PERFORMANCE ANALYSIS

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partment: CSE

urse Code & Title: UBA0449 Discrete Mathematics and cognitive application

Tutorials	Maximum Marks	Mark Scored
Unit 1 – Problems	150	140
GATE Questions in Unit 1	15	15
Unit 2 – Problems	150	140
GATE Questions in Unit 2	15	15
Unit 3 – Problems	150	130
GATE Questions in Unit 3	15	15
Unit 4 – Problems	150	140
GATE Questions in Unit 4	15	15
Unit 5 – Problems	150	140
GATE Questions in Unit 5	15	15
Total Marks	825	765
Percentage	92.73%	

8/10

Signature of the Course Faculty

1. Find PCNF of $(P \wedge Q) \vee (\neg P \wedge R)$ by using suitable concept map (i) using truth table (ii) without using truth table.

Solution: Given $(P \wedge Q) \vee (\neg P \wedge R)$

i) using truth table:-

P	Q	R	$\neg P$	$P \wedge Q$	$\neg P \wedge R$	$(P \wedge Q) \vee (\neg P \wedge R)$
T	T	T	F	T	F	T
T	T	F	F	T	F	T
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	T	T
F	T	F	T	F	F	F
F	F	T	T	F	T	T
F	F	F	T	F	F	F

Minterms:-

$(P \wedge Q \wedge R)$, $(P \wedge Q \wedge \neg R)$, $(\neg P \wedge Q \wedge R)$, $(\neg P \wedge \neg Q \wedge R)$

Maxterms:-

$(P \vee \neg Q \wedge R)$, $(P \vee \neg Q \vee \neg R)$, $(\neg P \vee Q \vee \neg R)$, $(\neg P \vee \neg Q \vee \neg R$

PCNF:-

PCNF = product of maxterms.

$= (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$

ii) without using truthtable:-

$(P \wedge Q) \vee (\neg P \wedge R)$

$\{P \wedge Q \wedge (R \vee \neg R)\} \vee \{\neg P \wedge R \wedge (Q \vee \neg Q)\}$

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge Q) \vee (\neg P \wedge R \wedge \neg Q)$

= sum of minterms

$\neg S = (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge \neg R) \vee$
$(\neg P \wedge \neg Q \wedge \neg R)$

$\neg\neg S = (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge$
$(P \vee Q \vee R)$

$S = (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge$
$(P \vee Q \vee R)$

= PCNF $\longrightarrow$ product of maxterms

PCNF $= (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge$
$(P \vee Q \vee R)$

2. Prove that the following argument is valid using suitable concept map: If a baby is hungry, then the baby cries. If the baby is not mad, then he does not cry. If a baby is mad, then he has a red face. Therefore, if a baby is hungry, then he has a red face.

Solution: From the above statement

Baby is hungry $\longrightarrow P$

Baby cries $\longrightarrow Q$

Baby is not mad $\longrightarrow R$

Baby has red face $\longrightarrow S$

Premises:-

conclusion.

$P \longrightarrow Q$, $R \longrightarrow \neg Q$, $\neg R \longrightarrow S$, $\underset{\uparrow}{P \longrightarrow S}$.

S.NO	Premises	Rule
1.	$P \longrightarrow Q$	Rule P
2.	$R \longrightarrow \neg Q$	Rule P
3.	$Q \longrightarrow \neg R$	logical equivalence
4	$P \longrightarrow \neg R$	from (1), (3)
5.	$\neg R \longrightarrow S$	Rule P
6.	$P \longrightarrow S$	from (4), (5)

Conclusion:-

If a baby is hungry then he has red face

$$P \longrightarrow S$$

$\therefore$ Hence proved.

3. Draw the concept map of logical equivalence and prove that $r \to s$ logically follows from the premises $p \to (q \to s)$, $(\neg r \vee p)$ and q.

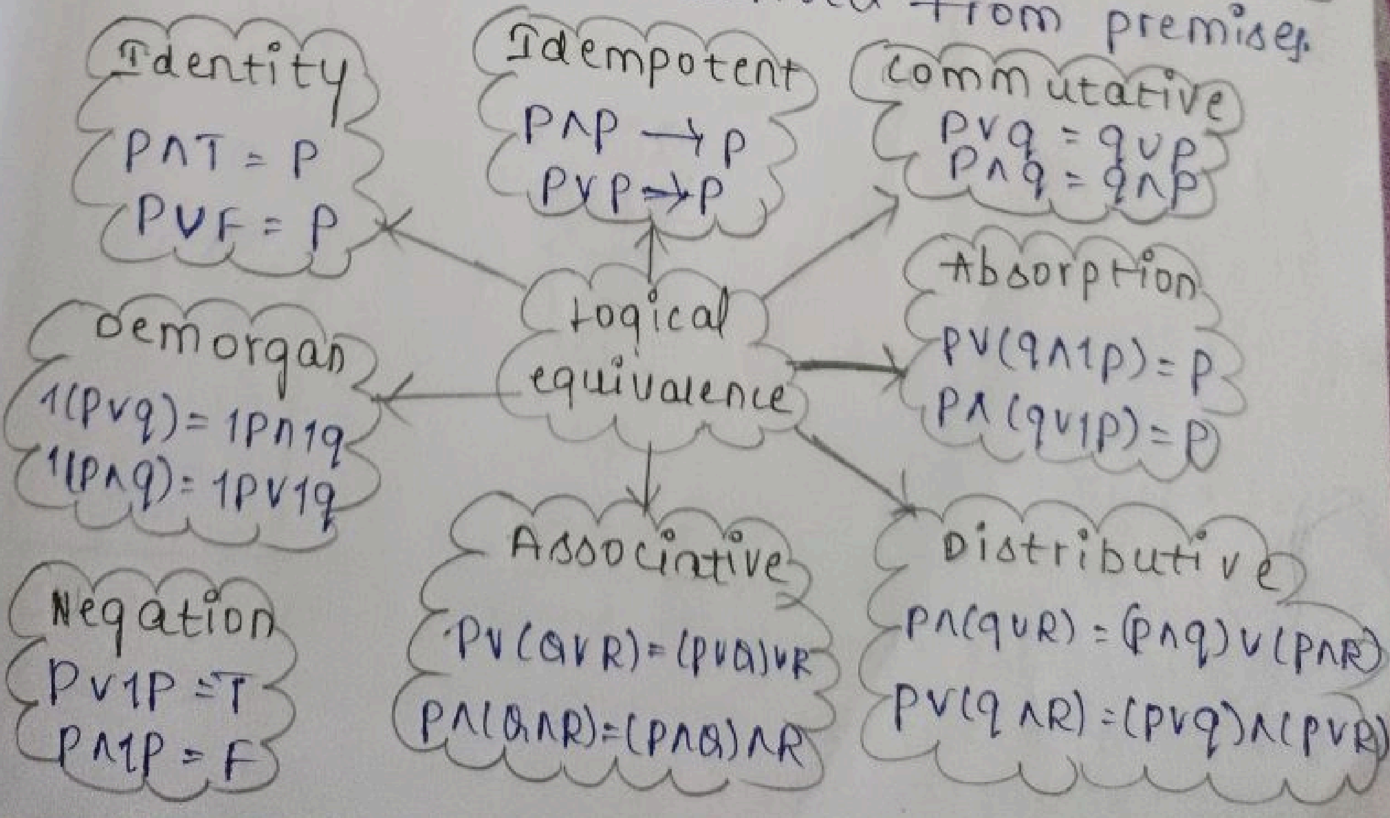
ution: Given,

premises $\longrightarrow$ $p \to (q \to s)$, $(\neg r \vee p)$, q.

conclusion $\longrightarrow$ $r \to s$.

S.NO	premises	Rule.
1.	R	assumed premise
2.	$\neg R \vee P$	Rule P
3.	$R \to P$	Rule T
4.	P	Rule T {1,3}
5.	$P \to (q \to s)$	Rule P
6.	$q \to s$	Rule T {4,5}
7.	q	Rule P
8.	s	Rule T {6,7}
9.	$r \to s$.	Rule T {1,8}

∴ Hence $r \to s$ is derived from premises.



4. Explain the direct and indirect method using suitable concept map and hence derive $p \to \sim s$ from the premises $p \to (q \vee r)$, $q \to \sim p$, $s \to \sim r$ and p.

Solution:

Given premises,

To use indirect method of proof, let us include $\neg(p \to \neg s)$ as additional premise.

Now,

$$\neg(p \to \neg s) = \neg[\neg p \vee \neg s] \Rightarrow \neg\neg p \wedge \neg\neg s$$
$$= p \wedge s$$

S.NO	premises	Rules
1.	$p \wedge s$	Additional premise
2.	p	Rule T {1}
3.	s	Rule T {2}
4.	$p \to (q \vee r)$	Rule P
5.	$q \vee r$	Rule T {4,2}
6.	$s \to \neg r$	Rule P
7.	$\neg r$	Rule T {3,6}
8.	$q \to \neg p$	Rule P
9	$\neg q \to r$	Rule T {5,8}
10	$\neg r \to q$ $\neg\neg(q)$	Rule T {7,9}
11.	q	Rule T
12	$\neg p$	Rule T {8,11}
13.	$p \wedge \neg p = F$	Rule T {2,12}

Direct method :- Truth of premise shows conclusion.

Indirect method :- Negation of the premise and the conclusion is taken to prove contradiction

Prove that the premises $a \to (b \to c)$, $d \to (b \wedge \neg c)$, and d ($a \wedge d$) are inconsistent using suitable concept map.

Solution:

Given

premises are $a \to (b \to c)$, $d \to (b \wedge \neg c)$, $(a \wedge d)$

conclusion:- False (inconsistent).

S.NO	premise	Rule.
1.	$a \to (b \to c)$	Rule P
2.	$d \to (b \wedge \neg c)$	Rule P
3.	$a \wedge d$	Rule P
4.	a	Rule T {3}
5.	d	Rule T {3}
6.	$b \to c$	Rule T {4, 1}
7.	$b \wedge \neg c$	Rule T {2, 5}
8.	b	Rule T {7}
9.	$\neg c$	Rule T {7}
10.	c	Rule T {6, 8}
11.	$c \wedge \neg c$	Rule T {9, 10}
12.	False	contradiction.

$\therefore$ $a \to (b \to c)$, $d \to (b \wedge \neg c)$, $(a \wedge d)$ are proved as inconsistent.

Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ & Q using suitable concept map.

Solution:

Given,

premises are $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$, Q

Let us assume that R as an additional premise.

conclusion:- $R \rightarrow S$

S.NO	premises	Rule.
1.	R	assumed premise.
2.	$\neg R \vee P$	Rule P
3	$R \rightarrow P$	Rule T {2}
4.	$\therefore P$	Rule T {1, 3}
5.	$P \rightarrow (Q \rightarrow S)$	Rule P
6.	$Q \rightarrow S$	Rule T {4, 5}
7.	Q	Rule P
8.	S	Rule T {6, 7}
9.	$R \rightarrow S$	from (1, 8)

$\therefore$ Hence $R \rightarrow S$ is derived from premises.

Explain tautology and contradiction using suitable concept map and solve the formula $Q \vee (p \wedge 7Q) \vee (7p \wedge 7Q)$ is a tautology.

Solution:

tautology:-

If the truthtable is true for all the combination values of a variable then it is said to be tautology.

contradiction:-

If the truthtable is false for all the combinational values of a variable then it is said to be contradiction.

given,

$$Q \vee (P \wedge 7Q) \vee (7P \wedge 7Q)$$

P	Q	7P	7Q	P∧7Q	Q∨(P∧7Q)	7P∧7Q	Q∨(P∧7Q)∨(7P∧7Q)
T	T	F	F	F	T	F	T
T	F	F	T	T	T	F	T
F	T	T	F	F	T	F	T
F	F	T	T	F	F	T	T

∴ Hence $Q \vee (P \wedge 7Q) \vee (7P \wedge 7Q)$ is a tautology. Because all the combinational values of a variables are true.

Obtain the principal disjunctive normal form of the formula $(P \vee R) \wedge (P \vee \neg Q)$ by truth table method using suitable concept map.

Solution:

principle disjunctive normal form:-

It is the sum minterms.

Minterm:-

The product in which each variable or its negation occurs only once is called minterm.

using truth table:-

$(P \vee R) \wedge (P \vee \neg Q)$

P	Q	R	$\neg Q$	$P \vee R$	$P \vee \neg Q$	$(P \vee R) \wedge (P \vee \neg Q)$
T	T	T	F	T	T	T - $P \wedge Q \wedge R$
T	T	F	F	T	T	T - $P \wedge Q \wedge \neg R$
T	F	T	T	T	T	T - $P \wedge \neg Q \wedge R$
T	F	F	T	T	T	T - $P \wedge \neg Q \wedge \neg R$
F	T	T	F	T	F	F - $\neg P \wedge Q \wedge R$
F	T	F	F	F	F	F - $\neg P \wedge Q \wedge \neg R$
F	F	T	T	T	T	T - $\neg P \wedge \neg Q \wedge R$
F	F	F	T	F	T	F - $\neg P \wedge \neg Q \wedge \neg R$

Minterms:-

$(P \wedge Q \wedge R)$, $(P \wedge Q \wedge \neg R)$, $(P \wedge \neg Q \wedge R)$, $(P \wedge \neg Q \wedge \neg R)$, $(\neg P \wedge \neg Q \wedge R)$

PDNF:-

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$

Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$ using suitable concept map.

