Assignment 4

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***Abstract*—This document explains the concept of computing the determinant of a matrix given.**

Download all python codes from

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https://github.com/harshachinta/EE5609−Matrix−Theory/tree/master/Assignments/Assignment4/code
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and latex-tikz codes from

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https://github.com/harshachinta/EE5609−Matrix−Theory/tree/master/Assignments/Assignment4
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1 Problem

If a, b and c are real numbers, and $\triangle = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0,$ Show that either $a+b+c=0$ or $a=b=c$.

2 Explanation

Given,

$$\triangle = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} \tag{2.0.1}$$

$$\xleftrightarrow{C_1 \leftarrow C_1+C_2+C_3} \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} \tag{2.0.2}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_1 \leftarrow R_1-R_2; R_2 \leftarrow R_2-R_3} 2(a+b+c) \begin{vmatrix} 0 & c-b & a-c \\ 0 & a-c & b-a \\ 1 & b+c & c+a \end{vmatrix} = 0 \tag{2.0.4}$$

On expanding determinant along first column from equation (2.0.4),

$$2(a+b+c)[(c-b)(b-a)-(a-c)^2] = 0$$

$$\implies 2(a+b+c)(a^2+b^2+c^2 - ab - bc - ca) = 0$$

$$\implies (a+b+c)(2a^2+2b^2+2c^2 - 2ab - 2bc - 2ca) = 0$$

$$\implies (a+b+c) [(a-b)^2+(b-c)^2+(c-a)^2] = 0 \tag{2.0.5}$$

From equation (2.0.5) we get 2 equations,

$$\implies \boxed{(a+b+c) = 0} \tag{2.0.6}$$

or,

$$\implies (a-b)^2+(b-c)^2+(c-a)^2 = 0 \tag{2.0.7}$$

Equation (2.0.7) is possible only when, $a = b = c$

$$\implies \boxed{a = b = c} \tag{2.0.8}$$

From equation (2.0.6) and (2.0.8) we can say that, $\triangle = 0$ if $a+b+c=0$ or $a=b=c$.

3 Solution

From equation (2.0.6) and (2.0.8) we can say that, $\triangle = 0$ if $a+b+c=0$ or $a=b=c$.