Challenge Problem

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Abstract—**This document explains the property of convolution.**

Download latex-tikz codes from

https://github.com/harshachinta/EE5609−Matrix−Theory/tree/master/Challenges/challenge_4

1 Problem

If

$$x_1(n) * h_1(n) = y(n)$$
$$x_2(n) * h_2(n) = y(n)$$

then is $h_1(n) = h_2(n)$?

2 Solution

A finite-length discrete-time signal is basically a sequence, say, $(x_0, \ldots, x_{m-1})$ which can be written as an m-length vector $\mathbf{x} \in R^m$.

Given two signals $(x_0, \ldots, x_{n-1})$ and $(h_0, \ldots, h_{m-1})$, the (linear) convolution of the two is a $m + n - 1$ length signal defined as

$$y(t) = (h * x)_t = \sum_{\tau=0}^{\tau=n-1} x_\tau h_{(t-\tau)} \tag{2.0.1}$$
$$0 \le t < m + n - 1$$

The above convolution can be written in the form of matrix as: $\mathbf{Y} = \mathbf{HX}$

$$Y = \begin{pmatrix} h_0 & 0 & 0 & . & . & 0 & 0 \\ h_1 & h_0 & 0 & . & . & 0 & 0 \\ h_2 & h_1 & h_0 & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ h_{n-1} & h_{n-2} & n_{n-3} & . & . & h_1 & h_0 \\ . & . & . & . & . & . & . \\ h_{m-1} & h_{m-2} & h_{m-3} & . & . & h_{m-n+1} & h_{m-n} \\ 0 & h_{m-1} & h_{m-2} & . & . & h_{m-n+2} & h_{m-n+1} \\ 0 & 0 & h_{m-1} & . & . & h_{m-n+3} & h_{m-n+2} \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & h_{m-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2. \\ . \\ . \\ x_{n-1} \end{pmatrix} \tag{2.0.2}$$

Therefore we can write equation (2.0.1) in matrix form as $\mathbf{Y} = \mathbf{HX}$ where

$$\mathbf{H} = \begin{pmatrix} h_0 & 0 & 0 & . & . & 0 & 0 \\ h_1 & h_0 & 0 & . & . & 0 & 0 \\ h_2 & h_1 & h_0 & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ h_{n-1} & h_{n-2} & n_{n-3} & . & . & h_1 & h_0 \\ . & . & . & . & . & . & . \\ h_{m-1} & h_{m-2} & h_{m-3} & . & . & h_{m-n+1} & h_{m-n} \\ 0 & h_{m-1} & h_{m-2} & . & . & h_{m-n+2} & h_{m-n+1} \\ 0 & 0 & h_{m-1} & . & . & h_{m-n+3} & h_{m-n+2} \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & h_{m-1} \end{pmatrix} \tag{2.0.3}$$

Therefore, from question we can rewrite convolution in form of matrices as,

$$x_1(n) * h_1(n) = y(n) \tag{2.0.4}$$
$$\implies \mathbf{Y} = \mathbf{H_1X} \tag{2.0.5}$$
$$x_2(n) * h_2(n) = y(n) \tag{2.0.6}$$
$$\implies \mathbf{Y} = \mathbf{H_2X} \tag{2.0.7}$$

Equating equations (2.0.5) and (2.0.7),

$$\mathbf{H_1X} = \mathbf{H_2X} \tag{2.0.8}$$

$\mathbf{H_1}$ and $\mathbf{H_2}$ are in the form of equation (2.0.3) and we can see by comparing both matrices that:

$$\implies \boxed{\mathbf{H_1} = \mathbf{H_2}} \tag{2.0.9}$$

Because the upper half of matrix is lower triangular and the values of both matrix will be equal. Similarly the matrix from below is upper triangular and both matrix will be equal by comparison.