MATHEMATICAL FORMULAE

Algebra

1. $(a+b)^2 = a^2 + 2ab + b^2;\ a^2 + b^2 = (a+b)^2 - 2ab$
2. $(a-b)^2 = a^2 - 2ab + b^2;\ a^2 + b^2 = (a-b)^2 + 2ab$
3. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
4. $(a+b)^3 = a^3 + b^3 + 3ab(a+b);\ a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
5. $(a-b)^3 = a^3 - b^3 - 3ab(a-b);\ a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
6. $a^2 - b^2 = (a+b)(a-b)$
7. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
9. $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + b^{n-1})$
10. $a^n = a.a.a \ldots n$ times
11. $a^m.a^n = a^{m+n}$
12. $\dfrac{a^m}{a^n} = a^{m-n}$ if $m > n$

    $= 1$ if $m = n$

    $= \dfrac{1}{a^{n-m}}$ if $m < n; a \in R, a \neq 0$
13. $(a^m)^n = a^{mn} = (a^n)^m$
14. $(ab)^n = a^n.b^n$
15. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
16. $a^0 = 1$ where $a \in R, a \neq 0$
17. $a^{-n} = \dfrac{1}{a^n}, a^n = \dfrac{1}{a^{-n}}$
18. $a^{p/q} = \sqrt[q]{a^p}$
19. If $a^m = a^n$ and $a \neq \pm 1, a \neq 0$ then $m = n$
20. If $a^n = b^n$ where $n \neq 0$, then $a = \pm b$
21. If $\sqrt{x}, \sqrt{y}$ are quadratic surds and if $a + \sqrt{x} = \sqrt{y}$, then $a = 0$ and $x = y$
22. If $\sqrt{x}, \sqrt{y}$ are quadratic surds and if $a + \sqrt{x} = b + \sqrt{y}$ then $a = b$ and $x = y$
23. If a, m, n are positive real numbers and $a \neq 1$, then $\log_a mn = \log_a m + \log_a n$
24. If a, m, n are positive real numbers, $a \neq 1$, then $\log_a \left(\dfrac{m}{n}\right) = \log_a m - \log_a n$
25. If a and m are positive real numbers, $a \neq 1$ then $\log_a m^n = n \log_a m$
26. If a, b and k are positive real numbers, $b \neq 1, k \neq 1$, then $\log_b a = \dfrac{\log_k a}{\log_k b}$
27. $\log_b a = \dfrac{1}{\log_a b}$ where a, b are positive real numbers, $a \neq 1, b \neq 1$
28. if a, m, n are positive real numbers, $a \neq 1$ and if $\log_a m = \log_a n$, then $m = n$

29. if $a + ib = 0 \quad$ where $i = \sqrt{-1}$, then $a = b = 0$
30. if $a + ib = x + iy, \quad$ where $i = \sqrt{-1}$, then $a = x$ and $b = y$
31. The roots of the quadratic equation $ax^2+bx+c = 0;\ a \neq 0$ are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

    The solution set of the equation is $\left\{ \dfrac{-b + \sqrt{\Delta}}{2a}, \dfrac{-b - \sqrt{\Delta}}{2a} \right\}$

    where $\Delta =$ discriminant $= b^2 - 4ac$
32. The roots are real and distinct if $\Delta > 0$.
33. The roots are real and coincident if $\Delta = 0$.
34. The roots are non-real if $\Delta < 0$.
35. If α and β are the roots of the equation $ax^2 + bx + c = 0, a \neq 0$ then
    i) $\alpha + \beta = \dfrac{-b}{a} = -\dfrac{\text{coeff. of } x}{\text{coeff. of } x^2}$
    ii) $\alpha \cdot \beta = \dfrac{c}{a} = \dfrac{\text{constant term}}{\text{coeff. of } x^2}$
36. The quadratic equation whose roots are α and β is $(x - \alpha)(x - \beta) = 0$

    i.e. $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

    i.e. $x^2 - Sx + P = 0$ where $S =$Sum of the roots and $P =$Product of the roots.
37. For an arithmetic progression (A.P.) whose first term is (a) and the common difference is (d).
    i) n^{th} term$= t_n = a + (n - 1)d$
    ii) The sum of the first (n) terms $= S_n = \dfrac{n}{2}(a + l) = \dfrac{n}{2}\{2a + (n - 1)d\}$

        where $l =$last term$= a + (n - 1)d$.
38. For a geometric progression (G.P.) whose first term is (a) and common ratio is (γ),
    i) n^{th} term$= t_n = a\gamma^{n-1}$.
    ii) The sum of the first (n) terms:

$$\begin{aligned} S_n &= \frac{a(1 - \gamma^n)}{1 - \gamma} && \text{if} \gamma < 1 \\ &= \frac{a(\gamma^n - 1)}{\gamma - 1} && \text{if } \gamma > 1 \\ &= na && \text{if } \gamma = 1 \end{aligned}.$$

39. For any sequence $\{t_n\}, S_n - S_{n-1} = t_n$ where $S_n =$Sum of the first (n) terms.
40. $\sum\limits_{\gamma=1}^{n} \gamma = 1 + 2 + 3 + \cdots + n = \dfrac{n}{2}(n + 1)$.
41. $\sum\limits_{\gamma=1}^{n} \gamma^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n}{6}(n + 1)(2n + 1)$.

42. $\sum_{\gamma=1}^{n} \gamma^3 = 1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2}{4}(n+1)^2$.
43. $n! = (1).(2).(3).\ldots.(n-1).n$.
44. $n! = n(n-1)! = n(n-1)(n-2)! = \ldots\ldots$
45. $0! = 1$.
46. $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \cdots + b^n, n > 1$.