JEE-MAIN EXAMINATION – JANUARY 2026

(23RD JANUARY 2026) **TIME: 9:00 A.M. TO 12:00 NOON**

MATHEMATICS TEST PAPER WITH SOLUTION

1. Among the statements:

I: If $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$,
then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{2}$, and

II: If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = px + q$, then $p^2 = 196q^2$

(1) only II is true
(2) only I is true
(3) both are true
(4) both are false

Answer (4)

Sol.

$$\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = 1\begin{pmatrix}(1-\cos^2\gamma) - \cos\alpha(\cos\alpha - \cos\beta\cos\gamma) \\ +\cos\beta(\cos\alpha\cos\gamma - \cos\beta)\end{pmatrix}$$

$$= 1 - \cos^2\gamma - \cos^2\alpha - \cos^2\beta + 2\cos\beta + 2 + \cos\alpha\cos\beta\cos\gamma$$

$$\begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix} = -\cos\alpha(-\cos\beta\cos\gamma) + \cos\beta(\cos\alpha\cos\gamma)$$

$$= 2\cos\alpha\cos\beta - \cos\gamma$$

$$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\because q = \begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 3 & -1 & -1 \end{vmatrix} = -10 - 2 = -12$$

And $-p + q = \begin{vmatrix} 0 & 0 & -3 \\ -2 & -3 & -6 \\ 2 & -3 & -3 \end{vmatrix} = -36$

$\Rightarrow p = q + 36 = 24$

$\Rightarrow p^2 \neq 196q^2$

Option (4) is correct.

2. The value of the integral $\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1+\sqrt[3]{\tan 2x}}$ is:

(1) $\frac{\pi}{3}$
(2) $\frac{\pi}{12}$
(3) $\frac{\pi}{18}$
(4) $\frac{\pi}{6}$

Answer (2)

Sol. $I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{1}{1+(\tan 2x)^{1/3}} dx$

$$\Rightarrow \quad I=\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{1}{1+\tan\left(2\left(\frac{5\pi}{24}+\frac{\pi}{24}-4\right)\right)^{1/3}}dx$$

$$I=\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{1}{1+\tan\left(\frac{\pi}{2}-2x\right)^{1/3}}dx(2)$$

From equation (1) and (2)

$$2I=\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{(\tan(2x))^{1/3}+1}{1+(\tan 2x)^{1/3}}dx$$

$$=(x)|_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \Rightarrow I=\frac{1}{2}\times\frac{4\pi}{24}=\frac{\pi}{12}$$

3. If α and $\beta(\alpha<\beta)$ are the roots of the equation $(-2+\sqrt{3})(|\sqrt{x}-3|)+(x-6\sqrt{x})+(9-2\sqrt{3})=0, x\geqslant 0$, then $\sqrt{\frac{\beta}{\alpha}}+\sqrt{\alpha\beta}$ is equal to:

(1) 8

(2) 10

(3) 9

(4) 11

Answer (2)

Sol. $(-2+\sqrt{3})(|\sqrt{x}-3|)+(x-6\sqrt{x}+9)-2\sqrt{3}=0, x\geq 0$ put $|\sqrt{x}-3|=t$

$\Rightarrow (-2+\sqrt{3})t+t^2-2\sqrt{3}=0$

$\Rightarrow t(t-2)+\sqrt{3}(t-2)=0$

$\Rightarrow (t+\sqrt{3})(t-2)=0$

$t+\sqrt{3}\neq 0\Rightarrow t=2$

$\Rightarrow |\sqrt{x}-3|=2$

$\sqrt{x}-3=\pm 2$

$\sqrt{x}=5,1$

$x=25,1$

$\Rightarrow \alpha=1,\beta=25$

$\Rightarrow \sqrt{\frac{\beta}{\alpha}}+\sqrt{\alpha\beta}=10$

Option (2) is Correct.

4. Let the line $y-x=1$ intersect the ellipse $\frac{x^2}{2}+\frac{y^2}{1}=1$ at the points A and B. Then the angle made by the line segment AB at the center of the ellipse is:

(1) $\pi-\tan^{-1}\left(\frac{1}{4}\right)$

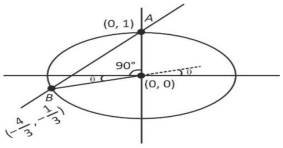
(2) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)$

(3) $\frac{\pi}{2}+2\tan^{-1}\left(\frac{1}{4}\right)$

(4) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

Answer (4)

Sol.



$$y = x + 1 \text{ intersects } \frac{x^2}{2} + y^2 = 1$$

$$\Rightarrow \frac{x^2}{2} + (x+1)^2 = 1 \Rightarrow x^2 + 2x^2 + 4x = 0$$

$$\Rightarrow x = 0, \frac{-4}{3}$$

$\Rightarrow$ Points A and B are

$(0,1), \left(\frac{-4}{3}, \frac{-1}{3}\right)$

$$\theta = \tan^{-1}\left(\frac{\frac{-1}{3}}{\frac{-4}{3}}\right) = \tan^{-1}\left(\frac{1}{4}\right)$$

$$\Rightarrow \angle AOB = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$$

5. Let $\vec{a} = -\hat{\imath} + \hat{\jmath} + 2\hat{k}$, $\vec{b} = \hat{\imath} - \hat{\jmath} - 3\hat{k}, \vec{c} = \vec{a} \times \vec{b}$ and $\vec{d} = \vec{c} \times \vec{a}$. Then $(\vec{a} - \vec{b}) \cdot \vec{d}$ is equal to:

(1) -4

(2) -2

(3) 4

(4) 2

Answer (2)

Sol. $(\vec{a} - \vec{b}) \cdot \vec{d} = \vec{a} \cdot \vec{d} - \vec{b} \cdot \vec{d}$

$\because \vec{d} = \vec{c} \times \vec{a} \Rightarrow \vec{a} \cdot \vec{d} = [\vec{a}\vec{c}\vec{a}] = 0$

And $\vec{b} \cdot \vec{d} = \vec{b} \cdot (\vec{c} \times \vec{a})$

$= \vec{b} \cdot ((\vec{a} \times \vec{b}) \times \vec{a})$

$\Rightarrow (\vec{a} - \vec{b}) \cdot \vec{d} = \vec{b} \cdot (\vec{a} \times (\vec{a} \times \vec{b}))$

$= \vec{b} \cdot [\vec{a} \cdot \vec{b}\vec{a} - \vec{a} \cdot \vec{a}\vec{b}]$

$= (\vec{a} \cdot \vec{b})^2 - (\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{b})$

$= (-8)^2 - 6 \times 11 = 64 - 66 = -2$

Option (2) is correct.

6. Let $A = \{-2, -1, 0, 1, 2, 3, 4\}$. Let R be a relation on A defined by xRy if and only if $2x + y \leqslant 2$. Let I be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations respectively. Then $I + m + n$ is equal to :

(1) 33

(2) 35

(3) 34

(4) 32

Answer (1)

Sol.

$\frac{x}{y}$	(-2)	(-1)	0	1	2	3	4
-2	P	P	P	P	P	A	A
-1	P	P	P	P	A	A	A
0	P	P	P	P	A		
1	P	P	P	A			
2	P	P	P		A		
3	P	P				A	
4	P	P					A

Here P implies, A implies absent

$\Rightarrow$ 10 elements to be added as $A = 10, P = 23$

$\Rightarrow A + P = 33$

7.Let the domain of the function $f(x) = \log_3 \log_5 \log_7 (9x - x^2 - 13)$ be the interval (m, n). Let the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ have eccentricity $\frac{n}{3}$ and the length of the latus rectum $\frac{8m}{3}$. Then $b^2 - a^2$ is equal to :

(1) 11

(2) 7

(3) 9

(4) 5

Answer (2)

Sol. $\log_5 \left(\log_7 (9x - x^2 - 13)\right) > 0$

$\Rightarrow 9x - x^2 - 13 > 7$

$\Rightarrow x \in (4,5) = (m, n)$

$\Rightarrow$ eccentricity $= \frac{5}{3}$

$L(LR) = \frac{32}{3}$

$\frac{25}{9} = 1 + \frac{b^2}{a^2}$(1)

$\frac{2b^2}{a} = \frac{32}{3} \Rightarrow b^2 = \frac{16a}{3}$

Stultify in (1)

$\frac{16}{9} = \frac{16a}{3a^2} \Rightarrow a = 3$

$b^2 = 16$

$\therefore b^2 - a^2 = 16 - 9$

$= 7$

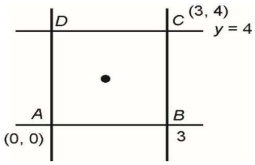
8. A rectangle is formed by the lines $x = 0, y = 0, x = 3$ and $y = 4$. Let the line L be perpendicular to $3x + y + 6 = 0$ and divide the area of the rectangle into two equal parts. Then the distance of the point $\left(\frac{1}{2}, -5\right)$ from the line L is equal to :

(1) $3\sqrt{10}$

(2) $2\sqrt{10}$
(3) $2\sqrt{5}$
(4) $\sqrt{10}$

Answer (2)

Sol.



