Algebra
Actual CAT Problems 1999-2005

CAT 1999

1. The number of positive integer valued pairs (x, y) satisfying $4x - 17y = 1$ and $x \leq 1000$ is
   a. 59 b. 57 c. 55 d. 58

2. Total expenses of a boarding house are partly fixed and partly varying linearly with the number of boarders. The average expense per boarder is Rs. 700 when there are 25 boarders and Rs. 600 when there are 50 boarders. What is the average expense per boarder when there are 100 boarders?
   a. 550 b. 580 c. 540 d. 570

3. If $|r - 6| = 11$ and $|2q - 12| = 8$, what is the minimum possible value of $\frac{q}{r}$?
   a. $-\frac{2}{5}$ b. $\frac{2}{17}$ c. $\frac{10}{17}$ d. None of these

Directions for questions 4 to 6: Answer the questions based on the following information.
A young girl Roopa leaves home with x flowers, goes to the bank of a nearby river. On the bank of the river, there are four places of worship, standing in a row. She dips all the x flowers into the river. The number of flowers doubles. Then she enters the first place of worship, offers y flowers to the deity. She dips the remaining flowers into the river, and again the number of flowers doubles. She goes to the second place of worship, offers y flowers to the deity. She dips the remaining flowers into the river, and again the number of flowers doubles. She goes to the third place of worship, offers y flowers to the deity. She dips the remaining flowers into the river, and again the number of flowers doubles. She goes to the fourth place of worship, offers y flowers to the deity. Now she is left with no flowers in hand.

4. If Roopa leaves home with 30 flowers, the number of flowers she offers to each deity is
   a. 30 b. 31 c. 32 d. 33

5. The minimum number of flowers that could be offered to each deity is
   a. 0 b. 15 c. 16 d. Cannot be determined

6. The minimum number of flowers with which Roopa leaves home is
   a. 16 b. 15 c. 0 d. Cannot be determined

Directions for questions 7 and 8: Answer the questions based on the following information.

There are blue vessels with known volumes $v_1, v_2 ..., v_m$, arranged in ascending order of volume, $v_1 > 0.5$ litre, and $v_m < 1$ litre. Each of these is full of water initially. The water from each of these is emptied into a minimum number of empty white vessels, each having volume 1 litre. The water from a blue vessel is not emptied into a white vessel unless the white vessel has enough empty volume to hold all the water of the blue vessel. The number of white vessels required to empty all the blue vessels according to the above rules was n.

7. Among the four values given below, which is the least upper bound on e, where e is the total empty volume in the white vessels at the end of the above process?

a. mv_m b. $m(1-v_m)$ c. mv_1 d. $m(1-v_1)$

8. Let the number of white vessels needed be n_1 for the emptying process described above, if the volume of each white vessel is 2 litres. Among the following values, which is the least upper bound on n_1?

a. $\frac{m}{4}$

b. Smallest integer greater than or equal to $\left(\frac{n}{2}\right)$

c. n

d. Greatest integer less than or equal to $\left(\frac{n}{2}\right)$

CAT 2000

9.

x	1	2	3	4	5	6
y	4	8	14	22	32	44

In the above table, for suitably chosen constants a, b and c, which one of the following best describes the relation between y and x?

a. $y = a + bx$ b. $y = a + bx + cx^2$ c. $y = e^{a+bx}$ d. None of these

10. If $a_1 = 1$ and $a_{n+1} = 2a_n + 5$, n = 1, 2 ..., then a_{100} is equal to

a. $(5 \times 2^{99} - 6)$ b. $(5 \times 2^{99} + 6)$ c. $(6 \times 2^{99} + 5)$ d. $(6 \times 2^{99} - 5)$

