JEE-MAIN EXAMINATION – JANUARY 2026

(24TH JANUARY 2026) **TIME: 9:00 A.M. TO 12:00 NOON**

PART-A : MATHEMATICS

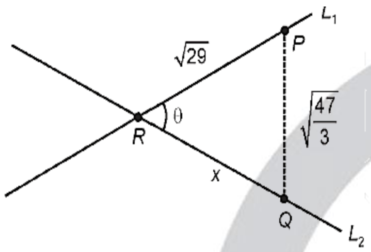
SECTION- I

1. Let the lines $L_1: \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k}), \lambda \in \mathbb{R}$ and $L_2: \vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k}), \mu \in \mathbb{R}$, intersect at the point R . Let P and Q be the points lying on lines L_1 and L_2, respectively, such that $|\overrightarrow{PR}| = \sqrt{29}$ and $|\overrightarrow{PQ}| = \sqrt{\frac{47}{3}}$. If the point P lies in the first octant, then $27(QR)^2$ is equal to

   1. 360
   2. 340
   3. 348
   4. 320

Ans : (1)

Sol :



$$|\cos\theta| = \frac{16+6+4}{\sqrt{4+9+16}\sqrt{25+4+1}} = \frac{20}{\sqrt{29}\sqrt{30}}$$

$$\Rightarrow \pm\frac{20}{\sqrt{29}\sqrt{30}} = \frac{29+x^2-\frac{47}{3}}{2\sqrt{29}x}$$

$$\Rightarrow x^2 \pm \frac{40}{\sqrt{30}}x + \frac{40}{3} = 0$$

$$\Rightarrow x = \pm\frac{20}{\sqrt{30}}$$

$$\because x > 0 \Rightarrow x = \frac{20}{\sqrt{30}}$$

$$\Rightarrow 27x^2 = 27 \times \frac{400}{30} = 360$$

$\Rightarrow$ Option (1) is correct.

2. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \vec{a} \times \vec{b}$. Let $\vec{d}$ be a vector such that $|\vec{d} - \vec{a}| = \sqrt{11}, |\vec{c} \times \vec{d}| = 3$ and the angle between $\vec{c}$ and $\vec{d}$ is $\frac{\pi}{4}$. Then $\vec{a} \cdot \vec{d}$ is equal to

   1. 11
   2. 1
   3. 3
   4. 0

Ans :(4)

Sol :

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$$

$$|\vec{c}| = \sqrt{4+4+1} = 3$$

$$|\vec{c} \times \vec{d}| = 3 = |\vec{c}||\vec{d}|\sin\frac{\pi}{4}$$

$$\Rightarrow |\vec{d}| = \sqrt{2}$$

$$|\vec{d} - \vec{a}| = \sqrt{11}$$

$$|\vec{d}|^2 + |\vec{a}|^2 - 2\vec{a}\cdot\vec{d} = 11$$

$$2 + 9 - 2\vec{a}\cdot\vec{d} = 11$$

$$\vec{a}\cdot\vec{d} = 0$$

3. Let $\alpha, \beta \in \mathbb{R}$ be such that the function $f(x) = \begin{cases} 2\alpha(x^2 - 2) + 2\beta x & , x < 1 \\ (\alpha + 3)x + (\alpha - \beta) & , x \geq 1 \end{cases}$ be differentiable at all $x \in \mathbb{R}$.

Then $34(\alpha + \beta)$ is equal to

1. 84
2. 24
3. 36
4. 48

Ans :(4)

Sol :

$f(x) = \begin{cases} 2\alpha(x^2 - 2) + 2\beta x & x < 1 \\ (\alpha + 3)x + (\alpha - \beta) & x \geq 1 \end{cases}$

$\because f(x)$ is diff $\Rightarrow f(x)$ is continuous at $x = 1$

$2\alpha(1 - 2) + 2\beta = (\alpha + 3) + (\alpha - \beta)$

$-2\alpha + 2\beta = 2\alpha - \beta + 3$

$3\beta - 4\alpha = 3$

$f'(x) = \begin{cases} 4\alpha x + 2\beta & x < 1 \\ (\alpha + 3) & x \geq 1 \end{cases}$

$\Rightarrow 4\alpha + 2\beta = \alpha + 3$

$3\alpha + 2\beta = 3$

By solving (i) & (ii)

$\alpha = \frac{3}{17}\ \beta = \frac{21}{17}$

$34[\alpha + \beta] = 2[3 + 21]$

$= 48$

4. The number of the real solutions of the equation:

$x|x + 3| + |x - 1| - 2 = 0$ is

1. 3
2. 2
3. 4
4. 5

Ans :(1)

Sol :

$x|x + 3| + |x - 1| - 2 = 0$

Case I: $x < -3$ then

$-x(x + 3) - (x - 1) - 2 = 0$

$-x^2 - 4x - 1 = 0$

$\therefore x^2 + 4x + 1 = 0$

$\therefore x = \frac{-4 \pm \sqrt{12}}{2} = -2 \pm \sqrt{3}$

$x = -2 - \sqrt{3}$ is a solution

Case II: $-3 \leq x < 1$, then

$\because x(x + 3) - x + 1 - 2 = 0$

Or, $x^2 + 2x - 1 = 0$

$\therefore x = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$

$x = -1 + \sqrt{2}, -1 - \sqrt{2}$ are two solutions.

Case III: $x \geq 1$ then

$x^2 + 3x + x - 1 - 2 = 0$

$x^2 + 3x + x - 1 - 2 = 0$

$x^2 + 4x - 3 = 0$

$x = \frac{-4 \pm \sqrt{16 + 12}}{2} = -2 \pm \sqrt{7}$

no solution is possible.

Total number of solutions is 3.

5. The mean and variance of a data of 10 observations are 10 and 2, respectively. If an observations α in this data is replaced by β, then the mean and variance become 10.1 and 1.99 , respectively. Then $\alpha+\beta$ equals
1. 20
2. 15
3. 5
4. 10

Ans : (1)

Sol :

$\Sigma x_i = 100$

$\frac{\Sigma x_i^2}{10} - (10)^2 = 2$

$\Rightarrow \Sigma x_i^2 = 1020$

$\mu' = \frac{\Sigma(x_i)-\alpha+\beta}{10} \Rightarrow 100-\alpha+\beta = 101$

$\Rightarrow \beta-\alpha = 1$

$\sigma' = \left(\frac{\Sigma x_i^2-\alpha^2+\beta^2}{10}\right) - \left(\frac{101}{10}\right)^2 = \frac{199}{100}$

$\Rightarrow \frac{1020-\alpha^2+\beta^2}{10} = \frac{199}{100} + \left(\frac{101}{100}\right)^2$

$= \frac{10400}{100} = 104$

$\Rightarrow 1020-\alpha^2+\beta^2 = 1040$

$\Rightarrow \beta^2-\alpha^2 = 20$

$\beta-\alpha = 1$

$\Rightarrow (\beta+\alpha)(\beta-\alpha) = 20$

$\Rightarrow \alpha+\beta = 20$

6. If the domain of the function
$f(x) = \log_{(10x^2-17x+7)}(18x^2-11x+1)$
is $(-\infty, a) \cup (b, c) \cup (d, \infty) - \{e\}$, then
$90(a+b+c+d+e)$ equals:
1. 316
2. 307
3. 170
4. 177

Ans : (1)

Sol :

$10x^2-17x+7 > 0$

$10x^2-17x+7 \neq 1$

$18x^2-11x+1 > 0$

$10x^2-17x+7 > 0$

$10x^2-10x-7x+7 > 0$

$10x(x-1)-7(x-1) > 0$

$\left(x-\frac{7}{10}\right)(x-1) > 0$

$x \in \left(-\infty, \frac{7}{10}\right) \cup (1, \infty)$ ... (a)

$10x^2-17x+6 \neq 0 \Rightarrow x \neq \frac{1}{2}, \frac{6}{5}$ ... (b)

$18x^2-11x+1 > 0$

$18x^2-9x-2x+1 > 0$

$9x(2x-1)-(2x-1) > 0$

$\left(x-\frac{1}{2}\right)\left(x-\frac{1}{9}\right) > 0$

$\Rightarrow x \in \left(-\infty, \frac{1}{9}\right) \cup \left(\frac{1}{2}, \infty\right)$ ... (c)

Intersection of(a),(b) and(c)

$x \in \left(-\infty, \frac{1}{9}\right) \cup \left(\frac{1}{2}, \frac{7}{10}\right) \cup (1, \infty) - \left\{\frac{6}{5}\right\}$

$\Rightarrow a = \frac{1}{9}, b = \frac{1}{2}, c = \frac{7}{10}, d = 1, e = \frac{6}{5}$

$\Rightarrow 90(a + b + c + d + e) = 316$

⇒ Option (1) is correct

7. From a lot containing 10 defective and 90 non-defective bulbs, 8 bulbs are selected one by one with replacement. Then the probability of getting at least 7 defective bulbs is

1. $\frac{81}{10^8}$
2. $\frac{7}{10^7}$
3. $\frac{73}{10^8}$
4. $\frac{67}{10^8}$

Ans :(3)

Sol :

$P(D) = \frac{1}{10} = p$

$P(ND) = \frac{9}{10} = q$

$P(x \geq 7) = P(x = 7) + P(x = 8)$

$= {}^8C_7 \left(\frac{1}{10}\right)^7 \left(\frac{9}{10}\right)^{8-7} + {}^8C_8 \left(\frac{1}{10}\right)^8 \left(\frac{9}{10}\right)^0$

$P(x \geq 7) = \frac{73}{10^8}$

8. If $\cot x = \frac{5}{12}$ for some $x \in \left(\pi, \frac{3\pi}{2}\right)$, then $\sin 7x \left(\cos\frac{13x}{2} + \sin\frac{13x}{2}\right) + \cos 7x \left(\cos\frac{13x}{2} - \sin\frac{13x}{2}\right)$ is equal to

1. $\frac{4}{\sqrt{26}}$
2. $\frac{6}{\sqrt{26}}$
3. $\frac{1}{\sqrt{13}}$
4. $\frac{5}{\sqrt{13}}$

Ans :(3)

Sol :

$\cot x = \frac{5}{12}, x \in \left(\pi, \frac{3\pi}{2}\right)$

$E = \sin(7x)\left[\cos\left(\frac{13x}{2}\right) + \sin\left(\frac{13x}{2}\right)\right]$

$+\cos 7x \left(\cos\left(\frac{13x}{2}\right) - \sin\left(\frac{13x}{2}\right)\right]$

$= \sin(7x)\cdot\cos\left(\frac{13x}{2}\right) - \cos(7x)\cdot\sin\left(\frac{13x}{2}\right)$

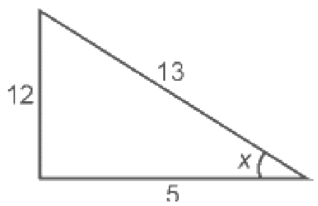
$+\sin(7x)\cdot\sin\left(\frac{13x}{2}\right) + \cos(7x)\cos\left(\frac{13x}{2}\right)$