Solution:

Given,

$$(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$$

To prove $S \vee R$ is a tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$ we use direct method.

S.NO	premises	Rule.
1.	$P \vee Q$	Rule P
2	$\neg P \rightarrow Q$	Rule T
3.	$\neg Q \rightarrow P$	logical equivalence
4.	$P \rightarrow R$	Rule P
5.	$\neg Q \rightarrow R$	From (4), (5)
6.	$\neg R \rightarrow Q$	logical equivivalence
7.	$Q \rightarrow S$	Rule P
8.	$\neg R \rightarrow S$	From (6, 7)
9.	$R \vee S$	
10.	$S \vee R$	conclusion.

$\therefore$

Hence $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$ is proved.

e down the contra positive, converse and inverse of the Implication "if it is raining, I get wet"
g suitable concept map and Construct the truth table for the following proposition
q)∧r↔(p∨q)→r.

tion:

Given,

$(p \vee q) \wedge r \longleftrightarrow (p \vee q) \rightarrow r$.

Truth table:-

P	Q	R	$p \vee q$	$(p \vee q) \wedge r$	$(p \vee q) \rightarrow r$	$(p \vee q) \wedge r \leftrightarrow (p \vee q) \rightarrow r$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	F	F	T	F	F	T
F	F	T	F	F	F	T
F	F	F	F	F	F	T

∴ Hence $(p \vee q) \wedge r \longleftrightarrow (p \vee q) \rightarrow r$

given statement:-

$P \longrightarrow$ It is raining $\neg P \longrightarrow$ It is not raining

$Q \longrightarrow$ I get wet $\neg Q \longrightarrow$ I won't get wet.

$P \rightarrow q \Rightarrow$ If it is raining, I get wet.

contrapositive

$\neg q \longrightarrow \neg P$

if not q then not P

If I won't get wet, then it is not raining

converse

$q \longrightarrow P$.

If q then P

If I get wet, then it is raining

Inverse

$\neg P \longrightarrow \neg q$

If not P then not q

If it is not raining, then I wont get wet

Obtain the principal conjunctive normal form and principal disjunctive normal form of $(\neg P \to R) \wedge (Q \leftrightarrow P)$ by using the concept map on logical equivalences.

n: Given,

$(\neg P \to R) \wedge (Q \leftrightarrow P)$

$(P \vee R) \wedge (Q \to P) \wedge (P \to Q)$

$(P \vee R) \wedge (\neg Q \vee P) \wedge (\neg P \vee Q)$

$= [P \vee R \vee (Q \wedge \neg Q)] \wedge [P \vee \neg Q \vee (R \wedge \neg R)] \wedge [\neg P \vee Q \vee (R \wedge \neg R)]$

$= (P \vee R \vee Q) \wedge (\underline{P \vee R \vee \neg Q}) \wedge (\underline{P \vee \neg Q \vee R}) \wedge (P \vee \neg Q \vee \neg R) \wedge$
$(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

$S = (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R)$
$\wedge (\neg P \vee Q \vee \neg R)$

= Product of max terms = PCNF.

$\therefore$ S represents PCNF.

$\neg S = (P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)$

$\neg\neg S = (\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$

(min) sum (min)

= PDNF.

$\therefore \neg\neg S$ represents the PDNF.

$\therefore$ PCNF =

$(P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \wedge R) \wedge (\neg P \vee Q \wedge \neg R)$

$\therefore$ PDNF =

$(\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$.

Show that: $(P \to Q) \wedge (R \to S), (Q \wedge M) \wedge (S \to N), \neg(M \wedge N)$ and $(P \to R) \Rightarrow \neg P$ using suitable concept map.

Solution: Given,

premises $\longrightarrow (P \to Q) \wedge (R \to S), (Q \wedge M) \wedge (S \to N), \neg(M \wedge N), (P \to R)$

conclusion $\longrightarrow \neg P$.

S.NO	premises	Rule.
1.	$(P \to Q) \wedge (R \to S)$	Rule P
2	$P \to Q$	Rule T {1}
3	$R \to S$	Rule T {1}
4.	$(Q \wedge M) \wedge (S \to N)$	Rule P
5.	$Q \wedge M$	Rule T {4}
6.	$S \to N$	Rule T {4}
7.	$P \to M$	from (2,5)
8.	$R \to N$	from (3,6)
9.	$P \to R$	Rule P
10.	$P \to N$	Rule T {7}
11	$\neg M \to \neg P$	Rule T {7}
12.	$P \to N$	from 10
13.	$\neg N \to \neg P$	Rule T {12}
14.	$(\neg M \vee \neg N) \to \neg P$	Rule T {11, 13}
15	$\neg(M \wedge N) \to \neg P$	Rule T
16	$\neg(M \wedge N)$	Rule P
17	$\neg P$	Rule T {15, 16}

$\therefore (P \to Q) \wedge (R \to S), (Q, M) \wedge (S \to N), \neg(M \wedge N), (P \to R) \Rightarrow \neg P$

Prove that the premises $P \to Q$, $Q \to R$, $R \to S$, $S \to \neg R$ and $P \wedge S$ are inconsistent using the **suitable concept map.**

on: Given,

premises :- $P \to Q$, $Q \to R$, $R \to S$, $S \to \neg R$, $P \wedge S$.

S.NO	premises	Rule
1.	$P \to Q$	Rule P
2.	$Q \to R$	Rule P
3.	$P \to R$	Rule T {1,2}
4.	$R \to S$	Rule P
5.	$P \to S$	Rule T {3,4}
6.	$S \to \neg R$	Rule P
7.	$P \to \neg R$.	Rule T {5.6}
8.	$P \wedge S$	Rule P
9.	P	Rule T {8}
10.	S	Rule T {8}
11.	$\neg R$	Rule T from (9,7)
12.	$\neg S$	Rule T from (4, 10)
13	$S \wedge \neg S$	from (10,12)
14	F	contradiction

∴ Hence proved that,

$P \to Q$, $Q \to R$, $R \to S$, $S \to \neg R$, $P \wedge S$ are inconsistent

Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q using suitable concept map.

Solution:

Given

premises are $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$, Q

conclusion $R \rightarrow S$.

let us assume that R is an additional premises and shows S.

S.NO	Premises	Rule
1.	R.	Assumed
2.	$\neg R \vee P$	Rule P
3.	$R \rightarrow P$	Rule T
4.	P	from (1,3)
5.	$P \rightarrow (Q \rightarrow S)$	Rule P
6.	$Q \rightarrow S$	Rule T {4,5}
7.	Q	Rule P
8.	S	Rule T {6,7}
9.	$R \rightarrow S$	Rule CP.

∴ Hence derived.

ow that the hypothesis " It is not sunny this afternoon and it is colder than yesterday', "we will go ning only if it is sunny", " If we do not go swimming, then we will take a canoe trip" and "If we conoe trip, then we will be home by sunset" lead to the conclusion " we will be home by sunset suitable concept map.

on: from the above statements

$P \longrightarrow$ It is sunny

$Q \longrightarrow$ It in coldes than yesterday.

$R \longrightarrow$ swimming

$S \longrightarrow$ Take a canoe trip.

$T \longrightarrow$ will be home by sunset.

premises:-

$\neg P \wedge Q,\ P \rightarrow R,\ \neg R \rightarrow S,\ S \rightarrow T$

conclusion:-

T

S.NO	Premises	Rule.
1	$\neg P \wedge Q$	Rule p
2	$\neg P$	Rule T {1}
3	Q	Rule T {1}
4.	$P \rightarrow R$.	Rule p
5.	$\neg R$	Rule T {4, 2}
6.	$\neg R \rightarrow S$	Rule p
7.	S	Rule T {5, 6}
8.	$S \rightarrow T$	Rule p
9.	T	Rule T {7, 8}

$\therefore$ conclusion is we will be home by sunset $\rightarrow T$

UNIT II – PREDICATE CALCULUS

Write the symbolic form and negate the following statements
(1) Everyone who is healthy can do all kinds of work.
(2) Some people are not advised by everyone.
(3) Everyone should keep his neighbours or his neighbours will not help him.
(4) Everyone agree with someone and someone agrees with everyone. Hence illustrate the procedure
ing a suitable concept map.

ion:

1) Everyone who is healthy can do all kinds of work

let

$H(x)$: x is healthy

$W(x)$: x can do all kinds of works

symbolic form:-

$\forall x\, [H(x) \longrightarrow W(x)]$

2) some people are not advised by everyone.

let

$A(x)$: x is advised

symbolic form:-

$\exists x\, [\neg A(x)]$

3) Everyone should keep his neighbours or his neighbours will not help him.

$H(x)$: x is help neigbours

$P(x)$: x is person

symbolic form:-

$\forall x\, [(P(x) \longrightarrow H(x)) \vee H(x) \longrightarrow \neg P(x)]$

4) Everyone argue with someone and someone argues with everyone. Hence illustrate the procedure.

$P(x)$: x argue with someone

$P(y)$: y argue with everyone.

symbolic form:-

$\forall x\, [P(x) \wedge P(y)]$

ove that $(x)(P(x) \to Q(x)) \wedge (x)(Q(x) \to R(x)) \Rightarrow (x)(P(x) \to R(x))$. Hence illustrate the procedure using a suitable concept map.

lution:

Given

$(x)(P(x) \to Q(x)) \wedge (x)(Q(x) \to R(x))$

premises:-

$(x)(P(x) \to Q(x))$, $(x)(Q(x) \to R(x))$

conclusion:-

$(x)(P(x) \to R(x))$

S.NO	Premises	Rule
1.	$(x)(P(x) \to Q(x))$	Rule P
2.	$P(y) \to Q(y)$	Rule US
3.	$(x)(Q(x) \to R(x))$	Rule P
4.	$Q(y) \to R(y)$	Rule US
5.	$P(y) \to R(y)$	Rule T $P(y) \to Q(y), Q(y) \to R(y) \Rightarrow P(y) \to R(y)$
6.	$x[P(x) \to R(x)]$	Rule UG

Hence proved that

$(x)(P(x) \to Q(x)) \wedge (x)\, Q(x) \to R(x)) \Rightarrow$

$(x)(P(x) \to R(x))$

Use the indirect method to prove that the conclusion $(\exists z)Q(z)$ follows from the premises $(\forall x)(P(x) \to Q(x))$ and $(\exists y)P(y)$. Hence illustrate the procedure by using a suitable concept map.

Solution:

Given,

$(\forall x)(P(x) \longrightarrow Q(x))$ and $(\exists y)P(y)$

premises:-

$(\forall x)(P(x) \longrightarrow Q(x))$, $(\exists y)P(y)$

conclusion:-

$(\exists z)Q(z)$

To solve the above premises we can take additional premise.

$\neg(\exists z)Q(z)$.

S.NO	premises	Rules.
1.	$\neg(\exists z)Q(z)$	Rule P
2.	$\forall z(\neg Q(z))$	Rule T, demorgan
3.	$\neg Q(a)$	Rule US
4.	$(\forall x)(P(x) \longrightarrow Q(x))$	Rule P
5.	$P(a) \longrightarrow Q(a)$	Rule US.
6.	$\neg P(a)$	Rule T {5,3}
7.	$(\exists y)P(y)$	Rule P.
8	$P(a)$	Rule ES
9	$\neg P(a) \wedge P(a)$ Con' F	Rule T {6,8} contradiction

∴ Hence the given precises are inconsistent

Verify the validity of the following arguments:
Lions are dangerous animals. There are lions. Therefore, there are dangerous animals. Hence illustrate the procedure by using a suitable concept map.

Solution:

Let

$L(x)$: x is a lion

$D(x)$: x is a dangerous animal

We need to show that,

$(\forall x)(L(x) \to D(x)), (\exists x) L(x) \Rightarrow (\exists x) D(x)$

S.NO	premises	Rule
1	$(\exists x)(L(x))$	Rule P
2	$L(a)$	Rule ES
3.	$(\forall x)[L(x) \to D(x)]$	Rule P
4.	$L(a) \to D(a)$	Rule US
5.	$D(a)$	Rule T {2,4}
6.	$\exists(x) D(x)$	Rule EG.

$\therefore$ The argument is valid.

5. Show that $\forall x P(x) \wedge \exists x Q(x)$ is equivalent to $\forall x \exists y (P(x) \wedge Q(y))$ Hence illustrate the procedure by using a suitable concept map.

Solution:

Given,

$$\forall x\, p(x) \wedge \exists x\, Q(x)$$

It doesn't matter if the variables are called x or y. Let us recall the variable in the second expression of conjunction.

$$\forall x\, p(x) \wedge \exists y\, Q(y)$$

$$[\forall x\, p(x) \wedge (\exists y) \cdot Q(y)]$$

$$\forall x(p(x) \wedge (\exists y\, Q(y)))$$

using commutative law

$$\forall x[(\exists y)\, Q(y) \wedge p(x)]$$

$$\forall x \exists y(Q(y) \wedge p(x))$$

using commutative law

$$\forall x \exists y(p(x) \wedge Q(y)]$$

$\therefore \forall x\, p(x) \wedge \exists x\, Q(x)$ is equivalent to $\forall x \exists y(p(x) \wedge Q(y))$.

