

JEE-MAIN EXAMINATION – JANUARY 2026

(21ST JANUARY 2026)

TIME: 9:00 A.M. TO 12:00 NOON

MATHEMATICS TEST PAPER WITH SOLUTION

1. If the domain of the function $f(x) = \cos^{-1}\left(\frac{2x-5}{11-3x}\right) + \sin^{-1}(2x^2 - 3x + 1)$ is the interval $[\alpha, \beta]$, then

 $\alpha + 2\beta$ is equal to :

- (1) 1 (2) 3
(3) 5 (4) 2

Ans. (2)

Sol. $-1 \leq \frac{2x-5}{11-3x} \leq 1$ and $-1 \leq 2x^2 - 3x + 1 \leq 1$

Upon solving

$$x \in \left(-\infty, \frac{11}{3}\right) \cup [6, 8) \text{ and } x \in \left(-\infty, \frac{16}{5}\right] \cup \left[\frac{11}{3}, 8\right)$$

$$\text{and } 2x^2 - 3x \leq 0, x(2x-3) \leq 0, x \in \left[0, \frac{3}{2}\right]$$

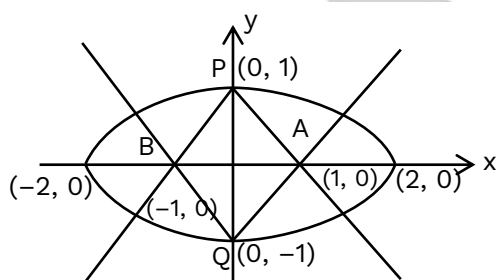
$$\alpha = 0, \quad \beta = \frac{3}{2}$$

2. The area of the region, inside the ellipse $x^2 + 4y^2 = 4$ and outside the region bounded by the curves $y = |x| - 1$ and $y = 1 - |x|$, is :

- (1) $2(\pi - 1)$ (2) $2\pi - \frac{1}{2}$
(3) $3(\pi - 1)$ (4) $2\pi - 1$

Ans. (1)

Sol. $\frac{x^2}{4} + \frac{y^2}{1} = 1$



Area of ellipse - Area of PAQB

$$\pi(2)(1) - 4 \times \frac{1}{2} \times 1 \times 1$$

$$2\pi - 2 = 2(\pi - 1)$$

3. The number of relations, defined on the set $\{a, b, c, d\}$, which are both reflexive and symmetric, is equal to:

- (1) 256 (2) 16
(3) 1024 (4) 64

Ans. (4)

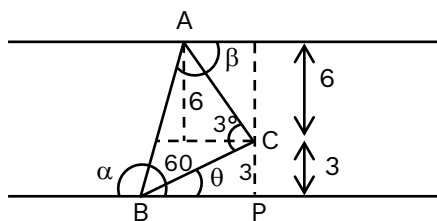
Sol. $2^{\frac{n(n-1)}{2}} = 2^{\frac{4 \times 3}{2}} = 2^6 = 64$

- 4** Let a point A lie between the parallel lines L_1 and L_2 such that its distances from L_1 and L_2 are 6 and 3 units, respectively. Then the area (in sq. units) of the equilateral triangle ABC, where the points B and C lie on the lines L_1 and L_2 , respectively, is :

- (1) $15\sqrt{6}$ (2) 27
(3) $21\sqrt{3}$ (4) $12\sqrt{2}$

Ans. (3)

Sol.



$$\sin \theta = \frac{3}{BC} = \frac{3}{a}, \quad \cos \theta = \sqrt{1 - \frac{9}{a^2}}$$

$$= \frac{\sqrt{a^2 - 9}}{a}$$

$$60^\circ + \theta + \alpha = 180^\circ$$

$$\alpha = 120^\circ - \theta$$

$$60^\circ + \beta = 120^\circ - \theta \Rightarrow \beta = 60^\circ - \theta$$

$$\sin \beta = \frac{6}{a} \Rightarrow \sin(60^\circ - \theta) = \frac{6}{a}$$

$$\Rightarrow \frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta = \frac{6}{a}$$

$$\Rightarrow \sqrt{3} \cdot \frac{\sqrt{a^2 - 9}}{a} - \frac{3}{a} = \frac{12}{a}$$

$$\Rightarrow \sqrt{3} \sqrt{a^2 - 9} = 15$$

$$\Rightarrow 3(a^2 - 9) = 15 \times 15$$

$$a^2 = 75 + 9 = 84$$

$$A = \frac{\sqrt{3}}{4} \times 84 = 21\sqrt{3}$$

- 5.** Let $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b} = 8\hat{i} + 7\hat{j} - 3\hat{k}$ and \vec{c} be a vector such that $\vec{a} \times \vec{c} = \vec{b}$. If $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$, then $|\vec{a} + \vec{c}|^2$ is equal to:

- (1) 33 (2) 30
(3) 35 (4) 27

Ans. (4)

Sol. $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{a} \times \vec{c} = \vec{b}$$

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 2 \\ x & y & z \end{vmatrix}$$

$$= \hat{i}(2z - 2y) - \hat{j}(-z - 2x) + \hat{k}(-y - 2x)$$

$$= 8\hat{i} + 7\hat{j} - 3\hat{k}$$

Now $2z - 2y = 8$ and

$$z + 2x = 7, y + 2x = 3$$

$$z - y = 4;$$

$$z + 2x = 7;$$

$$y + 2x = 3$$

$$\text{and } x + y + z = 4 = z - y$$

after solving these equation we get

$$x = -2y;$$

$$y - 4y = 3;$$

$$y = -1$$

$$x = 2$$

$$z = 3$$

$$\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{a} + \vec{c} = \hat{i} + \hat{j} + 5\hat{k}$$

$$\Rightarrow |\vec{a} + \vec{c}|^2 = 1 + 1 + 25 = 27$$

6. Let a_1, a_2, a_3, \dots be a G.P. of increasing positive terms such that $a_2 \cdot a_3 \cdot a_4 = 64$ and

$$a_1 + a_3 + a_5 = \frac{813}{7}. \text{ Then } a_3 + a_5 + a_7 \text{ is equal to :}$$

(1) 3256

(2) 3252

(3) 3244

(4) 3248

Ans. (2)

Sol. Let initial term 'a'

Common ratio is r

$$\text{Then } a_2 \cdot a_3 \cdot a_4 = 64$$

$$a_3 = 4 = ar^2$$

$$\text{and } a + ar^2 + ar^4 = \frac{813}{7}$$

$$\frac{4}{r^2} + 4 + 4r^2 = \frac{813}{7}$$

$$4\left(r^2 + \frac{1}{r^2}\right) = \frac{813}{7} - 4 = \frac{813 - 28}{7}$$

$$r^2 + \frac{1}{r^2} = \frac{785}{28} \text{ from here } r^2 = 28$$

Now

$$a_3 + a_5 + a_7$$

$$ar^2 + ar^4 + ar^6$$

$$= 4 + 4r^2 + 4r^4$$

$$= 4 + 4[r^2 + r^4]$$

$$= 4 + 4[28 + 28^2] = 4 + 3248 = 3252$$

7. Let \vec{c} and \vec{d} be vectors such that $|\vec{c} + \vec{d}| = \sqrt{29}$ and

$\vec{c} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{d}$. If λ_1, λ_2 ($\lambda_1 > \lambda_2$) are the possible values of $(\vec{c} + \vec{d}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$, then the equation

$$K^2 x^2 + (K^2 - 5K + \lambda_1) xy + \left(3K + \frac{\lambda_2}{2}\right)$$

$y^2 - 8x + 12y + \lambda_2 = 0$ represents a circle, for K equal to :

- (1) 4 (2) 1
(3) -1 (4) 2

Ans. (2)

Sol. $|\vec{c} + \vec{d}| = \sqrt{29}$

$$\vec{c} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{d}$$

$$\Rightarrow (\vec{c} + \vec{d}) \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = 0$$

$$\Rightarrow \vec{c} + \vec{d} = \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\therefore |\vec{c} + \vec{d}| = \sqrt{29} = \pm \lambda \sqrt{29} \Rightarrow \lambda = \pm 1.$$

$$\therefore \vec{c} + \vec{d} = 2\hat{i} + 3\hat{j} + 4\hat{k} \text{ or } \vec{c} + \vec{d} = -2\hat{i} - 3\hat{j} - 4\hat{k}$$

$$\lambda_1 = (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= -14 + 6 + 12 = 4$$

$$\lambda_2 = -(2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= -4$$

$$K^2 x^2 + (K^2 - 5K + 4)xy + (3K - 2)y^2$$

$$-8x + 12y - 4 = 0$$

$$\text{For circle: } K^2 - 5K + 4 = 0 \Rightarrow K = 1 \text{ or } 4$$

$$\text{Also } K^2 = 3K - 2$$

$$\Rightarrow K^2 - 3K + 2 = 0 \Rightarrow K = 1 \text{ or } 2$$

$$\therefore K = 1$$

Option (2)

8. Let $y = y(x)$ be the solution curve of the differential equation

$$(1 + x^2)dy + (y - \tan^{-1} x)dx = 0, y(0) = 1.$$

Then the value of $y(1)$ is :

(1) $\frac{2}{e^{\pi/4}} + \frac{\pi}{4} - 1$ (2) $\frac{2}{e^{\pi/4}} - \frac{\pi}{4} - 1$

(3) $\frac{4}{e^{\pi/4}} + \frac{\pi}{2} - 1$ (4) $\frac{4}{e^{\pi/4}} - \frac{\pi}{2} - 1$

Ans. (1)

Sol. $(1 + x^2)dy + (y - \tan^{-1} x)dx = 0$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{\tan^{-1} x}{1+x^2}$$

$$\text{IF} = e^{\int \frac{dx}{1+x^2}} = e^{\tan^{-1} x}$$

$$y \cdot e^{\tan^{-1} x} = \int \frac{\tan^{-1} x}{1+x^2} e^{\tan^{-1} x} dx$$

$$I_1 = \int \frac{\tan^{-1} x}{1+x^2} e^{\tan^{-1} x} dx, \tan^{-1} x = t$$

$$\Rightarrow \frac{dx}{1+x^2} = dt$$

$$= \int te^t = te^t - e^t + c$$

$$y.e^{\tan^{-1} x} = \tan^{-1} x e^{\tan^{-1} x} - e^{\tan^{-1} x} + C$$

$$y = \tan^{-1} x - 1 + 2e^{\tan^{-1} x}$$

$$x = 1, y = \frac{\pi}{4} - 1 + \frac{2}{e^{\pi/4}} \text{ option(1)}$$

9. The number of strictly increasing functions f from the set $\{1, 2, 3, 4, 5, 6\}$ to the set $\{1, 2, 3, \dots, 9\}$ such that $f(i) \neq i$ for $1 \leq i \leq 6$, is equal to :

- (1) 21 (2) 27
(3) 22 (4) 28

Ans. (4)

