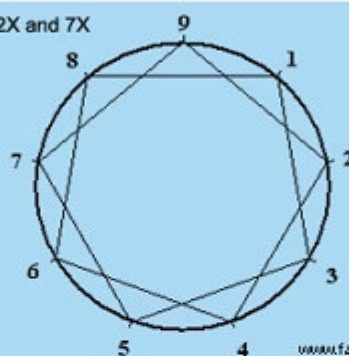
The pattern are shown below

PATTERN FOR 1X and 8X



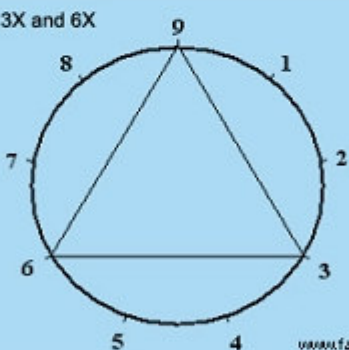
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PATTERN FOR 2X and 7X



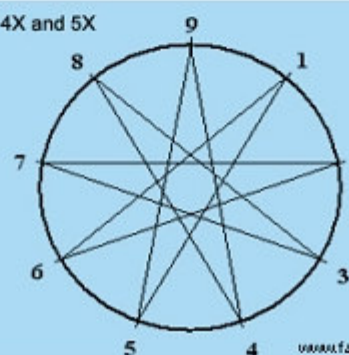
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PATTERN FOR 3X and 6X



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PATTERN FOR 4X and 5X



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Chapter 3 : Digital roots or Digital Sum of Numbers

3.5 Useful application of Digital sums

Checking the answers to addition and subtraction sums

3.5.1 Addition: Digital Sum Check

3.5.1.1 Sum Involving No Carriers

Example 1: Find $4352 + 342$ and check the answer using digit sum

$$\begin{array}{r} 4352 + \\ 342 \\ \hline 4694 \end{array}$$

Line the numbers up with the units under units. There are no carriers so we simply add in each column

$$2 + 2 = 4, \quad 5 + 4 = 9, \quad 3 + 3 = 6 \text{ and } 4 + 0 = 4$$

Digit sum of 4352 is $4 + 3 + 5 + 2 = 14$, again digit sum of 14 gives $1 + 4 = 5$

Digit sum of 342 is $3 + 4 + 2 = 9$

Sum of digital roots = $5 + 9 = 14$, again digit sum of 14 gives $1 + 4 = 5$

The answer should have a digit sum of 5

Verifying the digit sum of the answer $4 + 6 + 9 + 4 = 23$, Digit sum of 23 is $2 + 3 = 5$

Example 2. Find $32 + 12$ and check the answer using digit sum

$$\begin{array}{r} 32 + \\ 12 \\ \hline 44 \end{array}$$

Digit sum of 32 is $3 + 2 = 5$ and the digit sum of 12 is

$1+2 = 3$. The sum total of the digital sums is $5+3 = 8$. If the answer is correct the digit sum of the answer should be 8. i.e $4 + 4 = 8$.

3.5.1.2 Sum Involving Carriers

Example 1. Find $76 + 18$ and check the answer using digit sum

$$\begin{array}{r} 76 + \\ 18 \\ \hline 94 \end{array}$$

Carrying 1 over to the left gives 9 4

Add $8 + 6 = 14$, so write down 4 in the unit's column and 'carry ' 1 to the next column. Add this carry 1 to $7+1$ and write 9 in tens column.

Example 2: Add 375 and 108 and check the number

$$\begin{array}{r} 375 + \\ 108 \\ \hline 483 \end{array}$$

Digit sum of 375 is $3 + 7 + 5 = 15$, again $1+ 5 = 6$ and the digit sum of 208 is $2 + 0 + 8 = 10$ or 1. The sum total of the digital sums is $6 + 1 = 7$. If the answer is correct the digit sum of the answer should be 6. i.e $5+8+3 = 16$, again $1+ 6 =7$.

3.5.2 Subtraction: Digital Sum Check

Example 1: Find $57 - 22$ and check the answer using digit sum

$$\begin{array}{r} 57 - \\ 22 \\ \hline 35 \end{array}$$

Digit sum of 57 is $5 + 7 = 12$, again $1 + 2$, the digit sum is 3. The digit sum of 22 is $2 + 2 = 4$. The difference of the digital sums is $3 - 4 = 3 + 9 - 4 = 8$. If the answer is correct the digit sum of the answer should be 8. i.e $3 + 5 = 8$.

Example 2: Find $518 - 211$ and check the answer using digit sum

$$\begin{array}{r} 518 + \\ 211 \\ \hline 307 \end{array}$$

Digit sum of 518 is $5 + 1 + 8 = 14$, again $1 + 4 = 5$ and the digit sum of 211 is $2 + 1 + 1 = 4$. The difference of the digital sums is $5 - 4 = 1$. If the answer is correct the digit sum of the answer should be 1, i.e $3 + 0 + 7 = 10$, again $1 + 0 = 1$.

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Chapter 4 : Digital roots or Digital Sum of Numbers

3. 6 Assignments

Q1. Add the following and check your answers using digital roots

1. $34 + 46$
2. $54 + 27$
3. $198 + 276$
4. $555 + 77$
5. $4530 + 672$

Q2. Subtract the following and check your answers using digital roots

1. $62 - 27$
2. $812 - 344$
3. $503 - 274$
4. $6005 - 2739$
5. $9786 - 6879$

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Chapter 4 : Digital roots or Digital Sum of Numbers

3. 6 Assignments Answers

Q1. Add the following and check your answers using digital roots

1. $34 + 46 = 80$
2. $54 + 27 = 81$
3. $198 + 276 = 474$
4. $555 + 77 = 632$
5. $4530 + 672 = 5202$

Q2. Subtract the following and check your answers using digital roots

1. $62 - 27 = 35$
2. $812 - 344 = 468$
3. $503 - 274 = 229$
4. $6005 - 2739 = 3266$
5. $9786 - 6879 = 2907$

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Chapter 4 : Multiplication

4.1 Multiplication:

There is no change when any number is multiplied by 1.

When we multiply one number by another then it is increased and becomes further away from one. When 4 is multiplies by 5 it becomes 20 which is further away from 4 and 5.

Using our multiplication techniques, we relate each number very close to another number called base. The difference between the number and the base is termed as deviation.

Deviation may be positive or negative. Positive deviation is written without the positive sign and the negative deviation, is written using a bar or negative sign on the number.

Number	Base	Deviation
15	10	$15-10 = 5$
9	10	$9-10=-1$
98	100	$98-100=-2$
112	100	$112-100=12$
994	1000	$994-1000=-6$
1013	1000	$1013-1000=13$

Example 1: Find the deviation of 94 from base 100

Now deviation can be obtained by ?all from 9 and the last from 10? method i.e, the last digit 4 is subtracted from 10 gives 06 and remaining digit 9 is subtracted from 9 gives 00.

Deviation of 94 from base 100 is 06

Example 2: Find the deviation of 86 from base 100

The last digit 6 is subtracted from 10 gives 04 and remaining digit 8 from 9 gives 1.

Deviation of 86 from base 100 is 14

Assignments

Q1. Write down the deviation from nearest base for the following

- | | |
|------------------------|----------------------|
| 1. 88 from 100 | 5. 423 from 1000 |
| 2. 75 from 100 | 6. 902 from 1000 |
| 3. 8004 from 10000 | 7. 70503 from 100000 |
| 4. 123870 from 1000000 | 8. 9993 from 10000 |

Assignments Answers

Q1. Write down the deviation from nearest base for the following

- | | |
|-----------|----------|
| 1. 12 | 5. 577 |
| 2. 25 | 6. 098 |
| 3. 1996 | 7. 29497 |
| 4. 876130 | 8. 0007 |

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Chapter 1 : Multiplication

4.2 : Multiplication near to the base

4.2.1 Both the numbers are lower than the base.

4.2.1.1 Multiplication using a base of 10

Example 1: Multiply 7 by 8.

Consider the base number as 10 since it is near to both the numbers.

Step 1. Write the numbers one below the other.

$$\begin{array}{r} 7 \times \\ 8 \\ \hline \end{array}$$

Step 2. Take the deviations of both the numbers from the base and represent

$$\begin{array}{r} 7 \quad -3 \quad [\text{Base } 10] \\ 8 \quad -2 \\ \hline \end{array}$$

Remainders 3 and 2 implies that the numbers to be multiplied are both less than 10

Step 3. The product or answer will have two parts, one on the left side and the other on the right. A vertical or a slant line i.e. a slash may be drawn for the demarcation of the two parts.

$$\begin{array}{r} 7 \quad -3 \quad [\text{Base } 10] \\ 8 \quad -2 \\ \hline / \\ \hline \end{array}$$

Step4. The R.H.S. of the answer is the product of the deviations of the numbers. It contains the number of digits equal to number of zeroes in the base.

$$\begin{array}{r} 7 \quad -3 \quad [\text{Base } 10] \\ 8 \quad -2 \\ \hline / \quad (3 \times 2) \\ \hline \end{array}$$

Since base is 10, $3 \times 2 = 6$ can be taken as it is.

Step5. L.H.S of the answer is the sum of one number with

the deviation of the other. It can be arrived at in any one of the four ways.

- i) Cross-subtract deviation 2 on the second row from the original number 7 in the first row $7 - 2 = 5$.
- ii) Cross-subtract deviation 3 on the first row from the original number 8 in the second row $8 - 3 = 5$
- iii) Subtract the base 10 from the sum of the given numbers. $(7 + 8) - 10 = 5$
- iv) Subtract the sum of the two deviations from the base. $10 - (3 + 2) = 5$

Hence 5 is left hand side of the answer.

$$\begin{array}{r}
 7 \quad -3 \quad [\text{Base } 10] \\
 8 \quad -2 \\
 \hline
 5 \quad / \quad 6 \\
 \hline
 \end{array}$$

Step 6 : If R.H.S. contains less number of digits than the number of zeros in the base, the remaining digits are filled up by giving zero or zeroes on the left side of the R.H.S. If the number of digits are more than the number of zeroes in the base, the excess digit or digits are to be added to L.H.S of the answer.

The general form of the multiplication
Let N1 and N2 be two numbers near to a given base in powers of 10, and D1 and D2 are their respective deviations from the base. Then N1 X N2 can be represented as

$$\begin{array}{r}
 N1 \quad D1 \quad [\text{BASE}] \\
 N2 \quad D2 \\
 \hline
 (N1+D2) \text{ OR } (N2+D1) / (D1 \times D2)
 \end{array}$$

Chapter 4 : Multiplication

4.2 : Multiplication near to the base

4.2.1 Both the numbers are lower than the base.

4.2.1.2 Multiplication using a base of 100

Example. 1: Find 97 X 94.

Here base is 100

Deviation of 97 from 100 is -03

Deviation of 94 from 100 is -06

97 -03 [BASE 100]
94 -06

(97-06) or (94-03) / (3X6)

97 -03 [BASE 100]
94 -06

91 / 18

Answer is 9118

Example. 2: Find 98 X 97.

Deviation of 98 from 100 is -02

Deviation of 97 from 100 is -03

Here base is 100

98 -02 [BASE 100]
97 -03

(98-03) or (97-02) / (2X3)

98 -02 [BASE 100]

$$\begin{array}{r}
 97 \quad -03 \\
 \hline
 95 \quad / \quad 06
 \end{array}$$

Answer is 9506

Example. 3: Find 75 X 95.

Here base is 100

Deviation of 75 from 100 is -25

Deviation of 95 from 100 is -05

$$\begin{array}{r}
 75 \quad -25 \quad [\text{BASE } 100] \\
 95 \quad -05 \\
 \hline
 (75-05) \text{ or } (95-25) \quad / \quad (25 \times 5)
 \end{array}$$

$$\begin{array}{r}
 75 \quad -25 \quad [\text{BASE } 100] \\
 95 \quad -05 \\
 \hline
 70 \quad / \quad {}_125
 \end{array}$$

Since the base is 100, we write down 25 and carry 1 over to the left giving us $70 / {}_125 = (70+1) / 25$

Answer is 7125

Assignments

Find the following

Q1. 95 X 99

Q2. 93 X 98

Q3. 76 X 98

Q4. 96 X 98

Q5. 97 X 89

Q6. 98 X 91

Q7. 94 X 93

Q8. 92 X 97

Assignments Answers

Find the following

- Q1. $95 \times 99 = 9405$ Q2. $93 \times 98 = 9114$
Q3. $76 \times 98 = 7448$ Q4. $96 \times 98 = 9408$
Q5. $97 \times 89 = 8633$ Q6. $98 \times 91 = 8918$
Q7. $94 \times 93 = 8742$ Q8. $92 \times 97 = 8924$

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Chapter 4 : Multiplication

4.2 : Multiplication near to the base

4.2.1 Both the numbers are lower than the base.

4.2.1.3 Multiplication using a base of 1000

Example 1: Find 786×998

Here base is 1000

Complement of 786 is 214.

7 from 9 is 2 and 8 from 9 is 1 and 6 from 10 is 4 .

Complement of 998 is 002

$$\begin{array}{r} 786 \quad -214 \quad [\text{BASE } 1000] \\ 998 \quad -002 \end{array}$$

(786-002) or (998-214) / (214X2)

$$\begin{array}{r} 786 \quad -214 \quad [\text{BASE } 1000] \\ 998 \quad -002 \end{array}$$

$$784 \quad / \quad 428$$

Answer is 784428

Example. 2: Find 994 X 988.

Here base is 1000

$$\begin{array}{r} 994 \quad -006 \quad [\text{BASE } 1000] \\ 988 \quad -012 \\ \hline \end{array}$$

(786-002) or (998-214) / (214X2)

$$\begin{array}{r} 994 \quad -006 \quad [\text{BASE } 1000] \\ 988 \quad -012 \\ \hline 982 \quad / \quad 072 \end{array}$$

Answer is 982072

Example. 3: Find 750 X 995.

Here base is 1000

$$\begin{array}{r} 750 \quad -250 \quad [\text{BASE } 1000] \\ 995 \quad -005 \\ \hline \end{array}$$

(750-005) or (995-250) / (250X005)

$$\begin{array}{r} 750 \quad -250 \quad [\text{BASE } 1000] \\ 995 \quad -005 \\ \hline 745 \quad / \quad {}_1250 \end{array}$$

Since the base is 1000, we write down 250 and carry 1 over to the left giving us $745 / {}_1250 = (745+1) / 250$

Answer is 746250

Assignments

Find the following

Q1. 993 X 998

Q2. 815 X 998

Q3. 987 X 994

Q5. 995 X 999

Q7. 999 X 999

Q4. 985 X 998

Q6. 688 X 998

Q8. 872 X 998

Assignments Answers

Find the following

Q1. 993 X 998 =991014

Q2. 815 X 998 =813370

Q3. 987 X 994 =981078

Q4. 985 X 998 =983030

Q5. 995 X 999 =994005

Q6. 688 X 998 =686624

Q7. 999 X 999 =998001

Q8. 872 X 998 =870256

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Chapter 4 : Multiplication

4.2 : Multiplication near to the base

4.2.2 Both the numbers are higher than the base.

The method and rules : The only difference is the positive deviation. Instead of cross?subtract, we follow cross?add.

Example.1: Find 13X12.

Base is 10

$$\begin{array}{r} 13 \quad 3 \quad [\text{BASE } 10] \\ 12 \quad 2 \\ \hline \end{array}$$

$$(13 + 2) \text{ or } (12 + 3) / (3 \times 2)$$

$$\begin{array}{r} 13 \quad 3 \quad [\text{BASE } 10] \\ 12 \quad 2 \\ \hline 15 \quad 6 \end{array}$$

Answer is 156

Example.2: Find 18X14.

Base is 10

$$\begin{array}{r} 18 \quad 8 \quad [\text{BASE } 10] \\ 14 \quad 4 \\ \hline \end{array}$$

$$(18 + 4) \text{ or } (14 + 8) / (8 \times 4)$$

$$\begin{array}{r} 18 \quad 8 \quad [\text{BASE } 10] \\ 14 \quad 4 \\ \hline 22 \quad 32 \end{array}$$

Since the base is 10, we write down 2 and carry 3 over to the left giving us $22 /_{32} = (22+3) / 2$

Answer is 252

Example 3: Find 104 X 102

Here base is 100

$$\begin{array}{r} 104 \quad 04 \quad [\text{BASE } 100] \\ 102 \quad 02 \\ \hline \end{array}$$

$$(104 + 02) \text{ or } (102 + 04) / (04 \times 02)$$

$$\begin{array}{r}
 104 \ 04 \quad [\text{BASE } 100] \\
 102 \ 02 \\
 \hline
 106 \ / \ 08
 \end{array}$$

Answer is 10608

Example. 4: Find 1275 X 1004.

Here base is 1000

$$\begin{array}{r}
 1275 \ 275 \quad [\text{BASE } 1000] \\
 1004 \ 004 \\
 \hline
 (1275 + 004) \text{ or } (1004 + 275) \ / \ (275 \times 004)
 \end{array}$$

$$\begin{array}{r}
 1275 \ 275 \quad [\text{BASE } 1000] \\
 1004 \ 004 \\
 \hline
 1279 \ / \ _1100
 \end{array}$$

Since the base is 1000, we write down 100 and carry 1 over to the left giving us $1279 \ / \ _1100 = (1279+1) \ / \ 100$

Answer is 1280100

Assignments

Find the following

- | | |
|-------------------------|-------------------------|
| Q01. 11 X 14 | Q02. 15 X 10 |
| Q03. 12 X 13 | Q04. 11 X 11 |
| Q05. 101 X 104 | Q06. 121 X 104 |
| Q07. 107 X 103 | Q08. 134 X 102 |
| Q09. 1004 X 1009 | Q10. 1115 X 1004 |
| Q11. 1005 X 1003 | Q12. 1035 X 1002 |

Assignments Answers

Find the following

Q01. 154

Q03. 156

Q05. 10504

Q07. 11021

Q09. 1013036

Q11. 1008015

Q02. 150

Q04. 121

Q06. 12584

Q08. 13668

Q10. 1119460

Q12. 1037070

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Chapter 4 : Multiplication

4.2 : Multiplication near to the base

4.2.3.1.1 Bar Numbers and application

Look at the following Subtractions

$$9 = 10 - 1 = 10 - 1$$

$$8 = 10 - 2 = 10 - 2$$

$$7 = 10 - 3 = 10 - 3$$

$$6 = 10 - 4 = 10 - 4$$

$$5 = 10 - 5 = 10 - 5$$

$$4 = 10 - 6 = 10 - 6$$

9 is same as $10 - 1$, and this may be written as one ten in the ten's column and take away 1 in the units column.

8 is same as $10 - 2$, and this may be written as one ten in the ten's column and take away 2 in the units column.

More examples

—

$$98 = 100 \text{ ? } 2 = 102$$

$$196 = 200 \text{ ? } 4 = 204$$

$$32 = 30 \text{ ? } 2 = 28$$

$$145 = 140 \text{ ? } 5 = 135$$

A Viniculum Number OR Bar Number is a take away or minus number.

$$28 = 30 \text{ ? } 2 = 32 \text{ because 28 is 2 less than 30.}$$

Thirty viniculum two is 28.

Example 1.

Bar number of 47 can be found by $47 = 50 \text{ ? } 3 = 53$
One more than 4 is 5 and the complement of 7 is 3.

It is like telling the time when we say 'Five to six' instead of 5:55

To change a number back into its ordinary form, *write down the complement of the viniculum number and subtract 1 from the next digit to the left*

To convert Viniculum 53

Complement of 3 is 7 and $5 \text{ ? } 1$ is 4 give us the original number as 47.

