LESSON PLAN

Lesson No /Session No	Topics	No. of Hours
	UNIT-I: Multivariate Calculus	
1	Taylor's series and Maclaurin's series expansion of one variable	1Hr
2	Newton-Raphson method derivation and geometrical interpretation	1Hr
3	Taylor's series and Maclaurin's series expansion of two variables	1Hr
4	Solution of system of non-linear equations using Newton-Raphson method	1Hr
5	Maxima and minima of functions of two variables	1Hr
6	Lagrange's method of undetermined multipliers	1Hr
	UNIT-II: First order Ordinary Differential Equations	
7	Exact Differential Equations. Equations reducible to exact.	1Hr
8	Applications of ODE's to solve LR, RC circuits & Newton's Law of cooling	1Hr
9	Orthogonal trajectories.	1Hr
10	Numerical solution of ODE's using Taylor's series method	1Hr
11	Euler's & modified Euler's method	1Hr
12	Runge- Kutta method of fourth order	1Hr
	UNIT-III: Higher Order Linear Ordinary Differential Equations	
13	Introduction to LDE with constant coefficients. Solution of homogeneous LDE with constant coefficients.	1Hr
14	Method of finding Particular integral for standard cases	1Hr
15	Problems continued on L14.	1Hr
16	Cauchy's and Legendre's linear ordinary differential equations	1Hr
17	Method of variation of parameters	1Hr
18	Initial value problems, Boundary value problems.	
	UNIT-IV: Beta and Gamma functions and Laplace transforms – I	
19	Beta and Gamma functions	1Hr
20	Introduction to Laplace transform. Laplace transform of standard functions.	1Hr
21	Properties of Laplace transform	1Hr
22	Problems continued on L21	1Hr
23	Evaluation of integrals using Laplace transform	1Hr
	UNIT-V: Laplace Transforms -II	
24	Laplace transform of Periodic function	1Hr
25	Laplace transform of Heaviside Unit step function, Dirac-Delta function.	1Hr
26	Inverse Laplace Transforms	1Hr
27	Convolution theorem – Proof, problems on verification and finding inverse LT.	1Hr
28	Solution of ODE's and system of ODE's using Laplace transform technique.	1Hr

UNIT – I: Multivariate Calculus

TAYLOR'S & MACLAURIN'S SERIES EXPANSION OF FUNCTIONS OF ONE VARIABLE AND IT'S APPLICATIONS	
Two & Four marks questions	
1.	Write Taylor's series for the function of one variable.
2.	Write Maclaurin's series for the function of one variable.
3.	Obtain the first four terms of the Taylor's series of $\cos x$ about $x=\frac{\pi}{3}$.
4.	Expand $\sin^{-1} x$ in powers of x up to second degree term.
5.	Expand a^x in powers of x up to first three terms.
6.	Using Maclaurin's series expand $\sqrt{1+\sin 2x}$ up to the term containing x^4.
7.	Define algebraic & transcendental equations with an example.
8.	Write Newton-Raphson iterative formula.
9.	Derive Newton-Raphson method to compute the real root of the equation $f(x)=0$.
10.	Give the geometrical interpretation of Newton-Raphson iteration formula.
11.	Using NR method, find a root between 0 and 1 of $x^3 = 6x - 4$.
Seven marks questions	
12.	Find the positive root of $x=\cos x$ using Newton's method.
13.	Expand $\tan^{-1}x$ in powers of $(x-1)$ up to the term containing x^4.
14.	Using Maclaurin's series expand $\log(\sec x)$ up to the term containing x^5.
15.	Expand $e^{a\sin^{-1}x}$ in ascending powers of x up to the term containing x^4.
16.	Obtain the Maclaurin's expansion of $e^x \cos x$ up to x^4.
17.	Expand $f(x)=x^5-x^4+x^3-x^2+x-1$ in powers of $(x-1)$ and find $f(0.99)$
18.	Expand $f(x)=\sqrt{1+x+2x^2}$ in powers of $(x-1)$.
19.	Expand $\log(1+\sin 2x)$ in powers of x up to the term containing x^4.
20.	Find a real root of the following equations correct to three decimal places using Newton-Raphson method. (i) $2x^3-3x-6=0$ (ii) $2x-\log_{10}x=7$ (iii) $3x-\cos x-1=0$
21.	Using Newton-Raphson method, find the real root of $x\log_{10}x=1.2$ correct to four decimals.
22.	Find a negative root of the equation $xe^x-\sin x=0$ using Newton-Raphson method correct to four decimals.

23.	Find a negative root of the equation $x^2+4\sin x=0$ using Newton–Raphson method correct to four decimals.
24.	The bacteria concentration in a reservoir varies as $c=4e^{-2t}+e^{-(0.1)t}$. Using Newton-Raphson method, calculate the time required for the bacteria concentration to be 0.5.
25.	The current i in an electric circuit is given by $i=10e^{-t}\sin 2\pi t$, where t is in seconds. Using Newton-Raphson method, find the value of t correct to three decimals for $i = 2$ amp.
TAYLOR'S & MACLAURIN'S SERIES EXPANSION OF FUNCTIONS OF TWO VARIABLES AND IT'S APPLICATIONS	
Two & Four marks questions	
26.	State Taylor's series for the function of two variables.
27.	State Maclaurin's series for the function of two variables.
28.	Explain the Newton-Raphson method to find the solution of system of non-linear simultaneous equations.
Seven marks questions	
29.	Expand the following functions at the given point up to second degree terms: (i) $xy^2+\cos(xy)$ about $\left(1, \frac{\pi}{2}\right)$ (ii) $x^2y+3y-2$ about (1, -2) (iii) x^y about (1, 1).
30.	Expand the following functions in powers of x and y up to second degree terms: (i) $\sin x \sin y$ (ii) $e^x \sin y$ (iii) $e^x \log(1+y)$.
31.	Solve the following system of non-linear equations using Newton-Raphson method (Carry out two iterations) (i) $x^2+y=11,\ y^2+x=7$, given that $x_0=3.5$ and $y_0=-1.8$. (ii) $x^2+y^2=x,\ x^2-y^2=y$, given that $x_0=0.8$ and $y_0=0.4$. (iii) $x^2+y^2=16,\ x^2-y^2=4$, given that $x_0=y_0=2.828$. (iv) $x=2(y+1),\ y^2=3xy-7$, given that $x_0=-1.8$ and $y_0=-1.9$.
MAXIMA & MINIMA OF FUNCTIONS OF TWO VARIABLES AND LAGRANGE'S METHOD OF UNDETERMINED MULTIPLIERS	
Two & Four marks questions	
32.	Define maxima and minima for the function of two variables.
33.	Define stationary point and saddle point.
34.	Write the steps involved in finding the extreme values of $f(x,y)$.
35.	Write the steps involved in the Lagrange's method of undetermined multipliers.
36.	Examine x^3+y^3-3axy for extreme values.
37.	Show that $f(x,y)=xy(1-x-y)$ is maximum at the point $(1/3, 1/3)$.
38.	Show that minimum value of $f(x,y)=xy+\frac{a^3}{x}+\frac{a^3}{y}$ is $3a^2$.

Seven marks questions	
39.	Discuss the maxima and minima of $x^3y^2(1-x-y)$.
40.	Find the extreme values of $f(x,y)=\sin x\sin y\sin(x+y);\ 0<x<\frac{\pi}{2}, 0<y<\frac{\pi}{2}$.
41.	Find the minimum and maximum values of $x^3+3xy^2-3x^2-3y^2+4=0$.
42.	Find the extreme values of $u=x^3+y^3-63(x+y)+12xy$.
43.	Find the maximum value of $x^m y^n z^p$, when $x+y+z=a$.
44.	Find the maximum and minimum distances of the point $(1, 2, 3)$ from the sphere $x^2+y^2+z^2=56$.
45.	The temperature T at any point (x,y,z) in space is $T=400\,xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$.
46.	A wire of length b is cut into two parts which are bent in the form of a square and circle respectively. Find the least value of the sum of the areas so found.
47.	Show that the volume of the greatest parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $\frac{8abc}{3\sqrt{3}}$.
48.	A rectangular box open at the top is to have volume of 108 cubic ft. Find the dimension of the box if its total surface area is minimum.