Sol. ${}^8C_6 = 28$

10. Let $f: \mathbf{R} \rightarrow (0, \infty)$ be a twice differentiable function such that $f(3) = 18, f'(3) = 0$ and

$f''(3) = 4$. Then $\lim_{x \rightarrow 1} \log_e \left(\frac{f(2+x)}{f(3)} \right)^{\frac{18}{(x-1)^2}}$ is equal to :

- (1) 1 (2) 9
(3) 2 (4) 18

Ans. (3)

Sol. $\ln \left(\lim_{x \rightarrow 1} \left(\frac{f(x+2)}{f(3)} \right)^{\frac{18}{(x-1)^2}} \right) \rightarrow (1^\infty) \text{ form}$

$$\ln \left(e^{\lim_{x \rightarrow 1} \frac{18}{(x-1)^2} \left(\frac{f(x+2)}{f(3)} - 1 \right)} \right)$$

$$\ln \left(e^{\lim_{x \rightarrow 1} \frac{f'(x+2)}{2(x-1)}} \right)$$

$$\ln \left(e^{\lim_{x \rightarrow 1} \frac{f''(x+2)}{2}} \right)$$

$$\lim_{x \rightarrow 1} \frac{f''(3)}{2} = \frac{4}{2} = 2$$

11. Let the foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If the eccentricity of the hyperbola is 5, then the length of its latus rectum is :

- (1) 12 (2) 16
(3) $\frac{96}{\sqrt{5}}$ (4) $24\sqrt{5}$

Ans. (3)

Sol. $a^2e^2 = a^2 - b^2 = 36 - 16 = 20$

$$ae = \sqrt{20} = 2\sqrt{5}$$

$$S: (2\sqrt{5}, 0)$$

For hyperbola focus of hyperbola is $(2\sqrt{5}, 0)$

$$\therefore ae = 2\sqrt{5}$$

$$a \times 5 = 2\sqrt{5} \Rightarrow a = \frac{2\sqrt{5}}{5} = \frac{2}{\sqrt{5}}$$

$$e^2 = 1 + \frac{b^2}{a^2} \Rightarrow a^2e^2 = a^2 + b^2$$

$$\Rightarrow 20 = \frac{4}{5} + b^2 \Rightarrow b^2 = 20 - \frac{4}{5}$$

$$b^2 = \frac{96}{5}$$

$$\ell(L \cdot R) = \frac{2b^2}{a} = \frac{2 \times 96}{5 \times \frac{2}{\sqrt{5}}} = \frac{96}{\sqrt{5}}$$

12. The value of $\int_{-\pi/6}^{\pi/6} \left(\frac{\pi + 4x^{11}}{1 - \sin(|x| + \pi/6)} \right) dx$ is equal to:

(1) 2π

(2) 4π

(3) 8π

(4) 6π

Ans. (2)

Sol. $\int_{-\pi/6}^{\pi/6} \left(\frac{\pi + 4x^{11}}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} \right) dx$

$$= \int_0^{\pi/6} \frac{2\pi}{1 - \sin(x + \pi/6)} dx$$

$$= 2\pi \int_0^{\pi/6} \frac{1 + \sin(x + \pi/6)}{\cos^2(x + \pi/6)} dx$$

$$= 2\pi \int_0^{\pi/6} (\sec^2(x + \pi/6) + \tan(x + \pi/6)\sec(x + \pi/6)) dx$$

$$= 2\pi \left[\tan(x + \pi/6) + \sec(x + \pi/6) \right]_0^{\pi/6}$$

$$= 2\pi \left[\sqrt{3} + 2 - \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} \right]$$

$$= 2\pi \times 2 = 4\pi \text{ option (2)}$$

13. Let the mean and variance of 7 observations 2, 4, 10, x, 12, 14, y, ($x > y$), be 8 and 16 respectively. Two numbers are chosen from $\{1, 2, 3, x-4, y, 5\}$ one after another without replacement, then the probability, that the smaller number among the two chosen numbers is less than 4, is :

(1) $\frac{3}{5}$

(2) $\frac{4}{5}$

(3) $\frac{2}{5}$

(4) $\frac{1}{3}$

Ans. (2)

Sol. Mean = $8 = \frac{2+4+10+x+12+14+y}{7}$

$$\Rightarrow x + y = 14$$

Variance = $\frac{\sum xi^2}{7} - (\text{Mean})^2$

$$\Rightarrow x^2 + y^2 = 100$$

$$\Rightarrow x = 8, y = 6$$

Nos: {1, 2, 3, 4, 5, 6}

Required probability = $1 - P(\text{both has } > 4)$

$$= 1 - \frac{{}^3C_2}{{}^6C_2} = 1 - \frac{3}{15} = \frac{4}{5}$$

Option (2)

- 14.**
- Let
- (α, β, γ)
- be the co-ordinates of the foot of the perpendicular drawn from the point
- $(5, 4, 2)$
- on the line
- $\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$
- .

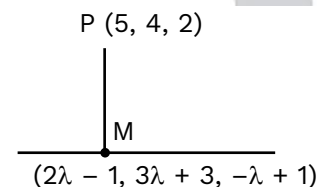
Then the length of the projection of the vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ on the vector $6\hat{i} + 2\hat{j} + 3\hat{k}$ is :

(1) $\frac{15}{7}$

(2) 4

(3) $\frac{18}{7}$

(4) 3

Ans. (3)**Sol.**

$$\vec{PM} = (2\lambda - 6)\hat{i} + (3\lambda - 1)\hat{j} + (-\lambda - 1)\hat{k}$$

$$\vec{PM} \perp (2\hat{i} + 3\hat{j} - \hat{k})$$

$$\Rightarrow \vec{PM} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 0$$

$$\Rightarrow 2(2\lambda - 6) + 3(3\lambda - 1) - 1(-\lambda - 1) = 0$$

$$\Rightarrow 14\lambda - 14 = 0 \Rightarrow \lambda = 1$$

$$\therefore \alpha = 1 \quad \beta = 6 \quad \gamma = 0$$

Projection of $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ on $6\hat{i} + 2\hat{j} + 3\hat{k}$

$$= \frac{6 + 12}{\sqrt{36 + 4 + 9}} = \frac{18}{7} \rightarrow (3)$$

- 15.**
- Let PQ and MN be two straight lines touching the circle
- $x^2 + y^2 - 4x - 6y - 3 = 0$
- at the points A and B respectively. Let O be the centre of the circle and
- $\angle AOB = \pi / 3$
- . Then the locus of the point of intersection of the lines PQ and MN is :

(1) $3(x^2 + y^2) - 18x - 12y + 25 = 0$

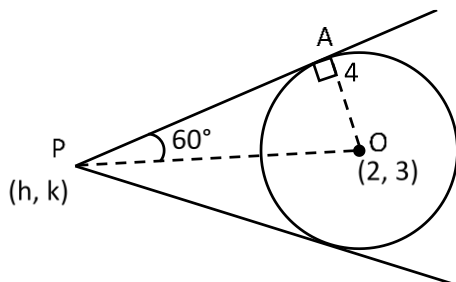
(2) $x^2 + y^2 - 12x - 18y - 25 = 0$

$$(3) x^2 + y^2 - 18x - 12y - 25 = 0$$

$$(4) 3(x^2 + y^2) - 12x - 18y - 25 = 0$$

Ans. (4)

Sol.



$$AP = \sqrt{(h-2)^2 + (k-3)^2 - 16}, \quad OP = \sqrt{(h-2)^2 + (k-3)^2}$$

$$\sin 60^\circ = \frac{4}{OP}$$

$$\Rightarrow 3\{(h-2)^2 + (k-3)^2\} = 64$$

$$\Rightarrow 3x^2 + 3y^2 - 12x - 18y - 25 = 0$$

16. If the coefficient of x in the expansion of $(ax^2 + bx + c)(1-2x)^{26}$ is -56 and the coefficients of x^2 and x^3 are both zero, then $a+b+c$ is equal to:

- (1) 1300 (3) 1500
(3) 1403 (4) 1483

Ans. (3)

Sol. $ax^2(1-2x)^{26} + bx(1-2x)^{26} + c(1-2x)^{26}$

$$b + c({}^{26}C_1(-2)) \Rightarrow b - 52c = -56 \dots (i)$$

$$a - b({}^{26}C_1(2)) + c({}^{26}C_2(2)^2) = 0$$

$$a - 52b + 26 \times 50c = 0 \dots (ii)$$

$$a(-52) + b \times 26 \times 50 - c({}^{26}C_3 \times (2)^3) = 0$$

$$-52a + b \times 26 \times 50 - c \times 26 \times 50 \times 16 = 0$$

$$a - 25b + 25 \times 16c = 0 \dots (iii)$$

Upon solving it

$$a = 1300, b = 100 \text{ and } c = 3$$

17. If $x^2 + x + 1 = 0$, then the value of $\left(x + \frac{1}{x}\right)^4 + \left(x^2 + \frac{1}{x^2}\right)^4 + \left(x^3 + \frac{1}{x^3}\right)^4 + \dots + \left(x^{25} + \frac{1}{x^{25}}\right)^4$ is:

- (1) 128 (2) 162
(3) 175 (4) 145

Ans. (4)

Sol. $x^2 + x + 1 = 0$ has roots ω and ω^2

Let $x = \omega$

$$\left(\omega + \frac{1}{\omega}\right)^4 + \left(\frac{1}{\omega^2} + \frac{\omega^2}{1}\right)^4 + \left(\omega^3 + \frac{1}{\omega^3}\right)^4 + \left(\omega^4 + \frac{1}{\omega^4}\right)^4$$

$$+ \left(\omega^5 + \frac{1}{\omega^5}\right)^4 + \left(\omega^6 + \frac{1}{\omega^6}\right)^4 + \dots + \left(\omega^{25} + \frac{1}{\omega^{25}}\right)^4$$

$$= 8[(-1)^4 + (-1)^4 + (2)^4] + (-1)^4$$

$$= 8 \times 18 + 1 = 144 + 1 = 145$$

18. The value of $\operatorname{cosec} 10^\circ - \sqrt{3} \sec 10^\circ$ is equal to :

- (1) 4 (2) 2
(3) 8 (4) 6

Ans. (1)

Sol.
$$= -\frac{\sqrt{3}}{\cos 10^\circ} + \frac{1}{\sin 10^\circ} = \frac{-\sqrt{3} \sin 10^\circ + \cos 10^\circ}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{2 \left(-\frac{\sqrt{3}}{2} \sin 10^\circ + \frac{1}{2} \cos 10^\circ \right)}{2 \sin 10^\circ \cos 10^\circ} \times 2$$

$$= \frac{2(-\sin 10^\circ \cos 30^\circ + \cos 10^\circ \sin 30^\circ)}{\sin 20^\circ} \times 2$$

$$= 4 \frac{\sin(20^\circ)}{\sin 20^\circ} = 4$$

19. The sum of all the roots of the equation

$$(x-1)^2 - 5|x-1| + 6 = 0, \text{ is}$$

- (a) 4 (b) 3
(c) 1 (4) 5

Ans. (1)

