JEE-MAIN EXAMINATION – JANUARY 2026

(23ST JANUARY 2026) **TIME: 2:00 P.M. TO 5:00 P.M**

MATHEMATICS TEST PAPER WITH SOLUTION

1. Let $\mathrm{I}(x)=\int \frac{3dx}{(4x+6)\left(\sqrt{4x^2+8x+3}\right)}$ and

$\mathrm{I}(0)=\frac{\sqrt{3}}{4}+20$. If $\mathrm{I}\left(\frac{1}{2}\right)=\frac{a\sqrt{2}}{b}+c$, where $a,b,c\in N,\gcd(a,b)=1$, then $a+b+c$ is equal to

(1) 30
(2) 31
(3) 29
(4) 28

Answer (2)

Sol. $I(x)=\int \frac{3dx}{(4x+6)\left(\sqrt{4x^2+8x+3}\right)}$

$=\frac{3}{2}\int \frac{dx}{(2x+3)\sqrt{4x^2+8x+3}}$

Let $t=\frac{1}{2x+3}$

$dt=\frac{-dt}{(2x+3)^2}$

$x=\frac{\frac{1}{t}-3}{2}$

Substitute

$=-\frac{3}{2}\int \frac{1}{\sqrt{1-2t}}dt$

$=\frac{3}{2}\sqrt{1-2t}$

$=\frac{3}{4}\sqrt{1-\frac{2}{2x+3}}+C$

$I(0)=\frac{\sqrt{3}}{4}+20=\frac{3}{4}\sqrt{1-\frac{2}{3}}+C$

$\Rightarrow C=20$

$I(x)=\frac{3}{4}\sqrt{1-\frac{2}{2x+3}}+20$

$I\left(\frac{1}{2}\right)=\frac{3}{4}\sqrt{1-\frac{2}{4}}+20$

$=20+\frac{3}{4\sqrt{2}}=20+\frac{3\sqrt{2}}{8}$

$a+b+c=20+3+8=31$

2. Let $\vec{a},\vec{b},\vec{c}$ be three vectors such that $\vec{a}\times\vec{b}=2(\vec{a}\times\vec{c})$. If $|\vec{a}|=1,|\vec{b}|=4,|\vec{c}|=2$, and the angle between $\vec{b}$ and $\vec{c}$ is 60°, then $|\vec{a}\cdot\vec{c}|$ is equal to

(1) 0
(2) 1
(3) 2
(4) 4

Answer (2)

Sol. $\vec{a}\times\vec{b}=2(\vec{a}\times\vec{c})$

$\Rightarrow \vec{a}\times(\vec{b}-2\vec{c})=0$

$\Rightarrow \vec{a}$ is parallel to $\vec{b}-2\vec{c}$

$\therefore\ \vec{a} = \lambda(\vec{b} - 2\vec{c})$

$\Rightarrow\ 1 = \lambda^2\left(16 + 16 - 4 \cdot 4 \cdot 2 \cdot \frac{1}{2}\right)$

$\Rightarrow \lambda = \pm\frac{1}{4}$

$\vec{a} \cdot \vec{c} = \lambda(\vec{b} \cdot c - 2|\vec{c}|^2) = \lambda\left(4 \cdot 2 \cdot \frac{1}{2} - 2 \cdot 4\right)$

$= \lambda(4 - 8)$

$|\vec{a} \cdot \vec{c}| = 1$

3. The system of linear equations

$x + y + z = 6$

$2x + 5y + az = 36$

$x + 2y + 3z = b$

has

(1) infinitely many solutions for $a = 8$ and $b = 14$

(2) unique solution for $a = 8$ and $b = 14$

(3) infinitely many solutions for $a = 8$ and $b = 16$

(4) unique solution for $a = 8$ and $b = 16$

Answer (1)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & a \\ 1 & 2 & 3 \end{vmatrix} = 8 - a$

for unique solution $a \neq 8$

if $a = 8$

$x + y + z = 6$ ...(i)

$2x + 5y + 8z = 36$ ...(ii)

$x + 2y + 3z = b$ ...(iii)

3(iii) – (i)

$2x + 5y + 8z = 3b - 6$ ...(iv)

To have infinite solution eq. (ii) and (iv) must be identical.

$3b - 6 = 36$

$b = 14$

4. Consider two sets $A = \{x \in \mathbb{Z}: |(|x-3|-3)^2| \leq 1\}$ and $B = \left\{x \in \mathbb{R} - \{1,2\}: \frac{(x-2)(x-4)}{x-1}\log_e(|x-2|) = 0\right\}$.

Then the number of onto functions $f: A \to B$ is equal to

(1) 81

(2) 79

(3) 62

(4) 32

Answer (3)

Sol. $|(|x-3|-3)| \leq 1$

$-1 \leq |x-3| - 3 \leq 1$

$2 \leq |x-3| \leq 4 \Rightarrow x \in [-1,1] \cup [5,7]$

$\Rightarrow A = \{-1,0,1,5,6,7\}$

$\frac{(x-2)(x-4)}{(x-1)}\log_e(|x-2|) = 0$

$\Rightarrow x = 2,4,3$

$\Rightarrow B = \{3,4\}$

$\Rightarrow$ Number of onto function $f: A \to B$

$= 2^6 - 2 = 62$

$\Rightarrow$ Option (3) is correct.

5. If the mean and the variance of the data

Class	$4-8$	$8-12$	$12-16$	$16-20$
Frequency	3	λ	4	7

are μ and 19 respectively, then the value of $\lambda + \mu$ is

(1) 21

(2) 20
(3) 19
(4) 18

Answer (3)

Sol. $\mu = \frac{3\times b+\lambda\times 10+4\times 14+7\times 18}{14\times\lambda} = \frac{10\lambda+2}{14+\lambda}$

$$19 = \sigma^2 = \frac{3\times 6^2+\lambda(10)^2+4(14)^2+7(18)^2}{14+\lambda} - \left(\frac{10\lambda+200}{14+\lambda}\right)^2$$

$\Rightarrow \lambda = 6$

$\mu = 13$

$\lambda + \mu = 19$

6. The sum of all the real solutions of the equation, $\log_{(x^2+3)}(6x^2+28x+30) = 5 - 2\log_{(6x+10)}(x^2+6x+9)$ is equal to

(1) 2
(2) 0
(3) 1
(4) 4

Answer (2)

Sol. $6x^2+28x+30 = 2(x+3)(3x+5)$

$x^2+6x+9 = (x+3)^2$

$\Rightarrow \log_{(x+3)}(2(x+3)(3x+5))$

$= 5 - 2\log_{2(3x+5)}(x+3)^2$

$\Rightarrow \log_{(x+3)}(x+3) + \log_{(x+3)}(2(3x+5))$

$= 5 - 4\log_{2(3x+5)}(x+3)$

put $\log_{(x+3)}(2(3x+5)) = t$

$\Rightarrow 1 + t = 5 - \frac{4}{t}$

$\Rightarrow t^2 - 4t + 4 = 0 \Rightarrow t = 2$

$\Rightarrow \log_{(x+3)}(2(3x+5)) = 2$

$\Rightarrow 2(3x+5) = (x+3)^2$

$\Rightarrow x^2+9+6x = 6x+10 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$\Rightarrow$ Sum of all real solution $= 1 - 1 = 0$

$\Rightarrow$ option (2) is correct.

7. The number of ways, in which 16 oranges can be distributed to four children such that each child gets at least one orange, is

(1) 455
(2) 384
(3) 403
(4) 429

Answer (1)

Sol. $x_1 + x_2 + x_3 + x_4 = 18$

$x_i \geq 1$

$X_i = x_i + 1$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 12$

$^{12+4-}C_{4-1} = {}^{15}C_3 = \frac{15!}{3!\,12!} = \frac{13\times 14\times 15}{3\times 2} = 455$

8. Let $\frac{\pi}{2} < \theta < \pi$ and $\cot\theta = -\frac{1}{2\sqrt{2}}$. Then the value of $\sin\left(\frac{15\theta}{2}\right)(\cos 8\theta + \sin 8\theta) + \cos\left(\frac{15\theta}{2}\right)(\cos 8\theta - \sin 8\theta)$ is equal to

(1) $\frac{\sqrt{2}-1}{\sqrt{3}}$

(2) $\frac{\sqrt{2}}{\sqrt{3}}$

(3) $-\frac{\sqrt{2}}{\sqrt{3}}$

(4) $\frac{1-\sqrt{2}}{\sqrt{3}}$

Answer (4)

Sol. $f(\theta)=\sin\left(\frac{15\theta}{2}\right)(\cos 8\theta+\sin 8\theta)$

$+\cos\left(\frac{15\theta}{2}\right)(\cos 8\theta-\sin 8\theta)$

$=\sin\left(\frac{15\theta}{2}\right)\cdot\cos 8\theta-\cos\left(\frac{15\theta}{2}\right)\cdot\sin 8\theta+\sin\left(\frac{15\theta}{2}\right)\sin 8\theta+\cos\left(\frac{15\theta}{2}\right)\cdot\cos 8\theta$

$=\sin\left(\frac{15\theta}{2}-8\theta\right)+\cos\left(8\theta-\frac{15\theta}{2}\right)=\cos\frac{\theta}{2}-\sin\frac{\theta}{2}$

$\theta\in\left(\frac{\pi}{2},\pi\right)$

$\frac{\theta}{2}\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

$\cot\theta=-\frac{1}{2\sqrt{2}}\Rightarrow\tan\theta=-2\sqrt{2}$

$\Rightarrow\cos\theta=-\frac{1}{3}=2\cos^2\frac{\theta}{2}-1$

$\Rightarrow 2\cos^2\frac{\theta}{2}=\frac{2}{3}\Rightarrow\cos\frac{\theta}{2}=\frac{1}{\sqrt{3}}$