$= \sin\left(7x - \frac{13x}{2}\right) + \cos\left(7x - \frac{13x}{2}\right)$

$= \sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)$

$x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

$\left|\sin\frac{\pi}{2}\right| > \left|\cos\frac{\pi}{2}\right|$



$E^2 = 1 + \sin x$

$E^2 = 1 - \frac{12}{13} = \frac{1}{13}$

$E = \frac{1}{\sqrt{13}}$

9. Let R be a relation defined on the set $\{1,2,3,4\} \times \{1,2,3,4\}$ by $R = \{((a,b),(c,d)): 2a + 3b = 3c + 4d\}$. Then the number of elements in R is

1. 18
2. 6
3. 15
4. 12

Ans : (4)

Sol :

$R = \{\{(a,b),(c,d)\}: 2a + 3b = 3c + 4d\}$

$R: \{((2,1),(1,1)), ((2,2),(2,1)), ((1,3),(1,2)), ((4,1),$
$(1,2)), ((2,3),(3,1)), ((1,4),(2,2)), ((4,2),(2,2)), ((3,3),$
$(1,3)), ((2,4),(4,1)), ((4,3),(3,2)), ((3,4),(2,3)), ((4,4),$
$(4,2))\}$

$n(R) = 12$

10. Let $A(1,0), B(2,-1)$ and $C\left(\frac{7}{3},\frac{4}{3}\right)$ be three points. If the equation of the bisector of the angle ABC is $\alpha x + \beta y = 5$, then the value of $\alpha^2 + \beta^2$ is

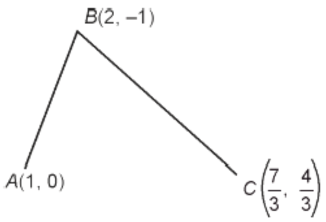
iptions

1. 13
2. 5
3. 10
4. 8

Ans : (3)

Sol : $AB: -1 = \dfrac{y}{x-1} \Rightarrow x + y = 1$

$BC: \dfrac{7/3}{1/3} = \dfrac{y+1}{x-2}$



$\Rightarrow 7x - 14 = y + 1$

$\Rightarrow 7x - y - 15 = 0$

Equation of angle bisector $\left|\frac{x+y-1}{\sqrt{2}}\right| = \left|\frac{7x-y-15}{\sqrt{50}}\right|$

$\Rightarrow x + y - 1 = \pm\left(\frac{7x-y-15}{5}\right)$

$\Rightarrow 5x + 5y - 5 = \pm(7x - y - 15)$

taking + sign:

$x - 3y = 5$

$\alpha = 1, \beta = -3 \quad \therefore \alpha^2 + \beta^2 = 10$

11. Let each of the two ellipses $E_1: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ and $E_2: \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1, (A < B)$ have eccentricity $\frac{4}{5}$. Let the lengths of the latus recta of E_1 and E_2 be l_1 and l_2, respectively, such that $2l_1^2 = 9l_2$. If the distance between the foci of E_1 is 8 , then the distance between the foci of E_2 is

1. $\frac{96}{5}$
2. $\frac{32}{5}$
3. $\frac{16}{5}$
4. $\frac{8}{5}$

Ans : (2)

So : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$

$I_1^2 = 1 - \frac{b^2}{a^2} \qquad I_2^2 = 1 - \frac{B^2}{A^2}$

$\frac{16}{25} = 1 - \frac{b^2}{a^2} \ldots(1) \quad \frac{16}{25} = 1 - \frac{B^2}{A^2} \ldots(2)$

Now $2l_1^2 = 9l_2$

$2\left(\frac{2b^2}{a}\right)^2 = 9\left(\frac{2B^2}{A}\right)$

$8\frac{b^4}{a^2} = 18\frac{B^2}{A}$

$\frac{b^4}{a^2} = \frac{9}{4}\frac{B^4}{A}$

Also given: $2ae = 8$

$2 \times \frac{4}{5}a = 8$

$a = 5$

$\Rightarrow b = 3$

Now $\frac{81}{25} = \frac{9}{4}\frac{B^2}{A}$

$\frac{36}{25}A = B^2$

Sub in (2)

$\frac{36}{25}\frac{A}{A^2} = \frac{9}{25}$

$A = 4$

Now 2Ae

$= 2 \times 4 \times \frac{4}{5}$

$= \frac{32}{5}$

12. Consider an A.P.: $a_1, a_2, \ldots, a_n; a_1 > 0$. If $a_2 - a_1 = \frac{-3}{4}, a_n = \frac{1}{4}a_1$, and $\sum_{i=1}^{n} a_i = \frac{525}{2}$, then $\sum_{i=1}^{17} a_i$ is equal to

1. 136
2. 952
3. 476
4. 238

Ans : (4)

Sol :

$\because a_1, a_2, a_3, \dots a_n$ are in A.P.
Given that $a_2 - a_1 = -\frac{3}{4}$ = common difference (d).
$d = -\frac{3}{4}$
and also given that $a_n = \frac{1}{4}a_1$
$\therefore S_n = \frac{n}{2}(a_1 + a_n) = \frac{525}{2}$
$\therefore n\left(a_1 + \frac{1}{4}a_1\right) = 525$
$\therefore a_1 n = 420$
Now $\frac{n}{2}\left\{2a_1 + (n-1)\left(-\frac{3}{4}\right)\right\} = \frac{525}{2}$

$8 \times 420 - 3n^2 + 3n = 2100$
$\therefore 3n^2 - 3n - 1260 = 0$
$\therefore n^2 - n - 420 = 0$
$(n-21)(n+20) = 0$
$\therefore n = 21$
$\therefore a_1 = 20$
$\sum_{i=1}^{17} a_i = \frac{17}{2}\left\{40 + 16 \times -\frac{3}{4}\right\} = 238$

13. The value of $\frac{\sqrt{3}\operatorname{cosec}20^\circ - \sec 20^\circ}{\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}$ is equal to
1. 64
2. 12
3. 16
4. 32

Ans :(1)

Sol : $\sqrt{3}\cos ec20^0 - \sec 20^0$

$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$

$= 4 \cdot \frac{\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ}{2\sin 20^\circ \cos 20^\circ}$

$= 4 \cdot \frac{\sin 40^\circ}{\sin 40^\circ}$

$= 4$

and $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ \cdot \cos 60^\circ$

$= \frac{1}{4}\cos 60^\circ \cdot \cos 60^\circ$

$= \frac{1}{16}$

$\therefore \frac{\sqrt{3}\operatorname{cosec}20^\circ - \sec 20^\circ}{\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ \cdot \cos 60^\circ} = \frac{4}{\frac{1}{16}}$

$= 64$

14. If the function $f(x) = \frac{e^x\left(e^{\tan x - x} - 1\right) + \log_e(\sec x + \tan x) - x}{\tan x - x}$ is continuous at $x = 0$, then the value of $f(0)$ is equal to
1. 2
2. $\frac{2}{3}$
3. $\frac{1}{2}$
4. $\frac{3}{2}$

Ans :(4)

Sol :

$f(0) = \text{Lim}_{x\to 0} f(x)$

$= \text{Lim}_{x\to 0} e^{x}\left(\frac{e^{\tan x - x} - 1}{\tan x - x}\right) + \text{Lim}_{x\to 0}\frac{\log_e(\sec x + \tan x) - x}{\tan x - x}$

$I_1 = \text{Lim}_{x\to 0} e^{x}\frac{(e^{\tan x - x} - 1)}{\tan x - x} = 1$

$I_2 = \text{Lim}_{x\to 0}\frac{\log_e(\sec x + \tan x) - x}{\tan x - x}$, Form: $\frac{0}{0}$

Using L-H Rule

$= \text{Lim}_{x\to 0}\frac{\frac{1}{(\sec x + \tan x)}\times(\sec x\tan x + \sec^2 x) - 1}{\sec^2 x - 1}$

$= \text{Lim}_{x\to 0}\frac{\sec x - 1}{\sec^2 x - 1} = \text{Lim}_{x\to 0}\frac{1}{\sec x + 1} = \frac{1}{2}$

$\Rightarrow f(0) = 1 + \frac{1}{2} = \frac{3}{2}$

Option (4) is correct

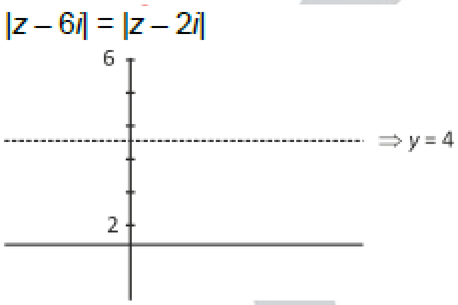
15. Let $S = \left\{z \in \mathbb{C}: \left|\frac{z-6i}{z-2i}\right| = 1 \text{ and } \left|\frac{z-8+2i}{z+2i}\right| = \frac{3}{5}\right\}$.

Then $\sum_{z\in S}|z|^2$ is equal to

1. 423
2. 385
3. 413
4. 398

Ans : (2)

Sol :



$z = x + iy$

$5|(x-8) + (y+2)| = 3|(x+0)^2 + (y+2)^2|$

$\Rightarrow 25(x-8)^2 + 25(y+2)^2$

$= 9(x^2) + 9(y+2)^2$

$\Rightarrow 25x^2 - 16\times 25x + 25\times 64 + 25y^2 + 100y + 100$

$= 9x^2 + 9y^2 + 36y + 36$

$\Rightarrow 16x^2 + 16y^2 - 400x + 64y + 166y = 0$

$\Rightarrow x^2 + y^2 - 25x + 4y + 104 = 0$

This circle intersects lines $y = 4$

at $x^2 + 16 - 25x + 16 + 104 = 0$

$x^2 - 25x + 136 = 0 \Rightarrow x = 8, 17$

$\Rightarrow z$ can be (17,4) and (8,4)

$\Rightarrow \Sigma|z|^2 = \left(\sqrt{8^2+4^2}\right)^2 + \left(\sqrt{4^2+17^2}\right)^2$

$= 64 + 16 + 16 + 289 = 385$

16. Let $f(t) = \int\left(\frac{1-\sin(\log_e t)}{1-\cos(\log_e t)}\right)dt, t > 1$.

If $f\left(e^{\pi/2}\right) = -e^{\pi/2}$ and $f\left(e^{\pi/4}\right) = \alpha e^{\pi/4}$, then α equals

1. $1+\sqrt{2}$
2. $-1+\sqrt{2}$
3. $-1-2\sqrt{2}$
4. $-1-\sqrt{2}$

Ans :(4)

Sol :

$\int \frac{1-\sin(\ln t)}{1-\cos(\ln t)} dt$

Let Int $= x$

$t = e^x$

$dt = e^x dx$

$\int e^x \frac{(1-\sin x)}{1-\cos x} dt$

$\Rightarrow \int e^x \left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx$

$\Rightarrow \int e^x \left(\frac{1}{2} - \operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right) dx$

$\Rightarrow \int e^x \left[\frac{-\cot\frac{x}{2}+\frac{1}{2}\operatorname{cosec}c^2\frac{x}{2}}{r(x)}\right]$