Example 2

To convert 75

Complement of 5 is 5 and $7-1$ gives 6
The original number is 65

To change a tens column digit into a Viniculum we use same method. The digit is replaced by its complement and the digit to the left is increased by 1

Example 3

Convert Viniculum 174

The complement of 7 is 3 and $1+1 = 2$

This is saying that one hundred seven tens and four units is the same as two hundreds minus three tens and 4 units.

Viniculum $17\bar{4}$ is 234

To change a tens column digit into a viniculum we use same method. The digit is replaced by its complement and the digit to the left is increased by 1

Example 4

Convert Viniculum $63\bar{2}$

The complement of 3 is 7 and $6 + 1 = 7$. Answer is 572

Some Numbers may have more than one Viniculum number

$$332458\bar{2} = 27\bar{2}4422$$

4.2.3.1.2 Adding and subtracting Viniculum numbers

Viniculum numbers are added or subtracted just like ordinary numbers.

$$\bar{3} + \bar{2} = \bar{5}$$

$$\bar{5} - \bar{2} = \bar{3}$$

$$12 + 3\bar{ } = 9$$

Assignments

Find the following

Q1. Change units digit into viniculum number for 46

Q2. Change the following numbers back to ordinary

form

1. $\overline{12}$ 2. $\overline{51}$ 3. $\overline{42}$ 4. $\overline{85}$

Q3. Change tens digit into viniculum number for 621

Q4. Change the following numbers back to ordinary form

1. $\overline{4131}$ 2. $\overline{3333}$ 3. $\overline{7151}$ 4. $\overline{65321}$

Q5. Find the following

1. $\overline{3} + \overline{2}$ 2. $\overline{8} - \overline{4}$ 3. $\overline{2} - \overline{2}$ 4. $\overline{6} + \overline{2}$

Assignments Answers

Find the following

Q1. $\overline{54}$

Q2. Change the following numbers back to ordinary form

1. 8 2. 49 3. 38 4. 75

Q3. $\overline{781}$

Q4. Change the following numbers back to ordinary form

1. 3929 2. 2727 3. 6949 4. 55281

Q5. Find the following

1. $\overline{5}$ 2. $\overline{4}$ 3. 0 4. $\overline{4}$
-

Chapter 4 : Multiplication

4.1 Multiplication:

There is no change when any number is multiplied by 1.

When we multiply one number by another then it is increased and becomes further away from one. When 4 is multiplies by 5 it becomes 20 which is further away from 4 and 5.

Using our multiplication techniques, we relate each number very close to another number called base. The difference between the number and the base is termed as deviation.

Deviation may be positive or negative. Positive deviation is written without the positive sign and the negative deviation, is written using a bar or negative sign on the number.

Number	Base	Deviation
15	10	$15-10 = 5$
9	10	$9-10=-1$
98	100	$98-100=-2$
112	100	$112-100=12$
994	1000	$994-1000=-6$
1013	1000	$1013-1000=13$

Example 1: Find the deviation of 94 from base 100

Now deviation can be obtained by ?all from 9 and the last from 10? method i.e, the last digit 4 is subtracted from 10 gives 06 and remaining digit 9 is subtracted from 9 gives 00.

Deviation of 94 from base 100 is 06

Example 2: Find the deviation of 86 from base 100

The last digit 6 is subtracted from 10 gives 04 and remaining digit 8 from 9 gives 1.

Deviation of 86 from base 100 is 14

Assignments

Q1. Write down the deviation from nearest base for the following

- | | |
|-------------------------------|-----------------------------|
| 1. 88 from 100 | 5. 423 from 1000 |
| 2. 75 from 100 | 6. 902 from 1000 |
| 3. 8004 from 10000 | 7. 70503 from 100000 |
| 4. 123870 from 1000000 | 8. 9993 from 10000 |

Assignments Answers

Q1. Write down the deviation from nearest base for the following

- | | |
|------------------|-----------------|
| 1. 12 | 5. 577 |
| 2. 25 | 6. 098 |
| 3. 1996 | 7. 29497 |
| 4. 876130 | 8. 0007 |

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Chapter 1 : Multiplication

4.2 : Multiplication near to the base

4.2.1 Both the numbers are lower than the base.

4.2.1.1 Multiplication using a base of 10

Example 1: Multiply 7 by 8.

Consider the base number as 10 since it is near to both the numbers.

Step 1. Write the numbers one below the other.

$$\begin{array}{r} 7 \times \\ 8 \\ \hline \end{array}$$

Step 2. Take the deviations of both the numbers from the base and represent

$$\begin{array}{r} 7 \quad -3 \quad [\text{Base } 10] \\ 8 \quad -2 \\ \hline \end{array}$$

Remainders 3 and 2 implies that the numbers to be multiplied are both less than 10

Step 3. The product or answer will have two parts, one on the left side and the other on the right. A vertical or a slant line i.e. a slash may be drawn for the demarcation of the two parts.

$$\begin{array}{r} 7 \quad -3 \quad [\text{Base } 10] \\ 8 \quad -2 \\ \hline / \\ \hline \end{array}$$

Step4. The R.H.S. of the answer is the product of the deviations of the numbers. It contains the number of digits equal to number of zeroes in the base.

$$\begin{array}{r} 7 \quad -3 \quad [\text{Base } 10] \\ 8 \quad -2 \\ \hline / \quad (3 \times 2) \\ \hline \end{array}$$

Since base is 10, $3 \times 2 = 6$ can be taken as it is.

Step5. L.H.S of the answer is the sum of one number with

the deviation of the other. It can be arrived at in any one of the four ways.

- i) Cross-subtract deviation 2 on the second row from the original number 7 in the first row $7 - 2 = 5$.
- ii) Cross-subtract deviation 3 on the first row from the original number 8 in the second row $8 - 3 = 5$
- iii) Subtract the base 10 from the sum of the given numbers. $(7 + 8) - 10 = 5$
- iv) Subtract the sum of the two deviations from the base. $10 - (3 + 2) = 5$

Hence 5 is left hand side of the answer.

$$\begin{array}{r}
 7 \quad -3 \quad [\text{Base } 10] \\
 8 \quad -2 \\
 \hline
 5 \quad / \quad 6 \\
 \hline
 \end{array}$$

Step 6 : If R.H.S. contains less number of digits than the number of zeros in the base, the remaining digits are filled up by giving zero or zeroes on the left side of the R.H.S. If the number of digits are more than the number of zeroes in the base, the excess digit or digits are to be added to L.H.S of the answer.

The general form of the multiplication
Let N1 and N2 be two numbers near to a given base in powers of 10, and D1 and D2 are their respective deviations from the base. Then N1 X N2 can be represented as

$$\begin{array}{r}
 N1 \quad D1 \quad [\text{BASE}] \\
 N2 \quad D2 \\
 \hline
 (N1 + D2) \text{ OR } (N2 + D1) / (D1 \times D2)
 \end{array}$$

Chapter 4 : Multiplication

4.2 : Multiplication near to the base

4.2.1 Both the numbers are lower than the base.

4.2.1.2 Multiplication using a base of 100

Example. 1: Find 97 X 94.

Here base is 100

Deviation of 97 from 100 is -03

Deviation of 94 from 100 is -06

97 -03 [BASE 100]
94 -06

(97-06) or (94-03) / (3X6)

97 -03 [BASE 100]
94 -06

91 / 18

Answer is 9118

Example. 2: Find 98 X 97.

Deviation of 98 from 100 is -02

Deviation of 97 from 100 is -03

Here base is 100

98 -02 [BASE 100]
97 -03

(98-03) or (97-02) / (2X3)

98 -02 [BASE 100]

$$\begin{array}{r}
 97 \quad -03 \\
 \hline
 95 \quad / \quad 06
 \end{array}$$

Answer is 9506

Example. 3: Find 75 X 95.

Here base is 100

Deviation of 75 from 100 is -25

Deviation of 95 from 100 is -05

$$\begin{array}{r}
 75 \quad -25 \quad [\text{BASE } 100] \\
 95 \quad -05 \\
 \hline
 (75-05) \text{ or } (95-25) \quad / \quad (25 \times 5)
 \end{array}$$

$$\begin{array}{r}
 75 \quad -25 \quad [\text{BASE } 100] \\
 95 \quad -05 \\
 \hline
 70 \quad / \quad {}_125
 \end{array}$$

Since the base is 100, we write down 25 and carry 1 over to the left giving us $70 / {}_125 = (70+1) / 25$

Answer is 7125

Assignments

Find the following

Q1. 95 X 99

Q2. 93 X 98

Q3. 76 X 98

Q4. 96 X 98

Q5. 97 X 89

Q6. 98 X 91

Q7. 94 X 93

Q8. 92 X 97

Assignments Answers

Find the following

- Q1. $95 \times 99 = 9405$ Q2. $93 \times 98 = 9114$
Q3. $76 \times 98 = 7448$ Q4. $96 \times 98 = 9408$
Q5. $97 \times 89 = 8633$ Q6. $98 \times 91 = 8918$
Q7. $94 \times 93 = 8742$ Q8. $92 \times 97 = 8924$

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Chapter 4 : Multiplication

4.2 : Multiplication near to the base

4.2.1 Both the numbers are lower than the base.

4.2.1.2 Multiplication using a base of 100

Example. 1: Find 97×94 .

Here base is 100

Deviation of 97 from 100 is -03

Deviation of 94 from 100 is -06

$$\begin{array}{r} 97 \quad -03 \quad [\text{BASE } 100] \\ 94 \quad -06 \\ \hline \end{array}$$

(97-06) or (94-03) / (3X6)

$$\begin{array}{r} 97 \quad -03 \quad [\text{BASE } 100] \\ 94 \quad -06 \\ \hline \end{array}$$

91 / 18

Answer is 9118

Example. 2: Find 98×97 .

Deviation of 98 from 100 is -02

Deviation of 97 from 100 is -03

Here base is 100

98 -02 [BASE 100]
97 -03

(98-03) or (97-02) / (2X3)

98 -02 [BASE 100]
97 -03

95 / 06

Answer is 9506

Example. 3: Find 75 X 95.

Here base is 100

Deviation of 75 from 100 is -25

Deviation of 95 from 100 is -05

75 -25 [BASE 100]
95 -05

(75-05) or (95-25) / (25X5)

75 -25 [BASE 100]
95 -05

70 / ₁25

Since the base is 100, we write down 25 and carry 1 over to the left giving us $70 / _1 25 = (70+1) / 25$

Answer is 7125

Assignments

Find the following

Q1. 95×99

Q3. 76×98

Q5. 97×89

Q7. 94×93

Q2. 93×98

Q4. 96×98

Q6. 98×91

Q8. 92×97

Assignments Answers

Find the following

Q1. $95 \times 99 = 9405$

Q3. $76 \times 98 = 7448$

Q5. $97 \times 89 = 8633$

Q7. $94 \times 93 = 8742$

Q2. $93 \times 98 = 9114$

Q4. $96 \times 98 = 9408$

Q6. $98 \times 91 = 8918$

Q8. $92 \times 97 = 8924$

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Chapter 4 : Multiplication

4.2 : Multiplication near to the base

4.2.1 Both the numbers are lower than the base.

4.2.1.3 Multiplication using a base of 1000

Example 1: Find 786×998

Here base is 1000

Complement of 786 is 214.

7 from 9 is 2 and 8 from 9 is 1 and 6 from 10 is 4 .

Complement of 998 is 002

$$\begin{array}{r} 786 \quad -214 \quad [\text{BASE } 1000] \\ 998 \quad -002 \\ \hline \end{array}$$

(786-002) or (998-214) / (214X2)

$$\begin{array}{r} 786 \quad -214 \quad [\text{BASE } 1000] \\ 998 \quad -002 \\ \hline 784 \quad / \quad 428 \end{array}$$

Answer is 784428

Example. 2: Find 994 X 988.

Here base is 1000

$$\begin{array}{r} 994 \quad -006 \quad [\text{BASE } 1000] \\ 988 \quad -012 \\ \hline \end{array}$$

(786-002) or (998-214) / (214X2)

$$\begin{array}{r} 994 \quad -006 \quad [\text{BASE } 1000] \\ 988 \quad -012 \\ \hline 982 \quad / \quad 072 \end{array}$$

Answer is 982072

Example. 3: Find 750 X 995.

Here base is 1000

$$\begin{array}{r} 750 \quad -250 \quad [\text{BASE } 1000] \\ 995 \quad -005 \\ \hline \end{array}$$

(750-005) or (995-250) / (250X005)

$$\begin{array}{r} 750 \quad -250 \quad [\text{BASE } 1000] \\ 995 \quad -005 \\ \hline 745 \quad / \quad 1250 \end{array}$$

Since the base is 1000, we write down 250 and carry 1 over

to the left giving us $745 / 1250 = (745+1) / 250$

Answer is 746250

Assignments

Find the following

Q1. 993×998

Q3. 987×994

Q5. 995×999

Q7. 999×999

Q2. 815×998

Q4. 985×998

Q6. 688×998

Q8. 872×998

Assignments Answers

Find the following

Q1. $993 \times 998 = 991014$

Q3. $987 \times 994 = 981078$

Q5. $995 \times 999 = 994005$

Q7. $999 \times 999 = 998001$

Q2. $815 \times 998 = 813370$

Q4. $985 \times 998 = 983030$

Q6. $688 \times 998 = 686624$

Q8. $872 \times 998 = 870256$

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Chapter 4 : Multiplication

4.2 : Multiplication near to the base

4.2.2 Both the numbers are higher than the base.

The method and rules : The only difference is the positive deviation. Instead of cross?subtract, we follow cross?add.

Example.1: Find 13X12.

Base is 10

$$\begin{array}{r} 13 \quad 3 \quad [\text{BASE } 10] \\ 12 \quad 2 \\ \hline \end{array}$$

$(13 + 2) \text{ or } (12 + 3) / (3 \times 2)$

$$\begin{array}{r} 13 \quad 3 \quad [\text{BASE } 10] \\ 12 \quad 2 \\ \hline 15 \quad / \quad 6 \end{array}$$

Answer is 156

Example.2: Find 18X14.

Base is 10

$$\begin{array}{r} 18 \quad 8 \quad [\text{BASE } 10] \\ 14 \quad 4 \\ \hline \end{array}$$

$(18 + 4) \text{ or } (14 + 8) / (8 \times 4)$

$$\begin{array}{r} 18 \quad 8 \quad [\text{BASE } 10] \\ 14 \quad 4 \\ \hline 22 \quad / \quad 32 \end{array}$$

Since the base is 10, we write down 2 and carry 3 over to the left giving us $22 /_{32} = (22+3) / 2$

Answer is 252

Example 3: Find 104 X 102

Here base is 100

$$\begin{array}{r} 104 \quad 04 \quad [\text{BASE } 100] \\ 102 \quad 02 \\ \hline \end{array}$$

$$(104 + 02) \text{ or } (102 + 04) / (04 \times 02)$$

$$\begin{array}{r} 104 \quad 04 \quad [\text{BASE } 100] \\ 102 \quad 02 \\ \hline \end{array}$$

$$106 / 08$$

Answer is 10608

Example. 4: Find 1275 X 1004.

Here base is 1000

$$\begin{array}{r} 1275 \quad 275 \quad [\text{BASE } 1000] \\ 1004 \quad 004 \\ \hline \end{array}$$

$$(1275 + 004) \text{ or } (1004 + 275) / (275 \times 004)$$

$$\begin{array}{r} 1275 \quad 275 \quad [\text{BASE } 1000] \\ 1004 \quad 004 \\ \hline \end{array}$$

$$1279 / {}_1100$$

Since the base is 1000, we write down 100 and carry 1 over to the left giving us $1279 / {}_1100 = (1279+1) / 100$

Answer is 1280100

Assignments

Find the following

Q01. 11×14

Q02. 15×10

Q03. 12×13

Q04. 11×11

Q05. 101 X 104
Q07. 107 X 103
Q09. 1004 X 1009
Q11. 1005 X 1003

Q06. 121 X 104
Q08. 134 X 102
Q10. 1115 X 1004
Q12. 1035 X 1002

Assignments Answers

Find the following

Q01. 154

Q02. 150

Q03. 156

Q04. 121

Q05. 10504

Q06. 12584

Q07. 11021

Q08. 13668

Q09. 1013036

Q10. 1119460

Q11. 1008015

Q12. 1037070

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Chapter 4 : Multiplication

4.2 : Multiplication near to the base

4.2.3.1.1 Bar Numbers and application

Look at the following Subtractions

$$9 = 10 \text{ ? } 1 = 11 \overline{}$$

$$8 = 10 \text{ ? } 2 = 12 \overline{}$$

$$7 = 10 \text{ ? } 3 = 13 \overline{}$$

$$6 = 10 \text{ ? } 4 = 14 \overline{}$$

—

$$5 = 10 \text{ ? } 5 = 15$$

$$4 = 10 \text{ ? } 3 = 16$$

9 is same as $10 - 1$, and this may be written as one ten in the ten's column and take away 1 in the units column.

8 is same as $10 - 2$, and this may be written as one ten in the ten's column and take away 2 in the units column.

More examples

$$98 = 100 \text{ ? } 2 = 102$$

$$196 = 200 \text{ ? } 4 = 204$$

$$32 = 30 \text{ ? } 2 = 28$$

$$145 = 140 \text{ ? } 5 = 135$$

A Viniculum Number OR Bar Number is a take away or minus number.

$$28 = 30 \text{ ? } 2 = 32 \text{ because } 28 \text{ is } 2 \text{ less than } 30.$$

Thirty viniculum two is 28.

Example 1.

Bar number of 47 can be found by $47 = 50 \text{ ? } 3 = 53$
One more than 4 is 5 and the complement of 7 is 3.

It is like telling the time when we say 'Five to six' instead of 5:55

To change a number back into its ordinary form, *write down the complement of the viniculum number and subtract 1 from the next digit to the left*

To convert Viniculum 53

Complement of 3 is 7 and $5 \text{ ? } 1$ is 4 give us the original number as 47.

Example 2

To convert 75

Complement of 5 is 5 and 7-1 gives 6
The original number is 65

To change a tens column digit into a Viniculum we use same method. The digit is replaced by its complement and the digit to the left is increased by 1

Example 3

Convert Viniculum $17\overline{4}$

The complement of 7 is 3 and $1+1 = 2$

This is saying that one hundred seven tens and four units is the same as two hundreds minus three tens and 4 units.

Viniculum $17\overline{4}$ is 234

To change a tens column digit into a viniculum we use same method. The digit is replaced by its complement and the digit to the left is increased by 1

Example 4

Convert Viniculum $63\overline{2}$

The complement of 3 is 7 and $6 - 1 = 5$. Answer is 572

Some Numbers may have more than one Viniculum number

$$332458\overline{2} = 27\overline{2}4422$$

4.2.3.1.2 Adding and subtracting Viniculum numbers

Viniculum numbers are added or subtracted just like ordinary numbers.

$$\overline{3} + \overline{2} = \overline{5}$$

$$\overline{5} - \overline{2} = \overline{3}$$

$$1\overline{2} + 3\overline{=} 9$$

Assignments

Find the following

Q1. Change units digit into vinculum number for 46

Q2. Change the following numbers back to ordinary form

1. $1\overline{2}$ 2. $5\overline{1}$ 3. $4\overline{2}$ 4. $8\overline{5}$

Q3. Change tens digit into vinculum number for 621

Q4. Change the following numbers back to ordinary form

1. $4\overline{1}3\overline{1}$ 2. $3\overline{3}3\overline{3}$ 3. $7\overline{1}5\overline{1}$ 4. $6\overline{5}3\overline{2}1$

Q5. Find the following

1. $\overline{3} + 2\overline{}$ 2. $8\overline{-} - 4\overline{-}$ 3. $2\overline{-} - 2\overline{-}$ 4. $6\overline{+} 2$

Assignments Answers

Find the following

Q1. $5\overline{4}$

Q2. Change the following numbers back to ordinary form

1. 8 2. 49 3. 38 4. 75

Q3. $7\overline{8}1$

Q4. Change the following numbers back to ordinary form

1. 3929 2. 2727 3. 6949 4. 55281

Q5. Find the following

1. $\overline{5}$ 2. $\overline{4}$ 3. 0 4. $\overline{4}$

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Chapter 4 : Multiplication

4.2 : Multiplication near to the base

4.2.3.1.1 Bar Numbers and application

Example.1: Find 13 X 7.

