Challenge Problem

Sri Harsha CH

***Abstract*—This document explains the proof of linear independence of orthogonal vectors.**

Download latex-tikz codes from

https://github.com/harshachinta/EE5609−Matrix−Theory/tree/master/Challenges/challenge_3

1 Problem

Prove that an orthogonal set of nonzero vectors is linearly independent.

2 Explanation

Let us consider a set of orthogonal vectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, ...\mathbf{v_n}$ such that $\mathbf{v_i}^T\mathbf{v_j} = 0$.
Let $a_1, a_2, ..a_n$ be constants, then the linear combination of these vectors is,

$$a_1\mathbf{v_1} + a_2\mathbf{v_2} + a_3\mathbf{v_3} + ... + a_n\mathbf{v_n} = 0 \tag{2.0.1}$$

Let us consider an orthogonal vector $\mathbf{v_j}$ from the above set. Then the dot product of $\mathbf{v_j}$ with equation (2.0.1) is,

$$a_1\mathbf{v_1}^T\mathbf{v_j} + a_2\mathbf{v_2}^T\mathbf{v_j} + a_3\mathbf{v_3}^T\mathbf{v_j} + ... + a_n\mathbf{v_n}^T\mathbf{v_j} = 0 \tag{2.0.2}$$

As we know $\mathbf{v_i}^T\mathbf{v_j} = 0$, equation (2.0.2) becomes,

$$a_j \left\|\mathbf{v_j}\right\|^2 = 0 \tag{2.0.3}$$

$$\implies a_j = 0 \tag{2.0.4}$$

From equation (2.0.4), this holds true for all the vectors in the orthogonal set, and we can say in general that,

$$a_1 = a_2 = a_3 = ... = a_n = 0 \tag{2.0.5}$$

From equation (2.0.5), we can say that the set of orthogonal vectors are linearly independent.

3 Solution

From equation (2.0.5), we can say that the set of orthogonal vectors are linearly independent.