For L to bisect area, line must pass through the centre of rectangle $\left(\frac{3}{2}, 2\right)$

$\Rightarrow L \equiv (y-2) = \frac{1}{3}\left(x - \frac{3}{2}\right)$

$\Rightarrow 3y - 6 = x - \frac{3}{2} \Rightarrow 6y - 12 = 2x - 3$

$6y - 2x - 9 = 0$

The distance from L of $\left(\frac{1}{2}, -5\right)$ is

$$\left|\frac{6(-5) - 2\left(\frac{1}{2}\right) - 9}{\sqrt{40}}\right| = \sqrt{40}$$

$= 2\sqrt{10}$

9. The value of $\frac{^{100}C_{50}}{51} + \frac{^{100}C_{51}}{52} + \cdots . + \frac{^{100}C_{100}}{101}$ is :

(1) $\frac{2^{100}}{100}$
(2) $\frac{2^{101}}{100}$
(3) $\frac{2^{101}}{101}$
(4) $\frac{2^{100}}{101}$

Answer (4)

Sol. $(1+x)^{100} = {}^{100}C_0 + {}^{100}C_{1x} + {}^{100}C_{2x^2} + \cdots \ldots . + {}^{100}C_{100x^{100}}$

$$\sum_{r=50}^{100} \frac{^{100}C_r}{r+1} = \frac{1}{101}\sum_{r=50}^{100} {}^{101}C_{r+1}$$

$$\Rightarrow \frac{1}{101}\sum_{r=51}^{100} {}^{101}C_r$$

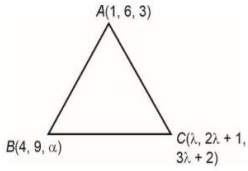
$$\Rightarrow \frac{2^{100}}{101}$$

10. The vertices B and C of a triangle ABC lie on the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{z-2}{3}$. The coordinates of A and B are (1,6,3) and (4,9, α) respectively and C is at a distance of 10 units from B. The area (in sq. units) of $\triangle$ ABC is :

(1) $5\sqrt{13}$
(2) $10\sqrt{13}$
(3) $15\sqrt{13}$
(4) $20\sqrt{13}$

Answer (1)

Sol.



$L: \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$

Any point $(\lambda, 2\lambda+1, 3\lambda+2)$

Also B lies on L

$\Rightarrow \frac{4}{1} = \frac{\alpha-2}{3} \Rightarrow \alpha = 14 \ \therefore B(4,9,14)$

$C \equiv \langle 4,9,14\rangle \pm \frac{10}{\sqrt{14}}\langle 1,2,3\rangle$

Area of $\triangle ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \overrightarrow{AB} = \langle 3,3,11\rangle$

$\overrightarrow{AC} = \overrightarrow{AB} + \frac{10}{\sqrt{11}}\langle 1,2,3\rangle$

Cross product of $\overrightarrow{AB}$ to eliminate parallel component

So, area $= \frac{1}{2} \times \frac{10}{\sqrt{14}}(\overrightarrow{AB} \times \langle 1,2,3\rangle)$

$\overrightarrow{AB} \times \langle 1,2,3\rangle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 11 \\ 1 & 2 & 3 \end{vmatrix}$

$= \langle -13,2,3\rangle$

Area $= \frac{1}{2} \times \frac{10}{\sqrt{14}} \cdot \sqrt{182} = 5\sqrt{13}$

11. Let the direction cosines of two lines satisfy the equations: $4l + m - n = 0$ and $2mn + 10nl + 3lm = 0$. Then the cosine of the acute angle between these lines is :

(1) $\frac{10}{7\sqrt{38}}$

(2) $\frac{10}{\sqrt{38}}$

(3) $\frac{20}{3\sqrt{38}}$

(4) $\frac{10}{3\sqrt{38}}$

Answer (4)

Sol. $n = 4l + m$

$$n(2m + 10l) + 3lm = 0$$
$$\Rightarrow 2(4l + m)(m + 5l) + 3lm = 0$$
$$\Rightarrow 8lm + 2m^2 + 10ml + 40R + 3lm = 0$$
$$\Rightarrow 40R + 21ml + 2m^2 = 0$$
$$\Rightarrow 40R + 16ml + 5ml + 2m^2 = 0$$
$$\Rightarrow (8l + m)(5l + 2m) = 0$$
$$m = -8l$$
$$n = -4l \ \Rightarrow$$

$$D \cdot R_1 = (l_1, m_1, n_1)$$
$$= (-1, -8, -4)$$
$$DR_2 = (l_2, m_2, n_2)$$

$= (2, -5, 3)$

$\cos\theta = \left|\frac{2 + 40 - 12}{9 \cdot \sqrt{38}}\right|$

$\cos\theta = \frac{10}{3\sqrt{38}}$

i.e. option (1) is correct

12. Let $S = \{z: 3 \le |2z - 3(1+i)| \le 7\}$ be a set of complex numbers.

Then $\text{Min}_{z\in S}\left|\left(z + \frac{1}{2}(5+3i)\right)\right|$ is equal to:

(1) 2

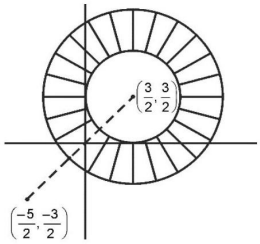
(2) $\frac{1}{2}$

(3) $\frac{5}{2}$

(4) $\frac{3}{2}$

Answer (4)

Sol. $\frac{3}{2} \le \left|z - \left(\frac{3}{2} + \frac{3}{2}i\right)\right| \le \frac{7}{2}$



Min distance $= \sqrt{16+9} - \frac{7}{2}$

$= 5 - \frac{7}{2}$

$= \frac{3}{2}$

13. Let $f(x) = \begin{cases} \frac{ax^2+2ax+}{4x^2+4x-3} & , x \ne -\frac{3}{2}, \frac{1}{2} \\ b & , x = -\frac{3}{2}, \frac{1}{2} \end{cases}$

be continuous at $x = -\frac{3}{2}$. If $f \circ f(x) = \frac{7}{5}$, then x is equal to:

(1) 0

(2) 1

(3) 1.4

(4) 2

Answer (2)

Sol. $\lim_{x\to\frac{-3}{2}} \frac{ax^2+2ax+}{4x^2+4x-3} = b$

$ax^2 + 2ax + 3 = 0$ at $n = \frac{-3}{2}$

$\Rightarrow \frac{9a}{4} - 3a + 3 = 0$

$\Rightarrow \frac{3a}{4} = 3$

$\Rightarrow a = 4$

$\lim_{x\to\frac{-3}{2}} \frac{4x^2 + 8ax + 3}{4x^2 + 4x - 3}$

$= \lim_{x\to\frac{-3}{2}} \frac{(2x+1)(2x+3)}{(2x-1)(2x+3)}$

$= \frac{-2}{-4} = \frac{1}{2} = b$

$\therefore f(x) = \frac{2x+1}{2x-1}$

$f(f(x)) = \frac{7}{5}$

$\therefore \frac{2f(x)+1}{2f(x)-1}=\frac{7}{5}$

$\Rightarrow f(x)=3$

$\frac{2x+1}{2x-1}=3$

$\Rightarrow x=1$

14. Let $f(x)=\int \frac{(2-x^2)\cdot e^x}{(\sqrt{1+x})(1-x)^{\frac{3}{2}}}dx$. If $f(0)=0$, then $f\left(\frac{1}{2}\right)$ is equal to :

(1) $\sqrt{2e}+1$

(2) $\sqrt{3e}+1$

(3) $\sqrt{2e}-1$

(4) $\sqrt{3e}-1$

Answer (4)

Sol. $f(x)=\int e^x\left[\frac{1}{\sqrt{1-x^2}(1-x)}+\frac{(1-x^2)}{\sqrt{1-x^2}(1-x)}\right]dx$

$=\int e^x\left(\frac{1}{(1-x)\sqrt{1-x^2}}+\frac{\sqrt{(1-x^2)}}{\sqrt{(1-x^2)}}\right)$

$=\int e^x(\underbrace{\frac{1}{(1-x)\sqrt{1-x^2}}}_{f'(x)}+\underbrace{\sqrt{\frac{1+x}{1-x}}}_{f(x)})$

$=e^x\sqrt{\frac{1+x}{1-x}}+c$

$\because f(0)=0\Rightarrow 0=1+c\Rightarrow c=-1$

$f\left(\frac{1}{2}\right)\Rightarrow e^{\frac{1}{2}}\sqrt{3}-1$

15. Number of solutions of $\sqrt{3}\cos 2\theta+8\cos\theta+3\sqrt{3}=0,\ \theta\in[-3\pi,2\pi]$ is:

(1) 4

(2) 0

(3) 3

(4) 5

Answer (4)

