END-SEMESTER EXAM

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Abstract

The main objectives of the question are:

- The problem is to find the antenna currents in a half-wave dipole antenna
- We find the value of current through an Antenna using different expressions for Magnetic field H
- One of the expression for the current can be obtained from Ampere's law and other from differentiation of potential vector
- Compare the I value obtained from the above with actual expression, by plotting them.

Dividing the wire

As we are evaluating current in the different parts of the wire we divide the wire of half length l into **2N** pieces, i.e, N pieces on each half and and store them in an array named z. The formula for length of each division and value of each index is given as

$$dz = l/N$$

$$z = i * dz, -N < i < N$$

We use the following python code to perform the above operation for us. We also modify the matrix to get another matrix which doesn't include the pieces containing known values of current

```
dz = l/N
i = 0
z_1 = zeros(2*N +1)
while i<2*N +1:
    z_1[i] = (i-N)*(dz)
```

```
    i+=1
z = z_1
z =delete(z_1,N)
z_1 = z_1[1:-1]
u = z_1 #Locations of unknown currents
```

H from ampere's law

From ampere's law we can get H_ϕ which is given by the equation

$$2 * \pi * a * H_\phi(z_i) = I_i$$

H can be expressed in a matrix form given by $H = MJ$ where J is the current matrix of unknown current values, here we calculate the value of H at r=a, We get the matrix M from the following python code snippet

```
def Mat(N,a):
    M = (1/(2*pi*a))*identity(2*N -2)
    return M
```

Computation of Vector potential

We can get the expression for magnetic field H from vector potential A by taking curl of the vector potential and dividing it by permeability, while computing the expressions for Vector potentials and Magnetic field we need to calculate the distance between observer point and the current element. In the assignment we are asked to define two matrices(vectors) Rz and Ru which are distances between observer and the source while the former computes distances including distances to known currents, while Ru is a vector of distances to unknown currents. The following python code snippet helps in doing computation.

```
zi, zj = meshgrid(z, z)
Rz = sqrt((zi - zj) ** 2 + ones([2 * N + 1, 2 * N + 1], dtype=complex) * a ** 2)
ui, uj = meshgrid(u, u)
Ru = sqrt((ui - uj) ** 2 + ones([2 * N - 2, 2 * N - 2], dtype=complex) * a ** 2)
RiN = delete(Rz[N], [0, N, 2 * N])
```

The expression for Vector potential A can be approximately simplified into a matrix equation which is given by $\sum_j P_{ij} I_j + P_B I_N$, where P_B is vector potential due to contribution of current I_N and P_{ij} due to the unknown current elements , the matrix equations for the matrices can be computed using the following python code snippet.

```
P = (mu0 / (4 * pi)) * (exp(-k * Ru * j) / Ru) * dz
P_B = (mu0 / (4 * pi)) * (exp(-k * RiN * j) / RiN) * dz
```

Computation H from Vector potential

As mentioned earlier H can be computed from vector potential A by taking curl and dividing it by the permeability. Here H is in the direction ofϕ and the expression for H_ϕ can be approximated to a matrix equation given by $\sum_j Q_{ij}J_j + Q_B I_m$, and thus we have obtained another expression for H_ϕ, now equating the right hand side of the two equations we can compute the current vector and thus values of currents. The following python code snippet helps in computing the matrices for both Q and Q_B

```
Q = -(a / mu0) * P * ((-k * j / Ru) + (-1 / Ru ** 2))
Qb = -(a / mu0) * P_B * ((-k * j / RiN) + (-1 / RiN ** 2))
```

Final equation to compute J

The final equation to compute the matrix J can be obtained as mentioned earlier and is given by

$$MJ = QJ + Q_B I_M$$

$$(M - Q)J = Q_B I_M$$

The above equations can be easily solved and we get the matrix J and to obtain complete matrix for I we add the known current elements as well to get it.

```
J = matmul(inv(Mat(N,a)-Q),Qb)*(Im)
I = J
I = insert(I,0,0)
I = append(I,0)
I = insert(I,N,1)
```

