Assignment 8 : The Digital Fourier Transform

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Abstract

The main aim of the assignment 8 is

- Understand what is DFT,FFT and implement in python
- Obtaining DFT of various functions
- Observe how close DFT is to CTFT

We find the spectrum of various functions in this assignment.

Question 1:

Here we are asked to repat the examples given in the assignment. The different functions given are $f(t) = sin(5t)$ and $f(t) = (1+0.1\cos(t))\cos(10t)$

Spectrum of $sin(5t)$

As already mentioned in the assignemnt the following code snippet is used to get spectrum of $sin(5t)$

```
t_1 = linspace(0,2*pi,129); t_1 = t_1[:-1]
y_1 = sin(5*t_1)
Y_1 = fftshift(fft(y_1))/128
w_1 = linspace(-64,63,128)
figure(num=0,figsize=(7,7))
subplot(2,1,1)
plot(w_1,abs(Y_1))
title(r"Spectrum of $\sin(5t)$",loc='left')
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
```

```
xlim([-10,10])
grid(True)
subplot(2,1,2)
plot(w_1,angle(Y_1),'ro')
ii = where(abs(Y_1)>1e-3)
plot(w_1[ii],angle(Y_1[ii]),'go')
title(r"Phase of $\sin(5t)$",loc = 'left')
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-10,10])
grid(True)
show()
```

As expected we get 2 peaks at +5 and -5 with height 0.5. The phases of the peaks at $\frac{\pi}{2}$ and $-\frac{\pi}{2}$ are also expected based on the expansion of a sine wave ie:

$$sin(5t) = 0.5(\frac{e^{5t}}{j} - \frac{e^{-5t}}{j}) \tag{1}$$

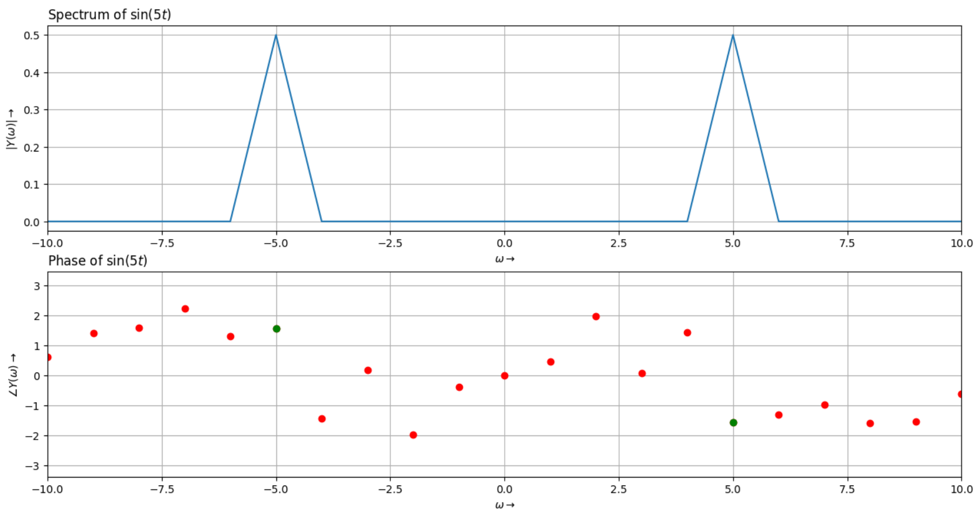

Figure 1: Spectrum of sin(5t)

Spectrum of Amplitude Modulated Wave

The function we want to analyse is

$$f(t) = (1 + 0.1\cos(t))\cos(10t) \tag{2}$$

The following code snippet helps to get the spectrum of given function

```
t_2 = linspace(-4*pi,4*pi,513); t_2 = t_2[:-1]
y_2 = (1 + 0.1*cos(t_2))*cos(10*t_2)
Y_2 = fftshift(fft(y_2))/512
w_2 = linspace(-64,64,513); w_2 = w_2[:-1]
figure(num=1,figsize=(7,7))
subplot(2,1,1)
plot(w_2,abs(Y_2))
title(r"Spectrum of $(1 + 0.1*cos(t))*cos(10*t)$",loc='left')
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-15,15])
grid(True)
subplot(2,1,2)
plot(w_2,angle(Y_2),'ro')
title(r"Phase of $(1 + 0.1*cos(t))*cos(10*t)$",loc='left')
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-15,15])
grid(True)
show()
```