Sol. $\sqrt{3}\cos 2\theta+8\cos\theta+3\sqrt{3}=0,\theta\in[-3\pi,2\pi]$

$\Rightarrow \sqrt{3}(2\cos^2\theta-1)8\cos\theta+3\sqrt{3}=0$

$\Rightarrow 2\sqrt{3}\cos^2\theta+8\cos\theta+2\sqrt{3}=0$

$\Rightarrow 2\sqrt{3}\cos^2\theta+6\cos\theta+2\cos\theta+2\sqrt{3}=0$

$\Rightarrow 2\sqrt{3}\cos\theta(\cos\theta+\sqrt{3})+2(\cos\theta+\sqrt{3})=0$

$\therefore (2\sqrt{3}\cos\theta+2)(\cos\theta+\sqrt{3})=0$

$\Rightarrow \cos\theta=\frac{-2}{2\sqrt{3}}$

$=-\frac{1}{\sqrt{3}}$

Total 5 solutions

Option (4) is correct

16 A building construction work can be completed by two masons A and B together in 22.5 days. Mason A alone can complete the construction work in 24 days less than mason B alone. Then mason A alone will complete the construction work in :

(1) 36 days

(2) 30 days

(3) 42 days
(4) 24 days

Answer (1)

Sol. Let B's time = x days

$\Rightarrow$ As time $= (x-24)$ day

Rate of A $= \frac{1}{x-2}$

Rate of $B = \frac{1}{x}$

$\frac{1}{x-24}+\frac{1}{x}=\frac{1}{22.5}$

$\frac{2x-24}{x(x-24)}=\frac{1}{22.5}$

$45x-540=x^2-24x$

$x^2-69x+540=0$

$(x-60)(x-9)=0$

$\Rightarrow x=60$

At time $= 60-24$

$= 36$ days

17. Let the mean and variance of 8 numbers $-10,-7,-1, x, y, 9,2,16$ be $\frac{7}{2}$ and $\frac{293}{4}$, respectively.

Then the mean of 4 numbers $x, y, x+y+1, |x-y|$ is

(1) 12
(2) 10
(3) 11
(4) 9

Answer (3)

Sol. Sol. Mean $= \frac{7}{2}$

$\frac{-10-7-1+x+y+16+2+9}{8}=\frac{7}{2}$

$\Rightarrow x+y+9=28$

$\Rightarrow x+y=19$

Variance $= \frac{293}{4}$

$\frac{100+49+1+x^2+y^2+256+4+81}{8}-\frac{49}{4}=\frac{293}{4}$

$\frac{491+x^2+y^2}{8}=\frac{342}{4}$

$x^2+y^2=193$

$\Rightarrow (x,y)=(7,12)$ or $(12,7)$

Mean $= \frac{x+y+x+y+1+(x-y)}{4}$

$=\frac{19+20+5}{4}$

= 11

18. Let $y=y(x)$ be the solution of the differential equation $x^4\,dy+(4x^3y+2\sin x)dx=0,\ x>0, y\left(\frac{\pi}{2}\right)=0$.

Then $\pi^4 y\left(\frac{\pi}{3}\right)$ is equal to :

(1) 81
(2) 92
(3) 64
(4) 72

Answer (1)

Sol. $x^4dy + (4x^3y + 2\sin\ x)dx = 0$

$$\frac{dy}{dx} + \frac{4x^3}{x^4}y = \frac{-2\sin\ x}{x^4}$$
$$\frac{dy}{dx} + \frac{4}{x}y = \frac{-2\sin\ x}{x^4}$$
$$\text{IF } = e^{\int \frac{4}{x}dx = e^{\ln\ x^4} = x^4}$$
$$x^4\frac{dy}{dx} + 4x^3y = -2\sin\ x$$
$$\frac{d}{dx}(x^4y) = -2\sin\ x$$
$$\Rightarrow x^4y = 2\cos\ x + c$$

$\therefore x^4y = \cos\ x$

$\Rightarrow y = \frac{2\cos\ x}{x^4}$

$y\left(\frac{\pi}{3}\right) = \frac{2\cos\left(\frac{\pi}{3}\right)}{\left(\frac{\pi}{3}\right)^4} = \frac{1}{\left(\frac{\pi}{3}\right)^4} = \frac{81}{\pi^4}$

$\pi^4 y\left(\frac{\pi}{3}\right) = 81$

19. The sum of all possible values of $n \in N$ so that the coefficients of x, x^2 and x^3 in the expansion of $(1 + x^2)^2(1 + x)^n$, are in arithmetic progression is :

(1) 3

(2) 9

(3) 12

(4) 7

Answer (2)

Sol. $(1 + x^2)^2(1 + x^n)$

$(1 + x^4 + 2x^2)(1 + x)^n$

Coeff of x = ${}^nC_1$

Coeff of $x^2 = {}^nC_2 + 2.\ {}^nC_0$

Coeff of $x^3 = {}^nC_3 + 2.\ {}^nC_1$

$\because$ Coeff of $x, x^2, x^3 \rightarrow$ AP

$\Rightarrow 2({}^nC_2 + 2) = {}^nC_1 + {}^nC_3 + 2n$

${}^2nC_2 + 4 = {}^nC_3 + 3n$

$\Rightarrow n^2 - 9n^2 + 26n - 24 = 0$

$\Rightarrow n = 2,3,4$

$\Rightarrow$ Sum of all value of n is 9

20. Let α and β respectively be the maximum and the minimum values of the function

$f(\theta) = 4\left(\sin^4\left(\frac{7\pi}{2} - \theta\right) + \sin^4(11\pi + \theta)\right)$

$-2\left(\sin^6\left(\frac{3\pi}{2} - \theta\right) + \sin^6(9\pi - \theta)\right), \theta \in \mathbf{R}$.

Then $\alpha + 2\beta$ is equal to :

(1) 5

(2) 3

(3) 4

(4) 6

Answer (1)

Sol. $f(\theta) = 4\left(\sin^{-1}\left(\frac{7\pi}{2} - \theta\right) + \sin^4(11\pi + \theta)\right)$

$$-2\left(\sin^6\left(\frac{3\pi}{2}-\theta\right)+\sin^6(9\pi-\theta)\right)$$
$$=4(\cos^4\theta+\sin^4\theta\cos^2\theta)-2(\cos^2\theta+\sin^6\theta)$$
$$=4(1-2\sin^2\theta\cos^2\theta)$$
$$-2(1)(\sin^4\theta+\cos^4\theta-\sin^2\theta\cos^2\theta)$$
$$=4-8\sin^2\theta\cos^2\theta-2(1-3\sin^2\theta\cos^2\theta)$$
$$=2-2\sin^2\theta\cos^2\theta$$
$$=2-\frac{(\sin 2\theta)^2}{2}$$
$$f(\theta)_{\max}=2=\alpha\, f(\theta)_{\min}=2-\frac{1}{2}=\frac{3}{2}=\beta$$
$$\alpha+2\beta=2+3=5$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let the area of the region bounded by the curve $y=\max\{\sin x,\cos x\}$, lines $x=0, x=\frac{3\pi}{2}$, and the x-axis be A. Then, $A+A^2$ is equal to

Answer (12)

Sol. $f(x)=\max(\sin x,\cos x)$

$$=\begin{cases}\cos x, x\in\left(0,\frac{\pi}{4}\right)\\ \sin x, x\in\left(\frac{\pi}{4},\frac{5\pi}{4}\right)\\ \cos x, x\in\left(\frac{5\pi}{4},\frac{3\pi}{2}\right)\end{cases}$$

$$\Rightarrow$$

$$\Rightarrow\int_0^{\frac{3\pi}{4}}|f(x)|dx=\int_0^{\frac{\pi}{4}}(\cos x)dx+\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x)dx$$
$$+\int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}(\cos x)dx$$
$$=3\Rightarrow A=3$$
$$A+A^2=12$$

22. Let f be a twice differentiable non-negative function such that $(f(x))^2=25+\int_0^x((f(t))^2+(f'(t))^2)dt$.

Then the mean of $f(\log_e(1)), f(\log_e(2)), \ldots\ldots, f(\log_e(625))$ is equal to ___.

Answer (1565)

Sol. Let $y=f(x)$

$y^2=25+\int_0^x(f(t)^2+f'(t)^2)dt$

Differentiating w.r.t. x

$$2y\frac{dy}{dx}=y^2+\left(\frac{dy}{dx}\right)^2$$
$$\Rightarrow\left(\frac{dy}{dx}-y\right)^2=0\Rightarrow\frac{dy}{y}=dx$$
$$\Rightarrow\ln|y|=x+c$$

Such that

$f(0)^2 = 25 + \int_0^0 (f^2(t) + f'(t)^2)dt$
$\Rightarrow f(0) = 5$
$\Rightarrow \ln 5 = c$
$\Rightarrow \ln |y| = x + \ln 5$
$\Rightarrow y = 5e^x$
$f(\ln k) = 5e^{\ln k} = 5k$
Mean of $\{5,10, \ldots 625 \times 5\}$
$= \frac{(5 + 10 + \cdots 5 \times 625)}{625} = \frac{5(625)(626)}{625 \times 2}$
$= 5 \times 313$
= 1565

23. The number of 4-letter words, with or without meaning, which can be formed using the letters PQRPQRSTUVP, is ___ .

Answer (1422)

Sol. Letter frequency

P:3
R, Q: 2
S, T, U, V : 1
4 letter words can be of type
$\Rightarrow$ ABCD or AABC, AABB, AAAB
$\Rightarrow {}^7C_4 \cdot 4! + ({}^3C_1) \cdot {}^6C_2 \cdot \frac{4!}{2!}$

$$+ {}^3C_2 \cdot \frac{4!}{2!\,1!} + ({}^1C_1) \cdot {}^6C_1 \cdot \frac{4!}{3!}$$

$= 1422$

24. Let $|A| = 6$, where A is a 3×3 matrix. If $\left|\text{adj}\left(3\text{adj}(A^2 \cdot \text{adj}(2A))\right)\right| = 2^m \cdot 3^n, m, n \in \boldsymbol{N}$, then $m + n$ is equal to ___ .