Printing all matrices for N=4

It is asked in the question to print all the vectors that are being calculated for N=4, the following images , contain the printed matrices.

```
z =
[-0.5  -0.38 -0.25 -0.12  0.    0.12  0.25  0.38  0.5 ]


u =
[-0.38 -0.25 -0.12  0.12  0.25  0.38]


Rz =
[[0.01+0.j 0.13+0.j 0.25+0.j 0.38+0.j 0.5 +0.j 0.63+0.j 0.75+0.j 0.88+0.j
  1.  +0.j]
 [0.13+0.j 0.01+0.j 0.13+0.j 0.25+0.j 0.38+0.j 0.5 +0.j 0.63+0.j 0.75+0.j
  0.88+0.j]
 [0.25+0.j 0.13+0.j 0.01+0.j 0.13+0.j 0.25+0.j 0.38+0.j 0.5 +0.j 0.63+0.j
  0.75+0.j]
 [0.38+0.j 0.25+0.j 0.13+0.j 0.01+0.j 0.13+0.j 0.25+0.j 0.38+0.j 0.5 +0.j
  0.63+0.j]
 [0.5 +0.j 0.38+0.j 0.25+0.j 0.13+0.j 0.01+0.j 0.13+0.j 0.25+0.j 0.38+0.j
  0.5 +0.j]
 [0.63+0.j 0.5 +0.j 0.38+0.j 0.25+0.j 0.13+0.j 0.01+0.j 0.13+0.j 0.25+0.j
  0.38+0.j]
 [0.75+0.j 0.63+0.j 0.5 +0.j 0.38+0.j 0.25+0.j 0.13+0.j 0.01+0.j 0.13+0.j
  0.25+0.j]
 [0.88+0.j 0.75+0.j 0.63+0.j 0.5 +0.j 0.38+0.j 0.25+0.j 0.13+0.j 0.01+0.j
  0.13+0.j]
 [1.  +0.j 0.88+0.j 0.75+0.j 0.63+0.j 0.5 +0.j 0.38+0.j 0.25+0.j 0.13+0.j
  0.01+0.j]]
```

Figure 1: Vector values calculated

```
Ru =
[[0.01+0.j 0.13+0.j 0.25+0.j 0.5 +0.j 0.63+0.j 0.75+0.j]
 [0.13+0.j 0.01+0.j 0.13+0.j 0.38+0.j 0.5 +0.j 0.63+0.j]
 [0.25+0.j 0.13+0.j 0.01+0.j 0.25+0.j 0.38+0.j 0.5 +0.j]
 [0.5 +0.j 0.38+0.j 0.25+0.j 0.01+0.j 0.13+0.j 0.25+0.j]
 [0.63+0.j 0.5 +0.j 0.38+0.j 0.13+0.j 0.01+0.j 0.13+0.j]
 [0.75+0.j 0.63+0.j 0.5 +0.j 0.25+0.j 0.13+0.j 0.01+0.j]]


P_B =
[0.-0.j 0.-0.j 0.-0.j 0.-0.j 0.-0.j 0.-0.j]


P*1e8 =
[[124.94-3.93j   9.2 -3.83j   3.53-3.53j  -0.  -2.5j   -0.77-1.85j
   -1.18-1.18j]
 [  9.2 -3.83j 124.94-3.93j   9.2 -3.83j   1.27-3.08j  -0.  -2.5j
   -0.77-1.85j]
 [  3.53-3.53j   9.2 -3.83j 124.94-3.93j   3.53-3.53j   1.27-3.08j
   -0.  -2.5j ]
 [ -0.  -2.5j    1.27-3.08j   3.53-3.53j 124.94-3.93j   9.2 -3.83j
    3.53-3.53j]
 [ -0.77-1.85j  -0.  -2.5j    1.27-3.08j   9.2 -3.83j 124.94-3.93j
    9.2 -3.83j]
 [ -1.18-1.18j  -0.77-1.85j  -0.  -2.5j    3.53-3.53j   9.2 -3.83j
  124.94-3.93j]]
```