$\sin\frac{\theta}{2}=\sqrt{1-\frac{1}{3}}=\frac{\sqrt{2}}{\sqrt{3}}$

$\therefore f(\theta)=\frac{1}{\sqrt{3}}-\frac{\sqrt{2}}{\sqrt{3}}=\frac{1-\sqrt{2}}{\sqrt{3}}$

9. If the points of intersection of the ellipses $x^2+2y^2-6x-12y+23=0$ and $4x^2+2y^2-20x-12y+35=0$ lie on a circle of radius r and centre (a,b), then the value of $ab+18r^2$ is

(1) 55

(2) 53

(3) 51

(4) 52

Answer (1)

Sol. $x^2+2y^2-6x-12y+23=0$------(1)

$4x^2+2y^2-20x-12y+35=0$-----(2)

$(x^2+2y^2-6x-12y+23)+\lambda(4x^2+2y^2-20x-12y+35)=0$

$(1+4\lambda)x^2+(2+2\lambda)y^2-16+20\lambda)x$

$-(12+12\lambda)y+23+35\lambda=0$

For this to be circle

$1+4\lambda=2+2\lambda$

$\lambda=\frac{1}{2}$

Substituting back

$3x^2+3y^2-16x-18y+\frac{81}{2}=0$

$x^2+y^2-\frac{16}{3}x-\frac{18}{3}y+\frac{81}{6}=0$

$(a,b)\equiv\left(\frac{8}{3},3\right)$

$r=\sqrt{\frac{64}{9}+9-\frac{81}{6}}$

$r^2=\frac{47}{18}$

$ab+18r^2=8+47=55$

10. Let $A(1,2)$ and $C(-3,-6)$ be two diagonally opposite vertices of a rhombus, whose sides AD and BC are parallel to the line $7x-y=14$. If $\mathrm{B}(\alpha,\beta)$ and $\mathrm{D}(\gamma,\delta)$ are the other two vertices, then $|\alpha+\beta+\gamma+\delta|$ is equal to

(1) 1

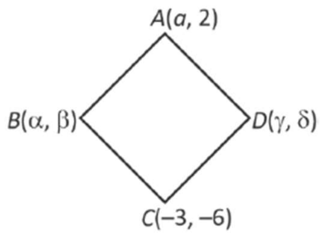
(2) 6

(3) 9

(4) 3

Answer (2)

Sol.



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$\because AC$ is parallel to $y = 2x + 14$

$\Rightarrow \dfrac{8}{\alpha + 3} = 2 \Rightarrow \alpha = 1$

Now $\frac{1-3}{2} = \frac{\alpha+\gamma}{2} \Rightarrow \alpha + \gamma = -2$

$\dfrac{2-6}{2} = \dfrac{\beta + \delta}{2} \Rightarrow \beta + \delta = -4$

$|\alpha + \beta + \gamma + \delta| = 6$

11. The area of the region enclosed between the circles $x^2 + y^2 = 4$ and $x^2 + (y-2)^2 = 4$ is

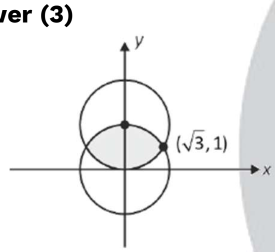
(1) $\frac{4}{3}(2\pi - \sqrt{3})$

(2) $\frac{4}{3}(2\pi - 3\sqrt{3})$

(3) $\frac{2}{3}(4\pi - 3\sqrt{3})$

(4) $\frac{2}{3}(2\pi - 3\sqrt{3})$

Answer (3)



Sol.

$$A = 2\int_0^{\sqrt{3}} \left(\sqrt{4 - x^2} - \left(2 - \sqrt{4 - x^2}\right)\right) dx$$

$$= 2\left(\left[x\sqrt{4 - x^2} + 4\sin^{-1}\left(\frac{x}{2}\right)\Big|_0^{\sqrt{3}} - 2(\sqrt{3})\right]\right)$$

$$= 4\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) \text{ sq. units}$$

12. Let $A = \{0,1,2,\ldots,9\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $x - y$ is a multiple of 3 .

Given below are two statements:

Statement I: $n(R) = 36$.

Statement II: R is an equivalence relation.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is correct but Statement II is incorrect

(2) Both Statement I and Statement II are correct

(3) Both Statement I and Statement II are incorrect

(4) Statement I is incorrect but Statement II is correct

Answer (4)

Sol. $A_1 = \{0,3,6,9\}, A_2 = \{1,4,7\}, A_3 = \{2,5,8\}$

$n(R) = 4 \times 4 + 3 \times 3 + 3 \times 3 = 34$

Reflexive: for any $x \in A, |x - x| = 0$, Since 0 is multiple of 3, $(x, x) \in R$, True

Symmetric: if $(x, y) \in R$, then $|x - y|$ is multiple of 3 .

Since $|y - x| = |x - y|$, then $|y - x|$ is multiple of 3 .

$\Rightarrow (y, x) \in R$, True

Transitive: if $(x, y) \in R, (y, z) \in R$
$|x-y| = 3k, |y-z| = 3m$
$\Rightarrow x-y = \pm 3k, y-z = \pm 3m$
$\Rightarrow x-z = 3(\pm k \pm m)$
$\Rightarrow |(x-z)|$ is multiple of 3 , True
$\Rightarrow R$ is an equivalence relation
$\Rightarrow$ Option (4) is correct.

13. Let $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}, \vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}, \vec{c} = \lambda\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{v} = \vec{a} \times \vec{b}$. If $\vec{v} \cdot \vec{c} = 11$ and the length of the projection of $\vec{b}$ on $\vec{c}$ is p, then $9p^2$ is equal to
(1) 12
(2) 4
(3) 9
(4) 6

Answer (1)

Sol. $\vec{v} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{vmatrix} = -\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$

$\vec{v} \cdot \vec{c} = 11 \Rightarrow \left(-\hat{\imath} + 7\hat{\jmath} + 5\hat{k}\right) \cdot \left(\lambda\hat{\imath} + \hat{\jmath} + \hat{k}\right) = 11$
$-\lambda + 7 + 5 = 11$
$\Rightarrow \lambda = 1$

Projection of $\vec{b}$ on $\vec{c} = \frac{\vec{b} \cdot \vec{c}}{|\vec{c}|}$

$$P = \frac{\left(2\hat{\imath} + \hat{\jmath} - \hat{k}\right) \cdot \left(\hat{\imath} + \hat{\jmath} + \hat{k}\right)}{\sqrt{3}}$$

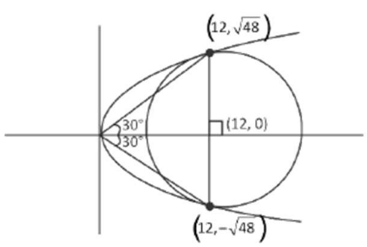
$$\Rightarrow \frac{2+1-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$9P^2 = 9 \times \frac{4}{3} = 12$$

14. An equilateral triangle OAB is inscribed in the parabola $y^2 = 4x$ with the vertex O at the vertex of the parabola. Then the minimum distance of the circle having AB as a diameter from the origin is
(1) $4(3-\sqrt{3})$
(2) $2(3+\sqrt{3})$
(3) $2(8-3\sqrt{3})$
(4) $4(6+\sqrt{3})$

Answer (1)

Sol.



$\Rightarrow$ circle is
$(x-12)^2 + (y-\sqrt{48})(y+\sqrt{48}) = 0$
$\Rightarrow (x-12)^2 + y^2 = 48 \Rightarrow$ radius $= 4\sqrt{3}$
Minimum distance is $12 - \sqrt{48} = 4(3-\sqrt{3})$

15. Let PQ be a chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, perpendicular to the x-axis such that OPQ is an equilateral triangle, O being the centre of the hyperbola. If the eccentricity of the hyperbola is $\sqrt{3}$, then the area of the triangle OPQ is
(1) $\frac{11}{5}$
(2) $\frac{8\sqrt{3}}{5}$

(3) $\frac{9}{5}$
(4) $2\sqrt{3}$
Answer (2)
Sol. $\because e=\sqrt{3}$
$\Rightarrow b^2=4(e^2-1) \Rightarrow b^2=4\times 2=8$
Let $m_{OP}=\tan\left(\pm\frac{\pi}{6}\right)=\pm\frac{1}{\sqrt{3}}$
$\Rightarrow P$ is intersection point of $y=\pm\frac{1}{\sqrt{3}}x$ and the hyperbola
$\Rightarrow \dfrac{x^2}{4}-\dfrac{x^2}{3\times 8}=1$
$\Rightarrow 6x^2-x^2=24$
$\Rightarrow x=\pm\sqrt{\dfrac{24}{5}} \Rightarrow P\left(\pm\sqrt{\dfrac{24}{5}},\pm\sqrt{\dfrac{24}{15}}\right)$
$a^2=OP^2=\dfrac{24}{5}+\dfrac{24}{15}=\dfrac{96}{15}$
Area $=\frac{\sqrt{3}}{4}\times\frac{96}{15}=\frac{8\sqrt{3}}{5}$ square units
$\Rightarrow$ Option (2) is correct.

