Challenge Problem

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***Abstract*—This document shows for what type of matrices V spectral decomposition produces a matrix P that is orthogonal.**

Download latex-tikz codes from

https://github.com/harshachinta/EE5609−Matrix−Theory/tree/master/Challenges/challenge_5

1 Problem

$\mathbf{V} = \mathbf{PDP^T}$, with $\mathbf{P^TP} = \mathbf{I}$. So $\mathbf{P}$ is an orthogonal matrix. For what matrices $\mathbf{V}$ do you get this kind of decomposition where $\mathbf{P}$ is an orthogonal ?

2 Explanation

Let us consider a matrix $\mathbf{V}$ and given $\mathbf{P}$ is an orthogonal matrix.
So spectral decomposition of matrix $\mathbf{V}$ can be written as,

$$\mathbf{P}^T\mathbf{VP} = \mathbf{D} \tag{2.0.1}$$

Since $\mathbf{P}$ is orthogonal, $\mathbf{PP^T} = \mathbf{P^TP} = \mathbf{I}$

$$\implies \mathbf{V} = \mathbf{PDP}^T \tag{2.0.2}$$

Applying transpose to equation (2.0.2),

$$\mathbf{V^T} = (\mathbf{PDP^T})^\mathbf{T} \tag{2.0.3}$$

$$\implies \mathbf{V^T} = (\mathbf{P^T})^\mathbf{T}\mathbf{D^TP^T} \tag{2.0.4}$$

$$\implies \mathbf{V^T} = \mathbf{PDP^T} \tag{2.0.5}$$

Comparing equations (2.0.2) and (2.0.5),

$$\implies \mathbf{V^T} = \mathbf{V} \tag{2.0.6}$$

Hence $\mathbf{V}$ is a symmetric matrix.

3 Solution

$\mathbf{V}$ is a symmetric matrix.