Figure 2: Vector values calculated

```
Q =
[[9.952e+01-0.j 5.000e-02-0.j 1.000e-02-0.j 0.000e+00-0.j 0.000e+00-0.j
  0.000e+00-0.j]
 [5.000e-02-0.j 9.952e+01-0.j 5.000e-02-0.j 0.000e+00-0.j 0.000e+00-0.j
  0.000e+00-0.j]
 [1.000e-02-0.j 5.000e-02-0.j 9.952e+01-0.j 1.000e-02-0.j 0.000e+00-0.j
  0.000e+00-0.j]
 [0.000e+00-0.j 0.000e+00-0.j 1.000e-02-0.j 9.952e+01-0.j 5.000e-02-0.j
  1.000e-02-0.j]
 [0.000e+00-0.j 0.000e+00-0.j 0.000e+00-0.j 5.000e-02-0.j 9.952e+01-0.j
  5.000e-02-0.j]
 [0.000e+00-0.j 0.000e+00-0.j 0.000e+00-0.j 1.000e-02-0.j 5.000e-02-0.j
  9.952e+01-0.j]]


Qb =
[0.  -0.j 0.01-0.j 0.05-0.j 0.05-0.j 0.01-0.j 0.  -0.j]


J =
[-0.+0.j -0.+0.j -0.+0.j -0.+0.j -0.+0.j -0.+0.j]


I =
[ 0.+0.j -0.+0.j -0.+0.j -0.+0.j  1.+0.j -0.+0.j -0.+0.j -0.+0.j  0.+0.j]


I_actual =
[0.   0.38 0.71 0.92 1.   0.92 0.71 0.38 0.  ]
```

Figure 3: Vector values calculated

Plotting the calculated and actual values of I

Now we check the dissimilarities between the calculated value of I from the above equations and the original formula of I by plotting them. The original value(s) for I are given by the following equations

$$I = I_m \sin(k(l - z)) 0 \leq z \leq l$$

$$I = I_m \sin(k(l + z)) - l \leq z \leq 0$$

The following python code snippet helps in calculating the actual values of I and plot them,

```
I_act = zeros(2*N + 1)
i=0
while i<2*N +1:
    if z[i]<=0:
        I_act[i] = Im*sin(k*(l+z[i]))
    elif z[i]>0:
        I_act[i] = Im*sin(k*(l-z[i]))
    i+=1
figure(num=1,figsize=(7,7))
plot(z,I_act,color='r',label="Iactual")
xlabel(r'$z$')
ylabel(r'$I$')
title("Actual current calculated from formula ")
legend()
grid(True)
```

Now the actual current plot and approximated current looks as shown below

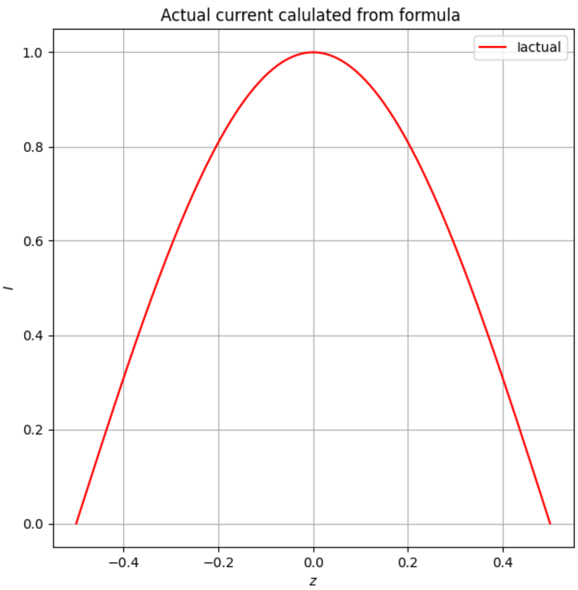

Figure 4: Actual Current

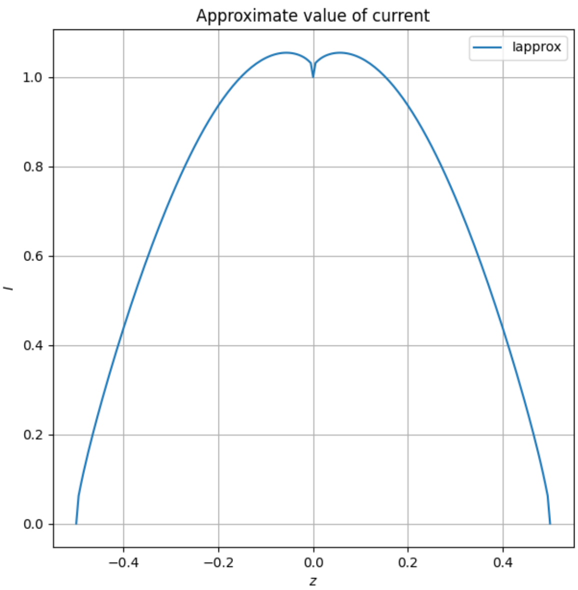

Figure 5: Approx calculated current

Conclusion

The actual plot of current and the Approximately calculated plot of current both are nearly same, the approximated plot has values slightly above the actual values at a given z , this issue might be resolved to a good extent if the number of parts in which wire is divided is increased.