11. What is the value of the following expression?

$$\left(\frac{1}{(2^2-1)}\right)+\left(\frac{1}{(4^2-1)}\right)+\left(\frac{1}{(6^2-1)}\right)+\cdots+\left(\frac{1}{(20^2-1)}\right)$$

a. $\frac{9}{19}$ b. $\frac{10}{19}$ c. $\frac{10}{21}$ d. $\frac{11}{21}$

12. A truck travelling at 70 km/hr uses 30% more diesel to travel a certain distance than it does when it travels at a speed of 50 km/hr. If the truck can travel 19.5 km/L of diesel at 50 km/hr, how far can the truck travel on 10 L of diesel at a speed of 70 km/hr?

a. 130 km b. 140 km c. 150 km d. 175 km

13. If $x > 2$ and $y > -1$, then which of the following statements is necessarily true?

a. $xy > -2$ b. $-x < 2y$ c. $xy < -2$ d. $-x > 2y$

14. If the equation $x^3 - ax^2 + bx - a = 0$ has three real roots, then it must be the case that

a. $b = 1$ b. $b \neq 1$ c. $a = 1$ d. $a \neq 1$

CAT 2001

15. If $x > 5$ and $y < -1$, then which of the following statements is true?

a. $(x + 4y) > 1$ b. $x > -4y$ c. $-4x < 5y$ d. None of these

16. m is the smallest positive integer such that for any integer $n \geq m$, the quantity $n^3 - 7n^2 + 11n - 5$ is positive. What is the value of m?

a. 4 b. 5 c. 8 d. None of these

17. Every 10 years the Indian Government counts all the people living in the country. Suppose that the director of the census has reported the following data on two neighbouring villages Chota Hazri and Mota Hazri.

Chota Hazri has 4,522 fewer males than Mota Hazri.
Mota Hazri has 4,020 more females than males.
Chota Hazri has twice as many females as males.
Chota Hazri has 2,910 fewer females than Mota Hazri.
What is the total number of males in Chota Hazri?

a. 11,264 b. 14,174 c. 5,632 d. 10,154

18. Three classes X, Y and Z take an algebra test.

The average score in class X is 83.
The average score in class Y is 76.
The average score in class Z is 85.
The average score of all students in classes X and Y together is 79.
The average score of all students in classes Y and Z together is 81.
What is the average for all the three classes?

a. 81 b. 81.5 c. 82 d. 84.5

19. At a certain fast food restaurant, Brian can buy 3 burgers, 7 shakes, and one order of fries for Rs. 120 exactly. At the same place it would cost Rs. 164.5 for 4 burgers, 10 shakes, and one order of fries. How much would it cost for an ordinary meal of one burger, one shake, and one order of fries?

a. Rs. 31 b. Rs. 41 c. Rs. 21 d. Cannot be determined

20. For a Fibonacci sequence, from the third term onwards, each term in the sequence is the sum of the previous two terms in that sequence. If the difference in squares of 7th and 6th terms of this sequence is 517, what is the 10th term of this sequence?

a. 147 b. 76 c. 123 d. Cannot be determined

21. Let x and y be two positive numbers such that x + y = 1.

Then the minimum value of $\left(x+\frac{1}{x}\right)^2+\left(y+\frac{1}{y}\right)^2$ is

a. 12 b. 20 c. 12.5 d. 13.3

Directions for questions 22 and 23: Answer the questions based on the following infirmation.

The batting average (BA) of a Test batsman is computed from runs scored and innings played — completed innings and incomplete innings (not out) in the following manner:

r_1 = Number of runs scored in completed innings
n_1 = Number of completed innings
r_2 = Number of runs scored in incomplete innings
n_2 = Number of incomplete innings

$$BA = \frac{r_1 + r_2}{n_1}$$

To better assess a batsman's accomplishments, the ICC is considering two other measures MBA_1 and MBA_2 defined as follows:

$$MBA_1 = \frac{r_1}{n_1} + \frac{n_2}{n_1} \max\left[0, \left(\frac{r_2}{n_2} - \frac{r_1}{n_1}\right)\right]$$

$$MBA_2 = \frac{r_1 + r_2}{n_1 + n_2}$$

22. Based on the above information which of the following is true?

a. $MBA_1 \le BA \le MBA_2$ b. $BA \le MBA_2 \le MBA_1$
c. $MBA_2 \le BA \le MBA_1$ d. None of these

