JEE-Main Examination – JANUARY 2026

(28TH JANUARY 2026) **TIME: 3:00 P.M. TO 6:00 P.M**

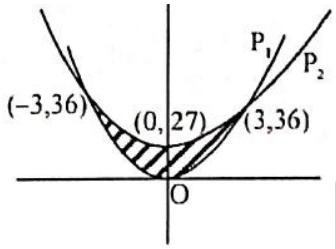
MATHEMATICS TEST PAPER WITH SOLUTION

1. Let $P_1 : y = 4x^2$ and $P_2 : y = x^2 + 27$ be two parabolas. If the area of the bounded region enclosed between P_1 and P_2 is six times the area of the bounded region enclosed between the line $y = \alpha x,\ \alpha > 0$ and P_1, then α is equal to:

(1) 15 (2) 6 (3) 8 (4) 12

Ans: (4)

Sol:



Area bounded between $P_1 \& P_2$ is

$$\int_{-3}^{3}\left(\left(x^2+27\right)-\left(4x^2\right)\right)dx \qquad \text{(P.O.I. of } P_1 \& P_2 \text{ is } x = \pm 3\text{)}$$

$$= 2\int_{0}^{3}\left(27-3x^2\right)dx = 2\left[27x - x^3\right]_0^3$$

$$= 2\left[81-27\right] = 108$$

$\therefore$ Area bounded between $P_1 \& L$ is 18 sq. units

(Area between $x^2 = 4ay \&$ line $x = my$) is $\dfrac{8a^2}{3m^3}$

$\therefore$ Area between $x^2 = \dfrac{y}{4} \& x = \dfrac{y}{\alpha}$ is

$$\frac{8.\left(\frac{1}{16}\right)^2}{3.\left(\frac{1}{\alpha}\right)^3} = 18$$

$$\Rightarrow \frac{\frac{8}{16.16}}{\frac{3}{\alpha^3}} = 18 \Rightarrow \alpha^3 = 2^6.3^3$$

$\Rightarrow \alpha = 12$

2. Let the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ be $\frac{5}{16}, a > 2$. If α is such that $a, 4, \alpha, b$ are in A.P., then the equation $\alpha x^2 - \alpha x + 2(\alpha - 2b) = 0$ has:

(1) One root in (1, 4) and another in (-2, 0)
(2) One root in (0, 2) and another in (-4, -2)
(3) Complex roots of magnitude less than 2
(4) Both roots in the interval (-2, 0)

Ans: (1)

Sol: $4 + \alpha = a + b$

$8 = a + \alpha \Rightarrow a = 8 - \alpha$

$2\alpha = 4 + b \Rightarrow b = 2\alpha - 4$

$$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{5}{16} \Rightarrow 8(a+b) = 5ab$$

$$\Rightarrow 8(4+\alpha) = (8-\alpha)(2\alpha - 4)$$

$$\Rightarrow 5\alpha^2 - 46\alpha + 96 = 0$$

$$\Rightarrow \alpha = 6, \frac{16}{5}$$

For $\alpha = 6, a = 2$. So $\alpha = 6$ rejected

For $\alpha = \frac{16}{5}$, the quadratic equation becomes

$$2x^2 - 3x - 2 = 0$$

$$\Rightarrow x = 2, \frac{-1}{2}$$

3. Given below are two statements:

Statement I: The function $f : \mathrm{R} \to \mathrm{R}$ defined by $\mathrm{f}(x) = \frac{x}{1 + |x|}$ is one-one.

Statement II: The function $f : \mathrm{R} \to \mathrm{R}$ defined by $f(x) = \frac{x^2 + 4x - 30}{x^2 - 8x + 18}$ is many-one.

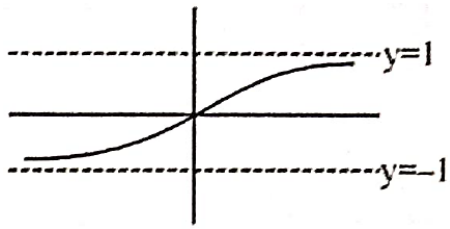
In the light of the above statements, choose the **correct answer** from the options given below:
(1) Both Statement I and Statement II are true.
(2) Statement I is true but Statement II is false.
(3) Statement I is false but Statement II is true.
(4) Both Statement I and Statement II are false.

Ans: (1)

Sol:

Statement I: $f(x)=\frac{x}{1+|x|}$

$$f(x)=\begin{cases}\frac{x}{1+x} & x\geq 0\\ \frac{x}{1-x} & x<0\end{cases}$$



$f(x)$ is one-one.

Statement 2: $f(x)=\frac{x^2+4x-30}{x^2-8x+18}$, $f(0)=\frac{-30}{18}=\frac{-5}{3}$

$$\frac{-5}{3}=\frac{x^2+4x-30}{x^2-8x+18}$$

On solving $x=0,-1$

$$\Rightarrow f(0)=f(-1)=\frac{-5}{3}$$

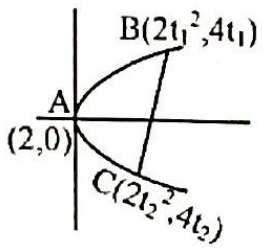
$\therefore f(x)$ is many-one.

4. Let A be the focus of the parabola $y^2=8x$. Let the line $y=mx+c$ intersect the parabola at two distinct points B and C. If the centroid of the triangle ABC is $\left(\frac{7}{3},\frac{4}{3}\right)$, then $(BC)^2$ is equal to:

(1) 32 (2) 80 (3) 89 (4) 41

Ans: (2)

Sol:



Coordinates of centroid of triangle ABC are

$$\frac{2}{3}\left(t_1^2+t_2^2+1\right)=\frac{7}{3}\Rightarrow t_1^2+t_2^2=\frac{5}{2}$$

$$\frac{4}{3}\left(t_1+t_2\right)=\frac{4}{3}\Rightarrow t_1+t_2=1$$

$$\left(t_1+t_2\right)^2=t_1^2+t_2^2+2t_1t_2\Rightarrow t_1t_2=\frac{-3}{4}$$

$$\left(t_1-t_2\right)^2=\left(t_1+t_2\right)^2-4t_1t_2=4$$

$$\left(BC\right)^2=4\left(t_1^2-t_2^2\right)^2+16\left(t_1-t_2\right)^2$$

$$\Rightarrow\left(BC\right)^2=80$$

5. Let

$A=\left\{z\in\mathbb{C}:|z-2|\le 4\right\}$ and

$B=\left\{z\in\mathbb{C}:|z-2|+|z+2|=5\right\}.$

Then the max $\left\{|z_1-z_2|:z_1\in A \text{ and } z_2\in B\right\}$ is

(1) 8 (2) $\frac{15}{2}$ (3) $\frac{17}{2}$ (4) 9

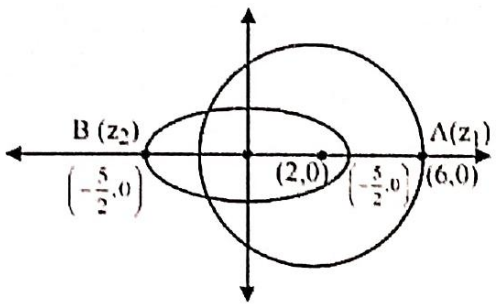
Ans: (3)

Sol:

$$|z-2|\le 4\Rightarrow(x-2)+y^2\le 16$$

$$|z-2|+|z+2|=5\Rightarrow\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\Rightarrow\frac{4x^2}{25}+\frac{4y^2}{9}=1$$



Maximum value of $|z_1-z_2|=6+\frac{5}{2}=\frac{17}{2}$

6. Let the ellipse E: $\frac{x^2}{144}+\frac{y^2}{169}=1$ and the hyperbola $H:\frac{x^2}{16}-\frac{y^2}{\lambda^2}=-1$ have the same foci. If e and L respectively denote the eccentricity and the length of the latus rectum of H, then the value of 24(e + L) is:

(1) 148 (2) 296 (3) 126 (4) 67

Ans: (2)