UNIT – II: First Order Ordinary Differential Equations

EXACT DIFFERENTIAL EQUATIONS AND APPLICATIONS OF FRIST ORDER FIRST DEGREE ODEs	
Two & Four marks questions	
1.	Define exact differential equations.
2.	State the condition for exactness of the differential equation $M(x,y)dx + N(x,y)dy = 0$ and also write its general solution.
3.	Define integrating factor with an example.
4.	Test the exactness of the equation $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$
5.	Test the exactness of the equation $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$
6.	Test the exactness of the equation $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xy\, e^{xy^2} - 3y^2\right)dy = 0$
7.	Solve exactness of the equation $\left(1 + 2xy\cos x^2 - 2xy\right)dx + \left(\sin x^2 - x^2\right)dy = 0$
8.	Obtain the differential equation of the closed circuit involving *R* and *C* along with a constant voltage source *E*.
9.	Form the differential equation of the closed circuit involving *L* and *C* both in series without applied e.m.f.
10.	Form the differential equation of the closed circuit involving *L* and *R* in series with applied e.m.f.
11.	Define orthogonal trajectories.
12.	Write the steps involved in finding the orthogonal trajectories of the curve in polar form.
13.	Write the steps involved in finding the orthogonal trajectories of the curve in Cartesian form.
14.	Define self-orthogonality of family of curves.
15.	The equation of an L-R circuit is given by $L\frac{di}{dt} + Ri = 10\sin t$. If i=0 at t = 0, then express i as a function of t.
16.	A coil having a resistance of 15 ohms and an inductance of 10 henrys is connected to 90-volt supply. Determine the value of current after 2 seconds.
17.	If a voltage of $20\cos 5t$ to a series circuit consisting of 10 ohm resister and 2 henry inductor, determine the current at any time t>0.
18.	Show that $r = b\sin\theta$ is the orthogonal trajectories of the family of curves $r = b\cos\theta$.
19.	If $\frac{dr}{d\theta} = r\cot\left(\frac{\theta}{2}\right)$ is the differential equation of the family of cardioids $r = a(1-\cos\theta)$, then find its orthogonal trajectory.

20.	If $\dfrac{xy}{a^2-x^2}+\dfrac{dy}{dx}=0$ is the differential equation of the family of curves $f(x,y,c)=0$, then find its orthogonal trajectory.
21.	If the stream lines of the flow in the channel are $\phi = xy = k$, then find the orthogonal trajectories of the stream lines.
22.	If $x^2+y^2-2a^2\log x = c$ is the orthogonal trajectory of given family of curves then find the differential equation of that family if a is a fixed constant.
23.	Test for self-orthogonality of $r^n = a\sin n\theta$ where a is the parameter.
24.	Solve the following differential equations:

a.	$(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$	d.	$y(y^2-2x^2)dx+x(2y^2-x^2)dy=0$
b.	$(y^3-3xy^2)dx+(2x^2y-xy^2)dy=0$	e.	$(x^4+y^4)dx-xy^3dy=0$
c.	$y^2dx+(x^2-xy-y^2)dy=0$	f.	$x^2ydx-(x^3+y^3)dy=0$

25.	Find the value of λ so that the differential equation $(xy^2+\lambda x^2y)dx+(x+y)x^2dy=0$ is exact. Solve the equation for this value of λ.
26.	A voltage Ee^{-at} is applied at $t=0$ to a circuit of inductance L and resistance R. Show that the current at time t is $\dfrac{E}{R-aL}\left(e^{-at}-e^{\frac{-Rt}{L}}\right)$ if initial current is zero.
27.	An emf $e=200e^{-5t}$ is applied to a series circuit consisting of 20 ohm resister and 0.01 F capacitor. Find the charge and current at any time assuming that there is no initial charge on capacitor.
28.	A circuit consisting of resistance R and a condenser of capacity C is connected in series with a voltage E. Assuming that there is no charge on condenser at t=0, find the value of current i, voltage and charge q at any time t.
29.	According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 30^0 C and the substance cools from 100^0 C to 70^0 C in 15 minutes, find when the temperature will be 40^0 C.
30.	Find the O.T of the family of curves $r=a(1+\sin\theta)$.
31.	Show that the family of curves $x^3-3xy^2=a$ & $y^3-3yx^2=b$ are O.T of each other.
32.	Given $y=ke^{-2x}+3x$, find member of the orthogonal trajectory passing through (0,3).
33.	Show that the family of parabolas $y^2=4a(x+a)$ is self-orthogonal.
34.	Show that the family of curves $r=a(\sin\theta+\cos\theta)$ and $r=b(\sin\theta-\cos\theta)$ intersect each other orthogonally.
35.	Show that the family of curves $\dfrac{x^2}{a^2+\lambda}+\dfrac{y^2}{b^2+\lambda}=1$ is self-orthogonal, where λ is the parameter.

Q Find the orthogonal trajectories of family of coaxial circles $x^2+y^2+2gx + c = 0$ where g is the parameter.

36.	In the electric field between two concentric cylinders the equipotential lines are circles given $u(x,y)=x^2+y^2=k$ then find the orthogonal trajectories of the curves of electric force.
37.	If the isotherms in a body are $T(x,y)=2x^2+y^2=k$, where k is constant, then find the orthogonal trajectories of the curves of constant temperature.
38.	Find the orthogonal trajectories of the family of the curves $r^n \sin n\theta = a^n$.
NUMERICAL SOLUTIONS OF ODEs	
Two & Four marks questions	
39.	Write Euler's formula to solve first order and first degree ODE.
40.	Write Euler's modified formula to solve first order and first degree ODE.
41.	Write any two disadvantages of Euler's method.
42.	Write Runge-Kutta formula of fourth order to solve first order and first degree ODE.
43.	Using Euler's method, solve $\frac{dy}{dx}-2y=3e^x;\ y(0)=0\ at\ x=0.2$ by taking h=0.1.
Seven marks questions	
44.	Using Taylor's series method solve the IVP $\frac{dy}{dt}=e^{-2t}-2y;\ y(0)=\frac{1}{10}\ at\ t=0.1\ and\ 0.2$ considering derivatives up to 4th degree.
45.	Using Tayler's series method, find the solution of $y'=xy^{1/3};y(1)=1$ upto the term containing x^4 and hence find $y(1.1)$
46.	Solve the following initial value problems by Euler's method. a) $\frac{dy}{dx}=-xy^2;\ y(2)=1$ at $x=2.4$ taking $h=0.1$ b) $\frac{dy}{dx}=\log_{10}(x+y);\ y(0)=1$ at $x=0.6$ taking $h=0.2$ c) $\frac{dy}{dx}-y^2=0;\quad y(0)=1$ at $x=0.1,\ 0.2,\ 0.3$
47.	Solve the following initial value problems using Modified Euler's method (Carry out three iterations at each stage) a) $\frac{dy}{dx}=\frac{-y}{1+x};\ y(3)=2$ at $x=3.4$ taking $h=0.2$ b) $\frac{dy}{dx}=x+\left\|\sqrt{y}\right\|;\quad y(0)=1$ at $x=0.4$ taking $h=0.2$
48.	Solve the following initial value problems using Runge-Kutta method of fourth order a) $\frac{dy}{dx}=-2xy^2;y(0)=1$ at $x=0.2$ taking $h=0.1$ b) $\frac{dy}{dx}=xy+y^2;y(0)=1$ at $x=0.4$ taking $h=0.2$

