Assignment 3

Sri Harsha CH

***Abstract*—This document explains the concept of finding the modulus and argument of the complex number.**

Download all python codes from

https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment3/code

and latex-tikz codes from

https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment3

1 Problem

Find the modulus and argument of the complex number $\frac{\begin{pmatrix}1\\2\end{pmatrix}}{\begin{pmatrix}1\\-3\end{pmatrix}}$.

2 Explanation

In general, any complex number can be expressed in matrix representation as follows:

$$\begin{pmatrix}a1\\a2\end{pmatrix} = \begin{pmatrix}a1 & -a2\\a2 & a1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.1}$$

Converting complex number to matrix form:

$$\frac{\begin{pmatrix}1\\2\end{pmatrix}}{\begin{pmatrix}1\\-3\end{pmatrix}} = \begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}\begin{pmatrix}1 & 3\\-3 & 1\end{pmatrix}^{-1}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.2}$$

$$\begin{pmatrix}1 & 3\\-3 & 1\end{pmatrix}^{-1} = \begin{pmatrix}1/10 & -3/10\\3/10 & 1/10\end{pmatrix} \tag{2.0.3}$$

Sub (2.0.3) in (2.0.2),

$$\frac{\begin{pmatrix}1\\2\end{pmatrix}}{\begin{pmatrix}1\\-3\end{pmatrix}} = \begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}\begin{pmatrix}1/10 & -3/10\\3/10 & 1/10\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}\begin{pmatrix}1/10\\3/10\end{pmatrix} \tag{2.0.5}$$

$$= \begin{pmatrix}-5/10\\5/10\end{pmatrix} \tag{2.0.6}$$

$$\implies \boxed{\frac{\begin{pmatrix}1\\2\end{pmatrix}}{\begin{pmatrix}1\\-3\end{pmatrix}} = \begin{pmatrix}-1/2\\1/2\end{pmatrix}} \tag{2.0.7}$$

From (2.0.7),
The modulus and argument of the complex number is,

$$r = \left\|\begin{pmatrix}-1/2\\1/2\end{pmatrix}\right\| = \frac{1}{\sqrt{2}} \tag{2.0.8}$$

$$\tan\theta = -1 \implies \theta = 180^\circ - 45^\circ = 135^\circ \tag{2.0.9}$$

3 Solution

From (2.0.8) and (2.0.9), the modulus of the complex number is $\frac{1}{\sqrt{2}}$ and the argument of the complex number is 135°.