$\Rightarrow -e^x \cot\frac{x}{2} + c$

$f(t) = -t\cot\left(\frac{\ln t}{2}\right) + c$

$f(e^{\pi/2}) = -e^{\pi/2} + c = -e^{-\pi/2}$

$\Rightarrow c = 0$

$f(e^{\pi/4}) = -e^{\pi/4}\cot\left(\frac{\pi}{8}\right)$

$= -e^{\pi/4}[\sqrt{2}+1]$

17. Let $729, 81, 9, 1, \ldots$ be a sequence and P_n denote the product of the first n terms of this sequence. If $2\sum_{n=1}^{40}(\mathrm{P}_n)^{\frac{1}{n}} = \frac{3^{\alpha}-1}{3^{\beta}}$ and $\gcd(\alpha,\beta) = 1$, then $\alpha+\beta$ is equal to

1. 76
2. 73
3. 75
4. 74

Ans : (2)

Sol :

$3^6, 3^4, 3^2, 3^0, \ldots\ldots$

$P_n = 3^{6+4+2+\cdots n \text{ terms}}$

$= 3^{\frac{n}{2}[2\times 6+(n-1)(-2)]} = 3^{n(6-n+1)} = 3^{n(7-n)}$

$\Rightarrow \sum_{n=1}^{40}(P_n)^{\frac{1}{n}} = \sum_{n=1}^{40} 3^{7-n} = 3^7 \times \frac{1}{3}\left(\frac{1-\frac{1}{3^{40}}}{1-\frac{1}{3}}\right)$

$= 3^7\left(\frac{3^{40}-1}{2\times 3^{40}}\right) = \frac{3^{40}-1}{2\cdot 3^{33}}$

$\Rightarrow \alpha+\beta = 73$

18. Let $\mathrm{S} = \frac{1}{25!} + \frac{1}{3!23!} + \frac{1}{5!21!} + \cdots$ up to 13 terms. If $13\,\mathrm{S} = \frac{2^k}{n!}, k \in \mathrm{N}$, then $n+k$ is equal to

1. 49
2. 52
3. 50
4. 51

Ans :(1)

Sol :

$\frac{1}{25!}+\frac{1}{23!\,3!}+\frac{1}{21!\,5!}+\cdots\ldots$ till 13 term $=S$

$26!\,S=\frac{26!}{25!\,1!}+\frac{26!}{23!\,3!}+\frac{26!}{21!\,5!}+\cdots\ldots$

$={}^{26}C_1+{}^{26}C_3+{}^{26}C_5+\cdots\ldots+{}^{26}C_{25}$

$26!\,S=2^{25}$

$S=\frac{2^{25}}{26!}$

$13S=13\times\frac{2^{25}}{26\times 25!}$

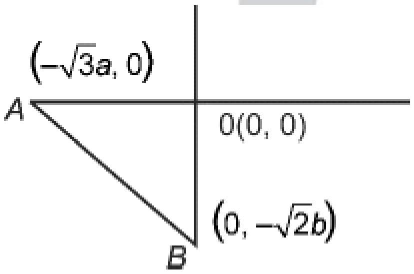
$=\frac{2^{24}}{25!}\Rightarrow k=24\ n=25\Rightarrow k+n=49$

19. Let a circle of radius 4 pass through the origin O , the points $A(-\sqrt{3}a,0)$ and $B(0,-\sqrt{2}b)$, where a and b are real parameters and $ab\neq 0$. Then the locus of the centroid of $\triangle OAB$ is a circle of radius

1. $\frac{11}{3}$
2. $\frac{5}{3}$
3. $\frac{7}{3}$
4. $\frac{8}{3}$

Ans :(4)

Sol :



Radius $=\frac{\sqrt{(-\sqrt{2}b)^2+(-\sqrt{3}a)^2}}{2}=4$

$\Rightarrow 2b^2+3a^2=64$

Let the centroid of the triangle is (h,k)

$\Rightarrow\ h=\frac{-\sqrt{3}a}{3}$ and $k=\frac{-\sqrt{2}b}{3}$

$\Rightarrow\ a=\frac{-3h}{\sqrt{3}}$ and $b=\frac{-3k}{\sqrt{2}}$

$2\left(\frac{9k^2}{2}\right)+3\left(\frac{9h^2}{3}\right)=64$

$\Rightarrow 9k^2+9h^2=64$

Locus is $x^2+y^2=\frac{64}{9}$

$\therefore$ Radius $=\frac{8}{3}$

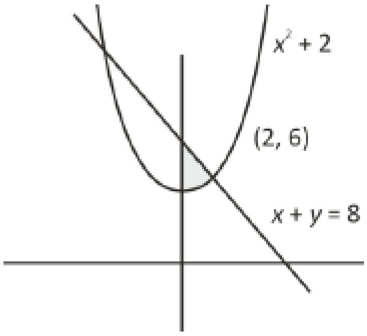
20. Let A_1 be the bounded area enclosed by the curves $y=x^2+2, x+y=8$ and y-axis that lies in the first quadrant. Let A_2 be the bounded area enclosed by the curves $y=x^2+2, y^2=x, x=2$, and y-axis that lies in the first quadrant. Then A_1-A_2 is equal to

1. $\frac{2}{3}(\sqrt{2}+1)$
2. $\frac{2}{3}(2\sqrt{2}+1)$

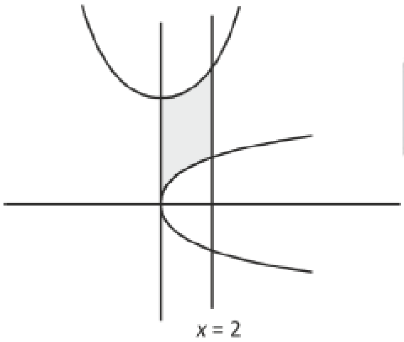
3. $\frac{2}{3}(3\sqrt{2}+1)$
4. $\frac{2}{3}(4\sqrt{2}+1)$

Ans :(2)

Sol :



$$A_1 = \int_0^2 ((8-x)-(x^2+2))dx = \frac{22}{3}$$



$$A_2 = \int_0^2 \left((x^2+2)-(\sqrt{x})\right)dx$$
$$= \frac{x^3}{3} + 2x - \frac{2x^{3/2}}{3}\Big]_0^2$$
$$= \frac{8}{3} + 4 - \frac{2(2)^{3/2}}{3}$$
$$= \frac{20}{3} - \frac{4\sqrt{2}}{3}$$
$$A_1 - A_2 = \frac{22}{3} - \frac{20}{3} + \frac{4\sqrt{2}}{3} = \frac{2+4\sqrt{2}}{3}$$

SECTION- II

21. Let a differentiable function f satisfy the equation $\int_0^{36} f\left(\frac{tx}{36}\right)dt = 4\alpha f(x)$.
If $y = f(x)$ is a standard parabola passing through the points $(2,1)$ and $(-4,\beta)$, then β^{α} is equal to

Ans :(64)

Sol :

$$\int_0^{36} f\left(\frac{tx}{36}\right)dt = 4\alpha f(x)$$
$$\frac{tx}{36} = p \Rightarrow dt = \frac{36}{x}dp$$
$$\int_0^x f(p)(36)\left(\frac{dp}{x}\right) = 4\alpha f(x)$$
$$\Rightarrow \int_0^x f(p)(dp) = \frac{x\alpha}{9} f(x)$$

Differentiating,

$f(x)=\frac{\alpha}{9}f(x)+\frac{\alpha x}{9}f'(x)$

Let $y=f(x)$

$y\left(1-\frac{\alpha}{9}\right)=\frac{\alpha x}{9}\frac{dy}{dx}$

$\Rightarrow\left(\frac{9-\alpha}{\alpha}\right)\frac{dx}{x}=\frac{dy}{y}$

$\Rightarrow\left(\frac{9-\alpha}{9\times\alpha}\right)\ln|(x)|=\ln|y|+\ln(k)$

$x\left(\frac{9-\alpha}{\alpha}\right)=yk$

$\Rightarrow\frac{9-\alpha}{\alpha}=2\Rightarrow\alpha=3$

$\Rightarrow x^2=yk$

$\Rightarrow x^2=4y$

$\Rightarrow 4\beta=16$

$\Rightarrow\beta=4$

$\Rightarrow\beta^\alpha=4^3=64$

22. The number of numbers greater than 5000 , less than 9000 and divisible by 3 , that can be formed using the digits $0,1,2,5,9$, if the repetition of the digits is allowed, is ___

Ans :(42)

Sol :

As number is more than 5000 and less than 9000 then thousand place must be 5 .

5	a	b	c

For $(a,b,c)=(0,0,1)\rightarrow 3$ ways
$(0,1,9)\rightarrow 6$ ways
$(0,2,5)\rightarrow 6$ ways
$(0,2,2)\rightarrow 3$ ways
$(0,5,5)\rightarrow 3$ ways
$(1,1,2)\rightarrow 3$ ways
$(1,1,5)\rightarrow 3$ ways
$(1,9,9)\rightarrow 3$ ways
$(2,2,9)\rightarrow 3$ ways
$(2,5,9)\rightarrow 6$ ways
$(5,5,9)\rightarrow 3$ ways

Total 42 numbers are possible.

23. Let a line L passing through the point $P(1,1,1)$ be perpendicular to the lines $\frac{x-4}{4}=\frac{y-1}{1}=\frac{z-1}{1}$ and $\frac{x-17}{1}=\frac{y-71}{1}=\frac{z}{0}$. Let the line L intersect the $yz-$ plane at the point Q . Another line parallel to L and passing through the point $S(1,0,-1)$ intersects the yz-plane at the point R . Then the square of the area of the parallelogram PQRS is equal to ___ .