49.	Use Euler's modified method to solve $y' = 4e^{0.8x} - 0.5y$ for $x = 0.2(0.2)0.4$, given that $y(0) = 2$.
50.	Solve initial value problem $\frac{dy}{dx} = \frac{x^3 + xy^2}{e^x}$; $y(0) = 1$ at $x = 0.2$ and 0.4 using Runge - Kutta method of fourth order.
51.	Using Runge-Kutta method, find y for $0 < x \leq 0.4$ taking $h = 0.2$ if $10\frac{dy}{dx} = x^2 + y^2$; $y(0) = 1$.
52.	A bungee jumper with a mass of 68.1 Kgs leaps from a stationary hot air balloon. Use $\frac{dv}{dt} = g - \frac{cv^2}{m}$ where $g = 9.8\, m/s^2$, $c = 0.25\, Kg/m$ to compute velocity for the first five seconds of free fall by Euler's method in steps of 1 second. Also compare the result with the exact value.
53.	Solve the following initial value problems using Modified Euler's method $\frac{dy}{dx} = \sqrt{x^2 + y}$; $y(1) = 0.5$ at $x = 1.4$ by taking $h = 0.2$. Carry out three iterations at each stage.
54.	Solve the following initial value problems using Runge - Kutta method of fourth order $\frac{dy}{dx} = \log\left(\frac{y^x}{xy}\right)$; $y(1) = 1$ at $x = 1.4$ by taking $h = 0.2$

UNIT – III: Linear Differential Equations of Higher Order

HIGHER ORDER LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS	
Two & Four marks questions	
1.	Define Linear and non-linear differential equations with example.
2.	Define linearly independent solutions of a differential equation with an example
3.	Define homogeneous and non-homogeneous linear differential equations with constant coefficients.
4.	Write the complementary function of the fourth order LDE $f(D)y=0$, if the roots of the auxiliary equation are imaginary and repeated.
5.	The free oscillations of a galvanometer needle, as affected by the viscosity of the surrounding air which varies directly as the angular velocity of the needle, are determined by the equation $\theta''+k\theta'+\mu\theta=0$, where θ is the angular deflection of the needle at time t. Obtain θ in terms of t.
6.	Find the general solution of a homogeneous equation whose auxiliary equation is $\lambda^3(\lambda+4)^2(\lambda^2+2\lambda+5)^2=0$.
7.	If $k>0$, then show that the general solution of $y^{iv}-k^4y=0$ can be expressed as $y=C_1\cos kx+C_2\sin kx+C_3\cosh kx+C_4\sinh kx$.
8.	Solve the following differential equations: (i) $(D^2-6D+9)y=0$, (ii) $y'''-8y'+8y=0$, (iii) $4y'''+4y''+y'=0$, (iv) $\dfrac{d^4y}{dx^4}+4y=0$.
9.	What is an initial value problem?
10.	What is a boundary value problem?
11.	Solve the following initial value problems and boundary value problems i) $y''-y=0,\ y(0)=3,\ y'(0)=-3$; (ii) $y''+y=0,\ y(0)=2,\ y\left(\frac{\pi}{2}\right)=-2$; (iii) $y''-9y=0,\ y(0)=2,\ y\left(\frac{1}{3}\right)=\frac{2}{e}$.
12.	If $D=\frac{d}{dx}$ and $X=X(x)$, then prove that $\frac{1}{D}X=\int X(x)dx$.
13.	If $D=\frac{d}{dx}$ and $X=X(x)$, then prove that $\frac{1}{D+a}X=e^{-ax}\int Xe^{ax}dx$.
14.	If $D=\frac{d}{dx}$ and $X=X(x)$, then prove that $\frac{1}{D-a}X=e^{ax}\int Xe^{-ax}dx$.
15.	Explain the working rule to find the P.I. of the LDE $f(D)y=X$, when $X=e^{ax}$.
16.	Explain the working rule to find the P.I. of the LDE $f(D)y=X$, when $X=\sin(ax+b)$ or $\cos(ax+b)$.

17.	Explain the working rule to find the P.I. of the LDE $f(D)y = X$, when X is a polynomial function of degree n.
18.	Explain the working rule to find the P.I. of the LDE $f(D)y = X$, when $X = e^{ax}V(x)$ and $V(x)$ is any function of x.
19.	Explain the working rule to find the P.I. of the LDE $f(D)y = X$, when $X = xV(x)$ and $V(x)$ is any function of x.
20.	Find the P.I of the following differential equations: (i) $(D^2-2D)y = x^2$, (ii) $(D^2+4)y = \sin 2x$ (iii) $(D^2-6D+5)y = e^x$. (iv) $(D^3+3D^2+3D+1)y = 2e^{-x}$

Seven marks questions

21.	Solve the following differential equations:			
	a) $(D^3-3D^2+3D-1)y = e^x + 2e^{-x} + 4$		b)	$y''-4y'+7y = e^x \cosh 2x + 2^x$
	c) $(D^3-1)y = (e^x+1)^2$		d)	$(D^3+2D^2-D-2)y = 2\cosh x$
	e) $(D^3-3D^2+3D-1)y = \sinh(x+2)$		f)	$(D^3-6D^2+2D+36)y = \cosh(x-1)$
22.	Solve the following differential equations:			
	a) $(D^2-4D+3)y = \sin 3x.\cos 2x$		b)	$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y = \frac{\sin^2 x}{4}$
	c) $(D^3+D^2-D)y = 2\cos^2 x$		d)	$y''+9y = \cos 2x.\cos x$
	e) $(D^2-D)y = \cos^3 x$		f)	$(D^2+1)y = \sin(3x).\sin(4x)$
23.	Solve the following differential equations:			
	a) $y'''-2y''+y = x^4+2x+5$		b)	$y''+y'+3 = x^2+2x$
	c) $(D^2-2D+1)y = x^2-3x$		d)	$(3D^2+3D-1)y = x+3$
24.	Solve the following differential equations:			
	a) $y'''+4y''+3y'-4 = e^{-x}+\sin 2x$		b)	$(D^3+2D^2+D)y = x^2e^{2x}+\sin^2 x$
	c) $(D^2+1)y = x\cos x$		d)	$(D^2-2D+1)y = xe^x \sin x$
	e) $(D^2-4)y = 8xe^x$		f)	$(D^2+4D+5)y = x^2e^{2x}$
	g) $(D^2-2D+4)y = e^x \cos x$		h)	$(D^3-7D-6)y = e^{2x}(1+x)$
	i) $(D^2-4D+4)y = 8x^2e^{2x}\sin 2x$		j)	
25.	Solve the following IVP/BVP: a) $y''-2y'+y = x$, $y(0)=0, y(1)=3$ b) $y''+4y'+4y=0$, $y(1)=0, y'(0)=-1$ c) $(D^2+D)y = 2+2x+x^2$, $y(0)=8, y'(0)=-1$ d) $y'''-y''+100y'-100y=0, y(0)=4, y'(0)=11, y''(0)=-299$.			

HIGHER ORDER LINEAR DIFFERENTIAL EQUATIONS WITH VARIABLE COEFFICIENTS AND METHOD OF VARIATION OF PARAMETERS	
Two & Four marks questions	
26.	Write a second order general Legendre's linear differential equation.
27.	Write the steps involved in solving Cauchy's LDE.
28.	Write a second order general Cauchy's linear differential equation.
29.	Write the steps involved in solving Legendre's LDE.
30.	Reduce the following into a linear differential equation with constant coefficients: a) $x^2y''+9xy'+25y=0$, b) $(x+1)^2y''+2(x+1)-y=0$ $(2x+3)^2y''+2(2x+3)-12y=0.$
31.	Solve the following linear differential equations: a) $(x^2D^2+7xD+9)y=0$, b) $(1-2x)^2y''-2(1-2x)y'=0$, c) $xy''+4y'=0$. d) $(x+1)^2y''+2(x+1)y'-12y=0$, e) $4x^2y''-4xy'+3y=0.$
32.	Write the steps involved in solving the LDE by the method of variation of parameters.
Seven marks questions	
33.	Solve the following differential equations: a) $x^2y''+xy'+y=\log x\sin(\log x)$ b) $x^2y''-xy'+2y=x\sin(\log x)$ c) $x^2y''-4xy'+6y=\cos[2\log x]$ d) $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\log x}{x^2}$ e) $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$ f) $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+y=(\log x)^2$ g) $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x\log x$ h) $(2x+1)^2y''-2(2x+1)y'-12y=6x+5$ i) $(3x+2)^2y''+3(3x+2)y'-36y=8x^2+4x+1$
34.	Solve the following differential equations by the method of variation of parameters: a) $y''+y=\operatorname{cosec}x$ b) $y''+a^2y=\tan ax$ c) $y''+a^2y=\sec ax$ d) $y''-2y'+y=\frac{e^x}{x}$ e) $y''-2y'+2y=e^x\tan x$ f) $y''+3y'+2y=e^{e^x}$ g) $y''-y=\frac{2}{1+e^x}$ h) $y''-3y'+2y=\cos(e^{-x})$ i) $x^2y''+xy'-y=x^2e^x$ j) $y''-6y'+9y=\frac{e^{3x}}{x^2}$ k) $y''+y=\frac{1}{1+\sin x}$

