

# Assignment 9

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**Abstract**—This document explains the method of performing QR decomposition on a  $2 \times 2$  matrix. where,

Download all python codes from

<https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment9/code>

and latex-tikz codes from

<https://github.com/harshachinta/EE5609-Matrix-Theory/tree/master/Assignments/Assignment9>

## 1 PROBLEM

Find the QR decomposition of the given matrix.

$$\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix} \quad (1.0.1)$$

## 2 EXPLANATION

QR decomposition of a square matrix is given by,

$$\mathbf{A} = \mathbf{QR} \quad (2.0.1)$$

where  $\mathbf{Q}$  is an orthogonal matrix and  $\mathbf{R}$  is an upper triangular matrix.

Given matrix,

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix} \quad (2.0.2)$$

The column vectors of the matrix is given by,

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} \quad (2.0.3)$$

Equation (2.0.2) can be written in form of (2.0.3) as,

$$(\mathbf{a} \ \mathbf{b}) = (\mathbf{q}_1 \ \mathbf{q}_2) \begin{pmatrix} u_1 & u_3 \\ 0 & u_2 \end{pmatrix} = \mathbf{QR} \quad (2.0.4)$$

$$u_1 = \|\mathbf{a}\| = \sqrt{1^2 + 2^2} = \sqrt{5} \quad (2.0.5)$$

$$\mathbf{q}_1 = \frac{\mathbf{a}}{u_1} = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix} \quad (2.0.6)$$

$$u_3 = \frac{\mathbf{q}_1^T \mathbf{b}}{\|\mathbf{q}_1\|^2} = \left( \frac{1}{\sqrt{5}} \quad \frac{2}{\sqrt{5}} \right) \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \frac{-2}{\sqrt{5}} \quad (2.0.7)$$

$$\mathbf{q}_2 = \frac{\mathbf{b} - u_3 \mathbf{q}_1}{\|\mathbf{b} - u_3 \mathbf{q}_1\|} = \begin{pmatrix} \frac{2}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} \end{pmatrix} \quad (2.0.8)$$

$$u_2 = \mathbf{q}_2^T \mathbf{b} = \left( \frac{2}{\sqrt{5}} \quad -\frac{1}{\sqrt{5}} \right) \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \frac{6}{\sqrt{5}} \quad (2.0.9)$$

Substituting equation (2.0.5) to (2.0.9) in (2.0.4),

$$\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & -\frac{2}{\sqrt{5}} \\ 0 & \frac{6}{\sqrt{5}} \end{pmatrix} \quad (2.0.10)$$

## 3 SOLUTION

The QR decomposition is,

$$\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & -\frac{2}{\sqrt{5}} \\ 0 & \frac{6}{\sqrt{5}} \end{pmatrix} \quad (3.0.1)$$