Assignment 20

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***Abstract*—This document explains the representation of transformations by matrix.**

Download all python codes from

```
https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment20
    /code
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and latex-tikz codes from

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https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment20
```

1 Problem

Let $\mathbf{N_1}$ and $\mathbf{N_2}$ be 6×6 nilpotent matrices over the field $\mathbf{F}$. Suppose that $\mathbf{N_1}$ and $\mathbf{N_2}$ have the same minimal polynomial and the same nullity. Prove that $\mathbf{N_1}$ and $\mathbf{N_2}$ are similar. Show that this is not true for 7×7 nilpotent matrices.

2 Explanation

Refer Table 0.

3 Solution

Refer Table 0.

Given	Derivation
Given	$\mathbf{N_1}$ and $\mathbf{N_2}$ be 6×6 nilpotent matrices. $\mathbf{N_1}$ and $\mathbf{N_2}$ have the same minimal polynomial and the same nullity. To prove $\mathbf{N_1}$ and $\mathbf{N_2}$ are similar.
From given statement	Two matrices are similar if they have the same Jordan Canonical form. 1. As $\mathbf{N_1}$ and $\mathbf{N_2}$ are nilpotent matrices, 0 is the only eigen value. 2. As minimal polynomial is same, $p\mathbf{N_1} = p\mathbf{N_2}$, the two matrices should have the same maximum block size. 3. As they have same nullity, they will have same total number of blocks. Let us consider all the possibilities for the dimensions of the block matrices for both Jordan forms.
Matrix size - 6 Jordan size - 6	If Jordan form $\mathbf{J_1}$ consists of one block of dimension 6, then by (3) above $\mathbf{J_2}$ also has one block of dimension 6.
Matrix size - 6 Jordan size - 5 + 1	If Jordan form $\mathbf{J_1}$ consists of one block of dimension 5 and other 1, then by (2), $\mathbf{J_2}$ also has same maximum block of dimension 5 and by (3) have other block of size 1.
Matrix size - 6 Jordan size - 4+2 Jordan size - 4+1+1	Although there are two different possibilities for Jordan blocks, with first block matrix of dimension 4, both $\mathbf{J_1}$ and $\mathbf{J_2}$ will be same because of (3).
Matrix size - 6 Jordan size - 3+3 Jordan size - 3+2+1 Jordan size - 3+1+1+1	Although there are three different possibilities for Jordan blocks, with first block matrix of dimension 3, both $\mathbf{J_1}$ and $\mathbf{J_2}$ will be same because of (3).
Matrix size - 6 Jordan size - 2+2+2 Jordan size - 2+2+1+1 Jordan size - 2+1+1+1+1	Although there are three different possibilities for Jordan blocks, with first block matrix of dimension 2, both $\mathbf{J_1}$ and $\mathbf{J_2}$ will be same because of (3), where, the total number of blocks will be same.
Matrix size - 6 Jordan size-1+1+1+1+1+1	$\mathbf{J_1}$ and $\mathbf{J_2}$ will be same because of (3). As $\mathbf{J_1}$ and $\mathbf{J_2}$ are same, $\mathbf{N_1}$ and $\mathbf{N_2}$ are similar.
7×7 Nielpotent matrix	Let us take a counter example, Matrix size - 7, Jordan block size - 3+3+1 or 3+2+2 $$\mathbf{J_1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathbf{J_2} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (2.0.1)$$ From above, $\mathbf{J_1}$ and $\mathbf{J_2}$ are not same.

TABLE 0: Explanation