Assignemnt 5: Laplace equation

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1 Abstract

The main aim of the assignment on Laplace equation is

- Understanding the Laplace equation
- Solving the Resistor problem using Laplace equation
- Implementing iterative method to solve Laplace equation
- Analysing the errors

2 Introduction

- A cylindrical wire is soldered to the middle of copper plate and its voltage is held at 1V. One side of the plate is grounded, while the remaining are floating. The plate is 1 cm by 1 cm in size.
- The continuity equation
$$\nabla.\vec{J} = -\frac{\partial \rho}{\partial t} \tag{1}$$
- Ohms law in dfferential form
$$\vec{J} = \sigma \vec{E} \tag{2}$$
- Taking potential as gradient of electric field and using above equations we get
$$\nabla^2 \phi = \frac{1}{\sigma}\frac{\partial \rho}{\partial t} \tag{3}$$
- For DC currents RHS is zero, hence we get
$$\nabla^2 \phi = 0 \tag{4}$$
- Simplifying the Laple equation we get
$$\phi_{i,j} = \frac{\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}}{4} \tag{5}$$

3 Defining potential array and initialising it

We define the parameteres N_x and N_y to be 25 and take the number of iterations to be 1500, if user gives the input then we take parameter values from the user, the following python code snippet is used to do it.

```
if len(sys.argv) == 5:
    Nx = int(sys.argv[1])
    Ny = int(sys.argv[2])
    radius = int(sys.argv[3])
    Niter  = int(sys.argv[4])
else:
    Nx = 25
    Ny = 25
    radius = 8
    Niter = 1500
```

Now we initialise the matrix phi(ϕ) using the following python code snippet

```
phi = zeros((Ny,Nx))
x = linspace(-0.5,0.5,Nx)
y = linspace(-0.5,0.5,Ny)
Y,X = meshgrid(y,x)
R = (X**2) + (Y**2)
ii = where(R <= (0.35*0.35))
phi[ii] = 1.0
```

The plot of initial potential contour is as shown:

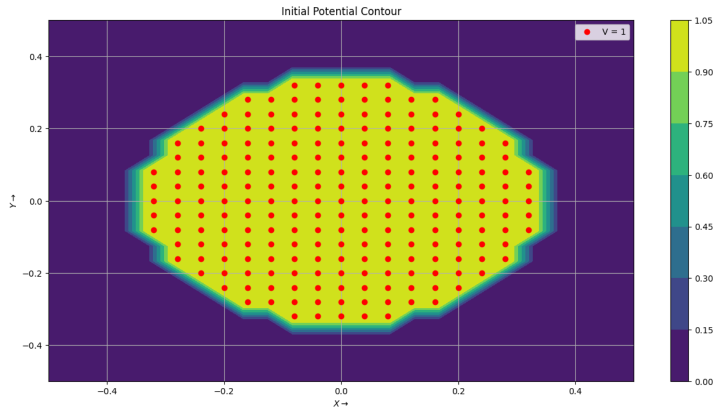

Figure 1: Data Plot

4 Updating nad iterating potential

Using equation(5) we iterate the potential array and then we add boundary conditions to it and calculate the error, we use the following python code snippet to do it.

```
#Updating the potential
for i in range(Niter):
    phiold = phi.copy()
    phi[1:-1,1:-1] = 0.25*(phiold[1:-1,0:-2] + phiold[1:-1,2:] +
                                         phiold[0:-2,1:-1] + phiold[2:,1:-1])
#Set appropriate boundary conditions
    phi[:,0]=phi[:,1]
    phi[:,Nx-1]=phi[:,Nx-2]
    phi[0,:]=phi[1,:]
    phi[Ny-1,:]=0
    phi[ii] = 1
    errors[i]=(abs(phi-phiold)).max()
```

5 Error Calculations

We plot the errors which we calculated as shown in above code snippet and plot it on both log-log and semilog scale.
The plots are:

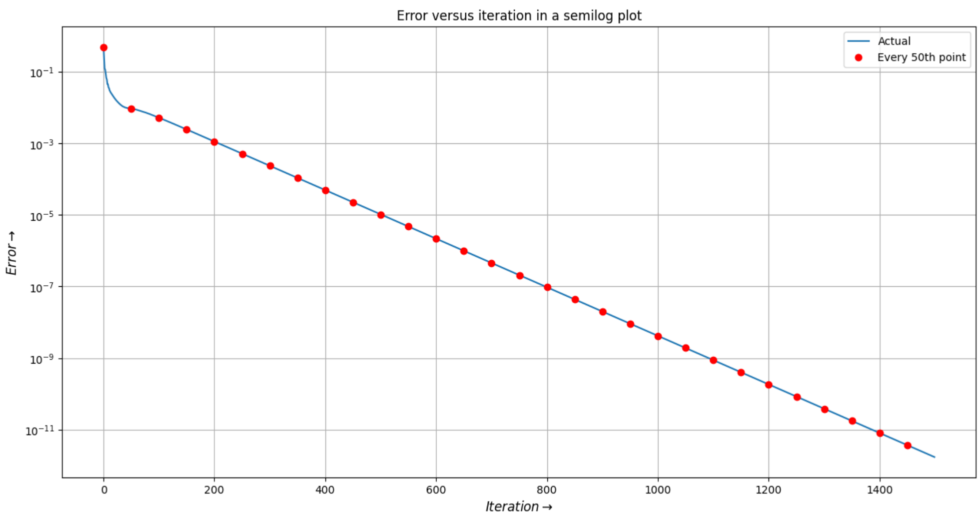

Figure 2: Error Plot

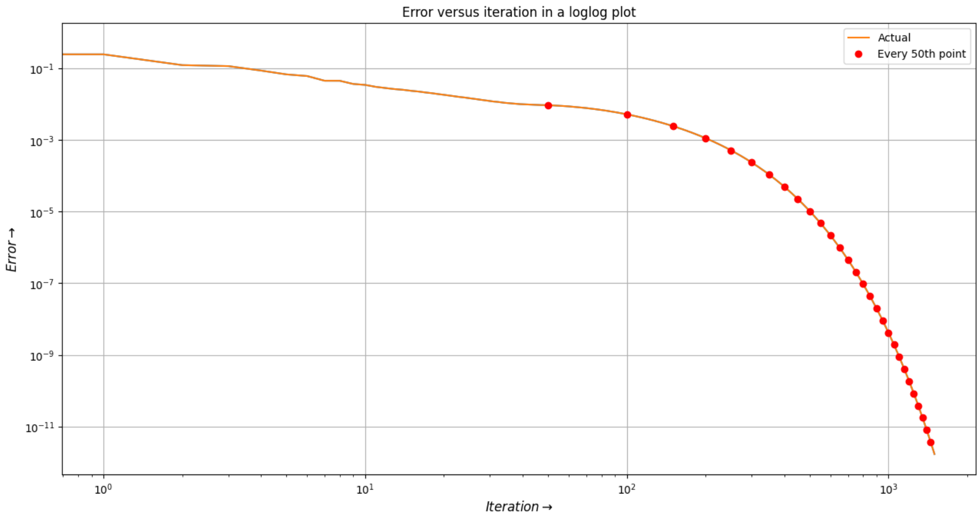

Figure 3: Error Plot

6 Best fit for error

We use the least squares method to calculate the errors and plot them along side with actual errors, for large number of iterations the error decays exponentially. In one case we consider all iterations and in other case we consider iterations above 500, the following python code snippet helps us to do it

```
log_y_mat = log(transpose(errors))
x_mat = c_[arange(0,Niter),ones(shape = [Niter,1])]
E1 = lstsq(x_mat,log_y_mat,rcond=None)[0]
E2 = lstsq(x_mat[500:-1],log_y_mat[500:-1],rcond=None)[0]
```

Now we plot the above plots:

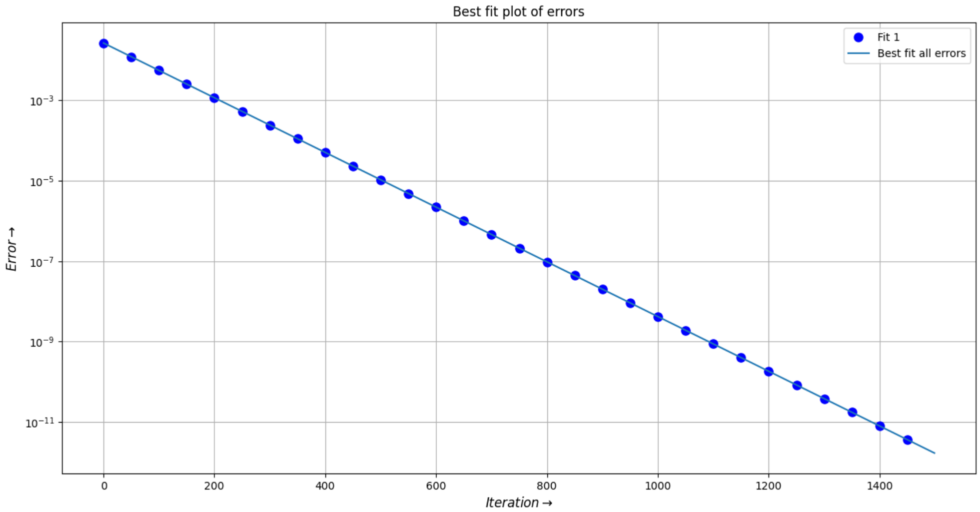

Figure 4: Least squares Error Plot

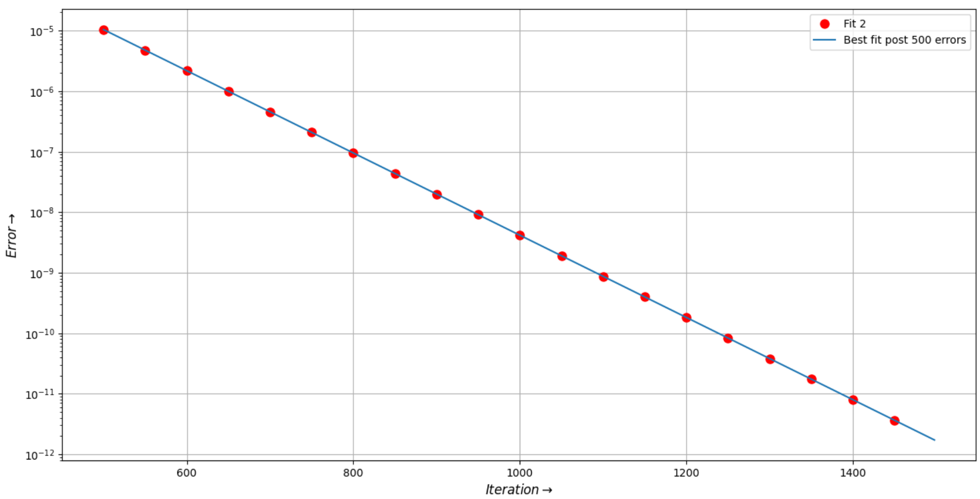

Figure 5: Least squares Error Plot

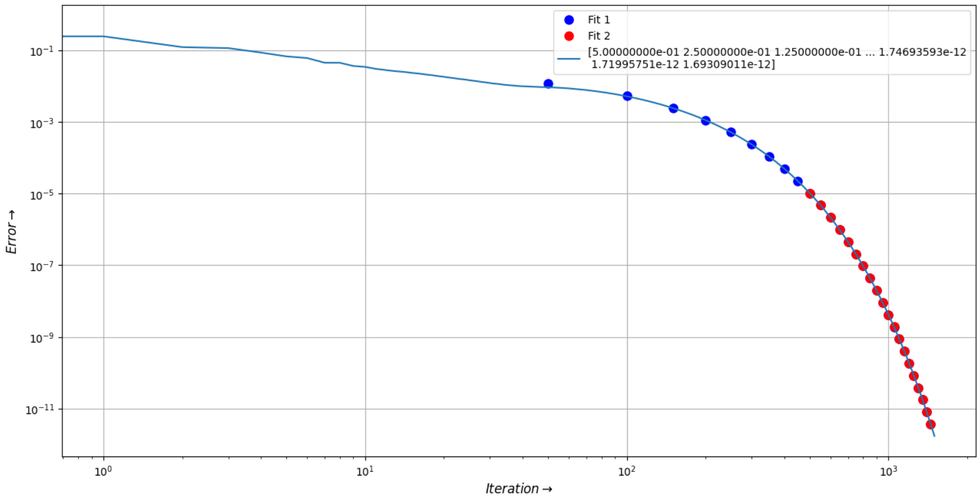

Figure 6: Error Plot

7 Plotting *phi*

Now we plot both the 2-D contour and 3-D surface plot of the potential.