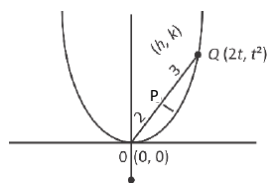
Sol. Let $|x-1| = t$
then $t^2 - 5t + 6 = 0$
 $t = 2; 3$
 $|x-1| = 2; 3$
 $x-1 = 2, -2, 3, -3$
 $x = 3, -1, 4 \text{ and } -2$
required sum = 4

20. Let O be the vertex of the parabola $x^2 = 4y$ and Q be any point on it. Let the locus of the point P, which divides the line segment OQ internally in the ratio 2 : 3 be the conic C. Then the equation of the chord of C, which is bisected at the point (1,2), is :

- (1) $5x - y - 3 = 0$ (2) $4x - 5y + 6 = 0$
(3) $x - 2y + 3 = 0$ (4) $5x - 4y + 3 = 0$

Ans (4)

Sol.



$$h = \frac{4t}{5}, k = \frac{2t^2}{5}$$

$$\Rightarrow \frac{2t^2}{5} = t, \frac{2t^2}{5} = t^2$$

$$\frac{5k}{2} = \left(\frac{5h}{4}\right)^2$$

Replace (h, k) with (x, y)

$$\frac{5y}{2} = \frac{25x^2}{16} =$$

$$C: 8y = 5x^2$$

chord with given middle point

$$T = S_1$$

$$5xx_1 - 4(y + y_1) = 5x_1^2 - 8y_1$$

$$5x - 4(y + 2) = 5 - 16$$

$$5x - 4y - 8 = -11$$

$$5x - 4y + 3 = 0$$

- 21.** Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function such that the quadratic equation. $f(x)m^2 - 2f'(x)m + f''(x) = 0$ in m , has two equal roots for every $x \in \mathbb{R}$. If $f(0) = 1$, $f'(0) = 2$, and (α, β) is the largest interval in which the function $f(\log_e x - x)$ is increasing, then $\alpha + \beta$ is equal to _____

Ans. (1)

Sol. $D = 0$

$$\Rightarrow 4(f'(x))^2 - 4f(x)f''(x) = 0$$

$$\Rightarrow \frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$$

$$\Rightarrow \ln f'(x) = \ln f(x) + \ln c$$

$$\Rightarrow f'(x) = cf(x)$$

at $x = 0$

$$2 = C$$

$$\Rightarrow f(x) = 2f(x)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = 2$$

$$\Rightarrow \ln f(x) = 2x + d$$

at $x = 0$

$$0 = 0 + d$$

$$\Rightarrow f(x) = e^{2x}$$

$$g(x) = f(\log_e x - x) = e^{2\log_e x - 2x}$$

$$= \frac{x^2}{e^{2x}}$$

$$g'(x) = \frac{e^{2x}(2x) - x^2 \cdot 2e^{2x}}{(e^{2x})^2} = \frac{2x(1-x)}{e^{2x}}$$

$$g'(x) > 0 \Rightarrow x \in (0, 1)$$

$$\therefore \alpha = 0, \beta = 1$$

$$\Rightarrow \alpha + \beta = 1$$

- 22.** Let $a_1 = 1$ and for $n \geq 1$, a_{n+1}

$$= \frac{1}{2}a_n + \frac{n^2 - 2n - 1}{n^2(n+1)^2}. \text{ Then } \left| \sum_{n=1}^{\infty} \left(a_n - \frac{2}{n^2} \right) \right| \text{ is equal to}$$

Ans. (2)

Sol. $a_{n+1} = \frac{1}{2}a_n + \frac{2n^2 - (n+1)^2}{n^2(n+1)^2}$

$$a_{n+1} = \frac{1}{2}a_n + \frac{2}{(n+1)^2} - \frac{1}{n^2}$$

$$2a_{n+1} = \left(a_n - \frac{2}{n^2}\right) + \frac{4}{(n+1)^2}$$

$$2a_{n+1} - \frac{4}{(n+1)^2} = a_n - \frac{2}{n^2}$$

$$2a_2 - \frac{4}{2^2} = a_1 - \frac{2}{1^2}$$

$$2a_3 - \frac{4}{3^2} = a_2 - \frac{2}{2^2}$$

$$2a_4 - \frac{4}{4^2} = a_3 - \frac{2}{3^2}$$

Let

$$a_1 + a_2 + a_3 + \dots = \lambda \quad \frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \dots = \mu$$

$$2(\lambda - a_1) - 2(\mu) + 4 = \lambda - \mu$$

$$2(\lambda - \mu) - 2 + 2 = \lambda - \mu$$

$$\lambda - \mu = -2$$

$$\sum_{n=1}^{\infty} a_n - \frac{2}{n^2} = -2$$

$$\text{Ans.} = 2$$

- 23.** Let $S = \{(m, n) : m, n \in \{1, 2, 3, \dots, 50\}\}$. If the number of elements (m, n) in S such that $6^m + 9^n$ is a multiple of 5 is p and the number of elements (m, n) in S such that $m + n$ is a square of a prime number is q , then $p + q$ is equal to

Ans. (1333)

Sol. $6^m + 9^n$ is a multiple of 5 then

$$m \in \{1, 2, 3, \dots, 50\}$$

$$n \in \{1, 3, 5, \dots, 49\}$$

$$\text{Then } P = 50 \times 25 = 1250$$

$$\text{For } \Rightarrow m + n = 4$$

$$\therefore \text{Number of solution} = 3$$

$$m + n = 9$$

$$\therefore \text{Number of solution} = 8$$

$$m + n = 25$$

$$\therefore \text{Number of solution} = 24$$

$$m + n = 49$$

$$\therefore \text{Number of solution} = 48$$

$$q = 3 + 8 + 24 + 48 = 83$$

$$p + q = 1250 + 83 = 1333$$

- 24.** For some $\alpha, \beta \in \mathbb{R}$, let $A = \begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix}$ and $B = A = \begin{bmatrix} 1 & 1 \\ 1 & \beta \end{bmatrix}$ be such that $A^2 - 4A + 2I = B^2 - 3B + I = O$.

Then $(\det(\text{adj})(A^3 - B^3)))^2$ is equal to

Ans. (225)

Sol. $A = \begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix} \quad A^2 - 4A + 2I = 0$

$$\Rightarrow \text{Tr}(A) = 4 = \alpha + 2 \Rightarrow \alpha = 2$$

$$B = \begin{bmatrix} 1 & 1 \\ 1 & \beta \end{bmatrix} \quad B^2 - 3B + I = 0$$

$$\text{Tr}(B) = 1 + \beta = 3 \Rightarrow \beta = 2$$

$$A^2 = 4A - 2I$$

$$A^3 = 4A^2 - 2A = 4(4A - 2I) - 2A$$

$$= 14A - 8I = \begin{bmatrix} 20 & 28 \\ 14 & 20 \end{bmatrix}$$

$$B^2 = 3B - I$$

$$\Rightarrow B^3 = 3B^2 - B$$

$$= 3(3B - I) - B$$

$$= 8B - 3I = \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$$

$$\therefore A^3 - B^3 = \begin{bmatrix} 15 & 20 \\ 6 & 7 \end{bmatrix}$$

$$|A^3 - B^3| = -15$$

$$\det(\text{adj}(A^3 - B^3))^2$$

$$= (15)^2 = 225$$

25. $6 \int_0^{\pi} |\sin 3x + \sin 2x + \sin x| dx$ is equal to _____

Ans. (17)

Sol. $6 \int_0^{\pi} |\sin 3x + \sin 2x + \sin x| dx$

$$= 6 \left\{ \int_0^{\pi/2} (\sin 3x + \sin 2x + \sin x) dx \right.$$

$$- \int_{\pi/2}^{2\pi/3} (\sin 3x + \sin 2x + \sin x) dx$$

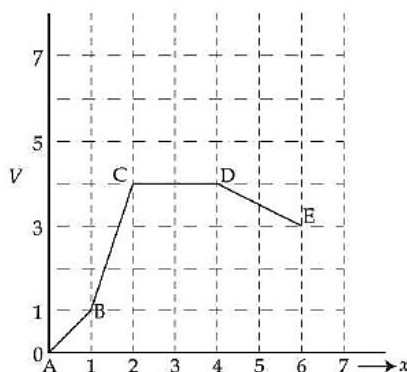
$$\left. + \int_{2\pi/3}^{\pi} (\sin 3x + \sin 2x + \sin x) dx \right\}$$

$$= 6 \left\{ \frac{7}{3} + \frac{1}{12} + \frac{5}{12} \right\} = 17$$

PHYSICS TEST PAPER WITH SOLUTION

26. Potential energy (V) versus distance (x) is given by the graph. Rank various regions as per the magnitudes of the force (F) acting on a particle from high to low.

[Work, Power & Energy]



(1) $F_{BC} > F_{CD} > F_{DE} > F_{AB}$

(2) $F_{CD} > F_{AB} > F_{BC} > F_{DE}$

(3) $F_{CD} > F_{DE} > F_{AB} > F_{BC}$

(4) $F_{BC} > F_{AB} > F_{DE} > F_{CD}$

Ans. (4)

Sol. Slope of potential energy v/s position curve gives negative of force

$$\therefore F_{BC} > F_{AB} > F_{DE} > F_{CD} \quad |F| = \left| \frac{dV}{dx} \right|$$

27. A gas based geyser heats water flowing at the rate of 5.0 litres per minute from 27°C to 87°C. The rate of consumption of the gas is _____ g/s.

(Take heat of combustion of gas = 5.0×10^4 J/g) specific heat capacity of water = 4200 J/kg. °C.

[Heat & Thermodynamics]

(1) 2.1

(2) 4.2

(3) 0.42

(4) 0.21

Ans. (3)

Sol. Water flow rate = $5 \ell / \text{min} = \frac{5}{60} \text{ kg/s}$

\therefore Power of heater

$$= \frac{dm}{dt} S \Delta T = \frac{1}{12} \times 4200 \times 60 \text{ W}$$

\therefore Let rate of consumption of gas be x g/s

$$\therefore x \times 50 \times 10^4 = \frac{1}{12} \times 4200 \times 60$$

$$\Rightarrow x = 4200 \times 10^{-4} = 0.42 \text{ g/s}$$

28. A conducting circular loop of area 1.0 m^2 is placed perpendicular to a magnetic field which varies as $B = \sin(100 t)$ Tesla. If the resistance of the loop is 100Ω , then the average thermal energy dissipated in the loop in one period is _____ J.