Answer (62)

Sol. $(2A)(\text{adj}2A) = |2A|/3 \times 3$

$= 2^3 \cdot |\,A| I_{3\times 3}$
$= 48I$
$\Rightarrow A\text{adj}(2\,A) = 24I$
$\Rightarrow A^2(\text{adj}2\,A) = 24\,A$
$|\text{adj}(3\text{adj}(24\,A))| = |3\text{adj}(24\,A)|^{3-1}$
$= (3^3)^2 |(\text{adj}(24\,A))|^2$
$= 3^6 \cdot (|24\,A|^2)^2$
$= 3^6 |24\,A|^4$
$= 3^6 \cdot [(24)^3 |\,A|]^4$
$= 3^6 \cdot 24^{12} \cdot |\,A|^4 = 3^6 \cdot 24^{12} \cdot 6^4$
$= 3^6 \cdot (2^3)^{12} \cdot 3^{12} \cdot 3^4 \cdot 2^4$
$= 2^{40} \cdot 3^{22}$
$\Rightarrow m + n = 62$

25. From the first 100 natural numbers, two numbers first a and then b are selected randomly without replacement. If the probability that $a - b \geqslant 10$ is $\frac{m}{n}, gcd(m,n) = 1$, then m + n is equal to ___.

Answer (311)

Sol. Let $b = 1$

$\Rightarrow a > 11 \Rightarrow a$ can take 90 values from $\{11,12, \ldots 100\}$

Let $b = 2 \Rightarrow a > 12 \Rightarrow 8 >$ values and so on till $b = 99 \Rightarrow$ a can only take $a = 100$, only 1 value $\Rightarrow$ favourable cases $= 1 + 2 + 3 + \cdots . + 90$

$$= \frac{\frac{90 \times 91}{2}}{100 \times 99} = \frac{91}{220} \Rightarrow m + n = 311$$

PHYSICS TEST PAPER WITH SOLUTION

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer :

26. In hydrogen atom spectrum, ($R \rightarrow$ Rydberg's constant)

A. the maximum wavelength of the radiation of Lyman series is $\frac{4}{3R}$

B. the Balmer series lies in the visible region of the spectrum

C. the minimum wavelength of the radiation of Paschen series is $\frac{9}{R}$

D. the minimum wavelength of Lyman series is $\frac{5}{4R}$

Choose the correct answer from the options given below:

(1) A, B Only

(2) A, B and C Only

(3) A, B and D Only

(4) B, D Only

Answer (2)

Sol. λ is maximum for minimum energy change and vice-versa.

$$\frac{1}{\lambda} = Rz^2\left\{\frac{1}{n_1^2} - \frac{1}{n_2^2}\right\}$$

A. $\frac{1}{\lambda} = R\left(1 - \frac{1}{4}\right)$

$$\lambda = \frac{4}{3R}$$

B. Wavelength varies between 3800Å to 7600Å for Balmer series.

C. $\frac{1}{\lambda} = R\left(\frac{1}{9}\right)$

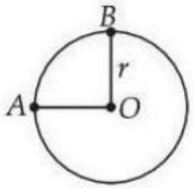
$$\lambda = \frac{R}{9}$$

D. $\frac{1}{\lambda} = R\left(\frac{1}{1}\right)$

$$\lambda = \frac{1}{R}$$

A, B, C are correct

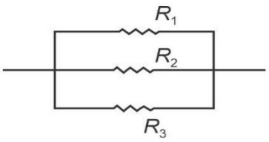
27. A wire of uniform resistance $\lambda\Omega/\text{m}$ is bent into a circle of radius r and another piece of wire with length $2r$ is connected between points A and $B(AOB)$ as shown in figure. The equivalent resistance between points A and B is ___ Ω.



(1) $\frac{6\pi\lambda r}{3\pi+1}$
(2) $(\pi+1)2r\lambda$
(3) $2\pi\lambda r$
(4) $\frac{3\pi\lambda r}{8}$

Answer (1)

Sol.

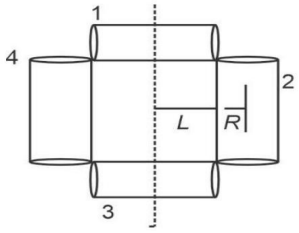


$R_1 = \frac{\lambda 2\pi r}{4} = \frac{\lambda\pi r}{4} = \frac{\lambda\pi r}{2}$
$R_2 = \lambda 2R$
$R_3 = \frac{3}{4}\lambda 2\pi r = \frac{3\lambda\pi r}{2}$
$\frac{1}{R} = \frac{2}{\lambda\pi r} + \frac{1}{\lambda 2r} + \frac{2}{3\lambda\pi r}$
$\frac{1}{R} = \frac{12+3\pi+4}{6\lambda\pi r}$
$R = \frac{6\pi\lambda r}{3\pi+16}$

28. The moment of inertia of a square loop made of four uniform solid cylinders, each having radius R and length $L(R<L)$ about an axis passing through the mid points of opposite sides, is (Take the mass of the entire loop as M) :
(1) $\frac{3}{8}MR^2 + \frac{7}{12}ML^2$
(2) $\frac{3}{4}MR^2 + \frac{7}{12}ML^2$
(3) $\frac{3}{4}MR^2 + \frac{1}{6}ML^2$
(4) $\frac{3}{8}MR^2 + \frac{1}{6}ML^2$

Answer (Bonus)

Sol.



$I = \sum I_i$
$= 2I_1 + 2I_2$

$2\left(\frac{MR^2}{4} + \frac{ML^2}{12}\right) + 2\left(\frac{MR^2}{2} + M\left(\frac{L}{2}+R\right)^2\right)$
$= \frac{7}{2}MR^2 + \frac{2}{3}ML^2 + 2MLR$
Now $M \to \frac{M}{4}$
$\frac{7}{8}MR^2 + \frac{1}{6}ML^2 + \frac{MLR}{2}$

29. A thin prism with angle 5° of refractive index 1.72 is combined with another prism of refractive index 1.9 to produce dispersion without deviation. The angle of second prism is ___ .

(1) 5°
(2) 4°
(3) 6°
(4) 4.5°

Answer (2)

Sol. $S_1 + S_2 = 0$

$(1.72 - 1)5^\circ + A(1.9 - 1) = 0$

$A(0.9) = -5 \times 0.72$

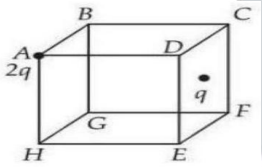
$A = -4^\circ$

30. Two point charges $2q$ and q are placed at vertex A and centre of face CDEF of the cube as shown in figure. The electric flux passing through the cube is:

(1) $\frac{3q}{2\varepsilon_0}$
(2) $\frac{q}{\varepsilon_0}$
(3) $\frac{3q}{\varepsilon_0}$
(4) $\frac{3q}{4\varepsilon_0}$

Answer (4)

Sol. $Q = Q_1 + Q_2$



$$= \frac{2q}{8\epsilon_0} + \frac{q}{2\epsilon_0}$$
$$= \frac{3q}{4\epsilon_0}$$

31. Consider light travelling from a medium A to medium B separated by a plane interface. If the light undergoes total internal reflection during its travel from medium A to B and the speed of light in media A and B are 2.4×10^8 m/s and 2.7×10^8 m/s, respectively, then the value of critical angle is:

(1) $\sin^{-1}\left(\frac{9}{8}\right)$
(2) $\cot^{-1}\left(\frac{3}{\sqrt{13}}\right)$
(3) $\cos^{-1}\left(\frac{8}{9}\right)$
(4) $\tan^{-1}\left(\frac{8}{\sqrt{17}}\right)$

Answer (4)

Sol. $\mu = \frac{c}{v}$

$\Rightarrow \mu_A = \frac{5}{4}, \mu_B = \frac{10}{9}$

$\theta_C = \sin^{-1}\left(\frac{10/9}{5/4}\right) = \sin^{-1}\frac{8}{9}$

$\sin^{-1}\frac{8}{9} = \tan^{-1}\frac{8}{\sqrt{17}}$

32. In a screw gauge, the zero of the circular scale lies 3 divisions above the horizontal pitch line when their metallic studs are brought in contact. Using this instrument thickness of a sheet is measured. If pitch scale reading is 1 mm and the circular scale reading is 51 then the correct thickness of the sheet is ___ mm .