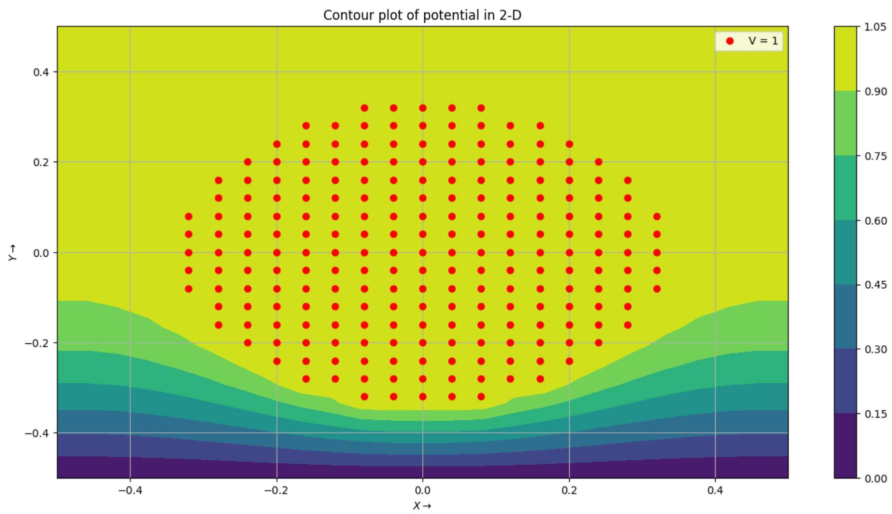

Figure 7: 2-D contour plot of potential

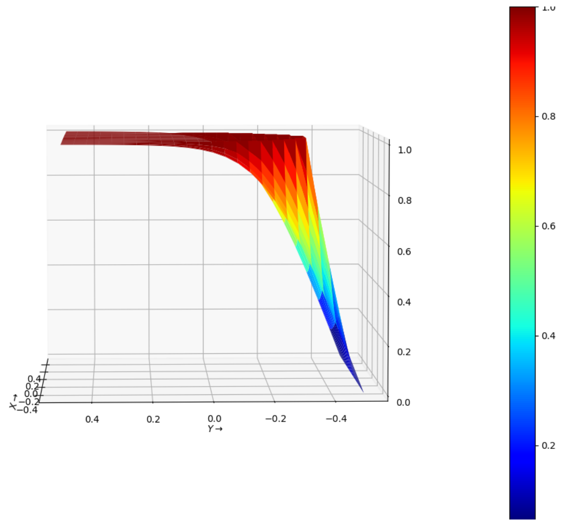

Figure 8: 3-D surface plot of potential

8 Plotting Current density

Current density profiles are given by

$$J_x = -\frac{\partial \phi}{\partial x} \tag{6}$$

$$J_y = -\frac{\partial \phi}{\partial y} \tag{7}$$

We use the following equations in our code:

$$J_{x,ij} = \frac{1}{2}(\phi_{i,j-1} - \phi_{i,j+1}) \tag{8}$$

$$J_{y,ij} = \frac{1}{2}(\phi_{i-1,j} - \phi_{i+1,j}) \tag{9}$$

The Python code snippet to find them is

```
Jx,Jy = (1/2*(phi[1:-1,0:-2]-phi[1:-1,2:]),1/2*(phi[:-2,1:-1]-phi[2:,1:-1]))
```

The vector plot of current flow is:

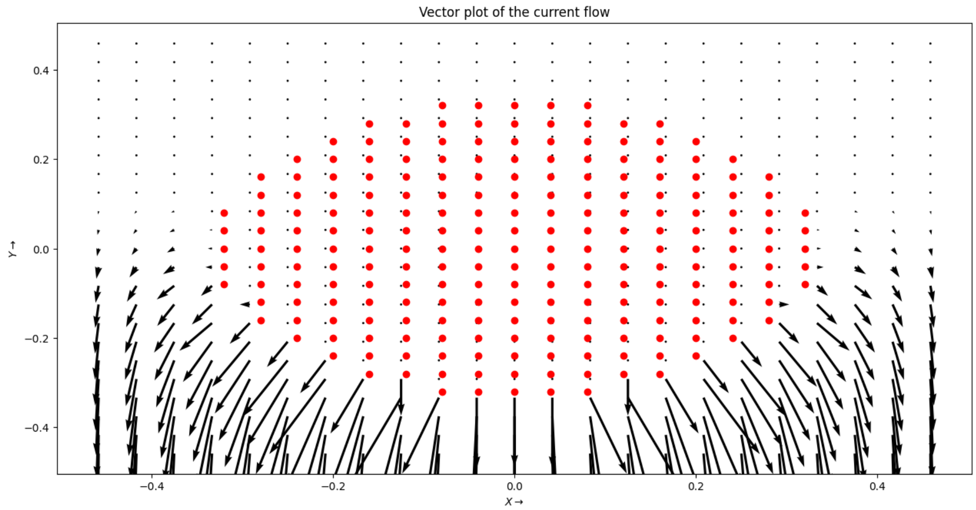

Figure 9: Vector Plot of current flow

9 Conclsion

The current is almost zero in the uppermost part of the plate, the bottom part of the plate gets hotter and temperature increases in down region of the plate.we observe that the best method to solve this is to increase N_x and N_y to very high values (100 or ≥ 100) and increase the number of iterations too, so that we get accurate currents, but with a tradeoff of more computation time and slow decrease in errors with iterations answers i.e currents in the resistor.