Sol : Equation of hyperbola: $\frac{y^2}{\lambda^2}-\frac{x^2}{16}=1$

Equation of ellipse: $\frac{x^2}{144}+\frac{y^2}{169}=1$

$e'=\sqrt{1-\frac{144}{169}}=\frac{5}{13}$

Focus $\Rightarrow(0,5)$

$\Rightarrow\lambda\sqrt{1+\frac{16}{\lambda^2}}=5$

$\Rightarrow\lambda^2+16=25$

$\lambda=3$

Eccentricity of hyperbola $=\sqrt{1+\frac{16}{\lambda^2}}=\frac{5}{3}$

Length of latus rectum of hyperbola $=\frac{2(16)}{3}=\frac{32}{3}$

$24(e+\ell)=24\left[\frac{5}{3}+\frac{32}{3}\right]=8\times37=296$

7. The sum of the coefficients of x^{499} and x^{500} in $(1+x)^{100}+x(1+x)^{999}+x^2(1+x)^{998}+\ldots\ \ldots+x^{1000}$ is

(1) ${}^{1001}C_{501}$ (2) ${}^{1002}C_{501}$ (3) ${}^{1000}C_{501}$ (4) ${}^{1002}C_{500}$

Ans : (4)

Sol : $S=(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+\ldots+x^{1000}$

$=(1+x)^{1000}\frac{\left(1-\left(\frac{x}{1+x}\right)^{1001}\right)}{1-\frac{x}{1+x}}$

$=(1+x)^{1001}-x^{1001}$

Required sum $={}^{1001}C_{499}+{}^{1001}C_{500}={}^{1002}C_{500}$

8. The sum of all the elements in the range of $f(x)=Sgn(\sin x)+Sgn(\cos x)+Sgn(\tan x)+Sgn(\cot x)$, $x\neq\frac{n\pi}{2}$, $n\in Z$, where $Sgn(t)=\begin{cases}1, & \text{if } t>0\\ -1 & \text{if } t<0\end{cases}$, is

(1) 2 (2) 4 (3) 0 (4) -2

Ans : (1)

Sol : $x\in(0,\pi/2)\Rightarrow y=1+1+1+1=4$

$x\in(\pi/2,\pi)\Rightarrow y=1-1-1-1=-2$

$x\in(\pi,3\pi/2)\Rightarrow y=-1-1+1+1=0$

$x\in(3\pi/2,2\pi)\Rightarrow y=-1+1-1-1=-2$

$\therefore$ Range of y is $\{-2,0,4\}$

Required sum $=-2+0+4=2$

9. Considering the principal values of inverse trigonometric functions, the value of the expression $\tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right)-2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$ is equal to:

(1) $\frac{33}{56}$ (2) $\frac{16}{63}$ (3) $-\frac{16}{63}$ (4) $-\frac{33}{56}$

Ans : (1)

Sol : Let $\sin^{-1}\frac{2}{\sqrt{13}}=0$ & $\cos^{-1}\frac{3}{\sqrt{10}}=\phi$

$\sin\theta=\frac{2}{\sqrt{13}}\ \&\ \cos\phi=\frac{3}{\sqrt{10}}$

$\tan(2\theta-2\phi)=\frac{\tan 2\theta-\tan 2\phi}{1+\tan 2\theta\tan 2\phi}$

$\left(\because \tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$=\frac{\frac{12}{5}-\frac{3}{4}}{1+\frac{12}{5}\cdot\frac{3}{4}}$

$= \frac{33}{56}$

10. Let the circle s $x^2 + y^2 = 4$ intersect x-axis at the points $A(a,0),\ a > 0$ and B (b, 0). Let $P(2\cos\alpha, 2\sin\alpha), 0 < \alpha < \frac{\pi}{2}$ and $Q(2\cos\beta, 2\sin\beta)$ be two points such that $(\alpha - \beta) = \frac{\pi}{2}$. Then the point of intersection of AQ and BP lies on:

(1) $x^2 + y^2 - 4x - 4y - 4 = 0$ (2) $x^2 + y^2 - 4y - 4 = 0$

(3) $x^2 + y^2 - 4x - 4 = 0$ (4) $x^2 + y^2 - 4x - 4y = 0$

Ans : (2)

Sol : Let point of intersection R(h, k)

$$m_{BR} = m_{BP} \Rightarrow \frac{k}{h+2} = \frac{2\sin\alpha}{2\cos\alpha + 2} \Rightarrow \frac{k}{h+2} = \tan\frac{\alpha}{2}$$

$$m_{AR} = m_{AQ} \Rightarrow \frac{k}{h-2} = \frac{2\sin\beta}{2\cos\beta - 2} = \frac{\sin\beta}{\cos\beta - 1} = -\cot\frac{\beta}{2}$$

$$\frac{\alpha}{2} - \frac{\beta}{2} = \frac{\pi}{4}$$

$$\tan\left(\frac{\alpha}{2} - \frac{\beta}{2}\right) = \tan\frac{\pi}{4} = 1$$

$$\frac{\tan\frac{\alpha}{2} - \tan\frac{\beta}{2}}{1 + \tan\frac{\alpha}{2}\tan\frac{\beta}{2}} = 1$$

$$\frac{\frac{k}{h+2} + \frac{h-2}{k}}{1 + \left(\frac{k}{h+2}\right)\left(\frac{2-h}{k}\right)} = 1 \Rightarrow \frac{k^2 + h^2 - 4}{\frac{k(h+2)}{4}} = 1$$

$$\frac{h^2 + k^2 - 4}{4k} = 1$$

$$x^2 + y^2 - 4y - 4 = 0$$

11. Let [.] denote the greatest integer function. Then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{12(3 + [x])}{3 + [\sin x] + [\cos x]}\right) dx$ is equal to:

(1) $12\pi + 5$ (2) $11\pi + 2$ (3) $15\pi + 4$ (4) $13\pi + 1$

Ans : (2)

Sol : $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{12(3+[x])dx}{3+[\sin x]+[\cos x]}$

$$I=\int_{-\frac{\pi}{2}}^{-1}\frac{12(1)dx}{2}+\int_{-1}^{0}\frac{12(2)dx}{2}+\int_{-1}^{0}\frac{12(3)dx}{3}+\int_{1}^{\frac{\pi}{2}}\frac{12(4)dx}{3}$$

$$I=6\left(\frac{\pi}{2}-1\right)+12(0+1)+12(1-0)+16\left(\frac{\pi}{2}-1\right)$$

$$I=3\pi-6+12+12+8\pi-16$$

$$I=11\pi+2$$

12. Let $f(x)=\lim\limits_{0\to 0}\left(\frac{\cos\pi x-x^{\left(\frac{2}{\theta}\right)}\sin(x-1)}{1+x^{\left(\frac{2}{\theta}\right)}(x-1)}\right)$, $x\in R$. Consider the following two statements:

Consider the following two statements:

(I) $f(x)$ is discontinuous at $x=1$.

(II) $f(x)$ is continuous at $x=-1$.

Then,

(1) Only (II) is True

(2) Neither (I) nor (II) is True

(3) Both (I) and (II) are True

(4) Only (I) is True

Ans : (2)

Sol : $f(x)=\begin{cases}\cos\pi x & x\to 1^-\\ \frac{-\sin(x-1)}{(x-1)} & x\to 1^+\end{cases}$

RHL $=\lim\limits_{x\to 1}\frac{-\sin(x-1)}{(x-1)}=-1$

LHL $=\lim\limits_{x\to 1}\cos\pi x=-1, f(1)=-1$

$f(x)$ is continuous at $x=1$

$$f(x)=\begin{cases}\frac{-\sin(x-1)}{-(x-1)} & x\to -1^-\\ \cos\pi x & x\to -1^+\end{cases}$$

RHL $=\lim\limits_{x\to -1}\cos\pi x=-1$

LHL $=\lim\limits_{x\to -1}\frac{-\sin(x-1)}{(x-1)}=\frac{\sin 2}{-2}$

$f(x)$ is discontinuous at $x=-1$

13. The probability distribution of a random variable X is given below:

X	$4k$	$\frac{30}{7}k$	$\frac{32}{7}k$	$\frac{34}{7}k$	$\frac{36}{7}k$	$\frac{38}{7}k$	$\frac{40}{7}k$	$6k$
$P(X)$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

