(28 JANUARY 2026) **TIME: 9:00 A.M. TO 12:00 NOON**

MATHEMATICS TEST PAPER WITH SOLUTION

1. Let $S=\left\{x^3+ax^2+bx+c : a,b,c \in N \text{ and } a,b,c \le 20\right\}$ be a set of polynomials. Then the number of polynomials in S, which are divisible by x^2+2, is

(1) 6 (2) 20 (3) 10 (4) 120

Ans. (3)

$$x^3+ax^2+bx+c=\left(x^2+2\right)\left(x+\frac{c}{2}\right)$$

$$x^2 : a=\frac{c}{2}$$

$x : b = 2$

$$b=2, a=\frac{c}{2} c \in \{2,4,...,20\}$$

Number of polynomials in 'S' will be 10.

2. The mean and variance of 10 observations are 9 and 34.2, respectively. If 8 of these observations are 2, 3, 5, 10, 11, 13, 15, 21, then the mean deviation about the median of all the 10 observations is

(1) 4 (2) 5 (3) 6 (4) 7

Ans. (2)

$$\frac{2+3+5+10+11+13+15+21+a+b}{10}=9$$

$$\frac{80+a+b}{10}=9 \Rightarrow a+b=10 \quad ...(1)$$

$$\frac{\Sigma x_i^2}{10}-\left(\frac{\Sigma x_i}{10}\right)^2=34.2$$

$$\frac{2^2+3^2+5^2+10^2+11^2+13^2+15^2+21^2+a^2+b^2}{10}-(9)^2=34.2$$

$$1094+a^2+b^2-810=342$$

$$a^2+b^2=58 \quad ...(2)$$

$a = 7, b = 3$

or $a = 3, b = 7$

Number $\rightarrow$ 2, 3, 5, 7, 10, 11, 13, 15, 21

$$\text{Mean} = \frac{7+10}{2}=8.5$$

$$\text{M.D.} = \frac{6.5+5.5+5.5+3.5+1.5+1.5+2.5+4.5+6.5+12.5}{10}=\frac{50}{5}=5$$

3. If $\frac{\tan(A-B)}{\tan A}+\frac{\sin^2 C}{\sin^2 A}=1, A,B,C \in \left(0,\frac{\pi}{2}\right)$, then

(1) tan A, tan B, tan C are in A.P. (2) tan A, tan C, tan B are in G.P.

(3) tan A, tan B, tan C are in G.P. (4) tan A, tan C, tan B are in A.P.

Ans. (3)

$$\frac{\tan A-\tan B}{(1+\tan A \tan B)\tan A}+\frac{1+\cot^2 A}{1+\cot^2 C}=1$$

Put tanA = x, tanB = y, tanC = z

$$\therefore \frac{x-y}{(1+xy)x}+\frac{(x^2+1)z^2}{x^2(z^2+1)}=1$$

$$\therefore x(x-y)(z^2+1)+z^2(1+x^2)(1+xy)$$

$$=(1+xy)x^2(1+z^2)$$

After solving we get

$z^2 = xy \quad \because 1+x^2 \neq 0$

$\therefore \tan^2 C = \tan A.\tan B$

tanA, tanC, tanB are in G.P.

4. A bag contains 10 balls out of which k are red and (10 – k) are black, where $0 \le K \le 10$. If three balls are drawn at random without replacement and all of them are found to be black, then the probability that the bag contains 1 red and 9 black balls is:

(1) $\frac{7}{11}$ (2) $\frac{7}{110}$ (3) $\frac{14}{55}$ (4) $\frac{7}{55}$

Ans. (3)

$$\text{Probability} = \frac{{}^1C_0.{}^9C_3}{\sum_{k=0}^{10} {}^kC_0.{}^{10-k}C_3}$$

$$=\frac{{}^9C_3}{{}^{10}C_3+{}^9C_3+{}^8C_3+\ldots{}^3C_3}$$

$$=\frac{{}^9C_3}{{}^{11}C_4}=\frac{14}{55}$$

5. Let y = y(x) be the solution of the differential equation $x\frac{dy}{dx}-\sin 2y = x^3(2-x^3)\cos^2 y, \quad x \neq 0$.

If y(2) = 0, then tan(y(1)) is equal to

(1) $-\frac{7}{4}$ (2) $-\frac{3}{4}$ (3) $\frac{7}{4}$ (4) $\frac{3}{4}$

Ans. (3)

$$x\frac{dy}{dx}-\sin 2y = x^3(2-x^3)\cos^2 y$$

$$\sec^2 y\frac{dy}{dx}-2\tan y.\frac{1}{x}=x^2(2-x^3)$$

$$\tan y = t \Rightarrow \sec^2 y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}-\frac{2t}{x}=x^2(2-x^3) \qquad \text{(LDE)}$$

$$\text{I.F.} = e^{\int \frac{2}{x}dx} = e^{-2\ell nx} = \frac{1}{x^2}$$

$$\therefore \frac{t}{x^2}=\int \frac{1}{x^2}x^2(2-x^3)dx+C$$

$$\frac{\tan y}{x^2}=2x-\frac{x^4}{4}+C$$

$$y(2)=0 \Rightarrow 0=4-4+C \Rightarrow C=0$$

$$\tan y = 2x^3-\frac{1}{4}x^6$$

$x = 1 \Rightarrow \tan y = 2 - \frac{1}{4} = \frac{7}{4} \Rightarrow (2)$

6. Let y = x be the equation of a chord of the circle C_1 (in the closed half-plane $x \geq 0$) of diameter 10 passing through the origin. Let C_2 be another circle described on the given chord as its diameter. If the equation of the chord of the circle C_2, which passes through the point (2, 3) and is farthest from the center of C_2, is x + ay + b = 0 then a − b is equal to

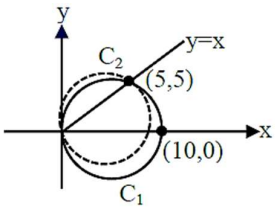
(1) 6 (2) −6 (3) 10 (4) −2

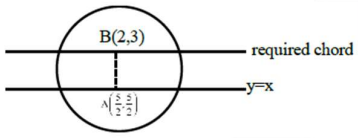
Ans. (4)

Equation of circle C_2 is

$x^2 + y^2 - 5x - 5y = 0$

Its centre is $\left(\frac{5}{2}, \frac{5}{2}\right)$





$m_{AB} = -1$

$\therefore$ Slope of required chord = 1

$\therefore$ equation of required chord is x − y + 1 = 0

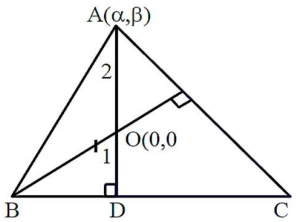
$\therefore$ a = −1, b = 2

$\therefore$ a − b = −2

7. Let ABC be an equilateral triangle with orthocenter at the origin and the side BC on the line $x + 2\sqrt{2}y = 4$. If the co-ordinates of the vertex A are (α, β), then the greatest integer less than or equal to $\left|\alpha + \sqrt{2}\beta\right|$ is

(1) 2 (2) 5 (3) 3 (4) 4

Ans. (4)



$\because m_{BC} . m_{AD} = -1$

$\Rightarrow \left(-\frac{1}{2\sqrt{2}}\right)\left(\frac{\beta}{\alpha}\right) = -1$

$\Rightarrow \beta = 2\sqrt{2}\alpha$ ...(1)

$\because OD = \left|\frac{-4}{\sqrt{1+8}}\right| = \frac{4}{3} \Rightarrow AO = \frac{8}{3}$

So, $AD = \frac{8}{3} + \frac{4}{3} = 4$

$$\Rightarrow \frac{\left|\alpha+2\sqrt{2}\beta-4\right|}{3}=4 \Rightarrow \alpha=\frac{16}{9} \text{ or } -\frac{8}{9} \{\because A(\alpha,\beta) \text{ \& } (0,0) \text{ lies on same side of given line}\}$$

$$\therefore (\alpha,\beta)=\left(\frac{16}{9},\frac{32\sqrt{2}}{9}\right); \text{ (Rejected)}$$

$$\text{So, } (\alpha,\beta)=\left(-\frac{8}{9},\frac{-16\sqrt{2}}{9}\right)$$

$$=\left[\left|\alpha+\sqrt{2}\beta\right|\right]=\left[\left|\frac{-8-32}{9}\right|\right]=4$$

8. The value of $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{k(k+1)}{k!}\right)$ is

(1) $\sqrt{e}$ (2) $\frac{1}{e}$ (3) $\frac{e}{2}$ (4) $\frac{2}{e}$

Ans. (2)

$$T_k=(-1)^{k+1}.\frac{k(k+1)}{\lfloor k}=(-1)^{k+1}\left(\frac{k(k-1)+2k}{\lfloor k}\right)$$

$$\therefore \text{ Sum } =\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\lfloor k-2}+\sum_{k=1}^{\infty}\frac{2(-1)^{k+1}}{\lfloor k-1}$$

$$=\left(\frac{1}{\lfloor -1}+\frac{1}{\lfloor 0}+\frac{1}{\lfloor 1}-\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}\ldots\right)+\left(\frac{2}{\lfloor 0}-\frac{2}{\lfloor 1}+\frac{2}{\lfloor 2}-\frac{2}{\lfloor 3}\ldots\right)=\frac{1}{e}$$

9. If α,β, where $\alpha<\beta$, are the roots of the equation $\lambda x^2-(\lambda+3)x+3=0$ such that $\frac{1}{\alpha}-\frac{1}{\beta}=\frac{1}{3}$, then the sum of all possible values of λ is