UNIT – IV: Beta & Gamma Functions and Laplace Transform-I

	BETA & GAMMA FUNCTIONS
Two & Four marks questions	
1.	Define beta function and hence write the trigonometric form of beta function.
2.	Write the relation between beta and gamma function.
3.	Define Gamma function. Find the value of $\Gamma(-3.5)$.
4.	Prove that $\Gamma(n+1) = n\Gamma(n)$.
5.	Evaluate a) $\Gamma\left(\frac{-1}{2}\right)$ b) $\beta\left(\frac{1}{3},\frac{2}{3}\right)$ c) $\Gamma(3.5)$ d) $\Gamma(-2.5)$
6.	Prove that $\int_0^1 x^m \left[\log\left(\frac{1}{x}\right)\right]^n dx = \frac{\Gamma(n+1)}{(m+1)^{n+1}}$.
7.	Evaluate the following: a) $\int_0^\infty x^3 e^{-x^2}\, dx$ b) $\int_0^\infty e^{-ax^{1/n}}\, dx$. c) $\int_0^{\pi/2} \sin^{1/2}\theta \cos^{3/2}\theta\, d\theta$. d) $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$.
Seven marks questions	
8.	Show that $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}} dx \times \int_0^\infty e^{-x^4} x^2\, dx = \frac{\pi}{4\sqrt{2}}$.
9.	Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$.
	LAPLACE TRANSFORM OF ELEMENTARY FUNCTIONS AND PROPERTIES
Two & Four marks questions	
10.	Define Laplace transform of a function.
11.	State the sufficient conditions for the existence of the Laplace transform of a function $f(t)$
12.	Prove that $L\{a\} = \frac{a}{s}$.
13.	Obtain the Laplace transform of (i) e^{at} (ii) $\sin at$ (iii) $\cos at$ (iv) $\sinh at$ (v) $\cosh at$ (vi) t^n
14.	Write shifting property of Laplace transform.
15.	Write the formula of (i) $L\left\{\frac{f(t)}{t}\right\}$ (ii) $L\left\{\int_0^t f(t)\, dt\right\}$.
16.	Write the formula for: (i) $L\{f'(t)\}$ (ii) $L\{f''(t)\}$ (iii) $L\{f'''(t)\}$.
17.	If $L\{f(t)\} = F(s)$, then prove that $L(e^{at}f(t)) = F(s-a)$.

18.	If $L\{f(t)\} = F(s)$, then prove that $L\{\sinh at\, f(t)\} = \frac{1}{2}[F(s-a) - F(s+a)]$
19.	If $L\{f(t)\} = F(s)$, then prove that $L\{\cosh at\, f(t)\} = \frac{1}{2}[F(s-a) + F(s+a)]$
20.	If $L\{f(t)\} = F(s)$, then prove that $L\{t\, f(t)\} = -\frac{d}{ds}(F(s))$.
21.	If $L\{f(t)\} = F(s)$, then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\,ds$.
22.	If $L\{f(t)\} = F(s)$, then prove that $L\left\{\int_0^t f(t)dt\right\} = \frac{F(s)}{s}$.
23.	If $L\{f(t)\} = F(s)$, then show that $L\{f'(t)\} = sF(s) - f(0)$.
24.	If $L\{f'(t)\} = sF(s) - f(0)$, then prove that $L\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$
25.	Find Laplace transform of the following: a) $L\{(t^2+4)^3\}$ b) $L\{e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t\}$ c) $L\{\sin \omega t - \omega t \cos \omega t\}$ d) $L\{1 + 2\sqrt{t} + 3/\sqrt{t}\}$ e) $L\{\sinh 2t \sin 4t\}$ f) $\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^3$.
26.	Obtain the Laplace transform of i) $e^{-3t}\cos 5t \sin 5t$ ii) $L\left\{t^4 e^{-\frac{3}{2}t}\right\}$
27.	Given $L\{Sin\sqrt{t}\} = \frac{\sqrt{\pi}}{4s^{3/2}}e^{-1/4s}$, then find $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$.
28.	Find i) $L\{te^{-4t}\sin 3t\}$ ii) $L\{t^2 e^{-2t}\sin 3t\}$
Seven marks questions	
29.	If $L\{f(t)\} = F(s)$, then prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\{F(s)\}$, where n is a positive integer.
30.	If $f(t)$ and its $(n-1)$ derivatives are continuous then prove that $L\{f^{(n)}(t)\} = s^n L\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots\ldots\ldots\ldots - f^{n-1}(0)$.
31.	Find i) $L\left\{\frac{\cos at - \cos bt}{t}\right\}$ ii) $L\left\{\frac{\sin t \sin 3t}{t}\right\}$ iii) $L\left\{\frac{t - \sinh at}{t}\right\}$
32.	Find i) $L\left\{\int_0^t te^{-t}\cos t\,dt\right\}$ ii) $L\left\{t\int_0^t \frac{e^{-t}\sin t}{t}dt\right\}$ iii) $L\left\{e^t \int_0^t \frac{\sin t}{t}dt\right\}$

33.	Evaluate $\int_0^\infty t^2 \cos t \, dt$ using Laplace transform.
34.	Evaluate $\int_0^\infty e^{-2t} t^2 \sin t \, dt$ using Laplace transform.
35.	Evaluate i) $\int_0^\infty e^{6t} t \cos 2t \, dt$ ii) $\int_0^\infty e^{-3t} t \sin t dt$

UNIT – V: Laplace Transforms-II

LAPLACE TRANSFORMS OF PERIODIC FUNCTION

Two & Four marks questions

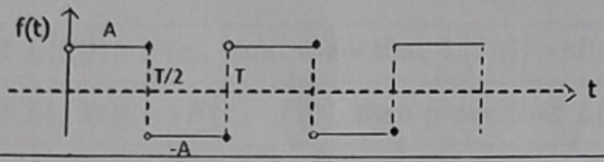
1. Define a periodic function with an example.

2. Draw the graph of the following periodic functions:

a) $f(t)=\begin{cases} t, & 0\le t\le a \\ 2a-t, & a\le t\le 2a \end{cases}$ b) $f(t)=\begin{cases} \sin\omega t, & 0<t<\pi/\omega \\ 0, & \pi/\omega<t<2\pi/\omega \end{cases}$ c) $f(t)=\begin{cases} 1 & where\ 0\le t\le 1 \\ -1 & where\ 1\le t\le 2 \end{cases}$

3. If $f(t)$ is a periodic function of period T then prove $L\{f(t)\}=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt$.

4. Obtain the Laplace transform of the rectangular wave $f(t)$ given by following figure:



5. Draw the graph of the periodic function $f(t)=\begin{cases} t, & 0<t<\pi \\ \pi-t, & \pi<t<2\pi \end{cases}$ and find its Laplace transform.

Seven marks questions

6. A periodic function of period $2\pi/\omega$ is defined by $f(t)=\begin{cases} E\sin(\omega t) & for\ 0<t<\pi/\omega \\ 0 & for\ \pi/\omega<t<2\pi/\omega \end{cases}$

where E and ω are positive constants, then show that $L\{f(t)\}=\frac{E\omega}{(s^2+\omega^2)(1-e^{-\pi s/\omega})}$.

7. A periodic function of period $2a$ is defined by $f(t)=\begin{cases} E & for\ 0\le t\le a \\ -E & for\ a\le t\le 2a \end{cases}$, find $L\{f(t)\}$.