If $E(X)=\frac{263}{15}$, then $P(X<20)$ is equal to:

(1) $\frac{8}{15}$ (2) $\frac{3}{5}$ (3) $\frac{14}{15}$ (4) $\frac{11}{15}$

Ans : (4)

Sol : $E(X)=\sum X_iP(X_i)=\frac{526k}{15\times 7}=\frac{263}{15}\Rightarrow k=\frac{7}{2}$

X	14	15	16	17	18	19	20	21
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

$$P(X<20)=1-P(X=20)-P(X=21)=\frac{11}{15}$$

14. Let $y=y(x)$ be the solution of the differential equation $x\frac{dy}{dx}-y=x^2\cot x,\ x\in(0,\pi)$. If $y\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$, then $6y\left(\frac{\pi}{6}\right)-8y\left(\frac{\pi}{4}\right)$ is equal to:

(1) -3π (2) 3π (3) $-\pi$ (4) π

Ans : (3)

Sol : $xdy-ydx=x^2\cot x\ dx$

$$x^2d\left(\frac{y}{x}\right)=x^2\cot x\,dx$$

$$d\left(\frac{y}{x}\right)=\cot x\ dx$$

$$\int d\left(\frac{y}{x}\right)=\int\cot x\ dx$$

$$\frac{y}{x}=\log_e\sin x+C$$

Given $y\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$

$\Rightarrow c=1$

$y=x\left(\log_e \sin x+1\right)$

$y\left(\frac{\pi}{6}\right)=\frac{\pi}{6}\left[-\log_e 2+1\right]$

$y\left(\frac{\pi}{4}\right)=\frac{\pi}{4}\left[-\frac{1}{2}\log_e 2+1\right]$

$6y\left(\frac{\pi}{6}\right)-8y\left(\frac{\pi}{4}\right)$

$=\pi\left[\left(-\log_e 2+1\right)+2\left(\frac{1}{2}\log_e 2-1\right)\right]$

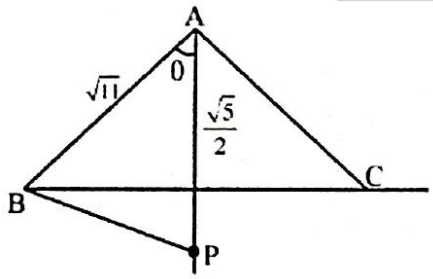
$=\pi[1-2]=-\pi$

15. Let P be a point in the plane of the vector $\overrightarrow{AB}=3\hat{i}+j-k$ and $\overrightarrow{AC}=\hat{i}-j+3k$ such that P is equidistant from the lines AB and AC. If $\left|\overrightarrow{AP}\right|=\frac{\sqrt{5}}{2}$,

(1) $\frac{\sqrt{26}}{4}$ (2) $\frac{\sqrt{30}}{4}$ (3) $\frac{3}{2}$ (4) 2

Ans : (2)

Sol : $\cos 2\theta=\frac{3-1-3}{\sqrt{11}\sqrt{11}}=-\frac{1}{11}$



$1-2\sin^2\theta=-\frac{1}{11}\Rightarrow 2\sin^2\theta=\frac{12}{11}\Rightarrow \sin\theta=\sqrt{\frac{6}{11}}$

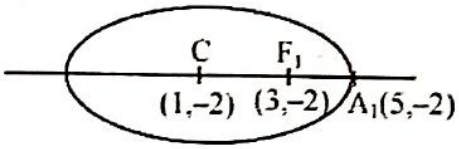
$\therefore$ Area $\left(\Delta APB\right)=\frac{1}{2}\times\sqrt{11}.\frac{\sqrt{5}}{2}.\sqrt{\frac{6}{11}}=\frac{\sqrt{30}}{4}$

16. An ellipse has its center at (1, -2), one focus at (3, -2) and one vertex at (5, -2). Then the length of its latus rectum is:

(1) $6\sqrt{3}$ (2) $4\sqrt{3}$ (3) 6 (4) $\frac{16}{\sqrt{3}}$

Ans : (3)

Sol :



$CA_1 = a = 4$

$CF_1 = ac = 2$

$e = \frac{1}{2}$

$LR = 2e\left(\frac{a}{e} - ae\right)$

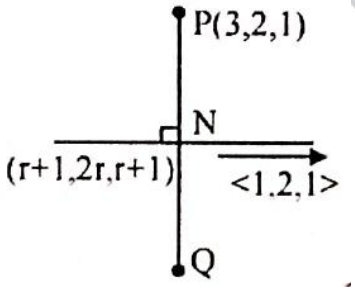
$= 2 \times \frac{1}{2} \times \left(\frac{4}{1/2} - 2\right) = 6$

17. Let Q(a, b, c) be the image of the point P(3, 2, 1) in the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$. Then the distance of Q from the line $\frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2}$ is

(1) 5 (2) 6 (3) 7 (4) 8

Ans : (3)

Sol :



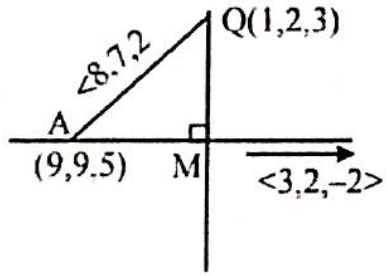
drs of PN $= < r-2, 2r-2, r >$

$1.(r-2) + 2(2r-2) + 1.(r) = 0$

$6r = 6 \Rightarrow r = 1$

$\therefore N \equiv (2,2,2)$

$\Rightarrow Q \equiv (1,2,3)$



$$AQ = \sqrt{64+49+4} = \sqrt{117}$$

$$AM = \left|\frac{24+14-4}{\sqrt{9+4+4}}\right| = \frac{34}{\sqrt{17}} = 2\sqrt{17}$$

$$\therefore QM = \sqrt{117-68} = \sqrt{49} = 7$$

18. Let $f(x) = \int \frac{dx}{x^{\left(\frac{2}{3}\right)} + 2x^{\left(\frac{1}{2}\right)}}$ be such that $f(0) = -26 + 24\log_e(2)$. If $f(1) = a + b\log_e(3)$ where $a, b \in Z$, then $a+b$ is equal to:

(1) -5 (2) -26 (3) -11 (4) -18

Ans : (3)

Sol : $f(x) = \int \frac{dx}{x^{2/3} + 2x^{1/2}}$

Put $x = t^6 \Rightarrow dx = 6t^5 dt$

$$= \int \frac{6t^5 dt}{t^4 + 2t^3} = 6\int \frac{\left(t^2-4\right)+4}{t+2} dt$$

$$= 6\left[\int (t-2)dt + 4\int \frac{1}{t+2} dt\right]$$

$$= 6\left[\frac{t^2}{2} - 2t + 4\ell n(t+2)\right] + C$$

$$= 3x^{1/3} - 12x^{1/6} + 24\ell n\left(x^{1/6}+2\right) + C$$

$f(0) = 24\ell n2 + C = -26 + 24\ell n2$ (given)

$\Rightarrow C = -26$

Now

$f(1) = -35 + 24\ell n3 = a + b\ell n3$

$\Rightarrow a = -35$ & $b = 24$

$\Rightarrow a + b = -11$

19. $\frac{6}{3^{26}}+\frac{10.1}{3^{25}}+\frac{10.2}{3^{24}}+\frac{10.2^2}{3^{23}}+...+\frac{10.2^{24}}{3}$ is equal to

(1) 2^{26} (2) 2^{25} (3) 3^{26} (4) 3^{25}

Ans : (1)

Sol : $S=\frac{6}{3^{26}}+\frac{10}{3^{25}}\left[\frac{(6)^{25}-1}{6-1}\right]$

$S=\frac{6}{3^{26}}+\frac{10}{3^{25}}\left[\frac{6^{25}-1}{5}\right]$

$S=\frac{2}{3^{25}}+2\left[2^{25}-\frac{1}{3^{25}}\right]$

$S=2^{26}$

20. Given below two statements:

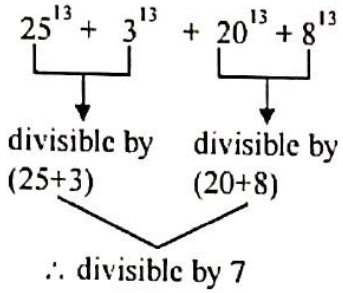
Statement I: $25^{13}+20^{13}+8^{13}+3^{13}$ is divisible by 7.