23. An experienced cricketer with no incomplete innings has BA of 50. The next time he bats, the innings is incomplete and he scores 45 runs. It can be inferred that

a. BA and MBA_1 will both increase
b. BA will increase and MBA_2 will decrease
c. BA will increase and not enough data is available to assess change in MBA_1 and MBA_2
d. None of these

24. Ujakar and Keshab attempted to solve a quadratic equation. Ujakar made a mistake in writing down the constant term. He ended up with the roots (4, 3). Keshab made a mistake in writing down the coefficient of x. He got the roots as (3, 2). What will be the exact roots of the original quadratic equation?

a. (6, 1) b. (–3, –4) c. (4, 3) d. (–4, –3)

CAT 2002

25. If $f(x) = \log\left\{\frac{(1+x)}{(1-x)}\right\}$, then f(x) + f(y) is

a. f(x + y) b. $f\left\{\frac{(x+y)}{(1+xy)}\right\}$ c. $(x+y)f\left\{\frac{1}{(1+xy)}\right\}$ d. $\frac{f(x)+f(y)}{(1+xy)}$

26. The nth element of a series is represented as

$X_n = (-1)^n X_{n-1}$

If $X_0 = x$ and x > 0, then which of the following is always true?

a. X_n is positive if n is even b. X_n is positive if n is odd

c. X_n is negative if n is even d. None of these

27. If x, y and z are real numbers such that x + y + z = 5 and xy + yz + zx = 3, what is the largest value that x can have?

a. $\frac{5}{3}$ b. $\sqrt{19}$ c. $\frac{13}{3}$ d. None of these

28. Let S denotes the infinite sum $2+5x+9x^2+14x^3+20x^4+\ldots$, where |x| < 1 and the coefficient of x^{n-1} is $\frac{1}{2}n(n+3), (n = 1, 2, \ldots)$. Then S equals:

a. $\frac{2-x}{(1-x)^3}$ b. $\frac{2-x}{(1+x)^3}$ c. $\frac{2+x}{(1-x)^3}$ d. $\frac{2+x}{(1+x)^3}$

29. If $x^2+5y^2+z^2 = 2y(2x+z)$, then which of the following statements is(are) necessarily true?

A. x = 2y B. x = 2z C. 2x = z

a. Only A b. B and C c. A and B d. None of these

30. Amol was asked to calculate the arithmetic mean of 10 positive integers, each of which had 2 digits. By mistake, he interchanged the 2 digits, say a and b, in one of these 10 integers. As a result, his answer for the arithmetic mean was 1.8 more than what it should have been. Then b – a equals

a. 1 b. 2 c. 3 d. None of these

31. Suppose for any real number x, [x] denotes the greatest integer less than or equal to x. Let L(x, y) = [x] + [y] + [x + y] and R(x, y) = [2x] + [2y]. Then it is impossible to find any two positive real numbers x and y for which

a. L(x, y) = R(x, y) b. L(x, y) $\neq$ R(x, y) c. L(x, y) < R(x, y) d. L(x, y) > R(x, y)

32. The number of real roots of the equation $\frac{A^2}{x}+\frac{B^2}{x-1}=1$, where A and B are real numbers not equal to zero simultaneously, is

a. None　　b. 1　　c. 2　　d. 1 or 2

33. A piece of string is 40 cm long. It is cut into three pieces. The longest piece is three times as long as the middle-sized and the shortest piece is 23 cm shorter than the longest piece. Find the length of the shortest piece.