8. Find the Laplace transform function of period 2π, given by $f(t)=\begin{cases} t & for\ 0<t<\pi \\ \pi-t & for\ \pi<t<2\pi \end{cases}$.

9. Find the Laplace transform of saw-tooth wave function of period T, given by $f(t)=\frac{t}{T}$ for $0<t<T$.

10. Find the Laplace transform of the triangular wave function of period $2a$, given by $f(t)=\begin{cases} t & for\ 0\le t\le a \\ 2a-t & for\ a\le t\le 2a \end{cases}$.

11. Find the Laplace transform of square wave function of period a defined by $f(t)=\begin{cases} 1 & for\ 0<t<a/2 \\ -1 & for\ a/2<t<a \end{cases}$.

UNIT-STEP & UNIT-IMPULSE FUNCTIONS	
Two & Four marks questions	
12.	Define unit step function and represent graphically
13.	Define the Dirac-Delta function and sketch its graph
14.	Write Laplace Transform of Heaviside's function $H(t-a)$
15.	Write Laplace Transform of Dirac-Delta function and find $L\{\delta(t)\}$
16.	Prove that $L\{H(t-a)\} = \frac{e^{-as}}{s}$.
17.	Prove that $L\{f(t)\delta(t-a)\} = e^{-as} f(a)$.
18.	Prove that $L\{f(t-a)H(t-a)\} = e^{-as} F(s)$.
19.	Find the Laplace transform of $(t^2+1)H(t-1)$
20.	Find the Laplace transform of $\cos t\, u(t-\pi)$
21.	Find $L\{(t^2-8t+16)\, e^{-(t-4)}\, u(t-4)\}$
22.	Find $L\{\, e^{-t}\sin t\; u(t-\pi)\}$
23.	Find the Laplace transform of the following function: (a) $(t^2-5t+3)u(t+3)$ (b) $(\sin t+\cos t)\, u(t-\pi)$ (c) $(t^2+3t+5)\, u(t+2)$
24.	Express the following functions in terms of Heaviside function (a) $f(t)=\begin{cases} A(t), & a<t<b \\ B(t), & t>b \end{cases}$ (b) $f(t)=\begin{cases} A(t), & a<t<b \\ B(t), & c<t<d \\ C(t), & t>d \end{cases}$
Seven marks questions	
25.	Express the following function in terms of the Heaviside function and hence find its Laplace transform: a) $f(t)=\begin{cases} 5+t, & 0<t<2, \\ t^2+2t-2, & t>2. \end{cases}$ b) $f(t)=\begin{cases} \sin t, & 0<t<\pi/2, \\ \cos t, & t>\pi/2. \end{cases}$ c) $f(t)=\begin{cases} t^2, & 0<t<2, \\ t-1, & 2<t<4, \\ 7, & t>4. \end{cases}$ d) $f(t)=\begin{cases} \sin t, & 0<t<\pi, \\ \sin 2t, & \pi<t<2\pi, \\ \sin 3t, & t>2\pi. \end{cases}$
26.	Express the function $f(t)=\begin{cases} t-1, & 1<t<2, \\ 3-t, & 2<t<3 \end{cases}$ in terms of the Heaviside function and hence find its Laplace transform
27.	Express the function $f(t)=\begin{cases} 2, & 0<t\le 1, \\ 0.5t^2, & 1<t\le\pi \\ \cos t, & t>\pi \end{cases}$ in terms of the Heaviside function and hence find its Laplace transform.

28.	Express the function $f(t)=\begin{cases}\cos t, & 0<t\le\pi,\\ \cos 2t, & \pi<t\le 2\pi\\ \cos 3t, & t>2\pi\end{cases}$ in terms of the Heaviside function and hence find its Laplace transform
29.	Express the function $f(t)=\begin{cases}t, & 0<t<2,\\ t-1, & 2<t<3,\\ 7, & t>3.\end{cases}$ in terms of the Heaviside function and hence find its Laplace transform.
	INVERSE LAPLACE TRANSFORM
Two & Four marks questions	
30.	Define inverse Laplace transform
31.	Write the formula of (a) $L^{-1}\left\{\frac{F(s)}{s}\right\}$ (b) $L^{-1}\{k\}$
32.	Define convolution of two functions.
33.	State convolution theorem.
34.	Find the inverse Laplace transform of the following: a) $\frac{1}{s\sqrt{s}}+\frac{3}{s^2\sqrt{s}}-\frac{8}{\sqrt{s}}$ b) $\frac{s}{(s-2)^5}$ c) $\frac{3s+1}{(s+1)^4}$ d) $\frac{s^3+6s^2+12s+8}{s^6}$ e) $\frac{4s+12}{s^2+8s+16}$ f) $\frac{se^{-s/2}+\pi e^{-s}}{s^2+\pi^2}$ g) $\frac{1+e^{-3s}}{s^2+2s+1}$ h) $\frac{\cosh 2s}{e^{3s}s^2}$ i) $\frac{s}{(s+3)^2+4}$ j) $\frac{se^{-as}+1}{s^2+\omega^2}$ k) $\frac{e^{3s}}{(s+25)^2}$ l) $\frac{e^{-3s}}{(s-4)^2}$ m) $\frac{s}{(s-4)^2+5}$ n) $\frac{se^{-2s}}{s^2+\omega^2}$ o) $\frac{s+8}{s^2-6s+13}$
35.	Show that convolution of two functions is commutative.
Seven marks questions	
36.	Evaluate the following: a) $L^{-1}\left\{\log\left(\frac{s^2+b^2}{s^2+a^2}\right)\right\}$ b) $L^{-1}\left\{\cot^{-1}\left(\frac{s}{a}\right)\right\}$ c) $L^{-1}\left\{\tan^{-1}(2/s)\right\}$ d) $L^{-1}\left\{\frac{e^{-s}(s+2)}{(s+1)^2}\right\}$
37.	Find the inverse Laplace transform by the method of partial fraction: a) $\frac{5s+3}{(s-1)(s^2+2s+5)}$ b) $\frac{s}{(s+1)(s+2)(s+3)}$ c) $\frac{s+4}{s(s^2+4)(s-1)}$ d) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$
38.	State and prove convolution theorem.

e Heaviside function and

isiside function and hence

d) $\frac{s^3+6s^2+12s+8}{s^6}$

) $\frac{\cosh 2s}{e^{3s}s^2}$

$\frac{e^{-3s}}{(s-4)^2}$

$n^{-1}(2/s)\}$

$\frac{\ -4}{)(s-1)}$

39.	Verify convolution theorem for the following pair of functions: a) $f_1(t)=t$ and $f_2(t)=te^{-t}$ b) $f_1(t)=\cos t$ and $f_2(t)=e^{-t}$ c) $\phi(t)=\cos at$ and $\psi(t)=\cos bt$ d) $f_1(t)=\sin 2t$ and $f_2(t)=e^{-t}$
40.	Using convolution theorem find the inverse Laplace transform of the following: a) $\frac{s^2}{(s^2+9)(s^2+16)}$ b) $\frac{1}{(s^2+4)(s+1)^2}$ c) $\frac{s^2}{(s^2+a^2)^2}$ d) $\frac{s}{(s+2)(s^2+9)}$
	SOLUTION OF DIFFERENTIAL EQUATIONS BY LAPLACE TRANSFORMS
Two & Four marks questions	
41.	Explain the procedure of solving initial value problem using Laplace transforms.
42.	Explain the procedure of solving simultaneous differential equations using Laplace transforms.
43.	Solve the following using Laplace transform: a) $y''-y=0;\ y(0)=3,\ y'(0)=-3$ b) $y'-5y=e^{5t};\ y(0)=0$ c) $y'-4y=1;\ y(0)=1$
Seven marks questions	
44.	Solve the following differential equation by the method of Laplace transform: a) $\frac{dy}{dt}+y=\cos 2t,\ y(0)=1$ b) $x''+9x=\cos t$; given $x(0)=1,\ x'(0)=-1$. c) $y''-6y'+9y=1$ given that $y(0)=0,\ y'(0)=0$. d) $y''+4y'+8y=1$ given that $y(0)=0,\ y'(0)=1$. e) $y'''+2y''-y'-2y=0$ given $y(0)=0,\ y'(0)=0, y''(0)=6$ f) $y''+y=H(t-1)$; given $y(0)=0,\ y'(0)=1$.
45.	Using Laplace transform, solve the following simultaneous differential equations: a) $\frac{dx}{dt}-2y=\cos 2t;\ \frac{dy}{dt}+2x=\sin 2t$, given $x(0)=1,\ y(0)=0$. b) $\frac{dx}{dt}-y=e^t;\ \frac{dy}{dt}+x=\sin t$, given $x(0)=1,\ y(0)=0$. c) $\frac{dx}{dt}+5x-2y=t,\quad \frac{dy}{dt}+2x+y=0$ given $x(0)=0, y(0)=0$.
46.	A voltage Ee^{-at} is applied at $t=0$ to a circuit of inductance L and resistance R. Show that the current at time t is $\frac{E}{R-aL}\left(e^{-at}-e^{-Rt/L}\right)$ using Laplace transforms
47.	The coordinates (x,y) of a particle moving along a plane curve at any point t, are given by $y'+2x=\sin 2t;\ x'-2y=\cos 2t\ ;t>0$. If at $t=0,\ x=1$ and $y=0$. show by using Laplace transforms that the particle moves along the curve $4x^2+4xy+5y^2=4$.