Statement II: The integral part of $\left(7+4\sqrt{3}\right)^{25}$ is an odd number.

In the light of the above statements, choose the correct answer from the options given below:
(1) Both Statement I and Statement II are true.
(2) Statement I is false but Statement II is true.
(3) Both Statement I and Statement II are false.
(4) Statement I is true but Statement II is false.

Ans : (1)

Sol : Statement I:



Statement II: $R=\left(7+4\sqrt{3}\right)^{25}=1+f$

$R'=\left(7-4\sqrt{3}\right)^{25}=f'$

$\therefore R+R'=2\left[{}^{25}C_0 7^{25}+{}^{25}C_2 7^{23}\left(4\sqrt{3}\right)^2+...\right]$

$1+f+f'=$ even integer

$\therefore 1 =$ odd integer

$\because 0 < f + f' < 2 \Rightarrow f + f' = 1$

$\Rightarrow$ Both the statements are correct

SECTION – B

21. If $\sum_{r=1}^{25}\left(\frac{r}{r^4+r^2+1}\right)=\frac{p}{q}$, where p and q are positive integers such that gcd $(p,q)=1$, then p + q is equal to _____.

Ans : (976)

Sol : $S=\sum\frac{r}{\left(r^2+r+1\right)\left(r^2-r+1\right)}$

$$=\frac{1}{2}\sum_{r=1}^{25}\left(\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}\right)$$

$$=\frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\ldots+\left(\frac{1}{601}-\frac{1}{651}\right)\right]$$

$$=\frac{1}{2}\left[\frac{1}{1}-\frac{1}{651}\right]$$

$$=\frac{1}{2}\left[\frac{650}{651}\right]=\frac{325}{651}$$

$$\frac{p}{q}=\frac{325}{651}\Rightarrow p+q=976$$

22. If the distance of the point $P(43,\ \alpha,\ \beta), \beta<0$ from the line $\vec{r}=4\hat{i}-k+\mu\left(2\hat{i}+3k\right),\ \mu\in R$ along a line with direction ratios $3,-1,0$ is $13\sqrt{10}$, then $\alpha^2+\beta^2$ is equal to _____.

Ans : (170)

Sol : $\frac{x-43}{3}=\frac{y-\alpha}{-1}=\frac{z-\beta}{0}\Rightarrow P_1\left(43+3\lambda,\alpha-\lambda,\beta\right)$

$$\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}\Rightarrow P_1\left(2\mu+4,0,3\mu-1\right)$$

$$\therefore\ \mu=\frac{3\lambda+39}{2},\ \alpha=\lambda,\ \beta=\frac{9\lambda-115}{2}$$

$$P\left(43,\alpha,\beta\right),P_1\left(43,+3\alpha,0,\beta\right)$$

$$\left(PP_1\right)^2=1690=10\alpha^2,\therefore\alpha=13,\beta=1$$

$\therefore \alpha^2+\beta^2=170$

23. Let f be a differentiable function satisfying $f(x)=1-2x+\int_0^x e^{(x-t)}\ f(t)\,dt, x\in R$ and let

$$g(x)=\int_0^x (f(t)+2)^{15}(t-4)^6(t+12)^{17}\,dt,\ x\in R.$$

If p and q are respectively the points of local minima and local maxima of g, then the value of $|p+q|$ is equal to _____.

Ans : (9)

Sol : $f(x)=1-2x+e^x\int_0^x e^{-1}f(t)dt$

$$e^{-x}f(x)=(1-2x)e^{-x}+\int_0^x e^{-1}f(t)dt$$

$$e^{-x}f'(x)=e^{-x}f(x)=-2e^{-x}+(1-2x)e^{-x}(-1)+e^{-x}f(x)$$

$$f'(x)-2f(x)=2x-3$$

$$\frac{dy}{dx}-2y=2x-3$$

$$\Rightarrow y.e^{-2x}=\int e^{-2x}(2x-3)dx$$

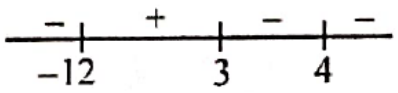
On solving we get

$$f(x)=1-x$$

$$g(x)=\int_0^x (3-t)^{15}(t-4)^6(t+12)^{17}\,dt$$

$$g'(x)=(3-x)^{15}(x-4)^6(x+12)^{17}$$

$$=-(x-3)^{15}(x-4)^6(x+12)^{17}$$



Local maxima $\Rightarrow q=3$

Local minima $\Rightarrow p=-12 \Rightarrow |p+q|=9$

24. Let $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ and B be two matrices such that $A^{100}=100B+I$. Then the sum of all the elements of B^{100} is _____.

Ans : (0)

Sol : $A = I + \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$, let $M = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$

$$M^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = M^3 = M^4 = \ldots = M^{100}$$

$$A^{100} = (I + M)^{100} = \sum_{r=0}^{100} \binom{100}{r} M^r . I$$

$$A^{100} = I + 100M = I + 100B$$

$$\therefore M = B \Rightarrow M^{100} = B^{100} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

25. Three persons enter in a lift at the ground floor. The lift will go upto 10th floor. The number of ways, in which the three persons can exit the lift at three different floors, if the lift does not stop at first, second and third floors, is equal to _____.

Ans : (210)

Sol : ${}^7C_3 \times \lfloor 3$

$= 210$

PHYSICS TEST PAPER WITH SOLUTION

PHYSICS – SECTION – A

26. Number of photons of equal energy emitted per second by a 6 mW laser source operating at 663 nm is _____. (Given: $h = 6.63 \times 10^{-34}$ J.s and $c = 3 \times 10^8 m/s$)

(1) 5×10^{15} (2) 5×10^{16} (3) 10×10^{15} (4) 2×10^{16}

Ans : (4)

Sol : $P = \dfrac{nhC}{\lambda}$

$$6 \times 10^{-3} = \frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^8}{663 \times 10^{-9}}$$

$n = 2 \times 10^{16}$ photons.

27. The time period of a simple harmonic oscillator is $T = 2\pi\sqrt{\dfrac{k}{m}}$. The measured value of mass (m) of the object is 10 g with an accuracy of 10 mg, and time for 50 oscillations of the spring is found to be 60 s using a watch of 2 s resolution. Percentage error in determination of spring constant (k) is _____%.

(1) 7.60 (2) 3.35 (3) 6.76 (4) 3.43

Ans : (3)

Sol : $\frac{\Delta K}{K}=\frac{2\Delta T}{T}+\frac{\Delta m}{m}$

$T=\frac{60}{50}=1.2\,\text{sec}$

$\Delta T=\frac{2}{50}$

$\therefore \frac{\Delta K}{K}=\frac{2\times 2}{50\times 1.2}+\frac{10\times 10^{-3}}{10}=0.0676$

$\therefore$ % Error $=6.76\%$

28. In an experiment, a set of reading are obtained -1.24 mm, 1.25 mm, 1.23 m, 1.21 mm. The expected least count of the instrument used in recording these readings is _____mm.

(1) 0.1 (2) 0.001 (3) 0.05 (4) 0.01

Ans : (4)

Sol : Difference of measurements is 0.01.

29. For a transparent prism, if the angle of minimum deviation is equal to its refracting angle, the refractive index n of the prism satisfies.

(1) $n\geq 2$ (2) $\sqrt{2}<n<2\sqrt{2}$ (3) $1<n<2$ (4) $\sqrt{2}<n<2$

Ans : (3)

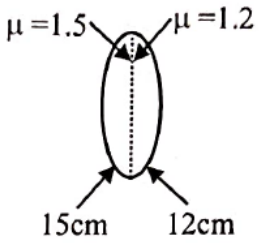
Sol : $\delta_{\min}=2i-A\Rightarrow i=\delta_{\min}=A$

Also, $\mu=\frac{\sin\left(\frac{\delta_{\min}+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\Rightarrow \mu=\frac{\sin A}{\sin\frac{A}{2}}=2\cos\left(\frac{A}{2}\right)$

Therefore, $1<\mu<2$.

