Assignment 5

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Abstract—**This document explains one of the property of triangles.**

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1 Problem

The line-segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.

2 Explanation

Let us consider a $\triangle ABC$, and let **D**,**E** and **F** be the mid- points of sides AB,BC and CA respectively.
Let us consider a line-segment joining the points **D** and **F** which are midpoints of line AB and CA.
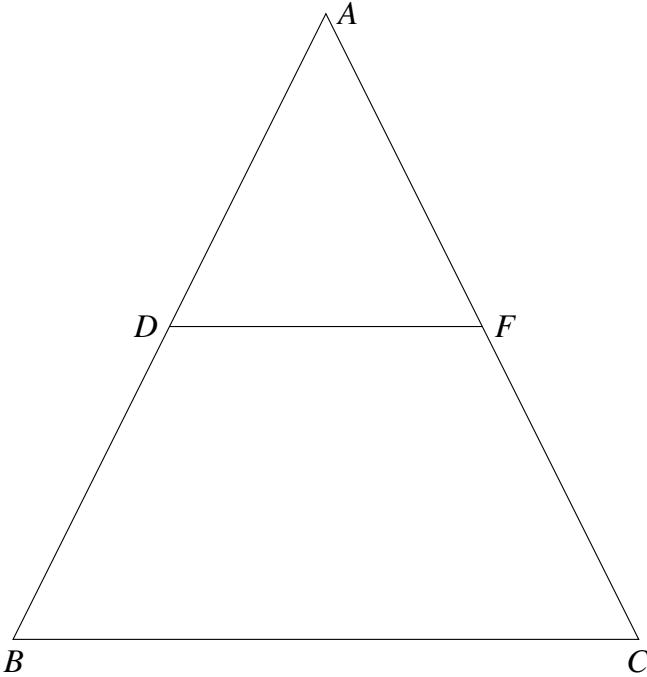

Fig. 1: Line segment DF joining mid-points of 2 sides of $\triangle ABC$

As **D** is midpoint of line AB, **E** is midpoint of line BC and **F** is midpoint of line CA, they can be written as follows:

$$\mathbf{D} = \frac{A + B}{2} \tag{2.0.1}$$

$$\mathbf{F} = \frac{C + A}{2} \tag{2.0.2}$$

The line DF can be written in the form of direction vector as,

$$\begin{aligned}
\mathbf{m}_{DF} &= \mathbf{D} - \mathbf{F} \\
&= \frac{A + B}{2} - \frac{C + A}{2} \\
&= \frac{\mathbf{B} - \mathbf{C}}{2} \\
&= \frac{\mathbf{m}_{BC}}{2}
\end{aligned} \tag{2.0.3}$$

where $\mathbf{m}_{BC}$ is the direction vector of line BC.
Consider equation (2.0.3),

$$\implies \mathbf{D} - \mathbf{F} = \frac{\mathbf{B} - \mathbf{C}}{2} \tag{2.0.4}$$

Applying norm on both sides of equation (2.0.4), we get

$$\|\mathbf{D} - \mathbf{F}\| = \frac{1}{2} \|\mathbf{B} - \mathbf{C}\| \tag{2.0.5}$$

From equation(2.0.3), **DF** ‖ **BC** and from (2.0.5), the line-segment $\mathbf{DF} = \frac{1}{2}\mathbf{BC}$

3 Solution

From equation(2.0.3), **DF** ‖ **BC** and from (2.0.5), the line-segment $\mathbf{DF} = \frac{1}{2}\mathbf{BC}$