(1) 6 (2) 2 (3) 8 (4) 4

Ans. (1)

$$\frac{\beta-\alpha}{\alpha\beta}=\frac{1}{3},\ \alpha+\beta=\frac{\lambda+3}{\lambda},\ \alpha\beta=\frac{3}{\lambda}$$

$$\beta-\alpha=\frac{\alpha\beta}{3}=\frac{1}{\lambda}$$

On squaring

$$\alpha^2+\beta^2-2\alpha\beta=\frac{1}{\lambda^2} \ldots(1)$$

$$\alpha^2+\beta^2+2\alpha\beta=\frac{(\lambda+3)^2}{\lambda^2} \ldots(2)$$

(2) – (1)

$$4\alpha\beta=\frac{(\lambda+3)^2-1}{\lambda^2}$$

$$\frac{12}{\lambda}=\frac{\lambda^2+6\lambda+8}{\lambda^2}$$

$$\Rightarrow \lambda^2-6\lambda^2+8\lambda=0$$

$$\Rightarrow \lambda=0,2,4$$

Sum of possible values of λ is = 6

10. If $g(x)=3x^2+2x-3$, $f(0)=-3$ and $4g(f(x))=3x^2-32x+72$, then $f(g(2))$ is

(1) $\frac{7}{2}$ (2) $-\frac{7}{2}$ (3) $\frac{25}{6}$ (4) $-\frac{25}{6}$

Ans. (1)

$g(2) = 13$

$f(g(2)) = f(13)$

Now $4g(f(x)) = 3x^2 - 32x + 72$

$4[3f^2(x) + 2f(x) - 3] = 3x^2 - 32x + 72$

Let $f(x) = t$

$12t^2 + 8t - (3x^2 - 32x + 84) = 0$

$$f(x) = \frac{-8 \pm \sqrt{64 + 48(3x^2 - 32x + 84)}}{24}$$

$$f(x) = \frac{-8 \pm 4(3x - 16)}{24}$$

$\because f(0) = -3 \ \therefore$ we take +ve sign

$$\therefore f(x) = \frac{-8 + 4(3x - 16)}{24}$$

$$\therefore f(13) = \frac{-8 + 4 \times 23}{24} = \frac{84}{24} = \frac{7}{2}$$

11. Let A, B and C be three 2 × 2 matrices with real entries such that $B = (I + A)^{-1}$ and A + C = I. If $BC = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}$ and $CB = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$, then $x_1 + x_2$ is

(1) 4 (2) 0 (3) −2 (4) 2

Ans. (2)

$B = (I + A)^{-1}, A + C = I$

$\Rightarrow B(I + A) = (I + A)B = I$

$\Rightarrow$ B + BA = B + AB

$\Rightarrow$ B + B(I − C) = B + (I − C)B

$\Rightarrow$ 2B − BC = 2B − CB

$\Rightarrow$ BC = CB

$$\therefore CB\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 12 \\ -6 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 2 & 5 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 32 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \ \therefore x_1 + x_2 = 0$$

12. Let z be a complex number such that |z − 6| = 5 and |z + 2 − 6i| = 5. Then the value of $z^3 + 3z^2 - 15z + 141$ is equal to

(1) 61 (2) 42 (3) 50 (4) 37

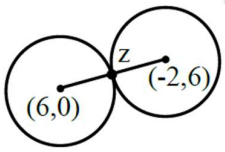
Ans. (3)

Center of first circle $C_1(6,0), r_1 = 5$

Center of second circle $C_2(-2,6), r_2 = 5$

$\because C_1C_2 = r_1 + r_2$

$\therefore$ common point Z is mid point of C_1 & C_2



$\therefore$ $z = 2 + 3i$

$\therefore$ $z^2 = 4z - 13$

$\therefore$ $z^3 = 3z - 52$

$\therefore$ $z^3 + 3z^2 - 15z + 141 = 50$

13. Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9). Let x be the number of 9-digit numbers formed using the digits of the set S such that only one digit is repeated and it is repeated exactly twice. Let y be the number of 9-digit numbers formed using the digits of the set S such that only two digits are repeated and each of these is repeated exactly twice. Then,

(1) 21x = 4y (2) 29x = 5y (3) 45x = 7y (4) 56x = 9y

Ans. (1)

S = {1, 2, 3, ..., 9}

$$x = {}^{9}C_{1}.\,{}^{8}C_{7} \times \frac{9!}{2} = \frac{9 \times 8 \times 9!}{2}$$

$$y = {}^{9}C_{2}.\,{}^{7}C_{5} \times \frac{9!}{2! \times 2!} = \frac{9 \times 8}{2} \times \frac{7 \times 6}{2} \times \frac{9!}{2! \times 2!}$$

$$\Rightarrow \frac{x}{y} = \frac{4}{21}$$

21x = 4y

14. If $\int\left(\frac{1-5\cos^2 x}{\sin^5 x \cos^2 x}\right)dx = f(x) + C$, where C is the constant of integration, then $f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{1}{\sqrt{3}}\left(26+\sqrt{3}\right)$ (2) $\frac{1}{\sqrt{3}}\left(26-\sqrt{3}\right)$ (3) $\frac{2}{\sqrt{3}}\left(4+\sqrt{6}\right)$ (4) $\frac{4}{\sqrt{3}}\left(8-\sqrt{6}\right)$

Ans. (2)

$$\int \frac{dx}{\sin^5 \cos^2 x} - 5\int \frac{dx}{\sin^5 x}$$

$$\int \frac{\sec^2 x dx}{\sin^5 x} - 5\int \frac{dx}{\sin^5 x}$$

By IBP

$$= \frac{\tan x}{\sin^5 x} - \int -\frac{5}{\sin^6 x}.\cos x.\tan x dx - 5\int \frac{dx}{\sin^5 x}$$

$$= \frac{\tan x}{\sin^5 x} + c$$

$$f(x) = \frac{\tan x}{\sin^5 x}$$

$$f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right) = \frac{2^5}{\sqrt{3}} - \left(\sqrt{2}\right)^5 = 4\sqrt{2} - \frac{32}{\sqrt{3}}$$

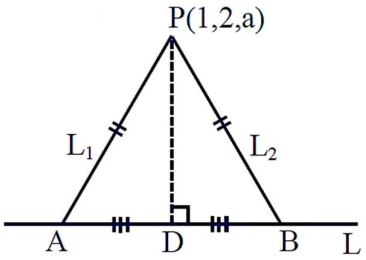
$$= \frac{32}{\sqrt{3}} - 4\sqrt{2} = \frac{4}{\sqrt{3}}\left(8-\sqrt{6}\right)$$

15. If the distances of the point (1, 2, a) from the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$ along the lines

$L_1 : \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b}$ and $L_2 : \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c}$ are equal, then a + b + c is equal to

(1) 7 (2) 4 (3) 5 (4) 6

Ans. (1)



$L : \frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$

$L_1 : \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b} = \lambda$

$L_2 : \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c} = \mu$

Let $A(3\lambda+1, 4\lambda+2, b\lambda+a)$

It lies on L

$\therefore \frac{3\lambda}{1} = \frac{4\lambda+2}{2} = \frac{b\lambda+a-1}{1}$

$\Rightarrow \boxed{\lambda = 1}$ and $a + b - 1 = 3$

$\Rightarrow$ A(4, 6, 4), $a + b = 4$ ...(1)

Let $B(\mu + 1, 4\mu + 2, c\mu + a)$

It also lies on L

$\frac{\mu}{1} = \frac{4\mu+2}{2} = \frac{c\mu+a-1}{1}$

$\Rightarrow 2\mu = 4\mu + 2$

$\Rightarrow \boxed{\mu = -1}$

$a - c - 1 = -1$

$\Rightarrow \boxed{a = c}$ ...(2) & B(0, −2, 0)

Also PA = PB, P(1, 2, a), A(4, 6, 4)

$\Rightarrow 9 + 16 + (a - 4)^2 = 1 + 16 + a^2$

$\Rightarrow 16 + 8 = 8a$

$\boxed{a = 3}$ $\therefore$ c = 3, b = 1

$\therefore \boxed{a + b + c = 7}$

16. For three unit vectors $\vec{a}, \vec{b}, \vec{c}$ satisfying $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$ and $|2\vec{a} + k\vec{b} + k\vec{c}| = 3$, the positive value of k is

(1) 6 (2) 3 (3) 4 (4) 5

Ans. (4)

$|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{3}{2}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{b} + \vec{c} = -\vec{a}$

$|2\vec{a} + k(\vec{b} + \vec{c})| = 3$

$\left|\vec{a}(2-k)\right| = 3$

K = 5 or –1

Positive value of k is 5

17. The value of $\lim\limits_{x\to 0}\dfrac{\log_e\left(\sec(ex)\cdot\sec\left(e^2x\right)\cdot\ldots\cdot\sec\left(e^{10}x\right)\right)}{e^2-e^{2\cos x}}$ is equal to

(1) $\dfrac{\left(e^{10}-1\right)}{2\left(e^2-1\right)}$ (2) $\dfrac{\left(e^{10}-1\right)}{2e^2\left(e^2-1\right)}$ (3) $\dfrac{\left(e^{20}-1\right)}{2e^2\left(e^2-1\right)}$ (4) $\dfrac{\left(e^{20}-1\right)}{2\left(e^2-1\right)}$

Ans. (4)

$\Rightarrow \lim\limits_{x\to 0}\dfrac{\ln(\sec(ex))+\ln(\sec(e^2x))+\ldots.\ell n(\sec(e^{10}x))}{e^{2\cos x}\left(\dfrac{e^{2-2\cos x}-1}{2-2\cos x}\right)\times\dfrac{2-2\cos x}{x^2}\times x^2}$

$\Rightarrow \lim\limits_{x\to 10}\dfrac{\ell n(\sec(ex))+\ell n(\sec(e^2x))+\ldots..\ell n(\sec(e^{10}x))}{e^2\ x^2}$

Using L'H rule

$\Rightarrow \lim\limits_{x\to 10}\dfrac{e\tan ex+e^2\tan e^2x+\ldots..+e^{10}\tan e^{10}x}{2e^2x}$