Taking more samples here gives us more clear spectrum

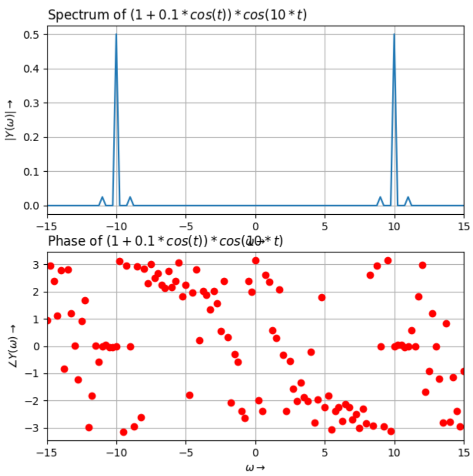

Figure 2: Spectrum of Spectrum of $f(t) = (1 + 0.1\cos(t))\cos(10t)$

Spectrum of $sin^3(t)$

This signal can be expressed as a sum of sine waves using this identity:
$\sin^3(t) = \frac{3}{4}\sin(t) - \frac{1}{4}\sin(3t)$
The following python code snippet helps to plot the spectrum of $sin^3(t)$

```
t_3 = linspace(-4*pi,4*pi,513); t_3 = t_3[:-1]
y_3 = (3*sin(t_3) - sin(3*t_3))/4
Y_3 = fftshift(fft(y_3))/512
w_3 = linspace(-64,64,513); w_3 = w_3[:-1]
figure(num=2,figsize=(7,7))
subplot(2,1,1)
plot(w_3,abs(Y_3))
title(r"Spectrum of $sin^{3}(t)$",loc='left')
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-15,15])
grid(True)
subplot(2,1,2)
plot(w_3,angle(Y_3),'ro')
title(r"Phase of $sin^{3}(t)$",loc='left')
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-15,15])
grid(True)
show()
```

We expect 2 peaks at frequencies 1 and 3, and phases similar to that expected from a sum of sinusoids.

