Assignment 7

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Abstract—**This document explains the method of finding the equation of circle.**

1 PROBLEM

Find the equation of the circle passing through $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and making intercepts a and b on the coordinate axes.

2 EXPLANATION

The equation of a circle can be expressed as,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

where **c** is the center.
Given the circle makes intercepts a and b on the coordinate axis. Let intercept on x-axis be a and the intercept on y-axis be b.
Therefore the points are,

$$\mathbf{P_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{P_2} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad \mathbf{P_3} = \begin{pmatrix} 0 \\ b \end{pmatrix} \tag{2.0.2}$$

Substituting $\mathbf{P_1}$ from equation (2.0.2) in (3.0.1) we get,

$$\begin{pmatrix} 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} - 2\begin{pmatrix} c1 & c2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} + f = 0 \tag{2.0.3}$$

$$\implies \boxed{f = 0} \tag{2.0.4}$$

Substituting $\mathbf{P_2}$ from equation (2.0.2) in (3.0.1) we get,

$$\begin{pmatrix} a & 0 \end{pmatrix}\begin{pmatrix} a \\ 0 \end{pmatrix} - 2\begin{pmatrix} c1 & c2 \end{pmatrix}\begin{pmatrix} a \\ 0 \end{pmatrix} + 0 = 0 \tag{2.0.5}$$

$$a^2 - 2(ac_1) = 0 \tag{2.0.6}$$

$$\implies \boxed{c_1 = \frac{a}{2}} \tag{2.0.7}$$

Substituting $\mathbf{P_3}$ from equation (2.0.2) in (3.0.1) we get,

$$\begin{pmatrix} 0 & b \end{pmatrix}\begin{pmatrix} 0 \\ b \end{pmatrix} - 2\begin{pmatrix} c1 & c2 \end{pmatrix}\begin{pmatrix} 0 \\ b \end{pmatrix} + 0 = 0 \tag{2.0.8}$$

$$b^2 - 2(bc_2) = 0 \tag{2.0.9}$$
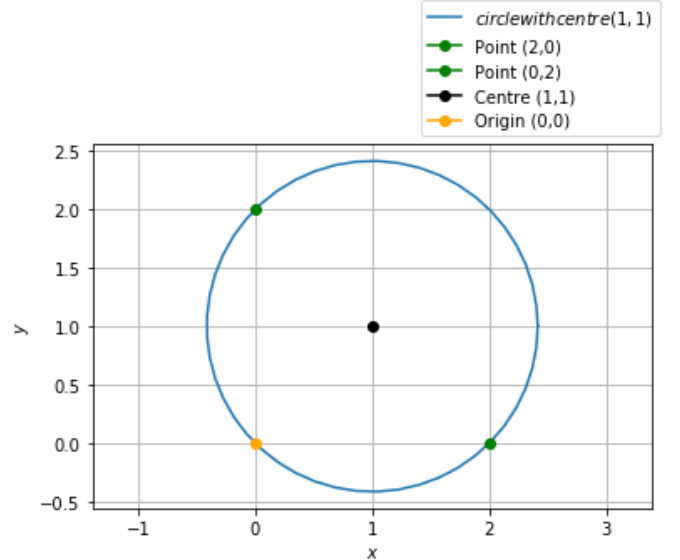
$$\implies \boxed{c_2 = \frac{b}{2}} \tag{2.0.10}$$

Fig. 1: Circle passing through origin and making intercept on coordinate axes

Thus, from equation (2.0.7) and (2.0.10),

The center of circle is,

$$\implies \boxed{\mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a \\ b \end{pmatrix}} \qquad (2.0.11)$$

The radius of the circle is,

$$r = \sqrt{\mathbf{c}^T\mathbf{c} - f} \qquad (2.0.12)$$

$$\implies \boxed{r = \sqrt{\frac{a^2 + b^2}{4}}} \qquad (2.0.13)$$

Therefore,
The equation of circle is,

$$\implies \boxed{\mathbf{x}^T\mathbf{x} - \begin{pmatrix} a & b \end{pmatrix}\mathbf{x} = 0} \qquad (2.0.14)$$

3 SOLUTION

The equation of circle is,

$$\boxed{\mathbf{x}^T\mathbf{x} - \begin{pmatrix} a & b \end{pmatrix}\mathbf{x} = 0} \qquad (3.0.1)$$