[Electromagnetic Induction]

(1) $\frac{\pi}{2}$

(2) 2π

(3) π

(4) π^2

Ans. (3)**Sol.** Area of the loop = 1 m^2

$$B = \sin(100t)$$

$$\therefore \phi = BA = \sin(100t)$$

$$= \frac{d\phi}{dt} = 100 \cos(100t)$$

$$\therefore P = \frac{V^2}{R} = \frac{10^4 \cos^2(100t)}{100}$$

\therefore Thermal energy dissipated in 1 time period

$$= \int_0^T P dt = \int_0^T 100 \cos^2(100t) dt$$

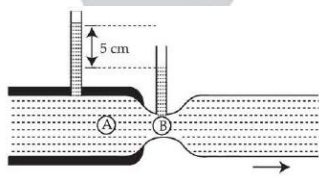
$$T = \frac{2\pi}{100} = \frac{\pi}{50} \text{ sec}$$

$$\therefore Q = 100 \int_0^{\pi/50} \cos^2(100t) dt$$

$$= 100 \int_0^{\pi/50} \frac{1 + \cos 200t}{2} dt$$

$$= 100 \left[\frac{\pi}{100} \right] = \pi$$

- 29.** Water flows through a horizontal tube as shown in the figure. The difference in height between the water column in vertical tubes is 5 cm and the area of cross – sections at A and B are 6 cm^2 and 3 cm^2 respectively. The rate of flow will be _____ cm^3/s . (take $g = 10 \text{ m/s}^2$)

[FLUID MECHANICS]

(1) $\frac{200}{\sqrt{3}}$

(2) $200\sqrt{6}$

(3) $200\sqrt{3}$

(4) $100\sqrt{3}$

Ans. (3)**Sol.** From continuity equation

$$A_A V_A = A_B V_B \Rightarrow 6V_A = 3V_B \Rightarrow V_B = 2V_A$$

Applying Bernoulli's equation between A and B

$$P_A + \frac{1}{2} \rho V_A^2 = P_B + \frac{1}{2} \rho V_B^2$$

$$\Rightarrow \rho g \times 0.05 = \frac{1}{2} \rho [V_B^2 - V_A^2] = \frac{1}{2} \rho (3V_A^2)$$

$$\Rightarrow V_A = \sqrt{\frac{2g \times 0.05}{3}} \text{ m/s}$$

$$= \frac{1}{\sqrt{3}} \text{ m/s}$$

$$= \frac{100}{\sqrt{3}} \text{ cm/s}$$

$$\Rightarrow \text{Volume flow rate} = A_A V_A$$

$$= \frac{6 \times 100}{\sqrt{3}} \text{ cm}^3/\text{sec}$$

$$= 200\sqrt{3} \text{ cm}^3/\text{sec}$$

- 30.** In an experiment the values of two spring constants were measured as $k_1 = (10 \pm 0.2) \text{ N/m}$ and $k_2 = (20 \pm 0.3) \text{ N/m}$. If these springs are connected in parallel, then the percentage error in equivalent spring constant is : **[Error]**

- (1) 2.67 % (2) 2.33 %
(3) 1.33 % (4) 1.67 %

Ans. (4)

Sol. For parallel combination of spring,

$$K_{eq} = K_1 + K_2 = 30 \text{ N/m}$$

$$\Delta K_{eq} = \Delta K_1 + \Delta K_2 = 0.2 + 0.3 = 0.5 \text{ N/m}$$

$$\therefore \% \text{ Error in } K = \frac{0.5}{30} \times 100 = 1.67\%$$

- 31.** A 4 kg mass moves under the influence of a force $\vec{F} = (4t^3\hat{i} - 3t\hat{j})\text{N}$ where t is the time in second. If mass starts from origin at $t = 0$. The velocity and position after $t = 2 \text{ s}$ will be : **[Newton's law of Motion]**

(1) $\vec{v} = 3\hat{i} + \frac{3}{2}\hat{j}$ $\vec{r} = \frac{6}{5}\hat{i} + \hat{j}$

(2) $\vec{v} = 4\hat{i} - \frac{3}{2}\hat{j}$ $\vec{r} = \frac{8}{5}\hat{i} - \hat{j}$

(3) $\vec{v} = 4\hat{i} + \frac{5}{2}\hat{j}$ $\vec{r} = \frac{8}{5}\hat{i} + 2\hat{j}$

(4) $\vec{v} = 4\hat{i} - \frac{3}{2}\hat{j}$ $\vec{r} = \frac{6}{5}\hat{i} - \hat{j}$

Ans. (2)

Sol. $\vec{F} = 4t^3\hat{i} - 3t\hat{j}$

$$\vec{a} = \frac{\vec{F}}{m} = t^3\hat{i} - \frac{3}{4}t\hat{j}$$

$$a_x = t^3$$

$$\frac{dv_x}{dt} = t^3$$

$$\int_{v_x=0}^{v_{x_2}} dv_x = \int_{t=0}^{t=2} t^3 dt$$

$$v_{x_2} - 0 = \left[\frac{t^4}{4} \right]_0^2$$

$$v_{x_2} = 4$$

$$\vec{v}_2 = 4\hat{i} - \frac{3}{2}\hat{j}$$

$$v_x = \frac{t^4}{4}$$

$$a_y = -\frac{3}{4}t$$

$$\frac{dv_y}{dt} = -\frac{3}{4}$$

$$\int_0^{v_{y_2}} dv_y = \int_0^2 -\frac{3}{4}t dt$$

$$v_{y_2} = -\frac{3}{4} \left[\frac{t^2}{2} \right]_0^2$$

$$v_{y_2} = -\frac{3}{2}$$

$$v_y = -\frac{3}{8}t^2$$

$$\int_0^{x_2} dx = \int_0^2 \frac{t^4}{4} dt \quad \int_0^{y_2} dx = \frac{-3}{8} t^2 dt$$

$$x_2 - 0 = \left[\frac{t^5}{20} \right]_0^2 \quad y_2 - 0 = \frac{-3}{8} \left[\frac{t^3}{3} \right]_0^2$$

$$x_2 = \frac{8}{5} \quad y_2 = -1$$

$$\vec{r} = \frac{8}{5} \hat{i} - \hat{j}$$

32. Consider a modified Bernoulli equation.

$$\left(P + \frac{A}{Bt^2} \right) + \rho g(h + Bt) + \frac{1}{2} \rho V^2 = \text{constant}$$

If t has the dimension of time then the dimensions of A and B are _____.
respectively.

[Unit and Dimensions]

- (1) $[ML^0T^{-1}]$ and $[M^0LT]$
 (2) $[ML^0T^{-1}]$ and $[M^0LT^{-1}]$
 (3) $[ML^0T^{-2}]$ and $[M^0LT^{-2}]$
 (4) $[ML^0T^{-2}]$ and $[M^0LT^{-1}]$

Ans. (2)

Sol. $[P] = \left[\frac{A}{Bt^2} \right] \dots(i)$

$\Rightarrow [h] = [Bt] \dots(ii)$

$\Rightarrow [B] = \left[\frac{h}{t} \right] = \left[\frac{L}{T} \right] = [LT^{-1}]$

Putting B in equation (1)

$$ML^{-1}T^{-2} = \left[\frac{A}{LT^{-1} \times T^2} \right]$$

$[A] = [ML^0T^{-1}]$

33. A current carrying solenoid is placed vertically and a particle of mass m with charge Q is released from rest. The particle moves along the axis of solenoid. If g is acceleration due to gravity then the acceleration (a) of the charged particle will satisfy : [Magnetic Effect of Current]

- (1) $a = g$
 (2) $a > g$
 (3) $a = 0$
 (4) $0 < a < g$

Ans. (1)

Sol. Since the solenoid is placed vertically the magnetic field inside the solenoid will be either along $-y$ or $+y$ axis

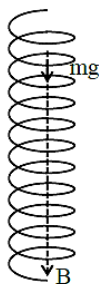
\Rightarrow Particle will gain velocity along $-y$ axis

$\Rightarrow \vec{F}_B = q(\vec{v} \times \vec{B})$

$\Rightarrow \vec{F}_B = 0$

$\Rightarrow \vec{F}_{\text{net}} = m\vec{g}$

$\Rightarrow a_{\text{net}} = g$



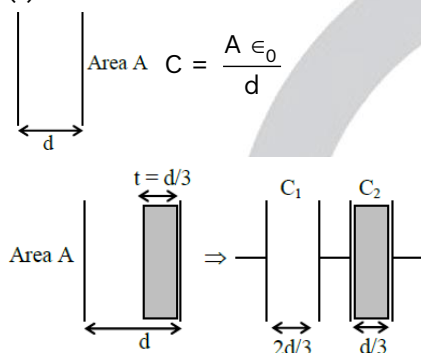
- 34.** A parallel plate capacitance C , when there is vacuum within the parallel plates. A sheet having thickness $\left(\frac{1}{3}\right)^{rd}$ of the separation between the plates and relative permittivity K is introduced between the plates. The new capacitance of the system is :

[Capacitor]

- (1) $\frac{3KC}{2K+1}$ (2) $\frac{CK}{2+K}$
 (3) $\frac{3CK^2}{(2K+1)^2}$ (4) $\frac{4KC}{3K-1}$

Ans. (1)

Sol.



$$C_1 = \frac{3A\epsilon_0}{2d} \quad C_2 = \frac{3A\epsilon_0 \times K}{d}$$

$$C_1 = \frac{3}{2}C \quad C_2 = 3KC$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{3}{2}C \times 3KC}{\frac{3}{2}C + 3KC}$$

$$C_{eq} = \frac{\frac{9}{2}KC^2}{\frac{3}{2}C(2K+1)} = \frac{3KC}{2K+1}$$

- 35.** The electric field in a plane electromagnetic wave is given by :

$$E_y = 69 \sin [0.6 \times 10^3 x - 1.8 \times 10^{11} t] \text{ V/m.}$$

The expression for magnetic field associated with this electromagnetic wave is _____ T.

[EM Waves]

- $B_z = 2.9 \times 10^{-7} \sin[0.6 \times 10^3 x - 1.8 \times 10^{11} t]$
- $B_z = 2.3 \times 10^{-7} \sin[0.6 \times 10^3 x - 1.8 \times 10^{11} t]$
- $B_y = 69 \sin[0.6 \times 10^3 x + 1.8 \times 10^{11} t]$
- $B_y = [2.3 \times 10^{-7} \sin [0.6 \times 10^3 x - 1.8 \times 10^{11} t]$

Ans. (1)

Sol. $\hat{B} = \hat{c} \times \hat{E}$

$\Rightarrow \vec{c} = \hat{i}$ because phase of electric field is function of x .

$$\Rightarrow \hat{E} = \hat{j} \quad \hat{E} = \hat{j} b$$

$$\Rightarrow \hat{B} = \hat{i} \times \hat{j} = \hat{k}$$

$$|B| = \frac{|E|}{c} = \frac{69 \times 0.6 \times 10^3}{1.8 \times 10^{11}} = \frac{69}{3 \times 10^8}$$

$$|B| = 2.9 \times 10^{-7}$$

$$\vec{B}_2 = 2.9 \times 10^{-7} \sin(0.6 \times 10^3 x - 1.8 \times 10^{11} t)$$

(Phase is same as that of electric field)

- 36.** In a double slit experiment the distance between the slits is 0.1 cm and the screen is placed at 50 cm from the slits plane. When one slit is covered with a transparent sheet having thickness t and refractive index $n (= 1.5)$, the central fringe shifts by 0.2 cm. The value of t is _____ cm.