Ans :(06)

Sol :

$\vec{l}\|(4\hat{\imath}+\hat{\jmath}+\hat{k})\times(\hat{\imath}+\hat{\jmath})$

$\Rightarrow\vec{l}\|(\hat{\imath}-\hat{\jmath}-3\hat{k})$

$\Rightarrow L:\frac{x-1}{-1}=\frac{y-1}{-1}=\frac{z-1}{-3}$

At $y-z$ plane $x=0$

$\Rightarrow -1=\frac{y-1}{-1}=\frac{z-1}{-3}\Rightarrow$

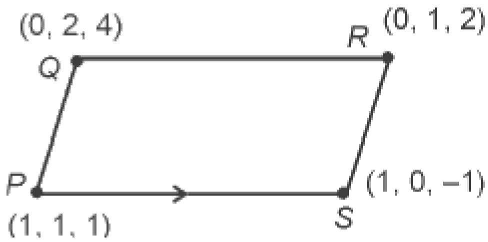
$Q(0,2,4)$

$L_2\Rightarrow\frac{x-1}{-1}=\frac{y}{-1}=\frac{z+1}{-3}$

At $y-z$ again $x=0$

$-1 = \frac{y}{-1} = \frac{z+1}{-3}$

$R(0,1,2)$



Area $|\overrightarrow{PS} \times \overrightarrow{PQ}|$

$= |(\hat{j} + 2\hat{k}) \times (\hat{\imath} - \hat{j} - 3\hat{k})|$

$\Rightarrow = |\hat{\imath} - 2\hat{j} + \hat{k}|$

$\Rightarrow$ Area $= \sqrt{6}$

$\Rightarrow$ (Area $)^2 = 6$

24. Let $(2\alpha, \alpha)$ be the largest interval in which the function $f(t) = \frac{|t+1|}{t^2}, t < 0$, is strictly decreasing. Then the local maximum value of the function $g(x) = 2\log_e(x-2) + \alpha x^2 + 4x - \alpha, x > 2$, is ___

Ans :(04)

Sol :

$$f(t) = \frac{|t+1|}{t^2}, t < 0$$

$$= \begin{cases} \frac{t+1}{t^2}, & t \geq -1 \\ \frac{-t-1}{t^2}, & t < -1 \end{cases}$$

$$= \begin{cases} \frac{1}{t} + \frac{1}{t^2}, & t \geq -1 \\ -\frac{1}{t} - \frac{1}{t^2}, & t \leq -1 \end{cases}$$

$$f'(t) = \begin{cases} -\frac{1}{t^2} - \frac{2}{t^3}, & t \geq -1 \\ +\frac{1}{t^2} + \frac{2}{t^3}, & t \leq -1 \end{cases}$$

$$= \begin{cases} \frac{-2-t}{t^3}, & t \in [-1,0) \\ \frac{t+2}{t^3}, & t \in (-\infty, -1) \end{cases}$$

$\Rightarrow \frac{t+2}{t^3} \leq 0$

$\Rightarrow t \in (-2,0)$ but for $t \in (-\infty, -1)$

$\Rightarrow t \in (-2,-1) \Rightarrow \alpha = -1$

$g(x) = 2\ln(x-2) - x^2 + 4x + 1, x > 2$

$g'(x) = \frac{2}{x-2} - 2x + 4 = \frac{-2x^2+4x+4x-8+2}{x-2}$

$= \frac{(-2x^2+8x-6)}{x-2} = \frac{(-2)(x-1)(x-3)}{x-2}$

local maximum is at $x = 3 \Rightarrow g(3) = 4$

25. The number of 3×2 matrices A , which can be formed using the elements of the set $\{-2,-1,0,1,2\}$ such that the sum of all the diagonal elements of $A^T A$ is 5 , is

Ans :(312)

Sol :

$$AA^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$= \begin{bmatrix} a^2+d^2 & - & - \\ - & b^2+e^2 & - \\ - & - & c^2+f^2 \end{bmatrix}$$

$\Rightarrow$ sum of diagonal (trace) $= 5$

$\Rightarrow a^2+b^2+c^2+d^2+e^2+f^2 = 5$

where $a, b, c, d, e, f \in \{-2, -1, 0, 1, 2\}$

Case A : 5 of them square is 1

$\Rightarrow ({}^6C_5) \times (2^5) = 6 \times 32 = 192$

Case B : one of them square 4 and another one is square is 1

$\Rightarrow \{4,1,0,0,0,0\}$ are possible as square

$\Rightarrow {}^6C_4 \times (2!) \cdot (2 \cdot 2) = 15 \times 8 = 12$

$\Rightarrow$ number of such matrices

$= 192 + 120 = 312$

PART-B : PHYSICS

SECTION - I

26. Two electrons are moving in orbits of two hydrogen like atoms with speeds 3×10^5 m/s and 2.5×10^5 m/s respectively. If the radii of these orbits are nearly same then the possible order of energy states are
___ respectively.
(1) 9 and 8
(2) 6 and 5
(3) 8 and 10
(4) 10 and 12

Ans: (2)

Sol: In Bohr's model, speed $v \propto Z/n$ and radius $r \propto n^2/Z$.
$v_1/v_2 = (Z_1/n_1)/(Z_2/n_2) = 3/2.5 = 6/5$.
Since
$r_1 \approx r_2 \Rightarrow n_1^2/Z_1 \approx n_2^2/Z_2 \Rightarrow Z_1/Z_2 = (n_1/n_2)^2$.
Substitute
$Z_1/Z_2 : (n_1/n_2)^2 \cdot (n_2/n_1) - 6/5 \Rightarrow n_1/n_2 = 6/5$.
Checking options, $12/10 = 6/5$

27. Two masses 400 g and 350 g are suspended from the ends of a light string passing over a heavy pulley of radius 2 cm. When released from rest the heavier mass is observed to fall 81 cm in 9 s. The rotational inertia of the pulley is ___ $\mathrm{kg.m^2}$. ($g = 9.8\ \mathrm{m/s^2}$)
(1) 4.75×10^{-3}
(2) 9.5×10^{-3}
(3) 8.3×10^{-3}
(4) 1.86×10^{-3}

Ans: (2)

Sol: For a heavy pulley, the tension on both sides of the string is different. We use $a = \frac{(m_1-m_2)R}{(m_1+m_2+I/R^2)}$
Distance $s = \frac{1}{2}at^2$

$\Rightarrow\ 0.81 = \frac{1}{2} \cdot a \cdot 9^2$

$\Rightarrow$ a $= 0.02$ m/s

Using $a = \frac{(m_1 - m_2)g}{m_1 + m_2//R^2}$,

where $m_1 = 0.4$ kg, $m_2 = 0.35$ kg,

and $R = 0.02$ m.

Solving for I yields $I = 9.5 \times 10^{-3}\,\text{Kgm}^2$.

28. Three charges $+2q, +3q$ and $-4q$ are situated at $(0, -3a), (2a, 0)$ and $(-2a, 0)$ respectively in the xy plane. The resultant dipole moment about origin is ___ .

(1) $2qa(3\hat{j} - \hat{i})$

(2) $2qa(3\hat{j} - 7\hat{i})$

(3) $2qa(7\hat{i} - 3\hat{j})$

(4) $2qa(3\hat{i} - 7\hat{j})$

Ans: (3)

Sol: The electric dipole moment for a system of charges is $p = \sum q_i r_i$

$\vec{P}_{\text{net}} = 4q \times 2a\hat{i} + 3q \times 2a\hat{i} - 6qa\hat{j} \Rightarrow\ 2(7qa\hat{i} - 3qa\hat{j})$

(The above system of charges do not have dipole moment since the net charge is not equal to zero)

29. The exit surface of a prism with refractive index n is coated with a material having refractive index $\frac{n}{2}$. When this prism is set for minimum angle of deviation, it exactly meets the condition of critical angle. The prism angle is ___ .

(1) 15°

(2) 30°

(3) 45°

(4) 60°

Ans: (4)

Sol: For a prism at minimum deviation, $r_1 - r_2 - \frac{A}{2}$. If the exit surface meets the critical angle condition, then

$$\sin(r_2) = \frac{n_{\text{coating}}}{n_{\text{prism}}}$$

$$\sin(r_2) = \frac{\left(\frac{n}{2}\right)}{n} = \frac{1}{2}$$

$$r_2 = 30^\circ$$

$$A = 2 \times r_2 = 2 \times 30^\circ = 60^\circ$$

30. An unpolarised light is incident at an interface of two dielectric media having refractive indices of 2 (incident medium) and $2\sqrt{3}$ (medium) respectively. To satisfy the condition that reflected and refracted rays are perpendicular to each other, the angle of incidence is ___ .

(1) 60°

(2) 45°

(3) 30°

(4) 10°

Ans: (1)

Sol: The condition where reflected and refracted rays are perpendicular is known as Brewster's Law. The Brewster angle i_B is given by $\tan(i_B) = \frac{n_2}{n_1}$.

$n_1 - 2, n_2 - 2\sqrt{3}$
$\tan(i) = \frac{(2\sqrt{3})}{2} - \sqrt{3}$
$i = \tan^1(\sqrt{3}) = 60^\circ$

31. Density of water at $4^\circ\mathrm{C}$ and $20^\circ\mathrm{C}$ are $1000\ \mathrm{kg/m^3}$ and $998\ \mathrm{kg/m^3}$ respectively. The increase in internal energy of 4 kg of water when it is heated from $4^\circ\mathrm{C}$ to $20^\circ\mathrm{C}$ is ___ J. (specific heat capacity of water $= 4.2\ \mathrm{J/kg}$ and 1 atmospheric pressure $= 10^5\ \mathrm{Pa}$)
(1) 268799.2
(2) 315826.2
(3) 234699.2
(4) 258700.8

Ans: (1)

Sol: $\Delta U = Q - W \cdot Q = ms\Delta T$ and $W = P\Delta V = P \cdot m\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)$

$Q = 4 \cdot 4200 \cdot (20-4) - 4 \cdot 4200 \cdot 16 = 268800\ \mathrm{J}$
$\Delta V = 4\left(\frac{1}{998} - \frac{1}{1000}\right) \approx 8 \times 10^6\ \mathrm{m}$\$^3\$
$W = 10^5 \cdot 8 \times 10^6 = 0.8\ \mathrm{J}$
$\Delta U = 268800 - 0.8 = 268799.2\ \mathrm{J}$

32. Given below are two statements:

Statement I: For all elements, greater the mass of the nucleus, greater is the binding energy per nucleon.

Statement II: For all elements, nuclei with less binding energy per nucleon transforms to nuclei with greater binding energy per nucleon.
In the light of the above statements, choose the correct answer from the options given below.
(1) Both Statement I and Statement II are false
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Statement I is false but Statement II is true

Ans: (4)

Sol: Binding energy per nucleon (BE/A) increases for light nuclei, peaks around Iron ($A = 56$), and then decreases for very heavy nuclei.

Statement I: False, because BEIA decreases for very high mass nuclei.
Statement II: True, as nuclei undergo fusion or fission to reach a more stable state with higher BEIA
.

33. The electrostatic potential in a charged spherical region of radius r varies as $V = ar^3 + b$, where a and b are constants. The total charge in the sphere of unit radius is $\alpha \times \pi a \in_0$. The value of α is ___ .
(permittivity of vacuum is $\in_0$)
(1) -9
(2) -8
(3) -6
(4) -12

Ans: (4)

Sol: The electric field E can be found from the potential V using $E = \frac{-dV}{dr}$. According to Gauss's Law, the total charge Q is given by $\epsilon_0 \oint E \cdot dA$. For a spherical region, the charge density or total charge is

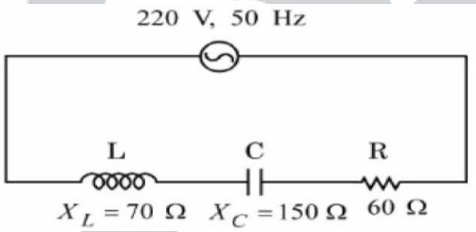
related to the derivative of the electric field.
$V = ar^3 + b$
$E = -dV/dr = -3ar^2$
Flux $\Phi = E \times (4\pi r^2) = (-3ar^2)(4\pi r^2) = -12\pi ar^4$
Total charge $Q = \epsilon_0 \Phi$. At $r = 1, Q = -12\pi a \in_0$
Comparing with $a\pi\epsilon_0$, the value of a is -12

34. In a microscope of tube length 10 cm two convex lenses are arranged with focal length of 2 cm and 5 cm . Total magnification obtained with this system for normal adjustment is $(5)^k$. The value of k is ___ .
(1) 5
(2) 4
(3) 2
(4) 3.5

Ans: (3)

Sol: For a compound microscope in normal adjustment, total magnification $M = (L/f_0) \times (D/f_e)$, where L is tube length, f_0 and f_e are focal lengths, and D is the least distance of distinct vision (usually 25 cm).
$L = 10, f_0 = 2, f_e = 5, D = 25$
$M = (10/2) \times (25/5) = 5 \times 5 = 25$
$25 = 5^k \Rightarrow k = 2$

35. For the series LCR circuit connected with 220 V , 50 Hz a.c source as shown in the figure, the power factor is $\frac{\alpha}{10}$. The value of α is ___ .