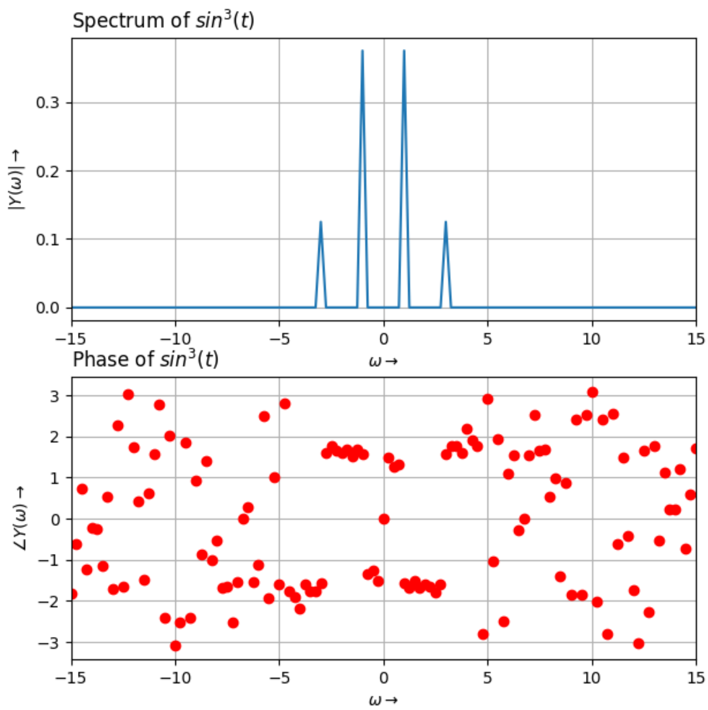

Figure 3: Spectrum of $f(t) = sin^3(t)$

Spectrum of $cos^3(t)$

This signal can be expressed as a sum of cosine waves using this identity:
$\cos^3(t) = \frac{3}{4}\cos(t) + \frac{1}{4}\cos(3t)$
The following python code snippet helps to plot the spectrum of $cos^3(t)$

```
y_4 = (3*cos(t_3) + cos(3*t_3))/4
Y_4 = fftshift(fft(y_4))/512; w_4 = w_3
figure(num=3,figsize=(7,7))
subplot(2,1,1)
plot(w_4,abs(Y_4))
title(r"Spectrum of $cos^{3}(t)$",loc='left')
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-15,15])
grid(True)
subplot(2,1,2)
plot(w_4,angle(Y_4),'ro')
title(r"Phase of $cos^{3}(t)$",loc='left')
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-15,15])
grid(True)
show()
```

We expect 2 peaks at frequencies 1 and 3, and phase=0 at the peaks.

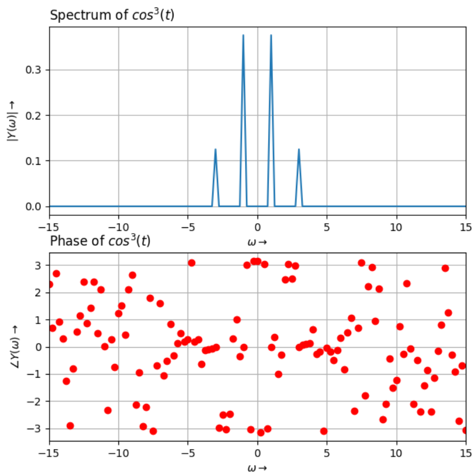

Figure 4: Spectrum of $f(t) = cos^3(t)$

Spectrum of Frequency Modulated Wave

Consider the signal:

$$f(t) = \cos(20t + 5\cos(t)) \tag{3}$$

The following code snippet is used to plot the spectrum of given function

```
y_5 = cos(20*t_3 + 5*cos(t_3))
Y_5 = fftshift(fft(y_5))/512; w_5 = w_4
figure(num=4,figsize=(7,7))
subplot(2,1,1)
plot(w_5,abs(Y_5))
title(r"Spectrum of $cos(20t + 5cos(t))$",loc='left')
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-40,40])
grid(True)
subplot(2,1,2)
ii = where(abs(Y_5)>1e-3)
plot(w_5[ii],angle(Y_5[ii]),'go')
title(r"Phase of $cos(20t + 5cos(t))$",loc = 'left')
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-40,40])
grid(True)
show()
```

The number of peaks have increased and energy in the side bands is comparable to that of the main signal.

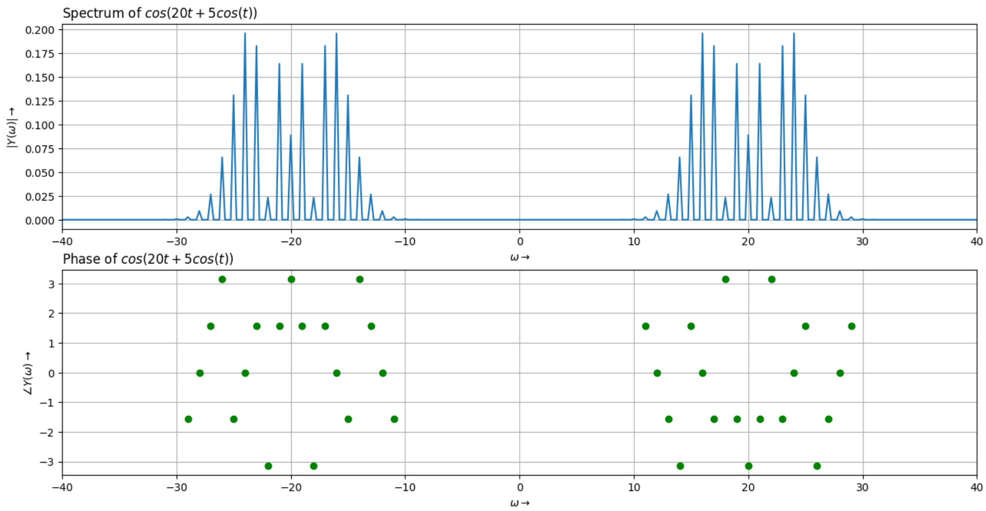

Figure 5: Spectrum of $f(t) = \cos(20t + 5\cos(t))$

Continuous time Fourier Transform of a Gaussian

For the given guassian we use a while loop and compare the calculkated function with a certain number of samples with actual fourier transform and The expression for the Gaussian is :