[Wave Optics]

- (1) 8×10^{-4}
 (2) 6.0×10^{-3}
 (3) 5.6×10^{-4}
 (4) 5.0×10^{-3}

Ans. (1)

Sol. $d \sin \theta = (\mu - 1)t$

$$d \left[\frac{x}{D} \right] = (\mu - 1)t$$

$$t = \frac{xd}{D(\mu - 1)}$$

$$= \frac{(0.2)(0.1)}{50(1.5 - 1)}$$

$$t = 8 \times 10^{-4} \text{ cm}$$

- 37.** A _____ light _____ wave _____ described _____ by
 $E = 60[\sin(3 \times 10^{15}t)]$ (in SI units) falls on a metal surface _____ eV.
 ($h = 6.6 \times 10^{-34}$ J.S. and $e = 1.6 \times 10^{-19}$ C)

- (1) 5.1 (2) 3.8
 (3) 6.0 (4) 7.8

Ans. (1)

Sol. $\omega_1 = 3 \times 10^{15} \text{ rad/sec}$

$$\omega_2 = 12 \times 10^{15} \text{ rad/sec}$$

$$\therefore \nu = \frac{\omega}{2\pi}$$

$$E_{\text{photon}} = h\nu = 6.6 \times 10^{-34} \times 1.91 \times 10^{15}$$

$$= 1.26 \times 10^{-18} \text{ J}$$

$$E_{\text{max}} = \frac{1.26 \times 10^{-18}}{1.6 \times 10^{-19}} \approx 7.9 \text{ eV}$$

$$K_{\text{max}} = E_{\text{max}} - \phi_0$$

$$= 7.9 - 2.8$$

$$K_{\text{max}} = 5.1 \text{ eV}$$

[Modern Physics]

38. If an alpha particle with energy 7.7 MeV is bombarded on a thin gold foil, the closest distance from nucleus it can reach is _____ m. (Atomic number of gold = 79 and $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ in SI units)

[Electrostatics]

- (1) 2.95×10^{-14}
 (2) 2.95×10^{-16}
 (3) 3.85×10^{-16}
 (4) 3.85×10^{-14}

Ans. (1)

Sol. Energy conservation

$$K_i + U_i = K_f + U_f$$

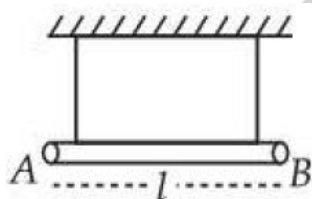
$$7.7 \times 10^6 \times 1.6 \times 10^{-19} + 0$$

$$= 0 + \frac{9 \times 10^9 (1.6 \times 10^{-19})(79 \times 1.6 \times 10^{-19})}{r}$$

$$r = 2.95 \times 10^{-14}$$

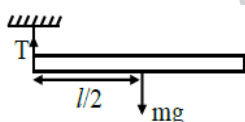
39. A uniform rod of mass m and length l is suspended by means of two identical inextensible light strings as shown in figure. Tension in one string, immediately after the other string is cut, is _____. (g acceleration due to gravity).

[Rotational Motion]



- (1) $mg/2$ (2) $mg/4$
 (3) $mg/3$ (4) mg

Ans. (2)



Sol.

$$mg \frac{l}{2} = \frac{ml^2}{3} \alpha$$

$$\alpha = \frac{3g}{2l} \quad \dots (1)$$

$$mg - T = ma_c$$

$$T = mg - ma_c$$

$$= mg - m \left(\frac{l}{2} \alpha \right)$$

$$= mg - m \left(\frac{l}{2} \cdot \frac{3g}{2l} \right)$$

$$T = \frac{mg}{4}$$

40. An aluminium and steel rods having same length and cross-sections are joined to make total length of 120 cm at 30°C . The coefficient of linear expansion of aluminium and steel are $24 \times 10^{-6}/^\circ\text{C}$ and $1.2 \times 10^{-5}/^\circ\text{C}$, respectively. The length of this composite rod when its temperature is raised to 100°C , is _____ cm.

[Thermal Expansion]

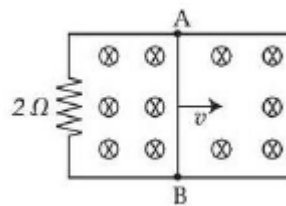
- (1) 120.20 (2) 120.15
(3) 120.03 (4) 120.06

Ans. (2)

Sol. $\ell_{\text{final}} = \ell_0(1 + \alpha_A \Delta T) + \ell_0(1 + \alpha_b \Delta T)$
 $= \ell_0[2 + (\alpha_A + \alpha_b)\Delta T]$
 $= 60[2 + (36 \times 10^{-6}) \times 70]$
 $= 60[2 + 0.0025]$
 $= 120.15 \text{ cm}$

- 41.** A 1 m long metal rod AB completes the circuit as shown in figure. The area of circuit is perpendicular to the magnetic field of 0.10 T. If the resistance of the total circuit is 2Ω then the force needed to move the rod towards right with constant speed (v) of 1.5 m/s is _____ N.

[Electromagnetic Induction]



- (1) 7.5×10^{-2} (2) 5.7×10^{-3}
(3) 7.5×10^{-2} (4) 7.5×10^{-3}
(4)

Ans.

Sol. To maintain constant speed

$$F_{\text{ext}} = F_B$$

$$\Rightarrow F_{\text{ext}} = IlB$$

$$= \left(\frac{vBl}{R} \right) lB$$

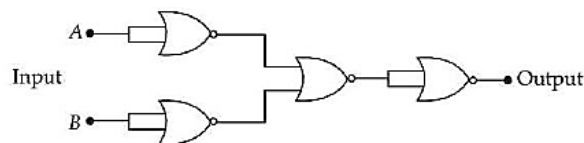
$$= \frac{B^2 l^2 v}{R}$$

$$= \frac{(0.1)^2 \times (1)^2 \times 1.5}{2}$$

$$= 7.5 \times 10^{-3} \text{ N}$$

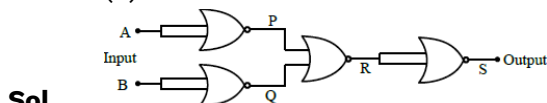
- 42.** The given circuit works as :

[Semiconductor]



- (1) AND gate
(2) NOR gate
(3) NAND gate
(4) OR gate

Ans. (3)



Sol.

$$P = \bar{A}$$

$$Q = \bar{B}$$

$$R = \overline{A + B} = \overline{AB} = AB$$

$$S = \overline{AB} \Rightarrow \text{NAND} = AB$$

- 43.** Two strings (A, B) having linear densities $\mu_A = 2 \times 10^{-4} \text{ kg/m}$ and, $\mu_B = 4 \times 10^{-4} \text{ kg/m}$ and lengths $L_A = 2.5 \text{ m}$ and $L_B = 1.5 \text{ m}$ respectively are joined. Free ends of A and B are tied to two rigid supports C and D, respectively creating a tension of 500 N in the wire. Two identical pulses, set from C and D ends, take time t_1 and t_2 , respectively, to reach the joint. The ratio t_1/t_2 is :

[Wave On Strings]

- (1) 1.08 (2) 1.90
(3) 1.67 (4) 1.18

Ans. (4)

Sol. Given $L_A = 2.5 \text{ m}$

$$L_B = 1.5 \text{ m},$$

$$T = 500 \text{ N}$$

$$v_A = \sqrt{\frac{T}{\mu_A}} = \sqrt{\frac{500}{2 \times 10^{-4}}} = 5\sqrt{10} \times 10^2 \text{ m/s}$$

$$v_B = \sqrt{\frac{T}{\mu_B}} = \sqrt{\frac{500}{4 \times 10^{-4}}} = 5\sqrt{5} \times 10^2 \text{ m/s}$$

$$t_1 = \frac{L_A}{v_A} = \frac{2.5}{5\sqrt{10}} \times 10^{-2} \text{ s}$$

$$t_2 = \frac{L_B}{v_B} = \frac{1.5}{5\sqrt{5}} \times 10^{-2} \text{ s}$$

$$\therefore \frac{t_1}{t_2} = \frac{2.5}{5\sqrt{10}} \times \frac{5\sqrt{5}}{1.5} = \frac{5}{3} \times \frac{1}{\sqrt{2}} \\ = \frac{1.66}{1.41} = 1.18$$

- 44.** Initially a satellite of 100 kg is in a circular orbit of radius $1.5 R_E$. This satellite can be moved to a circular orbit of radius $3R_E$ by supplying $\alpha \times 10^6 \text{ J}$ of energy. The value of α is _____. (Take Radius of Earth $R_E = 6 \times 10^6 \text{ m}$ and $g = 10 \text{ m/s}^2$)

[Gravitation]

- (1) 150 (2) 500
(3) 100 (4) 1000

Ans. (4)

Sol. Energy of a satellite in a circular orbits is given as $E = \frac{-GM_E m}{2r}$; r = radius of circular orbit

Required energy to be supplied

$$= \Delta E = E_f - E_i$$

$$\Delta E = \left(\frac{-GM_E m}{2(3R_E)} \right) - \left(\frac{-GM_E m}{2(1.5R_E)} \right)$$

$$= \frac{GM_E m}{6R_E}$$

$$\text{Now, } g = \frac{GM_E}{R_E^2} \Rightarrow \frac{GM_E}{R_E} = gR_E$$

$$\therefore \Delta E = \frac{1}{6} gmR_E$$

$$= \frac{1}{6} \times 10 \times 100 \times 6 \times 10^6$$

$$= 1000 \times 10^6$$

$$\alpha = 1000$$

45. A point charge of 10^{-8} C is placed at origin. The work done in moving a point charge $2 \mu\text{C}$ from point A(4, 4, 2) m to B(2, 2, 1) m is _____.

$$\left(\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ in SI units} \right) \text{ [Electrostatics]}$$

- (1) 45×10^{-6} (2) 0
(3) 30×10^{-6} (4) 15×10^{-6}

Ans. (3)

Sol. Work done by external agent :

$$W_{\text{ext}} = \Delta U;$$

$\Delta U \rightarrow$ Change in potential energy in taking the charge from initial to final configuration

$$\Rightarrow W_{\text{ext}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_f} - \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_i}$$

$$\text{Now, } r_f = \sqrt{(2-0)^2 + (2-0)^2 + (1-0)^2} = 3\text{m}$$

$$r_i = \sqrt{(4-0)^2 + (4-0)^2 + (2-0)^2} = 6\text{m}$$

$$\therefore W_{\text{ext}} = (9 \times 10^9) \times (10^{-8} \times 2 \times 10^{-6}) \left[\frac{1}{3} - \frac{1}{6} \right]$$

$$= 3 \times 10^{-5}$$

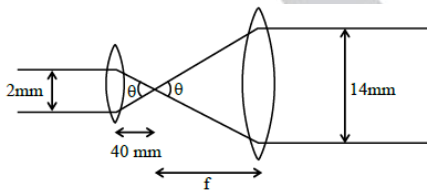
$$= 30 \times 10^{-6} \text{ J}$$

46. A collimated beam of light of diameter 2 mm is propagating 14 mm, using a system of two convex lenses. If first lens has focal length 40 mm, then the focal length of second lens is _____ mm.