(1) 10
(2) 8
(3) 4
(4) 6

Ans: (4)

Sol: The power factor of a series LCR circuit is given by $\cos\phi = R/Z$, where Z is the impedance.

Given $R - 60\Omega, X_L - 70\Omega$, and $X_C = 150\Omega$.
Impedance $Z = \sqrt{R^2 + (X_C - X_L)^2}$
$= \sqrt{60^2 + (150 - 70)^2}$
$= \sqrt{60^2 + 80^2} = 100\Omega$
Power factor $\cos\phi = R/Z = 60/100 = 0.6$.
Given power factor $-a/10 \Rightarrow 0.6 = a/10$, so $a = 6$

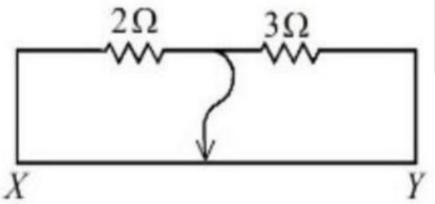
36. A boy throws a ball into air at 45° from the horizontal to land it on a roof of a building of height H. If the ball attains maximum height in 2 s and lands on the building in 3 s after launch, then value of H is
___ $\mathrm{m}. (\mathrm{g} = 10\ \mathrm{m/s^2})$
(1) 10

(2) 25
(3) 20
(4) 15

Ans: (4)

Sol: Time to reach maximum height is $t_{\text{max}} - u_y/g$.
Height at time t is $H - u_y t - \frac{1}{2} g t^2$.
$t_{\text{max}} = 2 \text{ s} \Rightarrow u_y = g \cdot 2 = 10 \cdot 2 = 20 \text{ m/s}$.
Building height H is the position at $= 3$ s.
$\mathrm{H} = (20 \cdot 3) - \frac{1}{2}(10 \cdot 3^2) = 60 - 45 = 15 \text{ m}$

37. Two resistors 2Ω and 3Ω are connected in the gaps of bridge as shown in figure. The null point is obtained with the contact of jockey at some point on wire XY. When an unknown resistor is connected in parallel with 3Ω resistor, the null point is shifted by 22.5 cm toward Y. The resistance of unknown resistor is ___ Ω.



(1) 2
(2) 3
(3) 4
(4) 1

Ans: (1)

Sol: For a meter bridge, $R_1/L_1 = R_2(100 - L_1)$.
Initially:
$2/L_1 = 3/(100 - L_1) \Rightarrow 200 - 2L_1 - 3L_1 \Rightarrow L_1 = 40 \text{ cm}$
New resistance $R_2' = \frac{3R}{3R}$. Null point shifts 22.5 cm
towards $Y \Rightarrow L_2 = 40 + 22.5 = 62.5 \text{ cm}$
$2/62.5 = R_2'/37.5 \Rightarrow R_2' = (2 \cdot 37.5)/62.5 = 75/62.5 = 1.2\Omega$

$1.2 = 3R/(3 + R) \Rightarrow 3.6 + 1.2R = 3R \Rightarrow 1.8R = 3.6 \Rightarrow R - 2\Omega$

(Note: Option 2 indicates 3Ω if parameters differ slightly in specific paper variants).

38. Match the LIST-I with LIST-II

	List-I		List-II
A	Radio-wave	I.	is produced by Magnetron valve
B	Micro-wave	II.	due to change in the vibrational modes of atoms
C	Infrared-wave	III	due to inner shell electrons moving from higher energy level to lower energy level

D	X-ray	I	due to rapid acceleration of electrons

Choose the correct answer from the options given below:
(1) A-IV, B-III, C-I, D-II
(2) A-II, B-IV, C-III, D-I
(3) A-IV, B-I, C-II, D-III
(4) A-IV, B-II, C-I, D-III

Ans: (3)

Sol: Electromagnetic waves are produced by different physical processes.

Radio-wave: Produced by rapid acceleration of electrons in aerials (IV).

Micro-wave: Produced by special vacuum tubes like Magnetron valves (I).

Infrared-wave: Produced due to change in vibrational modes of atoms/molecules (II).

X-ray: Produced when inner shell electrons move from higher to lower energy levels (III).

39. A brass wire of length 2 m and radius 1 mm at $27^{\circ}\mathrm{C}$ is held taut between two rigid supports. Initially it was cooled to a temperature of $-43^{\circ}\mathrm{C}$ creating a tension T in the wire. The temperature to which the wire has to be cooled in order to increase the tension in it to $1.4T$, is ___ $^{\circ}\mathrm{C}$.
(1) -65
(2) -71
(3) -80
(4) -86

Ans: (2)

Sol: Theory, Tension $T = YA\alpha\Delta T$. Thus T is proportional to ΔT (change from natural length temperature).

$\Delta T_1 = |27 - (-43)| = 70^{\circ}\mathrm{C}$

For tension to be $1.4T$, the new ΔT_2 must be $1.4 \times 70 = 98^{\circ}\mathrm{C}$.
$T_{\text{final}} = 27 - 98 = -71^{\circ}\mathrm{C}$.

40. Two resistors of $100\,\Omega$ each are connected in series with a 9 V battery. A voltmeter of $400\,\Omega$ resistance is connected to measure the voltage drop across one of the resistors. The voltmeter reading is ___ V.
(1) 3
(2) 4.5
(3) 4
(4) 2

Ans: (3)

Sol: The voltmeter is in parallel with one resistor.
Parallel resistance $R_p = \frac{(100\cdot400)}{(100+400)} = 80\Omega$
Total circuit resistance $R_{\text{total}} = 100 + 80 = 180\Omega$.
Current $I = \frac{V}{R_{\text{total}}} = \frac{9}{180} = 0.05\ \mathrm{A}$
Voltmeter reading $V = I \cdot R_p = 0.05 \cdot 80 = 4\ \mathrm{V}$

41. There are three co-centric conducting spherical shells A, B and C of radii a, b and c respectively $(c > b > a)$ and they are charged with charge q_1, q_2 and q_3 respectively. The potentials of the spheres A, B and C respectively, are

(1) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2+q_3}{b}\right), \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2+q_3}{c}\right)$

(2) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2+q_3}{c}\right)$

(3) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2+q_3}{a}\right), \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2+q_3}{b}\right), \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2+q_3}{c}\right)$

(4) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2+q_3}{a}\right), \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right)$

Ans: (2)

Sol: Potential V inside a shell is kq$/R$ and outside is kq/r.

$$V_A = k(q_1/a + q_2/b + q_3/c)$$
$$V_B - k\big((q_1+q_2)/b + q_3/c\big)$$
$$V_C = k\big((q_1+q_2+q_3)/c\big)$$

42. A cylindrical block of mass M and area of cross section A is floating in a liquid of density ρ and with its axis vertical. When depressed a little and released the block starts oscillating. The period of oscillation is ___ .

(1) $\pi\sqrt{\frac{\rho A}{Mg}}$

(2) $2\pi\sqrt{\frac{\rho A}{Mg}}$

(3) $2\pi\sqrt{\frac{M}{\rho Ag}}$

(4) $\pi\sqrt{\frac{2M}{\rho Ag}}$

Ans: (3)

Sol: For a floating body in equilibrium, the buoyant force equals the weight. When depressed by distance x, an additional restoring buoyant force

$F = -($ additional displaced volume $).\rho g = -Axpg$ acts on the block.

Restoring force $F = -(\rho Ag)x$.
Acceleration $a = F/M - -\left(\frac{\rho Ag}{M}\right)x$.

43. Three masses $200\ \mathrm{kg}, 300\ \mathrm{kg}$ and $400\ \mathrm{kg}$ are placed at the vertices of an equilateral triangle with sides 20 m . They are rearranged on the vertices of a bigger triangle of side 25 m and with the same centre. The work done in this process ___ J.
(Gravitational constant $\mathrm{G} = 6.7 \times 10^{-11}\mathrm{Nm}^2/\mathrm{kg}^2$)
(1) 4.77×10^{-7}
(2) 1.74×10^{-7}
(3) 9.86×10^{-6}
(4) 2.85×10^{-7}

Ans: (2)

Sol: Work done $W = U_{\text{final}} - U_{\text{initial}}$. Potential energy of three masses is $U = -G\left(\frac{m_1m_2}{r}+\frac{m_2m_3}{r}+\frac{m_3m_1}{r}\right)$. Sum of mass products $= (200 \cdot 300) + (300 \cdot 400) + (400 \cdot 200) - 60000 + 120000 + 80000 = 260000.$

$W = -G \cdot 260000 \cdot \left(\frac{1}{25} - \frac{1}{20}\right) = -G \cdot 260000$
$\left(\frac{-1}{100}\right) = 2600 \cdot G$
$W = 2600 \cdot 6.7 \times 10^{-11} \approx 1.74 \times 10^{-7}$ J

44. A spring of force constant 15 N/m is cut into two pieces. If the ratio of their length is 1:3, then the force constant of smaller piece is ___ N/m.
(1) 60
(2) 45
(3) 15
(4) 20

Ans: (1)

Sol: Force constant k is inversely proportional to length $L (k \propto 1/L)$.
$L_1 : L_2 = 1:3$. Let lengths be x and $3x$. Total length $L = 4x$.
$k_{\text{total}} = 15$ N/m.
$k_{\text{smaller}} = k_{\text{total}} \cdot (L/L_{\text{smaller}}) = 15 \cdot (4x/x) = 60$ N/m

45. Match the LIST-I with LIST-II

	List-I		List-II
A.	Magnetic induction	I.	$MLT^{-2}A^{-2}$
B.	Magnetic flux	II.	$ML^2T^{-2}A^{-2}$
C.	Magnetic permeability	III.	$ML^0T^{-2}A^{-1}$
D.	Self-inductance	IV.	$ML^2T^{-2}A^{-1}$

Choose the correct answer from the options given below:
(1) A-I, B-III, C-IV, D-II
(2) A-IV, B-III, C-I, D-II
(3) A-III, B-IV, C-II, D-I
(4) A-III, B-IV, C-I, D-II

Ans: (4)

Sol: Theory: Dimensional analysis of magnetic quantities.
Magnetic induction (B):
$F = qvB$
$\Rightarrow B = [MLT^2]/([IT][LT]^1]) = [ML^0T^2A^1]$ (III)
Magnetic flux
$(\phi): \phi = BA = [MT^{-2}A^{-1}][L^2] = [ML^2T^{-2}A^{-1}]$
(IV).