$\Rightarrow \dfrac{1}{2e^2}\left[e^2+e^4+e^6+\ldots.+e^{20}\right]$

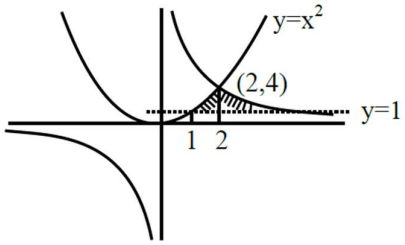
$\Rightarrow \dfrac{1}{2}\dfrac{e^2((e^2)^{10}-1)}{e^2(e^2-1)}$

$\Rightarrow \dfrac{1}{2}\dfrac{(e^{20}-1)}{(e^2-1)}$

18. The area of the region $R=\{(x,y): xy\le 8, 1\le y\le x^2, x\ge 0\}$ is

(1) $\dfrac{1}{3}\left(40\log_e(2)+27\right)$ (2) $\dfrac{2}{3}\left(24\log_e(2)-7\right)$ (3) $\dfrac{2}{3}\left(20\log_e(2)+9\right)$ (4) $\dfrac{1}{3}\left(49\log_e(2)-15\right)$

Ans. (2)



$A=\int_1^2\left(x^2-1\right)dx+\int_2^8\left(\dfrac{8}{x}-1\right)dx$

$A=8\log_e 4-\dfrac{14}{3}=16\log_e 2-\dfrac{14}{3}$

$=\dfrac{2}{3}\left(24\log_e 2-7\right)$

19. Let f be a polynomial function such that $f(x^2+1)=x^4+5x^2+2$, for all $x\in R$. Then $\int_0^3 f(x)dx$ is equal to

(1) $\dfrac{41}{3}$ (2) $\dfrac{33}{2}$ (3) $\dfrac{5}{3}$ (4) $\dfrac{27}{2}$

Ans. (2)

$\because f(x^2+1) = x^4 + 5x^2 + 2$ {put $x^2+1 = t$}

$\Rightarrow f(t) = (t-1)^2 + 5(t-1) + 2$

$\Rightarrow f(t) = t^2 + 3t - 2$

Now, $\int_0^3 f(t)dt = \int_0^3 (t^2 + 3t - 2)dt$

$\left[\frac{t^3}{3} + \frac{3t^2}{2} - 2t\right]_0^3$

$\left[\frac{27}{3} + \frac{27}{2} - 6\right] = \frac{33}{2}$

20. The common difference of the A.P.: $a_1, a_2, ..., a_m$ is 13 more than the common difference of the A.P. : $b_1, b_2, ..., b_n$. If $b_{31} = -277$, $b_{43} = -385$ and $a_{78} = 327$, then a_1 is equal to

(1) 24 (2) 16 (3) 21 (4) 19

Ans. (4)

Let common difference of A.P.'s are d_1 & d_2

$\therefore d_1 = 13 + d_2$

$b_1 + 30d_2 = -277$ ...(1)

$b_1 + 42d_2 = -385$ ...(2)

By (2) – (1)

$12d_2 = -108$

$d_2 = -9$

$\therefore \boxed{d_1 = 4}$

Now $a_{78} = 327$

$\Rightarrow a_1 + 77d_1 = 327$

$\Rightarrow a_1 + 308 = 327$

$a_1 = 19$

SECTION-B

21. In a G.P., if the product of the first three terms is 27 and the set of all possible values for the sum of its first three terms is R – (a, b), then $a^2 + b^2$ is equal to _____.

Ans. (90)

Let first three terms of G.P. are $\frac{A}{r}, A, Ar$

$\frac{A}{r}.A.Ar = 27$

A = 3

$3\left(\frac{1}{r} + 1 + r\right) = 3 + 3\left(r + \frac{1}{r}\right)$

We know, $r + \frac{1}{r} \geq 2$ or $r + \frac{1}{r} \leq -2$

$S \in R - (-3, 9)$

$a^2 + b^2 = 9 + 81 = 90$

22. If $k = \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right) + \tan\left(\frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)\right)$, then the number of solutions of the equation $\sin^{-1}(kx - 1) = \sin^{-1} x - \cos^{-1} x$ is _____

Ans. (1)

Let $\theta = \frac{1}{2}\sin^{-1}\frac{2}{3}$, then $\frac{1}{2}\cos^{-1}\frac{1}{3} = \left(\frac{\pi}{4} - \theta\right)$

$k = \tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{2}{\sin 2\theta}$

$k = \frac{2}{\frac{2}{3}} = 3$

$\sin^{-1}(3x - 1) = \sin^{-1} x - \cos^{-1} x$

$\sin^{-1}(3x - 1) = \frac{\pi}{2} - 2\cos^{-1} x$

$3x - 1 = \sin\left(\frac{\pi}{2} - 2\cos^{-1} x\right)$

$3x - 1 = 2x^2 - 1 \Rightarrow x = 0, \frac{3}{2}$ (rejected)

No. of solution = 1

23. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, let the eccentricity and the length of the latus rectum of the hyperbola $x^2 - y^2\sec^2\theta = 8$ be e_1 and l_1, respectively, and let the eccentricity and the length of the latus rectum of the ellipse $x^2\sec^2\theta + y^2 = 6$ be e_2 and l_2, respectively. If $e_1^2 = e_2^2\left(\sec^2\theta + 1\right)$, then $\left(\frac{l_1 l_2}{e_1 e_2}\right)\tan^2\theta$ is equal to _____.

Ans. (8)

$\frac{x^2}{8} - \frac{y^2}{8\cos^2\theta} = 1,\ e_1 = \sqrt{1 + \frac{8\cos^2\theta}{8}}$

$\ell_1 = \frac{2b^2}{a} = \frac{2.(8\cos^2\theta)}{2\sqrt{2}}$

$\frac{x^6}{6} + \frac{y^2}{6\cos^2\theta} = 1;\ e_2 = \sqrt{1 - \frac{6\cos^2\theta}{6}} = \sin\theta$

$\ell_2 = \frac{2b^2}{a} = \frac{2.6\cos^2\theta}{\sqrt{6}}$

$e_1^2 = e_2^2\left(1 + \sec^2\theta\right)$

$1 + \cos^2\theta = \sin^2\theta\left(1 + \frac{1}{\cos^2\theta}\right)$

$1 + \cos^2\theta = \sin^2\theta + \tan^2\theta$

Solving we get $\theta = \frac{\pi}{4}$

$\ell_1 = 2\sqrt{2}$

$e_1 = \sqrt{\frac{3}{2}}$

$\ell_2 = \sqrt{6}$

$e_2 = \frac{1}{\sqrt{2}}$

$\left(\frac{\ell_1\ell_2}{e_1e_2}\right)\tan^2\theta = 8$ (By putting values)

24. The value of $\sum_{r=1}^{20}\left(\left|\sqrt{\pi\left(\int_0^r x|\sin\pi x|dx\right)}\right|\right)$ is _____

Ans. (210)

Let $I_r = \int_0^r x|\sin\pi x|dx$ ...(1)

Apply King Property

$= \int_0^r (r-x)|\sin\pi x|dx$ ...(2)

By (1) + (2)

$2I_r = \int_0^r r|\sin\pi x|dx \Rightarrow I_r = \frac{r}{2}\int_0^r |\sin\pi x|dx$

$I_1 = \frac{1}{2}\int_0^1 |\sin\pi x|dx = \frac{1}{2\pi}\int_0^\pi |\sin t|dt = \frac{1}{2\pi}(2)$

$I_2 = \frac{2}{2}\int_0^2 |\sin\pi x|dx = \frac{2}{2\pi}\int_0^{2\pi} |\sin t|dt = \frac{2}{2\pi}(4)$

$S = \sqrt{\pi.\frac{1}{2\pi}.2} + \sqrt{\pi.\frac{2}{2\pi}.4} + \sqrt{\pi.\frac{3}{2\pi}.6} + ... + \sqrt{\pi.\frac{20}{2\pi}(2.20)}$

$= 1 + 2 + 3 + ... + 20$

$= \frac{20\times 21}{2} = 210$

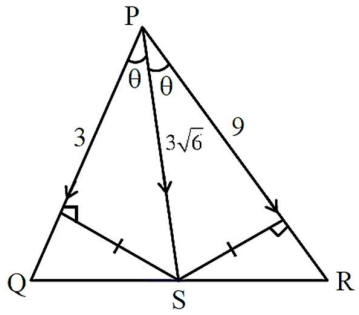
25. Let PQR be a triangle such that $\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{PR} = a\hat{i} + b\hat{j} - 4\hat{k}, a,b \in Z$. Let S be the point on QR, which is equidistant from the lines PQ and PR. If $|\overrightarrow{PR}| = 9$ and $\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$, then the value of 3a – 4b is _____

Ans. (37)

$\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{PR} = a\hat{i} + b\hat{j} - 4\hat{k}$ $(a,b \in Z)$

$\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$



$|\overrightarrow{PR}| = 9$

$a^2 + b^2 + 16 = 81$

$a^2 + b^2 = 65$ ...(1)

$$\cos\theta = \frac{\overrightarrow{PQ}.\overrightarrow{PS}}{\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PS}\right|}$$

$$= \frac{-2+7+4}{3.3\sqrt{6}} = \frac{9}{3.3\sqrt{6}} = \frac{1}{\sqrt{6}}$$

$$\frac{1}{\sqrt{6}} = \frac{\overrightarrow{PS}\,.\,\overrightarrow{PR}}{\left|\overrightarrow{PS}\right|\left|\overrightarrow{PR}\right|} = \frac{a-7b-8}{3\sqrt{6}.9}$$

$a - 7b = 35$ ...(2)

From (1) & (2)

$\Rightarrow$ $a = 7, b = -4$

$\therefore$ $3a - 4b = 21 + 16 = 37$

PHYSICS TEST PAPER WITH SOLUTION

26. The electric field of an electromagnetic wave travelling through a medium is given by $\vec{E}(x,t) = 25\sin(2.0\times10^{15}t - 10^{7}x)\hat{n}$ then the refractive index of the medium is _____.