Base is 10

$$\begin{array}{r}
 13 \quad 3 \quad [\text{BASE } 10] \\
 7 \quad -3 \\
 \hline
 10 \quad / \quad \overline{9}
 \end{array}$$

One deviation is positive and the other is negative. So the product of deviations becomes negative. So the right hand side of the answer obtained will therefore have to be subtracted.

Using ?To change a tens column digit into a Viniculum we use same method. The digit is replaced by its complement and the digit to the left is decreased by 1?

Complement of 9 is 1 and 10 is decreased by 1

$$\begin{array}{r} 13 \quad 3 \quad [\text{BASE } 10] \\ 7 \quad -3 \\ \hline \end{array}$$

$$(10-1) / \text{Complement of } 9 = 91$$

Answer is 91

Example.2: Find 108 X 94.

Base is 100

$$\begin{array}{r} 108 \quad 08 \quad [\text{BASE } 100] \\ 94 \quad -06 \\ \hline \end{array}$$

$$102 \quad / \quad \overline{48}$$

**Complement of 48 is 52 and 102 is decreased by 1
(102-1) / Complement of 48 = 10152**

Answer is 10152

Example.3: Find 998 X 1025.

Base is 1000

$$\begin{array}{r} 998 \quad -002 \quad [\text{BASE } 1000] \\ 1025 \quad 025 \\ \hline \end{array}$$

$$1023 \quad / \quad \overline{050}$$

**Complement of 50 is 950 and 1023 is decreased by 1
(1023-1) / Complement of 50 = 1022950**

Answer is 1022950

Assignments

Find the following

Q1. 9×13

Q2. 8×17

Q3. 97×106

Q4. 88×102

Q5. 997×1006

Q6. 989×1028

Assignments Answers

Find the following

Q1. $9 \times 13 = 117$

Q2. $8 \times 17 = 136$

Q3. $97 \times 106 = 10282$

Q4. $88 \times 102 = 78988$

Q5. $997 \times 1006 = 1002982$

Q6. $989 \times 1028 = 1016692$

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Ch4 : Multiplication

4.3 Squaring numbers that ends in 5

Example 1 : Find 75^2

$$\begin{array}{r} 75 \times \\ 75 \\ \hline \end{array}$$

8X7 / 5X5 => 5625

Simply multiply 7 the number before 5 by the next number up 8 . This gives $7 \times 8 = 56$ as the first part of the answer and the last part is $5 \times 5 = 25$ so the answer is 5625

Example 2 : Find 305^2

30 5

30 5

$30 \times 31 / 5 \times 5 \Rightarrow 93025$

Simply multiply 30 the number before 5 by the next number up 31 . This gives $30 \times 31 = 930$ as the first part of the answer and the last part is $5 \times 5 = 25$ so the answer is 93025

Assignments

Find the following

Q1. 35^2

Q2. 45^2

Q3. 105^2

Q4. 95^2

Assignments Answers

Find the following

Q1. 1225

Q2. 2025

Q3. 11025

Q4. 7225

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Ch4 : Multiplication

4.4 Multiplying numbers whose first figures are the same and whose last figures add up to 10, 100 etc

Example.1 : Find 43 X 47

Check for R.H.S : $3 + 7 = 10$, L.H.S. portion remains the same i.e., 4.

$$\begin{array}{r} 43 \\ \times 47 \\ \hline \end{array}$$

4X5 / 3X7 => 2021

Multiply 4 (the same figure in both the numbers) by the next number up 5. This gives $4 \times 5 = 20$ as the first part of the answer and the last part is $3 \times 7 = 21$ so the answer is 2021

Example.2 : Find 31 X 39

Check for R.H.S : $9 + 1 = 10$, L.H.S. portion remains the same i.e., 3.

$$\begin{array}{r} 31 \\ \times 39 \\ \hline \end{array}$$

3X4 / 1X9 => 1209

Multiply 3 (the same figure in both the numbers) by 4. This gives $3 \times 4 = 12$ as the first part of the answer and the last part is $1 \times 9 = 09$ so the answer is 1209

Example.3 : Find 127 X 123

Check for : $7 + 3 = 10$, L.H.S. portion remains the same i.e., 12.

$$\begin{array}{r} 127 \\ \times 123 \\ \hline \end{array}$$

12X13 / 7X3 => 15621

Answer is 15621

Example.4 : Find 395 X 395

Check for : $5 + 5 = 10$, L.H.S. portion remains the same
i.e., 39.

$$\begin{array}{r} 39 \ 5 \ X \\ 39 \ 5 \\ \hline \end{array}$$

 $39 \times 40 / 5 \times 5 \Rightarrow 156025$

Answer is 156025

Assignments

Find the following

Q1. 23×27

Q2. 34×36

Q3. 62×68

Q4. 136×134

Assignments Answers

Find the following

Q1. 621

Q2. 1224

Q3. 4216

Q4. 18224

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Ch4 : Multiplication

**4.5 Numbers of which the last 2 Or 3 Or 4 digits added up
give 100,1000,10000**

The same rule works when the sum of the last 2, last 3, last 4 digits added respectively equal to 100, 1000, 10000 .

Example.1 : Find 292 X 208

Here $92 + 08 = 100$, L.H.S portion is same i.e. 2

$$\begin{array}{r} 2 \ 92 \times \\ 2 \ 08 \\ \hline \end{array}$$

$$2 \times 3 / 92 \times 08 \Rightarrow 60 / 736$$

60 / 736 (for 100 raise the L.H.S. product by 0 i.e 6×10)

Answer is 60736.

Example.2 : Find 848 X 852

Here $48 + 52 = 100$, L.H.S portion is same i.e. 8

$$\begin{array}{r} 8 \ 48 \times \\ 8 \ 52 \\ \hline \end{array}$$

8X9 / 48X52

We can use our fastmaths technique to find the product of 48X52

$$\begin{array}{r} 48 \ -2 \ [\text{BASE } 50] \\ 52 \ 2 \\ \hline \end{array}$$

Half of (50) / complement of 04 $\Rightarrow (25-1) / 96 \Rightarrow$
2496

and write $848 \times 852 = 8 \times 9 / 48 \times 52$

$$\begin{aligned} &= 720 / 2496 \text{ (for 100 raise the L.H.S. product by 0 } \\ &\text{i.e } 72 \times 10) \\ &= (720+2)/496 = 722496 \end{aligned}$$

Since L.H.S product is to be multiplied by 10 and 2 to be carried over as the base is 100

Answer is 722496

Example.3 : Find 693 X 607

Check for : 93 + 07 = 100, L.H.S. portion remains the same i.e., 6

$$\begin{array}{r} 6 \ 93 \ X \\ 6 \ 07 \\ \hline \end{array}$$

6X7 / 93X07 => 2021

Now R.H.S product 93 X 07 can be obtained mentally.

$$\begin{aligned} 693 \times 607 &= 6 \times 7 / 93 \times 07 \\ &= 420 / 651 \text{ (for 100 raise the L.H.S. product by } 0 \text{ i.e. } 42 \times 10) \\ &= 420651. \end{aligned}$$

Answer is 420651.

Assignments

Find the Following

- Q1. 393 X 307**
- Q2. 696 X 604**
- Q3. 873 X 827**
- Q4. 188 X 112**
- Q5. 454 X 446**

Assignments Assignments

Find the Following

- Q1. 393 X 307 = 120/651 = 120651**
- Q2. 696 X 604 = 420384 = 420/384 = 420384**
- Q3. 873 X 827 = 720/1971 = 721971**
- Q4. 188 X 112 = 20/1056 = 21056**

Q5. $454 \times 446 = 200 /_2 484 = 202484$

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Ch4 : Multiplication

4.5 Numbers of which the last 2 Or 3 Or 4 digits added up give 100,1000,10000

The same rule works when the sum of the last 2, last 3, last 4 digits added respectively equal to 100, 1000, 10000 .

Example.1 : Find 292×208

Here $92 + 08 = 100$, L.H.S portion is same i.e. 2

$$\begin{array}{r} 2 \ 92 \times \\ 2 \ 08 \\ \hline \end{array}$$

2X3 / $92 \times 08 \Rightarrow 60 / 736$

60 / 736 (for 100 raise the L.H.S. product by 0 i.e 6×10)

Answer is 60736.

Example.2 : Find 848×852

Here $48 + 52 = 100$, L.H.S portion is same i.e. 8

$$\begin{array}{r} 8 \ 48 \times \\ 8 \ 52 \\ \hline \end{array}$$

8X9 / 48×52

We can use our fastmaths technique to find the product of 48×52

$$\begin{array}{r} 48 \quad -2 \quad [\text{BASE } 50] \\ 52 \quad 2 \\ \hline \end{array}$$

Half of (50) / complement of 04 $\Rightarrow (25-1)/ 96 \Rightarrow$
2496

and write $848 \times 852 = 8 \times 9 / 48 \times 52$

$$\begin{aligned} &= 720 / {}_2496 \text{ (for 100 raise the L.H.S. product by 0 } \\ &\text{i.e } 72 \times 10) \\ &= (720+2)/496 = 722496 \end{aligned}$$

Since L.H.S product is to be multiplied by 10 and 2 to be carried over as the base is 100

Answer is 722496

Example.3 : Find 693×607

Check for : $93 + 07 = 100$, L.H.S. portion remains the same
i.e., 6

$$\begin{array}{r} 6 \quad 93 \quad \times \\ 6 \quad 07 \\ \hline 6 \times 7 / 93 \times 07 \Rightarrow 2021 \\ \hline \end{array}$$

Now R.H.S product 93×07 can be obtained mentally.

$$\begin{aligned} 693 \times 607 &= 6 \times 7 / 93 \times 07 \\ &= 420 / 651 \text{ (for 100 raise the L.H.S. product by } \\ &0 \text{ i.e. } 42 \times 10) \\ &= 420651. \end{aligned}$$

Answer is 420651.

Assignments

Find the Following

Q1. 393×307

Q2. 696×604

- Q3. 873×827
 Q4. 188×112
 Q5. 454×446

Assignments Assignments

Find the Following

- Q1. $393 \times 307 = 120651$
 Q2. $696 \times 604 = 420384$
 Q3. $873 \times 827 = 721971$
 Q4. $188 \times 112 = 21056$
 Q5. $454 \times 446 = 202484$
-

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Ch4 : Multiplication

4.6 Multiplication using other bases

Example.1 : Find 568×998

Base is 1000

Complement of 568 is 432
 Complement of 998 is 002.

$$\begin{array}{r} 568 - 432 \quad [\text{BASE } 1000] \\ 998 - 002 \end{array}$$

$$568 - 2 / 864 \Rightarrow 566 / 864$$

Answer is 566864

Example.2 : Find 213 X 203

Base is 200

**Complement of 213 is 13
Complement of 203 is 03.**

**213 13 [BASE 200]
203 03**

213+3 / 39 => 216 x 2 / 39

since the base is 200 i e 2X100

**The numbers are close to 200 which is 100X 2 we multiply
only the left hand part of the answer by 2 to get 43239**

Answer is 43239

Example.3 : Find 29 X 28

Base is 30

**Complement of 29 is -1
Complement of 28 is -2.**

**29 -1 [BASE 30]
28 -2**

29 -2 / 2 => 27 x 3 / 2 since the base is 30 i.e 3X 10

**The numbers are close to 30 which is 10X 3 we multiply
only the left hand part of the answer by 3 to get 812**

Answer is 812

Example.4 : Find 43 X 44

Base is 40

Complement of 43 is 3
Complement of 44 is 4.

$$\begin{array}{r} 43 \quad 3 \quad [\text{BASE } 40] \\ 44 \quad 4 \end{array}$$

$$43 + 4 /_{12} \Rightarrow 47 \times 4 /_{12} \text{ since the base is } 40 \text{ i.e } 4 \times 10$$

The numbers are close to 40 which is 10×4 we multiply only the left hand part of the answer by 4 (before carrying 1 over to the left) to get $188 /_{12} = (188+1)/2$

Answer is 1892

Example.5 : Find 83×49

Same as $?(83 \times 98)$

$$\begin{array}{r} 83 \quad -17 \quad [\text{BASE } 100] \\ 98 \quad -02 \end{array}$$

$$83 - 02 /_{17 \times 2} \Rightarrow 81 /_{34} = 8134$$

$$\text{Answer} = ?(8134) = 4067$$

Answer is 4067

Example.6 : Find 9998×94

Numbers are close to different bases 10,000 and 100

$$\begin{array}{r} 9998 \quad -2 \quad [\text{BASE } 10000] \\ 94 \quad -6 \quad [\text{BASE } 100] \end{array}$$

$$9398 /_{12} \Rightarrow 939812$$

Note that 6 is not subtracted from 8, but from the 9 above the 4 in 94

Second column from left. So 9998 becomes 9398

Answer is 939812

Example.7: Find 10007 X 1003

Numbers are close to different bases 10,000 and 100

10007 007 [BASE 10000]

1000 003 [BASE 100]

10037 / 021 => 10037021

Note that 3 is not added to 7, but to the third column from left.

Answer = 10037021

Assignments

Find the Following

Q1. 314 X 304

Q2. 1014 X 998

Q3. 74 X 73

Q4. 93X 49

Q5. 9998 X 96

Assignments Answers

Find the Following

Q1. 314 X 304 = 95456

Q2. 1014 X 998 =1011972

Q3. 74 X 73 = 5402

Q4. 93 X 49 =4557

Q5. 9998 X 96 =959808

Ch4 : Multiplication

4.7 Multiplication by 5, 50 and 25

Example 1: Find 44 X 5

Multiply by 2 and divide by 2 gives

$$44 \times (5 \times 2) / 2 = 44 \times 10 / 2$$

Find 44 X 10 and divide by 2

$$440 / 2 = 220$$

Answer = 220

Example 2: Find 27 X 50

Multiply by 2 and divide by 2 gives

$$27 \times (50 \times 2) / 2 = 27 \times 100 / 2$$

Find 27 X 100 and divide by 2

$$2700 / 2 = 1350$$

Answer = 1350

Example.3: Find 82 X 25

Multiply by 4 and divide by 4 gives

$$82 \times (25 \times 4) / 4 = 82 \times 100 / 4$$

Find 82 X 100 and divide by 4

$$8200/4 = 2050$$

Answer = 2050

Assignments

Find the following

- Q1. 55×5**
- Q2. 55×25**
- Q3. 55×50**
- Q4. 98×50**
- Q5. 98×25**

Assignments Answers

Find the following

- Q1. $55 \times 5 = 275$**
 - Q2. $55 \times 25 = 1375$**
 - Q3. $55 \times 50 = 2750$**
 - Q4. $98 \times 50 = 4900$**
 - Q5. $98 \times 25 = 2450$**
-

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Ch4 : Multiplication

4.8 Multiplication by 9

Method:

- **Step 1. The left hand side digit is obtained by deduction 1 from the left side digit. To find 7×9 ; LHS. digit is $7 - 1 = 6$**
- **Step 2. The right hand side digit is the complement or difference between the multiplier and the left hand side digit. i.e. To find 7×9 , RHS is $9 - 6 = 3$.**

- **Step 3. The two numbers give the answer; i.e. $7 \times 9 = 63$.**

Example 1: Find 8×9

- **Step 1: $8 - 1 = 7$ (LHS. Digit)**
- **Step 2: $9 - 7 = 2$ (RHS. Digit)**
- **Step 3: The answer is 72**

Example 2: Find 15×99

- **Step 1: $15 - 1 = 14$**
- **Step 2: $99 - 14 = 85$ (or Complement of 15 , $100 - 15$)**
- **Step 3: $15 \times 99 =$ Answer is 1485**

Example 3: Find 24×99

- **Step 1: $24 - 1 = 23$**
- **Step 2: $99 - 23 = 76$ (or complement of 24, $100 - 24$)**
- **Step 3: $24 \times 99 =$ Answer is 2376**

Example 4: Find 356×999

- **Step 1: $356 - 1 = 355$**
- **Step 2: $999 - 355 = 644$ (or Complement of 356 , $1000 - 356$)**
- **Step 3: $356 \times 999 =$ Answer is 355644**

Example 5: Find 878×9999

- **Step 1: $878 - 1 = 877$**
- **Step 2: $9999 - 877 = 9122$ (or Complement of 878 , $10000 - 878$)**
- **Step 3: $878 \times 9999 =$ Answer is 8779122**

The multiplicand has to be reduced by 1 to obtain the LHS and the right side is obtained by the subtraction of the LHS from the multiplier.

Assignments

Find the products

- Q1. 64×99
- Q2. 723×999
- Q3. 3251×9999
- Q4. 443×999
- Q5. 256×9999
- Q6. 1857×99999

Assignments Answers

Find the products

- Q1. 6336
- Q2. 722277
- Q3. 32506749
- Q4. 442557
- Q5. 2559744
- Q6. 185698143

We have dealt the cases

a) When the multiplicand and multiplier both have the same number of digits

b) When the multiplier has more number of digits than the multiplicand.

In both the cases the same rule applies. But what happens when the multiplier has lesser digits?

i.e. for problems like 42×9 , 124×9 , 26325×99 etc

Multiplication table when both multiplicand and multiplier are of 2 digits.

	m	n
11×99	$= 10$	$89 = (11-1) / 99 - (11-1) = 1089$
12×99	$= 11$	$88 = (12-1) / 99 - (12-1) = 1188$
13×99	$= 12$	$87 = (13-1) / 99 - (13-1) = 1287$

$$18 \times 99 = 1782$$

$$19 \times 99 = 1881$$

$$20 \times 99 = 1980 = (20-1) / 99 - (20-1) = 1980$$

The rule mentioned in the case of above table also holds good here. Further we can state that the rule applies to all cases, where the multiplicand and the multiplier have the same number of digits.

Consider the following Tables.

Table A

	m	n
$11 \times 9 =$	9	9
$12 \times 9 =$	10	8
$13 \times 9 =$	11	7

$18 \times 9 =$	16	2
$19 \times 9 =$	17	1
$20 \times 9 =$	18	0

Table B

$21 \times 9 =$	18	9
$22 \times 9 =$	19	8
$23 \times 9 =$	20	7

$28 \times 9 =$	25	2
$29 \times 9 =$	26	1
$30 \times 9 =$	27	0

Table C

$35 \times 9 =$	31	5
$46 \times 9 =$	41	4
$53 \times 9 =$	47	7
$67 \times 9 =$	60	3
----- and so on.		

From the above tables the following points can be observed:

- 1) Table A has the multiplicands with 1 as first digit except the last one. Here LHS of products are uniformly 2 less than the multiplicands. So also with 20×9
- 2) Table B has the same pattern. Here LHS of products are uniformly 3 less than the multiplicands.
- 3) Table C is of mixed example and yet the same result. If 3 is first digit of the multiplicand then LHS of product is 4 less than the multiplicand; if 4 is first digit of the

multiplicand then, LHS of the product is 5 less than the multiplicand and so on.