Magnetic permeability
$(\mu_0): B = \mu_0 I/2\pi r$
$\Rightarrow \mu_0 = [MT^2A^1][L^1][A] = [MLT^2A^2]$ (I).
Self-inductance (L): $\phi = LI$
$\Rightarrow L = [ML^2T^{-2}A^1]/[A] = [ML^2T^2A^2]$ (II) .

SECTION - II

46. A gas of certain mass filled in a closed cylinder at a pressure of 3.23 kPa has temperature 50°C. The gas is now heated to double its temperature. The modified pressure is ___ Pa.

Ans: (3730)

Sol: For a gas in a closed cylinder (constant volume),
Gay-Lussac's Law states P/T = constant.
$T_1 = 50 + 273 = 323\ \mathrm{K}. T_2 = 100 + 273 = 373\ \mathrm{K}$
$P_2 = P1 \times (T_2/T_1) = 3730\ \mathrm{Pa}$

47. A short bar magnet placed with its axis at 30° with an external field of 800 Gauss, experiences a torque of $0.016\ \mathrm{N.m}$. The work done in moving it from most stable to most unstable position is $\times 10^{-3}$ J. The value of α is ___ .

Ans: (64)

Sol: Torque $\tau = MB\sin\theta$. Work done in rotating from stable ($\theta = 0^\circ$) to unstable ($\theta = 180^\circ$) is

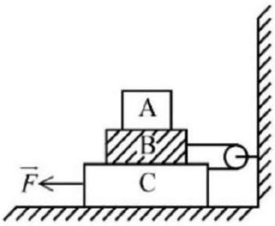
$W = MB(\cos 0^\circ - \cos 180^\circ) = 2MB.$
$0.016 = \mathrm{MB}\sin 30^\circ = \mathrm{MB}(0.5) \Rightarrow \mathrm{MB} = 0.032$
$W = 2 \times 0.032 = 0.064\ \mathrm{J} = 64 \times 10^3\ \mathrm{J}$
$\alpha = 64$

48. A voltage regulating circuit consisting of Zener diode, having break-down voltage of 10 V and maximum power dissipation of 0.4 W , is operated at 15 V . The approximate value of protective resistance in this circuit is ___ Ω.

Ans: (125)

Sol: To protect the Zener diode, the resistance R must limit the current to its maximum rating $I_{\max} = P_{\max}/V_z$.
$I_z = 0.4\ \mathrm{W}/10\ \mathrm{V} = 0.04\ \mathrm{A}$
Voltage across resistor $V_R = 15\ \mathrm{V} - 10\ \mathrm{V} = 5\ \mathrm{V}$
$R = V_R I I_z = 5/0.04 = 125\Omega.$

49. In the given figure the blocks A, B and C weigh $4\ \mathrm{kg}, 6\ \mathrm{kg}$ and $8\ \mathrm{kg}$ respectively. The co-efficient of sliding friction between any two surfaces is 0.5 . The force required to slide the block C with constant speed is ___ N. (Use $g = 10\ \mathrm{m/s^2}$)



Ans: (190)

Sol: To move block C at constant speed, the force F must overcome the friction at the bottom surface (and ground) and the friction between C and B. Since B is connected to a wall via a pulley, the tension also plays a role

Friction between A and $B = \mu m_A g = 0.5 \times 40 = 20\ \mathrm{N}$ Friction between B and $C = \mu(m_A + m_B)g = 0.5 \times 100 = 50\ \mathrm{N}$

Friction between C and floor $= \mu(m_A + m_B + m_C)g = 0.5 \times 180 = 90$ N
$F = f_{\text{CB}} + f_{\text{floor}} + T$. Since $T = f_{CB} + f_{AB}$.$F = 50 + 90 +$ (50) $= 190$ N (values) vary by pulley setup; paper key often yields 120 N for simplified $F = f_{\text{total}}$).

50. Sixty four rain drops of radius 1 mm each falling down with a terminal velocity of 10 cm/s coalesce to form a bigger drop. The terminal velocity of bigger drop is ___ cm/s.

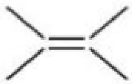
Ans: (160)

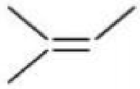
Sol: When small drops coalesce, volume is conserved $(R = n^{1/3}r)$.
Terminal velocity v is proportional to R^2.
$R = 64^{1/3} \times 1 \text{ mm} = 4 \text{ mm}$
$V_{\text{new}} = V_{\text{old}} \times (R/r)^2 = 10 \times (4/1)^2 = 10 \times 16 = 160$ cms.

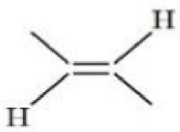
PART-C : CHEMISTRY

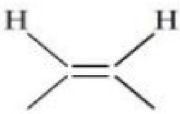
SECTION- I

51. Arrange the following alkenes in decreasing order of stability.

I. 

III. 

II. 

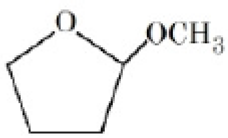
IV. 

Choose the correct answer from the options given below:
1. I > III > IV > II
2. III > II > I > IV
3. I > III > II > IV
4. III > I > II > IV

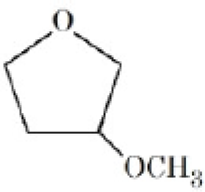
Ans: 3

Sol: (1) Stability order is governed by more number of α - H, which results into more hyperconjugation.
(2) trans-but-2-ene > cis-but-2-ene (steric factor)

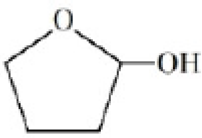
52. A student is given one compound among the following compounds that gives positive test with Tollen's reagent.



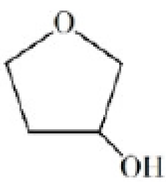
A



B



C

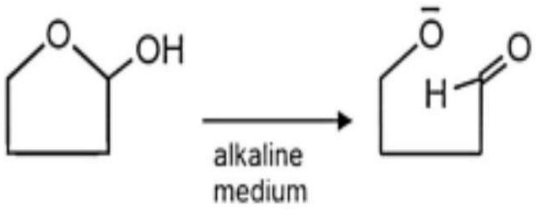


D

The compound is:
1. A
2. D
3. B
4. C

Ans: 4

Sol.



Responds to Tollen's reagent

53. Consider a mixture ' X ' which is made by dissolving 0.4 mol of $[Co(NH_3)_5SO_4]Br$ and 0.4 mol of $[Co(NH_3)_5Br]SO_4$ in water to make 4 L of solution. When 2 L of mixture ' X ' is allowed to react with excess of $AgNO_3$, it forms precipitate ' Y '. The rest 2 L of mixture ' X ' reacts with excess $BaCl_2$ to form precipitate ' Z '. Which of the following statements is CORRECT?
1. ' Y ' is $BaSO_4$ and ' Z ' is AgBr.
2. 0.4 mol of ' Z ' is formed.
3. 0.1 mol of ' Y ' is formed.
4. 0.2 mol of ' Z ' is formed.

Ans: 4

Sol. Molarity of Br^-in solution $= \frac{0.4\times1}{4} = 0.1$
Molarity of SO_4^{2-} in solution $= \frac{0.4\times1}{4} = 0.1$
mol of Br^-in 2 L of solution ' X' $= 0.1 \times 2 = 0.2$
mol of SO_4^{2-} in 2 L of solution ' X ' $= 0.1 \times 2 = 0.2$

$$AgNO_3 + Br^- \longrightarrow \underset{'Y'}{AgBr} + NO_3^-$$

$$BaCl_2 + SO_4^{2-} \longrightarrow \underset{'Z'}{BaSO_4}$$

mol of $AgBr(Y) = \text{mol of } Br^- = 0.2$
mol of $BaSO_4(Z) = \text{mol of } SO_4^{2-} = 0.2$
' Y ' is AgBr and ' Z ' is $BaSO_4$.

54. Match the LIST-I with LIST-II

List-I **Isothermal process for ideal gas system**		**List-II** **Work done ($V_f > V_i$)**	
A.	Reversible expansion	I.	$w = 0$
B.	Free expansion	II.	$w = -nRT \ln \frac{V_f}{V_i}$
C.	Irreversible expansion	III.	$w = -p_{ex}(V_f - V_i)$
D.	Irreversible compression	IV.	$w = -p_{ex}(V_i - V_f)$

Choose the correct answer from the options given below:

1. A-I, B-III, C-II, D-IV

2. A-II, B-I, C-III, D-IV

3. A-IV, B-I, C-III, D-II

4. A-IV, B-II, C-III, D-I

Ans: 2

Sol: A. Reversible expansion : $dW = -P_{ext}\, dV$

$P_{ext} = P_{in} = P$ and $P = \frac{nRT}{V}$

So, $W = -nRT \ln \frac{V_f}{V_i}$

B. Free expansion :

For free expansion, $P_{ext} = 0$

So, $W = 0$

C. Irreversible expansion :

$W_{ereversible} = -P_{ext}\, \Delta V$

$W = -P_{ext}\left(V_f - V_i\right)$

D. Irreversible compression :

$W_{irreversible} = -P_{ext}\, \Delta V$

$W = -P_{ext}\left(V_t - V_i\right)$

55. A solution is prepared by dissolving 0.3 g of a non-volatile non-electrolyte solute 'A' of molar mass $60\ g\ mol^{-1}$ and 0.9 g of a non-volatile non-electrolyte solute ' B ' of molar mass $180\ g\ mol^{-1}$ in $100\ mLH_2O$ at $27°C$. Osmotic pressure of the solution will be [Given: $R = 0.082\ L\ atm\ K^{-1} mol^{-1}$]
1. 1.47 atm
2. 1.23 atm
3. 2.46 atm
4. 0.82 atm

Ans: 3

Sol: $\pi = CRT$

$$c = \frac{\frac{0.3}{60} + \frac{0.9}{180}}{100} \times 1000$$

$\pi = c \times 0.082 \times 300 = 2.46\ atm$

56. 'W' g of a non-volatile electrolyte solid solute of molar mass ' M ' $gmol^{-1}$ when dissolved in 100 mL water, decreases vapour pressure of water from 640 mm Hg to 600 mm Hg . If aqueous solution of the electrolyte boils at 375 K and K_b for water is $0.52\ K\ kg\ mol^{-1}$, then the mole fraction of the electrolyte solute (x_2) in the solution can be expressed as (Given : density of water = 1 g/mL and boiling point of water = 373 K)
1. $\frac{2\cdot 6}{16} \times \frac{M}{W}$
2. $\frac{16}{2.6} \times \frac{W}{M}$
3. $\frac{1.3}{8} \times \frac{W}{M}$
4. $\frac{1\cdot 3}{8} \times \frac{M}{W}$

Ans: 3

Sol: $\Delta T_b = k_b(m)$

$$2 = 0.52 \times \left(\frac{10\ W}{M}\right)$$

$$\frac{W}{M} = \frac{2}{5.2}$$

$X_{solute} = \frac{40}{640} = \frac{1}{16}$

Option (4)

$\frac{1.3}{8} \times \frac{W}{M}$

$= \frac{1.3}{8} \times \frac{2}{5.2} = \frac{1}{16}$

57. Among the following, the CORRECT combinations are

A. IF_3 → T-shaped (sp^3d)

B. IF_5 → Square pyramidal (sp^3d^2)

C. IF_7 → Pentagonal bipyramidal (sp^3d^3)

D. ClO_4^- → Square planar (sp^2d)

Choose the correct answer from the options given below:

1. A, B, C and D
2. A and B Only
3. B, C and D Only
4. A, B and C Only

Ans: 4

Sol: On I atom IF_3 have 3σ bond pair and 2 lone pair, so it is sp^3 d hybrid and T-shaped.