(All given measurement are in SI units)

(1) 2 (2) 1.7 (3) 1.5 (4) 1.2

Ans. (3)

$\omega = 2\times10^{15}$ rad/s

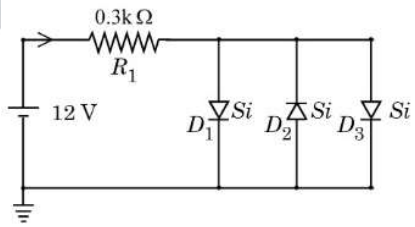
$k = 10^{7}m^{-1}$

$$V = \frac{2\pi}{k}\cdot\frac{\omega}{2\pi} = \frac{\omega}{k} = \frac{2\times10^{15}}{10^{7}} = 2\times10^{8} = \frac{C}{1.5}$$

$\Rightarrow \mu = 1.5$

27. Assuming in forward bias condition there is a voltage drop of 0.7 V across a silicon diode, the current through diode D_1 in the circuit is _____ mA.

(Assume all diodes in the given circuit are identical)



(1) 18.8 (2) 17.6 (3) 20.15 (4) 11.7

Ans. (1)

$12 - 0.3\times10^{3}I - 0.7 = 0$

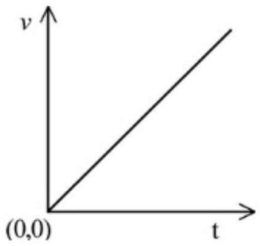
$$\frac{11.3}{0.3\times10^{3}} = I$$

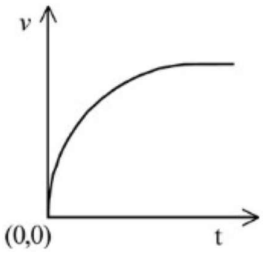
$37.66\times10^{-3}\,A = I$

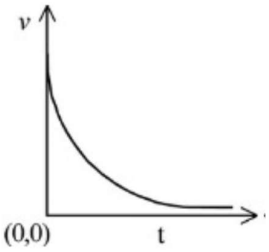
Current through diode $D_1, I_1 = \frac{I}{2}$

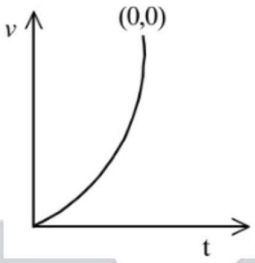
$I_1 = 18.83$ mA

28. A particle of mass m falls from rest through a resistive medium having resistive force, F = –kv, where v is the velocity of the particle and k is a constant. Which of the following graphs represents velocity (v) versus time (t)?

(1) 

(2) 

(3) 

(4) 

Ans. (2)

$$m\cdot\frac{dv}{dt}=mg-kv$$

$$\int_0^v\frac{dv}{mg-kv}=\int_0^t\frac{dt}{m}$$

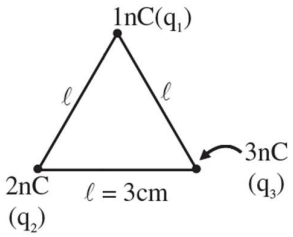
$$\frac{-1}{k}\ell n\left(\frac{mg-kv}{mg}\right)=\frac{t}{m}$$

$$v=\frac{mg}{k}\left(1-e^{-kt/m}\right)$$

29. Two point charges of 1 nC and 2 nC are placed at the two corners of equilateral triangle of side 3 cm. The work done in bringing a charge of 3 nC from infinity to the third corner of the triangle is _____ μJ. $\frac{1}{4\pi\epsilon_0}=9\times10^9\ N.m^2/C^2$

(1) 27 (2) 5.4 (3) 2.7 (4) 3.3

Ans. (3)



$$W=\left(\frac{kq_1}{\ell}+\frac{kq_2}{\ell}\right)q_3$$

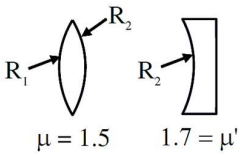
$$=\frac{9\times10^9}{3\times10^{-2}}\left(3\times10^{-9}\right)\times3\times10^{-9}$$

$$=27\times10^{-7}J=2.7\ \mu J$$

30. The magnitudes of power of a biconvex lens (refractive index 1.5) and that of a plano-concave lens (refractive index = 1.7) are same. If the curvature of plano-concave lens exactly matches with the curvature of back surface of the biconvex lens, then ratio of radius of curvature of front and back surface of the biconvex lens is _____.

(1) 12 : 5 (2) 5 : 2 (3) 2 : 5 (4) 5 : 12

Ans. (2)



$|P_A| = |P_B|$

$0.5\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{0.7}{R_2}$

$\frac{5}{R_1}=\frac{2}{R_2}$

$\frac{R_1}{R_2}=\frac{5}{2}$

31. When both jaws of vernier callipers touch each other, zero mark of the vernier scale is right to zero mark of main scale, 4th mark on vernier scale coincides with certain mark on the main scale. While measuring the length of a cylinder, observer observes 15 divisions on main scale and 5th division of vernier scale coincides with a main scale division. Measured length of cylinder is _____mm. (Least count of Vernier calliper = 0.1 mm)

(1) 15.1 (2) 15.4 (3) 15.5 (4) 15.9

Ans. (1)

Reading = MSR + (VSR × LC) – (zero Error)

= 15 mm + (5 × 0.1 mm) – (4 × 0.1 mm)

Reading = 15.1 mm

$\therefore \boxed{\ell = 15.1\,\text{mm}}$

32. The electric current in the circuit is given as $i = i_o(t/T)$. The r.m.s current for the period t = 0 to t = T is _____.

(1) i_o (2) $\frac{i_o}{\sqrt{6}}$ (3) $\frac{i_o}{\sqrt{3}}$ (4) $\frac{i_o}{\sqrt{2}}$

Ans. (3)

$$i_{rms}^2=\frac{\int_0^T\left(i_0^2t^2/T^2\right)dt}{\int_0^T dt}=\frac{i_0^2}{T^3}\cdot\frac{T^3}{3}=\frac{i_0^2}{3}$$

$$i_{rms}=\frac{i_0}{\sqrt{3}}$$

33. Water drops fall from a tap on the floor, 5 m below, at regular intervals of time, the first drop strikes the floor when the sixth drop begins to fall. The height at which the fourth drop will be from ground, at the instant when the first drop strikes the ground is _____m. ($g = 10\ m/s^2$)

(1) 4.0 (2) 4.2 (3) 2.5 (4) 3.8

Ans. (2)

Time to reach ground $=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times5}{10}}=1\,\text{sec}$

Five drops per second

Time between each drop = 0.2 sec.

Time of fall for 4th drop is 1 – 0.6 = 0.4 sec

Height fall of 4th drop is $=\frac{1}{2}\times10\times0.4^2=0.8\ \text{m}$

Height from ground = 5 – 0.8 = 4.2 m

34. The magnetic field at the centre of a current carrying circular loop of radius R is 16 μT. The magnetic field at a distance $x = \sqrt{3}\,R$ on its axis from the centre is _____ μT.

(1) 4 (2) 8 (3) 2 (4) $2\sqrt{2}$

Ans. (3)

$$\frac{\mu_0 I}{2R} = 16\mu T$$

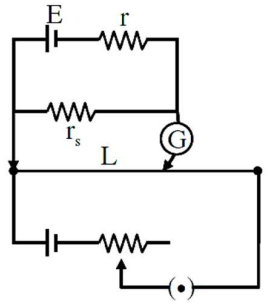
$$\frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 I R^2}{2\times 8R^3} = 2\mu T$$

35. In the potentiometer, when the cell in the secondary circuit is shunted with 4 Ω resistance, the balance is obtained at the length 120 cm of wire. Now when the same cell is shunted with 12 Ω resistance, the balance is shifted to a length of 180 cm. The internal resistance of cell is _____ Ω.

(1) 6 (2) 3 (3) 12 (4) 4

Ans. (4)

Let E is emf and r is internal resistance of cell.



$$\frac{E\cdot 4}{r+4} = 120\,K$$

$$\frac{E\cdot 12}{r+12} = 180\,K$$

$$\Rightarrow \frac{1}{3}\frac{r+12}{r+4} = \frac{2}{3}$$

$r + 12 = 2\,(r + 4)$

$\Rightarrow\ r = 4$

36. 10 kg of ice at –10 °C is added to 100 kg of water to lower its temperature from 25 °C. Consider no heat exchange to surroundings. The decrement to the temperature of water is _____ °C.
(specific heat of ice = 2100 J/Kg.°C, specific heat of water = 4200 J/Kg.°C, latent heat of fusion of ice = 3.36×10^5 J/Kg)

(1) 15 (2) 10 (3) 11.6 (4) 6.67

Ans. (2)

$10 \times 3.36 \times 10^5 + 10 \times 2100 \times 10 + 10 \times 4200 \times (T - 0)$

$= 100 \times 4200 \times (25 - T)$

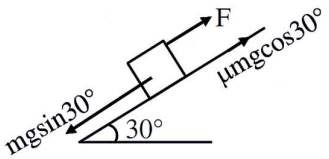
$\Rightarrow T = 15°C$

$\Delta T = 25 - 15 = 10°C$

37. A block of mass 5 kg is moving on an inclined plane which makes an angle of 30° with the horizontal. Friction coefficient between the block and inclined plane surface is $\frac{\sqrt{3}}{2}$. The force to be applied on the block so that the block will move down without acceleration is _____ N.
($g = 10\ m/s^2$)

(1) 7.5 (2) 25 (3) 12.5 (4) 15

Ans. (3)



