Assignment 7

Sri Harsha CH

***Abstract*—This document explains the method of finding the area of the region.**

Download all python codes from

```
https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/
    Assignment_7/code
```

and latex-tikz codes from

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https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/
    Assignment_7
```

1 Problem

Find the area of the region in the first quadrant enclosed by x-axis, line $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = 0$ and the circle $\mathbf{x}^T\mathbf{x}$=4.

2 Explanation

The equation of a circle can be expressed as,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

where $\mathbf{c}$ is the center.
Comparing equation (2.0.1) with the circle equation given,

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.2}$$

$$\implies \mathbf{c} = \begin{pmatrix}0\\0\end{pmatrix} \quad f = -4 \tag{2.0.3}$$

$$r = \sqrt{\mathbf{c}^T\mathbf{c} - f} = \sqrt{4} \tag{2.0.4}$$

$$\implies \boxed{r = 2} \tag{2.0.5}$$

From equation (2.0.5), the point at which circle touches x-axis is $\begin{pmatrix}2\\0\end{pmatrix}$.

The direction vector of x-axis is $\begin{pmatrix}1\\0\end{pmatrix}$.

The direction vector of the given line $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = 0$ is $\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$.

The angle that the line makes with the x-axis is given by,

$$\cos\theta = \frac{\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}}{\left\|\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\right\|\left\|\begin{pmatrix}1 & 0\end{pmatrix}\right\|} = \frac{\sqrt{3}}{2} \tag{2.0.6}$$

$$\implies \boxed{\theta = 30^\circ} \tag{2.0.7}$$

Using equation (2.0.5) and (2.0.7),the area of the sector is obtained as,

$$\implies \boxed{\frac{\theta}{360^\circ}\pi r^2 = \frac{30^\circ}{360^\circ}\pi(2)^2 = \frac{\pi}{3}} \tag{2.0.8}$$

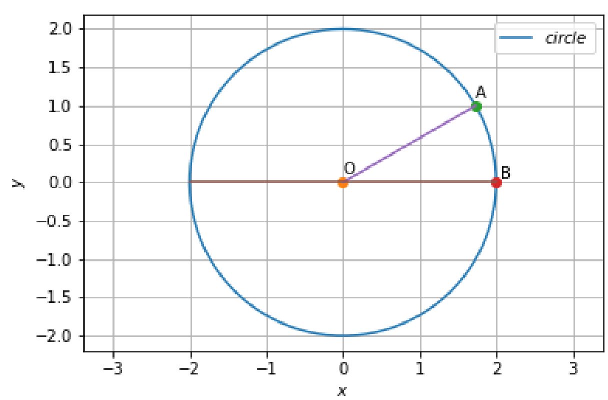

Fig. 1: Region enclosed by x-axis, line and circle

To find points $\mathbf{A}$ and $\mathbf{B}$,
The parametric form of x-axis is,

$$\mathbf{B} = \mathbf{q} + \lambda\mathbf{m} \tag{2.0.9}$$

$$= \begin{pmatrix}0\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.10}$$

From the intersection of circle and line, the value of λ can be found by,

$$\lambda^2 = \frac{-f_1 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.11}$$

$$= \frac{4-0}{1} = 4 \tag{2.0.12}$$

$$\implies \lambda = \pm 2 \tag{2.0.13}$$

Sub equation (2.0.13) in (2.0.10),

$$\mathbf{B} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2.0.14}$$

As given in question as first quadrant,

$$\implies \boxed{\mathbf{B} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}} \tag{2.0.15}$$

Similarly, to find point $\mathbf{A}$, The parametric form of line is,

$$\mathbf{A} = \mathbf{q} + \lambda \mathbf{m} \tag{2.0.16}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \tag{2.0.17}$$

$$\lambda^2 = \frac{-f_1 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.18}$$

$$= \frac{4-0}{4} = 1 \tag{2.0.19}$$

$$\implies \lambda = \pm 1 \tag{2.0.20}$$

$$\mathbf{A} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} -\sqrt{3} \\ -1 \end{pmatrix} \tag{2.0.21}$$

$$\implies \boxed{\mathbf{A} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}} \tag{2.0.22}$$

3 Solution

The area of the region is $\frac{\pi}{3}$.