a. 27　　b. 5　　c. 4　　d. 9

34. If pqr = 1, the value of the expression $\frac{1}{1+p+q^{-1}}+\frac{1}{1+q+r^{-1}}+\frac{1}{1+r+p^{-1}}$ is equal to

a. p + q + r　　b. $\frac{1}{p+q+r}$　　c. 1　　d. $p^{-1}+q^{-1}+r^{-1}$

CAT 2003 Leaked

35. The number of non-negative real roots of $2^x - x - 1 = 0$ equals

a. 0　　b. 1　　c. 2　　d. 3

36. When the curves $y = \log_{10}x$ and $y = x^{-1}$ are drawn in the x-y plane, how many times do they intersect for values $x \geq 1$?

a. Never　　b. Once　　c. Twice　　d. More than twice

37. Which one of the following conditions must p, q and r satisfy so that the following system of linear simultaneous equations has at least one solution, such that $p + q + r \neq 0$?

$x+ 2y - 3z = p$

$2x + 6y - 11z = q$

$x - 2y + 7z = r$

a. $5p - 2q - r = 0$　　b. $5p + 2q + r = 0$　　c. $5p + 2q - r = 0$　　d. $5p - 2q + r = 0$

38. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is Rs. 20 on a standard bag and Rs. 30 on a deluxe bag. Every bag must be processed on machine A and on Machine B. The processing times per bag on the two machines are as follows:

	Time required (Hours/bag)	
	Machine A	Machine B
Standard Bag	4	6
Deluxe Bag	5	10

The total time available on machine A is 700 hours and on machine B is 1250 hours. Among the following production plans, which one meets the machine availability constraints and maximizes the profit?

a. Standard 75 bags, Deluxe 80 bags　　b. Standard 100 bags, Deluxe 60 bags

c. Standard 50 bags, Deluxe 100 bags　　d. Standard 60 bags, Deluxe 90 bags

39. The sum of 3rd and 15th elements of an arithmetic progression is equal to the sum of 6th, 11th and 13th elements of the same progression. Then which element of the series should necessarily be equal to zero?

a. 1st b. 9th c. 12th d. None of the above

40. Let g(x) = max(5 – x, x + 2). The smallest possible value of g(x) is

a. 4.0 b. 4.5 c. 1.5 d. None of the above

41. The function f(x) = |x – 2| + |2.5 – x| + |3.6 – x|, where x is a real number, attains a minimum at

a. x = 2.3 b. x = 2.5 c. x = 2.7 d. None of the above

DIRECTIONS for Questions 42 and 43: Each question is followed by two statements, A and B. Answer each question using the following instructions.
Choose (a) if the question can be answered by one of the statements alone but not by the other.
Choose (b) if the question can be answered by using either statement alone.
Choose (c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.
Choose (d) if the question cannot be answered even by using both the statements together.

42. What are the unique values of b and c in the equation $4x^2 + bx + c = 0$ if one of the roots of the equation is (–1/2)?

A. The second root is 1/2.

B. The ratio of c and b is 1.

43. Is $\left(\frac{1}{a^2}+\frac{1}{a^4}+\frac{1}{a^6}+\cdots\right)>\left(\frac{1}{a}+\frac{1}{a^3}+\frac{1}{a^5}+\cdots\right)$?

A. $-3 \le a \le 3$

B. One of the roots of the equation $4x^2–4x+1 = 0$ is a

DIRECTIONS for Questions 44 to 67: Answer the questions independently of each other.

44. The 288th term of the series a,b,b,c,c,c,d,d,d,d,e,e,e,e,e,f,f,f,f,f,f… is

a. u b. v 3 w d. x

45. Let p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)\ x - \alpha - 1 = 0$. What is the minimum possible value of $p^2 + q^2$?

a. 0 b. 3 c. 4 d. 5

46. Let a, b, c, d be four integers such that a+b+c+d = 4m+1 where m is a positive integer. Given m, which one of the following is necessarily true?

