

Assignment 4

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Abstract—This document explains the concept of computing the determinant of a matrix given.

Download all python codes from

<https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment4/code>

and latex-tikz codes from

<https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment4>

1 PROBLEM

If a, b and c are real numbers, and

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0,$$

Show that either $a + b + c = 0$ or $a = b = c$.

2 EXPLANATION

Given,

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} \quad (2.0.1)$$

$$\xleftrightarrow{C_1 \leftarrow C_1 + C_2 + C_3} \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} \quad (2.0.2)$$

$$= 2(a+b+c) \begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix} \quad (2.0.3)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2; R_2 \leftarrow R_2 - R_3} 2(a+b+c) \begin{vmatrix} 0 & c-b & a-c \\ 0 & a-c & b-a \\ 1 & b+c & c+a \end{vmatrix} = 0 \quad (2.0.4)$$

On expanding determinant along first column from equation (2.0.4),

$$2(a+b+c)[(c-b)(b-a) - (a-c)^2] = 0 \quad (2.0.5)$$

$$\implies 2(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0 \quad (2.0.6)$$

$$\implies (a+b+c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca) = 0 \quad (2.0.7)$$

$$\implies (a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0 \quad (2.0.8)$$

From equation (2.0.8) we get 2 equations,

$$\implies \boxed{a+b+c=0} \quad (2.0.9)$$

or,

$$\implies (a-b)^2 + (b-c)^2 + (c-a)^2 = 0 \quad (2.0.10)$$

Equation (2.0.10) is possible only when, $a = b = c$

$$\implies \boxed{a=b=c} \quad (2.0.11)$$

From equation (2.0.9) and (2.0.11) we can say that, $\Delta = 0$ if $a + b + c = 0$ or $a = b = c$.

3 SOLUTION

From equation (2.0.9) and (2.0.11) we can say that, $\Delta = 0$ if $a + b + c = 0$ or $a = b = c$.