16. The least value of $(\cos^2\theta-6\sin\theta\cos\theta+3\sin^2\theta+2)$ is
(1) $4+\sqrt{10}$
(2) -1
(3) 1
(4) $4-\sqrt{10}$
Answer (4)
Sol. $\cos^2\theta-6\sin\theta\cos\theta+3\sin^2\theta+2$
$=3-3\sin 2\theta+2\sin^2\theta$
$=3-3\sin 2\theta+1-\cos 2\theta$
$=4-3\sin 2\theta-\cos 2\theta$
Least value is $4-\sqrt{10}$

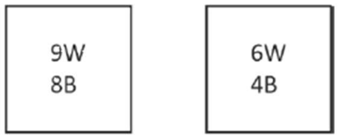
17. Let $\sum_{k=1}^{n}a_k=\alpha n^2+\beta n$. If $a_{10}=59$ and $a_6=7a_1$, then $\alpha+\beta$ is equal to
(1) 5
(2) 7
(3) 12
(4) 3
Answer (1)
Sol. $\because S_n=\alpha n^2+\beta n$
$\Rightarrow$ Sequence is in AP
$a_{10}=a+9d=5a$
$a_6=a+5d=7a \Rightarrow 5d=6a$
$\Rightarrow a=5\ d=6$
Now $S_n=\frac{n}{2}[10+(n-1)6]$
$=n(5+(n-1)3)$
$=n(3n+2)$
$\Rightarrow 3n^2+2n$
$\Rightarrow \alpha=3\ \beta=2$
$\alpha+\beta=5$

18. Bag A contains 9 white and 8 black balls, while bag B contains 6 white and 4 black balls. One ball is randomly picked up from the bag B and mixed up with the balls in the bag A . Then a ball is randomly drawn from the bag A . If the probability, that the ball drawn is white, is $\frac{p}{q}$, $\gcd(p,q)=1$, then $p+q$ is equal to
(1) 23
(2) 22

(3) 24
(4) 21

Answer (1)

Sol.



Bag A Bag B

A : probability of getting white from bag A
B_w : probability of getting white from bag B
B_b : probability of getting black from bag B

$$P\left(\frac{A}{B_w \cup B_b}\right) = P(B_w)P\left(\frac{A}{B_w}\right) + P(B_b)P\left(\frac{A}{B_b}\right)$$

$$= \frac{6}{10} \times \frac{10}{18} + \frac{4}{10} \times \frac{9}{18}$$

$$= \frac{60+36}{180} = \frac{96}{180} = \frac{8}{15}$$

19. If $f(x) = \begin{cases} \frac{a|x|+x^2-2(\sin\,|x|)(\cos\,|x|)}{x} & ,x \neq 0 \\ b & ,x = 0 \end{cases}$ is continuous at $x = 0$, then $a + b$ is equal to

(1) 4
(2) 0
(3) 1
(4) 2

Answer (4)

Sol. $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{a|x|+x^2-2(\sin|x|)(\cos|x|)}{x}$

$$\text{RHL} = \lim_{x\to 0^+} \frac{ax + x^2 - \sin 2x}{x}$$

$$\text{RHL} = a - 2$$

$$\text{LHL} = \lim_{x\to 0^-} \frac{-ax + x^2 - 2\sin(-x)\cdot\cos(-x)}{x}$$

$$= \lim_{x\to 0^-} \frac{-ax + x^2 + \sin 2x}{x}$$

$$= -a + 2$$

$$\text{RHL} = \text{LHL} = f(0)$$

$$a - 2 = -a + 2 = b$$

$$\Rightarrow a = 2, b = 0$$

$$a + b = 2$$

20. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}, i = \sqrt{-1}$, then $(z^{201} - i)^8$ is equal to

(1) 256
(2) -1
(3) 0
(4) 1

Answer (1)

Sol. $z = \frac{\sqrt{3}}{2} + \frac{i}{2} = e^{i\pi/6}$

$$(z^{201} - i)^8 = \left(e^{i\frac{201\pi}{6}} - i\right)^8$$

$$= \left(e^{i\frac{67\pi}{2}} - i\right)^8$$

$$= (-i - i)^8$$

$$= (-2i)^8$$

$$= 2^8$$

$$= 256$$

SECTION – B

21. The number of elements in the set $S = \{x: x \in [0,100] \text{ and } \int_0^x t^2 \sin(x-t)dt = x^2\}$ is ___

Answer (16)

Sol. $\int_0^x t^2 \sin(x-t)dt = x^2$

$\Rightarrow \int_0^x (x-t)^2 \sin t dt = x^2$

$\Rightarrow \int_0^x (x^2+t^2-2xt)\sin t dt = x^2$

$\Rightarrow x^2 \int_0^x \sin t dt - 2x \int_0^x t \cdot \sin t dt + \int_0^x t^2 \sin t dt = x^2$

$\Rightarrow x^2(-\cos t)|_0^x - 2x\left(t(-\cos t)|_0^x + \int_0^x \cos t dt\right) +$

$\Rightarrow x^2(1-\cos x) - 2x(-x\cos x + \sin x) +$

$\left[t^2(-\cos t)|_0^x + 2\int_0^x (t\cos t)dt\right] = x^2$

$\Rightarrow x^2(1-\cos x) + 2x^2\cos x - 2x\sin x - x^2\cos x$

$+2x\sin x + 2\cos x - 2 = x^2$

$\Rightarrow \quad x^2 + 2\cos x - 2 = x^2$

$\Rightarrow \quad \cos x = 1$

$x = 2n\pi, n \in I$

$\therefore \quad n = 0,1,2,\ldots,15$

$\therefore$ Number of elements $= 16$

22. Let S denote the set of 4-digit numbers abcd such that $a > b > c > d$ and P denote the set of 5-digit numbers having product of its digits equal to 20 . Then $n(S) + n(P)$ is equal to ___

Answer (260)

Sol. $n(S) = {}^{10}C_4 = 210$

Let 5 digit number is

$x_1x_2x_3x_4x_5 = 20$

One of them has to be 5

$\Rightarrow x_1x_2x_3x_4 = 4$

$5 \times 4 \times 1 \times 1 \times 1$ or $5 \times 2 \times 2 \times 1 \times 1$

$\Rightarrow \frac{5!}{3!} + \frac{5!}{2!\,2!} = 50$

$\Rightarrow n(S) + n(P) = 260$

23. Let $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and B be a matrix such that $B(I-A) = I + A$. Then the sum of the diagonal elements of B^TB is equal to ___

Answer (3)

Sol. $B(I-A) = (I+A)$

Since A is skew symmetric matrix

$\Rightarrow A^T = -A$ since, $B^T(I-A^T) = (I+A^T)$

$\Rightarrow B^T(I+A) = (I-A)$

$\Rightarrow BB^T(I+A) = B(I-A) = (I+A)$

$\Rightarrow BB^T(I+A)(I+A)^{-1} = (I+A)(I+A)^{-1}$

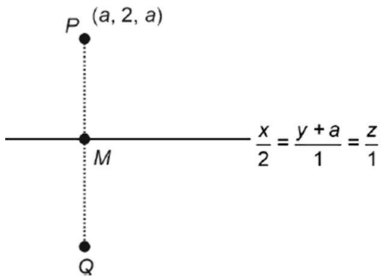
$= I_{3\times3}$

$\Rightarrow$ Trace $(BB^T) = 3$

24. If the image of the point $P(a,2,a)$ in the line $\frac{x}{2} = \frac{y+a}{1} = \frac{z}{1}$ is Q and the image of Q in the line $\frac{x-2b}{2} = \frac{y-a}{1} = \frac{z+2b}{-5}$ is P , then $a + b$ is equal to

Answer (3)

Sol.



Let M be the mid point of P and Q

$\Rightarrow \overrightarrow{PM} \| (2\hat{i}+\hat{j}+\hat{k}) \times (2\hat{i}+\hat{j}-5\hat{k})$

$\overrightarrow{PM} \| \hat{i}-2\hat{j}$

Let, $M=(2\lambda, \lambda-a, \lambda)$

$$\frac{2\lambda-a}{1}=\frac{\lambda-a-2}{-2}=\frac{\lambda-a}{0} \Rightarrow \lambda=a$$

$M=(2a, 0, a)$

$\overrightarrow{PM} \cdot (2\hat{i}+\hat{j}+\hat{k})=0$

$\Rightarrow 2(2a-a)+(1)(0-2)+1(a-a)=0$

$\Rightarrow 2a-2=0 \Rightarrow a=1$

$\Rightarrow P \equiv (1,2,1)$

$M \equiv (2,0,1)$

$\Rightarrow Q \equiv (3,-2,1)$

M lie on $\frac{x-2b}{2}=\frac{y-a}{1}=\frac{z+2b}{-5}$

$$\Rightarrow \frac{2-2b}{2}=\frac{-a}{1} \Rightarrow b=2$$

$\Rightarrow a+b=3$

25. If the solution curve $y=f(x)$ of the differential equation $(x^2-4)y'-2xy+2x(4-x^2)^2=0, x>2,$ passes through the point $(3,15)$, then the local maximum value of f is ___

Answer (16)

Sol. $(x^2-4)\frac{dy}{dx}-2xy+2x(4-x^2)^2=0, x>2$

$$\frac{dy}{dx}+\left(\frac{-2x}{x^2-4}\right)y=\frac{-2x(4-x^2)^2}{(x^2-4)}$$

$=(-2x)(x^2-4)$

I.F. $=e^{\int \frac{-2x}{(x^2-4)}dx}=e^{-\ln(x^2-4)}=\frac{1}{x^2-4}$

$$\Rightarrow y \times \frac{1}{(x^2-4)}=\int (-2x)dx=-x^2+C$$

$$\frac{15}{5}=-9+C \Rightarrow C=12$$

$\Rightarrow y=(12-x^2)(x^2-4)$

$=-48-x^4+16x^2$

$=16-(x^2-8)^2$

$\Rightarrow$ local maximum value is 16 at $x=2\sqrt{2}$

$$=6\left\{\frac{7}{3}+\frac{1}{12}+\frac{5}{12}\right\}=17$$

PHYSICS TEST PAPER WITH SOLUTION

26. The ratio of speeds of electromagnetic waves in vacuum and a medium, having dielectric constant $k=3$ and permeability of $\mu=2\mu_0$, is (μ_0 = permeability of vacuum)

