

# Assignment 6

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**Abstract**—This document explains the concept of finding the unknown value in an equation such that it is represented as two straight lines .

Download latex-tikz codes from

<https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment6>

## 1 PROBLEM

Find the value of  $h$  so that the equation  $6x^2 + 2hxy + 12y^2 + 22x + 31y + 20 = 0$  may represent two straight lines.

## 2 EXPLANATION

Given equation,

$$6x^2 + 2hxy + 12y^2 + 22x + 31y + 20 = 0 \quad (2.0.1)$$

is a second order equation.

The general equation of second degree is given by

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.2)$$

$$\Rightarrow \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.3)$$

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (2.0.4)$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \quad (2.0.5)$$

Equation (2.0.3) represents a pair of straight lines if,

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \quad (2.0.6)$$

Comparing equation (2.0.1) with (2.0.2), we can write in the form of (2.0.4) and (2.0.5) as,

$$\mathbf{V} = \begin{pmatrix} 6 & h \\ h & 12 \end{pmatrix} \quad (2.0.7)$$

$$\mathbf{u} = \begin{pmatrix} 11 \\ \frac{31}{2} \end{pmatrix} \quad (2.0.8)$$

$$f = 20 \quad (2.0.9)$$

As given in question, the equation represent two straight lines, substitute (2.0.7), (2.0.8), (2.0.9) in (2.0.6) to satisfy the equation.

$$\begin{vmatrix} 6 & h & 11 \\ h & 12 & \frac{31}{2} \\ 11 & \frac{31}{2} & 20 \end{vmatrix} = 0 \quad (2.0.10)$$

Expanding equation (2.0.10) along row 1 gives

$$\begin{aligned} \Rightarrow 6 \times (240 - \frac{961}{4}) - h \times (20h - \frac{341}{2}) + \\ 11 \times (\frac{31h}{2} - 132) = 0 \end{aligned}$$

$$\Rightarrow 20h^2 - 341h + \frac{2907}{2} = 0 \quad (2.0.11)$$

$$\Rightarrow h = \frac{17}{2} \quad (2.0.12)$$

$$\Rightarrow h = \frac{171}{20} \quad (2.0.13)$$

## 3 SOLUTION

If  $h = \frac{17}{2}$  or  $h = \frac{171}{20}$ , the equation given will represent two straight lines.

Sub  $h = \frac{17}{2}$  in equation (2.0.1) we get,

$$6x^2 + 17xy + 12y^2 + 22x + 31y + 20 = 0 \quad (3.0.1)$$

Equation (3.0.1) can be expressed as,

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 6 & \frac{17}{2} \\ \frac{17}{2} & 12 \end{pmatrix} \quad (3.0.2)$$

$$\mathbf{u} = \begin{pmatrix} 11 \\ \frac{31}{2} \end{pmatrix} \quad (3.0.3)$$

$$\mathbf{f} = 20 \quad (3.0.4)$$

The pair of straight lines are given by,

$$(\mathbf{n}_1^T \mathbf{x} - c_1)(\mathbf{n}_2^T \mathbf{x} - c_2) = 0 \quad (3.0.5)$$

The slopes of the lines are given by the roots of the polynomial:

$$cm^2 + 2bm + a = 0 \quad (3.0.6)$$

$$\Rightarrow m_i = \frac{-b \pm \sqrt{-\det(V)}}{c} \quad (3.0.7)$$

$$(3.0.8)$$

Substituting (3.0.1) in the equation (3.0.6),

$$12m^2 + 17m + 6 = 0 \quad (3.0.9)$$

$$m_i = \frac{-\frac{17}{2} \pm \sqrt{\frac{1}{4}}}{12} \quad (3.0.10)$$

$$\Rightarrow m_1 = \frac{-2}{3}, m_2 = \frac{-3}{4} \quad (3.0.11)$$

$$\mathbf{m}_1 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \mathbf{m}_2 = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad (3.0.12)$$

$$\Rightarrow \mathbf{n}_1 = \begin{pmatrix} -2 \\ -3 \end{pmatrix}, \mathbf{n}_2 = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \quad (3.0.13)$$

$c_1$  and  $c_2$  can be obtained as,

$$(\mathbf{n}_1 \quad \mathbf{n}_2) \begin{pmatrix} c_2 \\ c_1 \end{pmatrix} = -2\mathbf{u} \quad (3.0.14)$$

Substituting (3.0.13) in (3.0.14), the augmented matrix is,

$$\begin{pmatrix} -2 & -3 & -22 \\ -3 & -4 & -31 \end{pmatrix} \xrightarrow[R_1 \leftarrow \frac{-R_1 - 3R_2}{2}]{R_2 \leftarrow 2R_2 - 3R_1} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \end{pmatrix} \quad (3.0.15)$$

$$\Rightarrow c_1 = 4, c_2 = 5 \quad (3.0.16)$$

Substituting (3.0.13) and (3.0.16) in (3.0.5) we get,

$$\begin{aligned} \Rightarrow (-2x - 3y - 4)(3x - 4y - 5) &= 0 \\ \Rightarrow (2x + 3y + 4)(3x + 4y + 5) &= 0 \end{aligned} \quad (3.0.17)$$

Equation (3.0.17) represents equations of two straight lines.