$mg\sin 30° = F + \mu mg\cos 30°$

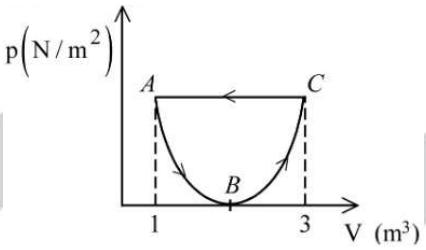
$F = 5\times 10\times\frac{1}{2} - \frac{\sqrt{3}}{2}\times 5\times 10\times\frac{\sqrt{3}}{2}$

$F = 25 - \frac{75}{2} = 25 - 37.5$

F = –12.5 N

∴ Force will be downward on incline of magnitude 12.5 N

38. In the following p-V diagram the equation of state along the curved path is given by $(V - 2)^2 = 4ap$ where a is a constant. The total work done in the closed path is



(1) $-\frac{1}{a}$ (2) $\frac{1}{2a}$ (3) $-\frac{1}{3a}$ (4) $+\frac{1}{3a}$

Ans. (3)

w = Area of parabola

$= \frac{2}{3}$ (Area of rectangle AC31A)

$= \frac{2}{3}P_0(3-1) = \frac{4P_0}{3}$

When V = 1

$(1-2)^2 = 4aP_0$

$P_0 = \frac{1}{4a}$

$w = \frac{4}{3}P_0 = \frac{4}{3}\frac{1}{4a} = \frac{1}{3a}$

$\boxed{w_{gas} = \frac{-1}{3a}}$

39. For the two cells having same EMF E and internal resistance r, the current passing through the external resistor 6 Ω is same when both the cells are connected either in parallel or in series. The value of internal resistance r is _____ Ω.

(1) 4 (2) 9 (3) 3 (4) 6

Ans. (4)

In series, $i_1 = \frac{2E}{6+2r}$

In parallel, $i_2 = \frac{E}{6+\frac{r}{2}}$

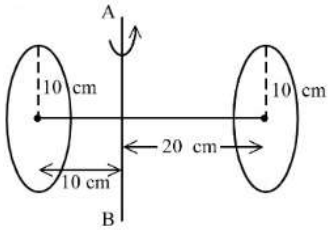
$$i_1 = i_2 \Rightarrow \frac{2E}{6+2r} = \frac{E}{6+\frac{r}{2}}$$

$12 + r = 6 + 2r$

$\boxed{r = 6\Omega}$

40. Two circular discs of radius each 10 cm are joined at their centres by a rod of length 30 cm and mass 600 gm as shown in figure.

If the mass of each disc is 600 gm and applied torque between two discs is 43×10^5 dyne.cm, the angular acceleration of the discs about the given axis AB is _____ rad/s^2.



(1) 27 (2) 11 (3) 22 (4) 100

Ans. (2)

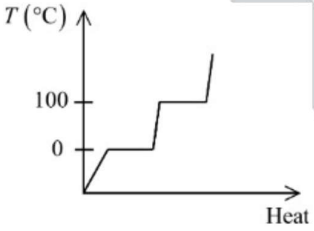
$$\alpha = \frac{\tau}{I}$$

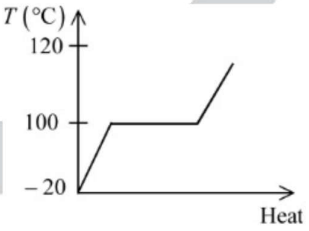
$$I = \frac{1}{4}mR^2 + mR^2 + \frac{1}{4}mR^2 + m(2R)^2 + \frac{m(3R)^2}{12} + m\left(\frac{R}{2}\right)^2$$

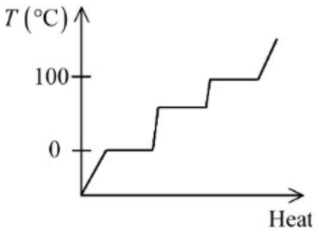
$$= \left(\frac{3}{2} + 4 + 1\right)mR^2 = \frac{13}{2}mR^2 = \frac{13}{2} \times 600 \times 10^2 = 39 \times 10^4$$

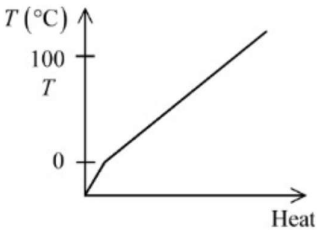
$$\alpha = \frac{43 \times 10^5}{39 \times 10^4}\text{rad/s}^2 = \frac{430}{39}\text{rad/s}^2 \approx 11\,\text{rad/s}^2$$

41. Which of the following best represents the temperature versus heat supplied graph for water, in the range of −20 °C to 120 °C?

(1)


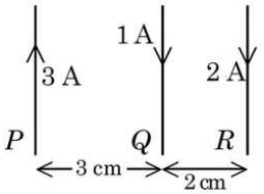
(2)


(3)


(4)


Ans. (1)

42. Three long straight wires carrying current are arranged mutually parallel as shown in the figure. The force experienced by 15 cm length of wire Q is _____.



$(\mu_o = 4\pi \times 10^{-7}\,T.m/A)$

(1) 6×10^{-6} N towards R

(2) 6×10^{-7} N towards P

(3) 6×10^{-6} N towards P

(4) 6×10^{-7} N towards R

Ans. (1)

$$F_{net} = \frac{\mu_0}{2\pi} I_0 \left(\frac{I_1}{d_1} + \frac{I_2}{d_2}\right)\ell$$

$$F_{net} = 2 \times 10^{-7} \times 1\left(\frac{3}{3} + \frac{2}{2}\right) \times \frac{15 \times 10^{-2}}{10^{-2}}$$

$$= 4 \times 15 \times 10^{-7}$$

$$F_{net} = 6 \times 10^{-6} N$$

43. An atom ${}^{8}_{3}X$ is bombarded by shower of fundamental particles and in 10 s this atom absorbed 10 electrons, 10 protons and 9 neutrons. The percentage growth in the surface area of the nucleons is recorded by:

(1) 150% (2) 250% (3) 900% (4) 225%

Ans. (4) Bonus

Surface area $x \propto A^{2/3}$

$X_i = 8^{2/3}K = 4K$

$X_f = (8 + 10 + 9)^{2/3}K = 9K$

% increase in surface area of nucleus

$$X_i = \frac{9K - 4K}{4K} \times 100 = 125\%$$

44. Given below are two statements:

Statement I: A plane wave after passing through prism remains as plane wave but passing through small pin hole may become spherical wave.

Statement II: The curvature of a spherical wave emerging from a slit will increase for increasing slit width.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Ans. (1)

Increasing the slit width 'a' decreases the diffraction angle $(\theta = \lambda / a)$ and reduces the spreading of the wave. A narrower slit produces a more pronounced spherical wave (high curvature) while a wider slit leads to a flatter, less curved wave.

45. Two wires A and B made of different materials of lengths 6.0 cm and 5.4 cm, respectively and area of cross sections 3.0×10^{-5} m^2 and 4.5×10^{-5} m^2, respectively are stretched by the same magnitude under a given load. The ratio of the Young's modulus of A to that of B is x : 3. The value of x is _____.

(1) 5 (2) 2 (3) 1 (4) 4

Ans. (1)

$$T = \frac{F/A}{\Delta\ell/\ell} \Rightarrow Y = \frac{F\ell}{A\Delta\ell}$$

$$\frac{Y_A}{Y_B} = \frac{\ell_A}{\ell_B}\left(\frac{A_B}{A_A}\right)$$

$$= \frac{6}{5.4}\left(\frac{4.5 \times 10^{-5}}{3 \times 10^{-5}}\right) = \frac{9}{5.4} = \frac{5}{3} \Rightarrow \frac{x}{3} = \frac{5}{3}$$

$\boxed{x = 5}$

SECTION-B

46. A solid sphere of radius 10 cm is rotating about an axis which is at a distance 15 cm from its centre. The radius of gyration about this axis is $\sqrt{n}$ cm. The value of n is

Ans. (265)

Let radius of gyration is k

$$\Rightarrow mk^2 = \frac{2}{3}mR^2 + md^2$$

$$k^2 = \frac{2}{3} \times 10^2 + 15^2 = 265$$

$$\left(\sqrt{n}\right)^2 = 265 \Rightarrow n = 265$$

47. A convex lens of refractive index 1.5 and focal length f = 18 cm is immersed in water. The difference in focal lengths of the given lens when it is in water and in air is $\alpha \times f$. The value of α is _____. (refractive index of water = 4/3)

Ans. (3)

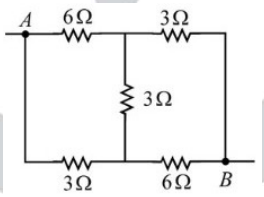
$$\frac{1}{f_{Air}} = \left(\frac{1.5-1}{1}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{f_{water}} = \left(\frac{1.5-4/3}{4/3}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

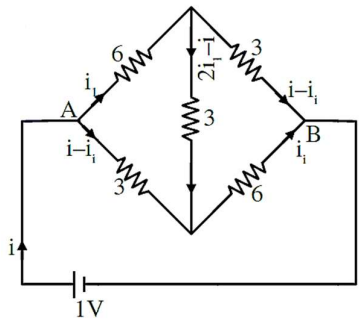
$$\frac{f_{water}}{f_{air}} = \frac{0.5}{0.5/4} = 4$$

$$\Rightarrow f_{water} - f_{air} = 3f$$

48. The equivalent resistance between the points A and B in the following circuit is $\frac{x}{5}\Omega$. The value of x is _____.