(1) $3:2$

(2) $6:1$

(3) $36:1$

(4) $\sqrt{6}:1$

Answer (4)

Sol. $\frac{c}{v} = n = \sqrt{\mu_r \varepsilon_r}$

$\Rightarrow \frac{c}{v} = \sqrt{6}$

27. The internal energy of a monoatomic gas is 3 nRT . One mole of helium is kept in a cylinder having internal cross section area of 17 cm^2 and fitted with a light movable frictionless piston. The gas is heated slowly by suppling 126 J heat. If the temperature rises by 4°C, then the piston will move ___ cm . (atmospheric pressure = 10^5 Pa)

(1) 1.45

(2) 1.55

(3) 14.5

(4) 15.5

Answer (4)

Sol. Given,

$\Delta Q = 126 \text{ J}$

$\Delta T = 4^\circ\text{C}$

$A = 17 \text{ cm}^2 = 17 \times 10^{-4} \text{ m}^2$

$P = 10^5 \text{ Pa}$

$\Delta U = nR\Delta T$

$\Delta U = 3 \times 8.314 \times 4 = 99.768 \text{ J}$

From first law of thermodynamics

$\Delta Q = \Delta U + W$

$W = 126 - 99.768 = 26.232 \text{ J}$

Work done by gas $= P\Delta V = PAh$

$h = \dfrac{26.232}{10^5 \times 17 \times 10^{-4}}$

$h = 1.543 \times 10^{-1} \text{ m}$

$h = 15.43 \text{ cm}$

28. A body of mass 14 kg initially at rest explodes and breaks into three fragments of masses in the ratio $2:2:3$. The two pieces of equal masses fly off perpendicular to each other with a speed of 18 m/s each. The velocity of the heavier fragment is ___ m/s.

(1) $10\sqrt{2}$

(2) $24\sqrt{2}$

(3) $12\sqrt{2}$

(4) 12

Answer (3)

Sol. Applying conservation of linear momentum.

$m_1:m_2:m_3 = 4:4:6$

$O = 4 \times 18\hat{\imath} + 4 \times 18\hat{\jmath} + 6\vec{v}_3$

$\vec{v}_3 = -12(\hat{\imath} + \hat{\jmath})$

$|\vec{v}_3| = 12\sqrt{2} \text{ m/s}$

29. A paratrooper jumps from an aeroplane and opens a parachute after 2 s of free fall and starts deaccelerating with 3 m/s^2. At 10 m height from ground, while descending with the help of parachute, the speed of paratrooper is 5 m/s. The initial height of the airplane is ___ m. ($g = 10 \text{ m/s}^2$)

(1) 92.5

(2) 62.5

(3) 82.5

(4) 20

Answer (3)

Sol. $h_1 = \frac{1}{2} \times 10 \times L^2 = 20 \text{ m}$

$v_2^2 = v_1^2 + 2ah_2$

$5^2 = 20^2 - 2 \times 3 \times h_2$

$h_2 = 62.5 \text{ m}$

$h_3 = 10 \text{ m}$

30. Suppose a long solenoid of 100 cm length, radius 2 cm having 500 turns per unit length, carries a current $I = 10\sin(\omega t)A$, where $\omega = 1000\text{rad./s}$. A circular conducting loop (B) of radius 1 cm coaxially slided through the solenoid at a speed $v = 1 \text{ cm/s}$. The r.m.s. current through the loop when the coil B is inserted 10 cm inside the solenoid is $\alpha/\sqrt{2}\mu \text{ A}$. The value of α is ___ .

[Resistance of the loop $= 10\Omega$]

(1) 280

(2) 197

(3) 100

(4) 80

Answer (1)

Sol. $\Phi = BA$

$$\varepsilon = \frac{dB}{dt}A$$

$$i = \frac{A}{R}\mu_0 n\, 10\omega \cos\, \omega t$$

$$i = \frac{500 \times \pi \times 4 \times 10^{-4} \times 4\pi \times 10^{-7}}{4 \times 10} \times 10 \times 1000 \cos\, \omega t$$

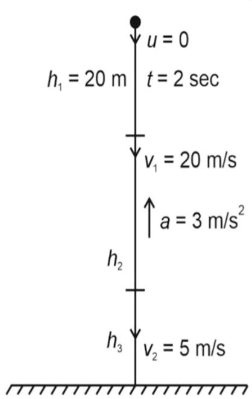
$$= 500 \times 16 \times 10^{-7} \cos\, \omega t$$

$$= \frac{8}{4} \times 10^{-4} < \cos\, \omega t >_{rms}$$

$$< \cos \omega\, t >_{\text{rms}} = \sqrt{\frac{\int \left(\frac{1 - \cos 2\,\omega t}{2}\right) dt}{T}}$$

$$= \frac{1}{\sqrt{2}}$$

$$i = \frac{8}{4} \times 10^{-4} \frac{1}{\sqrt{2}} \approx \frac{200}{\sqrt{2}} \mu \text{ A}$$



total height of aeroplane from ground is

$= h_1 + h_2 + h_3$

$= 92.5 \text{ m}$

31. A parallel plate capacitor with plate separation 5 mm is charged by a battery. On introducing a mica sheet of 2 mm and maintaining the connections of the plates with the terminals of the battery, it is found that it draws 25% more charge from the battery. The dielectric constant of mica is ___ .

(1) 1.0

(2) 1.5

(3) 2.0

(4) 2.5

Answer (3)

Sol. Given

$d = 5$ mm

$t = 2$ mm

$C_0 = \frac{\varepsilon_0 A}{d}$

$\frac{1}{C'} = \frac{d-t}{\varepsilon_0 A} + \frac{t}{\varepsilon_0 Ak}$

$\frac{1}{C'} = \frac{1}{\varepsilon_0 A}\left((d-t) + \frac{t}{k}\right)$

$C' = \frac{\varepsilon_0 Ak}{(d-t)k + t}$

$Q_f = 1.25 Q_i$

$C'v = 1.25 C_0 V$

$\frac{\varepsilon_0 Ak}{(d-t)k+t} = 1.25\frac{\varepsilon_0 A}{d}$

$\frac{k}{3k+2} = \frac{1.25}{5}$

$k = 2$

32. A circular loop of radius 7 cm is placed in uniform magnetic field of 0.2 T directed perpendicular to plane of loop. The loop is converted into a square loop in 0.5 s . The EMF induced in the loop is ___ mV .

(1) 13.2

(2) 6.6

(3) 1.32

(4) 8.25

Answer (3)

Sol. $r = 7$ cm

$2\pi r = 4I$

$I = \frac{2\pi \times 7}{4} = 10.99$ cm

Change in Area $\Delta A = (153.9 - 120.75)\text{cm}^2$

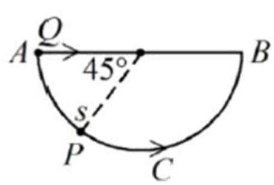
$\Delta A = 33.12\ \text{cm}^2$

e.m.f. $= \frac{BA}{t} = \frac{0.2 \times 33.12 \times 10^{-4}}{0.5}$

$= 13.248 \times 10^{-4}$ V

e.m.f. $= 1.3248$mV

33. A bead P sliding on a frictionless semi-circular string (ABC) and it is at point S at $t = 0$ and at this instant the horizontal component of its velocity is v. Another bead Q of the same mass as P is ejected from point A at $t = 0$ along the horizontal string AB, with the speed v, friction between the beads and the respective strings may be neglected in both cases. Let t_P and t_Q be the respective times taken by beads P and Q to reach the point B, then the relation between t_P and t_Q is



(1) $t_P < t_Q$

(2) $t_P = t_Q$

(3) $t_P > 1.25 t_Q$

(4) $t_P > t_Q$

Answer (1)

Sol. The velocity of Q remain the same horizontal as so force it acting on the horizontal dissection but in case P the horizontal velocity first increase reaches has a maximum at lowest point and then decrease this it always remain greater then v.

There force $t_P < t_Q$

34. One mole of an ideal diatomic gas expands from volume V to 2 V isothermally at a temperature 27°C and does W joule of work. If the gas undergoes same magnitude of expansion adiabatically from 27°C doing the same amount of work W, then its final temperature will be (close to) ___ °C. ($\log_e 2 = 0.693$)
(1) -117
(2) -189
(3) -56
(4) -30

Answer (1)

Sol. Given, $n = 1$ mole
$v_i = V$ and $v_f = 2V$
$T_i = 300$ K
$W_{\text{iso}} = |W_{\text{adi}}|$
$nRT_i \ln\left(\frac{v_2}{v_1}\right) = nC_v(T_1 - T_2)$
$nRT_i \ln(2) = n\frac{5}{2}R(T_1 - T_2)$
$300 \ln(2) = 2.5(300 - T_2)$
$T_2 = 216.82$ K
$T_2 = -56.17°\text{C}$

35. Which of the following pair of nuclei are isobars of the element?
(1) ${}_1^2\text{H}$ and ${}_1^3\text{H}$
(2) ${}_{80}^{198}\text{Hg}$ and ${}_{79}^{197}\text{Au}$
(3) ${}_1^3\text{H}$ and ${}_2^3\text{He}$
(4) ${}_{92}^{236}\text{U}$ and ${}_{92}^{238}\text{U}$

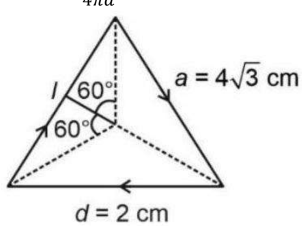
Answer (3)

Sol. Isobars are nuclei with the same mass number (A) but different atomic number (Z)
Both Tritium (${}_1^3\text{H}$) and Helium (${}_2^3\text{He}$) are isobars.

