Assignment 11

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Abstract—**This document explains the method of finding whether two system of linear equations are equivalent or not.**

Download all python codes from

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https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment11
    /code
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and latex-tikz codes from

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https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment11
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1 Problem

Are the following two systems of linear equations equivalent?

$$\begin{aligned} -x_1 + x_2 + 4x_3 &= 0 \\ x_1 + 3x_2 + 8x_3 &= 0 \\ \frac{1}{2}x_1 + x_2 + \frac{5}{2}x_3 &= 0 \end{aligned} \tag{1.0.1}$$

$$\begin{aligned} x_1 - x_3 &= 0 \\ x_2 + 3x_3 &= 0 \end{aligned} \tag{1.0.2}$$

2 Explanation

System of linear equations in (1.0.1) can be expressed in matrix form as,

$$\mathbf{A}\mathbf{x} = 0 \tag{2.0.1}$$

$$\begin{pmatrix} -1 & 1 & 4 \\ 1 & 3 & 8 \\ \frac{1}{2} & 1 & \frac{5}{2} \end{pmatrix} \mathbf{x} = 0 \tag{2.0.2}$$

System of linear equations in (1.0.2) can be expressed in matrix form as,

$$\mathbf{B}\mathbf{x} = 0 \tag{2.0.3}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.4}$$

Two system of linear equations are equivalent if one system can be expressed as a linear combination of other system.
Matrix $\mathbf{B}$ can be obtained from matrix $\mathbf{A}$ as,

$$\mathbf{B} = \mathbf{C}\mathbf{A} \tag{2.0.5}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \end{pmatrix} = \mathbf{C} \begin{pmatrix} -1 & 1 & 4 \\ 1 & 3 & 8 \\ \frac{1}{2} & 1 & \frac{5}{2} \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{C} = \begin{pmatrix} -1 & 1 & -2 \\ \frac{1}{2} & -\frac{1}{2} & 2 \end{pmatrix} \tag{2.0.7}$$

Now, writing equations in matrix-vector form,

$$x_1 - x_3 = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \mathbf{x}$$

$$\begin{aligned} \implies \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \mathbf{x} &= -1 \begin{pmatrix} -1 & 1 & 4 \end{pmatrix} \mathbf{x} \\ &+ 1 \begin{pmatrix} 1 & 3 & 8 \end{pmatrix} \mathbf{x} - 2 \begin{pmatrix} \frac{1}{2} & 1 & \frac{5}{2} \end{pmatrix} \mathbf{x} \end{aligned} \tag{2.0.8}$$

$$x_2 + 3x_3 = \begin{pmatrix} 0 & 1 & 3 \end{pmatrix} \mathbf{x}$$

$$\begin{aligned} \implies \begin{pmatrix} 0 & 1 & 3 \end{pmatrix} \mathbf{x} &= \frac{1}{2} \begin{pmatrix} -1 & 1 & 4 \end{pmatrix} \mathbf{x} \\ &- \frac{1}{2} \begin{pmatrix} 1 & 3 & 8 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{1}{2} & 1 & \frac{5}{2} \end{pmatrix} \mathbf{x} \end{aligned} \tag{2.0.9}$$

Equations (2.0.8) and (2.0.9) is same as multiplying $\mathbf{C}$ with $\mathbf{A}$ which is the linear combination of rows of matrix $\mathbf{A}$.
Thus each equation in second system can be expressed as linear combination of the equations in first system.
Therefore, the two system of linear equations are equivalent.

3 Solution

The two systems of linear equations are equivalent