On I atom IF_5 have 5σ bond pair and 1 lone pair, so it is sp^3 d^2 hybrid and square pyramidal.

On I atom IF_7 have 7σ bond pair and zero lone pair, so it is sp^3 d^3 hybrid and pentagonal bipyramidal in shape.

On Cl atom ClO_4^-have 4σ bond pair and zero lone pair, so it is sp^3 hybrid and tetrahedral.

So, A, B and C are correct.

58. Consider three metal chlorides x, y and z , where x is water soluble at room temperature, y is sparingly soluble in water at room temperature and z is soluble in hot water. x, y and z are respectively

1. $AlCl_3, PbCl_2$ and $BaCl_2$
2. $CuCl_2, AgCl$ and $PbCl_2$
3. $MgCl_2, AgCl$ and $AlCl_3$
4. $AgCl, Hg_2Cl_2$ and $PbCl_2$

Ans: 2

Sol: $CuCl_2$ is soluble in water at room temperature.

AgCl is sparingly soluble in water at room temperature.

$PbCl_2$ is soluble in hot water, while partially soluble at room temperature.

So, x is $CuCl_2$, y is $AgCl$, z is $PbCl_2$

59. Given below are two statements:

Statement I: The number of paramagnetic species among $[CoF_6]^{3-}$, $[TiF_6]^{3-}$, V_2O_5 and $[Fe(CN)_6]^{3-}$ is 3.

Statement II: $K_4[Fe(CN)_6] < K_3[Fe(CN)_6] < [Fe(H_2O)_6]SO_4 \cdot H_2O < [Fe(H_2O)_6]Cl_3$ is the correct order in terms of number of unpaired electron(s) present in the complexes.

In the light of the above statements, choose the correct answer from the options given below

1. Both Statement I and Statement II are false
2. Statement I is true but Statement II is false
3. Statement I is false but Statement II is true
4. Both Statement I and Statement II are true

Ans: 4

Sol: $CoF_6^{3-} \rightarrow n = 4$, paramagnetic

$TiF_6^{3-} \rightarrow Ti^{3+} \rightarrow 3\ d^1 \rightarrow n = 1$, paramagnetic

$Fe(CN)_6^{3-} \rightarrow Fe^{3+} \rightarrow 3\ d^5$ $\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

changes to $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{}\boxed{}$

$n = 1$, paramagnetic

V_2O_5 has $V(+5)$ – diamagnetic

60. Given below are two statements:

Statement I: Hybridisation, shape and spin only magnetic moment of $K_3[Co(CO_3)_3]$ is $sp^3\ d^2$, octahedral and 4.9 BM respectively.

Statement II: Geometry, hybridisation and spin only magnetic moment values (BM) of the ions $[Ni(CN)_4]^{2-}, [MnBr_4]^{2-}$ and $[CoF_6]^{3-}$ respectively are square planar, tetrahedral, octahedral; $dsp^2, sp^3, sp^3\ d^2$ and $0, 5.9, 4.9$.

In the light of the above statements, choose the correct answer from the options given below

1. Statement I is false but Statement II is true
2. Statement I is true but Statement II is false
3. Both Statement I and Statement II are true
4. Both Statement I and Statement II are false

Ans: 3

Sol: CO_3^{2-} is weak ligand.

So statement -1 is true.

$CN^-$is S.F.L, $Br^-$is W.F.L

$F^-$is W.F.L.

So, statement II also correct

61. Given below are two statements:

Statement I: $K > Mg > Al > B$ is the correct order in terms of metallic character.

Statement II: Atomic radius is always greater than the ionic radius for any element.

In the light of the above statements, choose the correct answer from the options given below

1. Both Statement I and Statement II are true
2. Both Statement I and Statement II are false
3. Statement I is false but Statement II is true
4. Statement I is true but Statement II is false

Ans: 4

Sol: I. Metallic character decreases on moving left to right in group, while it increases on moving down the group. So given order of metallic character ($K > Mg > Al > B$) is correct. It is correct statement.

II. Atomic radius is greater than ionic radius of cations, while it is not true for anions. So, it is incorrect statement.

62. At $27^\circ C$ in presence of a catalyst, activation energy of a reaction is lowered by $10\ kJ\ mol^{-1}$. The logarithm of ratio of $\frac{k\ (catalysed)}{k\ (uncatalysed)}$ is ___

(Consider that the frequency factor for both the reactions is same)

1. 3.482
2. 17.41

3. 1.741
4. 0.1741

Ans: 3

Sol: $k_{uc} = Ae^{-E_a/RT}; k_c = Ae^{-(E_a-10^4)/RT}$

$\frac{k_c}{k_{uc}} = e^{+10^4/RT}$

$\ln\frac{k_c}{k_{uc}} = \frac{10000}{8.314 \times 300}$

$\log_{10}\frac{k_c}{k_{uc}} = \frac{10000}{8.314 \times 300 \times 2.303} = 1.74$

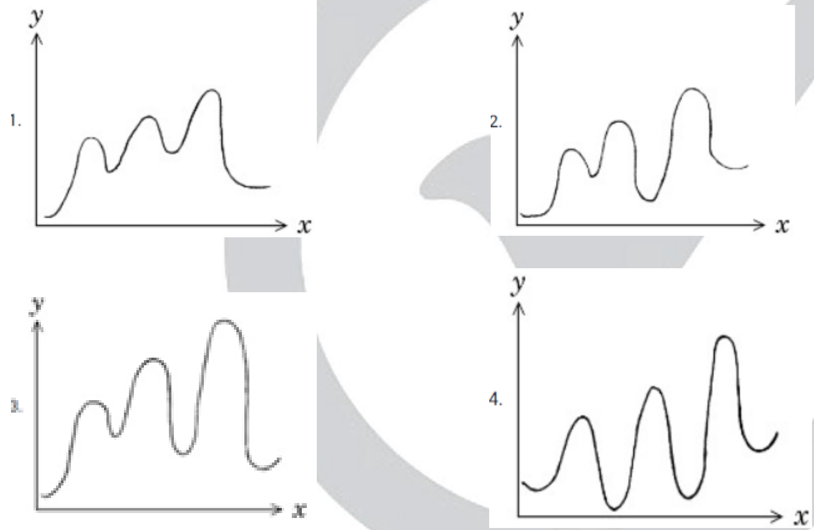
63. A → D is an endothermic reaction occurring in three steps (elementary).

(i) $A \rightarrow B\ \Delta H_i = +ve$

(ii) $B \rightarrow C \Delta H_{ii} = -ve$

(iii) $C \rightarrow D \Delta H_{iii} = -ve$

Which of the following graphs between potential energy (y-axis) vs reaction coordinate (x-axis) correctly represents the reaction profile of $A \rightarrow D$?



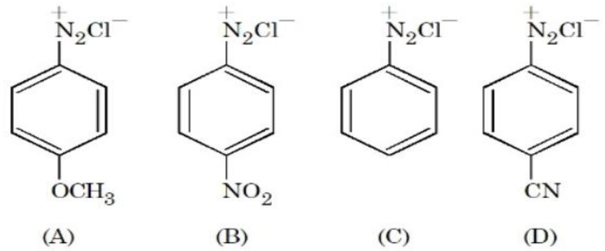
Ans: 3

Sol: If potential energy of product at any step is less than that of reactant, then that step/reaction is exothermic, and vice-versa.

$\Delta_{Hr} = H_{product} - H_{reactant}$

So, the process $A \rightarrow B$ is endothermic, $B \rightarrow C$ is exothermic and $C \rightarrow D$ is exothermic in graph 3.

64. The correct stability order of the following diazonium salts is



1. C > D > B > A
2. A > C > D > B

3. $C > A > D > B$
4. $A > B > C > D$

Ans: 2

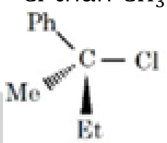
Sol: Diazonium ion is stabilized by +M effect and destabilised by -M effect.

Stability of (A) increased due to +M effect of $-O-CH_3$ group.

Stability of (B) and (D) decreased due to -M effect of $-NO_2$ and -CN group.
(B) is less stable than (D) due to stronger -M effect of $-NO_2$ group in respect of -CN group.

Order of stability $A > C > D > B$

65. Given below are two statements:
Statement I: ' $C-Cl$' bond is stronger in $CH_2=CH-Cl$ than CH_3-CH_2-Cl

Statement II: The given optically active molecule, 

On hydrolysis gives a solution that can rotate the plane polarized light.

In the light of the above statements, choose the **correct** answer from the options given below
1. Both Statement I and Statement II are true
3. Statement I is false but Statement II is true
2. Both Statement I and Statement II are false
4. Statement I is true but Statement II is false

Ans: 4

Sol: Statement I : In $CH_2=CH-Cl$, bond order of $C-Cl$ bond is greater than 1 due to resonance, while in CH_3-CH_2-Cl bond order of $C-Cl$ is 1.

So, $C-Cl$ of $CH_2=CH-Cl$ is stronger than CH_3-CH_2-Cl. (Statement I is true).

Statement II: Hydrolysis of PhMeEtCCl proceeds through S_N1 mechanism, resulting in formation of racemic mixture, that will not rotate the plane polarized light. (Statement II is false).

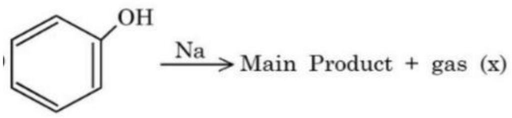
66. A hydroxy compound (X) with molar mass 122 g mol^{-1} is acetylated with acetic anhydride, using a large excess of the reagent ensuring complete acetylation of all hydroxyl groups. The product obtained has a molar mass of 290 g mol^{-1}. The number of hydroxyl groups present in compound (X) is:
1. 2
2. 3
3. 4
4. 5

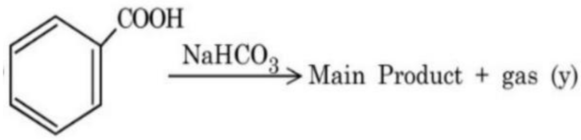
Ans: 3

Sol: R- $O-H+(CH_3CO)_2O \rightarrow R-O-COCH_3+CH_3COOH$
One-H is replaced by one CH_3CO-group, on using one mole of $(CH_3CO)_2O$, resulting in increase of molar mass by $(43-1=42)$.