36. The current passing through a conducting loop in the form of equilateral triangle of side $4\sqrt{3}$ cm is 2 A . The magnetic field at its centroid is $\alpha \times 10^{-5}$ T. The value of α is ___ .
(Given : $\mu_0 = 4\pi \times 10^{-7}$ SI units)
(1) $\sqrt{3}$
(2) $3\sqrt{3}$
(3) $2\sqrt{3}$
(4) $\frac{\sqrt{3}}{2}$

Answer (2)

Sol. $B = 3 \cdot \frac{4_0 I}{4\pi d} 2\sin 60°$



$$B = 3 \times \frac{4\pi \times 10^{-7} \times 2}{4\pi \times 2 \times 10^{-2}} \times 2 \times \frac{\sqrt{3}}{2}$$
$B = 3\sqrt{3} \times 10^{-5} T$

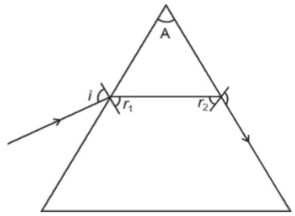
37. A prism of angle 75° and refractive index $\sqrt{3}$ is coated with thin film of refractive index 1.5 only at the back exit surface. To have total internal reflection at the back exit surface the incident angle must be ___ .($\sin 15° = 0.25$ and $\sin 25° = 0.43$)
(1) $< 15°$
(2) $15°$

(3) between 15° and 20°
(4) > 25°

Answer (Bonus)

Sol. $\sin\ Q_c = \frac{1.5}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

$Q = 60^\circ$
$r_1 + r_2 = A$
$r_1 = A - r_2$
$r_2 > 60^\circ$
$r_2 > A - 60^\circ$
$r_1 < 75^\circ - 60^\circ$
$r_1 < 15^\circ$
$1 \times \sin\ i = \sqrt{3}\sin\ r_1$
$i < \sin^{-1}\left(\sqrt{3}\sin\left(15^\circ\right)\right)$
$i < 25^\circ$



None of the options is matching.

38. When an unpolarized light falls at a particular angle on a glass plate (placed in air), it is observed that the reflected beam is linearly polarized. The angle of refracted beam with respect to the normal is___ .
($\tan^{-1}(1.52) = 57.7^\circ$, refractive indices of air and glass are 1.00 and 1.52 , respectively.)
(1) 32.3°
(2) 36.3°
(3) 39.6°
(4) 42.6°

Answer (2)

Sol. For Brewster angle, reflected beam is polarized when $\tan i_p = \mu, i_p + r = 90^\circ$

$\tan i_p = \mu$
$\tan(90^\circ - r) = 1.52$
$90^\circ - r = \tan^{-1}(1.52)$
$r = 90^\circ - 57.70$
$r = 32.3^\circ$

39. Two charges $7\mu C$ and $-2\mu C$ are placed at ($-9,0,0$) and ($9,0,0$) respectively in an external field $E = \frac{A}{r^2}\hat{r}$, where $A = 9 \times 10^5\ \mathrm{N/C.m^2}$.
Considering the potential at infinity is 0 , the electrostatic energy of the configuration is ___ J.
(1) 1.4
(2) 24.3
(3) 49.3
(4) -90.7

Answer (3)

Sol. $E = \frac{A}{r^2}\hat{r}$

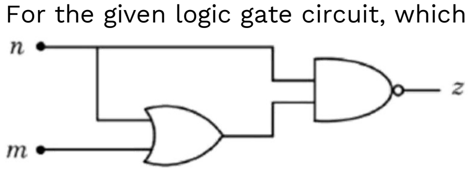
$$v = \frac{A}{r}$$

$$v_{\text{total}} = q_1 v_{ext}(r_1) + q_2 v_{ext}(r_2) + \frac{kq_1q_2}{r_{12}}$$

$$V_{\text{total}} = \frac{7 \times 10^{-6} \times 9 \times 10^5}{9 \times 10^{-2}} + \frac{(-2 \times 10^{-6}) \times 9 \times 10^5}{9 \times 10^{-2}} + \frac{9 \times 10^9 \times 7 \times (-2) \times 10^{-12}}{18 \times 10^{-2}}$$

$$V_{\text{total}} = 50 - 0.7 = 49.3\ \mathrm{J}$$

40. For the given logic gate circuit, which of the following is the correct truth table?



(1)

n	m	z
0	0	1
0	1	1
1	1	0
1	0	0
n	m	z

(2)

n	m	z
0	0	0
0	1	1
1	1	0
1	0	1
n	m	z

(3)

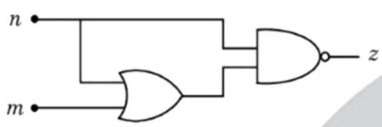
n	m	z
0	0	1
0	1	0
1	1	0
1	0	0
n	m	z

(4)

n	m	z
0	0	1
0	1	0
1	1	1
1	0	0
n	m	z

Answer (3)

Sol.



$z = \overline{n(n+m)}$

$z = \bar{n}$

41. A block is sliding down on an inclined plane of slope θ and at an instant $t = 0$ this block is given an upward momentum so that it starts moving up on the inclined surface with velocity u. The distance (S) travelled by the block before its velocity become zero, is ___ .

(g = gravitational acceleration)

(1) $\frac{u^2}{\sqrt{2} g\cos\theta}$

(2) $\frac{u^2}{2g\cos\theta}$

(3) $\frac{2u^2}{g\cos\theta}$

(4) $\frac{u^2}{4g\sin\theta}$

Answer (BONUS)

Sol. Question should state block was sliding down with uniform speed u before $t = 0$.

Such that $g\sin\theta = mg\cos\theta$

and when sliding up $s = \frac{v^2}{4g\sin\theta}$

(when $a = g\sin\theta + mg\cos\theta = 2g\sin\theta$) (Option 1)

Since it was not given it should be "BONUS"

42. A small metallic sphere of diameter 2 mm and density $10.5\ \mathrm{g/cm^3}$ is dropped in glycerine having viscosity 10 Poise and density $1.5\ \mathrm{g/cm^3}$ respectively. The terminal velocity attained by the sphere is ___ $\mathrm{cm/s}$.

($\pi = \frac{22}{7}$ and $g = 10\ \mathrm{m/s^2}$)

(1) 1.0

(2) 3.0

(3) 2.0

(4) 1.5

Answer (3)

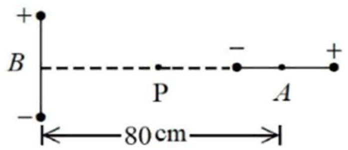
Sol. Terminal velocity is given by equation.

$$V_T = \frac{2g(\rho_s - \rho_l)}{9\eta} r^2$$

Given,

$r=\frac{d}{2}=1\ \mathrm{mm}=1\times10^{-3}\ \mathrm{m}$
$\rho_s=10.5\ \mathrm{g/cm^3}=10.5\times10^3\ \mathrm{kg/m^3}$
$\rho_l=1.5\ \mathrm{g/cm^3}=1.5\times10^3\ \mathrm{kg/m^3}$
$\eta=10$ poise $=1\ \mathrm{Pa.s}$
$V_T=\frac{2\times10\times(10.5-1.5)\times10^3\times(1\times10^{-3})^2}{9\times1}$
$V_T=0.02\ \mathrm{m/s}=2\ \mathrm{cm/s}$

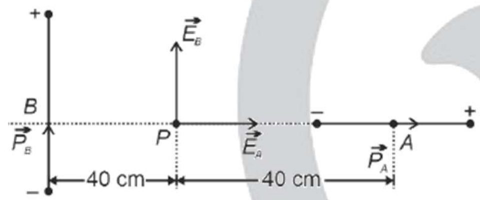
43. Two shorts dipoles (A,B), A having charges $\pm2\mu$C and length 1 cm and B having charges $\pm4\mu$C and length 1 cm are placed with their centres 80 cm apart as shown in the figure. The electric field at a point P, equi-distant from the centres of both dipoles is ___ N/C.



(1) $9\sqrt{2}\times10^4$
(2) $\frac{9}{16}\sqrt{2}\times10^4$
(3) $\frac{9}{16}\sqrt{2}\times10^5$
(4) $4.5\sqrt{2}\times10^4$

Answer (3)

Sol. $P_A=2\times10^{-6}\times1\times10^{-2}=2\times10^{-8}\ \mathrm{m-c}$
$P_B=4\times10^{-6}\times1\times10^{-2}=4\times10^{-8}\ \mathrm{m-c}$



$E_B=\frac{kP_B}{r^3},E_A=\frac{2kP_A}{r^3}$
$E_{\text{net}}=\frac{k}{r^3}\sqrt{P_B^2+4P_A^2}$
$E_{\text{net}}=\frac{9\times10^9}{(40\times10^{-2})^3}\sqrt{(4\times10^8)^2+4\times(2\times10^{-8})^2}$
$E_{\text{net}}=\frac{9}{16}\sqrt{2}\times10^4\ \mathrm{N/C}$

44. To compare EMF of two cells using potentiometer the balancing lengths obtained are 200 cm and 150 cm . The least count of scale is 1 cm . The percentage error in the ratio of EMFs is ___
(1) 1.45
(2) 1.55
(3) 1.65
(4) 1.75

Answer (BONUS)

Sol. $\frac{E_1}{E_2}=\frac{l_1}{l_2}$

% age error $=\left(\frac{\Delta l_1}{l_1}+\frac{\Delta l}{l_2}\right)\times100\%$
$=\left(\frac{1}{200}+\frac{1}{150}\right)\times100\%$
$=\frac{7}{6}\%$

% age error $=1.16\%$
None of the options are matching.

