Assignment 12

Sri Harsha CH

***Abstract*—This document explains the conditions of a matrix when it is invertible and not invertible.**

Download all python codes from

```
https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment12
    /code
```

and latex-tikz codes from

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https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment12
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1 Problem

Let $\mathbf{A}$ be an $n \times n$ (square) matrix, Prove the following two statements:

1) If $\mathbf{A}$ is invertible and $\mathbf{AB} = 0$ for some $n \times n$ matrix $\mathbf{B}$, then $\mathbf{B} = 0$.
2) If $\mathbf{A}$ is not invertible, then there exists an $n \times n$ matrix $\mathbf{B}$ such that $\mathbf{AB} = 0$ but $\mathbf{B} \neq 0$.

2 Explanation

1) If $\mathbf{A}$ is invertible and $\mathbf{AB} = 0$ for some $n \times n$ matrix $\mathbf{B}$, then $\mathbf{B} = 0$.

Given $\mathbf{A}$ is an invertible matrix and $\mathbf{AB} = 0$ then,

$$\mathbf{AB} = 0 \tag{2.0.1}$$
$$\implies \mathbf{A}^{-1}(\mathbf{AB}) = 0 \tag{2.0.2}$$
$$\implies (\mathbf{A}^{-1}\mathbf{A})\mathbf{B} = 0 \tag{2.0.3}$$
$$\implies \mathbf{IB} = 0 \quad [\because \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}] \tag{2.0.4}$$
$$\implies \mathbf{B} = 0 \tag{2.0.5}$$

2) If $\mathbf{A}$ is not invertible, then there exists an $n \times n$ matrix $\mathbf{B}$ such that $\mathbf{AB} = 0$ but $\mathbf{B} \neq 0$.

Since $\mathbf{A}$ is not invertible, $\mathbf{AX} = 0$ must have a non-trivial solution. Let the non-trivial solution be,

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \tag{2.0.6}$$

Let $\mathbf{B}$ which is an $n \times n$ matrix have all its columns as $\mathbf{y}$.

$$\mathbf{B} = \begin{pmatrix} \mathbf{y} & \mathbf{y} & \cdots & \mathbf{y} \end{pmatrix} \tag{2.0.7}$$

From equation (2.0.7), we can say that $\mathbf{B} \neq 0$ but $\mathbf{AB} = 0$

3 Solution

Equations (2.0.5) and (2.0.7) proves the problem.