4) The right hand side of the product is obtained by subtracting the RHS part of the multiplicand from 10.

Keeping these points in view we solve following problems:

Example1: Find 42×9

Step 1) Divide the multiplicand (42) of by a line '/' into a right hand portion consisting of as many digits as the multiplier.

i.e. 42 has to be written as 4/2

Step 2) Subtract from the multiplicand one more than the whole excess portion on the left. Left portion of multiplicand is 4.

one more than it $4 + 1 = 5$.

We have to subtract this from multiplicand
i.e. write it as

$$\begin{array}{r} 4 / 2 \\ -5 \\ \hline 3 / 7 \end{array}$$

This gives the LHS part of the product.

Step 3) Subtract the RHS part of the multiplicand. RHS of multiplicand is 2. Its complement is 8.
It gives the RHS of the product

i.e. answer is $3 / 7 / 8 = 378$.

Thus 42×9 can be represented as

$$\begin{array}{r} 4 / 2 \\ -5 / 8 \\ \hline 3 / 7 / 8 = 378. \end{array}$$

Example 2: Find 124×9

Step 1) Here Multiplier has one digit only. We

write 12 / 4

Step 2) $12 + 1 = 13$

i.e. 12 / 4

-1 / 3

Step 3) RHS of multiplicand is 4. Its complement is 6

124 X 9 is 12 / 4

-1 / 3 / 6

11 / 1 / 6 = 1116

The process can also be represented as

$$124 \times 9 = [124 - (12 + 1)] / (10 - 4) = (124 - 13) / 6 = 1116$$

Example 3: Find 15639×99

Since the multiplier has 2 digits, the answer is

$$[15639 - (156 + 1)] / (100 - 39) = (15639 - 157) / 61 = 1548261$$

Assignments

Find the products in the following cases.

- Q1. 58×9
- Q2. 62×9
- Q3. 427×99
- Q4. 832×9
- Q5. 24821×999
- Q6. 111011×99

Assignments Answers

Find the products in the following cases.

- Q1. 522
- Q2. 558

- Q3. 42273
- Q4. 7488
- Q5. 24796179
- Q6. 10990089

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Ch4 : Multiplication

4.8 Multiplication by 11

11 Multiplication table is easy to remember

**11 X 1 = 11
11 X 2 = 22
11 X 3 = 33
11 X 4 = 44
11 X 5 = 55
11 X 6 = 66
11 X 7 = 77
11 X 8 = 88
11 X 9 = 99**

Multiplying larger number by 11 is also easy.

Example 1. Find 52 X 11

52 X 11 is 5 72

Write down the number being multiplied and put the total of the digits between 2 digits

52 X 11 is [5 and 5+2=7 and 2], answer is 572

Example 2. Find 57 X 11

57 X 11 is [5 and 12 and 7], equals 627

$$5 / \text{ }_12 / \text{ }_7 = 627$$

The 1 in 12 is carried over to 5 to give 6

Example 3. Find 234×11

234×11 is [2 and 2+3 and 3+4 and 4] equals 2 5 7 4

Example 4. Find 777×11

777×11 is [7 and 7+7 and 7+7 and 7] simplifies to [7 and 14 and 14 and 7]

$$7 / \text{ }_14 / \text{ }_14 / \text{ }_7 = 8547$$

Answer is 8547. We simply carry the 1's over

Example 5. Find 13423×11

13423×11 is [1 and 3+1 and 3+4 and 4+2 and 2+3 and 3] simplifies to [1 and 4 and 7 and 6 and 5 and 3]

Answer is 147653

Assignments

Find the Following

Q1. 37×11

Q2. 137×11

Q3. 12337×11

Q4. 567×11

Q5. 98×11

Assignments Answers

Find the Following

Q1. 407

Q2. 1507

Q3. 135707

Q4. 6237

Q5. 1078

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Ch4 : Multiplication

4.9 Multiplication by 12

Multiplication table of 12 is easy to remember

**12 X 1 = 12
12 X 2 = 24
12 X 3 = 36
12 X 4 = 48
12 X 5 = 60
12 X 6 = 72
12 X 7 = 84
12 X 8 = 96
12 X 9 = 108**

Multiplication of large numbers with 12 is also easy. Just double the digit to the left before adding

Example 1. Find 52 X 12

52 X 12 is

Add 0 to the left and right as shown below

**0 5 2 0
0 5 12 4 [2X0 + 5, 2X5 + 2, 2X2+0]**

Answer is 624

Example 2. Find 234 X 12

234 X 12 is

Add 0 to the left and right as shown below

0 2 3 4 0
0 2 7 10 8 [2X0 + 2, 2X2 + 3, 2X3+4, 2X4+0]

Answer is 2808

Example 3. Find 65214 X 12

65214 X 12 is

Add 0 to the left and right as shown below

0 6 5 2 1 4 0
0 6 17 12 5 6 8 [2X0+6, 2X6 +5, 2X5+2, 2X2+1,
2X1+4, 2X4+0]

Answer is 782568

Assignments

Find the Following

- Q1. 98 X 12**
- Q2. 56 X 12**
- Q3. 134 X 12**
- Q4. 564 X 12**
- Q5. 123498 X 12**

Assignments Answers

Find the Following

- Q1. 1176**
 - Q2. 672**
 - Q3. 1608**
 - Q4. 6768**
 - Q5. 1481976**
-

Ch4 : Multiplication

4.10 Multiplication by vertically and Crosswire

We have seen all the multiplication sums had at least one of the numbers to be multiplied close to a particular base of 10,100 , 1000 etc. We learn a more general formula which can be used for all cases of multiplication.

Multiplying 2 two-digit Numbers

Remember the following diagram. Each dot represents a digit in the number and the lines joining the dots stand for digits to be multiplies

Step 1

Step 2

Step 3



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Example 1: Multiply 42 by 13

Step 1. Starting from left, multiply the two left handed most digits vertically. $4 \times 1 = 4$ and set the answer down underneath as the left most part of the answer.

$$\begin{array}{r} 42 \times \\ 13 \\ \hline 4 \end{array}$$

Step 2. Multiply 4 by 3 and 2 by 1, cross multiplying and add these two answers

together $4 \times 3 + 2 \times 1 = 14$. Set down 4 as the next answer digit and carry the 1 to the left.

$$\begin{array}{r} 4 \ 2 \times \\ 1 \ 3 \\ \hline 4 \ 14 \end{array}$$

Step 3. Multiply 2 by 3 vertically and set down the answer 6 as the right most answer digit.

$$\begin{array}{r} 4 \ 2 \times \\ 1 \ 3 \\ \hline 4 \ 14 \ 6 \end{array}$$

Step 4. Add to the carry digit to give the answer 546

This method can be started either from the right or from the left.

Example 2 Find 23×72

Step 1. Starting from right, multiply the two right-handed most digits vertically. $3 \times 2 = 6$ and set the answer down underneath as the right most part of the answer.

$$\begin{array}{r} 2 \ 3 \times \\ 7 \ 2 \\ \hline 6 \end{array}$$

Step 2. Multiply 2 by 2 and 3 by 7, cross multiplying and add these two answers together $2 \times 2 + 3 \times 7 = 4 + 21 = 25$. Set down 5 as the next answer digit and carry the 2 to the left.

$$\begin{array}{r} 2 \ 3 \times \\ 7 \ 2 \\ \hline 25 \ 6 \end{array}$$

Step 3. Multiply 2 by 7 vertically and set down the answer 14 as the left most answer digit.

$$\begin{array}{r} 2 \ 3 \times \\ 7 \ 2 \\ \hline \end{array}$$

$$\begin{array}{r}
 14 \ 25 \ 6 \\
 \hline
 1 \ 6 \ 5 \ 6
 \end{array}$$

Assignments

Find the Following

- Q1. 23 X 12
- Q2. 87 X 24
- Q3. 63 X 42
- Q4. 28 X 98
- Q5. 45 X 67

Assignments Answers

Find the Following

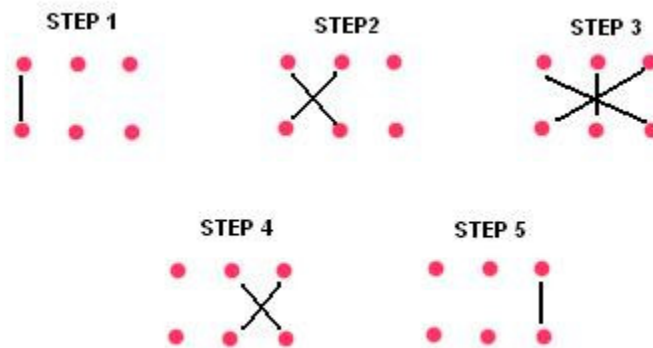
- Q1. 276
- Q2. 2088
- Q3. 2646
- Q4. 2744
- Q5. 3015

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Ch4 : Multiplication

4.11 Multiplying larger numbers

Each dot represents a digit in the number and the lines joining the dots stand for digits to be multiplied.



Example 1: Find 362×134

Step 1. Starting from left, multiply the two left handed most digits vertically. $3 \times 1 = 3$ and set the answer down underneath as the left most part of the answer.

$$\begin{array}{r} 3 \ 6 \ 2 \ X \\ 1 \ 3 \ 4 \\ \hline 3 \end{array}$$

Step 2. Multiply 3 by 3 and 6 by 1, cross multiplying and add these two answers together $3 \times 3 + 6 \times 1 = 15$. Set down 5 as the next answer digit and carry the 1 to the left.

$$\begin{array}{r} 3 \ 6 \ 2 \ X \\ 1 \ 3 \ 4 \\ \hline 3 \ 15 \end{array}$$

Step 3. Middle step is to add the cross product of all six digits as shown below

$$3 \times 4 + 2 \times 1 + 6 \times 3 = 32$$

$$\begin{array}{r} 3 \ 6 \ 2 \ X \\ 1 \ 3 \ 4 \\ \hline 3 \ 15 \ 32 \end{array}$$

Step 4. Sum of the products of the four hand most digits give

$$3 \times 2 + 6 \times 4 = 30$$

$$3 \ 6 \ 2 \ X$$

$$\begin{array}{r}
 1 \ 3 \ 4 \\
 \hline
 3 \ 15 \ 32 \ 30
 \end{array}$$

Step 5. The final step is the product of the two right hand most digits $2 \times 4 = 8$

$$\begin{array}{r}
 3 \ 6 \ 2 \ X \\
 1 \ 3 \ 4 \\
 \hline
 3 \ 15 \ 32 \ 30 \ 8
 \end{array}$$

Step 6. After adding up the carry digits the answer is 48508

Example 2 Find 498×289

Step 1. Starting from left, multiply the two left handed most digits vertically. $4 \times 2 = 8$ and set the answer down underneath as the left most part of the answer.

$$\begin{array}{r}
 4 \ 9 \ 8 \ X \\
 2 \ 8 \ 9 \\
 \hline
 8
 \end{array}$$

Step 2. Multiply 4 by 8 and 9 by 2, cross multiplying and add these two answers together $4 \times 8 + 9 \times 2 = 50$. Set down 0 as the next answer digit and carry the 5 to the left.

$$\begin{array}{r}
 4 \ 9 \ 8 \ X \\
 2 \ 8 \ 9 \\
 \hline
 8 \ 50
 \end{array}$$

Step 3. Middle step is to add the cross product of all six digits as shown below

$4 \times 9 + 8 \times 2 + 9 \times 8 = 36 + 16 + 72 = 124$. Set down 4 as the next answer digit and carry the 12 to the left.

$$\begin{array}{r}
 4 \ 9 \ 8 \ X \\
 2 \ 8 \ 9 \\
 \hline
 8 \ 50 \ 124
 \end{array}$$

Step 4. Sum of the products of the four right hand most digits give

$9 \times 9 + 8 \times 8 = 81 + 64 = 145$ Set down 5 as the next answer digit and carry the 14 to the left.

$$\begin{array}{r} 498 \\ \times 289 \\ \hline 850124145 \end{array}$$

Step 5. The final step is the product of the two right hand most digits $8 \times 9 = 72$

$$\begin{array}{r} 498 \\ \times 289 \\ \hline 85012414572 \end{array}$$

Step 6. After adding up the carry digits

$$131218122$$

Step 7. After adding up the carry digits the answer is 143922

Assignments

Find the Following

- Q1. 147×477**
- Q2. 270×131**
- Q3. 427×47**
- Q4. 353×566**
- Q5. 777×220**

Assignments Answers

Find the Following

- Q1. 70119**
 - Q2. 35370**
 - Q3. 20069**
 - Q4. 199798**
 - Q5. 170940**
-

Chapter 4 : Multiplication

4.12 Multiplication using Average

Consider the following example

Example 1 : Find 29×31

Since the average of 29 and 31 is 30

Find 30^2 and subtract the square of the difference of either number from the average. $900 - 1 = 899$

Square the average and subtract the square of the difference of either number from the average.

Example 2 : Find 26×34

Since the average of 26 and 34 is 30

Find 30^2 and subtract the square of the difference of either number from the average. $900 - 4^2 = 884$

Assignments

- Q1. Find 58×62 ?**
- Q2. Find 67×69 ?**
- Q3. Find 98×102 ?**
- Q4. Find 49×51 ?**
- Q5. Find 73×93 ?**

Assignments Answers

Q1. Find 58 X 62 ?

Since the average of 58 and 62 is 60

Find 60^2 and subtract the square of the difference of either number from the average. $3600 - 2^2 = 3596$

Q2. Find 67 X 69 ?

Since the average of 67 and 69 is 68

Find 68^2 and subtract the square of the difference of either number from the average. $4624 - 1^2 = 4623$

Q3. Find 98 X 102 ?

Since the average of 98 and 102 is 100

Find 100^2 and subtract the square of the difference of either number from the average. $10000 - 2^2 = 9996$

Q4. Find 49 X 51 ?

Since the average of 49 and 51 is 50

Find 50^2 and subtract the square of the difference of either number from the average. $2500 - 1^2 = 2499$

Q5. Find 73 X 93 ?

Since the average of 73 and 93 is 83

Find 83^2 and subtract the square of the difference of either number from the average. $6889 - 10^2 = 6789$

Chapter 5 : Division

5.1 Division:

Using one, there is no division. When one is divided into four, the answer four shows that four has not been divided at all. Division always start at two.

5.2.1. Simple Division

Find $1648 / 4$

- 4 Into 1 does not go, 4 into 16 = 4
- 4 into 4 = 1
- 4 into 8 = 2
- The answer is 412

5.2.1.Division with reminders

A division sum has 4 parts called Divisor, Divident, Quotient and Remainder.

The divisor is the number that divided the dividend, the answer is the quotient, the remainder's at the end.

In the conventional procedure for division, the process is of the following form.

Quotient	
Divisor) Dividend	OR Divisor) Dividend (Quotient
-----	-----
-----	-----
-----	-----
Remainder	Remainder

Find $2862 / 4$

- 4 Into 2 goes 0 remainder 2
- 4 goes into 28 = 7
- 4 into 6 goes 1 remainder 2
- 4 into 22 goes 5 remainder 2

Above example

$$\begin{array}{r} 715 \\ 4 \overline{) 2862} \\ \underline{28} \\ 6 \\ \underline{4} \\ 22 \\ \underline{20} \\ 2 \end{array}$$

**Divisor = 4
Divided = 2862
Quotient = 715
Remainder = 2**

But in the FastMaths Division process, the format is

Divisor) Dividend

Quotient / Remainder

The conventional method is always the same irrespective of the divisor. But FastMaths methods are different depending on the nature of the divisor.

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Chapter 5 : Division

5.3 Divisibility Tests

5.3.1 Divisibility by 2,5,10

Look at the following Series 2,4,6,8,10,12,14,16,???

All numbers ending in even numbers or zero must have 2 as a factor. We say the number is divisible by 2. Any number ending in an even number or zero is a multiple of 2.

Given number 36

2 is a factor of 36
36 is a multiple of 2
36 is divisible by 2

Look at the following Series 5,10,15,20,25 ,???
All numbers ending in 5 or zero are divisible by 5.

Look at the following Series 10,20,30,40,?..
All numbers ending in 0 are divisible by 10

All numbers ending in even numbers or zero are divisible by 2

All numbers ending in 5 or zero are divisible by 5.

All numbers ending in 0 are divisible by 10

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Chapter 5 : Division

5.3 Divisibility Tests

5.3.2 Divisibility by 3 and 9

Look at the following Series 9,18,27,36??.
Digital root or digit sum of the series 9,9,9,9?..

All numbers whose digit sum is 9 are divisible by 9

All numbers with a digit sum is 9 are divisible by 9

**Look at the following Series 3, 6, 9, 12, 15, 18,?..?.
Digital root or digit sum of the series 3, 6, 9, 3, 6, 9 ???..
All numbers with a digit sum of 3,6 or 9 are divisible by 3**

All numbers with a digit sum of 3,6 or 9 are divisible by 3

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Chapter 5 : Division

5.3 Divisibility Tests

5.3.3 Divisibility by 4

**Look at the following Series 4,8,12,16,20,24,28,32???.
If 4 divides into last 2 digits of a number then 4 divides into the whole number**

If 4 divides into last 2 digits of a number then 4 divides into the whole number

5.3.4 Divisibility by 6

Any number which is divisible by 6 must also divisible by 2 and by 3. So the test for divisibility by 6 ; it must pass the test for both 2 and 3

78 is divisible by 2, but also by 3 (as its digit sum is 6) so 78 is divisible by 6. All numbers divisible by both 2 and 3 are divisible by 6

All numbers divisible by both 2 and 3 are divisible by 6

5.3.4 Divisibility by 8

A 3 digit number is divisible by 8 if the ultimate plus 2 times the penultimate plus 4 times the pen-penultimate is divisible by 8.

Is 134 is divisible by 8 ?

Find we can divide $4 + 2 \times 3 + 8 \times 1$ by 8 :

$$4 + 6 + 8 = 18$$

18/8 gives 2 and remainder 2 ,

8 does not divide exactly 134 so 134 is not divisible by 8

A 3 digit number is divisible by 8 if the ultimate plus 2 times the penultimate plus 4 times the pen-penultimate is divisible by 8.

If a number is divisible by 8 it must also be divisible by 2 and 4.

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Chapter 5 : Division

5.3 Divisibility Tests

5.3.5 Divisibility by 11

Add all the digits in the ODD position and all digits in the EVEN position and subtract the smaller result from the larger result. If we get 0 or 11 or any multiples of 11 , then the number is divisible by 11

Example : 7282231

Sum of odd digits : $7+8+2+1 = 18$

Sum of Even digits : $2 +2+3 = 7$

$18-7 = 11$. the number 7282231 is divisible by 11

Add all the digits in the ODD position and all digits in the EVEN position and subtract the smaller result from the larger result. If we get 0 or 11 or any multiples of 11 , then the number is divisible by 11

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Chapter 5 : Division

5.3 Divisibility Tests

5.3.5 Divisibility by 15

Any number which is divisible by 15 must also be divisible by 5 and by 3. So the test for divisibility by 15 ; it must pass the test for both 5 and 3

All numbers divisible by both 5 and 3 are divisible by 15

345 is divisible by 5, but also by 3 (as its digit sum is 3) so 345 is divisible by 15

All numbers divisible by both 5 and 3 are divisible by 15

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Chapter 5 : Division

5.3 Divisibility Tests

5.3.6 Summary

Divisibility Tests: Summary	
Number tested	Test
0,2	Is the last digit 0 or even?
3,6,9	Is the digit Sum 3, 6 or 9?
4	Is the 2 digit number on the end is divisible by 4?
5	Is the last digit is 0 or 5?
6	Is the number divisible by both 2 and 3?
8	If the ultimate plus 2 times penultimate plus four times the pen-penultimate is divisible by 8
9	Is the digit sum 9?
10	Is the last digit 0?
15	Is the number divisible by both 3 and 5?