45. An air bubble of volume 2.9 cm^3 rises from the bottom of a swimming pool of 5 m deep. At the bottom of the pool water temperature is $17^\circ\mathrm{C}$. The volume of the bubble when it reaches the surface, where the water temperature is $27^\circ\mathrm{C}$, is ___ cm^3. ($g = 10\ \mathrm{m/s^2}$, density of water = $10^3\ \mathrm{kg/m^3}$, and 1 atm pressure is 10^5 Pa)

(1) 3.0
(2) 2.0
(3) 4.2
(4) 4.5

Answer (4)

Sol. $\frac{PV}{T}$ = constant

$P_i = P_{\text{atm}} + \rho g h$

$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$

$T_1 = 273 + 17 = 290\text{k}$

$T_f = 273 + 27 = 300\text{k}$

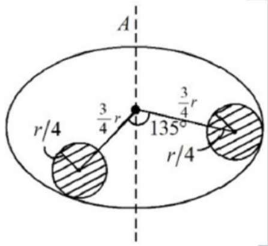
$V_i = 2.9\ \mathrm{cm}^3$

$$V_f = \frac{(1\times 10^5 + 10^3 \times 10 \times 5)\times 2.9}{1\times 10^5} \times \frac{300}{290}$$

$V_f = 4.5\ \mathrm{cm}^3$

SECTION – B

46. Suppose there is a uniform circular disc of mass M kg and radius r m shown in figure. The shaded regions are cut out from the disc. The moment of inertia of the remainder about the axis A of the disc is given by $\frac{x}{256} M r^2$. The value of x is ___ .



Answer (109)

Sol. $I = \frac{Mr^2}{2} - \left(\frac{\frac{M}{16}\times\frac{r^2}{16}}{2} + \frac{M}{16}\times\left(\frac{3r}{4}\right)^2\right)\times 2$

$$\Rightarrow I = \frac{Mr^2}{2} - \left(\frac{Mr^2}{2} + 9Mr^2\right)\frac{2}{16\times 16}$$

$$\Rightarrow I = \frac{Mr^2}{2} - \left(\frac{19Mr^2}{16\times 16}\right)$$

$$\Rightarrow \frac{(128-19)Mr^2}{16\times 16} = \frac{109Mr^2}{256}$$

47. The size of the images of an object, formed by a thin lens are equal when the object is placed at two different positions 8 cm and 24 cm from the lens. The focal length of the lens is ___ cm .

Answer (16)

Sol. $m = \frac{f}{f+u}$

$$\frac{f}{f-24} = \frac{-f}{f-8}$$

$f - 8 = -f + 24$

$f = 16$ cm

48. The average energy released per fission for the nucleus of $^{235}_{92}\mathrm{U}$ is 190 MeV . When all the atoms of 47 g pure $^{235}_{92}\mathrm{U}$ undergo fission process, the energy released is $\alpha \times 10^{23}$MeV. The value of α is ___ .

(Avogadro Number = 6×10^{23} per mole)

Answer (228)

Sol. 1 nuclei of $U^{235} \longrightarrow 190\text{Mev}$

Now we need to calculate total number of nucleid in 47 g .

$235 \text{ g} = 6 \times 10^{23}$

$47 \text{ g} \equiv \dfrac{6 \times 10^{23}}{235} \times 47 = \dfrac{6}{5} \times 10^{23}$

$\therefore$ Energy released $= 190 \times \frac{6}{5} \times 10^{23}\text{Mev}$

$\Rightarrow 228 \times 10^{23}\text{Mev}$

49. A ball of radius r and density ρ dropped through a viscous liquid of density σ and viscosity η attains its terminal velocity at time t, given by $t = A\rho^a r^b \eta^c \sigma^d$, where A is a constant and a, b, c and d are integers. The value of $\frac{b+c}{a+d}$ is ___ .

Answer (1)

Sol. $\mathrm{T}^1 = [\mathrm{ML}^{-3}]^{\mathrm{a}}[\mathrm{L}]^{\mathrm{b}}[\mathrm{ML}^{-1}\ \mathrm{T}^{-1}]^{\mathrm{c}}[\mathrm{ML}^{-3}]^{\mathrm{d}}$

$a + c + d = 0$

$-3a + b - c - 3d = 0$

$-c = 1$

$\Rightarrow a + d = 1$

$a + d = \dfrac{b - c}{3} = \dfrac{b + 1}{3} = 1$

$b = 2$

$\Rightarrow \dfrac{2 - 1}{1} = 1$

50. The velocity of sound in air is doubled when the temperature is raised from $0^\circ\mathrm{C}$ to $\alpha^\circ\mathrm{C}$. The value of α is ___ .

Answer (819)

Sol. $v \propto \sqrt{T}$

$\Rightarrow \sqrt{\alpha} = 2\sqrt{273}$

$\alpha(k) = 1092$

$t^\circ\mathrm{c} = 1092 - 273$

$= 819$

CHEMISTRY TEST PAPER WITH SOLUTION

51. The work functions of two metals ($\mathrm{M_A}$ and $\mathrm{M_B}$) are in the $1:2$ ratio. When these metals are exposed to photons of energy 6 eV , the kinetic energy of liberated electrons of $M_A : M_B$ is in the ratio of 2.642 : 1. The work functions (in eV) of $\mathrm{M_A}$ and $\mathrm{M_B}$ are respectively.

(1) 3.1,6.2

(2) 2.3,4.6

(3) 1.4,2.8

(4) 1.5,3.0

Answer (2)

Sol. Let incident energy for both metals =6; work functions are Y and 2Y for A and B and Kinetic energies are 2.642X and X.

For metal $A \Rightarrow 6 = Y + 2.642x$

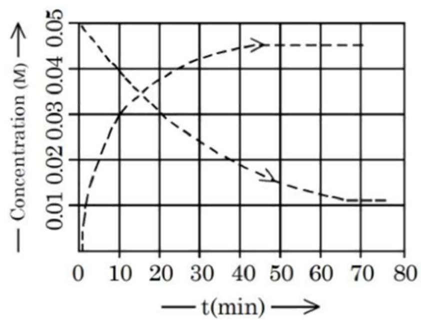
For metal $\mathrm{B} \Rightarrow 6 = 2\mathrm{Y} + \mathrm{x}$

Simplifying both equations we get; $\mathrm{Y} = 1.642\mathrm{x}$

Now put Y in second equation

$6 = 2(1.642x) + x$, on solving $x = 1.4, Y = 1.642 \times 1.4 = 2.3$

52.



Given above is the concentration vs time plot for a dissociation reaction : $A \rightarrow nB$.

Based on the data of the initial phase of the reaction (initial 10 min), the value of n is ___ .

(1) 2

(2) 5

(3) 4

(4) 3

Answer (4)

Sol. $\frac{-dA}{dt}=\frac{1}{n}\frac{dB}{dt}$

So, $\frac{0.05-0.04}{10}=\frac{1}{n}\left[\frac{0.03-0}{10}\right]$

$n=3$

53. Iodoform test can differentiate between

A. Methanol and Ethanol

B. CH_3COOH and CH_3CH_2COOH

C. Cyclohexene and cyclohexanone

D. Diethyl ether and Pentan-3-one

E. Anisole and acetone

Choose the correct answer from the options given below:

(1) A & D only

(2) A, B & E only

(3) A & E only

(4) B, C & E only

Answer (3)

Sol. In A ethanol gives positive idoforom test and in E acetone gives positive iodoform test.

54. A student has been given a compound " x " of molecular formula C_6H_7N. ' x ' is sparingly soluble in water. However, on addition of dilute mineral acid, ' x ' becomes soluble in water. ' x ' when treated with $CHCl_3$ and $KOH(alc)$, ' y ' is produced. ' y ' has a specific unpleasant smell. On treatment with benzensulphonyl chloride, ' x ' gives a compound ' z ' which is soluble in alkali. The number of different " H " atoms present in ' z ' is

(1) 7

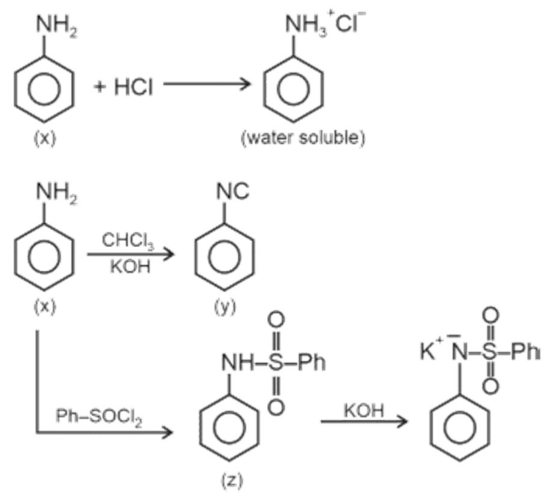
(2) 5

(3) 8

(4) 4

Answer (1)

Sol. Degree of unsaturation for 'X' is 4.Based on the given below reactions all it indicates X is a primary amine and based on degree of unsaturation it indicates it is aniline . Number of different types of H atoms present



55. Observe the following reactions at $T(K)$.

I. $A \rightarrow$ products.

II. $5Br(aq) + BrO_3^-(aq) + 6H^+(aq) \rightarrow 3Br_2(aq) + 3H_2O(l)$

Both the reactions are started at 10.00 am . The rates of these reactions at 10.10 am are same. The value of $-\frac{\Delta[Br^-]}{\Delta t}$ at 10.10 am is 2×10^{-4} mol $L^{-1}min^{-1}$. The concentration of A at 10.10 am is $10^{-2} molL^{-1}$.

What is the first order rate constant (in min^{-1}) of reaction I?

