Assignment 6

Sri Harsha CH

***Abstract*—This document explains the concept of finding the unknown value in an equation such that it is represented as two straight lines .**

Download latex-tikz codes from

https://github.com/harshachinta/EE5609−Matrix−Theory/tree/master/Assignments/Assignment6

1 Problem

Find the value of h so that the equation $6x^2+2hxy+12y^2+22x+31y+20 = 0$ may represent two straight lines.

2 Explanation

Given equation,

$$6x^2 + 2hxy + 12y^2 + 22x + 31y + 20 = 0 \quad (2.0.1)$$

is a second order equation.
The general equation of second degree is given by

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.2)$$

and can be expressed as

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \quad (2.0.3)$$

where

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (2.0.4)$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \quad (2.0.5)$$

Equation (2.0.3) represents a pair of straight lines if

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \quad (2.0.6)$$

Comparing equation (2.0.1) with (2.0.2), we can write in the form of (2.0.4) and (2.0.5) as,

$$\mathbf{V} = \begin{pmatrix} 6 & h \\ h & 12 \end{pmatrix} \quad (2.0.7)$$

$$\mathbf{u} = \begin{pmatrix} 11 \\ \frac{31}{2} \end{pmatrix} \quad (2.0.8)$$

and

$$f = 20 \quad (2.0.9)$$

As given in question, the equation represent two straight lines, substitute (2.0.7), (2.0.8), (2.0.9) in (2.0.6) to satisfy the equation.

$$\begin{vmatrix} 6 & h & 11 \\ h & 12 & \frac{31}{2} \\ 11 & \frac{31}{2} & 20 \end{vmatrix} = 0 \quad (2.0.10)$$

Expanding equation (2.0.10) along row 1 gives

$$\implies 6 \times (240 - \frac{961}{4}) - h \times (20h - \frac{341}{2}) + 11 \times (\frac{31h}{2} - 132) = 0$$

$$\implies 20h^2 - 341h + \frac{2907}{2} = 0 \quad (2.0.11)$$

$$\implies \boxed{h = \frac{17}{2}} \quad (2.0.12)$$

$$\implies \boxed{h = \frac{171}{20}} \quad (2.0.13)$$

3 Solution

If $h = \frac{17}{2}$ or $h = \frac{171}{20}$, the equation given will represent two straight lines.

Sub $h = \frac{17}{2}$ in equation (2.0.1) we get,

$$6x^2 + 17xy + 12y^2 + 22x + 31y + 20 = 0 \quad (3.0.1)$$

From equation (3.0.1) consider,

$$6x^2 + 17xy + 12y^2 \quad (3.0.2)$$

$$\implies 6x^2 + 8xy + 9xy + 12y^2 \quad (3.0.3)$$

$$\implies 2x(3x + 4y) + 3y(3x + 4y) \quad (3.0.4)$$

$$\implies (2x + 3y)(3x + 4y) \quad (3.0.5)$$

Equation (3.0.5) can be modified as,

$$\implies (2x+3y+l)(3x+4y+m)=0$$
$$\implies 6x^2+17xy+12y^2+(2m+3l)x+ (3m+4l)y+lm=0 \quad (3.0.6)$$

Comparing equations (3.0.1) and (3.0.6),

$$2m+3l=22$$
$$3m+4l=31$$

Solving the two equations we get,

$$l=4 \quad (3.0.7)$$
$$m=5 \quad (3.0.8)$$

Substituting equations (3.0.7) and (3.0.8) in (3.0.6),

$$\implies \boxed{(2x+3y+4)(3x+4y+5)=0} \quad (3.0.9)$$

Equation (3.0.9) represents equations of two straight lines.

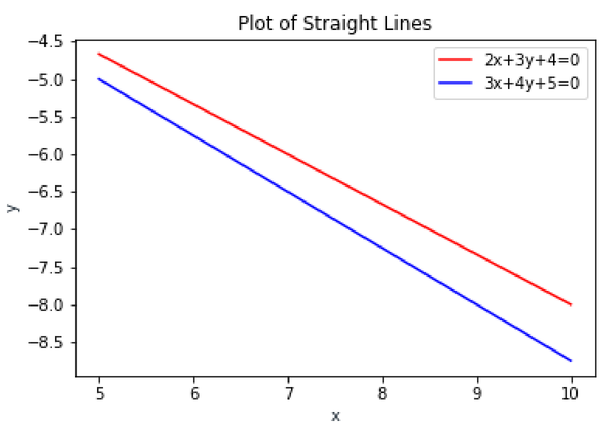

Fig. 1: Plot of Straight lines when $h=\frac{17}{2}$

Similarly, Sub $h=\frac{171}{20}$ in equation (2.0.1) we get,

$$20x^2+57xy+40y^2+\frac{220}{3}x+\frac{310}{3}y+\frac{200}{3}=0 \quad (3.0.10)$$

From equation (3.0.10) consider,

$$20x^2+57xy+40y^2 \quad (3.0.11)$$
$$\implies 20x^2+32xy+25xy+40y^2 \quad (3.0.12)$$
$$\implies 4x(5x+8y)+5y(5x+8y) \quad (3.0.13)$$
$$\implies (4x+5y)(5x+8y) \quad (3.0.14)$$

Equation (3.0.14) can be modified as,

$$\implies (4x+5y+l)(5x+8y+m)=0$$
$$\implies 20x^2+57xy+40y^2+(4m+5l)x+ (5m+8l)y+lm=0 \quad (3.0.15)$$

Comparing equations (3.0.10) and (3.0.15),

$$4m+5l=\frac{220}{3}$$
$$5m+8l=\frac{310}{3}$$

Solving the two equations we get,

$$l=\frac{20}{3} \quad (3.0.16)$$
$$m=10 \quad (3.0.17)$$

Substituting equations (3.0.16) and (3.0.17) in (3.0.15),

$$\implies \boxed{(4x+5y+\frac{20}{3})(5x+8y+10)=0} \quad (3.0.18)$$

Equation (3.0.18) represents equations of two straight lines.

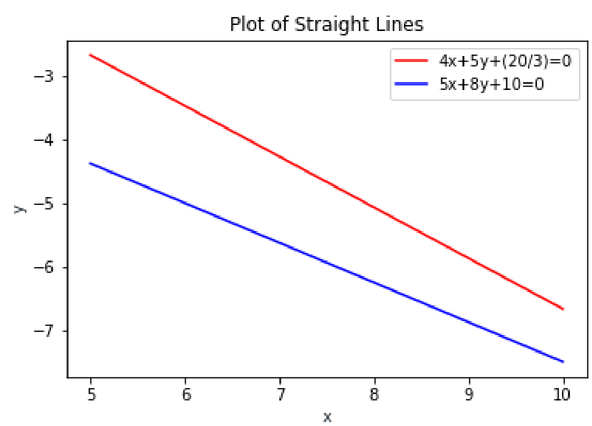

Fig. 2: Plot of Straight lines when $h=\frac{171}{20}$