Assignments

- Q1. Is 4662 divisible by 18 ?**
Q2. Is 1848 divisible by 24 ?
Q3. Is 3444 Divisible by 12 ?
Q4. Is 2772 Divisible by 36 ?
Q5. Is 7341 Divisible by 52?

Assignments Answers

Q1. Is 4662 divisible by 18 ?

Since $18 = 2 \times 9$, a number will be divisible by 18 if it is divisible by both 2 and 9

4662 can be easily divisible by 2

4662 digit sum is 9, so divisible by 9 also

4662 is divisible by 18

Q2. Is 1848 divisible by 24 ?

Since $24 = 6 \times 4$, Do not use 6 and 4 since they are not relatively prime.

$24 = 3 \times 8$, use 3 and 8 and verify 1848 can be divided by both 3 and 8.

1848 can be easily divisible by 8 since ultimate plus 2 times penultimate plus four times the pen-penultimate is $8 + 2 \times 4 + 4 \times 8 = 48$ is divisible by 8

1848 digit sum is 3, so divisible by 3 also

1848 is divisible by 24

Q3. Is 3444 Divisible by 12 ?

Since $12 = 3 \times 4$, verify 3444 can be divided by both 3 and 4.

3444 can be easily divisible by 3 since the digit sum is 6

3444 can be divided by 4 since the last 2 digit sum 44 can be divided by 4

3444 is divisible by 12

Q4. Is 2772 Divisible by 36 ?

Since $36 = 6 \times 6$, verify 2772 can be divided by 6, ie divisible by 2 and 3

2772 can be easily divisible by 3 since the digit sum is 9

2772 can be divided by 2

2772 is divisible by 36

Q5. Is 7341 Divisible by 52?

7341 is an odd number cannot be divided by an even number 52

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Chapter 5 : Division

5.4. Division techniques

5.4.1. Dividing by 9

Consider some two digit numbers (dividends) and same divisor 9.

Observe the following example.

i) $15 \div 9$ The quotient (Q) is 1, Remainder (R)

is 6.

$$\begin{array}{r} \text{since } 9 \overline{) 15} \text{ (1} \\ \underline{9} \\ 6 \end{array}$$

ii) $34 \div 9$, Q is 3, R is 7.

iii) $60 \div 9$, Q is 6, R is 6.

iv) $80 \div 9$, Q is 8, R is 8.

Each number to be divided has been separated into two parts by a diagonal stroke. The left-hand part gives the first part of the answer and right-hand side gives the remainder.

Steps

1) Separate off the last digit of the dividend with a diagonal stroke.

2) Put the first digit of the dividend as it is under the horizontal line. Put the same digit under the right hand part for the remainder, add the two and place the sum i.e., sum of the digits of the numbers as the remainder.

Example 1: Find $13/9$, $34/9$ and $80/9$

$$\begin{array}{r} 1 / 3 \\ 1 \\ \hline 1 / 4 \end{array} \quad , \quad \begin{array}{r} 3 / 4 \\ 3 \\ \hline 3 / 7 \end{array} \quad , \quad \begin{array}{r} 8 / 0 \\ 8 \\ \hline 8 / 8 \end{array}$$

$13 \div 9$ gives Q = 1, R = 4

$34 \div 9$ gives Q = 3, R = 7

$80 \div 9$ gives Q = 8, R = 8

Example 2: Find $21 \div 9$

$$9 \overline{) 21}$$

$$\underline{18} \quad 3$$

i.e Q=2, R=3

Example 3: Find 43 ? 9

$$9) 4 / 3$$

$$4 / 7$$

$$\text{i.e } Q = 4, R = 7.$$

In the division of two digit numbers by 9, we can take the first digit down for the quotient-column and by adding the quotient to the second digit, we get the remainder.

Consider the following examples

1) Find 104 / 9

9) 104 (11		9) 10 / 4
99		1 / 1
?????	as	??????
5		11 / 5

2) Find 212 / 9

9) 212 (23		9) 21 / 2
207		2 / 3
?????	as	??????
5		23 / 5

3) Find 401 / 9

9) 401 (44		9) 40 / 1
396		4 / 4
?????	as	??????
5		44 / 5

Note that the remainder is the sum of the digits of the dividend. The first digit of the dividend from left is added to the second digit of the dividend to obtain the second digit of the quotient. This digit added to the third digit sets the remainder. The first digit of the dividend remains as the first digit of the quotient.

Division by 9 rules:

- The remainder is the sum of the digits of

the dividend.

- The first digit of the dividend from left is added to the second digit of the dividend to obtain the second digit of the quotient.
- This digit is added to the third digit set the remainder.
- The first digit of the dividend remains as the first digit of the quotient.

Example 4: Find $511 / 9$

Add the first digit 5 to second digit 1 getting $5 + 1 = 6$. Hence Quotient is 56. Now second digit of 56 i.e., 6 is added to third digit 1 of dividend to get the remainder i.e., $1 + 6 = 7$

$$9 \text{) } 51 / 1$$

$$56 / 7$$

Q is 56, R is 7.

Example 5: Find $1204 / 9$

The first digit 1 is set down as the first answer digit . Take this 1 and add the next digit 2. This gives 3 as the next digit. Working this way $3+0 = 3$, and the remainder is $3+4 = 7$

$$9 \text{) } 120 / 4$$

$$133 / 7$$

$$Q = 133, R = 7$$

Example 6: Find $13210 / 9$

The first digit 1 is set down as the first answer digit . Take this 1 and add the next digit 3. This gives 4 as the next digit. Working this way $4+2 = 6$, $6+1 = 7$ and the remainder is $7+0 = 7$

$$9 \text{) } 1321 / 0$$

$$1467 / 7$$

$$Q = 1467, R = 7$$

Assignments

Q1. Find $235471 / 9$

Q2 Find $42111 / 9$

Q3. Find $214091 / 9$

Q4. Find $112 / 9$

Q5. Find $1022 / 9$

Assignments Answers

Q1. Find $234571 / 9$

The first digit 2 is set down as the first answer digit . Take this 2 and add the next digit 3. This gives 5 as the next digit. Working this way $5+4 =9$, $9+5 =14$, $14+7 =21$ and the remainder is $21+1 =22$

9) 234 5 7 / 1
259 ₁4 ₂1/ 22

26061 / 22

The remainder 22 is larger than 9 , the divisor and so divide by 9 giving 2 and remainder 4. This 2 is carried over to the left giving answer are 26063/4

Q= 26063, R = 4

Q2. Find $42111 / 9$

The first digit 4 is set down as the first answer digit . Take this 4 and add the next digit 2. This gives 6 as the next digit. Working this way $6+1 =7$, $7+1 =8$, and the remainder is $8+1 =9$

9) 4211 / 1
4678 / 9

4678 / 9

The remainder 9 is equal to 9 , the divisor and so divide by 9 giving 1 and remainder 0. This 1 is carried over to the left

giving answer are 4679/0

Q= 4679, R = 0

Q3. Find 214091/9

**9) 2140 9 / 1
2377 16/ 17

23786 / 17**

The remainder 17 is larger than 9 , the divisor and so divide by 9 giving 1 and remainder 8. This 1 is carried over to the left giving answer are 23787/8

Q= 23787, R = 8

Q4. Find 112/9

**9) 11 / 2
12 / 4

12 / 4**

Q= 12, R = 4

Q5. Find 1022/9

**9) 102 / 2
113 / 5

113 / 5**

Q= 113, R = 5

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Chapter 5 : Division

5.4. Division techniques

5.4.2 Dividing by 5, 50 and 25

Example 1: Find 85 / 5

Find double of 85 and divide by 10

$$85 \times 2 / 5 \times 2$$

$$170 / 10 = 17$$

$$\text{Answer} = 17$$

Example 2: Find 750 / 50

Find double of 750 and divide by 100

$$750 \times 2 / 50 \times 2$$

$$1500 / 100 = 15$$

$$\text{Answer} = 15$$

Example 3: Find 82 / 25

Double 82 twice and divide by 100

$$82 \times 4 / 25 \times 4$$

$$82 \times 2 \times 2 / 100$$

Double 82 gives 164 and doubling this gives 328

$$\text{Answer} = 328 / 100 = 3.28$$

Assignments

Q1. Find 250 / 5

Q2. Find $343 / 25$

Q3. Find $765 / 50$

Assignments Answers

Q1. Find $250 / 5$

Can be written as $250 \times 2 / 10$

Find double of 250 and divide by 10

$$250 \times 2 / 10 = 500 / 10 = 50$$

Answer = 50

Q2. Find $343 / 25$

Can be written as $343 \times 4 / 100$

Double 343 gives 686 and doubling this gives 1372

$$1372 / 100 = 13.72$$

Answer = 13.72

Q3. Find $765 / 50$

Can be written as $765 \times 2 / 100$

Double 765 gives 1530

$$1530 / 100 = 15.3$$

Answer = 15.3

Chapter 5 : Division

5.4.3 Division Techniques

5.4.3 Division with any base

Example 1: Consider the division $1235 \div 89$.

Conventional method:

$$\begin{array}{r} 89 \overline{) 1235} \text{ (13} \\ \underline{89} \\ 345 \\ \underline{267} \\ 78 \end{array}$$

Thus $Q = 13$ and $R = 78$.

FastMaths method:

This method is useful when the divisor is nearer and less than the base. Since for 89, the base is 100 we can apply the method.

Step (i): Write the dividend and divisor as in the conventional method. Obtain the modified divisor (M.D.) applying the complement formula. Write M.D. just below the actual divisor. Thus for the divisor 89, the M.D. obtained by using complement is 11 in the last from 10 and the rest from 9. Now Step 1 gives

$$\begin{array}{r} 89 \overline{) 1235} \\ 11 \end{array}$$

Step (ii): Bifurcate the dividend by a slash so that RHS of dividend contains the number of digits equal to that of M.D. Here M.D. contains 2 digits hence

$$89 \text{) } 12 / 35$$

11

Step (iii): Multiply the M.D. with first column digit of the dividend. Here it is 1. i.e. $11 \times 1 = 11$. Write this product place wise under the 2nd and 3rd columns of the dividend.

$$\begin{array}{r} 89 \text{) } 12/35 \\ 11 \quad 1 \quad 1 \\ \hline 1 \end{array}$$

Step (iv): Add the digits in the 2nd column and multiply the M.D. with that result i.e. $2+1=3$ and $11 \times 3=33$. Write the digits of this result column wise as shown below, under 3rd and 4th columns. i.e.

$$\begin{array}{r} 89 \text{) } 12/35 \\ 11 \quad 1 \quad 1 \\ \quad 33 \\ \hline 13 / \end{array}$$

Step (v): Add the digits in the 3rd column $3+1+3=7$. Add the digits in the 4th column $5+3=8$.

$$\begin{array}{r} 89 \text{) } 12/35 \\ 11 \quad 1 \quad 1 \\ \quad 33 \\ \hline 13/78 \end{array}$$

Now the division process is complete, giving $Q = 13$ and $R = 78$.

Example 2: Find Q and R for $121134 \div 8988$.

Steps (1+2):

$$8988 \text{) } 12 / 1134$$

1012

Step (3):

$$\begin{array}{r}
 8988 \) \ 12 \ / \ 1134 \\
 1012 \quad 1 \quad 012 \\
 \hline
 1
 \end{array}$$

Step (4):

$$\begin{array}{r}
 8988 \) \ 12 \ / \ 1134 \\
 1012 \quad 1 \quad 012 \\
 \quad \quad \quad 3036 \ [\ 2 + 1 = 3 \text{ and } 3 \times 1012 = \\
 3036 \] \\
 \hline
 13 \ /
 \end{array}$$

Now final Step

$$\begin{array}{r}
 8988 \) \ 12 \ / \ 1134 \\
 1012 \quad 1 \quad 012 \\
 \quad \quad \quad 3036 \ (\text{Column wise addition}) \\
 \hline
 13 \ / \ 4290
 \end{array}$$

Thus $121134 / 8988$ gives $Q = 13$ and $R = 4290$.

In all the cases mentioned above, the remainder is less than the divisor.

What about the case when the remainder is equal or greater than the divisor?

Example 3.

$$\begin{array}{cc}
 9 \) \ 3 \ / \ 6 & 9 \) \ 24 \ / \ 6 \\
 \quad 3 & \quad 2 \ / \ 6 \\
 \quad \text{??????} & \text{or} \quad \text{????????} \\
 \quad 3 \ / \ 9 \ (\text{equal}) & 26 \ / \ 12 \ (\text{greater}).
 \end{array}$$

We proceed by re-dividing the remainder by 9, carrying over this Quotient to the quotient side and retaining the final remainder in the remainder side.

$$\begin{array}{cc}
 9 \) \ 3 \ / \ 6 & 9 \) \ 24 \ / \ 6 \\
 \quad / \ 3 & \quad 2 \ / \ 6 \\
 \quad \text{???????} & \text{????????} \\
 \quad 3 \ / \ 9 & \quad 26 \ / \ 12 \\
 \quad \text{???????} & \text{????????} \\
 \quad 4 \ / \ 0 & \quad 27 \ / \ 3 \\
 \\
 Q = 4, R = 0 & Q = 27, R = 3.
 \end{array}$$

Example 4. Find 113/89

Get the complement of 89 as 11. Set off the 2 digits from the right as the remainder consists of 2 digits. Further while carrying the added numbers to the place below the next digit, we have to multiply by this 11.

89) 1 / 13

11 / 11
???????
 1 / 24

Q = 1, R = 24.

Example 5. Find 10015 / 89

Get the complement of 89 as 11. Set off the 2 digits from the right as the remainder consists of 2 digits. Further while carrying the added numbers to the place below the next digit, we have to multiply by this 11.

89) 100 / 15
11 11 / first digit 1 x 11
 1 / 1 total second is 0+1=1, 1x11
 /22 total of 3rd digit is 0+1+1=2, 2x11=22

 112 / 47
Q = 112, R = 47.

Example 6 : What is 10015 ? 98 ?

Get the complement as 100 - 98 = 02. Set off the 2 digits from the right as the remainder consists of 2 digits. While carrying the added numbers to the place below the next digit, multiply by 02.

Thus

98) 100 / 15
02 02 / i.e., 10015 ? 98 gives
 0 / 0 Q = 102, R = 19
 / 04
??????????

102 / 19

Example 7: Find $11422 \div 897$?

Complement of 897 is 103

```
897 ) 11 / 422
103   1 / 03
      / 206
      ????????
      12 / 658      Answer is Q = 12,
R=658.
```

Example 8: Find $1374 \div 878 = ?$

Step1. Separate off the last 3 digit of the dividend 1374 with a diagonal stroke

Step2. Write the complement of 878 ie 122 underneath 878.

```
878 ) 1 / 374
122
```

Step3. Bring down the first digit.

```
878 ) 1 / 374
122
      1
```

Step4. Multiply this 1 by the complement 122 and write $1 \times 122 = 122$ underneath the next dividend digit

```
878 ) 1 / 374
122   122
      1
```

Step5. Add up the second column. $374 + 122 = 496$ and this is the next quotient digit.

$$\begin{array}{r}
 878 \) \quad 1 / 374 \\
 122 \qquad \quad 122 \\
 \hline
 1 / 496
 \end{array}$$

The answer is 1 remainder 496

Assignments

Find The following

Q1) $3116 \div 98$

Q2) $120012 \div 9$

Q3) $1135 \div 97$

Q4) $113401 \div 997$

Q5) $11199171 \div 99979$

Assignments Answers

Q1 Find $3116 \div 88$

Step1. Separate off the last 2 digit of the dividend 3116 with a diagonal stroke

Step2. Write the complement of 88 ie 12 underneath 98.

$$\begin{array}{r}
 88 \) \quad 31 / 16 \\
 12
 \end{array}$$

Step3. Bring down the first digit.

$$\begin{array}{r}
 88 \) \quad 31 / 16 \\
 12 \qquad \quad 3
 \end{array}$$

Step4. Multiply this 3 by the complement 12 and write $3 \times 12 = 36$ underneath the next dividend digit

$$88 \) \quad 31 / 16$$

$$\begin{array}{r} 12 \quad 3 / 6 \\ 3 \end{array}$$

Step5. Add up the second column. $3 + 1 = 4$ and this is the next quotient digit.

$$\begin{array}{r} 88) \quad 31 / 16 \\ 12 \quad 3 / 6 \\ 34 \end{array}$$

Step6. Multiply this 4 by the complement 12 and write $4 \times 12 = 48$ underneath the next dividend digit

$$\begin{array}{r} 88) \quad 31 / 16 \\ 12 \quad 3 / 6 \\ 48 \end{array}$$

$$34 / 124$$

**Remainder is 124 and it is greater than 88.
Divide 124 by 88 gives 1 and remainder 36.
Carry over 1 to left and gives 35/36**

The answer is 35 remainder 36.

Q2) $120012 \div 9 = 13334$ and remainder 6

Q3) $1135 \div 97 = 11$ and remainder 68

Q4) $113401 \div 997 = 113$ and remainder 740

Q5) $11199171 \div 99979 = 112$ and remainder 11623

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Chapter 5 : Division

5.4.3 Division Techniques

5.4.4 Vulgar fractions whose denominators are numbers ending in 9 :

Consider examples of $1/a9$, where $a = 1, 2, \dots, 9$. In the

conversion of such vulgar fractions into recurring decimals

Example.1: Division Method: Find the value of $1 / 19$.

The numbers of decimal places before repetition is the difference of numerator and denominator, i.e., $19-1=18$ places.

For the denominator 19, the previous is 1.

Hence one more than the previous is $1 + 1 = 2$.

The method of division is as follows:

Step.1: Divide numerator 1 by 20.

$1 / 20 = 0.1 / 2 = .10$ (0 times, 1 remainder)

Step.2: Divide 10 by 2

0.0_05 (5 times, 0 remainder)

Step.3: Divide 5 by 2

0.05_12 (2 times, 1 remainder)

Step.4: Divide 12 by 2

0.0526 (6 times, No remainder)

Step.5: Divide 6 by 2

0.05263 (3 times, No remainder)

Step. 6: Divide 3 by 2

0.05263_11 (1 time, 1 remainder)

Step.7: Divide $_11$ i.e., 11 by 2

0.052631_5 (5 times, 1 remainder)

Step.8: Divide $_15$ i.e., 15 by 2

0.0526315_7 (7 times, 1 remainder)

Step.9: Divide $_17$ i.e., 17 by 2

0.05263157_8 (8 times, 1 remainder)

Step.10: Divide $_18$ i.e., 18 by 2

0.0526315789 (9 times, No remainder)

Step.11: Divide 9 by 2

0.0526315789_4 (4 times, 1 remainder)

Step.12: Divide $_14$ i.e., 14 by 2

0.052631578947 (7 times, No remainder)

Step.13: Divide 7 by 2

0.052631578947₁3 (3 times, 1 remainder)

Step. 14: Divide ₁3 i.e., 13 by 2

0.052631578947₃6 (6 times, 1 remainder)

Step.15: Divide ₁6 i.e., 16 by 2

0.052631578947368 (8 times, No remainder)

Step.16: Divide 8 by 2

0.0526315789473684 (4 times, No remainder)

Step.17: Divide 4 by 2

0.05263157894736842 (2 times, No remainder)

Step.18: Divide 2 by 2

0.052631578947368421 (1 time, No remainder)

Now from Step 19, i.e., dividing 1 by 2, Step 2 to Step 18 repeats thus giving

$$1 / 19 = 0.052631578947368421$$

Note that we have completed the process of division only by using 2's. Nowhere the division by 19 occurs.