Use of rule of inference to prove that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book". Hence illustrate the procedure by using a suitable concept map.

Solution:

from the above conditions

universe $\longrightarrow$ set of all students in college

$C(x) \longrightarrow$ x is in class

$B(x) \longrightarrow$ x has read book

$P(x) \longrightarrow$ x passed first exam.

premises

$$\exists x[C(x) \wedge \neg B(x)], \forall x(C(x) \longrightarrow P(x))$$

conclusion.

$$\exists x[P(x) \wedge \neg B(x)]$$

S.NO	premises	Rule
1.	$\exists x[C(x) \wedge \neg B(x)]$	Rule P
2	$C(a) \wedge \neg B(a)$	Rule ES
3.	$C(a)$	Rule T {2}
4.	$\neg B(a)$	Rule T {2}
5.	$\forall x[C(x) \rightarrow P(x)]$	Rule P
6	$C(a) \longrightarrow P(a)$	Rule US
7.	$P(a)$	Rule T {6,3}
8.	$P(a) \wedge \neg B(a)$	Rule T {4,7}
9.	$\exists x[P(x) \wedge \neg B(x)]$	Rule EG

∴ Hence proved.

. State and explain the proof methods. Hence illustrate the procedure by using a suitable concept map.

Solution:

Proof Methods :-

Mathematical proof is an argument. We give logically to validate a mathematical statement. In order to validate a statement we consider statement and logical operator.

↑

Proof Methods

Direct proof	Indirect proof
A direct proof is a proof in which truth of the premise directly shows the truth of the conclusion.	In indirect proof, we assume the negation of the statement and proceed to conclude the proof.
	⇒ The conclusion for the inconsistent will be of contradiction which means false.

Rules

Rule P   Rule T

3. Show that the hypotheses, "It is not sunny this afternoon and it is colder than yesterday", "We will go swimming only if it sunny", "If we do not go swimming then we will take a canoe trip" and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset". Hence illustrate the procedure by using a suitable concept map.

Solution:

From the above statement

$P \longrightarrow$ It is sunny

$Q \longrightarrow$ It is colder than yesterday

$R \longrightarrow$ We will go swimming

$S \longrightarrow$ We will take canoe trip.

$T \longrightarrow$ Will be home by sunset

premises:-

$\neg P \wedge Q,\ P \rightarrow R,\ \neg R \rightarrow S,\ S \rightarrow T$

conclusion:-

T

S.NO	premises	Rule.
1.	$\neg P \wedge Q$	Rule P
2.	$P \rightarrow R$	Rule P
3.	$\neg P$	Rule T {1}
4.	Q	Rule T {1}
5.	$P \rightarrow R$	Rule P
6.	$\neg R$	Rule T {3,5}
7.	$\neg R \rightarrow S$	Rule P
8.	S	Rule T {6,7}
9.	$S \rightarrow T$	Rule P
10.	T	Rule T {8,9)

∴ Hence conclusion is T.

T implies we will be home by sunset.

Verify the validity of the following argument. Every living thing is a plant or an animal. John's gold fish is alive and it is not a plant. All animals have hearts. Therefore, John's gold fish has a heart. Hence illustrate the procedure by using a suitable concept map.

Solution:

from the above conditions,

$P(x) \rightarrow x$ is a plant

$A(x) \rightarrow x$ is an animal

$H(x) \rightarrow x$ has hearts

$g \rightarrow$ John's gold heart

premises:-

$\forall x[P(x) \vee A(x)]$, $\neg P(g)$, $\forall x[A(x) \rightarrow H(x)]$

conclusion:-

$H(g)$

S.NO	premises	Rule
1.	$\forall x[P(x) \vee A(x)]$	Rule P
2.	$P(g) \vee A(g)$	Rule US
3.	$\neg P(g)$	Rule P
4.	$A(g)$	Rule T $\{\neg P, P \vee Q = Q\}$
5.	$\forall x[A(x) \rightarrow H(x)]$	Rule P
6.	$A(g) \rightarrow H(g)$	Rule US
7.	$H(g)$	Rule T $\{4, 6\}$

∴ conclusion:- $H(g) \rightarrow$ gold fish has a hear

∴ The argument is valid.

0. Show that $(\forall x)(P(x) \rightarrow Q(x)), (\exists y)P(y) \rightarrow (\exists x)Q(x)$. Hence illustrate the procedure by using a suitable concept map.

Solution:

Given,

$(\forall x)(P(x) \rightarrow Q(x)), (\exists y)P(y) \rightarrow (\exists x)Q(x)$

premises:-

$(\forall x)(P(x) \rightarrow Q(x)), (\exists y)P(y)$

conclusion:-

$(\exists x)Q(x)$

S.NO	premises	Rules
1	$(\forall x)(P(x) \rightarrow Q(x))$	Rule P
2	$P(a) \rightarrow Q(a)$	Rule US
3.	$(\exists y)P(y)$	Rule P
4.	$P(a)$	Rule ES
5.	$Q(a)$	from {2,4}
6.	$(\exists x)Q(x)$	Rule EG

∴ Hence proved

$(\forall x)(P(x) \rightarrow Q(x)), (\exists y)P(y) \Rightarrow (\exists x)Q(x)$

11. Verify the validation of the following inference.
If one person is more successful than another, then he has worked harder to deserve success. John s not worked harder than Peter. Therefore, John is not more successful than peter. Hence illustrate the ocedure by using a suitable concept map.

ution:

from the above conditions

let

$S(x,y)$: x is more successful than y

$H(x,y)$: x works harder than y to deserve success

a = John b = peter

premises :-

$(\forall x), (\forall y)[S(x,y) \longrightarrow H(x,y)], \neg H(a,b)$

conclusion :-

$\neg S(a,b)$

S.NO	premises	Rule
1.	$(\forall x)(\forall y)[S(x,y) \longrightarrow H(x,y)$	Rule P
2	$(\forall y)[S(a,y) \longrightarrow H(a,y)]$	Rule US
3.	$S(a,b) \longrightarrow H(a,b)$	Rule US
4.	$\neg H(a,b)$	Rule P
5.	$\neg S(a,b)$	Rule T {3,4}

∴ conclusion is $\neg S(a,b)$

∴ John is not more successful than peter.

Use indirect method of proof to prove that $(\forall x)(P(x) \vee Q(x)) \rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$. Hence trate the procedure by using a suitable concept map.

tion:

Given,

$$(\forall x)(P(x) \vee Q(x)) \rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$$

To prove the above statement let us assume the additional precise.

$$\neg[(\forall x)P(x) \vee (\exists x)Q(x)]$$

S.NO	premises	Rule.
1.	$\neg[(\forall x)P(x) \vee (\exists x)Q(x)]$	Rule P.
2.	$\neg[\forall x\, P(x)] \wedge \neg[(\exists x)Q(x)]$	Rule T, demorgan
3.	$(\exists x)\neg P(x) \wedge (\forall x)\neg Q(x)$	Rule T {2}
4.	$(\exists x)\neg P(x)$	Rule T {3}
5.	$(\forall x)\neg Q(x)$	Rule T {3}
6.	$\neg Q(a)$	Rule US
7.	$\neg P(a)$	Rule ES
8.	$\neg Q(a) \wedge \neg P(a)$	from {6, 7}
9.	$\neg[Q(a) \vee P(a)]$	Demorgan's
10.	$\forall x[P(x) \vee Q(x)]$	Rule UG.

∴ Hence proved

$$(\forall x)(P(x) \vee Q(x)) \rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$$

...ow that the premises "One student in this class knows how to write programs in Java" and ...yone who knows how to write program in Java can get a high paying job" imply the conclusion ...eone in this class can get a high paying job". Hence illustrate the procedure by using a suitable ...pt map.

...on:

from the above conditions:-

$C(x) \rightarrow x$ is in class

$J(x) \rightarrow x$ knows how to write java program

$H(x) \rightarrow x$ gets highly paid job.

precises:-

$\exists x\,[C(x) \wedge J(x)],\ \forall x\,[J(x) \rightarrow H(x)]$

conclusion:-

$\exists x\,[C(x) \wedge H(x)]$

S.NO	premises	Rules
1.	$(\exists x)[C(x) \wedge J(x)]$	Rule P
2	$C(a) \wedge J(a)$	Rule ES
3.	$C(a)$	Rule T {2}
4.	$J(a)$	Rule T {2}
5.	$\forall x[J(x) \rightarrow H(x)]$	Rule P
6.	$J(a) \rightarrow H(a)$	Rule US
7.	$H(a)$	Rule T {4,6}
8.	$C(a) \wedge H(a)$	Rule T {3,7}
9.	$\exists x[C(x) \wedge H(x)]$	Rule EG.

∴ Hence the conclusion of the premise is someone in class can get highly paid job.

rove that $(\forall x)(P(x) \to Q(y) \wedge R(x)), (\exists x)P(x) \to Q(y) \wedge (\exists x)(P(x) \wedge R(x))$. Hence illustrate the dure by using a suitable concept map.

ion:

Given

$(\forall x)(P(x) \to Q(y) \wedge R(x)), (\exists x)P(x) \to Q(y) \wedge (\exists x)(P(x) \wedge R(x))$

precises

$(\forall x)(P(x) \to Q(y) \wedge R(x)), (\exists x)P(x)$

conclusion:

$Q(y) \wedge (\exists x)(P(x) \wedge R(x))$

S.NO	premise	Rule
1.	$(\forall x)P(x) \to (Q(y) \wedge R(x))$	Rule P
2.	$P(a) \to Q(y) \wedge R(a)$	Rule US
3.	$(\exists x)P(x)$	Rule P
4.	$P(a)$	Rule ES
5.	$Q(y) \wedge R(a)$	Rule T {2,4}
6.	$Q(y)$	Rule T {5}
7.	$R(a)$	Rule T {5}
8.	$P(a) \wedge R(a)$	Rule T {4,7}
9.	$(\exists x)(P(x) \wedge R(x))$	Rule EG
10.	$Q(y) \wedge (\exists x)P(x) \wedge R(x)$	Rule T {6,9}

∴ Hence proved

$(\forall x)(P(x) \to Q(y) \wedge R(x)), (\exists x)P(x) \to Q(y) \wedge (\exists x)(P(x) \wedge R(x))$

Show that the conclusion $(\forall x)P(x) \to \neg Q(x)$ follows from the premises $(\exists x)(P(x)\wedge Q(x)) \to (\forall y)(R(y) \to S(y))$ and $(\exists y)(R(y)\wedge \neg S(y))$. Hence illustrate the procedure by using a suitable concept map.

Solution:

Given,

precises:-

$$(\exists x)(P(x)\wedge Q(x)) \to (\forall y)(R(y) \to S(y))$$

conclusion:-

$$(\exists y)(R(y)\wedge \sim S(y)]$$

S.NO	premises	Rule
1.	$(\exists y)(R(y)\wedge \neg S(y))$	Rule P.
2	$R(a)\wedge \neg S(a)$	Rule ES
3.	$\neg[\neg R(a) \vee S(a)]$	Demorgans
4.	$\neg(\exists y)[\neg R(y) \vee S(y)]$	Rule EG
5.	$\forall(y)[R(y)\wedge \neg S(y)]$	Demorgans law
6.	$\neg \exists(y)[\neg R(y) \vee (S(y))]$	Rule T {5}
7	$\neg \exists(y)[R(y) \to S(y)]$	Rule T {6}
6.	$\neg(\forall y)(R(y) \to S(y)]$	Rule T, Demorgans
7.	$(\exists x)(P(x)\wedge Q(x)) \to (\forall y)(R(y) \to S(y))$	Rule P
8	$\neg(\exists x)(P(x)\wedge Q(x))$	Rule T {7, 6}
9.	$\forall x[\neg P(x) \vee \neg Q(x)]$	Rule T, demorgans
10.	$\forall x[P(x) \to \neg Q(x)]$	Rule T {9}

∴ Hence proved.

n a discrete mathematics class every student is a major in computer science or mathematics, or both. e number of students having computer science as a major is 25; the number of students having thematics as a major is 13; and the number of students majoring in both computer science and thematics is 8. How many students are in this class? Hence illustrate the procedure by using a suitable ncept map.

ution:

Given,

students having computer science,

$n(C) \longrightarrow 25$

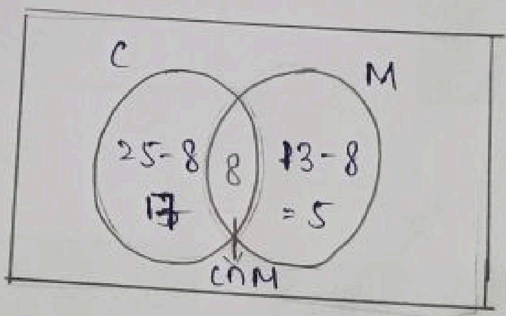
students having mathematics

$n(m) \longrightarrow 13$

students in both computer science and mathematics

$n(C \cap m) \longrightarrow 8.$

from the above conditions



We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(C \cup M) = n(C) + n(M) - n(C \cap M)$

$= 25 + 13 - 8$

$= 38 - 3$

$\boxed{n(C \cup M) = 30}$

∴ No. of students in class are 30.

2. State and prove De Morgan's law in set theory using Venn diagram. Hence illustrate the procedure by using a suitable concept map.

