Assignment 14

Sri Harsha CH

Abstract—**This document explains the conditions to check for a sub space.**

Download all python codes from

```
https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment14
    /code
```

and latex-tikz codes from

```
https://github.com/harshachinta/EE5609−Matrix−
    Theory/tree/master/Assignments/Assignment14
```

1 Problem

Let $\mathbf{V}$ be the vector space of all functions from $\mathbf{R}$ into $\mathbf{R}$; let $\mathbf{V_e}$ be the subset of even functions, $f(-x) = f(x)$; let $\mathbf{V_o}$ be the subset of odd functions, $f(-x) = -f(x)$.

1) Prove that $\mathbf{V_e}$ and $\mathbf{V_o}$ are subspaces of $\mathbf{V}$
2) Prove that $\mathbf{V_e} + \mathbf{V_o} = \mathbf{V}$
3) Prove that $\mathbf{V_e} \cap \mathbf{V_o} = \{0\}$

2 Explanation

1) Prove that $\mathbf{V_e}$ and $\mathbf{V_o}$ are subspaces of $\mathbf{V}$.

A non-empty subset $\mathbf{W}$ of $\mathbf{V}$ is a subspace of $\mathbf{V}$ if and only if for each pair of vectors α, β in $\mathbf{W}$ and each scalar c in $\mathbf{F}$ the vector $c\alpha + \beta$ is again in $\mathbf{W}$.

Let $\mathbf{u}$, $\mathbf{v} \in \mathbf{V_e}$ and $c \in \mathbf{R}$ and let $\mathbf{h} = c\mathbf{u} + \mathbf{v}$. Then,

$$\begin{aligned} \mathbf{h}(-x) &= c\mathbf{u}(-x) + \mathbf{v}(-x) \\ &= c\mathbf{u}(x) + \mathbf{v}(x) \\ &= \mathbf{h}(x) \end{aligned} \tag{2.0.1}$$

From (2.0.1)

$$\implies \mathbf{h}(-x) = \mathbf{h}(x) \tag{2.0.2}$$
$$\implies \mathbf{h} \in \mathbf{V_e} \tag{2.0.3}$$

Let $\mathbf{u}$, $\mathbf{v} \in \mathbf{V_o}$ and $c \in \mathbf{R}$ and let $\mathbf{h} = c\mathbf{u} + \mathbf{v}$. Then,

$$\begin{aligned} \mathbf{h}(-x) &= c\mathbf{u}(-x) + \mathbf{v}(-x) \\ &= -c\mathbf{u}(x) - \mathbf{v}(x) \\ &= -\mathbf{h}(x) \end{aligned} \tag{2.0.4}$$

From (2.0.4)

$$\implies \mathbf{h}(-x) = -\mathbf{h}(x) \tag{2.0.5}$$
$$\implies \mathbf{h} \in \mathbf{V_o} \tag{2.0.6}$$

From (2.0.3) and (2.0.6), $\mathbf{V_e}$ and $\mathbf{V_o}$ are subspaces of $\mathbf{V}$.

2) Prove that $\mathbf{V_e} + \mathbf{V_o} = \mathbf{V}$.

Let $\mathbf{u} \in \mathbf{V}$

$$\mathbf{u_e}(x) = \frac{\mathbf{u}(x) + \mathbf{u}(-x)}{2} \tag{2.0.7}$$
$$\mathbf{u_o}(x) = \frac{\mathbf{u}(x) - \mathbf{u}(-x)}{2} \tag{2.0.8}$$

Equation equation (2.0.7) and (2.0.8), $\mathbf{u_e}$ is even and $\mathbf{u_o}$ is odd. Adding both the equations,

$$\mathbf{u} = \mathbf{u_e} + \mathbf{u_o} \tag{2.0.9}$$

3) Prove that $\mathbf{V_e} \cap \mathbf{V_o} = \{0\}$.

Let $\mathbf{u} \in \mathbf{V_e} \cap \mathbf{V_o}$

$$\mathbf{u} \in \mathbf{V_e} \implies \mathbf{u}(-x) = \mathbf{u}(x) \tag{2.0.10}$$
$$\mathbf{u} \in \mathbf{V_o} \implies \mathbf{u}(-x) = -\mathbf{u}(x) \tag{2.0.11}$$

Equating (2.0.10) and (2.0.11),

$$\mathbf{u}(x) = -\mathbf{u}(x) \tag{2.0.12}$$
$$\implies 2\mathbf{u}(x) = 0 \tag{2.0.13}$$
$$\implies \mathbf{u} = 0 \tag{2.0.14}$$

3 Solution

Equations (2.0.3), (2.0.6), (2.0.9), (2.0.14) proves 1, 2 and 3.