[Assume least count is 0.01 mm]
(1) 1.48
(2) 1.54
(3) 1.50
(4) 1.51

Answer (1)

Sol. $\text{Z.E} = 3\text{CSD} = 3 \times 0.01$

$$\begin{aligned}\text{Reacting} &= 1\text{ mm} + 51\text{CSD} - \text{Z.E}\\ &= 1\text{ mm} + (51-3) \times 0.01\\ &= 1.48\end{aligned}$$

33. Given below are two statements: one is labelled as

Assertion (A) and the other is labelled as Reason (R).

Consider a ferromagnetic material:

Assertion (A): The individual atoms in a ferromagnetic material possess a magnetic dipole moment and interact with one another in such a way that they spontaneously align themselves forming domains.

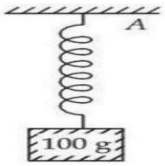
Reason (R): At high enough temperature, the domain structure of ferromagnetic material disintegrates. Thus, magnetization will disappear at high enough temperature known as Curie temperature.

In the light of the above statements, choose the correct answer from the options given below:
(1) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(2) (A) is false but (R) is true
(3) (A) is true but (R) is false
(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Answer (1)

Sol. Domains are reason for large induced magnetic dipole moment. Domains break down at high temperature.

34. Two blocks with masses 100 g and 200 g are attached to the ends of springs A and B as shown in figure. The energy stored in A is E. The energy stored in B, when spring constants k_A, k_B of A and B, respectively satisfy the relation $4k_A = 3k_B$, is



(1) $2E$
(2) $4E$
(3) $\frac{4}{3}E$
(4) $3E$

Answer (4)

Sol. $E = \frac{T^2}{2K}$

$$\frac{E_A}{E_B} = \left(\frac{100}{200}\right)^2 \frac{K_B}{KA}$$
$$\frac{E}{E_B} = \frac{1}{4} \times \frac{4}{3}$$
$$E_B = 3E$$

35. A 20 m long uniform copper wire held horizontally is allowed to fall under the gravity ($g = 10\ \text{m/s}^2$) through a uniform horizontal magnetic field of 0.5 Gauss perpendicular to the length of the wire. The induced EMF across the wire when it travells a vertical distance of 200 m is ___ mV .

(1) $20\sqrt{10}$

(2) $200\sqrt{10}$

(3) $2\sqrt{10}$

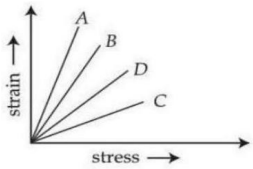
(4) $0.2\sqrt{10}$

Answer (1)

Sol. $\varepsilon = B/v$

$$= 0.5 \times 10^{-4} \times 20 \times \sqrt{20 \times 200}$$
$$= 0.5 \times 10^{-4} \times 20 \times 20\sqrt{10}$$
$$\varepsilon = 20\sqrt{10}\text{mV}$$

36. The strain-stress plot for materials A, B, C and D is shown in the figure. Which material has the largest Young's modulus?



(1) B

(2) D

(3) C

(4) A

Answer (3)

Sol. $\frac{\sigma}{\varepsilon} = Y$

for same strain, σ is highest for C

37. Match List - I with List - II.

	List-I Relation		List-II Law
A.	$\oint \dot{E} \cdot \overrightarrow{dl} = -\frac{d}{dt}\oint \dot{B} \cdot \overrightarrow{da}$	I.	Ampere's circuital law
B.	$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0\left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$	II.	Faraday's laws of electromagne tic induction
C.	$\oint \vec{E} \cdot \overrightarrow{da} = \frac{1}{\varepsilon_0}\int_v \rho \text{dv}$	III.	Ampere Maxwell law
D.	$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 l$	iv.	Gauss's law of electrostatics

Choose the correct answer from the options given below:

(1) A-I, B-IV, C-III, D-II

(2) A-II, B-III, C-I, D-IV

(3) A-II, B-III, C-IV, D-I

(4) A-IV, B-I, C-II, D-III

Answer (3)

Sol. A is Faraday's law
B is Modified ampere's law
C is Gauss's law
D is Ampere's law

38. A simple pendulum of string length 30 cm performs 20 oscillations in 10 s . The length of the string required for the pendulum to perform 40 oscillations in the same time duration is ___ cm.
[Assume that the mass of the pendulum remains same.]
(1) 7.5
(2) 0.75
(3) 15
(4) 120

Answer (1)
Sol. $T_1 = \frac{10}{20} = \frac{1}{2}$ S
$T_2 = \frac{10}{40} = \frac{1}{4}S$
$T = 2\pi\sqrt{\frac{L}{g}}$
$\frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{4}\right)^2} = \frac{30}{l}$
$I = \frac{30}{4}$
$I = 7.5$ cm

39. The de Broglie wavelength of an oxygen molecule at 27°C is $x \times 10^{-12}$ m. The value of x is
(take Planck's constant $= 6.63 \times 10^{-34}$ J.s, Boltzmann constant $= 1.38 \times 10^{-23}$ J/K, mass of oxygen molecule $= 5.31 \times 10^{-26}$ kg)
(1) 30
(2) 20
(3) 26
(4) 24

Answer (3)

Sol. $\lambda = \frac{h}{mv} v = \sqrt{\frac{3kT}{m}}$
$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.38 \times 10^{-23} \times 300 \times 5.31 \times 10^{-26}}}$
$\lambda = 0.258 \times 10^{-1}$ m

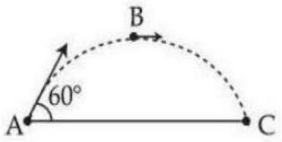
40. Four persons measure the length of a rod as 20.00 cm, 19.75 cm, 17.01 cm and 18.25 cm . The relative error in the measurement of average length of the rod is
(1) 0.08
(2) 0.18
(3) 0.06
(4) 0.24

Answer (3)

Sol. $\langle x \rangle = \frac{\Sigma x_i}{n}$

$$R_{eq} = \frac{\sum |(x - < x >)|}{n}$$
$$\text{Required} = \frac{(1.25 + 1 + 1.75 + 0.5)}{4 \times 18.75}$$
$$= 0.06$$

41. An object is projected with kinetic energy K from a point A at an angle 60° with the horizontal. The ratio of the difference in kinetic energies at points B and C to that at point A (see figure), in the absence of air friction is

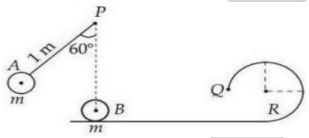


(1) $3:4$
(2) $2:3$
(3) $1:2$
(4) $1:4$

Answer (1)

Sol. $K_C = K_A$
$K \propto V^2$
$\Rightarrow K_B = \cos^2 60 K_A$
Required $= \frac{K_C - K_B}{KA} = 1 - \frac{1}{4} = \frac{3}{4}$

42. A small bob A of mass m is attached to a massless rigid rod of length 1 m pivoted at point P and kept at an angle of 60° with vertical as shown in figure. At distance of 1 m below point P, an identical bob B is kept at rest on a smooth horizontal surface that extends to a circular track of radius R as shown in figure. If bob B just manages to complete the circular path of radius R upto a point Q after being hit elastically by bob A, then radius R is ___ m.



(1) $\frac{2-\sqrt{3}}{5}$
(2) $\frac{3}{5}$
(3) $\frac{2+\sqrt{3}}{5}$
(4) $\frac{1}{5}$

Answer (4)

Sol. After collision speed of B
$v = \sqrt{2gl(1 - \cos 60)} = \sqrt{10}$
Also, $\sqrt{10} = \sqrt{50R} (= \sqrt{5}\ \text{g}/)$
$R = \frac{1}{5}$

43. In a perfectly inelastic collision, two spheres made of the same material with masses 15 kg and 25 kg , moving in opposite directions with speeds of $10\ \text{m/s}$ and $30\ \text{m/s}$, respectively, strike each other and stick together. The rise in temperature (in $^\circ\text{C}$), if all the heat produced during the collision is retained by these spheres, is :
(specific heat of sphere material $31\text{cal/kg.}^\circ\text{C}$ and $1\text{cal} = 4.2\ \text{J}$)
(1) 1.15
(2) 1.95
(3) 1.75
(4) 1.44

Answer (4)

Sol. $\Delta T = \frac{\Delta K}{m_1 s_1 + m_2 s_2}$

$$= \frac{\frac{1}{2} \times 15 \times 10^2 + \frac{1}{2} \times 25 \times 30^2 - \frac{1}{2} 40 \times 15^2}{40 \times 31 \times 4.2}$$

$$\Delta T = \frac{750T + 11250 - 4500}{5208} = 1.44$$

44. Two small balls with masses m and 2 m are attached to both ends of a rigid rod of length d and negligible mass. If angular momentum of this system is L about an axis (A) passing through its centre of mass and perpendicular to the rod then angular velocity of the system about A is

(1) $\frac{4}{3}\frac{L}{md^2}$

(2) $\frac{3}{2}\frac{L}{md^2}$

(3) $\frac{2L}{md^2}$

(4) $\frac{2L}{5md^2}$

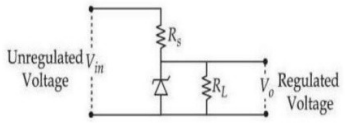
Answer (2)

Sol. $L = \mu\omega r^2$

$$L = \frac{2m}{3}\omega d^2$$

$$\omega = \frac{3L}{2md^2}$$

45. The following diagram shows a Zener diode as a voltage regulator. The Zener diode is rated at $V_z = 5$ V and the desired current in load is 5 mA . The unregulated voltage source can supply upto 25 V. Considering the Zener diode can withstand four times of the load current, the value of resistor R_s (shown in circuit) should be ___ Ω.