Solution:

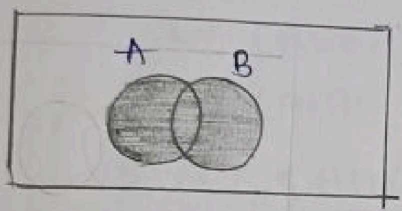
Demorgan's law are based on the complement of sets

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$

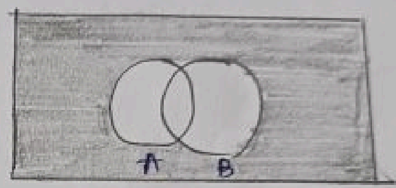
Let us prove the law by venn diagram.

a) $(A \cup B)' = A' \cap B'$

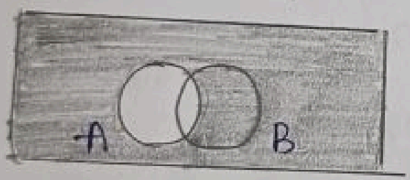
$A \cup B$



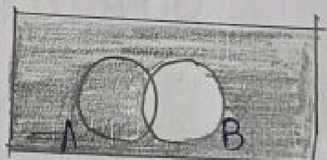
$(A \cup B)'$



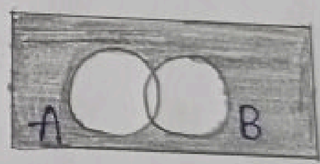
RHS:- A'



B'



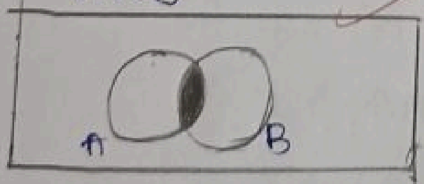
$A \cap B'$



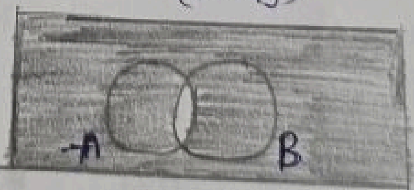
$\boxed{\therefore (A \cup B)' = A' \cap B'}$

b) $(A \cap B)' = A' \cup B'$

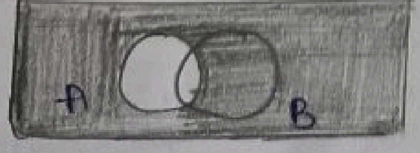
$A \cap B$



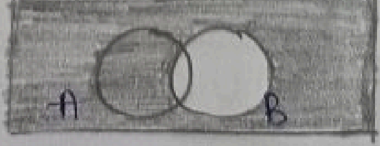
$(A \cap B)'$



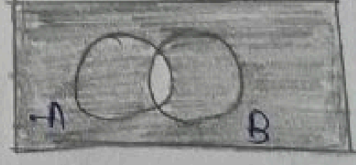
RHS $\Rightarrow A'$



B'



$A' \cup B'$



$\boxed{\therefore (A \cap B)' = A' \cup B'}$

3. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Hence illustrate the procedure by using a suitable concept map.

Solution: Given $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Let us take

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Let $x \in A \cup (B \cap C)$. If $x \in A \cup (B \cap C)$ then x either belongs to A (or) $(B \cap C)$

$x \in A$ (or) $x \in (B$ and $C)$

$x \in A$ (or) $\{x \in B$ and $x \in C\}$

$\{x \in A$ (or) $x \in B\}$ and $\{x \in A$ (or) $x \in C\}$

$\{x \in (A$ (or) $B)\}$ and $\{x \in (A$ or $C)\}$

$x \in (A \cup B)$ and $x \in A (\cup) C$

$x \in (A \cup B) \cap x \in (A \cup C)$

$x \in \cancel{\$}A \cup (B \cap C) \Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\boxed{\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A$ and $\{x \in B$ (or) $x \in C\}$

$\Rightarrow x \in A \cap B$ (or) $x \in A \cap C$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in A \cap (B \cup C)$

$\Rightarrow x \in (A \cap B) \cup x \in (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\boxed{\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

$\therefore$ Hence proved.

4. If $A=\{a,b\}, B=\{1,2,3\}$, then find $(A\times B)\cap(A\times B)$, $(A\times B)U(A\times B)$. Hence illustrate the procedure by using a suitable concept map.

Solution:

Given,

$A=\{a,b\}$

$B=\{1,2,3\}$

$(A\times B)=\{(a,1)\ (a,2)\ (a,3)\ (b,1)\ (b,2)\ (b,3)\}$

1) $(A\times B)\cap(A\times B)$

$=\{(a,1)\ (a,2)\ (a,3)\ (b,1)\ (b,2)\ (b,3)\}$

2) $(A\times B)\cup(A\times B)$

$=\{(a,1)\ (a,2)\ (a,3)\ (b,1)\ (b,2)\ (b,3)\}$

5. Let $X = \{1,2,3,4\}$ and $R = \{(x,y) | x > y\}$. Draw the graph of R and also give its matrix. Hence illustrate the procedure by using a suitable concept map.

Solution:

Given,

$x = \{1,2,3,4\}$

$R = \{(x,y) | x > y\}$

from x, we can write

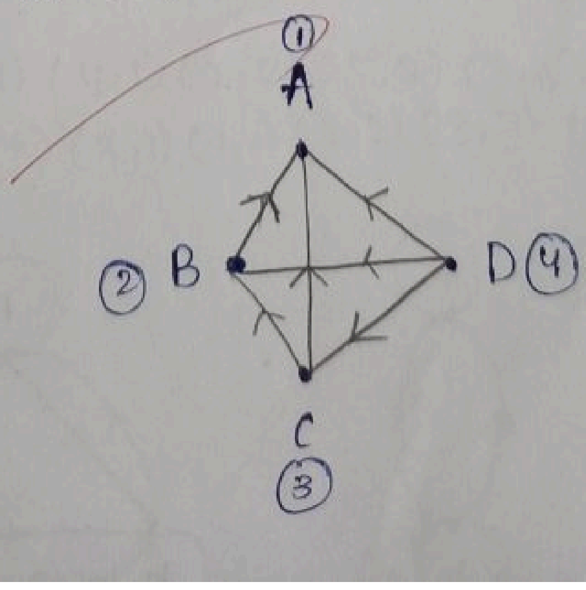
$P(X) = \{(1,2)(1,3)(1,1)(1,4)(2,1)(2,2)(2,3)(2,4)$
$(3,1)(3,2)(3,3)(3,4)(4,1)(4,2)(4,3)(4,4)\}$

$R = \{(2,1)(3,1)(3,2)(4,1)(4,2)(4,3)\}$

Matrix of R:-

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Graph of R:-



6. Let $X = \{1,2,3 \ldots,7\}$ and $R = \{(x,y) | x - y$ is divisible by3$\}$. Show that R is an equivalence relation. Draw the graph of R. Hence illustrate the procedure by using a suitable concept map.

Solution:

For any $a \in X$, $a-a$ is divisible by 3.

$\therefore (a,a) \in R$ for all $a \in X$

$\boxed{R \text{ is reflexive}}$

Equivalence

↓

Reflexive

symmetric

Transitive

Let $(a,b) \in R$; $a-b$ is divisible by 3

$b-a$ is divisible by 3

$(a,b) \in R \rightarrow (b,a) \in R$

$\therefore \boxed{R \text{ is symmetric}}$

Let $(a,b) \in R$ and $(b,c) \in R$

$(a-b)$ and $(b-c)$ are divisible by 3

i.e. $(a-b) = 3k$ and $(B-c) = 3m$

$(a-c) = (a-b) + (b-c) = 3k + 3m = 3(k+m)$

$\therefore$ It is divisible by 3.

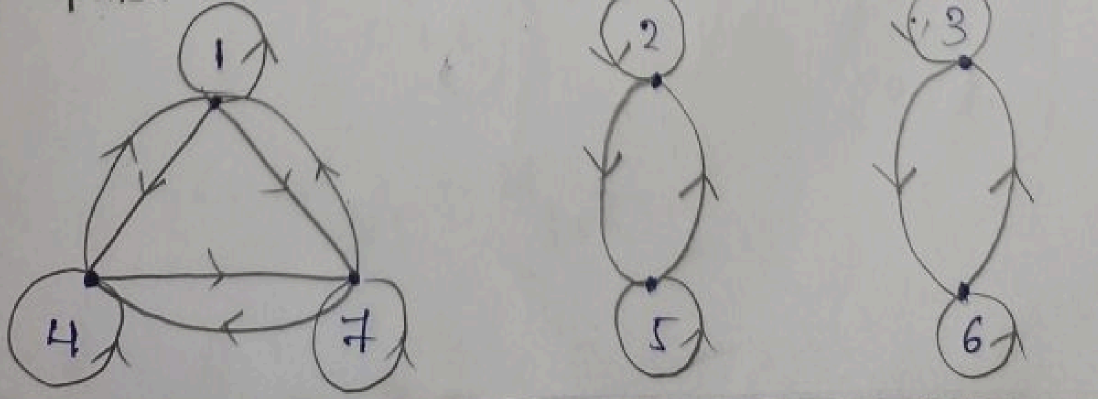
$\therefore \boxed{R \text{ is transitive}}$

$\therefore$ R has an equivalence relation

Given, $X = \{1,2,3,{}^{4,5,6}7\}$ $R = \{(x,y) | x-y \div 3\}$

$R = \{(1,4)(4,1)(2,5)(5,2)(3,6)(6,3)(4,7)(7,4),$
$(1,7)(7,1)(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)(7,7)\}$

graphs:-



7. Evaluate Given the relation matrices M_R and M_S as $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$. Find $M_{R\circ S}$, $M_{\bar{R}}$, $M_{\bar{S}}$, $M_{\overline{R\circ S}}$ and show that $M_{\overline{R\circ S}} = M_{\bar{S}\circ\bar{R}}$. Hence illustrate the procedure by using a suitable concept map.

Solution:

Given,

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$M_{\bar{R}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_{\bar{S}} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$M_{ROS} = M_R \cdot M_S$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M_{R\bar{O}S} = (\overline{M_{ROS}}) = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\dot{M}_{R\bar{O}S} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$M_S^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_R^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M^{-1}_{S \cdot R^{-1}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\therefore M_{R\bar{O}S} = M_{\bar{S}\circ\bar{R}}$$

8. Two equivalence relations R and S are given by their relation matrices M_R and M_S . $M_R = \begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}, M_S = \begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 1\\ 0 & 1 & 1\end{bmatrix}$. Draw the relation graph of $R \circ S$ & $S \circ R$ and find $M_{R \circ S}$ & $M_{S \circ R}$. Also, show that $R \circ S$ is not an equivalence relation. Hence illustrate the procedure by using a suitable concept map.

Solution:

Given,

$$M_R = \begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \qquad M_S \begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 1\\ 0 & 1 & 1\end{bmatrix}$$

$$M_{R \cdot S} = M_R \cdot M_S$$

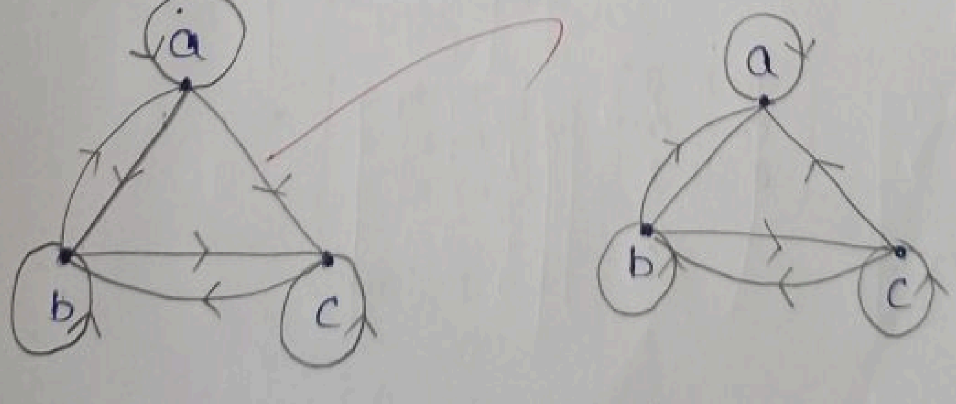
$$= \begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 1\\ 0 & 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 0 & 1 & 1\end{bmatrix}$$

$R \cdot S = \{(a,a)\ (a,b)\ (a,c)\ (b,a)\ (b,b)\ (b,c)\ (c,b)\ (c,c)\}$

$$M_{S \cdot R} = M_S \cdot M_R$$

$$= \begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 1\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$S \cdot R = \{(a,a)\ (a,b)\ (b,a)\ (b,b)\ (b,c)\ (c,a)\ (c,b)\ (c,c)\}$



∴ As c is not related to a RoS is not an equivalence relation.

9. For the given relation matrices M_R and M_S as $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Find the matrices of $R \cup S, R \cap S$, of $R \circ S$, $S \circ R$ and $R \oplus S$. Hence illustrate the procedure by using a suitable concept map.