Similarly, Sub  $h = \frac{171}{20}$  in equation (2.0.1) we get,

$$20x^2 + 57xy + 40y^2 + \frac{220}{3}x + \frac{310}{3}y + \frac{200}{3} = 0 \quad (3.0.18)$$

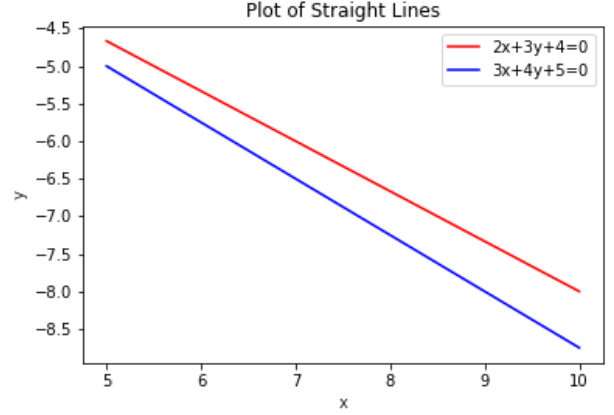


Fig. 1: Plot of Straight lines when  $h = \frac{17}{2}$

Equation (3.0.18) can be expressed as,

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 20 & \frac{57}{2} \\ \frac{57}{2} & 40 \end{pmatrix} \quad (3.0.19)$$

$$\mathbf{u} = \begin{pmatrix} \frac{220}{6} \\ \frac{310}{6} \end{pmatrix} \quad (3.0.20)$$

$$\mathbf{f} = \frac{200}{3} \quad (3.0.21)$$

The pair of straight lines are given by,

$$(\mathbf{n}_1^T \mathbf{x} - c_1)(\mathbf{n}_2^T \mathbf{x} - c_2) = 0 \quad (3.0.22)$$

Substituting (3.0.18) in the equation (3.0.6),

$$40m^2 + 57m + 20 = 0 \quad (3.0.23)$$

$$m_i = \frac{-\frac{57}{2} \pm \sqrt{\frac{49}{4}}}{40} \quad (3.0.24)$$

$$\Rightarrow m_1 = \frac{-5}{8}, m_2 = \frac{-4}{5} \quad (3.0.25)$$

$$\mathbf{m}_1 = \begin{pmatrix} 8 \\ -5 \end{pmatrix}, \mathbf{m}_2 = \begin{pmatrix} 5 \\ -4 \end{pmatrix} \quad (3.0.26)$$

$$\Rightarrow \mathbf{n}_1 = \begin{pmatrix} -5 \\ -8 \end{pmatrix}, \mathbf{n}_2 = \begin{pmatrix} -4 \\ -5 \end{pmatrix} \quad (3.0.27)$$

$c_1$  and  $c_2$  can be obtained as,

$$(\mathbf{n}_1 \quad \mathbf{n}_2) \begin{pmatrix} c_2 \\ c_1 \end{pmatrix} = -2\mathbf{u} \quad (3.0.28)$$

Substituting (3.0.27) in (3.0.28), the augmented

matrix is,

$$\begin{pmatrix} -5 & -4 & -\frac{220}{3} \\ -8 & -5 & -\frac{310}{3} \end{pmatrix} \xrightarrow[R_1 \leftarrow \frac{-R_1 - 4R_2}{5}]{R_2 \leftarrow \frac{5R_2 - 8R_1}{7}} \begin{pmatrix} 1 & 0 & \frac{20}{3} \\ 0 & 1 & 10 \end{pmatrix} \quad (3.0.29)$$

$$\Rightarrow c_1 = 10, c_2 = \frac{20}{3} \quad (3.0.30)$$

Substituting (3.0.27) and (3.0.30) in (3.0.22) we get,

$$\Rightarrow (5x + 8y + 10)(4x + 5y + \frac{20}{3}) = 0 \quad (3.0.31)$$

Equation (3.0.31) represents equations of two straight lines.

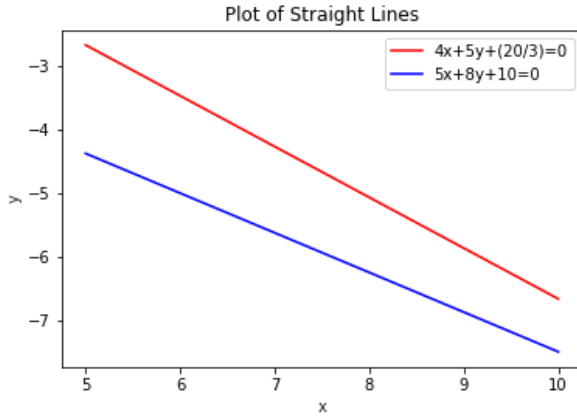


Fig. 2: Plot of Straight lines when  $h = \frac{171}{20}$