30. A biconvex lens is formed by using two thin planoconvex lenses, as shown in the figure. The refractive index and radius of curved surfaces are also mentioned in figure. When an object is placed on the left side of lens at a distance of 30 cm from the biconvex lens, the magnification of the image will be:



(1) +2.5 (2) -2.5 (3) -2 (4) +2

Ans : (3)

Sol : $\frac{1}{v}-\frac{1}{u}=\frac{1}{f_{net}}=\frac{1}{f_1}+\frac{1}{f_2}$

$$\frac{1}{v}+\frac{1}{30}=(1.5-1)\left(\frac{1}{15}-\frac{1}{\infty}\right)+(1.2-1)\left(\frac{1}{\infty}+\frac{1}{12}\right)$$

$$\frac{1}{v}+\frac{1}{30}=\frac{1}{30}+\frac{1}{60}$$

$$v=60$$

$$m=\frac{v}{u}=\frac{60}{-30}=-2$$

31. The speed of a longitudinal wave in a metallic bar is 400 m/s. If the density and Young's modulus of the bar material are increased by 0.5% and 1% respectively then the speed of the wave is changed approximately to _____ m/s.

(1) 399 (2) 402 (3) 398 (4) 401

Ans : (4)

Sol : $V_{\text{sound}}=\sqrt{\frac{Y}{\rho}}$

$$\frac{\Delta V}{V}\times 100=\frac{1}{2}\left(\frac{\Delta Y}{Y}\times 100\right)-\frac{1}{2}\left(\frac{\Delta \rho}{\rho}\times 100\right)$$

$$=\frac{1}{2}\times 1-\frac{1}{2}\times 0.5$$

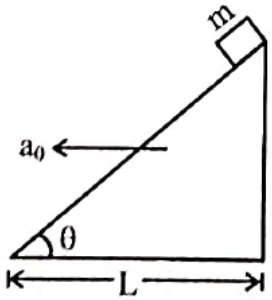
$$\frac{\Delta V}{V}\times 100=\frac{1}{4}$$

$$\Delta V=\frac{1}{4}\times\frac{V}{100}$$

$$\Delta V=1 m/s$$

$$V_{\text{final}}=400+1=401 m/s$$

32. A small block of mass m slides down from the top of a frictionless inclined surface, while the inclined plane is moving towards left with constant acceleration a_0. The angle between he inclined plane and ground is θ and its base length is L. Assuming that initially the small block is at the top of the inclined plane, the time it takes to reach the lowest point of the inclined plane is _____.



(1) $\sqrt{\frac{4L}{g\sin 2\theta - a_0(1+\cos 2\theta)}}$

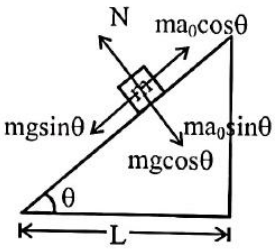
(2) $\sqrt{\frac{4L}{g\cos^2\theta - a_0\sin\theta\cos\theta}}$

(3) $\sqrt{\frac{2L}{g\sin 2\theta - a_0(1+\cos 2\theta)}}$

(4) $\sqrt{\frac{2L}{g\sin\theta - a_0\cos\theta}}$

Ans : (1)

Sol :



$$mg\sin\theta - ma_0\cos\theta = ma$$

$$a = g\sin\theta - a_0\cos\theta$$

Now using,

$$S = ut + \frac{1}{2}a_{\text{down}}t^2$$

$$\frac{L}{\cos\theta} = \frac{1}{2}(g\sin\theta - a_0\cos\theta)t^2$$

$$t = \sqrt{\frac{2L}{g\sin\theta\cos\theta - a_0\cos^2\theta}}$$

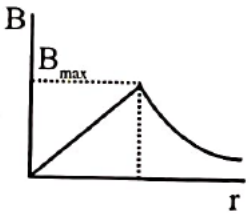
$$t = \sqrt{\frac{4L}{g\sin 2\theta - a_0(1+\cos 2\theta)}}$$

33. A long cylindrical conductor with large cross section carries an electric current distributed uniformly over its cross-section. Magnetic field due to this current is:

A. maximum at either ends of the conductor and minimum at the midpoint
B. maximum at the axis of the conductor
C. minimum at the surface of the conductor
D. minimum at the axis of the conductor
E. same at all points in the cross-section of the conductor
Choose the correct answer from the options given below:
(1) B, C only (2) D only (3) E only (4) A. D only

Ans : (2)

Sol : Solid cylinder



B_{max} at surface

B_{min} at Axis

34. Match List – I with List – II.

List – I	List – II
A. Coefficient of viscosity	I. $\left[ML^{-1}T^{-2}\right]$
B. Surface tension	II. $\left[ML^{2}T^{-2}\right]$
C. Pressure	III. $[ML^{0}T^{-2}]$
D. Surface energy	IV. $\left[ML^{-1}T^{-1}\right]$

Choose the correct answer from the options given below:
(1) A-IV, B-I, C-II, D-III (2) A-IV, B-III, C-I, D-II
(3) A-I, B-II, C-IV, D-III (4) A-I, B-III, C-II, D-IV

Ans : (2)

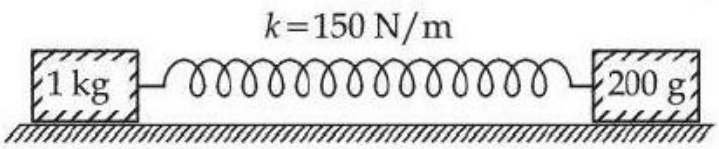
Sol : (A) $\eta = \frac{Fdr}{Adv} = \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]} = [ML^{-1}T^{-1}]$

(B) $S = \frac{F}{L} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}]$

(C) $P = \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

(D) $E = S \times A = [ML^{-2}][L^2] = [ML^2T^{-2}]$

35. As shown in the figure, a spring is kept in a stretched position with some extension by holding the masses 1 kg and 0.2 kg with a separation more than spring natural length and are released. Assuming the horizontal surface to be frictionless, the angular frequency (in SI unit) of the system is:



(1) 20 (2) 27 (3) 30 (4) 5

Ans : (3)

Sol : $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{1 \times 0.2}{1.2}$

$\mu = \frac{1}{6}$

$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{150}{1/6}} = 30$

36. A particle starts moving from time t = 0 and its coordinate is given as $x(t) = 4t^3 - 3t$.

A. The particle returns to its original position (origin) 0.866 units later
B. The particle is 1 unit away from origin at its turning point.
C. Acceleration of the particle is non-negative.
D. The particle is 0.5 units away from origin at its turning point.
E. Particle never turns back as acceleration is non-negative.
Choose the correct answer from the options given below:
(1) C, E only (2) A, C, D only (3) A, C only (4) A, B, C only

Ans : (4)

Sol : $x = 0 \Rightarrow t = 0, \frac{\sqrt{3}}{2}$

$v = 12t^2 - 3$

At turning point, $v = 0$

$t = \frac{1}{2} \Rightarrow x = \frac{4}{8} - \frac{3}{2} = -1$

$a = 24t$ (always positive)

37. Identify the correct statements:

A. Effective capacitance of a series combination of capacitors is always smaller than the smallest capacitance of the capacitor in the combination.
B. When a dielectric medium is placed between the charged plates of a capacitor, displacement of charges cannot occur due to insulation property of dielectric.
C. Increasing of area of capacitor plate or decreasing of thickness of dielectric is an alternate method to increase the capacitance.

D. For a point charge, concentric spherical shells centered at the location of the charge are equipotential surfaces.

Choose the correct answer from the options given below.

(1) A, B and C only (2) C and D only

(3) A, C and D only (4) B and D only

Ans : (3)

Sol : For series combination

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}$$

$\therefore C_{eq}$ is less than $C_1 \& C_2$.