Solution:

Given,

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$R = \{(a,b)(b,a)(b,b)(b,c)(c,a)\}$ $S = \{(a,b)(b,b)(b,c)(c,a)(c,b)(c,c)\}$

$R \cup S = \{(a,b)(b,a)(b,b)(b,c)(c,a)(c,b)(c,c)\}$

$$\underline{R \cup S} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \underline{R \cap S} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$R \circ S = M_R \cdot M_S$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$S \circ R = M_S \cdot M_R$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & \phi \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R \oplus S} = M_{R \cup S} - M_{R \cap S}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

10. For the set $\{1,2,3,4,5\}$ let $R = \{(1,2),(3,4),(2,2)\}$ and $S = \{(4,2),(2,5),(3,1),(1,3)\}$. Obtain the relation matrix for $S \circ R$, $R \circ S$. Hence illustrate the procedure by using a suitable concept map.

Solution:

Given,

$R = \{(1,2)\ (3,4)\ (2,2)\}$

$S = \{(4,2)\ (2,5)\ (3,1)\ (1,3)\}$

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

SOR

$SOR = \{(4,2)\ (3,2)\ (1,4)\}$

$$M_S = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Relation matrix:-

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_{S \cdot R} = M_S \cdot M_R$$

ROS

$ROS = \{(1,5)\ (3,2),\ (2,5)\}$

Matrix relation:-

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_{R \cdot S} = M_R \cdot M_S.$$

11. Find $R^2 = R \circ R$, R^3 where $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Hence illustrate the procedure by using a suitable concept map.

Given,

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R^2} = M_R \cdot M_R$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1\vee0\cup1 & 0\cup0\cup1 & 1\cup0\cup1 \\ 1\cup1\cup0 & 0\cup1\cup0 & 1\cup0\cup0 \\ 1\cup1\cup1 & 0\cup1\cup1 & 1\cup0\cup1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R^3 = R^2 \circ R$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1\cup1\cup1 & 0\cup1\cup1 & 1\cup0\cup1 \\ 1\cup1\cup1 & 0\cup1\cup1 & 1\cup0\cup1 \\ 1\cup1\cup1 & 0\cup1\cup1 & 1\cup0\cup1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore R_B^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad R_B^3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) Prove that the relation usual division on the set of positive integers is a partial order relation. Hence illustrate the procedure by using a suitable concept map.

(b) Prove that the relation usual less than or equal to on the set of real numbers is a partial order relation. Hence illustrate the procedure by using a suitable concept map.

ution:

Let us consider

$A = \{1, 2, 3, 4, [illegible]\}$

Given condition is R less than or equal to A

$R = \{(2,1)\ (3,1)\ (3,2)\ (4,1)\ (4,2)\ (4,3)\ (1,1)\ (2,2)\ (3,3)\ (4,4)\}$

Partial order

Reflexive | Antisymmetric | Transitive

Reflexive :-

A relation is said to be reflexive if $(a,a) \in R$.

i.e., $(1,1)\ (2,2)\ (3,3)\ (4,4)$

Antisymmetric.

A relation is said to be antisymmetric if $(a,b)\ \&\ (b,a) \in R$

$\Rightarrow a = b$

i.e $(1,1)\ (2,2)\ (3,3)\ (4,4)$

Transitive :-

A relation is said to be transitive if $(a,b)\ (b,c) \in R$

$\Rightarrow (a,c) \in R$

i.e $(4,2)\ \&\ (2,1) \in R$

$\Rightarrow (4,1) \in R$

$\therefore$ It is partial order relation.

Let L denote set of element (a, b) such that the relation is "a less than or equal to b" and D denote set elements having the relation "a divides b", where L and D are defined on the set $\{1,2,3,4,5\}$. Show both L and D are reflexive, antisymmetric and transitive. Also, find $L \cap D$. Hence illustrate the cedure by using a suitable concept map.

ution:

Given,

$A = \{1, 2, 3, 4, 5\}$

$L = \{(a, b) / a \leq b\}$ $\quad D = \{(a, b) / b \div a\}$

$P(A) = \{(1,1) (1,2) (1,3) (1,4) (1,5) (2,1) (2,2) (2,3) (2,4),$
$(2,5) (3,1) (3,2) (3,3) (3,4) (3,5) (4,1) (4,2)$
$(4,3) (4,4) (4,5) (5,1) (5,2) (5,3) (5,4) (5,5)\}$

$L = \{(1,1) (1,2) (1,3) (1,4) (1,5) (2,3) (2,4) (2,5) (3,4)$
$(3,5) (4,5) (2,2) (3,3) (4,4) (5,5)\}$

$D = \{(1,1) (2,2) (3,3) (4,4) (5,5) (2,4)\}$

Reflexive:

L, D are reflexive as

$\{(1,1) (2,2) (3,3) (4,4) (5,5)\}$ where $(a,a) \in A$

for every element of $\{1, 2, 3, 4, 5\} \in A$.

Antisymmetric:-

Let $a, b \in L, D$ $\quad a \leq b$ is true

$b \leq a$ is true ie $b = a$

$(b, a) \in R$ $\therefore$ L is antisymmetric.

Transitive:-

Let $a, b, c, \in A$, Let $(a, b) \in (L, D)$ & $(b, c) \in (L, D)$

$a \leq b$ and $b \leq c \Rightarrow a \leq c$ $\therefore (a, c) \in (L, D)$

$\therefore$ L and D are transitive

$L \cap D = (1,1) (2,2) (3,3) (4,4) (5,5) (2,4)\}$

r the set $\{1,2,3,4,5\}$ let $R = \{(1,2),(3,4),(2,2)\}$ and $S = \{(4,2),(2,5),(3,1),(1,3)\}$. Obtain the relation s for $S \circ R$, $R \circ S$. Hence illustrate the procedure by using a suitable concept map.

Given,

$R = \{(1,2)\ (3,4)\ (2,2)\}$ $S = \{(4,2)\ (2,5)\ (3,1)\ (1,3)\}$

$$M_R = \begin{bmatrix} 0&1&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix} \quad M_S = \begin{bmatrix} 0&0&1&0&0 \\ 0&0&0&0&1 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$$

$$M_{S \cdot R} = M_S \cdot M_R$$

$$= \begin{bmatrix} 0&0&1&0&0 \\ 0&0&0&0&1 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&0&0 \end{bmatrix} \begin{bmatrix} 0&1&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} 0&0&0&1&0 \\ 0&0&0&0&0 \\ 0&1&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$$

$\therefore S O R = \{(1,4)\ (3,2)\ (4,2)\}$

$$M_{R \cdot S} = M_R \cdot M_S$$

$$= \begin{bmatrix} 0&1&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix} \begin{bmatrix} 0&0&1&0&0 \\ 0&0&0&0&1 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} 0&0&0&0&1 \\ 0&0&0&0&1 \\ 0&1&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$$

$\therefore R O S = \{(1,5)\ (3,2)\ (2,5)\}$

nd $R^2 = R \circ R$, R^3 where $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Hence illustrate the procedure by using a suitable

pt map.

ion:

Given,

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R^2} = M_R \cdot M_R$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+1 & 0+0+1 & 0+0+1 \\ 1+1+0 & 0+1+0 & 1+0+0 \\ 1+1+1 & 0+1+1 & 1+0+1 \end{bmatrix}$$

$$M_{R^2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \{(a,a)(a,b)(a,c)(b,a)(b,b)(b,c)(c,a)(c,b)(c,c)\}$$

$$M_{R^3} = M_{R^2} \cdot M_R$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

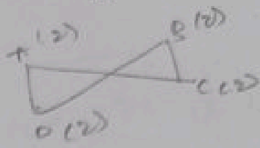
$$= \begin{bmatrix} 1+1+1 & 0+1+1 & 1+0+1 \\ 1+1+1 & 0+1+1 & 1+0+1 \\ 1+1+1 & 0+1+1 & 1+0+1 \end{bmatrix}$$

$$M_{R^3} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad R^3 = \{(a,a)(a,b)(a,c)(b,a)(b,b)(b,c)(c,a)(c,b)(c,c)\}$$

16/10/22

...e that a given connected graph is Eulerian if and only if all the vertices of G are of even degree. ...illustrate the proof by using a suitable concept map.

Let us consider a graph



Let us consider a graph with 4 vertices.

vertices $\longrightarrow$ A, B, C, D

degree of A = 2

degree of B = 2

degree of C = 2

degree of D = 2.

It satisfies the given condition that it has degree 2 which is even.

from the above graph,

graph consists of each edge exactly once. There is no repetition of edge

$\therefore$ Hence the above graph is eulerian for the vertices ~~of~~ G are of even degree.

...that a finite graph is bipartite if and only if it contains no cycles of odd length. Hence illustrate ...of by using a suitable concept map.

Bipartite graph:-

A complete bipartite graph $K_{m,n}$ is a graph that has its vertex set partioned into two subset v_1 of size m and v_2 of size n such that there is an edge from every vertex in v_1 to every vertex in v_2.

special graphs:-

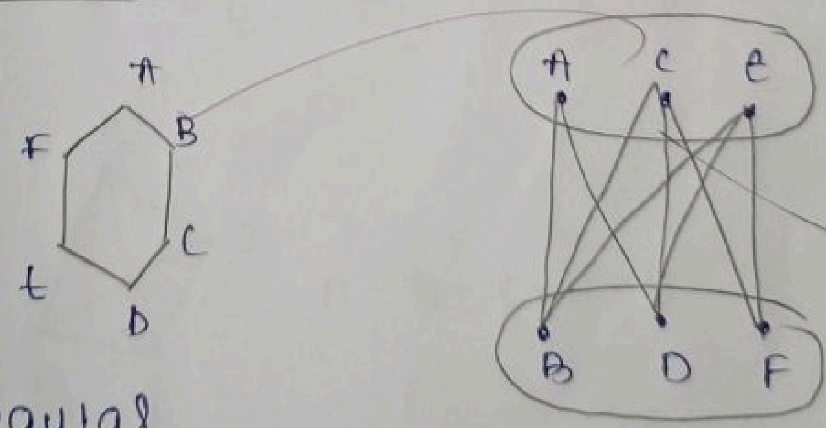
1) Bipartite graphs $K_{m,n}$:-

A graph where vertices can be divided into two sets such as adjacent of each vertex of one partition belongs to other partition.

Complete bipartite:-

$G(m+n), (m*n)$: Every vertex of one set is connected to each vertices of another set. It becomes regular graph where m=n.

Example:-



Regular graph

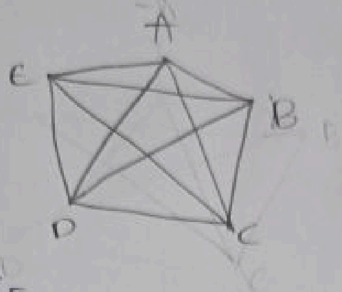
complete bipartite.

...the complete graph K_5 with vertices A, B, C, D, E. Draw all complete sub graph of K_5 with 4 ... Hence illustrate the procedure by using a suitable concept map.

Given,

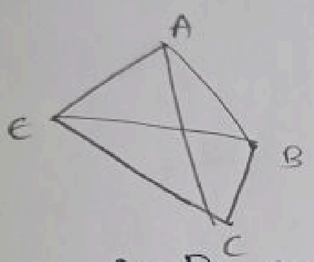
vertices = A, B, C, D, E

A graph with 5 vertices are

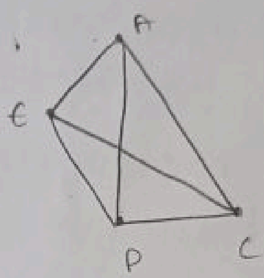


A - B - C - D - E - A

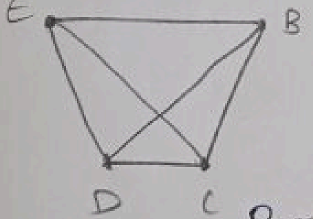
Subgraphs:-



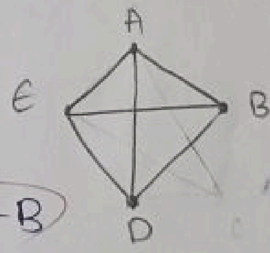
A - B - C - E



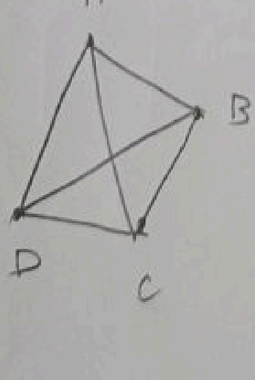
A - C - D - E - A



B - C - D - E - B



A - B - D - E - A



A - B - C - D - A

...prove hand shaking theorem. Show that the maximum number of edges in a simple graph ...ces is $\frac{n(n-1)}{2}$. Hence illustrate the proof by using a suitable concept map.

Handshaking theorem states that sum of degrees of the vertices of a graph is twice the no. of edges.

if $G = (V, E)$ be a graph with e edges then

$$\sum_{G} \deg(v) = 2e$$

proof:-

Since the graph is simple, it has no self loops and parallel edges.

since it has n vertices. Each vertex is adjacent to (n-1) vertices.

i.e., the degree of each vertex is (n-1)

sum of degree of all vertices is n(n-1)

W.K.T

sum of degree of all vertex = 2 × No. of edges

$$n(n-1) = 2 \times \text{no. of edges}$$

$$\text{no. of edges} = \frac{n(n-1)}{2}$$

∴ The maximum no. of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

...the maximum number of edges in a simple disconnected graph n with k vertices and ... is $\frac{(n-k)(n-k+1)}{2}$. Hence illustrate the proof by using a suitable concept map.

let $n_1, n_2, n_3, \ldots n_k$ be the no. of vertices in each of k components of graph G.