(1) 100

(2) 10

(3) 1000

(4) 4000

Answer (3)

Sol. $I_s = I_z + I_L$

$= (4 \times 5 + 5)\text{mA}$

$= 25$ mA

$V_s = 20\text{v}$

$20 = 25 \times 10^{-3} R_s$

$\frac{20}{25} \times 10 \times 100 = R_s$

$R_s = 800$

For $R_s = 4000$ and $V = 25$ V&$i_L = 5$ mA the Zener diode is just active therefore

$R_s = 1000$ should be most appropriate resistance

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. The equation of the electric field of an electromagnetic wave propagating through free space is given by: $E = \sqrt{377}\sin(6.27 \times 10^3 t - 2.09 \times 10^{-5} x)\text{N/C}$

The average power of the electromagnetic wave is $\left(\frac{1}{\alpha}\right)\text{W/m}^2$. The value of α is ___ (Take $\sqrt{\frac{\mu_0}{\varepsilon_0}} = 377$ in SI units)

Answer (2)

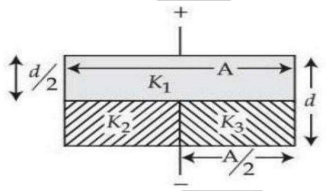
Sol. $I = \frac{\bar{E}AC\Delta t}{A\Delta t} = \bar{E}C$

$$= \frac{1}{2}\varepsilon_0 E_0^2 C$$

$$= \frac{1}{2}\varepsilon_0^2\sqrt{\frac{\varepsilon_0}{\mu_0}}$$

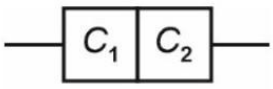
$$= \frac{377}{2 \times 377}\ \text{W/m}^2$$

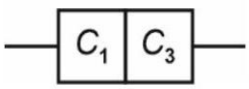
47. The space between the plates of a parallel plate capacitor of capacitance C (without any dielectric) is now filled with three dielectric slabs of dielectric constants $K_1 = 2, K_2 = 3$ and $K_3 = 5$ (as shown in figure). If new capacitance is $\frac{n}{3}C$ then the value of n is ___.



Answer (8)

Sol.





$$C = \frac{\varepsilon_0\frac{A}{2}}{\frac{d}{2k_1}+\frac{d}{2k_2}} + \frac{\varepsilon_0\frac{A}{2}}{\frac{d}{2k_1}+\frac{d}{2k_2}}$$

$$= \frac{\varepsilon_0 A}{d}\left\{\frac{1}{\frac{1}{2}+\frac{1}{3}} + \frac{1}{\frac{1}{2}+\frac{1}{5}}\right\}$$

$$C = C_0\left(\frac{6}{5}+\frac{10}{7}\right)$$

$$C = C_0\left(\frac{92}{35}\right) = \frac{NC_0}{3}$$

$$N = \frac{276}{35} \approx 8$$

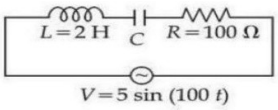
48. In two separate Young's double-slit experimental set-ups and two monochromatic light sources of different wavelengths are used to get fringes of equal width. The ratios of the slits separations and that of the wavelengths of light used are $2:1$ and $1:2$ respectively. The corresponding ratio of the distances between the slits and the respective screens (D_1/D_2) is ___。

Answer (4)

Sol. $B = \frac{\lambda D}{d}$

$$\frac{D_1}{D_2} = \frac{\frac{2}{1}}{\frac{1}{2}} = 4$$

49. Using a variable frequency a.c. voltage source the maximum current measured in the given LCR circuit is 50 mA for $V = 5\sin(100t)$. The values of L and R are shown in the figure. The capacitance of the capacitor (C) used is ___ μF.



Answer (50)

Sol. $L\omega = \frac{1}{\omega C}$

$$C = \frac{1}{100 \times 100 \times 2} \times 10^6 \mu F$$
$$= 50$$

50. A simple pendulum made of mass 10 g and a metallic wire of length 10 cm is suspended vertically in a uniform magnetic field of 2 T . The magnetic field direction is perpendicular to the plane of oscillations of the pendulum. If the pendulum is released from an angle of 60° with vertical, then maximum induced EMF between the point of suspension and point of oscillation is ___ mV . (Take $g = 10\ \mathrm{m/s^2}$)

Answer (100)

Sol. $E = Blv$

Where $V = \frac{l\omega}{2}$

$$E = \frac{2 \times 0.1\sqrt{2 \times 10 \times 0.05}}{2}$$
$$= 0.2$$
$$= 0.1\ \mathrm{V}$$

CHEMISTRY TEST PAPER WITH SOLUTION

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer :

51. Given below are two statements :

Statement I: $[CoBr_4]^{2-}$ ion will absorb light of lower energy than $[CoCl_4]^{2-}$ ion.

Statement II: In $[Col_4]^{2-}$ ion, the energy separation between the two set of d-orbitals is more than $[CoCl_4]^{2-}$ ion.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true
(2) Statement I is false but Statement II is true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are false

Answer (3)

Sol. Strength of ligand according to spectrochemical series $\Rightarrow F^- > Cl^- > Br > I^-$

- [CoBr4] $^{2-}$ ion will absorb light of lower energy than $[CoCl_4]^{2-}$
- Statement II is incorrect

52. The correct trend in the first ionization enthalpies of the elements in the 3^{rd} period of periodic table is :

(1) $S < Si < Al < P < Cl$
(2) $Si < S < Al < P < Cl$
(3) $Al < Si < S < P < Cl$
(4) $Al < S < P < Si < Cl$

Answer (3)

Sol. I. $E_1: Al < Si < S < P < Cl$

53. The correct statements from the following are:

A. Ionic radii of trivalent cations of group 13 elements decreases down the group.
B. Electronegativity of group 13 elements decreases down the group.
C. Among the group 13 elements, Boron has highest first ionisation enthalpy.
D. The trichloride and triiodide of group 13 elements are covalent in nature.

Choose the correct answer from the options given below :

(1) A and D Only
(2) B and D Only
(3) A and C Only
(4) C and D Only

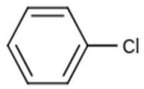
Answer (4)

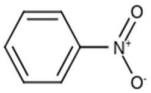
Sol. A. $B^{3+} < Al^{3+} < Ga^{3+} < In^{3+} < Tl^{3+}$ (Ionic radii)

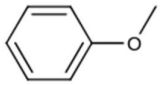
B. $\underset{}{B}\ \underset{1.5}{Al}\ \underset{1.6}{Ga}\ \underset{1.7}{In}\ \underset{1.8}{Tl}$ (Electronegativity)

C. $B\ \ Al\ \ Ga\ \ In\ \ Tl$(I.E in kJ/mol)

54. Consider the following compounds

a 

b 

c 

Arrange these compounds in the increasing order of reactivity with nitrating mixture.

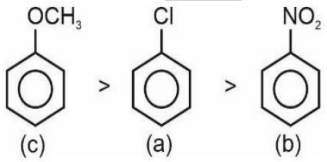
(1) $b < c < a$

(2) $c < a < b$

(3) b < a < c

(4) c < b < a

Answer (3)

Sol. 

(Order of reactivity with nitrating mixture)

55. Given below are two statements :

Statement I: Sublimation is used for the separation and purification of compounds with low melting point.

Statement II: The boiling point of a liquid increases as the external pressure is reduced.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Answer (2)

Sol. Statement-I and statement-II both are incorrect.

56. A cup of water at $5°C$ (system) is placed in a microwave oven and the oven is turned on for one minute during which the water begins to boil. Which of the following option is true?

(1) $q = -ve, w = -ve, \Delta U = -ve$

(2) $q = +ve, w = -ve, \Delta U = -ve$

(3) $q = +ve, w = 0, \Delta U = -ve$

(4) $q = +ve, w = -ve, \Delta U = +ve$

Answer (4)

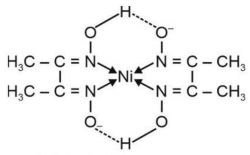
Sol. $q = +ve$
$w = -ve$
$\Delta U = +ve$

57. The statements that are incorrect about the nickel(II) complex of dimethylglyoxime are :
A. It is red in colour.
B. It has a high solubility in water at $pH = 9$.
C. The Ni ion has two unpaired d-electrons.
D. The $N - Ni - N$ bond angle is almost close to $90°$.
E. The complex contains four five-membered metallacycles (metal containing rings).