$$x(t) = e^{\frac{-t^2}{2}} \tag{4}$$

The CTFT is given by:

$$X(j\omega) = \frac{1}{\sqrt{2\pi}} e^{\frac{-\omega^2}{2}} \tag{5}$$

The following python code snippet helps to calculate best fit for number of samples.

```
T = 2*pi
N = 128
iter = 0
tolerance = 1e-15
error = tolerance + 1
while error > tolerance:
    t = linspace(-T/2,T/2,N+1)[:-1]
    w = N/T * linspace(-pi,pi,N+1)[:-1]
    y = exp(-0.5*t**2)
    iter = iter + 1

    Y = fftshift(fft(y))*T/(2*pi*N)
    Y_actual = (1/sqrt(2*pi))*exp(-0.5*w**2)
    error = mean(abs(abs(Y)-Y_actual))

    T = T*2
    N = N*2
print(" Error: %g \n Iteration: %g" % (error,iter))
print(" Best value of T: %g*pi \n Best value of N: %g"%(T/pi,N))
```

Best value of T: 16π
Best value of N: 1024
Now we plot the spectrum using the following python code snippet

```
figure(num=5,figsize=(7,7))
subplot(2,1,1)
plot(w,abs(Y))
title(r"Spectrum of Guassian $exp(-0.5t^2)$",loc='left')
ylabel(r"$|Y(\omega)|\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-10,10])
```

```
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro')
title(r"Phase of $exp(-0.5t^2)$",loc='left')
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlabel(r"$\omega\rightarrow$")
xlim([-10,10])
grid(True)
show()
```

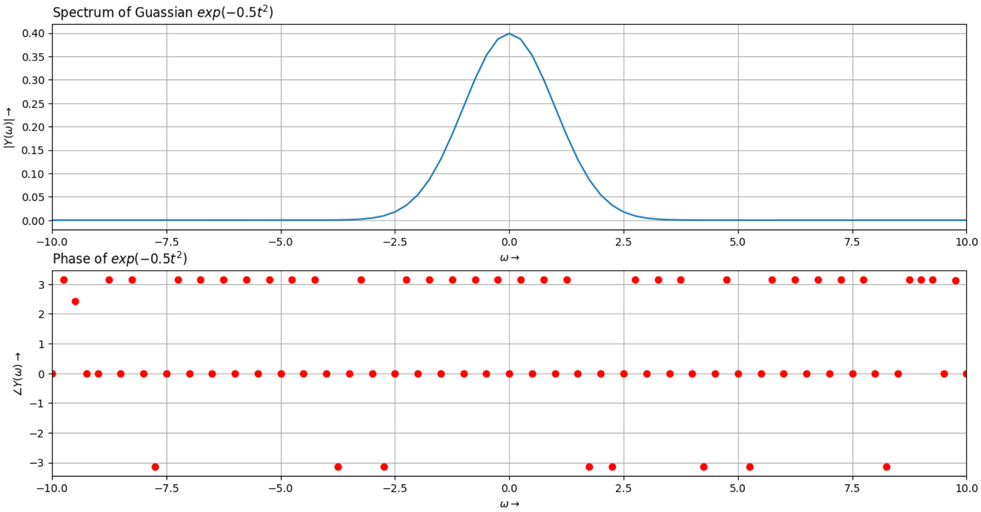

Figure 6: estimated CTFT of Gaussian

Conclusion

We analysed on how to find DFT for various kinds of signals and how to estimate nomalising factor for Guassian function, finding the sampling rate by minimising the error and using FFT (Fast Fourier Transform) to compute DFT