Then $n_1+n_2+\cdots+n_k = n = [V(G)]$

$$\sum_{i=1}^{n} n_i = n$$

now, $\sum_{i=1}^{k}(n_i-1) = (n_1-1)+(n_2-1)+\cdots(n_k-1)$

$$= \sum_{i=1}^{k} n_i - k$$

$$\sum_{i=1}^{k}(n_i-1) = n-k$$

squaring on both sides

$$\left[\sum_{i=1}^{k}(n_i-1)\right]^2 = (n-k)^2$$

$$(n_1-1)^2+(n_2-1)^2+\cdots+(n_k-1)^2 = n^2+k^2-2nk$$

$$n_1^2+1-2n_1+n_2^2+1-2n_2+\cdots n_k^2+1-2n_k = n^2+k^2-2nk$$

$$\sum_{i=1}^{k} n_i^2 + k - 2n \leq n^2+k^2-2nk$$

$$\sum_{i=1}^{k} n_i^2 = n^2+k^2-2nk+2n-k$$

$$\sum_{i=1}^{k} n_i^2 = n^2+k^2-k-k-2nk+2n$$

$$= n^2+k(k-1)-(k-1)2n$$

$$= n^2+(k-1)(k-2n)$$

$$\sum_{i=1}^{k} n_i^2 = n^2+(k-1)(k-2n)$$

since G is simple, the max no. of degree of G in components is $\frac{n_i(n_i-1)}{2}$

$\therefore$ Max no. of edges of G

$$= \sum_{i=1}^{K} \frac{n_i(n_i-1)}{2}$$

$$\sum_{i=1}^{K} \left[\frac{(n_i^2-n_i)}{2}\right]$$

$$= \frac{1}{2}\sum_{i=1}^{K} n_i^2 - \frac{1}{2}\sum_{i=1}^{K} n_i$$

$$\frac{1}{2}\left[n^2+(K-1)(K-2n)\right] - \frac{n}{2}$$

$$\frac{1}{2}\left[n^2-2nK+K^2+2n-K-n\right]$$

$$\frac{1}{2}\left[n^2-2nK+K^2+n-K\right]$$

$$\frac{1}{2}\left[(n-K)^2+(n-K)\right]$$

$$\frac{1}{2}(n-K)(n-K+1)$$

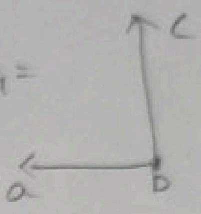
$$\left.\begin{array}{l}\text{Maximum}\\ \text{no. of edges of G}\end{array}\right\} = \frac{(n-K)(n-K+1)}{2}.$$

...ency matrices of two pairs of graph as given below. Examine the isomorphism of G and H by ...graphs of the matrices $A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Hence illustrate the procedure by using ...oncept map.

We know that,

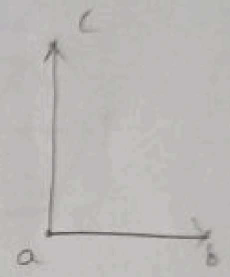
Two simple graphs are isomorphic if adjacency of A_G and A_H are related by

$$A_G = P^{-1} A_H P.$$

$A_G =$ 

where P is the permutation matrix

$$A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$A_H =$ 

Let us take $A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \text{ [interchanging } R_1 \,\&\, R_3]$$

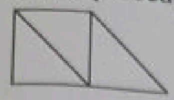
$$\sim \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ (interchanging } C_1 \,\&\, C_3)$$

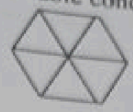
$$= A_H$$

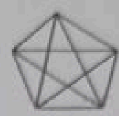
$\therefore$ A_G is similar to A_H.

$\therefore$ Their corresponding graphs are isomorphic.

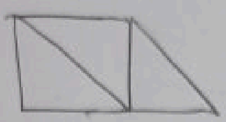
Euler path or an Euler circuit, if it exists in each of the three graphs below. If it does not exist, why? Hence illustrate the procedure by using a suitable concept map.

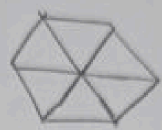


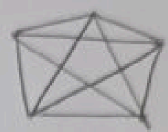




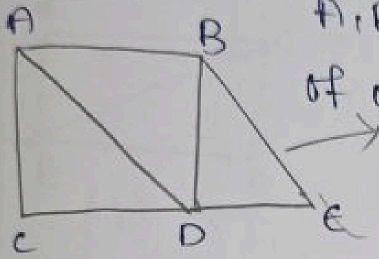
Given graphs are







Graph 1



A, B consists of odd degree

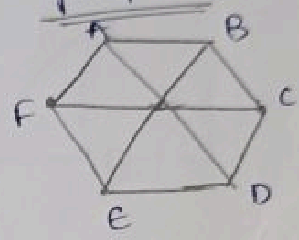
A - C - D - E - B - D - A - B

In graph 1 the vertex repetition is allowed so it is not hamiltonian

⇒ Edge repetition is not allowed

∴ so it is eulerian path.

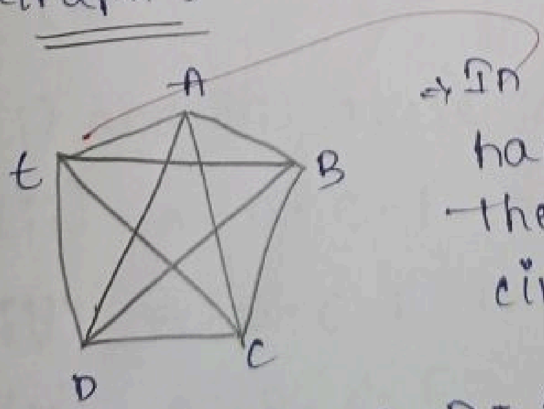
graph 2



In the above graph there are 6 vertices of odd degree and edge, vertex repetition is allowed

∴ It is neither an euler path or circuit

Graph 3.

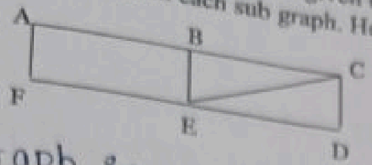


→ In graph 3, all the vertices has even degree, then there exists an euler circuit.

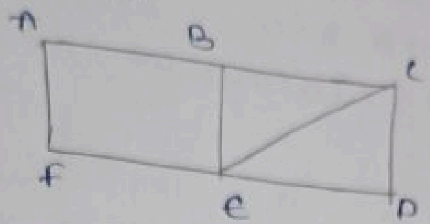
A - B - C - D - E - A - C - E - B - D - A

∴ It is an euler circuit.

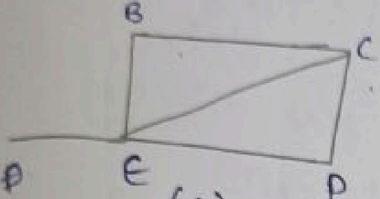
... all the connected subgraph obtained from the graph given in the following figure, by deleting ... List out the simple paths from A to in each sub graph. Hence illustrate the procedure by ...ble concept map.



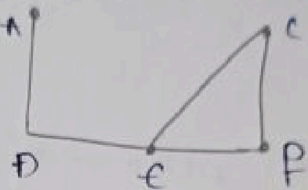
Given graph is



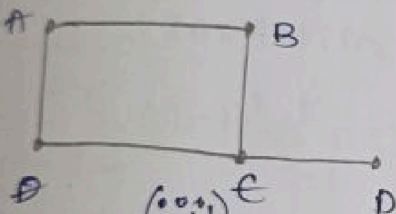
The subgraphs for the above graph is



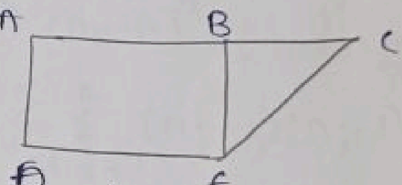
(i)



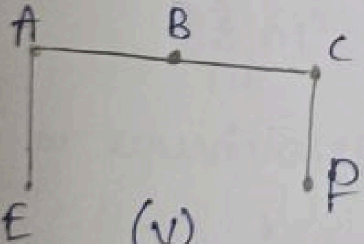
(ii)



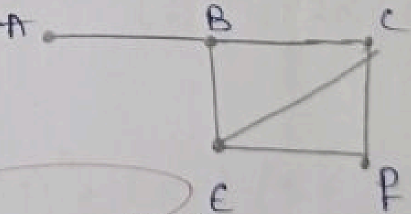
(iii)



(iv)



(v)



(vi)

In (i) and (ii) there is no path from A to F

In (iii) A - D - E - F and A - D - E - C - F

In (iv) A - D - E - F and A - B - E - F

In (v) A - B - C - F

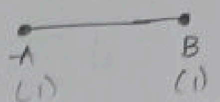
In (vi) A - B - E - F and A - B - C - F,

A - B - E - C - F, A - B - C - E - F

...h, either connected or disconnected, has exactly two vertices of odd degree, prove that there is ...ing these two vertices. Hence illustrate the procedure by using a suitable concept map.

proof:-

Let C be a connected component of x containing the vertex u. since the no. of vertices of odd degree in any graph is even, C has to contain other vertex of odd degree.



Lemma 3:-

Let $n_1, n_2 \ldots n_K \in N$ then

$$\sum_{i=1}^{K} n_i^2 \leq \left(\sum_{i=1}^{K} n_i\right)^2 - (K-1)\left(2\sum_{i=1}^{K} n_i - K\right)$$

proof:- We have,

$$(n_1-1)+(n_2-1)+\cdots+(n_K-1) = (n_1+n_2+n_3+\cdots n_K)-k$$

squaring on both sides,

$$[(n_1-1)+(n_2-1)+\cdots+(n_K-1)]^2 = (n_1+n_2+n_3+\cdots n_K)-K]^2$$

$$\sum_{i=1}^{K}(n_i-1)^2 + \left(\sum_{i=1}^{K} n_i\right) + K + \sum_{i=1}^{K}\sum_{j\neq 1}^{K}(n_i-1)(n_j-1) = \left(\sum_{i=1}^{R} n_i\right)^2 - 2K\left[\sum_{i=1}^{K} n_i\right) + k^2$$

Hence we have

$$\sum_{i=1}^{K} n_i^2 \leq 2\left[\sum_{i=1}^{K} n_i\right) - K + \left(\sum_{i=1}^{K} n_i\right)^2 - 2K\left(\sum_{i=1}^{K} n_i\right) + K^2$$

or equivalently,

$$\sum_{i=1}^{K} n_i^2 \leq \left(\sum_{i=1}^{K} n_i\right)^2 - (K-1)\left(2\sum_{i=1}^{K} n_i - K\right)$$

$$\sum_{i=1}^{K}(n_i^2 - 2n_i + 1) + \text{non negative} = (n-k)^2$$

$$\sum n_i^2 - 2\sum n_i + \sum 1 \leq (n-K)^2$$

$$\sum n_i^2 - 2n + K \leq (n-K)^2$$

$$\sum n_i^2 \leq (n-K)^2 + 2n - K \longrightarrow (3)$$

Max no. of edges in $G = \frac{1}{2}\left[\sum n_i^2 - n\right]$

$$\leq \left[(n-k)^2 + 2n - k - n\right]$$

$$\leq \frac{1}{2}\left[(n-k)^2 + (n-k)\right]$$

$$= \frac{1}{2}(n-k)(n-k+1)$$

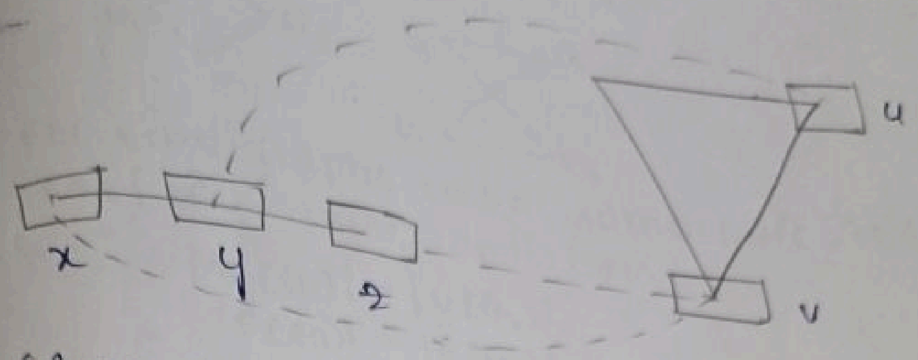
$$= \frac{(n-k)(n-k+1)}{2}$$

Hence in graph G the no. of edge cannot exceed is

$$\frac{(n-k)(n-k+1)}{2}.$$

cannot

...at the comp... ...disconnected graph is connected. Hence illustrate the proof by using a ...ncept map.



Given a vertex i(c_1) and vertex in c_2 there is a path from u to v in the compound path from x to y.

$$x\ to\ y = x - z - y$$

complement of disconnected graph = connected.

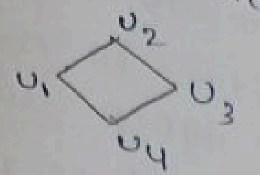
Let us assume we have a disconnected graph G. conside vertices x and y which are not adjacent in G_1 then they will be adjacent to G and we can find a trival x-y path.