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Chapter 5 : Division

5.4.3 Division Techniques

5.4.4 Multiplication Method: Find the value of $1 / 19$

As we recognize the right most digit of the repeating block of decimals for the type $1 / a^9$. For any fraction of the form

i.e., in whose denominator 9 is the digit in is 1, we continue in the case of $1/19$ as follows :

For $1/19$, ?previous? of 19 is 1 and one more than of it is $1 + 1 = 2$. In the case of $1/29$ we work with $2 + 1 = 3$, In the case of $1/49$ we work with $4 + 1 = 5$

Therefore 2 is the multiplier for the conversion. In all cases of multiplication, we write the right most digit in the block as 1 and follow steps leftwards. When there is more than one digit in that product, we set the last of those digits down there and carry the rest of it over to the next immediately preceding digit towards left.

Step. 1 : 1

Step. 2 : 21 (multiply 1 by 2, put to left)

Step. 3 : 421 (multiply 2 by 2, put to left)

Step. 4 : 8421 (multiply 4 by 2, put to left)

Step. 5 : 168421 (multiply 8 by 2 =16, 1 carried over, 6 put to left)

Step. 6 : 1368421 ($6 \times 2 = 12, +1$ [carry over]
= 13, 1 carried over, 3 put to left)

Step. 7 : 7368421 ($3 \times 2, = 6 +1$ [Carryover]
= 7, put to left)

Step. 8 : 147368421 (as in the same process)

Step. 9 : 947368421 (continue to step 18)

Step. 10 : 18947368421

Step. 11 : 178947368421

Step. 12 : 1578947368421

Step. 13 : 11578947368421

Step. 14 : 31578947368421

Step. 15 : 631578947368421

Step. 16 : 12631578947368421

Step. 17 : 52631578947368421

Step. 18 : 1052631578947368421

Now from step 18 onwards the same numbers and order towards left continue.

Thus $1 / 19 = 0.052631578947368421$

It is interesting to note that we have

- i) Not at all used division process**
- ii) Instead of dividing 1 by 19 continuously, just multiplied 1 by 2 and continued to multiply the resultant successively by 2.**

Observations:

a) For any fraction of the form $1/a9$ i.e., in whose denominator 9 is the digit in the units place and ?a? is the set of remaining digits, the value of the fraction is in recurring decimal form and the repeating block's right most digit is 1.

b) Whatever may be a9, and the numerator, it is enough to follow the said process with (a+1) either in division or in multiplication.

c) Starting from right most digit and counting from the right, we see (in the given example $1 / 19$)

$1 / 19 = 0.052631578947368421$

Sum of 1st digit + 10th digit = $1 + 8 = 9$

Sum of 2nd digit + 11th digit = $2 + 7 = 9$

Sum of 3rd digit + 12th digit = $4 + 5 = 9$

Sum of 9th digit + 18th digit = $9 + 0 = 9$

From the above observations, we conclude that if we find first 9 digits, further digits can be derived as complements of 9.

i) Thus at the step 8 in division process we have 0.052631517 and next step. 9 gives 0.052631578

Now the complements of the numbers
0, 5, 2, 6, 3, 1, 5, 7, 8 from 9
9, 4, 7, 3, 6, 8, 4, 2, 1 follow the right order

i.e., 0.052631578947368421

Now taking the multiplication process we have
Step. 8 : 147368421
Step. 9 : 947368421

Now the complements of 1, 2, 4, 8, 6, 3, 7, 4, 9
from 9

i.e., 8, 7, 5, 1, 3, 6, 2, 5, 0 precede in
successive steps, giving the answer.
0.052631578947368421.

d) When we get (Denominator ? Numerator)
as the product in the multiplication process,
half the work is done. We stop the
multiplication there and write the remaining
half of the answer by merely taking down
complements from 9.

e) Either division or multiplication process of
giving the answer can be put in a single line
form.

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Chapter 5 : Division

5.4.3 Division Techniques

Find the value of $1 / 49$

Here ?previous? is 4. ?One more than the previous? is $4 + 1$
= 5. Now by division right ward from the left by ?5?.

$1/49 = .10$ ---- (divide 1 by 50)
= .02 - - - - - (divide 2 by 5, 0 times, 2 remainder)
= .02₂0 - - - - - (divide 20 by 5, 4 times)

= .0204 - - - - (divide 4 by 5, 0 times, 4 remainder)
 = .0204₄0 -- -- (divide 40 by 5, 8 times)
 = .020408 - - - (divide 8 by 5, 1 time, 3 remainder)
 = .020408₃1 - -(divide 31 by 5, 6 times, 1 remainder)
 = .0204081₁ 6 - - - - - continue
 = .02040816₁3₃2₂6₁5306₁1₁2₂2₂4₄4₄8 - - - - -

On completing 21 digits, we get 48 [Denominator - Numerator = 49 ? 1 = 48] standing up before us. Half of the process stops here. The remaining half can be obtained as complements from 9.

Thus 1 / 49 = 0.020408163265306122448
979591836734693877551

Now finding 1 / 49 by process of multiplication left ward from right by 5, we get

1 / 49 = -----1
 = -----51
 = -----₂551
 = -----₂7551
 = -----₃77551
 4₈3₉4₇2₉4₅9₄1₁8₃3₃6₁7₂3₃4₆9₄3₃8₇2₇551

Denominator ? Numerator = 49 ? 1 = 48

When we get 45 + 3 = 48 half of the process is over. The remaining half is automatically obtained as complements of 9.

Thus 1 / 49 = -----979591836734693877551
 = 0.020408163265306122448979591836734693877551

Example 2: Find 1/39

Now by multiplication method, 3 + 1 = 4

1/39 = -----1
 = -----41
 = -----₁641
 = -----₂5641
 = -----₂25641
 = -----₁025641

Here the repeating block happens to be block of 6 digits. Now the rule predicting the completion of half of the computation does not hold. The complete block has to be computed.

Now continue and obtain the result.

$$1/39 = 0.025641025641025641025641...$$

Assignments

Find the recurring decimal form of the fractions

Q1. $1 / 29$

Q2. $1 / 59$

Q3. $1 / 69$

Q4. $1 / 79$

Q5. $1 / 89$

Assignments Answers

Find the recurring decimal form of the fractions

Q1. $1 / 29 = 0.034482758620689655172413793103448....$

Q2. $1 / 59 = 0.016949152542372881355932203389831...$

Q3. $1 / 69 = 0.0144927536231884057971.....$

Q4. $1 / 79 = 0.0126582278481.....$

Q5. $1 / 89 = 0.011235955056179775280898876404494$

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Chapter 5 : Division

5.4.3 Division Techniques

General Method: Straight division

Example 4: 43852 ? 54.

Step1:

Put down the first digit (5) of the divisor (54) in the divisor column as operator and the other digit (4) as flag digit. Separate the dividend into two parts where the right part has one digit. This is because the flag digit is single digit.

The representation is as follows.

**4 : 4 3 8 5 : 2
5**

Step2:

i) Divide 43 by the operator 5. Now $Q = 8$ and $R = 3$. Write this $Q=8$ as the 1st Quotient - digit and prefix $R=3$, before the next digit i.e. 8 of the dividend, as shown below. Now 38 becomes the gross-dividend (G.D.) for the next step.

**4 : 4 3 8 5 : 2
5 : 3
_____**
: 8

ii) Subtract the product of flag digit (4) and first quotient digit (8) from the G.D. (38) i.e. $38 - (4 \times 8) = 38 - 32 = 6$. This is the net - dividend (N.D) for the next step.

Step3:

Now N.D Operator gives Q and R as follows. 6 ? 5, $Q = 1$, $R = 1$. So $Q = 1$, the second quotient-digit and $R = 1$, the prefix for the next digit (5) of the dividend.

$$\begin{array}{r}
 4 : 4 \ 3 \ 8 \ 5 : 2 \\
 5 : \quad 3 \ 1 \\
 \hline
 : \quad 8 \ 1
 \end{array}$$

Step4:

Now G.D = 15; product of flag-digit (4) and 2nd quotient - digit (1) is $4 \times 1 = 4$ Hence N.D = $15 - 4 = 11$ divide N.D by 5 to get $11 \div 5$, Q = 2, R = 1. The representation is

$$\begin{array}{r}
 4 : 4 \ 3 \ 8 \ 5 : 2 \\
 5 : \quad 3 \ 1 : 1 \\
 \hline
 : \quad 8 \ 1 \ 2 :
 \end{array}$$

Step5:

Now the RHS part has to be considered. The final remainder is obtained by subtracting the product of flag-digit (4) and third quotient digit (2) from 12.

Final remainder = $12 - (4 \times 2) = 12 - 8 = 4$.

Thus the division ends into

$$\begin{array}{r}
 4 : 4 \ 3 \ 8 \ 5 : 2 \\
 5 : \quad 3 \ 1 : 1 \\
 \hline
 : \quad 8 \ 1 \ 2 : 4
 \end{array}$$

Thus $43852 \div 54$ gives Q = 812 and R = 4.

Example 5: Divide $237963 \div 524$

Step1:

We take the divisor 524 as 5, the operator and 24, the flag-digit and proceed as in the above example. We now separate the dividend into two parts where the RHS part contains two digits for Remainder.

$$\begin{array}{r}
 24 : 2 \ 3 \ 7 \ 9 : 63 \\
 5
 \end{array}$$

Step2:

i) $23 \div 5$ gives Q = 4 and R = 3, G.D = 37.

ii) N.D is obtained as

$$= 37 ? (4 \times 2 + 4 \times 0)$$

$$= 29.$$

Representation

$$24 : 2 \ 3 \ 7 \ 9 : 63$$

$$5 \quad \quad 3$$

$$: 4$$

Step3:

i) N.D ? Operator = 29 ? 5 gives Q = 5, R = 4
and G.D = 49.

ii) N.D is obtained as

$$= 49 ? (10 + 16)$$

$$= 49 ? 26$$

$$= 23.$$

i.e.,

$$24 : 2 \ 3 \ 7 \ 9 : 63$$

$$5 : \quad \quad 3 \ 4 :$$

$$: 4 \ 5 :$$

Step 4:

i) N.D ? Operator = 23 ? 5 gives Q = 4, R = 3
and G.D = 363.

Note that we have reached the remainder part,
thus 363 is total sub?remainder.

$$24 : 2 \ 3 \ 7 \ 9 : 63$$

$$5 : \quad 3 \ 4 \quad : 3$$

$$: \quad 4 \ 5 \ 4 :$$

Step 5:

We find the final remainder as follows.

Subtract the cross-product of the two, flag-digits [2,4] and two last quotient-digits [5,4] and then vertical product of last flag-digit with last quotient-digit from the total sub-remainder.

Note that 2, 4 are two flag digits: 5, 4 are two last quotient digits:

$$363 - [(8 + 20) / 16] = 363 - [28 / 16]$$

$$= 363 - 296 = 67$$

Thus the division $237963 \div 524$ gives $Q = 454$ and $R = 67$.

Assignment

Q1. Find $2465 \div 98$

Q2 . Find $1313 \div 867$

Q3 . Find $111 \div 76$

Q4 . Find $12034 \div 8877$

Q4. Find $166 \div 82$

Assignment Answers

Q1. Find $2465 \div 98$

Step1:

Put down the first digit (9) of the divisor (98) in the divisor column as operator and the other digit (8) as flag digit. Separate the dividend into two parts where the right part has one digit. This is because the flag digit is single digit.

The representation is as follows.

8 : 2 4 6 : 5
9

Step2:

i) Divide 24 by the operator 9. Now $Q = 2$ and $R = 6$. Write this $Q = 2$ as the 1st Quotient - digit and prefix $R = 6$, before the next digit i.e. 1 of the dividend, as shown below. Now 61 becomes the gross-dividend (G.D.) for the next step.

8 : 2 4 6 : 5
9 : 6

: 2

ii) Subtract the product of flag digit (8) and first quotient digit (2) from the G.D. (66) i.e. $66 - (2 \times 8) = 66 - 16 = 50$. This is the net - dividend (N.D) for the next step.

Step3:

Now N.D Operator gives Q and R as follows. 50 ? 9, Q = 5, R = 5. So Q = 5, the second quotient-digit and R = 5, the prefix for the next digit (5) of the dividend.

$$\begin{array}{r} 8 : 246 : 5 \\ 9 : \quad 6 \quad 5 \\ \hline : 25 : \end{array}$$

Step4:

Now G.D = 55; product of flag-digit (8) and 2nd quotient - digit (5) is $8 \times 5 = 40$ Hence N.D = $55 - 40 = 15$

$$\begin{array}{r} 8 : 246 : 5 \\ 9 : \quad 6 \quad 5 \\ \hline : 25 : 15 \end{array}$$

Thus 2465 ? 98 gives Q = 25 and R = 15.

Q2 . Find 1313 / 867

Step1:

We take the divisor 867 as 8, the operator and 67, the flag-digit and proceed as in the above example. We now separate the dividend into two parts where the RHS part contains two digits for Remainder.

$$\begin{array}{r} 67 : 13 : 13 \\ 8 \end{array}$$

Step2:

i) $13?8$ gives $Q = 1$ and $R = 5$

Representation

$67 : 13 : 13$
 $8 \quad 5$

$: 1$

Step 3: We find the final remainder as follows.

Subtract the cross-product from $513 - 67 =$

446

$67 : 13 : 13$
 $8 \quad 5$

$: 1 : 446$

Thus the division $1313 ? 867$ gives $Q = 1$ and $R = 446$.

Q3 . Find $111 / 76$

Step1:

We take the divisor 76 as 7, the operator and 6, the flag-digit and proceed as in the above example. We now separate the dividend into two parts where the RHS part contains one digit for Remainder.

$6 : 11 : 1$
 7

Step2:

i) $11?7$ gives $Q = 1$ and $R = 4$

Representation

$6 : 11 : 1$
 $7 \quad 4$

$: 1$

Step 3: G.D = 41. N.D is obtained as

= $41 ? (6 \times 1)$

= 35.

6 : 1 1 : 1
7 4

: 1 : 35

Thus the division $111 \div 67$ gives $Q = 1$ and $R = 35$.

Q4 . Find $12034 / 8877$

Step1:

We take the divisor 8877 as 88, the operator and 77, the flag-digit and proceed as in the above example. We now separate the dividend into two parts where the RHS part contains two digits for Remainder.

77 : 120 : 34
88

Step2:

i) $120 \div 88$ gives $Q = 1$ and $R = 32$

Representation

77 : 1 2 0 : 3 4
88 32

: 1

Step 3:

We find the final remainder as follows.
Subtract the cross-product from $3234 - 77 = 3157$

G.D = 3234. N.D is obtained as

= $3234 \div (77 \times 1)$
= $3234 - 77 = 3157$

77 : 1 2 0 : 3 4
88 32

$$: 1 \quad : 3157$$

Thus the division $12034 \div 8877$ gives $Q = 1$
and $R = 3157$

Q4. Find $166 \div 82$

Step1:

We take the divisor 82 as 8, the operator and 2, the flag-digit and proceed as in the above example. We now separate the dividend into two parts where the RHS part contains one digit for Remainder.

$$\begin{array}{r} 2 : 16 : 6 \\ 8 \end{array}$$

Step2:

i) $16 \div 8$ gives $Q = 2$ and $R = 0$, G.D = 6.

$$\begin{array}{r} 2 : 16 : 6 \\ 8 \quad 0 \end{array}$$

$$: 2 :$$

Remainder is calculated as $6 - 2 \times 2 = 2$

$$\begin{array}{r} 2 : 16 : 6 \\ 8 \quad 0 \end{array}$$

$$: 2 : 2$$

Thus the division $166 \div 82$ gives $Q = 2$ and $R = 0$.

Chapter 5 : Division

5.4. Divisibility Test for 7,13,19,39

5.4.1 Divisibility by 7

Is 33803 divisible by 7?

1/7 is same as 7/49 .

The test for 49 uses 5 (5 being one more than the 4 of 49). 5 is called the Osculator, $P = 5$. The process employed is called Osculation

**Multiply units place by Osculator $3 \times 5 = 15$.
Add the next digit, $15 + 0 = 15$**

Casting out 7's leaves $15 - 2 \times 7 = 1$ Repeat this process for other digits

$1 \times 5 = 5$, $5 + 8 = 13$. Casting out 7's leaves 6.

$6 \times 5 = 30$, $30 + 3 = 33$. Casting out 7's leaves 5.

$5 \times 5 = 25$, $25 + 3 = 28$

28 is divisible by 7 and therefore 33803 is divisible by 7.

Assignments

Check The following numbers are divisible by 7

Q1. 4523

Q2. 87339

Q3. 12349

Q4. 987

Assignments Answers

Check The following numbers are divisible by 7

Q1. 4527 is not divisible by 7

Q2. 87345 is divisible by 7

Q3. 12349 is not divisible by 7

Q4. 987 is divisible by 7

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Chapter 5 : Division

5.4. Divisibility Test for 7,13,19, 29 etc

5.4.2 Divisibility by 13

Is 42705 divisible by 13?

1/13 is same as 3/39 . So we use Osculator as 4

$5 \times 4 = 20$, $20 + 0 = 20$. Casting out 13's leaves 7.

$7 \times 4 = 28$, $28 + 7 = 35$. Casting out 13's leaves 9.

$9 \times 4 = 36$, $36 + 2 = 38$. Casting out 13's leaves 12.

$12 \times 4 = 48$, $48 + 4 = 52$

52 is divisible by 13 and therefore 42705 is divisible by 13.

Assignments

Check the following numbers are divisible by 13

Q1. 2908

Q2. 8723

Q3. 123567

Q4. 78351

Assignments Answers

Check the following numbers are divisible by 13

Q1. 2908 is not divisible by 13

Q2. 8723 is divisible by 13

Q3. 123567 is not divisible by 13

Q4. 78351 is not divisible by 13

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Chapter 5 : Division

5.4. Divisibility Test for 7,13,19, 29 etc

5.4.3 Divisibility by 19

The test for 19 uses 2 (2 being one more than the 1 of 19). 2 is called the Osculator, $P = 2$. The process employed is called Osculation

Is 548 divisible by 19?

Osculator is $1+1 = 2$

Multiply units place by Osculator $8 \times 2 = 16$. Add the next digit, $16 + 4 = 20$, and multiply by 2, $20 \times 2 = 40$. Add the next digit, $40 + 5 = 45$.

5	4	8
90	45	20

90 is not divisible by 19 and therefore 548 is also not divisible by 19.

Is 1015968 divisible by 19?

Osculator is $1+1 = 2$

Starting at the right-hand end, $8 \times 2 = 16$. Add the next digit, $16 + 6 = 22$. Cast out 19's, $22 \div 19 = 3$.

$3 \times 2 = 6$. Add the next digit, $6 + 9 = 15$.

$15 \times 2 = 30$. Cast out 19's = 11, add the 5 = 16.

$16 \times 2 = 32$, cast out 19 leaving 13 and add 1 = 14.

$14 \times 2 = 28$, cast out, = 9, and add 0, = 9.

$9 \times 2 = 18$, add 1 = 19.

19 is divisible by 19 and therefore 1015968 is divisible by 19.

Assignments

Check the following numbers are divisible by 19

Q1. 204567

Q2. 7866

Q3. 1234943

Q4. 987

Assignments Answers

Check the following numbers are divisible by 19

Q1. 204567 is not divisible by 19

Q2. 7866 is divisible by 19

Q3. 1234943 is divisible by 19

Q4. 987 is not divisible by 19

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Chapter 5 : Division

5.4. Divisibility Test for 7,13,19,39

5.4.4 Divisibility Test for 39

Is 161928 divisible by 39?