Ans. (21)



$$6i_1 + 3(2i_1 - i) = 3(i - i_1)$$

$$\Rightarrow 15i_1 = 6i \Rightarrow i_1 = \frac{2}{5}i \quad ...(1)$$

$$3(i - i_1) + 6i_1 = 1$$

$$3i + 3i_1 = 1$$

$$\left(3+\frac{6}{5}\right)i=1$$

$$\Rightarrow i=\frac{5}{21}A=\frac{1V}{R_{eq}}\Rightarrow R_{eq}=\frac{21}{5}\Omega$$

49. The ratio of de Broglie wavelength of a deutron with kinetic energy E to that of an alpha particle with kinetic energy 2E, is n : 1. The value of n is ____. (Assume mass of proton = mass of neutron):

Ans. (2)

$$\lambda=\frac{h}{mv}=\frac{h}{\sqrt{2m\cdot KE}}$$

$$\frac{\lambda_d}{\lambda_\alpha}=\sqrt{\frac{m_\alpha\cdot KE_\alpha}{m_d\cdot KE_d}}=\sqrt{\frac{4m\cdot 2E}{2m\cdot E}}=2:1$$

50. The displacement of a particle, executing simple harmonic motion with time period T, is expressed as x (t) = A sin ωt, where A is the amplitude. The maximum value of potential energy of this oscillator is found at t = T/2β. The value of β is _____.

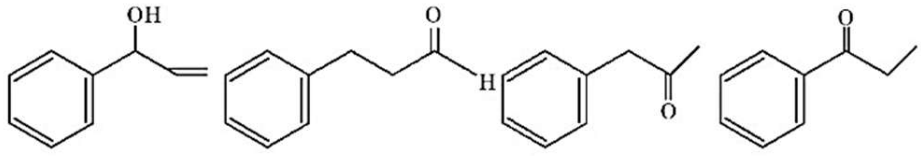
Ans. (2)

Potential energy is maximum at extreme position.

The particle starting at mean position reaches extreme position in time $\frac{T}{4}$.

CHEMISTRY TEST PAPER WITH SOLUTION

51. Given below are the four isomeric compounds (P, Q, R, S)



(P) (Q) (R) (S)

Identify correct statements from below.

A. Q, R and S will give precipitate with 2, 4-DNP.

B. P and Q will give positive Bayer's test.

C. Q and R will give sooty flame.

D. R and S will give yellow precipitate with I_2/NaOH.

E. Q alone will deposit silver with Tollen's reagent

Choose the correct option.

(1) A and E only (2) A, B, D and E only (3) A, C and E only (4) C and E only

Ans. (3)

A. Q, R, S all three give 2, 4 DNP test as they have Aldehyde/ketone group

C. Q & R gives sooty flame

E. Q gives Tollens reagent test

52. Given below are two statements:

Statement I: The number of pairs, from the following, in which both the ions are coloured in aqueous solution is 3.

$[Sc^{3+}, Ti^{3+}]$, $[Mn^{2+}, Cr^{2+}]$, $[Cu^{2+}, Zn^{2+}]$ and $[Ni^{2+}, Ti^{4+}]$

Statement II: Th^{4+} is the strongest reducing agent among Th^{4+}, Ce^{4+}, Gd^{3+} and Eu^{2+}.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Statement-I is true but Statement-II is false.

(2) Both Statement-I and Statement-II are true.

(3) Statement-I is false but Statement-II is true.

(4) Both Statement-I and Statement-II are false.

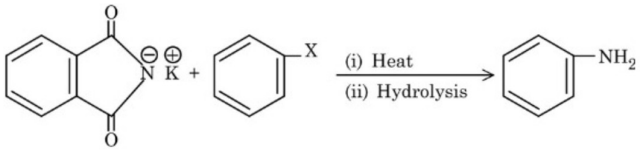
Ans. (4)

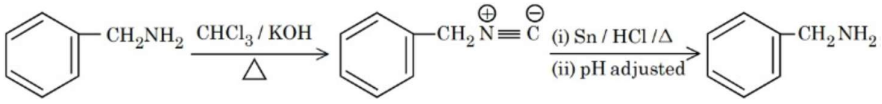
Sc^{3+}, Ti^{4+} and Zn^{2+} are colourless

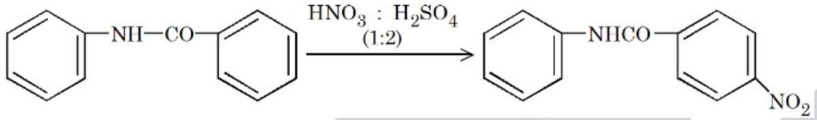
Th^{4+} cannot act as a reducing agent.

53. Consider the following reactions giving major product. Identify the correct reaction.

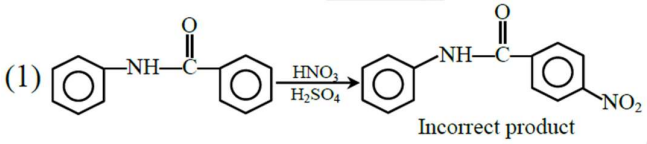
(1) $H_3CCH_2-\overset{\overset{O}{\|}}{C}-NH_2 + Br_2 + 4\,KOH\,(alc) \xrightarrow{\Delta} H_3CCH_2NH_2 + 2KBr + K_2CO_3 + 2H_2O$

(2) 

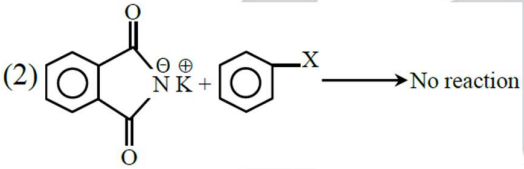
(3) 

(4) 

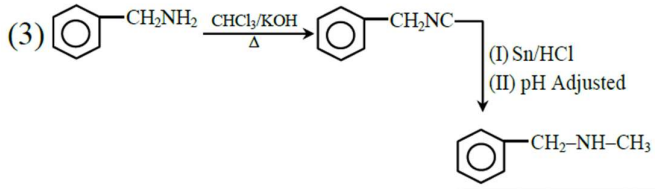
Ans. (1)

(1) 

Incorrect product

(2) 

Aromatic halide does not give Gabriel phthalimide reaction.

(3) 

(4) Hoffmann bromamide degradation

$CH_3-CH_2-\overset{\overset{O}{\|}}{C}-NH_2 \xrightarrow[4KOH]{Br_2} CH_3-CH_2-NH_2 + 2KBr + K_2CO_3 + 2H_2O$

54. The correct statement among the following is:

(1) $[Ni(CN)_4]^{2-}$ and $[NiCl_4]^{2-}$ are diamagnetic and $Ni(CO)_4$ is paramagnetic.

(2) $Ni(CO)_4$ is diamagnetic and $[NiCl_4]^{2-}$ and $[Ni(CN)_4]^{2-}$ are paramagnetic.

(3) $Ni(CO)_4$ and $[NiCl_4]^{2-}$ are diamagnetic and $[Ni(CN)_4]^{2-}$ is paramagnetic.

(4) $Ni(CO)_4$ and $[Ni(CN)_4]^{2-}$ are diamagnetic and $[NiCl_4]^{2-}$ is paramagnetic.

Ans. (4)

$[Ni(CN)_4]^{2-} \rightarrow 3d^8 \rightarrow$ diamagnetic $\rightarrow dsp^2$

$[Ni(CO)_4] \rightarrow 3d^{10} \rightarrow$ diamagnetic $\rightarrow sp^3$

$[NiCl_4]^{2-} \rightarrow 3d^8 \rightarrow e^{2,2}t_2^{2,1,1} \rightarrow sp^3 \rightarrow$ paramagnetic.

55. Consider a weak base 'B' of pK_b = 5.699. 'x' mL of 0.02 M HCl and 'y' mL of 0.02 M weak base 'B' are mixed to make 100 mL of a buffer of pH 9 at 25 °C. The values of 'x' and 'y' respectively are:
[Given: log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.699]

(1)

x	y
14.3	85.7

(2)

x	y
11.1	88.9

(3)

x	y
42.7	57.3

(4)

x	y
85.7	14.3

Ans. (1)

$$\underset{0.02M}{HCl} + \underset{0.02M}{B} \rightarrow BH^+ + Cl^-$$

	x ml	y ml	
t_f	0	$0.02y - 0.02x$	$0.02x$

$$pOH = pK_b + \log\left[\frac{Salt}{Base}\right]$$

$$5 = 5.699 + \log\left[\frac{Salt}{Base}\right]$$

$$\frac{x}{y-x} = \frac{1}{5}$$

$6x = y$

$7x = 100$

$$x = \frac{100}{7}ml \ \& \ y = \frac{600}{7}ml$$

56. An organic compound undergoes first order decomposition. The time taken for decomposition to $\left(\frac{1}{8}\right)^{th}$ and $\left(\frac{1}{10}\right)^{th}$ of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively. What is the value of $\frac{t_{1/8}}{t_{1/10}} \times 10$? (log 2 = 0.3)

(1) 3 (2) 30 (3) 0.9 (4) 9

Ans. (4)

$$t = \frac{1}{k}\ln\frac{A_0}{A_t}$$

$$t_{1/8} = \frac{1}{k}\ln\frac{A_0}{A_0/8} = \frac{1}{k}\ln 8$$

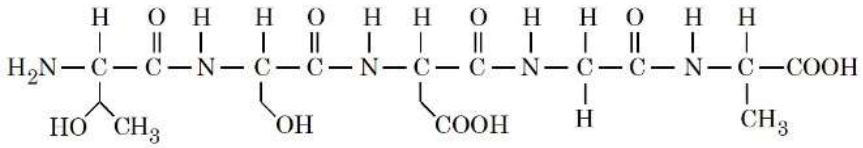
$$t_{1/10} = \frac{1}{k}\ln\frac{A_0}{A_0/10} = \frac{1}{k}\ln 10$$

$$\frac{t_{1/8}}{t_{1/10}} = \frac{\ln 8}{\ln 10} = \frac{\log 8}{\log 10}$$

$$\frac{t_{1/8}}{t_{1/10}} = \log 8 = 3\log 2 = 0.9$$

$$\frac{t_{1/8}}{t_{1/10}} \times 10 = 9$$

57. In the given pentapeptide, find out an essential amino acid (Y) and the sequence present in the pentapeptide:



Choose the correct answer from the options given below:

(1)

(Y)	(Sequence)
Threonine	Thr–Ser–Asp–Gly–Ala

(2)

(Y)	(Sequence)
Threonine	Ser–Thr–Asp–Gly–Ala

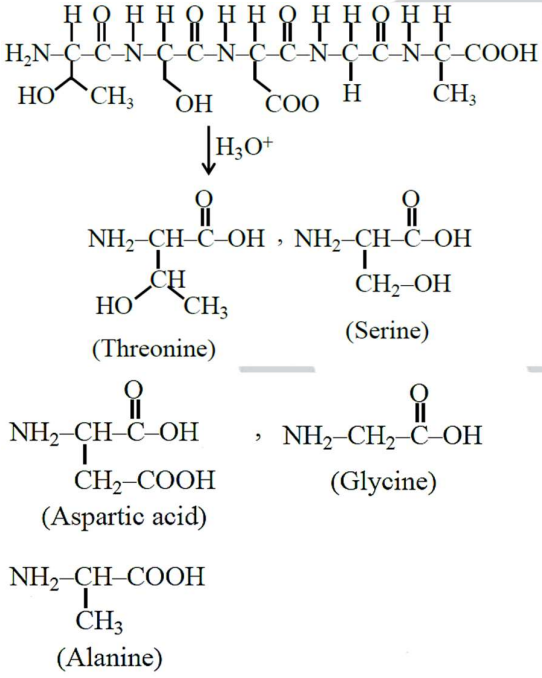
(3)

(Y)	(Sequence)
Serine	Ser–Asp–Thr–Ala–Gly

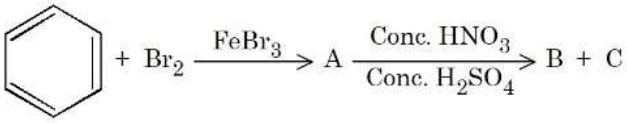
(4)

(Y)	(Sequence)
Serine	Thr–Ser–Asp–Ala–Gly

Ans. (1)

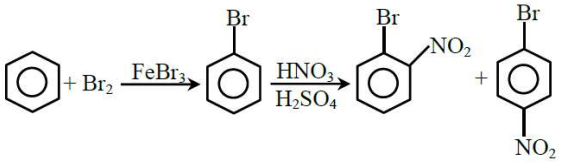


58. Method used for separation of mixture of products (B and C) obtained in the following reaction is



(1) fractional distillation
(2) sublimation
(3) steam distillation
(4) simple distillation

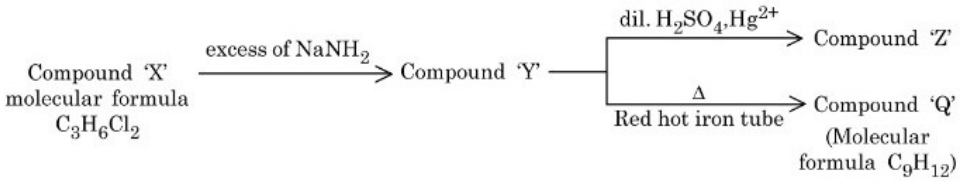
Ans. (1)



B & C separate by
Fractional Distillation method

Due to their different boiling point.

59. Given below are two statements for the following reaction sequence.



Statement I: Compound 'Z' will give yellow precipitate with NaOI.

Statement II: Compound 'Q' has two different types of 'H' atoms (aromatic: aliphatic) in the ratio 1:3.

In the light of the above statements, choose the **correct** answer from the options given below:

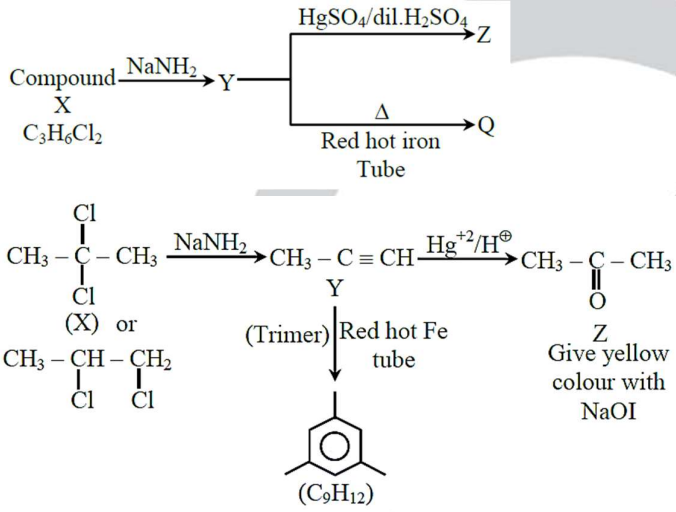
(1) Statement-I is false but Statement-II is true.

(2) Statement-I is true but Statement-II is false.

(3) Both Statement-I and Statement-II are true.

(4) Both Statement-I and Statement-II are false.

Ans. (3)



60. Given below are two statements:

Statement I: Griss-Ilosvay test is used for the detection of nitrite ion, which involves the use of sulphanilic acid and α-naphthylamine reagent.

Statement II: In the above test, sulphanilic acid is diazotized by the acidified nitrite ion, which on further coupling with α-naphthylamine forms an azo-dye.

In the light of the above statements, choose the **correct** answer from the options given below

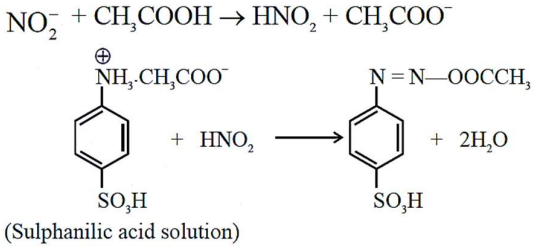
(1) Both Statement-I and Statement-II are true.

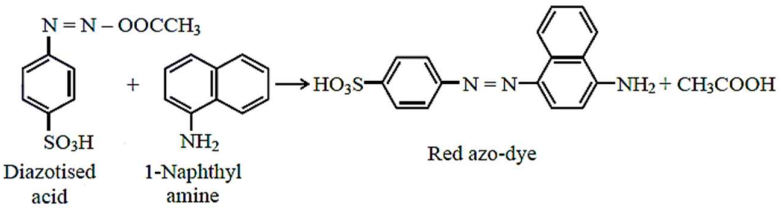
(2) Both Statement-I and Statement-II are false.

(3) Statement-I is false but Statement-II is true.

(4) Statement-I is true but Statement-II is false.

Ans. (1)





61. In period 4 of the periodic table, the elements with highest and lowest atomic radii are respectively.

(1) Rb & Br (2) K & Se (3) Na & Cl (4) K & Br

Ans. (4)

In a period moving from left to right atomic size decreases.

62. Consider the following reaction sequence

$$\text{Compound (x)}\ [76.6\%\ C,\ 6.38\%\ H,\ \text{vapour density } 47] \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CO_2,\ NaOH,\ 120°C,\ \text{high pressure}} \text{Compound (y) (Major Product)}$$

Compound (y) develops characteristic colour with neutral $FeCl_3$ solution.

Identify the **INCORRECT** statement from the following for the above sequence.

(1) Compound x is more acidic than compound y.

(2) Both compounds x and y will burn with sooty flame.

(3) Both compounds x and y will dissolve in NaOH.

(4) Compound y will dissolve in $NaHCO_3$ and evolve a gas.

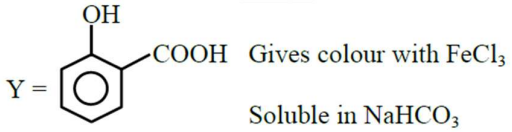
Ans. (1)

$(X) \xrightarrow[(2)\ H_3O^{\oplus}]{(1)\ CO_2/NaOH} Y_{Major}$

76.6%C

6.38% H

Vapour Density 47

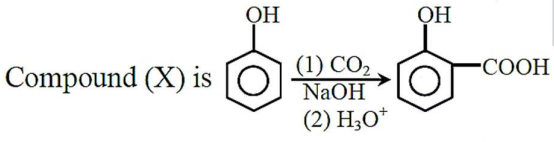


Gives colour with $FeCl_3$

Soluble in $NaHCO_3$

Salicylic acid

Soluble in NaOH



63. The wave numbers of three spectral lines of H atom are considered. Identify the set of spectral lines belonging to Balmer series. (R = Rydberg constant)

(1) $\frac{3R}{4}, \frac{3R}{16}, \frac{7R}{144}$ (2) $\frac{7R}{144}, \frac{3R}{16}, \frac{16R}{255}$ (3) $\frac{5R}{36}, \frac{8R}{9}, \frac{15R}{16}$ (4) $\frac{5R}{36}, \frac{3R}{16}, \frac{21R}{100}$

Ans. (4)

Balmer series line $\Rightarrow \bar{v} = R_H z^2 \left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

If n = 3 $\Rightarrow \bar{v} = R(1)^2 \left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$

If n = 4 $\Rightarrow \bar{v} = \frac{3R}{16}$

If n = 5 $\Rightarrow \bar{v} = \frac{21R}{100}$

64.

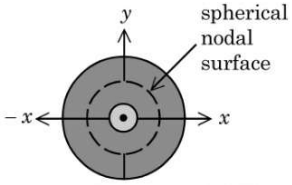


Figure 1. Electron probability density for 2s orbital

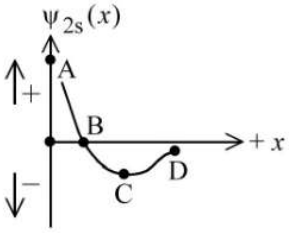


Figure 2. Wave function for 2s orbital

Which of the following point in Figure 2 most accurately represents the nodal surface as shown in Figure 1?