Note: In statement C, capacitor is assumed to be completely filled with dielectric then on decreasing thickness of dielectric capacitance will increase.

38. A plane electromagnetic wave is moving in free space with velocity $c=3\times10^8 m/s$ and its electric field is given as $\vec{E}=54\sin(kz-\omega t)\,j\ V/m,$ where j is the unit vector along y-axis. The magnetic field vector $\vec{B}$ of the wave is:

(1) $+1.8\times10^{-7}\sin(kz-\omega t)\hat{i}T$ (2) $-1.8\times10^{-7}\sin(kz-\omega t)\hat{i}T$

(3) $1.4\times10^{-7}\sin(kz-\omega t)\hat{i}T$ (4) $1.4\times10^{-7}\sin(kz-\omega t)kT$

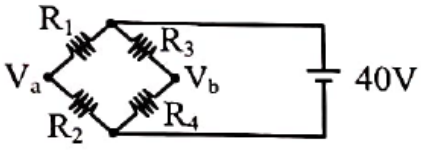
Ans : (2)

Sol : $B=C\times E=k\times j=-\hat{i}$

$$\therefore \vec{B}=\frac{54}{3\times10^8}\sin(kz-\omega t)(-\hat{i})$$

$$=-1.8\times10^{-7}\sin(kz-\omega t)\hat{i}$$

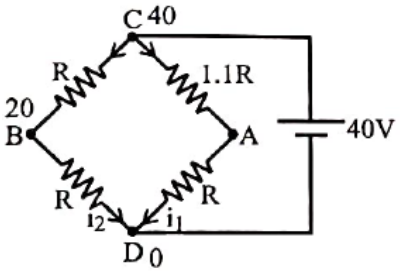
39. A Wheatstone bridge is initially at room temperature and all arms of the bridge have same value of resistances $(R_1=R_2=R_3=R_4)$. When R_3 resistance is heated to some temperature, its resistance value has gone up by 10%. The potential difference (V_a-V_b) (after R_3 is heated) is _____V.



(1) 0.95 (2) 0 (3) 2 (4) 1.05

Ans : (1)

Sol :



$$V_A = \frac{V}{2}$$

$$V_B = \frac{V}{2.1R} \times R = \frac{V}{2.1}$$

$$\therefore V_A - V_B = V\left[\frac{1}{2} - \frac{1}{2.1}\right]$$

$$V_A - V_B = \frac{0.1}{2 \times 2.1} \times 40$$

$$V_A - V_B = \frac{4}{4.2} = 0.95$$

40. Which one of the following is not a measurable quantity?

(1) Voltage
(2) Voltage difference
(3) Displacement current
(4) Resistance

Ans : (1)

Sol :

Here from voltage, question refers to potential. We can measure potential difference between two points but not potential at any point.

Note: If the potential of reference point is known then we can measure potential as well.

41. The mean free path of a molecule of diameter 5×10^{-10} m at the temperature 41°C and pressure 1.38×10^5 Pa, is given as _____ m.

(Given $k_B = 1.38 \times 10^{-23} J/K$).

(1) 2×10^{-8} (2) $2\sqrt{2} \times 10^{-10}$ (3) $10\sqrt{2} \times 10^{-8}$ (4) $2\sqrt{2} \times 10^{-8}$

Ans : (4)

Sol : $\lambda = \frac{k_B T}{\sqrt{2}\pi\sigma^2 P}$

$$= \frac{1.38 \times 10^{-23} \times (273+41) \times 100}{\sqrt{2} \times 3.14 \times (5 \times 10^{-10})^2 \times 1.38 \times 10^5} = 2\sqrt{2} \times 10^{-8}$$

42. Identify the correct statements:

A. Electrostatic field lines from closed loops.
B. The electric field lines point radially outward when charge is greater than zero.
C. The Gauss-Law is valid only for inverse-square force.
D. The work done in moving a charged particle in a static electric field around a closed path is zero.
E. The motion of a particle under Coulomb's force must take place in a plane.
Choose the correct answer from the options given below:
(1) A, B, C, D only (2) A, C, E only (3) A, B, D, E only (4) B, C, D, E only

Ans : (4)

Sol : Conceptual.

43. When the position vector $\vec{r}=x\hat{i}+yj+zk$ changes sign as $-\vec{r}$, which one of the following vector will not flip under sign change?

(1) Velocity
(2) Acceleration
(3) Linear momentum
(4) Angular momentum

Ans : (4)

Sol : $\vec{r}=x\hat{i}+yj+zk$

$$\vec{v}=\frac{d\vec{r}}{dt}=v_x\vec{i}+v_y\vec{j}+v_z\vec{k}$$

$$\vec{p}=m\vec{v}$$

$$\vec{L}=m\left(\vec{r}\times\vec{v}\right)$$

$$=\left(x\hat{i}+yj+zk\right)\times m\left(v_x\hat{i}+v_yj+v_zk\right)$$

When sign of $\vec{r}$ changes, $\vec{L}$ remains same.

44. A nucleus has mass number α and radius R_α. Another nucleus has mass number β and radius R_β. If $\beta=8\alpha$ then R_α/R_β is:

(1) 0.5 (2) 2 (3) 8 (4) 1

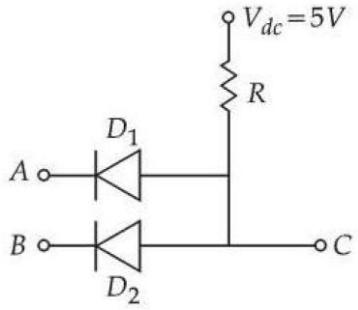
Ans : (1)

Sol : $R_\alpha=R_0\alpha^{1/3}$

$$R_\beta=R_0\beta^{1/3}$$

$$\frac{R_\alpha}{R_\beta}=\left(\frac{\alpha}{\beta}\right)^{1/3}=\frac{1}{2}$$

45. Two p-n junction diodes D_1 and D_2 are connected as shown in figure. A and B are input signals and C is the output. The given circuit will function as a _____.



(1) OR Gate (2) NOR Gate (3) NAND Gate (4) AND Gate

Ans : (4)

Sol : If either A or B is zero, n that case current flow and $v_c = 0$.

Hence the Gate will be AND Gate.

SECTION – B

46. Two tuning forks A and B are sounded together giving rise to 8 beats in 2s. When fork A is loaded with wax, the beat frequency is reduced to 4 beats in 2s. If the original frequency of tuning fork B is 380 Hz then original frequency of tuning fork A is _____Hz.

Ans : (384)

Sol : $|f_A - f_B| = 4$

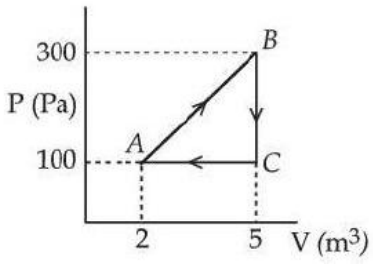
$|f_A - 380| = 4$

So, $f_A = 384$ Hz or 376 Hz

on loading with wax f_A decreases

so, $f_A = 384\,Hz$

47. A thermodynamic system is taken through the cyclic process ABC as shown in the figure. The total work done by the system during the cycle ABC is _____ J.



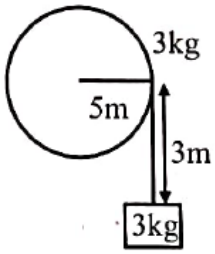
Ans : (300)

Sol : Work done = Area bounded by cycle $= \frac{1}{2} \times 3 \times 200 = 300\,J$

48. A fly wheel having mass 3 kg and radius 5 m is free to rotate about a horizontal axis. A string having negligible mass is wound around the wheel and the loose end of the string is connected to 3 kg mass. The mass is kept at rest initially and released. Kinetic energy of the wheel when the mass descends by 3 m is _____ J. (g = 10 m/s^2)

Ans : (30)

Sol :



$$mg\times 3=\frac{1}{2}.\frac{mR^2}{2}\omega^2+\frac{1}{2}mv^2 \qquad \text{... (i)}$$

$$\&v=\omega R \qquad \text{... (ii)}$$

From equation (i) & (ii)

$$g\times 3=\frac{3}{4}.v^2$$

K.E. of flywheel $=\frac{1}{2}\times\frac{mR^2}{2}\times\omega^2=\frac{1}{4}mv^2$

$=\frac{1}{4}\times 3\times 40=30$ Joule.