Choose the correct answer from the options given below.
(1) B, C and E Only
(2) C and D Only
(3) A, D and B Only
(4) C and E Only

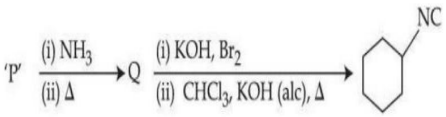
Answer (1)

Sol.

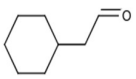


[Ni(dmg)] (Red in colour)
It contains 2 five-membered ring.
Ni^{2+} has no unpaired electron.

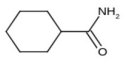
58. Compound ' P ' undergoes the following sequence of reactions :



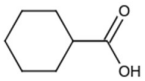
' P ' is
(1)



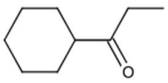
(2)



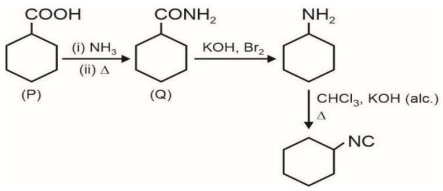
(3)



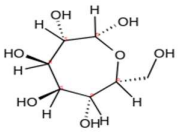
(4)

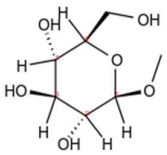


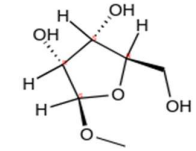
Answer (3)
Sol.

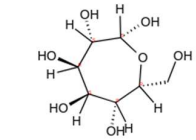


59. From the given following (A to D) cyclic structures, those which will not react with Tollen's reagent are

A 

B 

C 

D 

(1) B and D
(2) B and C
(3) A and D
(4) A and B

Answer (2)

Sol. Hemiacetals can reduce Tollen's reagent.

A, D ⇒ Reducing sugar

B, C ⇒ Non-reducing sugar

60. ' x ' is the product which is obtained from propanenitrile and stannous chloride in the presence of hydrochloric acid followed by hydrolysis. ' y ' is the product which is obtained from the but-2-ene by the ozonolysis followed by hydrolysis. From the following, which product is not obtained when one mole of ' x ' and one mole of ' y ' react with each other in the presence of alkali followed by heating?

(1) 2-Methylbut-2-enal
(2) 2-Methylpent-2-enal
(3) 3-Methylbut-2-enal
(4) Pent-2-enal

Answer (3)

Sol. $CH_3CH_2CN \xrightarrow[(ii)\ H_3O^+]{(i)\ SnCl_2/HCl} \underset{(x)}{CH_3CH_2CHO}$

$CH_3CH=CHCH_3 \xrightarrow[Zn,\ H_2O]{O_3} \underset{(y)}{CH_3CHO}$

$CH_3CH_2CHO + CH_3CHO \xrightarrow[OH^-]{dil}$

$CH_3CH=CHCHO$ (But-2-enal) + $CH_3-CH_2-CH=\underset{\underset{CH_3}{|}}{C}-CHO$ (2-Methylpent-2-enal)

+

$CH_3CH=\underset{\underset{CH_3}{|}}{C}-CHO$ (2-Methylbut-2-enal)

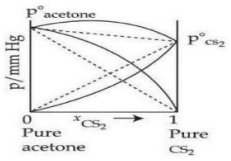
+

$CH_3CH_2CH=CH-CHO$

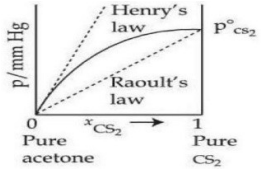
(Pent-2-enal)

61. Which one of the following graphs accurately represents the plot of partial pressure of CS_2 vs its mole fraction in a mixture of acetone and CS_2 at constant temperature?

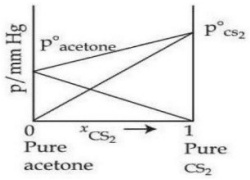
(1)



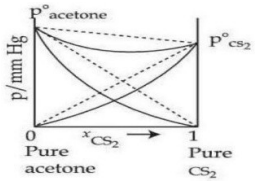
(2)



(3)



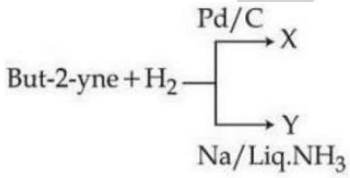
(4)



Answer (2)

Sol. A mixture of acetone and CS_2 exhibits a positive deviation from Raoult's law.

62. But-2-yne and hydrogen (one mole each) are separately treated with (i) Pd/C and (ii) $Na/liq.NH_3$ to give the products X and Y respectively.



Identify the incorrect statements.

A. X and Y are stereoisomers.

B. Dipole moment of X is zero.

C. Boiling point of X is higher than Y .

D. X and Y react with $O_3/Zn + H_2O$ to give different products.

Choose the correct answer from the options given below.

(1) B and C Only

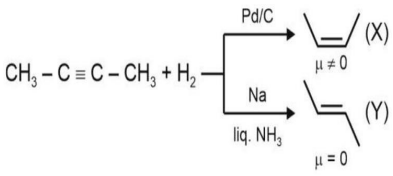
(2) A and C Only

(3) A and B Only

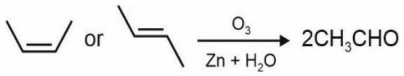
(4) B and D Only

Answer (4)

Sol.



B.P. : X > Y



63. Match List-I with List-II.

	List-I Functional group (detection)		List-II Change observed during detection
A.	Unsaturation (Baeyer's test)	I.	Red colour appears
B.	Alcoholic group (Ceric ammonium nitrate test)	II.	Silver mirror appears
C.	Aldehyde group (Tollen's reagent)	III.	Violet colour appears
D.	Phenolic group ($FeCl_3$ test)	IV.	Discharge of pink colour

Choose the correct answer from the options given below.

(1) A-III, B-IV, C-II, D-I

(2) A-IV, B-III, C-II, D-I

(3) A-III, B-IV, C-I, D-II

(4) A-IV, B-I, C-II, D-III

Answer (4)

Sol. A. Unsaturation (Baeyer's test) ⇒ Discharge of pink colour

B. Alcoholic group (ceric ammonium nitrate test) ⇒ Red colour appear

C. Aldehyde group (Tollen's reagent) ⇒ Violet colour appears

D. Phenolic group $(FeCl_3)$ ⇒ Violet colour appears

64. Consider the general reaction given below at 400 K $xA(g) \rightleftharpoons yB(g)$.

The values of K_p and K_c are studied under the same condition of temperature but variation in x and y.

(i) $K_p = 85.87$ and $K_c = 2.586$ appropriate units

(ii) $K_p = 0.862$ and $K_c = 28.62$ appropriate units

The values of x and y in (i) and (ii) respectively are:

(i)

(ii)

(1) 1,2 2,1

(2) 3,1 3,1

(3) 1,3 2,1

(4) 4,1 4,1

Answer (1)

Sol. (i) $\frac{K_P}{K_C} = (RT)^{y-x}$

$33.20 = (0.0821 \times 400)^{y-x}$

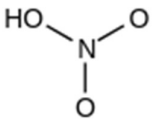
$\Rightarrow (y - x) = 1$

$x = 1$

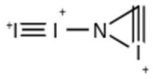
$y = 2$

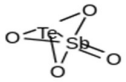
(ii) $\frac{K_P}{K_C} = (0.0821 \times 400)^{y-x}$

$$\frac{0.862}{28.62} = (32.84)^{y-x}$$
$$\frac{1}{33.20} = (32.84)^{y-x}$$
$$\Rightarrow (y-x) = -1$$
$$y = 1$$
$$x = 2$$

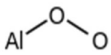


HNO_3





H_2SO_4



(10 lone pairs)

Bond angle → 102°

65. Which of the following statements regarding the energy of the stationary state is true in the following one - electron systems ?

(1) -2.18×10^{-18} J for third orbit of Li^{2+} ion

(2) $+8.72 \times 10^{-1}$ J for first orbit of He^{+}ion

(3) -1.09×10^{-18} J for second orbit of H atom.

(4) $+2.18 \times 10^{-18}$ J for second orbit of He^{+}ion

Answer (1)

Sol. $E_n = -2.18 \times 10^{-18}\left(\frac{Z^2}{n^2}\right)$ J

1. $n = 3, Z = 3 \Rightarrow E_n = -2.18 \times 10^{-18}$ J

2. $n = 1, Z = 2 \Rightarrow E_n = -8.72 \times 10^{-18}$ J

3. $n = 2, Z = 1 \Rightarrow E_n = -0.545 \times 10^{-18}$ J

4. $n = 2, Z = 1 \Rightarrow E_n = -2.18 \times 10^{-18}$ J

5. Identify the molecule (X) with maximum number of lone pairs of electrons (obtained using Lewis dot structure) among HNO_3, H_2SO_4, NF_3 and O_3.