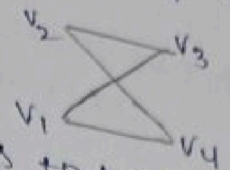
∴ If z be same vertex then xz & yz are not in G.

∴ complement of disconnected graph is connected.

...at any 2 simp... ...ected graphs with n vertices all of degree 2 are isomorphic. Hence ...proof by using a suitable concept map.

Let us consider two graphs:-





For two graphs to be isomorphic, it should satisfy three conditions.

$|V(G_1)| = |V(G_2)|$

$|E(G_1)| = |E(G_2)|$

degree of G_1 = degree of G_2

For graph 1

$|V(G_1)| = 4$

$|E(G_1)| = 4$

degrees = 2, 2, 2, 2

For graph 2

$|V(G_2)| = 4$

$|E(G_2)| = 4$

degrees = 2, 2, 2, 2.

Adjacency of G_1

$$\begin{array}{c|c} & \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} \\ \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Adjacency of G_2

$$\begin{array}{c|c} & \begin{array}{cccc} v_1 & v_3 & v_2 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_3 \\ v_2 \\ v_4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$\therefore$ adjacency of G_1 = Adjacency of G_2

$\therefore$ Above graphs are isomorphic.

We can conclude that,

2 simple connected graphs with n vertices of all degree 2 are isomorphic.

...vertices of an undirected graph are each of degree k, show that the number of edges of the ...multiple of k. Hence illustrate the procedure by using a suitable concept map.

We know that,

No. of vertices of odd degree in an undirection graph is even.

Let it be $2n$.

Let the no. of edges be n_e.

The by handshaking theorem

$$\sum_{i=1}^{2n} \deg(v_i) = 2 n_e$$

$$\sum_{i=1}^{2n} k = 2n_e \quad (\text{or}) \quad 2nk = 2n_e$$

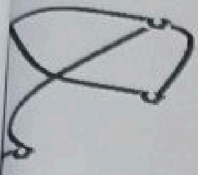
$$\cancel{2}nk = \cancel{2}n_e$$

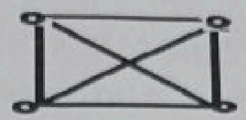
$$nk = n_e$$

$$n_e = nk$$

∴ No. of edges is a multiple of k.

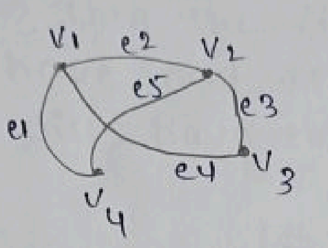
...nd the incidence matrix for the following graph. Hence illustrate the procedure by using a suitable ...pt map.





...ion:

Given graphs are

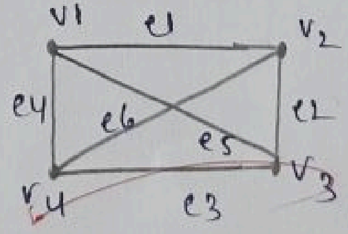


According to incidence matrix

$$B = [b_{ij}] = \begin{cases} 1 & \text{if } e \text{ incident on } v \\ 0 & \text{otherwise.} \end{cases}$$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

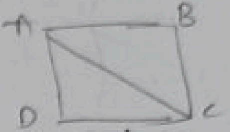
other graph :-



$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

...hat if a graph G has not more than two vertices of odd degree, then there can be Euler path in ... Illustrate the proof by using a suitable concept map.

proof:-



Let us assume that the graph G has an euler path but not a circuit. Everytime the path passes through a vertex, it contributes to the degree of vertex.

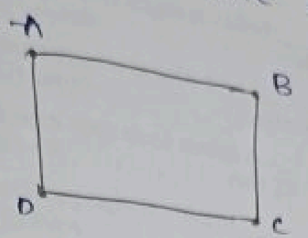
⇒ Then the first and the last vertices will have odd degree and all other vertices will have even degree.

Let us consides two vertices u and v has odd degree. If we connect these two vertices then every vertex will have even degree.

By theorem 1, there is an euler circuit in such a graph; if we remove the added edge {u,v} from the circuit, we will get euler path for original path.

...graph that is both Eulerian and Hamiltonian. Hence illustrate the procedure by using a ...cept map.

→ Let us consider the graph



A - B - C - D - A

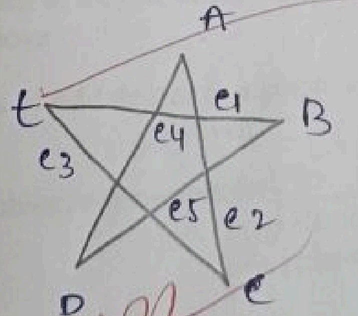
Graph 1

From the above graph,
the given graph is a eulerian because the graph contains all the edges exactly once.

The given graph is also a hamiltonian because the graph contains all the vertices exactly once.

∴ The above graph is both Eulerian and Hamiltonian.

⇒ Let us consider another graph:-



It consists of 5 edges and 5 vertices.

∴ each edge has taken only once.

∴ It consists of each vertex exactly once.

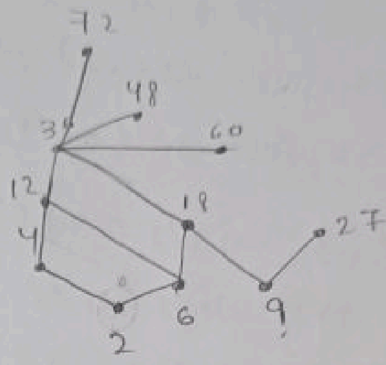
∴ It is a Eulerian and Hamiltanian.

the poset ({2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72},/).
i) Find all upper bounds of {2, 9}, ii) Find the least upper bound of {2, 9}, if it exists.
iii) Find all lower bounds of {60, 72}, iv) Find the greatest lower bound of {60, 72}, if it exists.
v) Find the least upper bound of {2, 27}, if it exists.
vi) Find the greatest lower bound of {60, 72}, if it exists. Hence illustrate the procedure by using concept map.

Given,

({2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72})

graph:-



i) upper bounds of {2, 9}

for 2 = 4, 6, 12, 18, 36, 48, 60, 72

for 9 = {18, 27, 36, 72}

for {2, 9}

= {18, 36, 72}

ii) Least upper bound of {2, 9} = {18}

iii) Lower bonds of {60, 72}

for 60 = {12, 4, 6, 2}

for 72 = {36, 12, 18, 4, 6, 9, 2}

iv) greatest lower bound of {60, 72} = {12}

(i) Show that least upper bound of a subset B in a poset $(A, \leq)$ is unique if it exists.
(ii) Show that every chain is a distributive lattice. Hence illustrate the procedure by a suitable map.

(i) Let $A = \{a_1, a_2\}$ Let u_1 and u_2 be the two Least upper bound of A.

By definition; $a_1 \leq u_1$ & $a_2 \leq u_1 \rightarrow (1)$

$a_1 \leq u_2$ & $a_2 \leq u_2 \rightarrow (2)$

from (1) and (2) $u_1 \neq u_2$ LUB

$u_1 \geq u_2$ & $u_2 \geq u_1 \Rightarrow u_1 = u_2$

$(A, \leq)$ is unique

ii) Let $(L, \leq)$ be a chain i.e., i) $a \leq b \leq c$ ii) $a \geq b \geq c$

To prove L is distributive

i.e, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Case 1:- $a \leq b \leq c$

LHS $= a \vee (b \wedge c) = a \vee b = b \rightarrow (1)$

RHS $= (a \vee b) \wedge (a \vee c) = b \wedge c = b \rightarrow (2)$

from (1) and (2) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

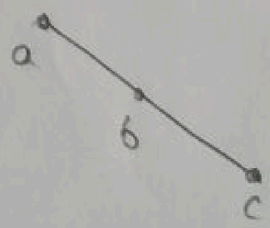
similarly $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Case 2:- $a \geq b \geq c$

LHS $= a \wedge (b \vee c) = a \wedge b = b \rightarrow (1)$

RHS $= (a \wedge b) \vee (a \wedge c) = (a \wedge b) \vee (a) = b \vee a$

$= b$,



$\therefore$ We can conclude that

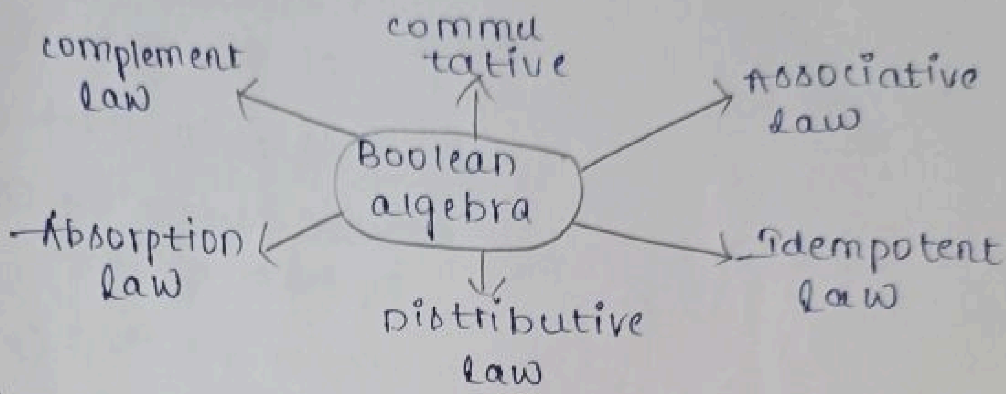
$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Boolean algebra and write the complement, associative, absorption, idempotent, distributive and
tive laws of Boolean algebra. Hence illustrate the concepts by a suitable map.

Boolean Algebra:-

A complemented distributive lattice is called boolean algebra.



Commutative law:-

$$A+B = B+A$$
$$A \cdot B = B \cdot A$$

Associative law:-

$$A+(B+C) = (A+B)+C$$
$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

Distributive law:-

$$A+(B \cdot C) = (A+B) \cdot (A+C)$$
$$A \cdot (B+C) = (A \cdot B) + (A \cdot C)$$

Idempotent law:-

$$A+A = A$$
$$A \cdot A = A$$

Absorption law:-

$$A+(A \cdot B) = A \qquad A \cdot (A+B) = A$$

Complement law:-

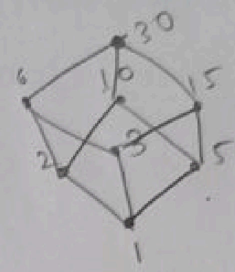
$$A \cdot \bar{A} = 0 \qquad A+\bar{A} = 1$$

D_{30}=1,2,3,5,6,10,15,30 with a relation $x \leq y$ if and only if x divides y. Find
(i) All lower bounds of 10 and 15, (ii) GLB of 10 and 15, (iii) All upper bounds of 10 and 15,
(iv) LUB of 10 and 15, (v) Draw the Hasse diagram of D_{30}. Hence illustrate the procedures by suitable maps.

tion:

Given,

$D_{30} = 1,2,3,5,6,10,15,30$



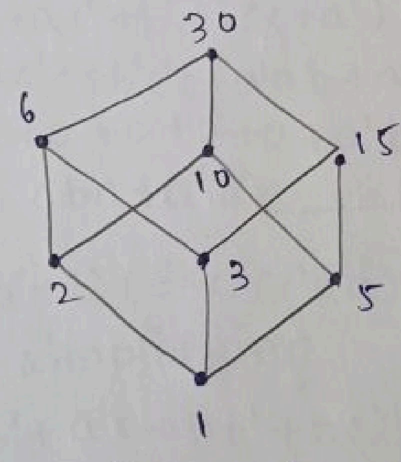
1) Lower bounds of 10 and 15 = 1,5

2) GLB of 10 & 15 = 5

3) All upper bounds of 10 and 15 = 30

4) LUB of 10 & 15 = 30

5) Hasse diagram :-



GLB of D_{30} is 1

LUB of D_{30} is 30.

(i) Simplify the Boolean expressions using Boolean algebra: $a'b(a'+c)+ab'(b'+c)$.

(ii) Show that algebraically in a Boolean algebra B: $(a+b')(b+c')(c+a')=(a'+b)(b'+c)(c'+a)$. Hence illustrate the procedures by suitable concept maps.

tion:

Given

(i) $a'b(a'+c)+ab'(b'+c)$

$a'a'b+a'bc+ab'b'+ab'c$ [distributive]

$a'b+a'bc+ab'+ab'c$ [a+a=1]

$(a'b+ab')+c(a'b+ab')$

$(a'b+ab')(c+1)$

$1\cdot(a'b+ab')$

$a \oplus b$.

ii) LHS $=(a+b')(b+c')(c+a')$

By simplifying

$(a+b')(b+c')(c+a')$

$(ab+ac'+bb'+b'c')(c+a')$

$(ab+ac'+b'c')(c+a')$

$abc+ac'c+b'c'c+a'ab+a'ac'+a'b'c'$

$abc+0+0+0+0+a'b'c$

$=abc+a'b'c \longrightarrow (1)$

RHS $=(a'+b)(b'+c)(c'+a)$

By simplifying

$(a'b'+a'c+bb'+bc)(c'+a)$

$a'b'c'+a'cc'+bb'c'+bcc'+aa'b+aa'c+abc$

$a'b'c'+0+0+0+0+0+abc$

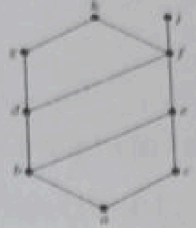
$a'b'c'+abc \longrightarrow (2)$

from eq (1) and (2)

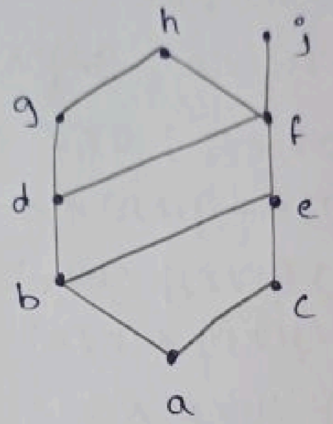
LHS = RHS

$\therefore$ Hence proved.