Osculator is $3+1 = 4$

Starting at the right-hand end, $8 \times 4 = 32$. Add the next digit, $32 + 2 = 34$.

$34 \times 4 = 136$. Add the next digit, $136 + 9 = 145$. Cast out 39's from 145, giving $145 - 3 \times 39 = 28$.

$28 \times 4 = 112$. Add the next digit, $112 + 1 = 113$. Cast out 39's = 35.

$35 \times 4 = 140$, Add the next digit 6, $140 + 6 = 146$. cast out 39 leaving 29

$29 \times 4 = 116$, and add 1, = 117.

117 is divisible by 39 and therefore 161928 is divisible by

39.

Assignments

Check the following numbers are divisible by 39

Q1. 34567

Q2. 976

Q3. 1287

Q4. 9876516

Assignments Answers

Check the following numbers are divisible by 39

Q1. 34567 is not divisible by 39

Q2. 976 is not divisible by 39

Q3. 1287 is divisible by 39

Q4. 9876516 is divisible by 39

Chapter 6 : Squaring Techniques

6.0 Square Numbers

			000	
		00	000	
	0	00	000	
	1	2	3	
Number of Squares	1	2	3	4
Number of counts	1	4	9	16

The numbers 1,4,9,16....are called Square Numbers because you can arrange the number of counters to form a Square. The 4 Counters are in 2 rows of 2. The 9 counters are in 3 rows and 3 columns.

$$\begin{aligned}1 \times 1 &= 1 \\2 \times 2 &= 4 \\3 \times 3 &= 9\end{aligned}$$

So if we square a number we multiply it by itself. 3 Squared is $3^2 = 3 \times 3 = 9$; 4 Squared is $4^2 = 4 \times 4 = 16$;

Square numbers always have an odd number of factors. All other numbers have an even number of factors

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Chapter 6 : Squaring Techniques

6.1 Squaring Techniques : Squares of numbers ending in 5 :

Consider the example 25^2 .

We have to find out the square of the number 25. For the number 25, the last digit is 5 and the "previous" is 2.

Hence 'one more than' previous is $2+1=3$. The method is 'to multiply the previous digit 2 by one more than itself, by 3'. It becomes the LHS of the result, $2 \times 3 = 6$. The RHS of the result is 5^2 , i.e., 25.

Thus $25^2 = (2 \times 3) / 25 = 625$.

In the same way,

$35^2 = 3 \times (3+1) / 25 = 3 \times 4 / 25 = 1225$;

$65^2 = 6 \times 7 / 25 = 4225$;

$105^2 = 10 \times 11 / 25 = 11025$;

$135^2 = 13 \times 14 / 25 = 18225$.

Assignments

Find the Squares of the following

Q1. 15

Q2. 125

Q3. 635

Q4. 1105

Q5. 2545.

Assignments Answers

Find the Squares of the following

Q1. $15^2 = 225$

Q2. $125^2 = 15625$

Q3. $635^2 = 403225$

Q4. $1105^2 = 1221025$

Q5. $2545^2 = 6477025$

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Chapter 6 : Squaring Techniques

6.2 Squares of numbers close to bases of powers of 10.

The Method is "what ever the deficiency subtract that deficit from the number and write along side the square of that deficit". This method can be applicable to obtain squares of numbers close to bases of powers of 10.

Method-1 : Numbers near and less than the bases of powers of 10.

Example 1: Find 9^2

Here base is 10. The answer is separated in to two parts by a '/'

Note that deficit is $10 - 9 = 1$

Multiply the deficit by itself or square it

$1^2 = 1$. As the deficiency is 1, subtract it from the number i.e., $9-1 = 8$.

Now put 8 on the left and 1 on the right side of the vertical line or slash i.e., 8/1. Hence 81 is answer.

Example. 2: Find 96^2

Here base is 100. Since deficit is $100 - 96 = 4$ and square of it is 16.

The deficiency subtracted from the number 96 gives $96 - 4 = 92$, we get the answer $92 / 16$ Thus $96^2 = 9216$.

Example 3: Find 994^2

Here base is 1000. Deficit is $1000 - 994 = 6$.

Square of 6 is 36. Deficiency subtracted from 994 gives $994 - 6 = 988$ Answer is $988 / 036$ [036 since base 1000 has 3 zero's]

Answer = 988036

Example 4: Find 9988^2

Base is 10,000. Deficit = $10000 - 9988 = 12$.

Square of deficit = $12^2 = 144$.

Deficiency subtracted from number = $9988 - 12 = 9976$.

Answer is $9976 / 0144$ [0144 since base 10,000 has 4 zero's].

Answer = 99760144

Example 5: Find 88^2

Base is 100. Deficit = $100 - 88 = 12$.

Square of deficit = $12^2 = 144$.

Deficiency subtracted from number = $88 - 12 = 76$.

Now answer is $76 / 144$ [since base is 100, keep 44 and carry over 1 to left]

Answer = $(76+1)/44 = 7744$

Assignments

Find the Squares of the following

1) 7

2) 98

- | | |
|----------|----------|
| 3) 987 | 4) 14 |
| 5) 116 | 6) 1012 |
| 7) 19 | 8) 475 |
| 9) 796 | 10) 108 |
| 11) 9988 | 12) 6014 |

Assignments Answers

Find the Squares of the following

- | | |
|--------------|--------------|
| 1) 49 | 2) 9604 |
| 3) 974169 | 4) 196 |
| 5) 13456 | 6) 1024144 |
| 7) 361 | 8) 225625 |
| 9) 633616 | 10) 11664 |
| 11) 99760144 | 12) 36168196 |

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Chapter 6 : Squaring Techniques

Method-2 : Numbers near and greater than the bases of powers of 10.

Example: Find 13^2 .

Instead of subtracting the deficiency from the number we add and proceed as in Method-1.

For 13^2 , base is 10, surplus is 3.

Surplus added to the number = $13 + 3 = 16$.

Square of surplus = $3^2 = 9$

Answer is $16 / 9 = 169$.

Example: Find 112^2 .

Base = 100, Surplus = 12,

Square of surplus = $12^2 = 144$

Add surplus to number = $112 + 12 = 124$.

Answer is $124 / 144 = (124+1)/44 = 12544$

Example 3: Find 10025^2

Base = 10000, Surplus = 25,

Square of surplus = $25^2 = 625$

Add surplus to number = $10025 + 25 = 10050$.

Answer is $10050 / 0625$ [since base is 10,000]

= 100500625.

Assignments

Find the Squares of the following

1) 7

2) 98

3) 987

4) 116

5) 1012

6) 9988

Assignments Answers

Find the Squares of the following

1) 49

2) 9604

3) 974169

4) 13456

5) 1024144

6) 99760144

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Chapter 6 : Squaring Techniques

Method 3 This is applicable to numbers which are near to multiples of 10, 100, 1000 etc.

Example 1: Find 388^2

Nearest base = 400.

$400 = 4 \times 100$. As the number is less than the base we proceed as follows

Number 388, deficit = $400 - 388 = 12$

**Since it is less than base, deduct the deficit
i.e. $388 - 12 = 376$.**

Multiply this result by 4 since base is $4 \times 100 = 400$.

$376 \times 4 = 1504$

Square of deficit = $12^2 = 144$.

Hence answer is $1504 / 144 = 150544$ [Since we have taken multiples of 100, write down 44 and carry 1 over to the left].

Example 2: Find 485^2

Nearest base = 500.

Treat 500 as 5×100 and proceed

Number 485, deficit = $500 - 485 = 15$

**Since it is less than base, deduct the deficit
i.e. $485 - 15 = 470$.**

Multiply this result by 5 since base is $5 \times 100 = 500$.

$470 \times 5 = 2350$

Square of deficit = $15^2 = 225$.

Hence answer is $2350 + 225$ [since we have taken multiples of 100].

Answer = 235255

Example 3: Find 67^2

Nearest base = 70

Number 67, deficit = $70 - 67 = 3$

**Since it is less than base, deduct the deficit
i.e. $67 - 03 = 64$.**

Multiply this result by 7 since base is $7 \times 10 = 70$.

$64 \times 7 = 448$

Square of deficit = $3^2 = 9$.

Hence answer is $448 + 9$ [since we have taken multiples of 10].

Answer = 4489

Example 4: Find 416^2

Nearest base = 400

Here surplus = 16 and $400 = 4 \times 100$

Number 416, deficit = $416 - 400 = 16$

**Since it is more than base, add the deficit i.e.
 $416 + 16 = 432$.**

Multiply this result by 4 since base is $4 \times 100 = 400$.

$432 \times 4 = 1728$

Square of deficit = $16^2 = 256$

Hence answer is $1728 / 256 = 173056$ [since we have taken multiples of 100].

Example 5: 5012^2

Nearest base is 5000

Here surplus = 12 and $5000 = 5 \times 1000$

Number 5012, surplus = $5012 - 5000 = 12$

Since it is more than base, add the deficit $5012 + 12 = 5024$.

Multiply this result by 5 since base is $5 \times 1000 = 5000$.

$5024 \times 5 = 25120$

Square of deficit = $12^2 = 144$

Hence answer is $25120 / 144 = 25120144$ [since we have taken multiples of 1000, write down 144 as it is].

Assignments

Find the Squares of the following

1) 7

2) 98

3) 14

4) 116

5) 1012

6) 475

7) 118

8) 6014

Assignments

Find the Squares of the following

- | | |
|------------|-------------|
| 1) 49 | 2) 9604 |
| 3) 196 | 4) 13456 |
| 5) 1024144 | 6) 225625 |
| 7) 13924 | 8) 36168196 |

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Chapter 6 : Squaring Techniques

6.3 Straight Squaring:

We have already noticed methods useful to find out squares of numbers. But the methods are useful under some situations and conditions only. Now we go to a more general formula.

The Duplex combination process is used in two different meanings.

They are a) by squaring

b) by cross-multiplying.

We use both the meanings of Duplex combination in the context of finding squares of numbers as follows:

We denote the Duplex of a number by the symbol D . We define

- for a single digit 'a', $D = a^2$.
- for a two digit number of the form 'ab', $D = 2(a \times b)$.
- for a 3 digit number like 'abc', $D = 2(a \times c) + b^2$.

- for a 4 digit number 'abcd', $D = 2(a \times d) + 2(b \times c)$ and so on.

If the digit is single central digit, D represents 'square'.

Consider the examples:

Number	Duplex D
3	$3^2 = 9$
6	$6^2 = 36$
23	$2(2 \times 3) = 12$
64	$2(6 \times 4) = 48$
128	$2(1 \times 8) + 2^2 = 16 + 4 = 20$
305	$2(3 \times 5) + 0^2 = 30 + 0 = 30$
4231	$2(4 \times 1) + 2(2 \times 3) = 8 + 12 = 20$
7346	$2(7 \times 6) + 2(3 \times 4) = 84 + 24 = 108$

For a n- digit number, the square of the number contains 2n or 2n-1 digits.

Thus in this process, we take extra Zeros to the left one less than the number of digits in the given numbers.

Examples:1 Find 62^2

Since number of digits = 2, we take one extra ZERO to the left. Thus

062

For 2, $D = 2^2 = 4$, Write down 4 as the right most digit

4

For 62, $D = 2(6 \times 2) = 24$, write down 4 and carry over 2 to the left

₂44

For 062, $D = 2(0 \times 2) + 6^2 = 36$

₃6₂44

Finally answer = 3844

$$62^2 = 3844.$$

Examples:2 Find 234^2

Number of digits = 3.

Extra ZEROS added to the left = Number of digits -1 = 2
Thus

00234

For 4, $D = 4^2 = 16$, Write down 6 as the right most digit and carry 1 over to left

₁6

For 34, $D = 2 (3 \times 4) = 24$, write down 4 and carry over 2 to the left

₂4₁6

For 234, $D = 2 (2 \times 4) + 3^2 = 16 + 9 = 25$, write down 5 and carry over 2 to the left

₂5₂4₁6

For 0234, $D = 2 (0 \times 4) + 2 (2 \times 3) = 0 + 12 = 12$, write down 2 and carry over 1 to the left

₁2₂5₂4₁6

For 00234, $D = 2 (0 \times 4) + 2 (0 \times 3) + 2^2 = 0 + 0 + 4 = 4$, write down 4 as it is

₄1₂2₅2₄1₆

Finally answer = 54756

$$234^2 = 54756.$$

Examples:3 1426^2 .

Number of digits = 4

Extra ZEROS = Number of digits -1 = 3 Thus

0001426

For 6, $D = 6^2 = 36$, Write down 6 as the right most digit and carry 3 over to left

${}_36$

For 26, $D = 2 (2 \times 6) = 24$, write down 4 and carry over 2 to the left

${}_24{}_36$

For 426, $D = 2 (6 \times 4) + 2^2 = 48 + 4 = 52$, write down 2 and carry over 5 to the left

${}_52{}_24{}_36$

For 1426, $D = 2 (1 \times 6) + 2 (2 \times 4) = 12 + 16 = 28$, write down 8 and carry over 2 to the left

${}_28{}_52{}_24{}_36$

For 01426, $D = 2 (0 \times 6) + 2 (1 \times 2) + 4^2 = 0 + 4 + 16 = 20$, write down 0 and carry over 2 to the left

${}_20{}_28{}_52{}_24{}_36$

For 001426, $D = 2 (0 \times 6) + 2 (0 \times 2) + 2 (1 \times 4) = 0 + 0 + 8 = 8$, write down 8 as it is

${}_8{}_20{}_28{}_52{}_24{}_36$

For 0001426, $D = 2 (0 \times 6) + 2 (0 \times 2) + 2 (0 \times 4) + 1^2 = 0 + 0 + 0 + 1 = 1$, write down 1 as it is

${}_18{}_20{}_28{}_52{}_24{}_36$

Finally answer = 2033476

$1426^2 = 2033476$

Assignments

Find the Squares of the following

Q1. 54

Q2. 123

Q3. 2051

Q4. 3146

Assignments Answers

Find the Squares of the following

Q1. 2916

Q2. 15129

Q3. 4206601

Q4. 9897316

Chapter 7 : Cubing Techniques

7.0 Cube Numbers

7.1 Cubing Technique : Find Cube of a two digit number

Example 1 : Find Cube of a two digit number : 14.

i) Find the ratio of the two digits i.e. 1:4

ii) Now write the cube of the first digit of the number i.e. 1^3

iii) Now write numbers in a row of 4 terms in such a way that the first one is the cube of the first digit and remaining three are obtained in a geometric progression with common ratio as the ratio of the original two digits (i.e. 1:4) i.e. the row is

1 4 16 64

iv) Write twice the values of 2nd and 3rd terms under the terms respectively in second row.

1 4 16 64
8 32 (2 x 4 = 8, 2 x 16 = 32)

v) Add the numbers column wise and follow carry over process.

First Column from Left: Bring down 4 and carry over 6

1 4 16 64
8 32

64

Second Column from Left: $16 + 32 + 6 = 54$. Bring down 4 and carry over 5

1 4 16 64
8 32

54 4

Third Column from Left: $4 + 8 + 5 = 17$. Bring down 7 and carry over 1

$$\begin{array}{r}
 1 \quad 4 \quad 16 \quad 64 \\
 \quad 8 \quad 32 \\
 \hline
 \quad 17 \quad 4 \quad 4
 \end{array}$$

Fourth Column from Left: $1+1 =2$. Write down 2

$$\begin{array}{r}
 1 \quad 4 \quad 16 \quad 64 \\
 \quad 8 \quad 32 \\
 \hline
 2 \quad 7 \quad 4 \quad 4
 \end{array}$$

This 2744 is the cube of the number 14

Chapter 7 : Cubing Techniques

7.1 Cubing Technique : Find Cube of a two digit number

Example 2 : Find Cube of a two digit number : 18.

i) Find the ratio of the two digits i.e. 1:8

ii) Now write the cube of the first digit of the number i.e. 1^3

iii) Now write numbers in a row of 4 terms in such a way that the first one is the cube of the first digit and remaining three are obtained in a geometric progression with common ratio as the ratio of the original two digits (i.e. 1:8) i.e. the row is

1 8 64 512.

iv) Write twice the values of 2nd and 3rd terms under the terms respectively in second row.

**1 8 64 512
 16 128 ($2 \times 8 = 16$, $2 \times 64 = 128$)**

v) Add the numbers column wise and follow carry over process.

First Column from Left: Bring down 2 and carry over 51

$$\begin{array}{r}
 1 \quad 8 \quad 64 \quad 512 \\
 \quad 16 \quad 128 \\
 \hline
 \quad \quad \quad 512
 \end{array}$$

Second Column from Left: $64+128+51 = 243$ wrote down 3 and carry over 24

$$\begin{array}{r}
 1 \quad 8 \quad 64 \quad 512 \\
 \quad 16 \quad 128 \\
 \hline
 \quad \quad 243 \quad 2
 \end{array}$$

Third Column from Left: $8+16+24 = 48$ write down 8 and carry over 3

$$\begin{array}{r}
 1 \quad 8 \quad 64 \quad 512 \\
 \quad 16 \quad 128 \\
 \hline
 \quad 48 \quad 243 \quad 2
 \end{array}$$

Fourth Column from Left: $4+1 = 5$, write down 5

$$\begin{array}{r}
 1 \quad 8 \quad 64 \quad 512 \\
 \quad 16 \quad 128 \\
 \hline
 5 \quad 8 \quad 3 \quad 2
 \end{array}$$

This 5832 is the cube of the number 18

Chapter 7 : Cubing Techniques

7.1 Cubing Technique : Find Cube of a two digit number

Example 3 : Find Cube of a two digit number : 33.

i) Find the ratio of the two digits 3:3 gives 1:1

ii) Now write the cube of the first digit of the number i.e. 3^3

iii) Now write numbers in a row of 4 terms in such a way that the first one is the cube of the first digit and remaining three are obtained in a geometric progression with common ratio as the ratio of the original two digits (i.e. 1:1) i.e. the row is

27 27 27 27

iv) Write twice the values of 2nd and 3rd terms under the terms respectively in second row.

27 27 27 27
 54 54 (2 x 27 = 54)

v) Add the numbers column wise and follow carry over process.

First Column from left : Bring down 7 as it is and carry over 2

27 27 27 27
 54 54

 ₂7

Second Column from left: 27+54+2 = 83 write down 3 and carry over 8.

27 27 27 27
 54 54

 ₈3 7

Third Column from left: 27 + 54 + 8 = 89 write down 9 and carry over 8

27 27 27 27
 54 54

 ₈9 3 7

Fourth Column from left: 27 + 8 = 35, write down 35

27 27 27 27
 54 54

 35 9 3 7

This 35937 is the cube of the number 33

Assignments

Find the cube of the following

Q1. 15

Q2. 18

Q3. 24

Q4. 36

Q5. 48

Assignments Answers

Find the cube of the following

Q1. 3375

Q2. 5832

Q3. 13824

Q4. 46656

Q5. 110592

Chapter 7 : Cubing Techniques

7.2 Cubing Technique : Find Cube of a 2 , 3 or 4 digit number

Example 1 : Find Cube of a 3 digit number : 106.

To find 106^3 .