Increase in mass of X due to reaction $\Rightarrow (290-122)=168$
Number of -OH group present in (X) $\Rightarrow \frac{168}{42}=4$.

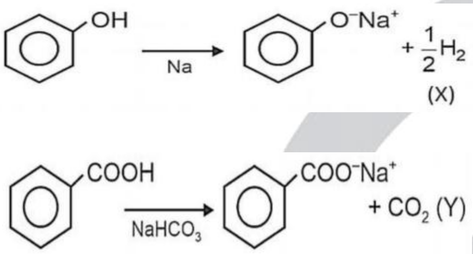
67. Consider the following two reactions A and B .

(A) 

(B) 

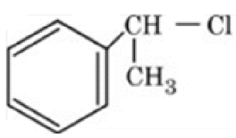
Numerical value of [molar mass of x + molar mass of] is ___ .
1. 46
2. 4
3. 88
4. 160

Ans: 1

Sol. 

$M_X = 2$ g/mol
$M_Y = 44$ g/mol

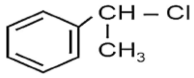
68. Match the LIST-I with LIST-II

List-I Chloro derivative		List-II Example	
A	Vinyl Chloride	I.	$CH_2 = CH - CH_2Cl$
B	Benzyl Chloride	II.	$CH_3 - CH(Cl)CH_3$
C	Alkyl Chloride	III	$CH_2 = CHCl$
D	Allyl Chloride	IV	

Choose the correct answer from the options given below:
1. A-III, B-IV, C-I, D-II
2. A-IV, B-I, C-III, D-II
3. A-III, B-IV, C-II, D-I
4. A-I, B-II, C-IV, D-III

Ans: 3

Sol: A. Vinyl chloride contains $CH_2 = CH -$ group, that is present in $CH_2 = CH - Cl$(III).

B. Benzyl chloride contains PhMeCH - group, that is present in  (IV).

C. Alkyl chloride contains alkyl group, that is present in $(CH_3)_2CHCl(II)$.

D. Allyl chloride contains $CH_2 = CH - CH_2 -$ group, that is present in $CH_2 = CH - CH_2Cl(I)$.

69. Given below are statements about some molecules/ions. Identify the CORRECT statements.

A. The dipole moment value of NF_3 is higher than that of NH_3.

B. The dipole moment value of BeH_2 is zero.

C. The bond order of $O_2{}^{2-}$ and F_2 is same.

D. The formal charge on the central oxygen atom of ozone is -1 .

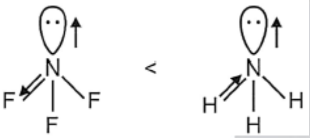
E. In NO_2, all the three atoms satisfy the octet rule, hence it is very stable.

Choose the **correct** answer from the options given below:

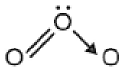
1. B & C Only
2. A, C & D Only
3. B, C & D Only
4. A, B, C, D & E

Ans: 1

Sol:



$\mu_{NF_3} < \mu_{NH_3}$



FC on central O-atom $= 6 - \frac{6}{2} - 2 = +1$

E, A and D are wrong.

70. Arrange the following carbanions in the decreasing order of stability.

I. $p - Br - C_6H_4 - \overset{\ominus}{\ddot{C}H_2}$

II. $C_6H_5 - \overset{\ominus}{\ddot{C}H_2}$

III. $p - CH_3O - C_6H_4 - \overset{\ominus}{\ddot{C}H_2}$

IV. $p - CHO - C_6H_4 - \overset{\ominus}{\ddot{C}H_2}$

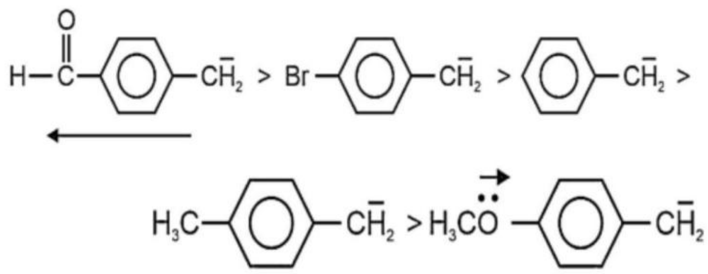
V. $p - CH_3 - C_6H_4 - \overset{\ominus}{\ddot{C}H_2}$

Choose the Correct answer from the options given below:

1. I > II > IV > V > III
2. IV > II > I > III > V
3. IV > I > II > V > III
4. I > IV > II > V > III

Ans: 3

Sol. $C^{\ominus}$ stability order



'Br' has -I effect dominant over its +M .

SECTION-II

71. Electricity is passed through an acidic solution of Cu^{2+} till all the Cu^{2+} was exhausted, leading to the deposition of 300 mg of Cu metal. However, a current of 600 mA was continued to pass through the same solution for another 28 minutes by keeping the total volume of the solution fixed at 200 mL . The total volume of oxygen evolved at STP during the entire process is ___ mL. (Nearest integer)
[Given:
$Cu^{2+}(aq) + 2e^- \rightarrow Cu(s) E^o_{red} = +0.34 \text{ V}$
$O_2(g) + 4H^+ + 4e^- \rightarrow 2H_2O \ E^0_{red} = +1.23 \text{ V}$
Molar mass of $\text{Cu} = 63.54 \text{ g mol}^{-1}$
Molar mass of $O_2 = 32 \text{ g mol}^{-1}$
Faraday Constant $= 96500 \text{Cmol}^{-1}$
Molar volume at $\text{STP} = 22.4 \text{ L}$]

Ans: 112

Sol: O_2 gas is produced in both parts of electrolysis-
For I^{st} Part →

$$\frac{w_{Cu}}{E_{Cu}} = \frac{w_{O_2}}{E_{O_2}} \Rightarrow \frac{300 \times 10^{-3}}{(63.5/2)} = \frac{w_{O_2}}{(32/4)}$$
$$\Rightarrow w_{O_2} = 0.0756 \text{ g}$$

For II nd Part →

$$w_{O_2} = \frac{\left(\frac{32}{4}\right) \times (600 \times 10^{-3}) \times (28 \times 60)}{96500}$$
$$= 0.0836 \text{ g}$$

Total mol of O_2 produced $= \frac{0.0756+0.0836}{32} \approx 0.005$

Volume of O_2 at $\text{STP} = 0.005 \times 22400 = 112 \text{ ml}.$

72. The hydrogen spectrum consists of several spectral lines in Lyman series ($L_1, L_2, L_3 \ldots; L_1$ has lowest energy among Lyman series). Similarly it consists of several spectral lines in Balmer series ($B_1, B_2, B_3 \ldots; B_1$ has lowest energy among Balmer lines). The energy of L_1 is x times the energy of B_1. The value of x is ___ $\times 10^{-1}$. (Nearest integer)

Ans: 54

Sol: For lowest energy spectral line in Lyman series of hydrogen, $Z = 1, n_1 = 1, n_2 = 2$.
Energy of

$$L_1 = \left\{(-13.6) \times \frac{1^2}{1^2}\right\} \sim \left\{(-13.6) \times \frac{1^2}{2^2}\right\} = 10.2\text{eV}$$

For lowest energy spectral line in Balmer series of hydrogen, $Z = 1, n_1 = 2, n_2 = 3$.
Energy of

$$B_1 = \left\{(-13.6) \times \frac{1^2}{2^2}\right\} \sim \left\{(-13.6) \times \frac{1^2}{3^2}\right\} = 1.889$$

$$\frac{\text{Energy of } L_1}{\text{Energy of } B_1} = \frac{10.2}{1.889} = 5.399 \approx 54 \times 10^{-1}$$

73. In Dumas method for estimation of nitrogen, 0.50 g of an organic compound gave 70 mL of nitrogen collected at 300 K and 715 mm pressure. The percentage of nitrogen in the organic compound is ___ %.
(Aqueous tension at 300 K is 15 mm).

Ans : 15

Sol: mol of $N_2 =$

$$\frac{P \cdot V}{R \cdot T} = \frac{\left(\frac{715}{760}\right) \times (70 \times 10^{-3})}{0.0821 \times 300} = 0.00267$$

Mass of nitrogen collected $\Rightarrow 0.00267 \times 28 = 0.0749$ g

% of N in organic compound = $\frac{0.0749 \times 100}{0.5} = 14.97 \approx 15$

74. Consider two Group IV metal ions X^{2+} and Y^{2+}.

A solution containing $0.01MX^{2+}$ and $0.01MY^{2+}$ is saturated with $H_2\,S$. The pH at which the metal sulphide YS will form as a precipitate is ___ . (Nearest integer)
(Given: $K_{sp}(XS) = 1 \times 10^{-22}$ at $25^\circ C, K_{sp}(YS) = 4 \times 10^{-16}$ at $25^\circ C$, $[H_2\,S] = 0.1M$ in solution, $K_{a1} \times K_{a2}(H_2\,S) = 1.0 \times 10^{-21}, \log 2 = 0.30,\ \log 3 = 0.48, \log 5 = 0.70)$

Ans: 4

Sol: $H_2\,S \rightarrow 2H^+ + S^{2-}, K_{a1} \times K_{a2} = 1.0 \times 10^{-21}$
Concentration of [S^{2-}] to start precipitation of

$$YS \Rightarrow \frac{K_{sp}(YS)}{[Y^{2+}]} = \frac{4 \times 10^{-16}}{0.01} = 4 \times 10^{-14}$$

For $H_2\,S - K_{a1} \times K_{a2} = \frac{[H^+]^2 \cdot [\,S^{2-}]^1}{H_2\,S}$

$$1.0 \times 10^{-21} = \frac{[H^+]^2 \times [4 \times 10^{-14}]}{0.1}$$

$$[H^+]^2 = \frac{1}{4} \times 10^{-8}$$

$$[H^+] = \frac{1}{2} \times 10^{-4}$$

$$\text{pH} = -\log\left[\frac{1}{2} \times 10^{-4}\right] = 4.3 \approx 4$$

75. X and Y are the number of electrons involved, respectively during the oxidation of I^- to I_2 and S^{2-} to S by acidified $K_2Cr_2O_7$. The value of $X + Y$ is ___ .

Ans : 12

Sol: $3 \times (2I^- \longrightarrow 1I_2 + 2e)$,

$1 \times (6e + K_2Cr_2O_7 \longrightarrow 2Cr^{3+})$
$K_2Cr_2O_7 + 6I^- \longrightarrow 3I_2 + 2Cr^{3+}$
$n = 6 \rightarrow X = 6$
$3 \times (S^{2-} \longrightarrow S + 2e)$
$1 \times (6e + K_2Cr_2O_7 \longrightarrow 2Cr^{3+})$
$3\ S^{2-} + K_2Cr_2O_7 \longrightarrow 3\ S + 2Cr^{3+}$
$n = 6 \rightarrow Y = 6$
$X + Y = 6 + 6 = 12$