[Geometrical optics]

Ans. (280)

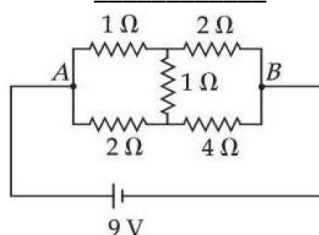
Sol.



$$\frac{40}{20} = \frac{f}{14}$$

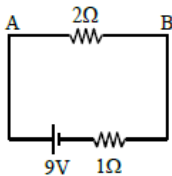
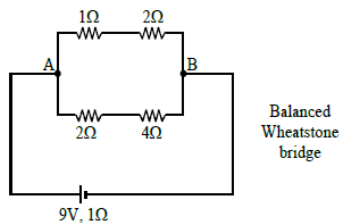
$$\Rightarrow f = 280 \text{ mm}$$

47. The heat generated in 1 minute between points A and B in the given circuit, when a battery of 9 V with _____ internal resistance _____ of 1Ω is connected across these points is _____ J. [Current Electricity]



Ans. 1080

Sol.

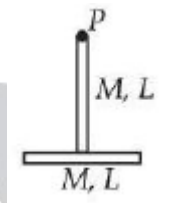


$$i = \frac{9}{3} = 3 \text{ A}$$

$$\therefore H_{AB} = i^2 R_{AB} t$$

$$= (3)^2 \times 2 \times 60 = 1080 \text{ J}$$

48. Two identical thin rods of mass M kg and length L m are connected as shown in figure. Moment of inertia of the combined rod system about an axis passing through point P and perpendicular to the plane of the rods is $\frac{x}{12} ML^2 \text{ kgm}^2$. The value of x is _____. [Rotational Motion]

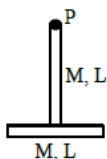


Ans. (17)

$$\begin{aligned} \text{Sol. } I &= \frac{ML^2}{3} + \left(\frac{ML^2}{12} + ML^2 \right) \\ &= \frac{4ML^2 + ML^2 + 12ML^2}{12} \end{aligned}$$

$$I = \frac{17}{12} ML^2$$

$$\therefore x = 17$$



49. 10 mole of oxygen is heated at constant volume from 30°C to 40°C . The change in the internal energy of the gas is _____ cal. (The molecular specific heat of oxygen at constant pressure, $C_p = 7 \text{ cal/mol } ^\circ\text{C}$ and $R = 2 \text{ cal./mol } ^\circ\text{C}$.)

[Heat and Thermodynamics]

Ans. 500

$$\begin{aligned} \text{Sol. } \Delta U &= nC_v\Delta T \\ &= n(C_p - R)\Delta T \\ &= 10(7 - 2)(40 - 30) \\ \Delta U &= 500 \end{aligned}$$

50. In a microscope the objective is having focal length $f_o = 2$ cm and eye – piece is having focal length $f_e = 4$. The tube length is 32 cm. The magnification produced by this microscope for normal adjustment is _____. **[Geometrical optics]**

Ans. 100

Sol.
$$m \approx \frac{LD}{f_o f_e}$$

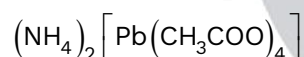
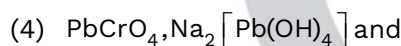
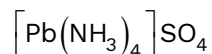
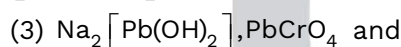
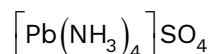
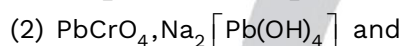
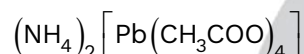
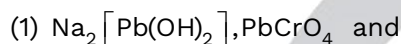
$$= \frac{32}{2} \times \frac{25}{4}$$

$$m = 100$$

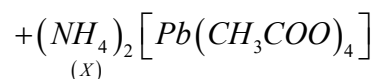
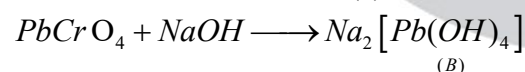
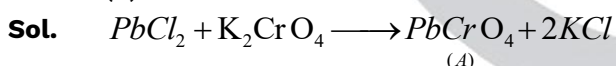
CHEMISTRY TEST PAPER WITH SOLUTION

51. Consider the following reactions.
 $\text{PbCl}_2 + \text{K}_2\text{CrO}_4 \rightarrow \text{A} + 2\text{KCl}$
 (Hot solution)
 $\text{A} + \text{NaOH} \rightleftharpoons \text{B} + \text{Na}_2\text{CrO}_4$
 $\text{PbSO}_4 + 4\text{CH}_3\text{COONH}_4 \rightarrow (\text{NH}_4)_2\text{SO}_4 + \text{X}$
 In the above reactions, A, B and X are respectively.

[Salt Analysis]



Ans. (4)



52. Which of the following represents the correct trend for the mentioned property?

(A) $\text{F} > \text{P} > \text{S} > \text{B}$ - First Ionization Energy

(B) $\text{Cl} > \text{F} > \text{S} > \text{P}$ - Electron Affinity

(C) $\text{K} > \text{Al} > \text{Mg} > \text{B}$ - Metallic character

(D) $\text{K}_2\text{O} > \text{Na}_2\text{O} > \text{MgO} > \text{Al}_2\text{O}_3$

- Basic character

Choose the correct answer from the options given below:

[Periodic Properties]

(1) A, B and D only

(2) A, B, C and D

(3) A and B only

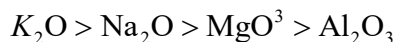
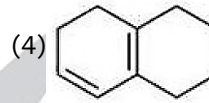
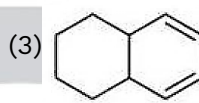
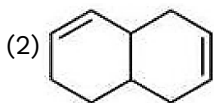
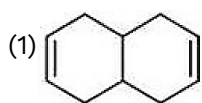
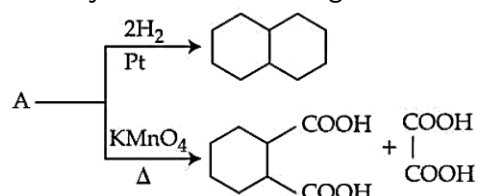
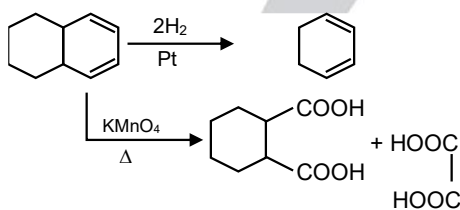
(4) B and C only

Ans.

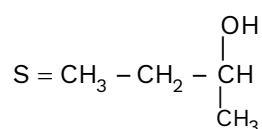
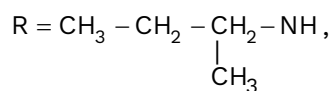
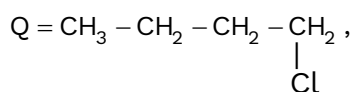
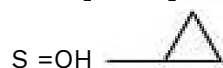
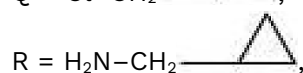
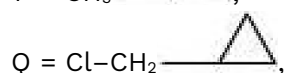
(1)

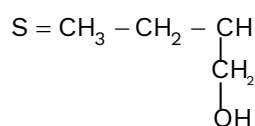
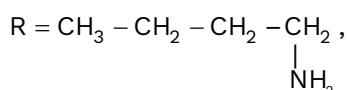
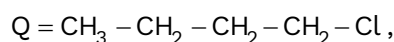
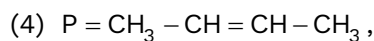
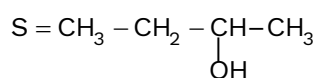
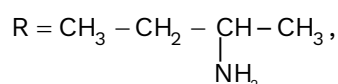
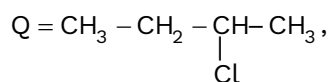
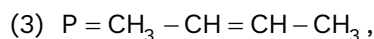
Sol.(A) Order of IE $F > P > S > B$ (B) Order of EA $U > F > S > P$ (C) Order of metallic $K > Mg > Al > B$

(D) Basic nature

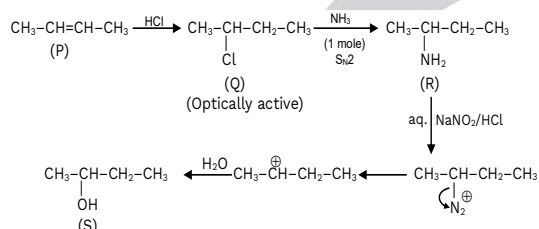
**53.** Identify A in the following reaction.**[Oxidation & Reduction]****Ans.** (3)**Sol.**

54. A hydrocarbon 'P' (C_4H_8) on reaction with HCl gives an optically active compound 'Q' (C_4H_9Cl) which on reaction with one mole of ammonia gives compound 'R' ($C_4H_{11}N$). 'R' on diazotization followed by hydrolysis gives 'S'. Identify P, Q, R and S.

[Hydrocarbon](1) $P = CH_3 - CH_2 - CH = CH_2$,(2) $P = CH_3 -$ 



Ans. (3)
Sol.



55. Given below are two statements:

Statement I: The number of pairs among $[\text{SiO}_2, \text{CO}_2]$, $[\text{SnO}, \text{SnO}_2]$, $[\text{PbO}, \text{PbO}_2]$ and $[\text{GeO}, \text{GeO}_2]$, which contain oxides that are both amphoteric is 2.

Statement II: BF_3 is an electron deficient molecule, can act as a Lewis acid, forms adduct with NH_3 and has a trigonal planar geometry.