a. The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2–2m+1$

b. The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2+2m+1$

c. The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2–2m+1$

d. The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2+2m+1$

47. There are 8436 steel balls, each with a radius of 1 centimeter, stacked in a pile, with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. The number of horizontal layers in the pile is

a. 34 b. 38 c. 36 d. 32

48. If $\log_3 2$, $\log_3 (2^x - 5)$, $\log_3 (2^x - 7/2)$ are in arithmetic progression, then the value of x is equal to

a. 5 b. 4 c. 2
d. 3

49. Given that $-1 \le v \le 1$, $-2 \le u \le -0.5$ and $-2 \le z \le -0.5$ and $w = vz/u$, then which of the following is necessarily true?

a. $-0.5 \le w \le 2$ b. $-4 \le w \le 4$ c. $-4 \le w \le 2$ d. $-2 \le w \le -0.5$

50. There are 6 boxes numbered 1,2,… 6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is

a. 5 b. 21 c. 33 d. 60

51. Consider the following two curves in the x-y plane:

$y = x^3 + x^2 + 5$
$y = x^2 + x + 5$

Which of following statements is true for $-2 \le x \le 2$?

a. The two curves intersect once. b. The two curves intersect twice.
c. The two curves do not intersect d. The two curves intersect thrice.

52. In a certain examination paper, there are n questions. For j = 1,2 …n, there are 2^{n-j} students who answered j or more questions wrongly. If the total number of wrong answers is 4095, then the value of n is

a. 12 b. 11 c. 10 d. 9

53. If x, y, z are distinct positive real numbers the $\dfrac{x^2(y+z)+y^2(x+z)+z^2(x+y)}{xyz}$ would be

a. greater than 4. b. greater than 5. c. greater than 6 d. None of the above.

54. A graph may be defined as a set of points connected by lines called edges. Every edge connects a pair of points. Thus, a triangle is a graph with 3 edges and 3 points. The degree of a point is the number of edges connected to it. For example, a triangle is a graph with three points of degree 2 each. Consider a graph with 12 points. It is possible to reach any point from any point through a sequence of edges. The number of edges, e, in the graph must satisfy the condition

a. $11 \le e \le 66$ b. $10 \le e \le 66$ c. $11 \le e \le 65$ d. $0 \le e \le 11$

CAT 2003 Retest

55. There are 12 towns grouped into four zones with three towns per zone. It is intended to connect the towns with a telephone lines such that every two towns are connected with three direct lines if they belong to the same zone, and with only one direct line otherwise. How many direct telephone lines are required?

a. 72 b. 90 c. 96 d. 144

56. If both a and b belong to the set {1, 2, 3, 4}, then the number of equations of the form $ax^2 + bx + 1 = 0$ having real roots is

a. 10 b. 7 c. 6 d. 12

57. If $\log_{10}x - \log_{10}\sqrt{x} = 2\log_x 10$, then the possible value of x is given by

a. 10 b. $\frac{1}{100}$ c. $\frac{1}{1000}$ d. None of these

58. If $\frac{1}{3}\log_3 M + 3\log_3 N = 1 + \log_{0.008} 5$, then

a. $M^9 = \frac{9}{N}$ b. $N^9 = \frac{9}{M}$ c. $M^3 = \frac{3}{N}$ d. $N^9 = \frac{3}{M}$

59. If x and y are integers, then the equation $5x + 19y = 64$ has

a. no solution for $x < 300$ and $y < 0$
b. no solution for $x > 250$ and $y > -100$
c. a solution for $250 < x < 300$
d. a solution for $-59 < y < -56$

60. What is the sum of 'n' terms in the series $\log m + \log\left(\frac{m^2}{n}\right) + \log\left(\frac{m^3}{n^2}\right) + \log\left(\frac{m^4}{n^3} + \right)\cdots$?

a. $\log\left[\frac{n^{(n-1)}}{m^{(n+1)}}\right]^{\frac{n}{2}}$ b. $\log\left[\frac{m^m}{n^n}\right]^{\frac{n}{2}}$ c. $\log\left[\frac{m^{(1-n)}}{n^{(1-m)}}\right]^{\frac{n}{2}}$ d. $\log\left[\frac{m^{(n+1)}}{n^{(n-1)}}\right]^{\frac{n}{2}}$

