Assignment 5

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***Abstract*—This document explains one of the property of triangles.**

Download latex-tikz codes from

https://github.com/harshachinta/EE5609−Matrix−Theory/tree/master/Assignments/Assignment5

1 Problem

The line-segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.

2 Explanation

Let us consider a $\triangle ABC$, and let $\mathbf{D}$,$\mathbf{E}$ and $\mathbf{F}$ be the mid- points of sides AB,BC and CA respectively.
Let us consider a line-segment joining the points $\mathbf{D}$ and $\mathbf{F}$ which are midpoints of line AB and CA.

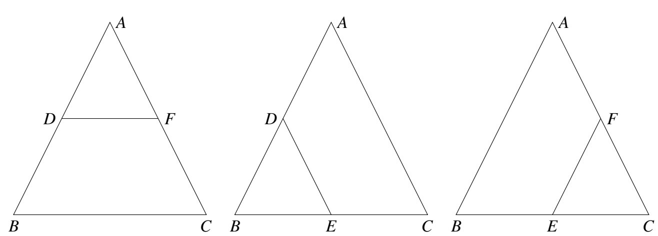

Fig. 1: Line segments joining mid-points of 2 sides of $\triangle ABC$

As $\mathbf{D}$ is midpoint of line AB, $\mathbf{E}$ is midpoint of line BC and $\mathbf{F}$ is midpoint of line CA, they can be written as follows:

$$\mathbf{D} = \frac{A+B}{2} \tag{2.0.1}$$

$$\mathbf{E} = \frac{B+C}{2} \tag{2.0.2}$$

$$\mathbf{F} = \frac{C+A}{2} \tag{2.0.3}$$

The line DF can be written in the form of direction vector as,

$$\mathbf{m}_{DF} = \mathbf{D} - \mathbf{F} \tag{2.0.4}$$

Substituting equation (2.0.1) and (2.0.3) in (2.0.4) we get,

$$\mathbf{m}_{DF} = \frac{A+B}{2} - \frac{C+A}{2} \tag{2.0.5}$$

$$\mathbf{m}_{DF} = \frac{B-C}{2} \tag{2.0.6}$$

$$\mathbf{m}_{DF} = \frac{\mathbf{m}_{BC}}{2} \tag{2.0.7}$$

where $\mathbf{m}_{BC}$ is the direction vector of line BC.
From equation (2.0.7) we can say that the direction vectors $\mathbf{m}_{DF}$ and $\mathbf{m}_{BC}$ are in same direction. Thus, line-segment DF joining the mid-points of two sides (AB and AC) of triangle is parallel to the third side (BC) of the triangle and is half of it.

This can be applied to other 2 sides as well, The line DE joining the mid-points D and E can be written in form of direction vector as:

$$\mathbf{m}_{DE} = \mathbf{D} - \mathbf{E} \tag{2.0.8}$$

Substituting equation (2.0.1) and (2.0.2) in (2.0.8) we get,

$$\mathbf{m}_{DE} = \frac{A+B}{2} - \frac{B+C}{2} \tag{2.0.9}$$

$$\mathbf{m}_{DE} = \frac{A-C}{2} \tag{2.0.10}$$

$$\mathbf{m}_{DE} = \frac{\mathbf{m}_{AC}}{2} \tag{2.0.11}$$

where $\mathbf{m}_{AC}$ is the direction vector of line AC.
Similarly, The line EF joining the mid-points E and F can be written in form of direction vector as:

$$\mathbf{m}_{EF} = \mathbf{E} - \mathbf{F} \tag{2.0.12}$$

Substituting equation (2.0.2) and (2.0.3) in (2.0.12) we get,

$$\mathbf{m}_{EF} = \frac{B+C}{2} - \frac{C+A}{2} \tag{2.0.13}$$

$$\mathbf{m}_{EF} = \frac{B-A}{2} \tag{2.0.14}$$

$$\mathbf{m}_{EF} = \frac{\mathbf{m}_{BA}}{2} \tag{2.0.15}$$

where $\mathbf{m}_{BA}$ is the direction vector of line BA.

3 Solution

From equations (2.0.7), (2.0.11) and (2.0.15), we can say that the line-segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it