49. An inductor stores 16 J of magnetic field energy and dissipates 32 W of thermal energy due to its resistance when an a.c. current of 2A (rms) and frequency 50 Hz flows through it. The ratio of inductive reactance to its resistance is _____. $(\pi=3.14)$

Ans : (314)

Sol : $\frac{1}{2}Li_{rms}^2=16\Rightarrow L=8$

$i^2R=32\Rightarrow R=8$

$x_L=\omega L\Rightarrow 2\times 3.14\times 50\times 8$

$\Rightarrow 800\times 3.14$

R = 8

$\frac{x_L}{R}=314$

50. A beam of light consisting of wavelengths 650 nm and 550 nm illuminates the Young's double slits with separation of 2 mm such that the interference fringes are formed on a screen, placed at a distance of 1.2 m from the slits. The least distance of a point from the central maximum, where the bright fringes due to both the wavelengths coincide, is _____ $\times 10^{-5}m.$

Ans : (429)

Sol : $y = n\frac{\lambda D}{d}$

$y_1 = y_2$

$n_1\lambda_1\frac{D}{d} = n_2\lambda_2\frac{D}{d}$

$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{550}{650} = \frac{11}{13}$

$y = 11\times\frac{\lambda_1 D}{d} = \frac{11\times650\times10^{-9}\times1.2}{2\times10^{-3}}$

$y = 429\times10^{-5}$

CHEMISTRY TEST PAPER WITH SOLUTION

CHEMISTRY – SECTION – A

51. $CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$

In above reaction 90 g $CaCO_3$ is added to 300 ml, 38 55% w/w, HCl solution with density 1.13 g/ml. Which of the following option is correct.

(1) 60.32 gm of HCl remains unreacted.

(2) 32.85 gm of $CaCO_3$ remains unreacted.

(3) 97.30 gm HCl reacted.

(4) 64.97 g HCl remain unreacted.

Ans : (4)

Sol : LR

$CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$

$\frac{90}{100}$ d = 1.13 g/ml

= 0.90 mol V = 300 ml

Wt. of solution = 339 g

Wt. of HCl $= 339\times\frac{38.55}{100} = 130.68$

Moles of HCl $= \frac{130.68}{36.5} = 3.58$

Moles of HCl remained = 1.78 mole

Mass of HCl remained = 64.97 g.

52. The correct order of acidic strength of the major products formed in the given reactions is:

A. $PhNH_2 \xrightarrow[(2)\ CuCN \\ (3)\ H_3O^+/\Delta]{(1)\ NaNO_2 + HCl\ (<5°C)} [A]$

B. $CH_3CH_2CHO \xrightarrow[\Delta]{[Ag(NH_3)_2]^+, OH^-} [B]$

C. $CH_4 + O_2 \xrightarrow[(ii)\ Na_2Cr_2O_7/H^+]{(i)\ Mo_2O_3} [C]$

D. $PhCH_2MgBr + CO_2 \xrightarrow[H_3O^+]{\text{Dry ether}} [D]$

(1) C > A > D > B (2) A > D > B > C (3) A > D > C > B (4) C > B > A > D

Ans : (1)

Sol : Product of I is PhCOOH

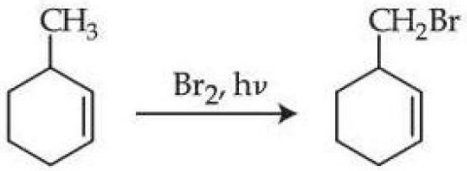
Product of II is HCOOH

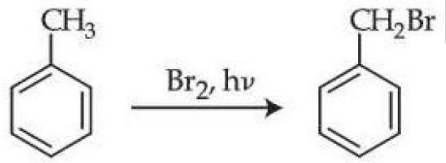
Product of III is $H_3C - CH_2COOH$

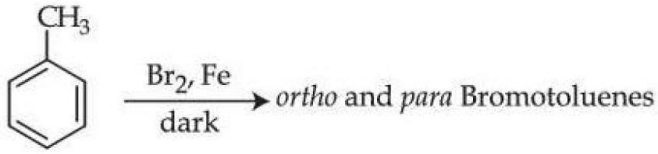
Product IV is $Ph - CH_2COOH$

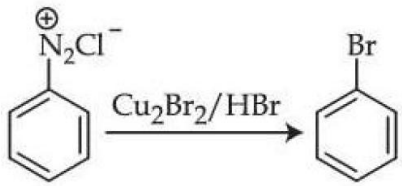
Order of acidic strength HCOOH > PhCOOH > $Ph - CH_2COOH$ > CH_3CH_2COOH

53. Which of the following reaction is NOT correctly represented?

1. 

2. 

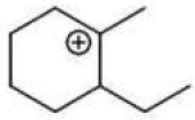
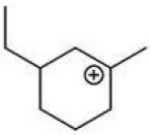
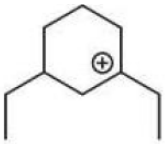
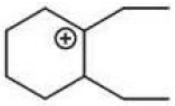
3. 

4. 

Ans : (1)

Sol : Conceptual

54. The cyclic cations having the same number of hyperconjugation are:

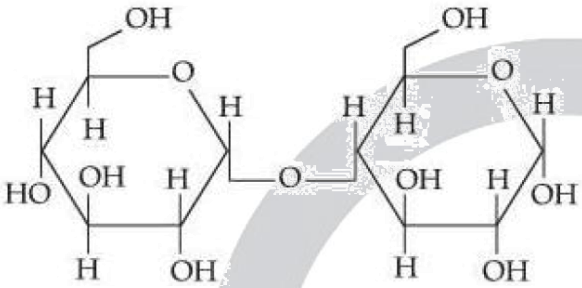
A.  B.  C.  D. 

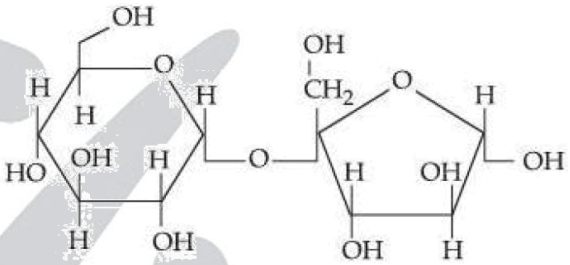
(1) B and C only (2) A, C and D (3) A and B only (4) A and C only

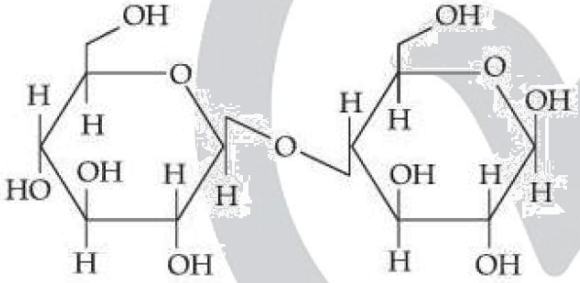
Ans : (4)

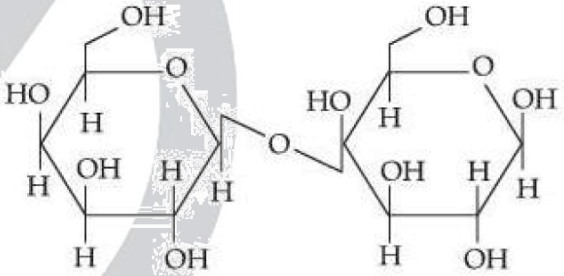
Sol : Number of hyperconjugation is directly related to number of $\alpha - H$ with respect to carbocation.

55. Structures of four disaccharides are given below. Among the given disachharides, the non-reducing sugar is:

1. 

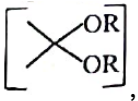
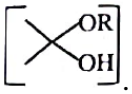
2. 

3. 

4. 

Ans: (2)

Sol:

For nonreducing sugar compound should have acetal linkage  , not hemiacetal linkage  .