61. If three positive real numbers x, y and z satisfy $y - x = z - y$ and $x\,y\,z = 4$, then what is the minimum possible value of y?

a. $2^{1/3}$ b. $2^{2/3}$ c. $2^{1/4}$ d. $2^{3/4}$

62. The infinite sum $1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \cdots$ equals

a. $\frac{27}{14}$ b. $\frac{21}{13}$ c. $\frac{49}{27}$ d. $\frac{256}{147}$

63. The number of roots common between the two equations $x^3 + 3x^2 + 4x + 5 = 0$ and $x^3 + 2x^2 + 7x + 3 = 0$ is

a. 0 b. 1 c. 2 d. 3

64. A real number x satisfying $1-\frac{1}{n} < x \le 3+\frac{1}{n}$, for every positive integer n, is best described by
a. $1 < x < 4$ b. $1 < x \le 3$ c. $0 < x \le 4$ d. $1 \le x \le 3$

65. If n is such that $36 \le n \le 72$, then $x = \frac{n^2 + 2\sqrt{n}(n+4)+16}{n+4\sqrt{n}+4}$ satisfies
a. $20 < x < 54$ b. $23 < x < 58$ c. $25 < x < 64$ d. $28 < x < 60$

66. If $13x + 1 < 2z$ and $z + 3 = 5y^2$, then
a. x is necessarily less than y
b. x is necessarily greater than y
c. x is necessarily equal to y
d. None of the above is necessarily true

67. If $|b| \ge 1$ and $x = -|a|b$, then which one of the following is necessarily true?
a. $a - xb < 0$ b. $a - xb \ge 0$ c. $a - xb > 0$ d. $a - xb \le 0$

Directions for questions 68 to 70: Answer the questions on the basis of the tables given below.
Two binary operations $\oplus$ and $*$ are defined over the set {a, e, f, g, h} as per the following tables:

$\oplus$	a	e	f	g	h
a	a	e	f	g	h
e	e	f	g	h	a
f	f	g	h	a	e
g	g	h	a	e	f
h	h	a	e	f	g

*	a	e	f	g	h
a	a	a	a	a	a
e	a	e	f	g	h
f	a	f	h	e	g
g	a	g	e	h	f
h	a	h	g	f	e

Thus, according to the first table $f \oplus g = a$, while according to the second table $g * h = f$, and so on. Also, let $f^2 = f * f$, $g^3 = g * g * g$, and so on.

68. What is the smallest positive integer n such that $g^n = e$?
a. 4 b. 5 c. 2 d. 3

69. Upon simplification, $f \oplus [f * \{f \oplus (f * f)\}]$ equals
a. e b. f c. g d. h

70. Upon simplification, $\{a^{10} * (f^{10} \oplus g^9)\} \oplus e^8$ equals
a. e b. f c. g d. h

71. Let x and y be positive integers such that x is prime and y is composite. Then,
a. $y - x$ cannot be an even integer
b. xy cannot be an even integer
c. $\frac{(x+y)}{x}$ cannot be an even integer
d. None of these

CAT 2004

72. If the sum of the first 11 terms of an arithmetic progression equals that of the first 19 terms, then what is the sum of the first 30 terms?

a. 0 b. –1 c. 1 d. Not unique

73. If f(x) = $x^3 - 4x + p$, and f(0) and f(1) are of opposite sings, then which of the following is necessarily true

a. $-1 < p < 2$ b. $0 < p < 3$ c. $-2 < p < 1$ d. $-3 < p < 0$

74. Suppose n is an integer such that the sum of digits on n is 2, and $10^{10} < n\ 10^n$. The number of different values of n is

a. 11 b. 10 c. 9 d. 8

75. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = r$ then r cannot take any value except.

a. $\frac{1}{2}$ b. –1 c. $\frac{1}{2}$ or -1 d. $-\frac{1}{2}$ or -1

76. Let $y = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+...}}}}$

What is the value of y?