48.	Using Laplace transform, solve the simultaneous differential equations $\frac{dx}{dt}=2x-3y;\ \frac{dy}{dt}=y-2x$ given $x(0)=8, y(0)=3$.
49.	The currents i_1 and i_2 in a mesh are given by the differential equations $i_1'-\omega i_2=a\cos pt$ $i_2'-\omega i_1=a\sin pt$. Find the currents i_1 and i_2 by Laplace transforms, if $i_1=i_2=0$ at $t=0$
50.	Solve the differential equation $y''+4y=f(t)$; $y(0)=y'(0)=0$, $f(t)=\begin{cases}0 & \text{If } t<3\\ t & \text{if } t\geq 3\end{cases}$ by the method of Laplace transform.
51.	An alternating emf $E\sin\omega t$ is applied to an inductance L and a capacitance C in series. Show by transform method, that the current in the circuit is $\frac{E\omega}{(p^2-\omega^2)L}(\cos\omega t-\cos pt)$, *where* $p^2=1/LC$.
52.	Solve the initial value problem for a damper mass-spring system acted upon by a sinusoidal force for some interval governed by $y''+2y'+2y=10\sin 2t$, if $0<t<\pi$ given that $y(0)=1,\ y'(0)=-5$
53.	Obtain the equation for the forced oscillation of a mass m attached to the lower end of an elastic spring whose upper end is fixed and whose stiffness is k, when the driving force is $F_0\sin at$. Using the Laplace Transform method, solve this equation when $a^2\neq k/m$, given that initial velocity and displacement (from the equilibrium position) are zero. (Hint: $\frac{d^2x}{dt^2}+\frac{k}{m}x=\frac{F_0}{m}\sin at,\ x=0$ *and* $\frac{dx}{dt}=0$ *when* $t=0$)

Tutorials

Tutorial-1

1.	Prove that $\log_e x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \ldots\ldots$ and hence evaluate $\log_e(1.1)$.
2.	Obtain the Maclaurin's expansion of $e^x \sin x$ up to x^5.
3.	Expand $\tan^{-1}(1+x)$ as far as the term containing x^3.
4.	Find a negative root of $\cos\left(\frac{\pi(x+1)}{8}\right) + 0.148x - 0.9062 = 0$ using Newton-Raphson method correct to 4 decimal places.
5.	Find a real root of the equation $xe^x - \cos x = 0$ using Newton-Raphson method correct to four decimal places.

Tutorial-2

1.	Expand $\tan^{-1}\left(\frac{y}{x}\right)$ about the point $(1, 1)$ up to second degree terms.
2.	Expand the function $x^2y + xy^2$ in powers of $(x-1)$ and $(y+3)$ up to second degree terms.
3.	Expand the function $e^{x^2-y^2}$ in powers of x and y up to second degree terms.
4.	Show that $\sin x \cos y = x - \frac{1}{6}(x^3 + 3xy^2)$ using Maclaurin's series.
5.	Solve the system of non-linear equations $2x^2 + 3xy + y^2 = 3,\ 4x^2 + 2xy + y^2 = 30$ using Newton-Raphson method, given that $x_0 = -3$ and $y_0 = 2$. Carry out two iterations.

Tutorial-3

1.	Find the minimum and maximum values of $2(x^2 - y^2) - x^4 + y^4$.
2.	Examine the function $xy + \frac{a^3}{x} + \frac{a^3}{y}$ for extreme values.
3.	Find the minimum value of x^2yz^3 subject to $2x + y + 3z = a$.
4.	Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.
5.	In a plane triangle ABC, find the maximum value of $CosA\, CosB\, CosC$.

Tutorial-4

1.	Test for exactness and solve $(2xy\cos x^2 - 2xy + 1)dx + (\sin x^2 - x^2 + 3)dy = 0$
2.	The temperature of a cup of coffee is 92^0 C, when freshly poured the room temperature being 24^0 C. In one minute it was cooled to 80^0 C. How long a period must elapse, before the temperature of the cup becomes 65^0 C?
3.	A resistance of 100 ohms, an inductance of 0.5 henry is connected in series with a battery of 20 volts. Find the current in a circuit as a function of t.

4.	Solve $\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx+\{x+\log x-x\sin y\}dy$

Tutorial-5

1.	Find the orthogonal trajectories of the family of co-axial circles $x^2+y^2+2gx+c=0$, where g is a parameter.
2.	The electric lines of force of two charges of the same strength at $(\pm 1,\ 0)$ are the circles through these points of the form $x^2+(y-c)^2=1+c^2$. Show that their equipotential lines(O.T's) are the circles of the form $(x+k)^2+y^2=k^2-1$.
3.	Find the O.T of the family of astroids $x^{2/3}+y^{2/3}=a^{2/3}$.
4.	Solve the initial value problem $\frac{dy}{dx}=1-2xy$; $y(0)=0$ at $x=0.2$ at $x=0.2$ by Taylor's series method by considering terms up to fourth degree.
5.	Using Euler's method solve $\frac{dy}{dx}-2y=3e^x$; $y(0)=0$ at $x=0.2$ by taking $h=0.1$

Tutorial-6

1.	Solve the initial value problem $\frac{dy}{dx}=\frac{2y}{x}+x^3$; $y(1)=0.5$ at $x=1.4$ by taking $h=0.2$ using Modified Euler's method.(Carry out three iterations at each stage)
2.	Using Euler's method solve $\frac{dy}{dx}=x-y^2$; $y(0)=1$ at $x=0.4$ by taking $h=0.2$
3.	Solve the initial value problem $\frac{dy}{dx}=\sin(x+y)$; $y(0)=1$ at $x=0.2$ by taking $h=0.1$ using Modified Euler's method.(Carry out three iterations at each stage)
4.	Solve the initial value problems using Runge - Kutta method of fourth order $\frac{dy}{dx}=y-x^2$; $y(0.6)=1.7379$ at $x=0.8$ by taking $h=0.1$
5.	Solve the initial value problems using Runge - Kutta method of fourth order $\frac{dy}{dx}=\sin(x+y)$; $y(0)=1$ at $x=0.2$ by taking $h=0.1$

Tutorial-7

1.	Solve: (i) $\frac{d^4y}{dx^4}+4y=0$ (ii) $(D^2+4)y=\sin 2x$
2.	Solve: $(D^3-1)y=(e^x+1)^2$
3.	Solve: $(D^2-4D+3)y=\sin 3x.\cos 2x$
4.	Solve: $y'''+2y''+y'=e^{-x}+\sin 2x$
5.	Solve: $(D^3+2D^2+D)y=x^2e^{2x}+\sin^2 x$

Tutorial-8

1.	Solve: $x^4\frac{d^3y}{dx^3}+2x^3\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=\sin(\log x)$.
2.	Solve: $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log x)$.

