Assignment 4

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***Abstract*—This document explains the concept of computing the determinant of a matrix given.**

Download all python codes from

```
https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment4/
    code
```

and latex-tikz codes from

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https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment4
```

1 Problem

If a, b and c are real numbers, and

$$\triangle = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0,$$

Show that either $a+b+c=0$ or $a=b=c$.

2 Explanation

Given,

$$\triangle = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0 \tag{2.0.1}$$

let us perform column operation on equation (2.0.1): $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} = 0 \tag{2.0.2}$$

Taking $2(a+b+c)$ common from C_1 in (2.0.2),

$$2(a+b+c)\begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix} = 0 \tag{2.0.3}$$

Let us perform row operations on equation (2.0.3), $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$2(a+b+c)\begin{vmatrix} 0 & c-b & a-c \\ 0 & a-c & b-a \\ 1 & b+c & c+a \end{vmatrix} = 0 \tag{2.0.4}$$

On expanding determinant along first column from equation (2.0.4),

$$2(a+b+c)[(c-b)(b-a)-(a-c)^2] = 0 \tag{2.0.5}$$

$$\implies 2(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = 0 \tag{2.0.6}$$

$$\implies (a+b+c)(2a^2+2b^2+2c^2-2ab-2bc-2ca) = 0 \tag{2.0.7}$$

$$\implies (a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2] = 0 \tag{2.0.8}$$

From equation (2.0.8) we get 2 equations,

$$\implies \boxed{(a+b+c)=0} \tag{2.0.9}$$

or,

$$\implies (a-b)^2+(b-c)^2+(c-a)^2 = 0 \tag{2.0.10}$$

Equation (2.0.10) is possible only when, $a=b=c$

$$\implies \boxed{a=b=c} \tag{2.0.11}$$

From equation (2.0.9) and (2.0.11) we can say that, $\triangle = 0$ if $a+b+c=0$ or $a=b=c$.

3 Solution

From equation (2.0.9) and (2.0.11) we can say that, $\triangle = 0$ if $a+b+c=0$ or $a=b=c$.