

# Assignment 6

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**Abstract**—This document explains the concept of finding and the unknown value in an equation such that it is represented as two straight lines .

Download latex-tikz codes from

<https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment6>

## 1 PROBLEM

Find the value of  $h$  so that the equation  $6x^2 + 2hxy + 12y^2 + 22x + 31y + 20 = 0$  may represent two straight lines.

## 2 EXPLANATION

Given equation,

$$6x^2 + 2hxy + 12y^2 + 22x + 31y + 20 = 0 \quad (2.0.1)$$

is a second order equation.

The general equation of second degree is given by

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.2)$$

and can be expressed as

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.3)$$

where

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (2.0.4)$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \quad (2.0.5)$$

Equation (2.0.3) represents a pair of straight lines if

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \quad (2.0.6)$$

Comparing equation (2.0.1) with (2.0.2), we can write in the form of (2.0.4) and (2.0.5) as,

$$\mathbf{V} = \begin{pmatrix} 6 & h \\ h & 12 \end{pmatrix} \quad (2.0.7)$$

$$\mathbf{u} = \begin{pmatrix} 11 \\ \frac{31}{2} \end{pmatrix} \quad (2.0.8)$$

$$f = 20 \quad (2.0.9)$$

As given in question, the equation represent two straight lines, substitute (2.0.7), (2.0.8), (2.0.9) in (2.0.6) to satisfy the equation.

$$\begin{vmatrix} 6 & h & 11 \\ h & 12 & \frac{31}{2} \\ 11 & \frac{31}{2} & 20 \end{vmatrix} = 0 \quad (2.0.10)$$

Expanding equation (2.0.10) along row 1 gives

$$\begin{aligned} \Rightarrow 6 \times (240 - \frac{961}{4}) - h \times (20h - \frac{341}{2}) + \\ 11 \times (\frac{31h}{2} - 132) = 0 \end{aligned}$$

$$\Rightarrow 20h^2 - 341h + \frac{2907}{2} = 0 \quad (2.0.11)$$

$$\Rightarrow \boxed{h = \frac{17}{2}} \quad (2.0.12)$$

$$\Rightarrow \boxed{h = \frac{171}{20}} \quad (2.0.13)$$

## 3 SOLUTION

If  $h = \frac{17}{2}$  or  $h = \frac{171}{20}$ , the equation given will represent two straight lines.