(1) 10^{-2}

(2) 10^{-3}

(3) 2×10^{-3}

(4) 4×10^{-3}

Answer (4)

Sol. $r = k[A]$ for the first reaction

$$r = \frac{1}{5}\frac{\Delta[Br^-]}{\Delta t} = \frac{1}{5} \times 2 \times 10^{-4}$$

for the second reaction ; $= 4 \times 10^{-5}$

$$4 \times 10^{-5} = k[10^{-2}]$$

$$k = 4 \times 10^{-3}\ min^{-1}$$

56. Elements X and Y belong to Group 15. The difference between the electronegativity values of ' X ' and phosphorus is higher than that of the difference between phosphorus and ' Y '. ' X ' & ' Y ' respectively

(1) As & Bi

(2) N & As

(3) Bi & N

(4) As & Sb

Answer (2)

Sol. EN of $N = 3.0$

$P = 2.1$

$As \Rightarrow 2.0$

$Sb = 1.9$

$Bi = 1.9$

57. The oxidation state of chromium in the final product formed in the reaction between KI and acidified $K_2Cr_2O_7$ solution is:

(1) +4

(2) +6

(3) +2

(4) +3

Answer (4)

Sol. $K_2Cr_2O_7 + 6KI + 7H_2SO_4 \rightarrow \overset{+3}{Cr_2}(SO_4)_3 + 3I_2 + 4\ K_2SO_4 + 7H_2O$

58. Both human DNA and RNA are chiral molecules. The chirality in DNA and RNA arises due to the presence of
(1) Base unit
(2) Chiral phosphate unit
(3) D-sugar component
(4) L- sugar component

Answer (3)

Sol. Chirality is due to D-sugar component

59. Which of the following statements are TRUE about Haloform reaction?
A. Sodium hypochlorite reacts with KI to give KOI .
B. KOI is a reducing agent.
C. α, β-unsaturated methylketone $\left(CH_3-CH=CH-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3\right)$will give iodoform reaction.
D. Isopropyl alcohol will not give iodoform test.
E. Methanoic acid will give positive iodoform test.
Choose the correct answer from the options given below.
(1) A, C & E Only
(2) B, D & E Only
(3) A, B & C Only
(4) A & C Only

Answer (4)

Sol. KOI is an oxidizing agent; α, β-unsaturated methylketone and isopropyl alcohol gives positive iodoform reaction

60. Given below are two statements:

Statement I: $(CH_3)_3\overset{\oplus}{C}$ is more stable than $\overset{\oplus}{C}H_3$ as nine hyperconjugation interactions are possible in $(CH_3)_3\overset{\oplus}{C}$.

Statement II: $\overset{\oplus}{C}H_3$ is less stable than $(CH_3)_3\overset{\oplus}{C}$ as only three hyperconjugation interactions are possible in $\overset{\oplus}{C}H_3$.

In the light of the above statements, choose the correct answer from the options given below
(1) Both Statement I and Statement II are false
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Statement I is false but Statement II is true

Answer (3)

Sol. No hyperconjugation possible in methyl carbocation, t-butyl carbocation has 9 alpha hydrogens which are responsible for stability .

61. Identify the INCORRECT statements from the following:
A. Notation ${}^{24}_{12}Mg$ represents 24 protons and 12 neutrons.
B. Wavelength of a radiation of frequency $4.5 \times 10^{15}\ s^{-1}$ is 6.7×10^{-8} m.
C. One radiation has wavelength $= \lambda_1(900\ nm)$ and energy $= E_1$. Other radiation has wavelength $= \lambda_2(300\ nm)$ and energy $= E_2$. $E_1 : E_2 = 3 : 1$.
D. Number of photons of light of wavelength 2000 pm that provides 1 J of energy is 1.006×10^{16}.
Choose the correct answer from the options given below:
(1) B and C Only
(2) A and B Only
(3) A and D Only
(4) A and C Only

Answer (4)

Sol. (A) $^{24}_{12}Mg$ represents 12 electrons, 12 protons and 24 nucleons (proton + neutron)

(B) $c = \nu\lambda;\ 4.5 \times 10^{15} x\, \lambda\ = 3.015 \times 10^{8}\ m/s$

(C) $\frac{E_1}{E_2} = \frac{\frac{hc}{\lambda_1}}{\frac{hc}{\lambda_2}} = \frac{\lambda_2}{\lambda_1} = \frac{300}{900} = \frac{1}{3}$

(D) Number of photons

$= \dfrac{E \times \lambda}{hc} = \dfrac{1 \times 2000 \times 10^{-1}}{6.626 \times 10^{-34} \times 3 \times 10^{8}} = 1.006 \times 10^{16}$

Therefore A and C are correct

62. Which statements are NOT TRUE about $XeO_2\ F_2$?

A. It has a see-saw shape.

B. Xe has 5 electron pairs in its valence shell in $XeO_2\ F_2$.

C. The $O - Xe - O$ bond angle is close to 180°.

D. The $F - Xe - F$ bond angle is close to 180°.

E. Xe has 16 valence electrons in $XeO_2\ F_2$.

Choose the correct answer from the options given below:

(1) B, D and E Only

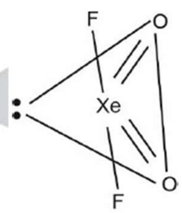
(2) A and D Only

(3) B, C and E Only

(4) B and D Only

Answer (3)

Sol.



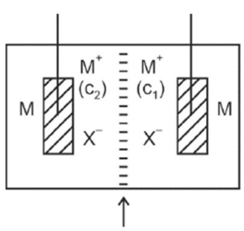
See-saw

$F - Xe - F$ bond angle is close to 180°.

Xe has 7 *electron pairs* .

$O - Xe - O$ bond angle is close to 120°.

63.



Semi permeable membrane

Consider the above electrochemical cell where a metal electrode (M) is undergoing redox reaction by forming $M^+ (M \rightarrow M^+ + e^-)$. The cation M^+ is present in two different concentrations c_1 and c_2 as shown above. Which of the following statement is correct for generating a positive cell potential?

(1) If c_1 is present at cathode, then $c_1 < c_2$

(2) If c_1 is present at anode, then $c_1 > c_2$

(3) If c_1 is present at anode, then $c_1 = c_2$

(4) If c_1 is present at cathode, then $c_1 > c_2$

Answer (4)

Sol. At anode : $M \rightarrow M^+_{(a)} + e^-$

At cathode: $M^+_{(c)} + e^- \rightarrow M$

$M^+_{(c)} \rightarrow M^+_{(a)}$

$E_{cell}\ = \dfrac{RT}{F} \ln \dfrac{M^+_{(c)}}{M^+_{(a)}}$

In order to have $E_{cell} > 0$; cathodic compartment concentration should be greater than anodic compartment concentration.

64. In Carius method 0.2425 g of an organic compound gave 0.5253 g silver chloride. The percentage of chlorine in the organic compound is

(1) 37.57%

(2) 34.79%

(3) 87.65%

(4) 53.58%

Answer (4)

Sol. % of $\mathrm{Cl} = \frac{35.5}{143.5} \times \frac{0.5253}{0.2425} \mathrm{X}100 = 53.58\%$

65. Given below are two statements:

Statement I: The second ionisation enthalpy of Na is larger than the corresponding ionisation enthalpy of Mg .

Statement II: The ionic radius of O^{2-} is larger than that of F^-.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Answer (4)

Sol. $\mathrm{Na} \xrightarrow{IE_1} \mathrm{Na}^+ \xrightarrow{IE_2} \mathrm{Na}^{2+}$; so IE_2 is high

$\mathrm{Mg} \xrightarrow[IE_1]{} \mathrm{Mg}^+ \rightarrow \mathrm{Mg}^{2+}$

$O^{2-}(1.40\text{Å})$ is larger than $F^-(1.33 - 1.36\text{Å})$

66. It is noticed that Pb^{2+} is more stable than Pb^{4+} but Sn^{2+} is less stable than Sn^{4+}. Observe the following reactions.

$PbO_2 + Pb \rightarrow 2PbO; \Delta_r G^\circ(1)$

$SnO_2 + Sn \rightarrow 2SnO; \Delta_r G^\circ(2)$

Identify the correct set from the following

(1) $\Delta_r G^\circ(1) > 0; \Delta_r G^\circ(2) < 0$

(2) $\Delta_r G^\circ(1) < 0; \Delta_r G^\circ(2) > 0$

(3) $\Delta_r G^\circ(1) > 0; \Delta_r G^\circ(2) > 0$

(4) $\Delta_r G^\circ(1) < 0; \Delta_r G^\circ(2) < 0$

Answer (2)

Sol. Due to inert pair effect ; As Pb^{2+} is more stable than Pb^{4+}

$(\Delta G^\circ)_1 < 0$

And as Sn^{2+} is less stable than Sn^{4+}

$(\Delta G^\circ)_2 > 0$

67. Identify the CORRECT set of details from the following:

A. $[Co(NH_3)_6]^{3+}$: Inner orbital complex; d^2sp^3 hybridized.

B. $[MnCl_6]^{3-}$: Outer orbital complex; sp^3d^2 hybridized.

C. $[CoF_6]^{3-}$: Outer orbital complex; d^2sp^3 hybridized.

D. $[FeF_6]^{3-}$: Outer orbital complex; sp^3d^2 hybridized.

E. $[Ni(CN)_4]^{2-}$: Inner orbital complex; sp^3 hybridized.

Choose the correct answer from the options given below.

(1) A, C & E Only

(2) A, B, C, D & E

(3) A, B & D Only

(4) C & D Only

Answer (3)

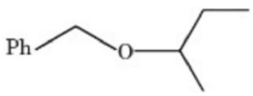
Sol.