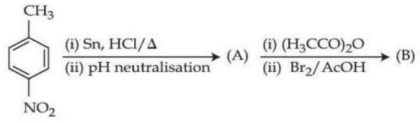
Choose the correct bond angle made by the central atom of the molecule (X).

(1) 107°
(2) 116°
(3) 102°
(4) 120°

Answer (3)

Sol.

67. Consider the following sequence of reactions.



4-Nitrotoluene

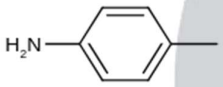
Assuming that the reaction proceeds to completion, then 137 mg of 4 -nitrotoluene will produce ___ mg of B.

(Given molar mass in $gmol^{-1}$ H: 1, C: 12 N: 14, O: 16Br: 80)

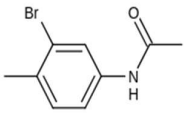
(1) 208
(2) 228
(3) 301
(4) 146

Answer (2)

Sol. A :



B :

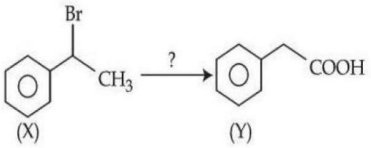


m. moles of reactant = 1

m . moles of product (B) = 1

mass of product $(B) = 228mg$

68.



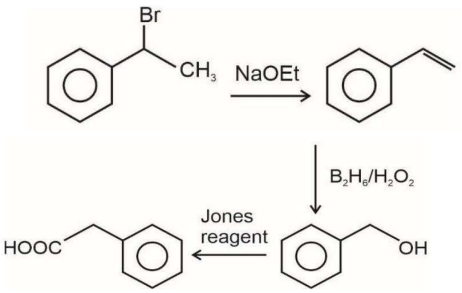
The correct sequence of reagents for the above conversion of X to Y is :

(1) (i) $NaOH(aq)$
(ii) Jones reagent
(iii) H_3O^+
(2) (i) Jones reagent
(ii) NaOEt
(iii) Hot $KMnO_4/KOH$
(3) (i) NaOEt
(ii) B_2H_6/H_2O_2
(iii) Jones reagent
(4) (i) B_2H_6/H_2O_2

(ii) NaOEt
(iii) Jones reagent

Answer (3)

Sol.



69. In the given electrochemical cell, $Ag(s)|AgCl(s)|FeCl_2(aq), FeCl_3(aq)$ | $Pt(s)$ at 298 K , the cell potential (E_{cell}) will increase when :

A. Concentration of Fe^{2+} is increased.
B. Concentration of Fe^{3+} is decreased.
C. Concentration of Fe^{2+} is decreased.
D. Concentration of Fe^{3+} is increased.
E. Concentration of $Cl^-$is increased.

Choose the correct answer from the options given below :
(1) A and B Only
(2) A and E Only
(3) B Only
(4) C, D and E Only

Answer (4)

Sol. C, D and E are correct

Anode: $Ag + Cl^- \rightarrow AgCl + e^-$

Cathode : $Fe^{3+} + e^- \rightarrow Fe^{2+}$

Net

$Ag + Cl^- + Fe^{3+} \rightarrow AgCl + Fe^{2+}$

$E_{cell} \uparrow$ if $Q \downarrow$

(C) $\Rightarrow Fe^{2+}$ is decreased $\Rightarrow Q \downarrow$
(D) $\Rightarrow Fe^{3+}$ is increased $\Rightarrow Q \downarrow$
(E) $\Rightarrow Cl^-$is increased $\Rightarrow Q \downarrow$

70. Given,

(A) $n = 5, m_l = -1$

(B) $n = 3, l = 2,\ m_l = -1,\ m_s = +\frac{1}{2}$

The maximum number of electron(s) in an atom that can have the quantum numbers as given in (A) and (B) respectively are :
(1) 8 and 1
(2) 2 and 4
(3) 4 and 1
(4) 26 and 1

Answer (1)

Sol. (A) $n = 5,\ m_l = -1 5p, 5\ d, 5f, 5\ g = 8$ electrons

(B) $n = 3, l = 2,\ m_l = -1,\ m_s = +\frac{1}{2}$

Only 1 electron is possible

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. Consider all the structural isomers with molecular formula $C_5H_{11}Br$ are separately treated with $KOH(aq)$ to give respective substitution products, without any rearrangement. The number of products which can exhibit optical isomerism from these is ___ .

Answer (3)

Sol. 2-Bromopentane
1-Bromo-2-methylbutane
2-Bromo-2-methylbutane

72. For the following gas phase equilibrium reaction at constant temperature,

$$NH_3(g) \rightleftharpoons 1/2\ N_2(g) + 3/2H_2(g)$$

if the total pressure is $\sqrt{3}$ atm and the pressure equilibrium constant (K_P) is 9 atm , then the degree of dissociation is given as $(x \times 10^{-2})^{-1/2}$. The value of x is ___ (nearest integer)

Answer (125)

Sol. $NH_3(g) \rightleftharpoons \frac{1}{2}\ N_2(g) + \frac{3}{2}H_2(g)$

$$1$$

$$\times \frac{1-\alpha}{1+\alpha}\ \frac{\alpha}{2(1+\alpha)}$$

$$P\ \left(\frac{1-\alpha}{1+\alpha}\right)\sqrt{3}\ \frac{\alpha}{2(1+\alpha)}\sqrt{3}$$

$$\text{Total Moles } = 1-\alpha+\frac{\alpha}{2}+\frac{3\alpha}{2}$$

$$1-\alpha+2\alpha=(1+\alpha)$$

$$\frac{}{\left(\frac{\sqrt{3}\alpha}{2(1+\alpha)}\right)^{1/2}\left(\frac{3\sqrt{3}\alpha}{2(1+\alpha)}\right)^{2/2}}$$

$$81 = \frac{\sqrt{3}\left(\frac{1+\alpha}{1+\alpha}\right)^{2(1+\alpha)}\Big\}^{1}\left\{\frac{3\sqrt{3}\alpha}{2(1+\alpha)}\right\}^{3}}{3\left(\frac{1-\alpha}{1+\alpha}\right)^{2}}$$

$$81 = \frac{27\times 9\times \alpha^4\times(1+\alpha)^2}{(2)(1+\alpha)(8)(1+\alpha)^3\times 3(1-\alpha)^2}$$

$$81 = \frac{27\times 3\times \alpha^4}{16(1+\alpha)^2(1-\alpha)^2}$$

$$\frac{3\alpha}{2(1+\alpha)}\sqrt{3}$$

$$1 = \frac{\alpha^4}{(16)(1-\alpha^2)^2}$$

$$1 = \frac{\alpha^2}{4(1-\alpha^2)} \Rightarrow 4 - 4\alpha^2 = \alpha^2$$

$$5\alpha^2 = 4$$

$$\alpha^2 = 0.8$$

$$\alpha^2 = 0.8$$

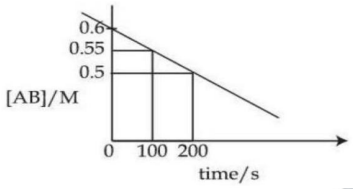
$$\alpha = (x \times 10^{-2})^{-1/2}$$

$$\alpha^2 = (x \times 10^{-2})^{-1}$$

$$\alpha^2 = \frac{1}{x \times 10^{-2}}$$

$$\alpha^2 = \frac{10^2}{x} = 0.8 \qquad x = \frac{100}{0.8} = 125$$

73. For the thermal decomposition of reactant $AB(g)$, the following plot is constructed.



The half life of the reaction is ' x ' min.

X = ___ min. (Nearest integer)

Answer (10)

Sol. $0.5 = 0.6 - kt$

$k(200) = 0.1$

$k = \frac{1}{2000}$

$t_{1/2} = \frac{C_0}{2k} = \frac{0.6}{2 \times 1} \times 2000$

$\Rightarrow 600\text{sec}$

$= 10$ min

74. The crystal field splitting energy of $[Co(\text{ oxalate })]_3]^{3-}$ complex is ' n ' times that of the $[Cr(\text{ oxalate }))_3]^{3-}$ complex. Here ' n ' is ___ . (Assume $\Delta_0 \gg P$)

Answer (2)

Sol. $Co^{3+} = t_{2g}^6 e_g^\circ$

$(\text{ CFSE })_1 = -0.4\Delta_\circ(6)$

$(\text{ CFSE })_2 = -0.4\Delta_o(3)$

$n = \frac{(CFSE)_1}{(CFSE)_2} = 2$

75. x mg of pure HCl was used to make an aqueous solution. 25.0 mL of $0.1 M Ba(OH)_2$ solution is used when the HCl solution was titrated against it. The numerical value of x is ___ $\times 10^{-1}$. (Nearest integer)

Given : Molar mass of HCl and $Ba(OH)_2$ are 36.5 and 171.0 g mol^{-1} respectively.

Answer (1825)

Sol. $\frac{x}{36.5} \times 1 = 25 \times 0.1 \times 2$

$x = 182.5$

$= 1825 \times 10^{-1}$

$= 1825$