

Assignment 7

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Abstract—This document explains the method of finding the area of the region.

Download all python codes from

https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment_7/code

and latex-tikz codes from

https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment_7

The angle that the line makes with the x -axis is given by,

$$\cos \theta = \frac{(\sqrt{3} \ 1) \begin{pmatrix} 2 \\ 0 \end{pmatrix}}{\|(\sqrt{3} \ 1)\| \|(2 \ 0)\|} = \frac{\sqrt{3}}{2} \quad (2.0.6)$$

$$\Rightarrow \boxed{\theta = 30^\circ} \quad (2.0.7)$$

Using equation (2.0.5) and (2.0.7), the area of the sector is obtained as,

$$\Rightarrow \boxed{\frac{\theta}{360^\circ} \pi r^2 = \frac{30^\circ}{360^\circ} \pi (2)^2 = \frac{\pi}{3}} \quad (2.0.8)$$

1 PROBLEM

Find the area of the region in the first quadrant enclosed by x -axis, line $(1 \ -\sqrt{3})\mathbf{x} = 0$ and the circle $\mathbf{x}^T \mathbf{x} = 4$.

2 EXPLANATION

The equation of a circle can be expressed as,

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{c}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

where \mathbf{c} is the center.

Comparing equation (2.0.1) with the circle equation given,

$$\mathbf{x}^T \mathbf{x} = 4 \quad (2.0.2)$$

$$\Rightarrow \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad f = -4 \quad (2.0.3)$$

$$r = \sqrt{\mathbf{c}^T \mathbf{c} - f} = \sqrt{4} \quad (2.0.4)$$

$$\Rightarrow \boxed{r = 2} \quad (2.0.5)$$

From equation (2.0.5), the point at which circle touches x -axis is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

The direction vector of x -axis is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

The direction vector of the given line $(1 \ -\sqrt{3})\mathbf{x} = 0$ is $\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$.

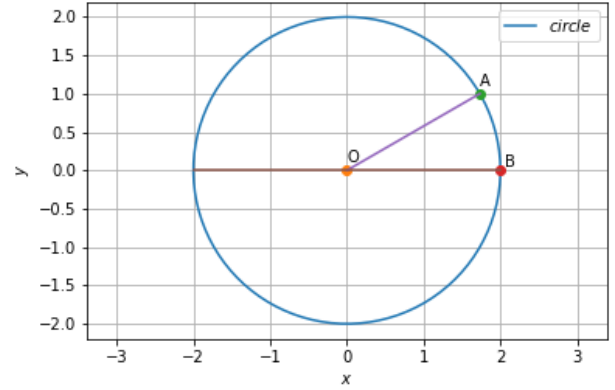


Fig. 1: Region enclosed by x -axis, line and circle

3 SOLUTION

The area of the region is $\frac{\pi}{3}$.