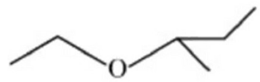
(A) $[Co(NH_3)_6]^{3+} \rightarrow d^2sp^3 \rightarrow$ Inner orbital complex

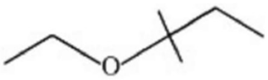
(B) $[MnCl_6]^{3-} \rightarrow Mn^{3+} \rightarrow 3\ d^4, Cl^- \rightarrow$ Weak Field Ligand $\rightarrow sp^3\ d^2$

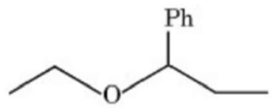
(C) $[CoF_6]^{3-}$;sp3d2 hybridised ; outer orbital complex
(D) $FeF_6^{3-} \rightarrow Fe^{3+} \rightarrow 3\ d^5,\ F^- \rightarrow \text{WeakField Ligand}, sp^3\ d^2$
(E) $Ni(CN)_4^{2-}$ is dsp^2 hybridised.square planar complex

68. A mixed ether (P), when heated with excess of hot concentrated hydrogen iodide produces two different alkyl iodides which when treated with aq. NaOH give compounds (Q) and (R). Both (Q) and (R) give yellow precipitate with NaOI . Identify the mixed ether (P):

(1) 

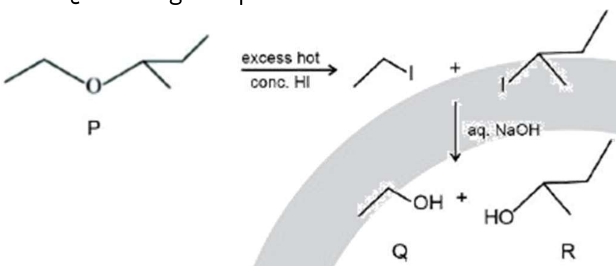
(2) 

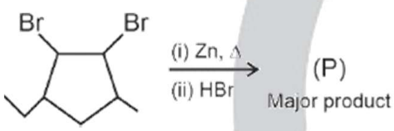
(3) 

(4) 

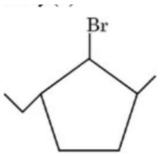
Answer (2)

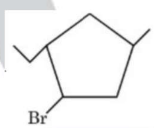
Sol. both Q and R gives positive iodoform test

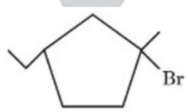


69. 

Identify (P)

(1) 

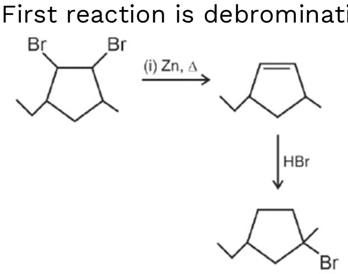
(2) 

(3) 

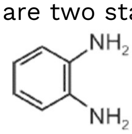
(4) 

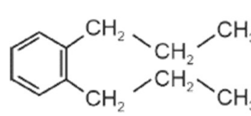
Answer (3)

Sol. First reaction is debromination by Zn; second reaction is HBr addition to alkene

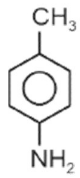
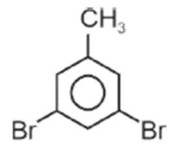


70. Given below are two statements:

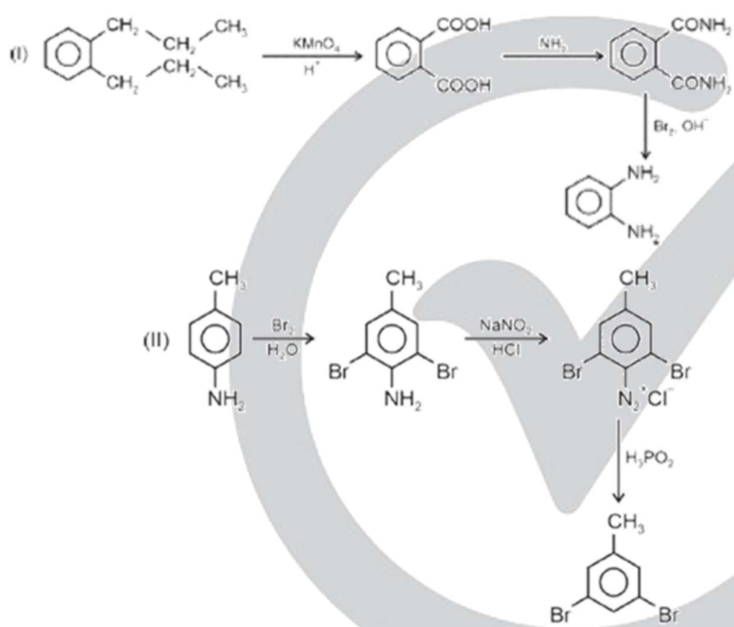
Statement I: 

can be synthesized from 

using simpler reagents in the order (i) Acidic $KMnO_4$, (ii) Ammonia, (iii) Bromine and alkali

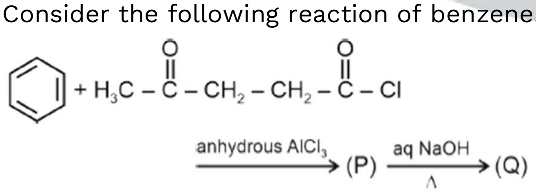
Statement II:  can be converted into  using reagents in the order (i) Bromine H_2O
(ii) $NaNO_2/HCl(0-5^{\circ}C)$
(iii) Aq. H_3PO_2.
In the light of above statements, choose the correct answer from the options given below.
(1) Statement I is true but statement II is false
(2) Both statement I and statement II are false
(3) Both statement I and statement II are true
(4) Statement I is false but statement II is true

Answer (3)
Sol.



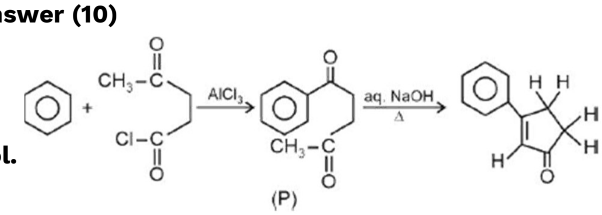
SECTION - B

71. Consider the following reaction of benzene.



In compound (Q), the percentage of oxygen is ___ %. (Nearest integer)

Answer (10)

Sol. 

Mass percentage of $O = \frac{16\times100}{158} = 10.126\%$

72. $X_2(g) + Y_2(g) \rightleftharpoons 2Z(g)$
$X_2(g)$ and $Y_2(g)$ are added to a 1 L flask and it is found that the system attains the above

equilibrium at $T(K)$ with the number of moles of $X_2(g), Y_2(g)$ and $Z(g)$ being 3,3 and 9 mol respectively (equilibrium moles). Under this condition of equilibrium, 10 mol of $Z(g)$ is added to the flask and the temperature is maintained at $T(K)$. Then the number of moles of $Z(g)$ in the flask when the new equilibrium is established is ___ . (Nearest integer)

Answer (15)

Sol. At equation $\underset{3\text{ mol}}{X_2(g)} + \underset{3\text{ mol}}{Y_2(g)} \underset{9\text{ mol}}{2Z(g)}$

$K_{eq} = \frac{(9)^2}{3 \times 3} = \frac{9 \times 9}{3 \times 3} = 9$

when 10 mol of Z is added reaction moves in backward direction

$\underset{3+x}{X_2(g)} + \underset{3+x}{Y_2(g)} \rightleftharpoons \underset{19-2x}{2Z}$

$9 = \frac{(19-2x)^2}{(3+x)^2}$

$3 = \frac{19-2x}{3+x}$

$9 + 3x = 19 - 2x$

$5x = 10$

$x = \frac{10}{5} = 2$

Moles of Z at new equilibrium

$= 19 - 2 \times 2 = 15$

73. Two liquids A and B form an ideal solution. At 320 K , the vapour pressure of the solution, containing 3 mol of A and 1 mol of B is 500 mm Hg . At the same temperature, if 1 mol of A is further added to this solution, vapour pressure of the solution increases by 20 mm Hg . Vapour pressure (in mm Hg) of B in pure state is ___ . (Nearest integer)

Answer (200)

Sol. $P_S = X_A P_A^\circ + X_B P_B^\circ$

$$500 = \frac{3}{4} \times P_A^O + \frac{1}{4} \times P_B^O$$

$2000 = 3P_A^O + P_B^O$(i)

After adding 1 mole of A,

$$520 = \frac{4}{5} P_A^O + \frac{P_B^O}{5}$$

$2600 = 4P_A^O + P_B^O$ (ii)

(ii) − (i) $\Rightarrow 600\text{ mm} = P_A^O$

$P_B^O = 2600 - 2400$

$= 200\text{ mmHg}$

74. Total number of unpaired electrons present in the central metal atoms/ions of $[Ni(CO)_4]$, $[NiCl_4]^{2-}$, $[PtCl_2(NH_3)_2]$, $[Ni(CN)_4]^{2-}$ and $[Pt(CN)_4]^{2-}$ is ___ .

Answer (2)

Sol.

$[Ni(CO)_4] \Rightarrow 3d^{10} \Rightarrow sp^3 \quad n = 0$

$[NiCl_4]^{2-} \Rightarrow 3d^8 \Rightarrow sp^3 \quad n = 2$

$[PtCl_2(NH_3)_2] \Rightarrow 3d^8 \Rightarrow dsp^2 \quad n = 0$

$[Ni(CN)_4]^{2-} \Rightarrow 3d^8 \Rightarrow dsp^2 \quad n = 0$

$[Pt(CN)_4]^{2-} \Rightarrow 3d^8 \Rightarrow dsp^2 \quad n = 0$

Total unpaired $e^- = 2$

75. 200 cc of $X \times 10^{-3}$M potassium dichromate is required to oxidise 750 cc of 0.6 M Mohr's salt solution in acidic medium.

Here $X =$ ___ .

Answer (375)

Sol. Number of equivalents of dichromate = Number of equivalents of Mohr's salt

$200 \times X \times 10^{-3} \times 6 = 750 \times 0.6 \times 1$

$X = 375$