Assignment 3

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***Abstract*—This document explains the concept of finding the modulus and argument of the complex number.**

Download all python codes from

https://github.com/harshachinta/EE5609−Matrix−Theory/tree/master/Assignments/Assignment3/code

and latex-tikz codes from

https://github.com/harshachinta/EE5609−Matrix−Theory/tree/master/Assignments/Assignment3

1 Problem

Find the modulus and argument of the complex number $\frac{\begin{pmatrix}1\\2\end{pmatrix}}{\begin{pmatrix}1\\-3\end{pmatrix}}$.

2 Explanation

In general, any complex number can be expressed in polar form as follows:

$$z = r\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{2.0.1}$$

where r and θ are the modulus and argument of complex number z.

Converting complex number $\begin{pmatrix}1\\2\end{pmatrix}$ into polar form, the modulus and argument are:

$$r = \left\|\begin{pmatrix}1\\2\end{pmatrix}\right\| = \sqrt{5} \tag{2.0.2}$$

$$\tan\theta = \frac{2}{1} \implies \theta = 63.43° \tag{2.0.3}$$

From equation (2.0.2) and (2.0.3), the polar form of $\begin{pmatrix}1\\2\end{pmatrix}$ is,

$$\boxed{\begin{pmatrix}1\\2\end{pmatrix} = \sqrt{5}\begin{pmatrix}\cos 63.43°\\ \sin 63.43°\end{pmatrix}} \tag{2.0.4}$$

Similarly, converting complex number $\begin{pmatrix}1\\-3\end{pmatrix}$ into polar form, the modulus and argument are:

$$r = \left\|\begin{pmatrix}1\\-3\end{pmatrix}\right\| = \sqrt{10} \tag{2.0.5}$$

$$\tan\theta = \frac{-3}{1} \implies \theta = -71.56° \tag{2.0.6}$$

From equation (2.0.5) and (2.0.6), the polar form of $\begin{pmatrix}1\\-3\end{pmatrix}$ is,

$$\boxed{\begin{pmatrix}1\\-3\end{pmatrix} = \sqrt{10}\begin{pmatrix}\cos 71.56°\\ -\sin 71.56°\end{pmatrix}} \tag{2.0.7}$$

Applying inverse to (2.0.7),

$$\boxed{\begin{pmatrix}1\\-3\end{pmatrix}^{-1} = \frac{1}{\sqrt{10}}\begin{pmatrix}\cos 71.56°\\ \sin 71.56°\end{pmatrix}} \tag{2.0.8}$$

In general, if

$$z_1 = r_1\begin{pmatrix}\cos\theta_1\\ \sin\theta_1\end{pmatrix}, z_2 = r_2\begin{pmatrix}\cos\theta_2\\ \sin\theta_2\end{pmatrix} \tag{2.0.9}$$

$$z_1 z_2 = r_1 r_2\begin{pmatrix}\cos(\theta_1+\theta_2)\\ \sin(\theta_1+\theta_2)\end{pmatrix} \tag{2.0.10}$$

From equation (2.0.10), the complex number can be rewritten as,

$$\frac{\begin{pmatrix}1\\2\end{pmatrix}}{\begin{pmatrix}1\\-3\end{pmatrix}} = \begin{pmatrix}1\\2\end{pmatrix}\begin{pmatrix}1\\-3\end{pmatrix}^{-1} \tag{2.0.11}$$

Sub (2.0.4) and (2.0.8) in (2.0.11),

$$\begin{pmatrix}1\\2\end{pmatrix}\begin{pmatrix}1\\-3\end{pmatrix}^{-1} = \frac{\sqrt{5}}{\sqrt{10}}\begin{pmatrix}\cos(63.43° + 71.56°)\\ \sin(63.43° + 71.56°)\end{pmatrix} \tag{2.0.12}$$

$$\begin{pmatrix}1\\2\end{pmatrix}\begin{pmatrix}1\\-3\end{pmatrix}^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix}\cos 135°\\ \sin 135°\end{pmatrix} \tag{2.0.13}$$

3 Solution

From (2.0.13), the modulus of the complex number is $\frac{1}{\sqrt{2}}$ and the argument of the complex number is $135°$.