...e lower and upper bound of the subsets {a,b,c}, {j,h} and {a,c,d,f} in the poset with the Hasse
... Hence illustrate the procedure by a suitable concept map.



...tion: 1) {a, b, c} Given,



1) {a, b, c}

Lower bounds = {a}

upper bounds = {e, f, j, h}

2) {j, h}

Lower bounds = {a, b, c, d, e, f}

upper bounds = nil

3) {a, c, d, f}

Lower bounds = {a}

upper bounds = {f, h, j}

...that in a complemented and distributive lattice, the complement of each element is unique. Hence ... the procedure by a suitable concept map.

Let us assume,

x, y are two complements of a

Since x is a complement of a we have

$a \wedge x = 0, \; a \vee x = 1 \longrightarrow (1)$

Since y is a complement of a we have

$a \wedge y = 0, \; a \vee y = 1 \longrightarrow (2)$

Now, $x = x \vee 0 \; [\because x \vee 0 = x]$

$= x \vee (a \wedge y) \longrightarrow (1)$

$(x \vee a) \wedge (x \vee y)$ (distributive)

$(a \vee x) \wedge (x \vee y)$ (commutative)

$1 \wedge (x \vee y)$ [from (1)]

$x = x \vee y \longrightarrow (3)$

$y = y \vee 0$

$= y \vee (a \wedge x)$ [using (2)]

$= (y \vee a) \wedge (y \vee x)$ (distributive)

$= (a \vee y) \wedge (y \vee x)$ (commutative)

$= 1 \wedge (y \vee x)$ [using (2)]

$y = y \vee x \longrightarrow (4)$

from (3) and (4)

$x = x \vee y \quad y = y \vee x.$

$x = x \vee y = y$

$\boxed{x = y}$

$\therefore$ complement is unique

...e disjunctive normal form of the Boolean expression y'+[z'+x+(yz)'](z+x'y) by truth table and algebraic method. Hence illustrate the concept map by a suitable concept map.

Given,

$$y' + [z' + x + (yz)'](z + x'y)$$

By algebraic method:-

$$y' + (z' + x + (yz)'](z + x'y)$$

$$y' + [z' + x + (y') + z'](z + x'y)$$

$$y' + (x + y' + z')(z + x'y)$$

$$y' + (xz) + y'z + zz' + xx'y + x'yy' + x'yz'$$

$$y' + xz + y'z + 0 + 0 + 0 + x'yz'$$

$$y' + xz + zy' + x'yz'$$

$$y'(x + x')(z + z') + xz(y + y') + zy'(x + x') + x'yz'$$

$$(xy' + x'y')(z + z') + xyz + xy'z + xzy' + x'y'z + x'yz'$$

$$\underline{xy'z} + x'y'z' + xyz + \underline{xy'z} + x'y'z + x'yz' + xy'z' + x'y'z$$

$$\rightarrow xy'z + \underline{x'y'z} + xyz + \underline{x'y'z} + x'yz' + x'y'z' + xy'z'$$

$= PDNF.$

Truth table:-

x	y	z	x'	y'	z'	yz	(yz)'	x'y	A	B	A·B	y+A·B	Minterms
0	0	0	1	1	1	0	1	0	1	0	0	1	x'y'z'
0	0	1	1	1	0	0	~~0~~1	0	1	1	1	1	x'y'z
0	1	0	1	0	1	0	1	1	1	1	1	1	x'yz'
0	1	1	1	0	0	1	0	1	0	1	0	0	
1	0	0	0	1	1	0	1	0	1	0	0	1	xy'z'
1	0	1	0	1	0	0	1	0	1	1	1	1	xy'z
1	1	0	0	0	1	0	1	0	1	0	0	0	—
1	1	1	0	0	0	1	0	0	1	1	1	1	xyz

Minterms :- $(x'y'z)$, $(x'y'z)$, $(x'yz')$, $(xy'z')$, $(xy'z)$, (xyz)

PDNF $\longrightarrow (x'y'z') + (x'y'z) + (x'yz') + (xy'z') + (xy'z) + (xyz)$

...e conjunctive normal form of the Boolean expression F(x,y,z) = (x+z)y by truth table method ...raic method. Hence illustrate the procedure by a suitable concept map.

$F(x,y,z) = (x+z)y$

By truth table:-

x	y	z	x+z	(x+z)y	Max terms	
0	0	0	0	0	$x+y+z$	→ Max
0	0	1	1	0	$x+y+z'$	→ Max
0	1	0	0	0	$x+y'+z$	→ Max
0	1	1	1	1	$x+y'+z'$	
1	0	0	1	0	$x'+y+z$	→ Max
1	0	1	1	0	$x'+y+z'$	→ Max
1	1	0	1	1	$x'+y'+z$	
1	1	1	1	1	$x'+y'+z'$	

Maxterms are:-

$(x+y+z), (x+y+z'), (x+y'+z), (x'+y+z)$
$(x'+y+z')$

conjunctive normal form:-

$(x+y+z) \cdot (x+y+z') \cdot (x+y'+z) \cdot (x'+y+z) \cdot$
$(x'+y+z')$

Algebraic method

$[(x+z)+(yy')] \cdot y+(z \cdot z')+(x \cdot x')]$

$(x+y+z)(x+z+y') \cdot [x+y+z] \cdot [x+y+z']$

$[(x+z+(y \cdot y')] \cdot [y+(xx')+(zz')]$

the product of max-terms of the Boolean expression F(x,y,z) = x+y+z by truth table method and method. Hence illustrate the procedure by a suitable concept map.

Given,

$F(x,y,z) = x+y+z$

By truth table:-

x	y	z	x+y+z	Max term
0	0	0	0	$x+y+z$ → $x+y+z$
0	0	~~0~~ 1	1	–
0	1	0	1	–
0	1	1	1	–
1	0	0	1	–
1	0	1	1	–
1	1	0	1	–
1	1	1	1	–

$F(x,y,z) = x + y + z$

Algebraic method:-

$F(x,y) = x+y+z$

∴ product of max terms $= x+y+z$

...the sum of min-terms of the Boolean expression $F(x,y,z) = x+y'z$ by truth table method and ...method. Hence illustrate the procedure by a suitable concept map.

Given,

$F(x,y,z) = x+y'z$

x	y	z	y'	$y'z$	$x+y'z$	Minterm
0	0	0	1	0	0	–
0	0	1	1	1	1	$x'y'z$
0	1	0	0	0	0	–
0	1	1	0	0	0	–
1	0	0	1	0	1	$xy'z'$
1	0	1	1	1	1	$xy'z$
1	1	0	0	0	1	xyz'
1	1	1	0	0	1	xyz

Minterms $= x'y'z + xy'z' + xy'z + xyz' + xyz$

PDNF = sum of minterms.

<u>Algebraic method:-</u>

$F(x,y,z) = x+y'z$

$\Rightarrow x(y+y')(z+z') + y'z(x+x')$

$= (xy+xy')(z+z') + xy'z + x'y'z$

$= xyz + \underline{xy'}z + xyz' + xy'z' + \underline{xy'}z + x'y'z$

$= xyz + xy'z + xyz' + xy'z' + x'y'z$

PDNF = sum of minterms

∴ Hence proved.

and prove De Morgan's law in Boolean algebra. Hence illustrate the proof by a suitable concept

Demorgan's law:-

$(a \vee b)' = a' \wedge b'$

$(a \wedge b)' = a' \vee b'$

$(A+B)' = A' \cdot B'$

$(A \cdot B)' = A' + B'$

Here, we need to prove two propositions are complement to each other.

we know that,

sum of variable and its complement is 1 & product is 0

$(A+B)(A' \cdot B') = 0$

$(A+B) + (A' \cdot B') = 1$

case 1:-

$(A+B)(A' \cdot B') = 0$

$LHS = (A+B)(A' \cdot B')$

$= A(A' \cdot B') + B(A' \cdot B')$

$= (AA')(B') + A'(BB')$

$= 0 + 0$

$= 0$

$LHS = RHS$

case 2:-

$(A+B) + (A' \cdot B') = 1$

$LHS = (A+B) + (A' \cdot B')$

$= (A+B+A') \cdot (A+B+B')$

$= (B+1) \cdot (A+1)$

$= 1 \cdot 1$

$= 1$

$LHS = RHS$

$\therefore$ Hence demorgan's law is proved.

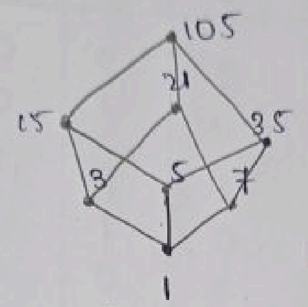
sider the lattice D_{105} with the partial ordered relation divides then
Draw the Hasse diagram of D_{105}, (ii) Find the complement of each elements of D_{105}. (iii) Find
of atoms of D_{105}, (iv) Find the number of sub algebras of D_{105}.
Hence illustrate the procedure by a suitable concept map.

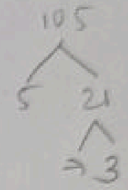
tion:

Given D_{105}

factors of 105 are 1, 3, 5, 7, 15, 21, 35, 105

i) Hasse diagram:-





ii) complement of each element:-

complement of 3 ⟶ 35
complement of 5 ⟶ 21
complement of 7 ⟶ 15
complement of 1 ⟶ 105
complement of 105 ⟶ 1

iii) Atoms of D_{105}:-

$A = \{3, 5, 7\}$

iv) No. of sub algebras:-

⇒ There can be only 1 two element sub algebra which consists of upper bound 105, lower bound

⇒ since D_{105} has 8 elements only 8 element subalgebra is D_{105}.

⇒ Any 4 sub elements subalgebra is of form $\{1, x, x', 105\}$ consists of upper and lower bounds and non bound elements and its complement.

the product of max-terms of the Boolean expression $F(x, y, z) = xyz$ by truth table method. Illustrate the procedure by a suitable concept map.

tion:

Given,

$F(x,y,z) = xyz$.

By truth table:-

x	y	z	xyz	Max terms
0	0	0	0	$x+y+z$
0	0	1	0	$x+y+z'$
0	1	0	0	$x+y'+z$
0	1	1	0	$x+y'+z'$
1	0	0	0	$x'+y+z$
1	0	1	0	$x'+y+z'$
1	1	0	0	$x'+y'+z$
1	1	1	1	—

Max terms:- $x+y+z$, $x+y+z'$, $x+y'+z$, $x+y'+z'$, $x'+y+z$, $x'+y+z'$, $x'+y'+z$

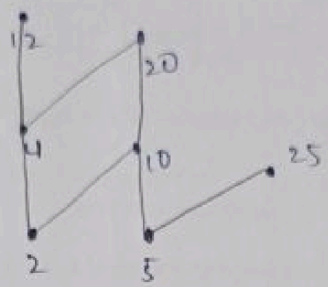
$\therefore$ product of max terms $= (x+y+z)\cdot(x+y+z')\cdot(x+y'+z)\cdot(x+y'+z')\cdot(x'+y+z)(x'+y+z')\cdot(x'+y'+z)$

Which elements of the poset $(\{2, 4, 5, 10, 12, 20, 25\}, /)$ are maximal and which are minimal?
In any distributive Lattice $(L, \wedge, \vee)$ $\forall a, b, c \in L$ prove that $a \vee b = a \vee c, a \wedge b = a \wedge c \Rightarrow b = c$.
ce illustrate the procedure by a suitable concept map.
tion:

i) Given $(\{2,4,5,10,12,20,25\})$

$R = \{(2,4)\ (2,10)\ (2,12)\ (2,20)\ (4,12)\ (4,20)\ (5,10),$
$(5,20)\ (5,25)\ (10,20)\}$

Hasse diagram:-



Max elements are 12, 20, 25
Min elements are 2, 5

ii) Given, $a \vee b = a \vee c, a \wedge b = a \wedge c \Rightarrow b = c$

consider,

$b = b \vee (b \wedge a)$ [absorption]
$= b \vee (a \wedge b)$ (commutative)
$= b \vee (a \wedge c)$ (condition)
$= (b \vee a) \wedge (b \vee c)$ (distributive)
$= (a \vee b) \wedge (c \vee b)$ (commutative)
$= (a \vee c) \wedge (c \vee b)$ [condition]
$= (c \vee a) \wedge (c \vee b)$ (commutative)
$= c \vee (a \wedge b)$ (distributive)
$= c \vee (a \wedge c)$ (condition)
$= c \vee (c \wedge a)$ (commutative)
$= c$ (absorption)

$\therefore b = c$

Hence proved.