(1) D (2) A (3) B (4) C

Ans. (3)

At spherical node

$\psi_r = 0$

65. Given below are two statements:

Statement I: The number of species among BF_4^-, SiF_4, XeF_4 and SF_4, that have unequal E-F bond lengths is two. Here, E is the central atom.

Statement II: Among O_2^-, O_2^{2-}, F_2 and O_2^+, O_2^- has the highest bond order.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both Statement-I and Statement-II are false.

(2) Both Statement-I and Statement-II are true.

(3) Statement-I is false but Statement-II is true.

(4) Statement-I is true but Statement-II is false.

Ans. (1)

In BF_4^-, SiF_4 and XeF_4 all bond lengths are identical

Molecules		**B.O.**
O_2^+	→	2.5
O_2^-	→	1.5
O_2^{2-}	→	1
F_2	→	1

66. CORRECT order of stability for the following is $CH_2 = CH^-, CH_3 - CH_2^-, CH \equiv C^-$

(1) $CH_2 = CH^- > CH \equiv C^- > CH_3 - CH_2^-$ (2) $CH_3 - CH_2^- > CH_2 = CH^- > CH \equiv C^-$

(3) $CH \equiv C^- > CH_2 = CH^- > CH_3 - CH_2^-$ (4) $CH \equiv C^- > CH_3 - CH_2^- > CH_2 = CH^-$

Ans. (3)

Stability $\propto$ %S

Order of stability

$CH \equiv \bar{C} > CH_2 = \bar{C}H > CH_3 - \bar{C}H_2$

67. 20.0 dm^3 of an ideal gas 'X' at 600 K and 0.5 MPa undergoes isothermal reversible expansion until pressure of the gas is 0.2 MPa. Which of the following option is correct?
(Given: log 2 = 0.3010 and log 5 = 0.6989)
(1) w = +4.1 kJ, $\Delta U = 0$, $\Delta H = 0$; q = −4.1 kJ (2) w = 9.1 J, ΔU = 9.1 J, $\Delta H = 0$; q = 0
(3) w = −3.9 kJ, $\Delta U = 0$, $\Delta H = 0$; q = 3.9 kJ (4) w = −9.1 kJ, $\Delta U = 0$, $\Delta H = 0$, q = 9.1 kJ

Ans. (4)

For isothermal reversible process $\Delta U = \Delta H = 0$

$$w_{iso} = -p_1 v_1 \ln \frac{P_1}{P_2}$$

$$w_{iso} = -0.5 \times 10^6 \times 20 \times 10^{-3} \ln \frac{0.5}{0.2}$$

$$w_{iso} = -0.5 \times 10^6 \times 20 \times 10^{-3} \times 2.303 \times (.6989 - .3010)$$

$$w \simeq -9.1\,kJ$$

$$q = -w = 9.1\,kJ$$

68. At T(K), 2 moles of liquid A and 3 moles of liquid B are mixed. The vapour pressure of ideal solution formed is 320 mm Hg. At this stage, one mole of A and one mole of B are added to the solution. The vapour pressure is now measured as 328.6 mm Hg. The vapour pressure (in mm Hg) of A and B are respectively:
(1) 400, 300 (2) 300, 200 (3) 500, 200 (4) 600, 400

Ans. (3)

2 moles of A + 3 moles of B

$$\Downarrow$$

$$X_A = 2/5,\ X_B = 3/5$$

$$P_S = X_A P_A^\circ + X_B P_B^\circ$$

$$320 = P_A^\circ \left(\frac{2}{5}\right) + P_B^\circ \left(\frac{3}{5}\right)$$

$$2P_A^\circ + 3P_B^\circ = 1600 \quad ...(I)$$

Now 1 mole of A & 1 mole of B is added

$$X_A^{'} = \frac{3}{7}, X_B^{'} = \frac{4}{7}$$

$$P_S^{'} = 328.6 = P_A^\circ \left(\frac{3}{7}\right) + P_B^\circ \left(\frac{4}{7}\right)$$

$$3P_A^\circ + 4P_B^\circ = 2300.2 \quad ...(II)$$

Now eq. (I) × 3 − eq. (II) × 2

$$6P_A^\circ + 9P_B^\circ = 4800$$

$$6P_A^\circ + 8P_B^\circ = 4600.4$$

$$P_B^\circ \simeq 200 \text{ mm of Hg}$$

$$P_A^\circ \simeq 500 \text{ mm of Hg}$$

69. $Ph - CH = CH_2 \xrightarrow[HBr]{(PhCOO)_2}$ Product

Consider the above reaction
A. The reaction proceeds through a more stable radical intermediate.
B. The role of peroxide is to generate $\dot{H}$ (Hydrogen radical).
C. During this reaction, benzene is formed as a byproduct.

D. 1-Bromo-2-phenylethane is formed as the minor product.

E. The same reaction in absence of peroxide proceeds via carbocation intermediate.

Identify the correct statements. Choose the **correct** answer from the options given below:

(1) A, B & D Only (2) A & E Only (3) A, C & E Only (4) C, D & E Only

Ans. (3)

$$Ph-CH=CH_2 \xrightarrow[HBr]{(PhCOO)_2} Ph-CH_2-CH_2-Br$$

Anti Markovnikov addition

- Reaction follow radical addition in presence of peroxide.
- In absence of peroxide follow carbocation mechanism
- Benzene also formed

$$Ph-\underset{\underset{O}{\|}}{C}-O-O-\underset{\underset{O}{\|}}{C}-Ph \rightarrow 2Ph-\underset{\underset{O}{\|}}{C}-\dot{O}$$

$$Ph-\underset{\underset{O}{\|}}{C}-\dot{O} \longrightarrow \dot{Ph} + CO_2$$

$$\dot{Ph} + HBr \longrightarrow Ph-H+\dot{Br}$$

$$Ph-CH=CH_2 + \dot{Br} \longrightarrow Ph-\dot{C}H-CH_2-Br \xrightarrow{HBr} Ph-CH_2-CH_2-Br$$

70. Regarding the hydrides of group 15 elements EH_3 (E = N, P, As, Sb), select the correct statement from the following:

A. The stability of hydrides decreases down the group.

B. The basicity of hydrides decreases down the group.

C. The reducing character increases down the group.

D. The boiling point increases down the group.

Choose the **correct** answer from the options given below:

(1) B & C only (2) A, B & C only (3) A & D only (4) A, B, C & D

Ans. (2)

Stability : $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

Basicity : $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

Reducing character : $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

Boiling point : $PH_3 < AsH_3 < NH_3 < SbH_3 < BiH_3$

SECTION-B

71. X is the number of geometrical isomers exhibited by $[Pt(NH_3)(H_2O)BrCl]$.

Y is the number of optically inactive isomer(s) exhibited by $[CrCl_2(ox)_2]^{3-}$

Z is the number of geometrical isomers exhibited by $[Co(NH_3)_3(NO_2)_3]$.

The value of X + Y + Z is _____.

Ans. (6)

Here

X = 3 (Two cis + one trans isomers)

Y = 1 (trans isomer)

Z = 2 (Fac- mer isomer)

X+Y+Z = 3 + 1 + 2 = 6

72. Consider the dissociation equilibrium of the following weak acid

$$HA \rightleftharpoons H^+(aq) + A^-(aq)$$

If the pKa of the acid is 4, then the pH of 10 mM HA solution is _____. (Nearest integer)

[Given: The degree of dissociation can be neglected with respect to unity]

Ans. (3)

$$pH = \frac{1}{2}\left[pK_a - \log c\right]$$

$$pH = \frac{1}{2}\left[4 - \log 10^{-2}\right]$$

pH = 3

73. 500 mL of 1.2 M KI solution is mixed with 500 mL of 0.2 M $KMnO_4$ solution in basic medium. The liberated iodine was titrated with standard 0.1 M $Na_2S_2O_3$ solution in the presence of starch indicator till the blue color disappeared. The volume (in L) of $Na_2S_2O_3$ consumed is _____. (Nearest integer)

Ans. (3)

$$MnO_4^- + I^- \rightarrow MnO_2 + I_2$$

$$I_2 + S_2O_3^{2-} \rightarrow S_4O_6^{2-} + I^-$$

Gram eq. of $KMnO_4$ = gram eq. of $Na_2S_2O_3$

$$0.2 \times \frac{500}{1000} \times 3 = 0.1 \times V \times 1$$

V = 3 L

74. 0.53 g of an organic compound (x) when heated with excess of nitric acid (concentrated) and then with silver nitrate gave 0.75 g of silver bromide precipitate. 1.0 g of (x) gave 1.32 g of CO_2 gas on combustion. The percentage of hydrogen in the compound (x) is _____ %. [Nearest Integer]
[Given: Molar mass in g mol^{-1} H : 1, C : 12, Br : 80, Ag : 108, O : 16; Compound (x) : $C_xH_yBr_z$]

Ans. (4)

$$\underset{1g}{C_xH_yBr_z} \rightarrow \underset{1.32\,g}{CO_2}$$

$$\%C = \frac{1.32 \times 12}{44 \times 1} \times 100 = 36\%$$

$$\underset{0.53\,g}{C_xH_yBr_z} \rightarrow \underset{0.75g}{AgBr}$$

$$\%Br = \frac{0.75 \times 80}{188 \times 0.53} \times 100 = 60.2\%$$

%H = 100 − (36 + 60.2)

$\%H \simeq 4\%$

75. Consider the following redox reaction taking place in acidic medium

$$BH_4^-(aq) + ClO_3^-(aq) \longrightarrow H_2BO_3^-(aq) + Cl^-(aq)$$

If the Nernst equation for the above balanced reaction is

$E_{cell} = E^\circ_{cell} - \frac{RT}{nF}\ln Q$, then the value of n is _____. (Nearest integer)

Ans. (24)

$$3BH_4^- + 4ClO_3^- \longrightarrow 4Cl^- + 3H_2BO_3^- + 3H_2O$$

n-factor = 8

moles = 3

$\therefore$ n = 3 × 8 = 24