

Assignment 5

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Abstract—This document explains one of the property of triangles.

Download latex-tikz codes from

<https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment5>

1 PROBLEM

The line-segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.

2 EXPLANATION

Let us consider a $\triangle ABC$, and let **D**, **E** and **F** be the mid- points of sides AB, BC and CA respectively. Let us consider a line-segment joining the points **D** and **F** which are midpoints of line AB and CA. As **D** is midpoint of line AB, **E** is midpoint of line BC and **F** is midpoint of line CA, they can be written as follows:

$$\mathbf{D} = \frac{A + B}{2} \quad (2.0.1)$$

$$\mathbf{E} = \frac{B + C}{2} \quad (2.0.2)$$

$$\mathbf{F} = \frac{C + A}{2} \quad (2.0.3)$$

The line DF can be written in the form of direction vector as,

$$\mathbf{m}_{DF} = \mathbf{D} - \mathbf{F} \quad (2.0.4)$$

Substituting equation (2.0.1) and (2.0.3) in (2.0.4) we get,

$$\mathbf{m}_{DF} = \frac{A + B}{2} - \frac{C + A}{2} \quad (2.0.5)$$

$$\mathbf{m}_{DF} = \frac{B - C}{2} \quad (2.0.6)$$

$$\mathbf{m}_{DF} = \frac{\mathbf{m}_{BC}}{2} \quad (2.0.7)$$

where \mathbf{m}_{BC} is the direction vector of line BC. From equation (2.0.7) we can say that the direction

vectors \mathbf{m}_{DF} and \mathbf{m}_{BC} are in same direction. Thus, line-segment DF joining the mid-points of two sides (AB and AC) of triangle is parallel to the third side (BC) of the triangle and is half of it.

This can be applied to other 2 sides as well, The line DE joining the mid-points D and E can be written in form of direction vector as:

$$\mathbf{m}_{DE} = \mathbf{D} - \mathbf{E} \quad (2.0.8)$$

Substituting equation (2.0.1) and (2.0.2) in (2.0.8) we get,

$$\mathbf{m}_{DE} = \frac{A + B}{2} - \frac{B + C}{2} \quad (2.0.9)$$

$$\mathbf{m}_{DE} = \frac{A - C}{2} \quad (2.0.10)$$

$$\mathbf{m}_{DE} = \frac{\mathbf{m}_{AC}}{2} \quad (2.0.11)$$

where \mathbf{m}_{AC} is the direction vector of line AC. Similarly, The line EF joining the mid-points E and F can be written in form of direction vector as:

$$\mathbf{m}_{EF} = \mathbf{E} - \mathbf{F} \quad (2.0.12)$$

Substituting equation (2.0.2) and (2.0.3) in (2.0.12) we get,

$$\mathbf{m}_{EF} = \frac{B + C}{2} - \frac{C + A}{2} \quad (2.0.13)$$

$$\mathbf{m}_{EF} = \frac{B - A}{2} \quad (2.0.14)$$

$$\mathbf{m}_{EF} = \frac{\mathbf{m}_{BA}}{2} \quad (2.0.15)$$

where \mathbf{m}_{BA} is the direction vector of line BA.

3 SOLUTION

From equations (2.0.7), (2.0.11) and (2.0.15), we can say that the line-segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it