**i) The base is 100 and excess is 6. In this context we double the excess and then add.
i.e. $106 + 12 = 118$. ($2 \times 6 = 12$)**

**This becomes the left - hand - most portion of the cube.
i.e. $106^3 = 118 / - - -$**

ii) Multiply the new excess by the initial excess

i.e. $18 \times 6 = 108$ (excess of 118 is 18)

Now this forms the middle portion of the product, of course 1 is carried over, 08 in the middle.

$$\text{i.e. } 1063 = 118 / \text{ }_1 08 / - - - -$$

iii) The last portion of the product is cube of the initial excess.

$$\text{i.e. } 6^3 = 216.$$

16 in the last portion and 2 carried over.

$$\text{i.e. } 106^3 = 118 / \text{ }_1 08 / \text{ }_2 16 = 1191016$$

Example 2: Find 1002^3 .

To Find 1002^3 .

i) Base = 1000. Excess = 2. Left-hand-most portion of the cube becomes $1002 + (2 \times 2) = 1006$.

ii) New excess X initial excess = $6 \times 2 = 12$.

Thus 012 forms the middle portion of the cube since the base is 1000.

iii) Cube of initial excess = $2^3 = 8$.

So the last portion is 008, since the base has 3 zero digits.

$$\text{Thus } 1002^3 = 1006 / 012 / 008 = 1006012008.$$

Example 3: Find 94^3 .

To Find 94^3 .

i) Base = 100, deficit = - 6. Left-hand-most portion of the cube becomes $94 + (2 \times -6) = 94 - 12 = 82$.

ii) New deficit X initial deficit = $-(100 - 82) \times (-6) = -18 \times -6 = 108$

Thus middle portion of the cube = 08 and 1 is carried over.

iii) Cube of initial deficit = $(-6)^3 = -216$

Now $94^3 = 82 / \text{108} / \text{216} = 83 / 06 / 16$
(since the carry 2 is subtracted from 8 to get 6)

Removing bar Number
= $83 / 05 / (100 - 16)$
= 830584.

Example 4: Find 998^3 .

998^3 Base = 1000; initial deficit = - 2.

$998^3 = (998 - [2 \times 2]) / (- 6 \times - 2) / (- 2)^3$
= $994 / 012 / 008$

Removing the bar number
= $994 / 011 / (1000 - 008)$
= $994 / 011 / 992$
= 994011992.

Assignments

Find the cube of the following

- | | |
|-----------|----------|
| Q1. 92 | Q2. 112 |
| Q3. 998 | Q4. 1003 |
| Q5. 10007 | Q6. 9992 |

Assignments Answers

Find the cube of the following

- | | |
|-------------------|------------------|
| Q1. 778688 | Q2. 1404928 |
| Q3. 994011992 | Q4. 1009027027 |
| Q5. 1002101470343 | Q6. 997601919488 |

Chapter 8 : Square Roots

8.0 Square Roots

8.1 Using straight Division

Basic Rules for extraction of Square Root

The given number is first arranged in two-digit groups from right to left; and a single digit if any left over at the left hand end is counted as a simple group itself

The number of digits in the square root will be the same as the number of digit-groups in the given number itself.

- **25 will count as one group**
- **144 will count as 2 groups**
- **1024 as two groups**

If the square root contains 'n' digits then square must contain $2n$ or $2n-1$ digits

If the given number has 'n' digits then square root will have $n/2$ or $(n+1)/2$ digits

The squares of the first nine natural numbers are 1, 4, 9, 16, 25, 36, 49, 64, 81 This means

An exact square cannot end in 2, 3, 7, or 8

- **That a complete square ending in 1 must have either 1 or 9 [mutual complements from 10] as the last digit of its square root.**
- **That a square can end in 4, only if the square root ends in 2 or 8**
- **That ending of a square in 5 or 0 means that its square root ends in 5 or 0 respectively**
- **That a square can end in 6, only if the square root ends in 4 or 6**
- **That a square can end in 9, only if the square root ends in 3 or 7**

We can see that

- 1,5,6 and 0 at the end of a number reproduce themselves as the last digits in the square.
- The squares of complements from ten have the same last digit . i.e 1^2 and 9^2 , 2^2 and 8^2 , 3^2 and 7^2 , 4^2 and 6^2 , 5^2 and 0^2 and 10^2 have the same ending.
- 2,3,7 and 8 cannot be a final digit of a perfect square.

Start with previous knowledge of the number of digits in the square root (N) and the first digit(F).

- 74562814 N=8 Digits in the square root is $8/2=4$ and the first digit will be 8
- 963106713 N=9 Digits in the square root is $(9+1)/2=5$ and the first digit will be 3
- $\text{Sqrt}(0.16) = 0.4$

Chapter 8 : Square Roots

8.1 Using straight Division

Basic Rules for extraction of Square Root

We use both the meanings of Duplex combination in the context of finding squares of numbers.

We denote the Duplex of a number by the symbol D.

We define

- for a single digit 'a', $D = a^2$.
- for a two digit number of the form 'ab', $D = 2(a \times b)$.
- for a 3 digit number like 'abc', $D = 2(a \times c) + b^2$.
- for a 4 digit number 'abcd', $D = 2(a \times d) + 2(b \times c)$ and so on. i.e.

If the digit is single central digit, D represents 'square'

Consider the examples:

Number	Duplex D
3	$3^2 = 9$
6	$6^2 = 36$
23	$2 (2 \times 3) = 12$
64	$2 (6 \times 4) = 48$
128	$2 (1 \times 8) + 2 \times 2 = 16 + 4 = 20$
305	$2 (3 \times 5) + 0 \times 2 = 30 + 0 = 30$
4231	$2 (4 \times 1) + 2 (2 \times 3) = 8 + 12 = 20$
7346	$2 (7 \times 6) + 2 (3 \times 4) = 84 + 24 = 108$

Example 1: Find the square root of 119716

Step 1 : Arrange the number as follows groups of 2 digits starting from right.

11 : 97 16
: :
: 3

Step 2: Find the perfect square less than the first group 11 . i.e 9 and its square root is 3. Write down this 3 and the remainder 2 as shown below

11 : 97 16
6 : : 2
: 3

New divisor is the exact double of the first digit of the quotient $3 \times 2 = 6$

Step 3 : Next gross dividend-unit is 29. Without subtracting anything from it, we divide 29 by the divisor 6 and put down the second Quotient digit 4 and the second remainder in their proper place .

11 : 97 16
6 : : 25
: 3 : 4

Step 4 : Third gross dividend-unit is 57. From 57 subtract 16 [Duplex value of the second quotient digit, $D(4) = 16$] , get 41 as the actual dividend. , divide it by 6 and set the

Quotient 6 and remainder 5 in their proper places

$$\begin{array}{r} 11 : 97 \ 16 \\ 6 : \quad : 25 \ 5 \\ : 3 : 46 \end{array}$$

Step 5 : Fourth gross dividend-unit is 51. From 51 subtract Duplex $D(46) = 48$ [because for 46 Duplex is $2(4 \times 6) = 48$] obtain 3 , divide this 3 by 6 and put down Quotient as 0 and remainder 3 in their proper places

$$\begin{array}{r} 11 : 97 \ 16 \\ 6 : \quad : 25 \ 53 \\ : 3 : 46 \ .0 \end{array}$$

Step 6 : Fifth gross dividend-unit is 36. From 36 subtract Duplex $(6) = 36$ [because for 6 Duplex is $6^2 = 36$] obtain 0 , This means the work is completed.

$$\begin{array}{r} 11 : 97 \ 16 \\ 6 : \quad : 25 \ 53 \\ : 3 : 46 \ .00 \end{array}$$

The given number is a perfect Square and 346 is the square root

A number cannot be an exact square when

- it ends in 2, 3, 7 or 8
- it terminates in an odd number of zeros
- its last digit is 6 but its penultimate digit is even
- its last digit is not 6 but its penultimate digit is odd
- its last 2 digits are not divisible by 4

Chapter 9 : Cube Roots

9.0 Cube Roots

Basic Rules for extraction of Cube Roots

The given number is first arranged in three-digit groups from right to left. A single digit if any left over at the left hand is counted as a simple group itself

The number of digits in the cube root will be the same as the number of digit-groups in the given number itself.

- 125 will count as one group
- 1000 will count as 2 groups
- 15625 as two groups

If the cube root contains 'n' digits , the cube must contain $3n$ or $3n-1$ digits

If the given number has 'n' digits the cube root will have $n/3$ or $(n+1)/3$ digits

The first digit of the Cube root will always be obvious from the first group in the cube .

For example a cube number with first group as 226 , the first digit of the cube root will be 6 since 6^3 is 216 which is a perfect cube closer to 226.

The Cubes of the first nine natural numbers are

$$1^3 = 1$$

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

$$5^3 = 125$$

$$6^3 = 216$$

$$7^3 = 343$$

$$8^3 = 512$$

$$9^3 = 729$$

This means, the last digit of the cube root of an exact cube is

- Cube ends in 1 , the Cube Root ends in 1
- Cube ends in 2 , the Cube Root ends in 8
- Cube ends in 3 , the Cube Root ends in 7
- Cube ends in 4 , the Cube Root ends in 4
- Cube ends in 5 , the Cube Root ends in 5
- Cube ends in 6 , the Cube Root ends in 6
- Cube ends in 7 , the Cube Root ends in 3
- Cube ends in 8 , the Cube Root ends in 2
- Cube ends in 9 , the Cube Root ends in 9

We can see that

- 1,4,5,6,9,0 repeat themselves in the cube ending
- 2,3,7 and 8 have their complements from 10, in the cube ending

Start with previous knowledge of the number of digits (N), first digit (F) and last digit (L) , in the cube root

Example 1: For 226981 , Find F, L and N

Write 226981 as 226, 981 , the number of 3 digit groups , N =2

Last digit of the cube is 1, the cube root also ends in 1, so L=1

The first group is 226 , the closest minimum exact cube to 226 is 216 which is nothing but 6^3

The first digit of the Cube root is 6. F=6

Example 2 For 1728 : Find F, L and N

Write 1278 as 1,278 , the number of 3 digit groups , N =2

Last digit of the cube is 8, the cube root ends in 2, so L=2

The first group is 1 , the closest minimum exact cube to 1 is 1 which is nothing but 1^3

The first digit of the CR is 1, F=1

Example 3: For 83453453 : Find F, L and N

Write 83453453 as 83,453, 453 the number of 3 digit groups , N =3

Last digit of the cube is 3, the cube root ends in 7, so L=7

The first group is 83 , the closest minimum exact cube to 83 is 64 which is nothing but 4^3

The first digit of the CR is 4, F=4

Assignments

Find F, L and N of the following

Q1. 1548816893

Q2. 4251528

Q3. 33076161

Q4. 1728

Q5. 6699961286208

Assignments Answers

Find F, L and N of the following

Q1. F = 1 , L =7 , N = 4

Q2. F = 1 , L =2 , N = 3

Q3. F = 3 , L =1 , N = 3

Q4. $F = 1$, $L = 2$, $N = 2$

Q5. $F = 1$, $L = 2$, $N = 5$

Chapter 9 : Cube Roots

9.0 Cube Roots

Basic Rules for extraction of Cube Roots

The given number is first arranged in three-digit groups from right to left. A single digit if any left over at the left hand is counted as a simple group itself

The number of digits in the cube root will be the same as the number of digit-groups in the given number itself.

- **125 will count as one group**
- **1000 will count as 2 groups**
- **15625 as two groups**

If the cube root contains 'n' digits , the cube must contain $3n$ or $3n-1$ digits

If the given number has 'n' digits the cube root will have $n/3$ or $(n+1)/3$ digits

The first digit of the Cube root will always be obvious from the first group in the cube .

For example a cube number with first group as 226 , the first digit of the cube root will be 6 since 6^3 is 216 which is a perfect cube closer to 226.

The Cubes of the first nine natural numbers are

$$\begin{aligned}
 1^3 &= 1 \\
 2^3 &= 8 \\
 3^3 &= 27 \\
 4^3 &= 64 \\
 5^3 &= 125 \\
 6^3 &= 216 \\
 7^3 &= 343 \\
 8^3 &= 512 \\
 9^3 &= 729
 \end{aligned}$$

This means, the last digit of the cube root of an exact cube is

- Cube ends in 1 , the Cube Root ends in 1
- Cube ends in 8 , the Cube Root ends in 2
- Cube ends in 27 , the Cube Root ends in 3
- Cube ends in 64 , the Cube Root ends in 4
- Cube ends in 125 , the Cube Root ends in 5
- Cube ends in 216 , the Cube Root ends in 6
- Cube ends in 343 , the Cube Root ends in 7
- Cube ends in 512 , the Cube Root ends in 8
- Cube ends in 729 , the Cube Root ends in 9

We can see that

- 1,4,5,6,9,0 repeat themselves in the cube ending
- 2,3,7 and 8 have their complements from 10, in the cube ending

Start with previous knowledge of the number of digits (N), first digit (F) and last digit (L) , in the cube root

Example 1: For 226981 , Find F, L and N

Write 226981 as 226, 981 , the number of 3 digit groups , N =2

Last digit of the cube is 1, the cube root also ends in 1, so L=1

The first group is 226 , the closest minimum exact cube to 226 is 216 which is nothing but 6^3

The first digit of the Cube root is 6. $F=6$

Example 2 For 1728 : Find F, L and N

Write 1278 as 1,278 , the number of 3 digit groups , $N = 2$

Last digit of the cube is 8, the cube root ends in 2, so $L=2$

The first group is 1 , the closest minimum exact cube to 1 is 1 which is nothing but 1^3

The first digit of the CR is 1, $F=1$

Example 3: For 83453453 : Find F, L and N

Write 83453453 as 83,453, 453 the number of 3 digit groups , $N = 3$

Last digit of the cube is 3, the cube root ends in 7, so $L=7$

The first group is 83 , the closest minimum exact cube to 83 is 64 which is nothing but 4^3

The first digit of the CR is 4, $F=4$

Assignments

Find F, L and N of the following

Q1. 1548816893

Q2. 4251528

Q3. 33076161

Q4. 1728

Q5. 6699961286208

Assignments Answers

Find F, L and N of the following

Q1. $F = 1$, $L = 7$, $N = 4$

Q2. $F = 1$, $L = 2$, $N = 3$

Q3. $F = 3$, $L = 1$, $N = 3$

Q4. $F = 1$, $L = 2$, $N = 2$

Q5. $F = 1$, $L = 2$, $N = 5$

<http://www.fastmaths.com>

Chapter 9 : Cube Roots

9.0 Cube Roots

General Method

Example 2: Find the cube root of 417 to 3 decimal places

Arrange the number as follows groups of 3 digits starting from right.

Step 1

$$417 : 0 \quad 0 \quad 0$$

**By inspection write down 7 and 74 as the first Q and R .
Since 343 is the perfect cube close to 417 and the
remainder from 417 is 74**

Step 2

$$\begin{array}{rcl} 417 & : & 0 \quad 0 \quad 0 \quad 0 \\ 147 & : & \quad : 74 \\ & : & 7 \quad : \end{array}$$

The dividend is found by multiplying the Quotient Squared by 3 , $7^2 \times 3 = 147$

Step 3

$$\begin{array}{r} 417 : 0 \quad 0 \quad 0 \quad 0 \\ 147 : \quad : 74 \quad 152 \\ : 7 : 4 \end{array}$$

The second gross dividend is 740 , Do not subtract anything from this, divide it by 147 and put down 4 as Quotient and 152 as Remainder.

Step 4

$$\begin{array}{r} 417 : 0 \quad 0 \quad 0 \quad 0 \\ 147 : \quad : 74 \quad 152 \quad 155 \\ : 7 : 4 \quad 7 \end{array}$$

The third gross dividend is 1520 , subtract $3ab^2$, $3 \times 7 \times 4^2 = 336$.

The third actual working Dividend is $1520 - 336 = 1184$.

Divide 1184 by 147 and put down 7 as Quotient and 155 as Remainder.

Step 5

$$\begin{array}{r} 417 : 0 \quad 0 \quad 0 \quad 0 \\ 147 : \quad : 74 \quad 152 \quad 155 \quad 163 \\ : 7 : 4 \quad 7 \quad 1 \end{array}$$

The 4th gross dividend is 1550 , subtract $6abc + b^3$, $6 \times 7 \times 4 \times 7 + 4^3 = 1176 + 64 = 1240$.

The 4th actual working Dividend is $1550 - 1240 = 310$.

Divide 310 by 147 and put down 1 as Quotient and 163 as Remainder.

Step 6

$$\begin{array}{r} 417 : 0 \quad 0 \quad 0 \quad 0 \\ 147 : \quad : 74 \quad 152 \quad 155 \quad 163 \quad 118 \\ : 7 : 4 \quad 7 \quad 1 \quad 1 \end{array}$$

The 5th gross dividend is 1630 , subtract $3ac^2 + 3b^2c$, $3 \times 7 \times 1^2 + 3 \times 4^2 \times 7 = 1029 + 336 = 1365$.

The 5th actual working Dividend is $1630 - 1365 = 265$.

Divide 265 by 147 and put down 1 as Quotient and 118 as Remainder.

The number of digits in the cube root will be 1 , so the cube root is 7.4711

Assignments

Find the cube root of the following up to 3 decimals

Q1. 250

Q2. 1500

Q3. 1728

Q4. 13824

Q5. 33076161

Q6. 30124

Q7. 83525660

Q8. 105820461

Assignments Answers

Find the cube root of the following up to 3 decimals

Q1. 6.2996

Q2. 11.4471

Q3. 12.000

Q4. 24.000

Q5. 321.000

Q6. 31.115

Q7. 437.126

Q8. 472.995

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Chapter 9 : Cube Roots

9.0 Cube Roots

General Method

The divisor should not be too small. The smallness will give rise to big quotients with several digits. This will lead to complications.

Another method is to multiply the given number by another small number cubed and find the cube root. Final answer is calculated by dividing the result by small number

Example 4: Find the cube root of 2

We multiply 2 by 5^3

The new Number becomes $2 \times 125 = 250$

Find the cube root of 250 and divide the answer by 5 [since we multiplied the original number by 5^3]

Step 1

250 : 0 0 0

By inspection write down 6 and 34 as the first Q and R .
Since 216 is the perfect cube close to 250 and the remainder from 250 is 34.

Step 2

250 : 0 0 0
108 : : 34
: 6 :

The dividend is found by multiplying the Quotient Squared by 3 , $6^2 \times 3 = 108$

Step 3

250 : 0 0 0
108 : : 34 124
: 6 : 2

The second gross dividend is 340 , Do not subtract anything from this, divide it by 108 and put down 2 as Quotient and

124 as Remainder.

Step 4

$$\begin{array}{r} 250 : \quad 0 \quad 0 \quad 0 \\ 108 : \quad : \quad 34 \quad 124 \quad 196 \\ : \quad 6 : \quad 2 \quad 9 \end{array}$$

The third gross dividend is 1240 , subtract $3ab^2$, $3 \times 6 \times 2^2 = 72$.

The third actual working Dividend is $1240 - 72 = 1168$.

Divide 1168 by 108 and put down 9 as Quotient and 196 as Remainder.

Step 5

$$\begin{array}{r} 250 : \quad 0 \quad 0 \quad 0 \\ 108 : \quad : \quad 34 \quad 124 \quad 196 \quad 332 \\ : \quad 6 : \quad 2 \quad 9 \quad 9 \end{array}$$

The 4th gross dividend is 1960 , subtract $6abc + b^3$, $6 \times 6 \times 2 \times 9 + 2^3 = 648 + 8 = 656$.

The 4th actual working Dividend is $1960 - 656 = 1304$.

Divide 1304 by 108 and put down 9 as Quotient and 332 as Remainder.

Step 6

The number of digits in the cube root will be 1 , so the cube root of 250 is 6.299

Cube root of 2 can be found by dividing 6.2999 by 5

The cube root of 2 is 1.259