In the light of the above statements, choose the correct answer from the options given below:

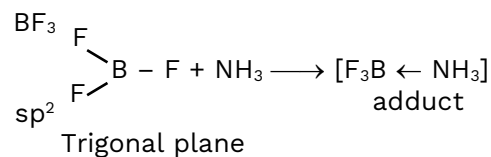
[Chemical Bonding]

- (1) Both Statement I and Statement II are true
- (2) Both Statement I and Statement II are false
- (3) Statement I is true but Statement II is false
- (4) Statement I is false but Statement II is true

Ans. (1)

Sol. $\text{CO}_2, \text{SiO}_2, \text{CrO}_2, \text{GeO} \rightarrow$ acidic oxide

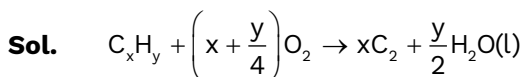
$\text{SnO}, \text{SnO}_2, \text{PbO}, \text{PbO}_2 \rightarrow$ amphoteric oxide



56. 80 mL of a hydrocarbon on mixing with 264 mL of oxygen in a closed U-tube undergoes complete combustion. The residual gases after cooling to 273 K occupy 224 mL. When the system is treated with KOH solution, the volume decreases to 64 mL. The formula of the hydrocarbon is: **[Mole Concept]**

- (1) C_2H_4 (2) C_4H_{10}
 (3) C_2H_2 (4) C_2H_6

Ans. (3)



Initial 80ml 264 ml 80x

$$V_{O_2} = 224 - 64 = 160$$

$$80x = 160 \Rightarrow x = 2$$

Unused $O_2 = 64$

$$\text{Used } O_2 = 200 = 80 \left(x + \frac{y}{4}\right)$$

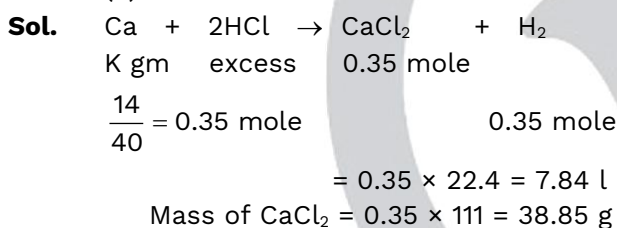
$$y = 2$$

- 57.** 14.0 g of calcium metal is allowed to react with excess HCl at 1.0 atm pressure and 273 K. Which of the following statements is incorrect?
 [Given: Molar mass in gmol^{-1} of Ca=40, Cl=35.5, H=1]

[Mole Concept]

- (1) 0.35 mol of H_2 gas is evolved.
 (2) 7.84 L of H_2 gas is evolved.
 (3) 33.3 g of $CaCl_2$ is produced.
 (4) The limiting reagent is calcium metal.

Ans. (3)



- 58.** In Carius method, 0.75 g of an organic compound gave 1.2 g of barium sulphate, find percentage of sulphur (molar mass 32gmol^{-1}). Molar mass of barium sulphate is 233gmol^{-1} .

[Mole Concept]

- (1) 4.55% (2) 10.30%
 (3) 21.97% (4) 16.48%

Ans. (3)

Sol. Mass of s = $\frac{1.2}{233} \times 32 = 0.1648 \text{ gm}$

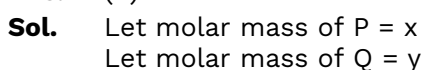
$$\%s = \frac{0.1648}{0.75} \times 100 = 21.97\%$$

- 59.** Elements P and Q form two types of non-volatile, non-ionizable compounds PQ and PQ_2 . When 1 g of PQ is dissolved in 50 g of solvent 'A', ΔT_b was 1.176 K while when 1 g of PQ_2 is dissolved in 50 g of solvent 'A', ΔT_b was 0.689 K. (K_b of 'A' = $5K \text{ kg mol}^{-1}$). The molar masses of elements P and Q (in gmol^{-1}) respectively, are:

[Liquid Solution]

- (1) 70,110 (2) 65,145
 (3) 60,25 (4) 25,60

Ans. (4)



$$1.176 = 5 \times \frac{\frac{1}{(x+y)}}{\frac{50}{1000}} - (1)$$

$$0.689 = 5 \times \frac{\frac{1}{(x+2y)}}{\frac{50}{1000}} - (2)$$

Form (1) and (2)

$$x = 25 \quad y = 60$$

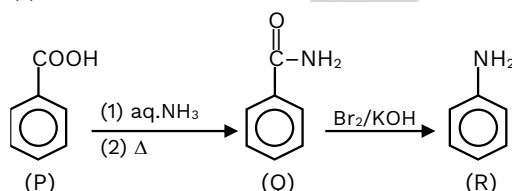
60. An organic compound (P) on treatment with aqueous ammonia under hot condition forms compound (Q) which on heating with Br_2 and KOH forms compound (R) having molecular formula $\text{C}_6\text{H}_7\text{N}$. Names of P, Q and R respectively are.

[Carboxylic Acid]

- (1) Benzoic acid, benzamide, aniline
 (2) Toluic acid, methylbenzamide, 2-methylaniline
 (3) Benzoic acid, 4-methylbenzamide, 4-methylaniline
 (4) Phenylethanoic acid, phenylethanamide, benzamine

Ans. (1)

Sol.



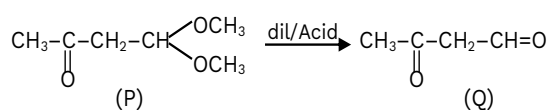
61. An organic compound "P" of molecular formula $\text{C}_6\text{H}_{12}\text{O}_3$ gives positive Iodoform test but negative Tollen's test. When "P" is treated with dilute acid, it produces "Q". "Q" gives positive Tollen's test and also iodoform test. The structure of "P" is:

[POC / Carbonyl Compound]

- (1) $\text{CH}_3 - \overset{\text{O}}{\underset{\text{O}}{\text{C}}} - \underset{\text{OCH}_3}{\text{CH}} - \underset{\text{OCH}_3}{\text{CH}_2}$
 (2) $\text{CH}_3 - \overset{\text{O}}{\underset{\text{O}}{\text{C}}} - \text{CH}_2 - \underset{\text{OCH}_3}{\underset{\text{OCH}_3}{\text{CH}}} - \text{OCH}_3$
 (3) $\text{H} - \overset{\text{O}}{\underset{\text{O}}{\text{C}}} - \text{CH}_2 - \text{CH}_2 - \underset{\text{OCH}_3}{\underset{\text{OCH}_3}{\text{CH}}} - \text{OCH}_3$
 (4) $\text{CH}_3 - \overset{\text{O}}{\underset{\text{O}}{\text{C}}} - \underset{\text{OCH}_3}{\underset{\text{OCH}_3}{\text{C}}} - \text{CH}_3$

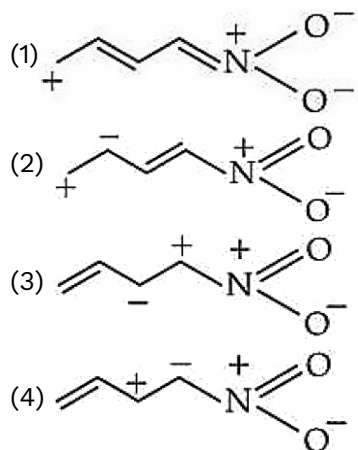
Ans. (2)

Sol.



62. From the following, the least stable structure is:

[General Organic Chemistry]



Ans. (3)

Sol. +ve charge on adjacent atoms, so it is least stable.

63. MnO_4^{2-} , in acidic medium, disproportionates to:

[d Block Elements]

- (1) Mn_2O_7 and MnO_2
- (2) MnO_4^- and MnO
- (3) MnO_4^- and MnO_2
- (4) Mn_2O_7 and MnO

Ans. (3)

Sol. $3\text{MnO}_4^{2-} + 4\text{H}^+ \longrightarrow 2\text{MnO}_4^- + \text{MnO}_2 + 2\text{H}_2\text{O}$

64. Given below are two statements:

Statement I:

The number of species among SF_4 , NH_4^+ , $[\text{NiCl}_4]^{2-}$, XeF_4 , $[\text{PtCl}_4]^{2-}$, SeF_4 and $[\text{Ni}(\text{CN})_4]^{2-}$, that have tetrahedral geometry is 3.

Statement II:

In the set $[\text{NO}_2]$, BeH_2 , BF_3 , AlCl_3 , all the molecules have incomplete octet around central atom.

In the light of the above statements, choose the correct answer from the options given below:

[Coordination Compounds]

- (1) Statement I is true but Statement II is false
- (2) Both Statement I and Statement II are false
- (3) Statement I is false but Statement II is true
- (4) Both Statement I and Statement II are true

Ans. (3)

Sol. $\text{SF}_4, \text{SeF}_4 \rightarrow$ See saw shape

$\text{NH}_4^+, [\text{NiCl}_4]^{2-} \rightarrow$ Tetrahedral

$\text{XeF}_4, [\text{PtCl}_4]^{2-}, [\text{Ni}(\text{CN})_4]^{2-} \rightarrow$ Sq. Planer

$\text{BeH}_2, \text{BF}_3, \text{AlCl}_3 \rightarrow$ Electron deficient molecules

$\text{NO}_2 \rightarrow$ Odd electron molecule

65. Given below are two statements:

Statement I: Among $[\text{Cu}(\text{NH}_3)_4]^{2+}$, $[\text{Ni}(\text{en})_3]^{2+}$, $[\text{Ni}(\text{NH}_3)_6]^{2+}$ and $[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$, $[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$ has the maximum number of unpaired electrons.

Statement II: The number of pairs among $\{[\text{NiCl}_4]^{2-}, [\text{Ni}(\text{CO})_4]\}$, $\{[\text{NiCl}_4]^{2-}, [\text{Ni}(\text{CN})_4]^{2-}\}$ and $\{[\text{Ni}(\text{CO})_4], [\text{Ni}(\text{CN})_4]^{2-}\}$ that contain only diamagnetic species is two.

In the light of the above statements, choose the correct answer from the options given below:

[Coordination Compounds]

- (1) Statement I is false but Statement II is true
- (2) Both Statement I and Statement II are true
- (3) Both Statement I and Statement II are false
- (4) Statement I is true but Statement II is false

Ans. (4)

Sol. $\text{Cu}(\text{NH}_3)_4^{2+}$ contains $\text{Cu}^{2+}(3d^9)$ which has one unpaired electron. $\text{Ni}(\text{en})_3^{2+}$ & $\text{Ni}(\text{NH}_3)_6^{2+}$ contains $\text{Ni}^{2+}(3d^8)$ which has 2 unpaired electrons.

$\text{Mn}(\text{H}_2\text{O})_6^{2+}$ contains $\text{Mn}^{2+}(3d^5)$ which has 5 unpaired electrons.

$\text{NiCl}_4^{2-} \rightarrow sp^3$, tetrahedral

$\text{Ni}(\text{CO})_4 \rightarrow sp^3$, tetrahedral

$\text{Ni}(\text{CN})_4^{2-} \rightarrow d_{sp^2}$, sq. planar

66. Identify correct statements from the following:
- A. Propanal and propanone are functional isomers.
 - B. Ethoxyethane and methoxypropane are metamers.
 - C. But-2-ene shows optical isomerism.
 - D. But-1-ene and but-2-ene are functional isomers.
 - E. Pentane and 2, 2-dimethyl propane are chain isomers.