3.	Solve: $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$
4.	Solve: $(x+1)^2y''+2(x+1)y'-12y=0,$
5.	Solve: $(2x-1)^2y''+(2x-1)y'-2y=8x^2-2x+3$.

Tutorial-9

1.	Solve: $y''-9y=0,\ y(0)=2,\ y\left(\frac{1}{3}\right)=\frac{2}{e}$.
2.	A particle moves along the x-axis according to the law $\frac{d^2x}{dt^2}+6\frac{dx}{dt}+25x=0$.If the particle is started at $x=0$ with an initial velocity of 12ft/sec to the left, determine x in terms of t.
3.	Solve the boundary value problem $y''+4y'+4y=8x^2$, given that $y(0)=1$ and $y(1)=1$.
4.	Solve the initial value problem $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=e^{-x}$ subject to the conditions $y(0)=1=y'(0)$.
5.	Solve: $y''-y=\frac{2}{1+e^x}$ by the method of variation of parameters.
6.	Solve: $y''+a^2y=\sec ax$ by the method of variation of parameters.

Tutorial-10

1.	Evaluate $\int_0^1\frac{x^2}{\sqrt{1-x^4}}dx\times\int_0^1\frac{dx}{\sqrt{1+x^4}}dx$
2.	Evaluate a) $\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta$. b) $\int_0^\infty x^5e^{-x^2}\,dx$
3.	Show that $\int_0^\infty e^{-a^2x^2}\,dx=\frac{\sqrt{\pi}}{2a}$
4.	Prove that $\int_a^b\frac{1}{(b-x)^{1-\alpha}(x-a)^\alpha}dx=\frac{\pi}{\sin\alpha\pi}$.; $0<\alpha<1$
5.	Find the Laplace transform of the functions a) t^3-3t^2+2 b) $(t^5+1)^2$ c) $sin^2 3t$ d) $3t^4+4e^{-2t}-2\,sin5t$ e) $cos^2 4t$ f) $\sin 6t\sin 2t$ g) $sin^2 t\cos t$

Tutorial-11

1.	Find $L(sin\sqrt{t})$ and $L(cos\sqrt{t})$
2.	Find i) $L\{t^2\sin 7t\}$ ii) $L\{t\sin^2 3t\}$ iii) $L\{t\sin 3t\cos 2t\}$
3.	Find a) $L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\}$ b) $L\left\{\int_0^t e^{-t}\cos t\,dt\right\}$

4.	Evaluate $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}dt$ using Laplace transform.

Tutorial-12

1.	Find the Laplace transform of saw-tooth wave function of period T, given by $f(t)=\frac{t}{T}$ for $0<t<T$.
2.	Express the function $f(t)=\begin{cases}\sin t, & 0<t<\pi,\\ \sin 2t, & \pi<t<2\pi,\\ \sin 3t, & t>2\pi.\end{cases}$ in terms of the Heaviside function and hence find its Laplace transform
3.	Prove that the following results: a) $L^{-1}\left\{\frac{1}{s}\cos\frac{1}{s}\right\}=1-\frac{t^2}{(2!)^2}+\frac{t^4}{(4!)^2}-......$ b) $L^{-1}\left\{\frac{1}{s}\sin\frac{1}{s}\right\}=\frac{t}{1!}-\frac{t^3}{(3!)^2}+\frac{t^5}{(5!)^2}-.......$
4.	Find (a) $L^{-1}\left\{s\log\left(\frac{s+4}{s-2}\right)\right\}$ (b) $L^{-1}\left\{\log\left(\frac{s^2+1}{(s-1)^2}\right)\right\}$

Tutorial-13

1.	Find the inverse Laplace transform the following by the method of partial fractions a) $\frac{s+2}{(s^2+4s+5)^2}$ b) $\frac{s}{(s+1)^2(s^2+1)}$
2.	Using convolution theorem find inverse Laplace transform the following a) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ b) $\frac{s^2}{(s^2+16)^2}$
3.	Verify convolution theorem for the pair of functions, $f(t)=e^{-2t}$ and $g(t)=\sin 3t$

Tutorial-14

1.	Solve the differential equation $y''+4y'+3y=e^t$; given $y(0)=0, y'(0)=2$ by the method of Laplace transform.
2.	Using Laplace transform method , solve the following simultaneous equations $\frac{dx}{dt}=2x-3y$; $\frac{dy}{dt}=y-2x$ given $x(0)=8, y(0)=3$
3.	Consider a beam of length L units, resting on two simple supports at the ends. Let x denote the position on the beam, and let $y(x)$ denote the deflection of the beam in the vertical direction satisfies the Euler-Bernoulli equation $\frac{d^4y}{dx^4}=-\delta\left(x-\frac{L}{3}\right)$. Determine the deflection of the beam with the initial conditions $y(0)=0=y'(0)$, $y''(0)=-\frac{1}{4}$ and $y'''(0)=\frac{1}{2}$.
4.	The current i and charge q in a series circuit containing an inductance L, capacitance C, e.m.f E, satisfy the differential equation $L\frac{di}{dt}+Ri=E$; $i=\frac{dq}{dt}$. Express i and q in terms of t given that L, C, E are constants and the value of i, q are both zero initially.

RAMAIAH Institute of Technology

Department of **MATHEMATICS**

DEPARTMENT OF MATHEMATICS

CIE MODEL QUESTION PAPER

Sub Code: MAE21 **Sub:** Differential equations and Laplace Transforms **Test:** 01

Semester: II **Term:** 01.06.2023 to 09.09.2023 **Marks:** 30

Note: Answer any TWO full questions. Each main question carries 15 marks

Q.No.		Questions	Blooms Level	CO's	Marks
1.	(a)	State Maclaurin's series for the function of one variable.	L1	CO1	2
	(b)	Test the exactness of the differential equation $\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$	L2	CO2	3
	(c)	Find a negative root of the equation $xe^x - \sin x = 0$ using Newton-Raphson method correct to four decimals.	L3	CO1	5
	(d)	Solve the initial value problem by Euler's modified method $\frac{dy}{dx} = \frac{y-x}{y+x}$; $y(0) = 1$ at $x = 0.2$ by taking $h = 0.1$	L5	CO2	5
2.	(a)	Write the steps involved in the Lagrange's method of undetermined multipliers.	L1	CO1	2
	(b)	Expand the function e^{x+y} in powers of x and y up to second degree terms	L2	CO1	3
	(c)	Solve the initial value problem $\frac{dy}{dx} = \frac{2y}{x} + x^3$; $y(1) = 0.5$ at $x = 1.4$ by taking $h = 0.2$ using Runge - Kutta method of fourth order	L3	CO2	5
	(d)	The charge q on the plate of condenser of capacity C charged through a resistance R by a steady voltage E satisfies differential equation $R\frac{dq}{dt} + \frac{q}{c} = E$. If $q = 0$ at $t = 0$ then show that $q = CE\left(1 - e^{\frac{-t}{RC}}\right)$. Find the current flowing into the plate.	L4	CO2	5
3.	(a)	Form the differential equation of the closed circuit involving L and C both in series without applied e.m.f.	L1	CO2	2
	(b)	Show that $f(x,y) = xy(1-x-y)$ is maximum at the point $(1/3, 1/3)$.	L2	CO1	3
	(c)	Show that the family of parabolas $y^2 = 4a(x+a)$ is self-orthogonal.	L3	CO2	5
	(d)	Solve the system of non-linear equations $x^2 + y^2 = 16$, $x^2 - y^2 = 4$, given that $x_0 = y_0 = 2.828$. using Newton-Raphson method.	L4	CO1	5

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DEPARTMENT OF MATHEMATICS
CIE MODEL QUESTION PAPER

Sub Code: MAE21 **Sub:** Differential equations and Laplace Transforms **Test:** 02