a. $\frac{\sqrt{11}+3}{2}$ b. $\frac{\sqrt{11}-3}{2}$ c. $\frac{\sqrt{15}+3}{2}$ d. $\frac{\sqrt{15}-3}{2}$

77. Let $f(x) = ax^2 - b|x|$, where a and b are constants. Then at x = 0, f(x) is

a. maximized whenever a > 0, b > 0 b. maximized whenever a > 0, b < 0
c. minimized whenever a > 0, b > 0 d. minimized whenever a > 0, b< 0

78. Each family in a locality has at most two adults, and no family has fewer than 3 children. Considering all the families together, there are more adults than boys, more boys than girls, and more girls than families, Then the minimum possible number of families in the locality is

a. 4 b. 5 c. 2 d. 3

79. The total number of integers pairs (x, y) satisfying the equation x + y = xy is

a. 0 b. 1 c. 2 d. None of the above

80. Consider the sequence of numbers a a_1, a_2, a_3, ... to infinity where a_1 = 81.33 and a_2 = –19 and a_j = a_{j-1} – a_{-2} for j ≥ 3. What is the sum of the first 6002 terms of this sequence?

a. –100.33 b. –30.00 c. 62.33 d. 119.33

81. Let u = $(\log_2 x)^2 - 6\log_2 x + 12$ where x is a real number. Then the equation $x^u = 256$, has

a. no solution for x
b. exactly one solution for x
c. exactly tow distinct solutions for x
d. exactly three distinct solutions for x

Directions for Questions 82 and 83: Answer the questions on the basis of the information given below. In an examination, there are 100 questions divided into three groups A, B and C such that each group contains at least one question. Each question in group a carries 1 mark, each question in group B carries 2 marks and each question in group C carries 3 marks. It is known that the questions in group A together carry at least 60% of the total marks.

82. If group B contains 23 questions, then how many questions are there in Group C?

a. 1 b. 2 c. 3 d. Cannot be determined

83. If group C contains 8 questions and group B carries at least 20% of the total marks, which of the following best describes the number of questions in group B?

a. 11 or 12 b. 12 or 13 c. 13 or 14 d. 14 or 15

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84. If $R = \dfrac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$, then

a. $0 < R \le 0.1$ b. $0.1 < R \le 0.5$ c. $0.5 < R \le 1.0$ d. R > 1.0

85. Let n! = 1 × 2 × 3 × … × n for integer $n \ge 1$. If p = 1! + (2 × 2!) + (3 × 3!) + … + (10 × 10!), then p + 2 when divided by 11! Leaves a remainder of

a.10 b. 0 c. 7 d. 1

86. If $a_1 = 1$ and $a_{n+1} - 3a_n + 2 = 4n$ for every positive integer n, then a_{100} equals

a. $3^{99} - 200$ b. $3^{99} + 200$ c. $3^{100} - 200$ d. $3^{100} + 200$

87. If x ≥ y and y > 1, then the value of the expression $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$ can never be

a. –1 b. –0.5 c. 0 d. 1

88. Let $x = \sqrt{4 + \sqrt{4 - \sqrt{4 + \sqrt{4 - \ldots \text{to infinity}}}}}$. Then x equals

a. 3 b. $\left(\frac{\sqrt{13} - 1}{2}\right)$ c. $\left(\frac{\sqrt{13} + 1}{2}\right)$ d. $\sqrt{13}$

89. A telecom service provider engages male and female operators for answering 1000 calls per day. A male operator can handle 40 calls per day whereas a female operator can handle 50 calls per day. The male and the female operators get a fixed wage of Rs. 250 and Rs. 300 per day respectively. In addition, a male operator gets Rs. 15 per call he answers and female operator gets Rs. 10 per call she answers. To minimize the total cost, how many male operators should the service provider employ assuming he has to employ more than 7 of the 12 female operators available for the job?

a. 15 b. 14 c. 12 d. 10