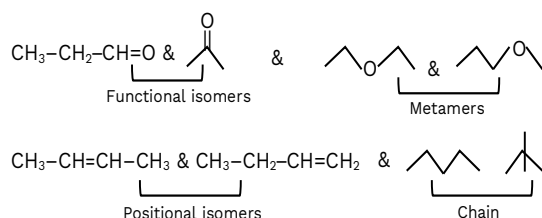
[Isomerism]

Choose the correct answer from the options given below:

- (1) B, C and D only
- (2) A, B and C only
- (3) A, B and E only
- (4) C, D and E only

Ans. (3)

Sol.



67. Identify the correct statements.
- A. Arginine and Tryptophan are essential amino acids.
 - B. Histidine does not contain heterocyclic ring in its structure.
 - C. Proline is a six membered cyclic ring amino acid.
 - D. Glycine does not have chiral centre.
 - E. Cysteine has characteristic feature of side chain as $\text{MeS--CH}_2\text{--CH}_2\text{--}$.

Choose the correct answer from the options given below: [Biomolecules]

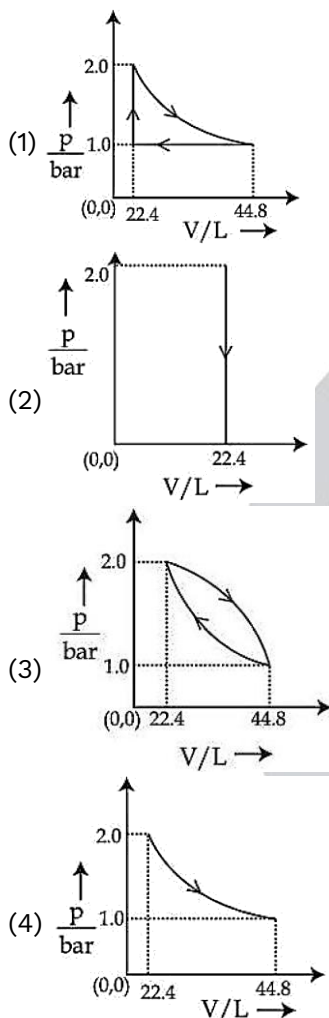
- (1) C and E Only (2) B and E Only
(3) C and D Only (4) A and D Only

Ans. (4)

Sol. $\text{H}_2\text{N}-\text{CH}_2-\text{COOH}$
Glycine

- 68.** Which of the following graphs between pressure 'p' versus volume 'V' represents the maximum work done?

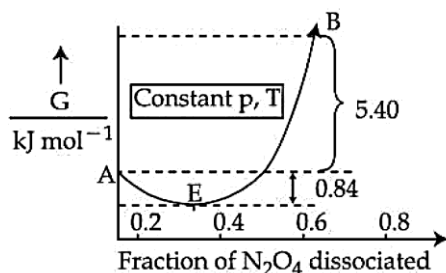
[Thermodynamics]



Ans. (4)

Sol. Maximum area enclosed is in 4th option

- 69.** For the reaction, $\text{N}_2\text{O}_4 \rightleftharpoons 2\text{NO}_2$, graph is plotted as shown below. Identify correct statements.
- A. Standard free energy change for the reaction is -5.40kJmol^{-1} .
 - B. As ΔG^\ominus in graph is positive, N_2O_4 will not dissociate into NO_2 at all.
 - C. Reverse reaction will go to completion.
 - D. When 1 mole of N_2O_4 changes into equilibrium mixture, value of $\Delta G^\ominus = -0.84\text{kJmol}^{-1}$
 - E. When 2 mole of NO_2 changes into equilibrium mixture, ΔG^\ominus for equilibrium mixture is -6.24kJmol^{-1} .



Choose the correct answer from the options given below:

[Thermodynamics & Thermochemistry]

- (1) D and E only
- (2) C and E only
- (3) A and D only
- (4) B and C only

Ans.

Sol. $\text{N}_2\text{O}_4 \rightleftharpoons 2\text{NO}_2$

(A) (B)

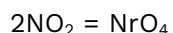
From the given graph:



$t = 0$ 1 mole

At eq^m $\Delta_r G^\circ = -0.84$ kJ/mole

and if reaction occurs in reverse direction



Initial 2 mole

At eq^m $\Delta_r G^\circ = -(5.4 + 0.84)$
 $= -6.25$ kJ/n

70. Given below are two statements:

Statement I: When an electric discharge is passed through gaseous hydrogen, the hydrogen molecules dissociate and the energetically excited hydrogen atoms produce electromagnetic radiation of discrete frequencies.

Statement II: The frequency of second line of Balmer series obtained from He^+ is equal to that of first line of Lyman series obtained from hydrogen atom.

In the light of the above statements, choose the correct answer from the options given below:

[Atomic Structure]

- (1) Both Statement I and Statement II are true
- (2) Both Statement I and Statement II are false
- (3) Statement I is false but Statement II is true
- (4) Statement I is true but Statement II is false

Ans.

(1)

Sol. 2nd line of Balmer series in $\text{He}^+ \Rightarrow \frac{1}{\lambda} = R \times 4 \left(\frac{1}{2^2} - \frac{1}{4^2} \right)$

1st line of Lyman in H $\Rightarrow \frac{1}{\lambda} = R \times 1 \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$

Both λ same

71. Pre-exponential factors of two different reactions of same order are identical. Let activation energy of first reaction exceeds the activation energy of second reaction by 20 kJ mol^{-1} . If k_1 and k_2 are

the rate constants of first and second reaction respectively at 300 K, then $\ln \frac{k_2}{k_1}$ will be ____.

(nearest integer)

$$[R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}]$$

[Chemical Kinetics]**Ans.** (8)

$$\text{Sol. } E_{a_1} = (E_{a_2} + 20)$$

$$k_2 = Ae^{-\frac{E_{a_2}}{R \times 300}}$$

$$k_1 = Ae^{-\frac{E_{a_1}}{R \times 300}}$$

$$\frac{k_2}{k_1} = e^{\frac{(E_{a_1} - E_{a_2})}{R \times 300}} \Rightarrow \ln\left(\frac{k_2}{k_1}\right) = \frac{20 \times 1000}{8.3 \times 3000} = 8$$

- 72.** The pH and conductance of a weak acid (HX) was found to be 5 and $4 \times 10^{-5} \text{ S}$, respectively. The conductance was measured under standard condition using a cell where the electrode plates having a surface area of 1 cm^2 were at a distance of 15 cm apart. The value of the limiting molar conductivity is ____ $\text{Sm}^2 \text{ mol}^{-1}$. (nearest integer)
(Given: degree of dissociation of the weak acid (α) $\ll 1$)

[Ionic Equilibrium]**Ans.** (6)

$$\text{Sol. } [\text{H}^+] = 10^{-5} = C \cdot \alpha \quad (1)$$

$$G = 4 \times 10^{-5}$$

$$G^* = \frac{15}{1} \text{ cm}^{-1}$$

$$k = 4 \times 10^{-5} \times 15 = 6 \times 10^{-4} \text{ S cm}^{-1}$$

$$k = 6 \times 10^{-2} \text{ S m}^{-1}$$

$$\Lambda_m = \frac{6 \times 10^{-2}}{1000 \times C} = \frac{6}{10^5 \times C}$$

$$\alpha = \frac{6}{105 \times C}$$

$$\Lambda_m^\infty$$

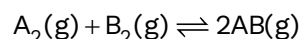
$$\text{From (1): } 10^{-5} = \frac{C \times 6}{10^5 \times C \times \Lambda_m^\infty}$$

$$\Lambda_m^\infty = 6 \text{ sm}^2 \text{ mol}^{-1}$$

- 73.** Use the following data:

Substance	$\frac{\Delta_f H^\ominus (500\text{K})}{\text{kJ mol}^{-1}}$	$\frac{S^\ominus (500\text{K})}{\text{J K}^{-1} \text{ mol}^{-1}}$
AB(g)	32	222
$\text{A}_2(\text{g})$	6	146
$\text{B}_2(\text{g})$	x	280

One mole each of $\text{A}_2(\text{g})$ and $\text{B}_2(\text{g})$ are taken in a 1 L closed flask and allowed to establish the equilibrium at 500 K .



The value of x (in kJ mol^{-1}) is _____. (Nearest integer)

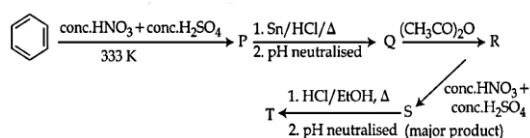
(Given: $\log K = 2.2$ $R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}$)

[Thermodynamics & Thermochemistry]

Ans. (70)

Sol. $\Delta_r G^\circ = 2.303 \times 8.3 \times 500 \log K$
 $= -21026.39 = -21.026 \text{ K 5/mole}$
 $\Delta_r S^\circ = (2 \times 222) - (146 + 280)$
 $= 18 \frac{\text{S}}{\text{K}}$
 $\Delta_r G^\circ = \Delta_r H^\circ - T \Delta_r S^\circ$
 $-21.026 = \Delta_r H^\circ - \frac{500 \times 18}{1000}$
 $\Delta_r H^\circ = -12.026 \frac{\text{KS}}{\text{mole}}$
 $-12.026 = (32 \times 2) - (6 + x)$
 $x = 70.026 = 70$

74. Consider the following reaction sequence

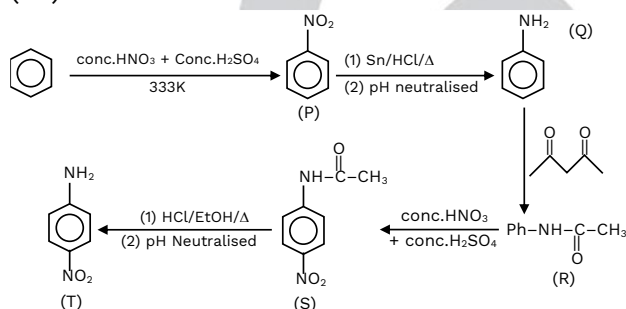


The percentage of nitrogen in product 'T' formed is ____%. (Nearest integer)
 (Given molar mass in g mol^{-1} H: 1, C: 12, N: 14, O: 16)

[Aromatic Compounds]

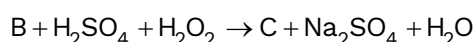
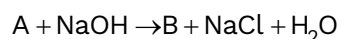
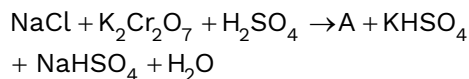
Ans. (20)

Sol.



$$\% \text{ of N} = \frac{28}{138} \times 100 = 20.29\%$$

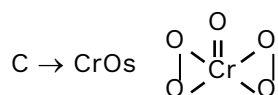
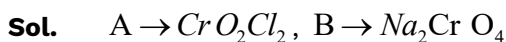
75. Consider the following reactions:



In the product 'C', 'X' is the number of O_2^{2-} units, 'Y' is the total number oxygen atoms present and 'Z' is the oxidation state of Cr. The value of $X + Y + Z$ is ____.

[Salt Analysis]

Ans. (13)



$$x = 2, y = 5, z = 6$$

$$(x + y + z) = 13$$

