

Assignment 6

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Abstract—This document explains the concept of finding the unknown value in an equation such that it is represented as two straight lines . and

Download latex-tikz codes from

<https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment6>

1 PROBLEM

Find the value of h so that the equation $6x^2 + 2hxy + 12y^2 + 22x + 31y + 20 = 0$ may represent two straight lines.

2 EXPLANATION

Given equation,

$$6x^2 + 2hxy + 12y^2 + 22x + 31y + 20 = 0 \quad (2.0.1)$$

is a second order equation.

The general equation of second degree is given by

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.2)$$

and can be expressed as

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.3)$$

where

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (2.0.4)$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \quad (2.0.5)$$

Equation (2.0.3) represents a pair of straight lines if

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \quad (2.0.6)$$

Comparing equation (2.0.1) with (2.0.2), we can write in the form of (2.0.4) and (2.0.5) as,

$$\mathbf{V} = \begin{pmatrix} 6 & h \\ h & 12 \end{pmatrix} \quad (2.0.7)$$

$$\mathbf{u} = \begin{pmatrix} 11 \\ \frac{31}{2} \end{pmatrix} \quad (2.0.8)$$

$$f = 20 \quad (2.0.9)$$

As given in question, the equation represent two straight lines, substitute (2.0.7), (2.0.8), (2.0.9) in (2.0.6) to satisfy the equation.

$$\begin{vmatrix} 6 & h & 11 \\ h & 12 & \frac{31}{2} \\ 11 & \frac{31}{2} & 20 \end{vmatrix} = 0 \quad (2.0.10)$$

Expanding equation (2.0.10) along row 1 gives

$$\begin{aligned} \Rightarrow 6 \times (240 - \frac{961}{4}) - h \times (20h - \frac{341}{2}) + \\ 11 \times (\frac{31h}{2} - 132) = 0 \end{aligned}$$

$$\Rightarrow 20h^2 - 341h + \frac{2907}{2} = 0 \quad (2.0.11)$$

$$\Rightarrow h = \frac{17}{2} \quad (2.0.12)$$

$$\Rightarrow h = \frac{171}{20} \quad (2.0.13)$$

3 SOLUTION

If $h = \frac{17}{2}$ or $h = \frac{171}{20}$, the equation given will represent two straight lines.

Sub $h = \frac{17}{2}$ in equation (2.0.1) we get,

$$6x^2 + 17xy + 12y^2 + 22x + 31y + 20 = 0 \quad (3.0.1)$$

Rearranging equation (3.0.1),

$$\begin{aligned} \Rightarrow 6x^2 + 8xy + 10x + 9xy + 12y^2 + 15y + \\ 12x + 16y + 20 = 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow 2x(3x + 4y + 5) + 3y(3x + 4y + 5) \\ + 4(3x + 4y + 5) = 0 \end{aligned}$$

$$\Rightarrow (2x + 3y + 4)(3x + 4y + 5) = 0 \quad (3.0.2)$$

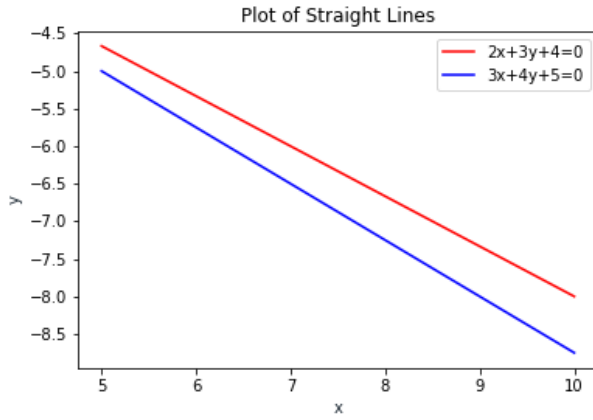


Fig. 1: Plot of Straight lines when $h = \frac{17}{2}$

Similarly, Sub $h = \frac{171}{20}$ in equation (2.0.1) we get,

$$20x^2 + 57xy + 40y^2 + \frac{220}{3}x + \frac{310}{3}y + \frac{200}{3} = 0 \quad (3.0.3)$$

Rearranging equation (3.0.3),

$$\begin{aligned} \Rightarrow & 20x^2 + 32xy + 40x + 25xy + 40y^2 + 50y + \\ & \frac{100}{3}x + \frac{160}{3}y + \frac{200}{3} = 0 \\ \Rightarrow & 4x(5x + 8y + 10) + 5y(5x + 8y + 10) \\ & + \frac{20}{3}(5x + 8y + 10) = 0 \\ \Rightarrow & \boxed{(4x + 5y + \frac{20}{3})(5x + 8y + 10) = 0} \quad (3.0.4) \end{aligned}$$

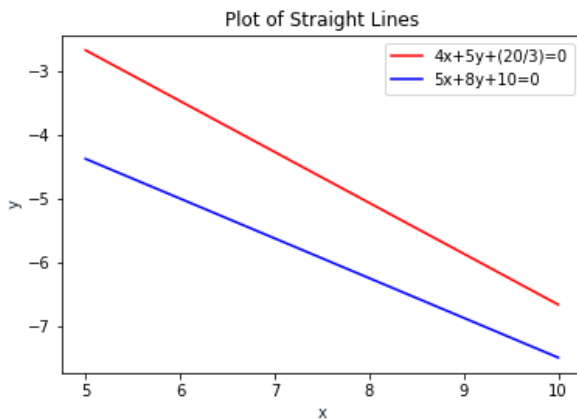


Fig. 2: Plot of Straight lines when $h = \frac{171}{20}$