Semester: II **Term:** 01.06.2023 to 09.09.2023 **Marks:** 30

Note: Answer any TWO full questions. Each main question carries 15 marks

Q.No.		Questions	Blooms Level	CO's	Marks
1.	(a)	What is an initial value problem?	L1	CO3	2
	(b)	Prove that $\Gamma(n+1)=n\Gamma(n)$.	L2	CO4	3
	(c)	Solve the following differential equation: $(D^3-3D^2+3D-1)y=e^x+2e^{-x}+4$	L3	CO3	5
	(d)	Evaluate $\int_0^\infty t^2 \cos t\, dt$ using Laplace transform.	L4	CO4	5
2.	(a)	If $L\{f(t)\}=F(s)$, then show that $L\{f'(t)\}=sF(s)-f(0)$.	L1	CO4	2
	(b)	Draw the graph of the periodic function $f(t)=\begin{cases} t, & 0<t<\pi \\ \pi-t, & \pi<t<2\pi \end{cases}$ and find its Laplace transform.	L2	CO5	3
	(c)	Given $L\{Sin\sqrt{t}\}=\frac{\sqrt{\pi}}{4s^{3/2}}e^{-1/4s}$, then find $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$.	L3	CO4	5
	(d)	Solve the following BVP: $y''-2y'+y=x$, $y(0)=0, y(1)=3$	L5	CO3	5
3.	(a)	Define a periodic function with an example.	L1	CO5	2
	(b)	Find the general solution of a homogeneous equation whose auxiliary equation is $\lambda^3(\lambda+4)^2(\lambda^2+2\lambda+5)^2=0$.	L2	CO3	3
	(c)	Solve the following differential equation $x^2y''+xy'+y=\log x\sin(\log x)$	L3	CO3	5
	(d)	Show that $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}dx \times \int_0^\infty e^{-x^4}x^2\,dx=\frac{\pi}{4\sqrt{2}}$.	L4	CO4	5

RAMAIAH Institute of Technology

Department of
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RAMAIAH INSTITUTE OF TECHNOLOGY
(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)
BANGALORE-560054

SEE MODEL QUESTION PAPER

Course & Branch:	B.E., Electrical & Electronics Engineering Stream	**Semester:**	II
Subject:	Differential Equations and Laplace transforms	**Max.Marks:**	100
Subject Code:	MAE21	**Duration:**	3Hrs

Instructions to the candidates: Answer ONE full question from each unit.

		UNIT – I		
1.	a.	Can you find a solution for the equation $xe^{-x}=0$ near $x=2$ using Newton-Raphson method. Justify your answer.	CO1	2
	b.	Expand a^x in powers of x up to first three terms.	CO1	4
	c.	Solve the following system of non-linear equations using Newton-Raphson method (Carry out two iterations). $x^2+y=11,\ y^2+x=7$, given that $x_0=3.5$ and $y_0=-1.8$	CO1	7
	d.	Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third may be maximum.	CO1	7
2.	a.	Explain the Newton-Raphson method to find the solution of system of non-linear simultaneous equations.	CO1	2
	b.	Show that $f(x,y)=xy(1-x-y)$ is maximum at the point $(1/3, 1/3)$.	CO1	4
	c.	The current i in an electric circuit is given by $i=10e^{-t}\sin 2\pi t$ where t is in seconds. Using Newton-Raphson method, find the value of t correct to three decimals for i = 2 amp.	CO1	7
	d.	Expand the function $xy^2+\cos(xy)$ about $\left(1, \frac{\pi}{2}\right)$ up to second degree terms.	CO1	7
		UNIT – II		
3.	a.	Define integrating factor with an example.	CO2	2
	b.	Solve $\dfrac{dy}{dx}+\dfrac{x+3y-4}{3x+9y-2}=0$	CO2	4
	c.	Using Taylor's series method solve the IVP $\frac{dy}{dt}=e^{-2t}-2y$; $y(0)=\frac{1}{10}$ at $t=0.1$ *and* 0.2 considering derivatives up to 4th degree.	CO2	7
	d.	If the isotherms in a body are $T(x,y)=2x^2+y^2=constant$ then find the orthogonal trajectories of the curves of constant temperature.	CO2	7
4.	a.	Using Euler's method solve $\frac{dy}{dx}-2y=3e^x$; $y(0)=0$ at $x=0.2$ by taking $h=0.1$.	CO2	2
	b.	If $\frac{dr}{d\theta}=r\cot\left(\frac{\theta}{2}\right)$ is the differential equation of the family of cardioids	CO2	4

		$r = a(1-\cos\theta)$ then find its orthogonal trajectory.		
	c.	Use Euler's modified method to solve $y' = 4e^{0.8x} - 0.5y$ for $x = 0.2(0.2)0.4$, given that $y(0) = 2$.	CO2	7
	d.	Solve $\frac{dy}{dx} = \frac{x^3+xy^2}{e^x}$; $y(0) = 1$ at $x = 0.2$ *and* 0.4 using R-K method of fourth order.	CO2	7
		UNIT – III		
5.	a.	Write the steps involved in solving Cauchy's LDE.	CO3	2
	b.	If $D = \frac{d}{dx}$ and $X = X(x)$, then prove that $\frac{1}{D+a}X = e^{-ax}\int Xe^{ax}dx$.	CO3	4
	c.	Solve $(D^2 - 2D + 1)y = xe^x \sin x$.	CO3	7
	d.	Solve $y'' - 3y' + 2y = \cos(e^{-x})$ by the method of variation of parameter.	CO3	7
6.	a.	Write the steps involved in solving the LDE by the method of variation of parameters.	CO3	2
	b.	Find the general solution of a homogeneous equation whose auxiliary equation is $\lambda^3(\lambda+4)^2(\lambda^2+2\lambda+5)^2 = 0$.	CO3	4
	c.	Solve $(x+1)^2 y'' + (x+1)y' + y = 4\cos[\log(x+1)]$.	CO3	7
	d.	Solve $y''' - y'' + 100y' - 100y = 0, y(0) = 4, y'(0) = 11, y''(0) = -299$.	CO3	7
		UNIT – IV		
7.	a.	Define Laplace transform of a function.	CO4	2
	b.	Evaluate $\Gamma\left(\frac{-1}{2}\right)$	CO4	4
	c.	Evaluate $\int_0^\infty e^{-2t}t^2 \sin t\, dt$ using Laplace transform.	CO4	7
	d.	Given $L\{Sin\sqrt{t}\} = \frac{\sqrt{\pi}}{4s^{3/2}}e^{-1/4s}$, then find $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$.	CO4	7
8.	a.	Define beta function and hence write the trigonometric form of beta function.	CO4	2
	b.	Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$	CO4	4
	c.	Show that $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}dx \times \int_0^\infty e^{-x^4}x^2\,dx = \frac{\pi}{4\sqrt{2}}$.	CO4	7
	d.	Evaluate $L\left\{\int_0^t \frac{e^{-t}\sin t}{t}dt\right\}$ using Laplace transforms.	CO4	7

UNIT – V

	a.	State convolution theorem.	CO5	2
	b.	Find the inverse Laplace transform of $\frac{1}{s\sqrt{s}}+\frac{3}{s^2\sqrt{s}}-\frac{8}{\sqrt{s}}$	CO5	4
	c.	Draw the graph of the periodic function $f(t)=\begin{cases} t, & 0<t<\pi \\ \pi-t, & \pi<t<2\pi \end{cases}$ and find its Laplace transform.	CO5	7
	d.	Solve using Laplace transforms $y''-6y'+9y=1$ given that $y(0)=0,\ y'(0)=0$.	CO5	7
10	a.	Define the Dirac-Delta function and sketch its graph	CO5	2
	b.	Express the function in terms of the Heaviside function and hence find its Laplace transform $f(t)=\begin{cases} 5+t, & 0<t<2, \\ t^2+2t-2, & t>2. \end{cases}$	CO5	4
	c.	A voltage Ee^{-at} is applied at $t=0$ to a circuit of inductance L and resistance R. Show that the current at time t is $\frac{E}{R-aL}\left(e^{-at}-e^{-Rt/L}\right)$ using Laplace transforms	CO5	7
	d.	Using Laplace transform, solve the simultaneous differential equation $\frac{dx}{dt}-2y=\cos 2t;\ \frac{dy}{dt}+2x=\sin 2t$, given $x(0)=1,\ y(0)=0$.	CO5	7

Note: **<u>Students shall not be under the impression that questions from model question paper will appear in SEE.</u>**

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