Assignment 13

Sri Harsha CH

***Abstract*—This document explains the conditions to check for a vector space.**

Download all python codes from

```
https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment13
    /code
```

and latex-tikz codes from

```
https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment13
```

1 Problem

Let $\mathbf{V}$ be the set of pairs (x, y) of real numbers and let $\mathbf{F}$ be the field of real numbers. Define

$$(x, y) + (x_1, y_1) = (x + x_1, 0)$$
$$c(x, y) = (cx, 0)$$

Is $\mathbf{V}$, with these operations, a vector space?

2 Explanation

$\mathbf{V}$ is a vector space if it satisfies all properties of the vector space. Let us consider the property of Existence of additive identity.
According to Existence of additive identity, there is a unique vector $\mathbf{0}$ in $\mathbf{V}$ called the zero vector, such that $\alpha + \mathbf{0} = \alpha$ for all α in $\mathbf{V}$.
Let $u = (x_1, y_1) \in \mathbf{V}$

$$\begin{aligned} u + \mathbf{0} &= (x_1, y_1) + (0, 0) \\ &= (x_1 + 0, 0) \\ &= (x_1, 0) \\ &\neq u \end{aligned} \tag{2.0.1}$$

From (2.0.1), there does not exist an additive identity for $\mathbf{V}$.
Hence $\mathbf{V}$ is not a vector space.

3 Solution

$\mathbf{V}$ is not a vector space.