Sucrose is non reducing sugar.

56. A student has been given 0.314 g of an organic compound and asked to estimate Sulphur. During the experiment, the student has obtained 0.4813 g of barium sulphate. The percentage of sulphur present in the compound is _____. (Given Molar mass in g mol^{-1} S: 32, $BaSO_4$: 233)

(1) 42.10% (2) 63.15% (3) 48.24% (4) 21.05%

Ans : (4)

Sol : Applying POAC as 's'

$$n_{BaSO_4} = \frac{0.4813}{233} = 0.0020$$

Moles of 's' = 0.0020

Mass of 's' in $BaSO_4 = 0.0020 \times 32 = 0.066\,g$

Same mass of 's' is present in 'OC'

% of 's' in OC $= \frac{0.066}{0.314} \times 100 = 20.9$

57. A student performed analysis of aliphatic organic compound 'X' which on analysis gave C = 61.01%.

H = 15.25%, N = 23.74%.
This compound, on treatment with HNO_2/H_2O produced another compound 'Y' which did not contain any nitrogen atom. However, the compound 'Y' upon controlled oxidation produced another compound 'Z' that responded to iodoform test.
The structure of 'X' is:

(1) $CH_3CH_2CH_2NH_2$

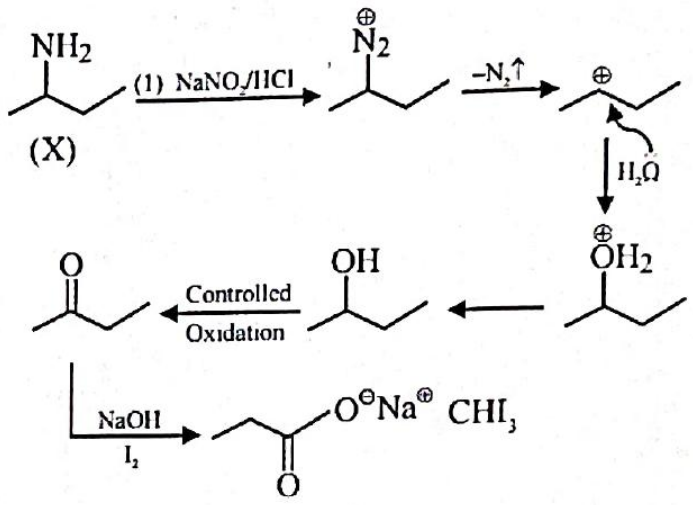
(2) $CH_3-\underset{CH_3}{\overset{}{CH}}-NH_2$

(3) $CH_3-CH_2\underset{NH_2}{\overset{}{CH}}-CH_3$

(4) $Ph-\underset{CH_3}{\overset{}{CH}}-NH_2$

Ans : (3)

Sol :



58. Given below are two statements:

Statement-I: The increasing order of boiling point of hydrogen halides is HCl < HBr < HI < HF.
Statement-II: The increasing order of melting point of hydrogen halides HCl < HBr < HF < HI.
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are false.
(3) Statement I is true but Statement II is false.
(4) Both Statement I and Statement II are true.

Ans : (4)

Sol :

B.P. HF > HI > HBr > HCl

M.P. HI > HF > HBr > HCl

59. Identify the correct statements:

The presence of $-NO_2$ group in benzene ring

A. Activates the ring towards electrophilic substitutions.
B. Deactivates the ring towards electrophilic substitutions.
C. Activates the ring towards nucleophilic substitutions.
D. Deactivates the ring towards nucleophilic substitutions.
Choose the correct answer from the options given below:
(1) B and D only (2) A and D only (3) C and A only (4) B and C only

Ans : (4)

Sol :

Nitro group is electron withdrawing group so it will decrease electron density from benzene ring.

So, it is deactivating towards electrophilic substitution reaction (ESR) and it increases the electrophilicity of benzene ring so it is activating towards nucleophilic substitution reaction (NSR).

60. Observe the following equilibrium in a 1 L flask.

$A(g) \rightleftharpoons B(g)$

At T(K), the equilibrium concentrations of A and B are 0.5 M and 0.375 M respectively. 0.1 moles of A is added into the flask and heated to T(K) to establish the equilibrium again. The new equilibrium concentrations (in M) of A and B are respectively
(1) 0.557, 0.418 (2) 0.367, 0.275 (3) 0.742, 0.557 (4) 0.53, 0.4

Ans : (1)

Sol :

$$\begin{array}{ccc} A & \rightleftharpoons & B \\ 0.5 & & 0.375 \end{array}$$

$$K_{eq} = \frac{[B]_{eq}}{[A]_{eq}} = \frac{0.375}{0.5} = 0.75$$

Now 0.1 mole of A is added so reaction will move in forward direction

$$\begin{array}{ccc} A & \rightleftharpoons & B \\ 0.6-x & & 0.375+x \end{array}$$

$$K_{eq} = 0.75 = \frac{0.375+x}{0.6-x}$$

$$0.45 - 0.75x = 0.375 + x$$

$1.75x = 0.075$

$X = \frac{0.075}{1.75} = \frac{3}{70} = 0.043$

Moles of A = 0.043 = 0.0557

Moles of B = 0.418

61. Consider following statements about manganate and permanganate ions. Identify the correct statements.

(A) The geometry of both manganate and permanganate ions is tetrahedral.
(B) The oxidation states of Mn in manganate and permanganate are +7 and +6, respectively.
(C) Oxidation of Mn(II) salt by peroxodisulphate gives manganate ion as the final product.
(D) Manganate ion is paramagnetic and permanganate ions is diamagnetic.
(E) Acidified permanganate ion reduces oxalate, nitrite and iodide ions.
Choose the correct answer from the options given below:
(1) A, C and D only (2) A, B and C only
(3) A and D only (4) A, D and E only

Ans : (4)

Sol : $Mn^{2+} + S_2O_8^{2-} \rightarrow MnO_4^-$

So, (C) is incorrect.

62. The correct increasing order of spin-only magnetic moment values of the complex ions

$[MnBr_4]^{2-}$ (A) , $\left[Cu\left(H_2O\right)_6\right]^{2+}$ (B), $\left[Ni\left(CN\right)_4\right]^{2-}$ (C) and $\left[Ni\left(H_2O\right)_6\right]^{2+}$ (D) is:
Select correct order of spin only magnetic moment among above complexes.
(1) A = B < D < C (2) C = D < B < A (3) A = B < C < D (4) C < B < D < A

Ans : (4)

Sol : $Mn^{2+} 3d^5 n = 5$

$Cu^{2+} 3d^9 t_g^{2.2.2}\ e_g^{2.1}\ n = 1$

$Ni^{2+} 3d^8$ square planar $n = 0$

$Ni^{2+} 3d^8$ tetrahedral $e^{2.2}\ t_2^{2.1.1}\ n = 2$

63. Consider the elements N, P, O, S, Cl and F. The number of valence electrons present in the elements with most and least metallic character from the above list is respectively.

(1) 5 and 6 (2) 7 and 5 (3) 6 and 7 (4) 5 and 7

Ans : (4)

Sol : Least metallic = F, valence electrons = 7

Most metallic = P, valence electrons = 5

64. The wavelength of photon 'A' is 400 nm. The frequency of photon 'B' is $10^{16} s^{-1}$. The wave number of photon 'C' is $10^4 cm^{-1}$. The correct order of energy of these photons is:

(1) C > B > A (2) A > C > B (3) A > B > C (4) B > A > C

Ans : (4)

Sol : (1) Wavelength of A = 400 nm.

(2) $v = \frac{C}{\lambda} \Rightarrow$ wavelength of $B(\lambda) = \frac{3\times10^8}{10^{16}}$

$= 3\times10^{-8} = 30\times10^{-9} = 30\,nm.$

(3) Wavelength of $C(\lambda) = \frac{1}{v} = \frac{1}{10^4} = 10^{-4} cm$

$= 10^{-6} m = 1000\,nm$

Here $\lambda_C > \lambda_A > \lambda_B$

$E \propto \frac{1}{\lambda}$

So $E_C < E_A < E_B$

65. Match List – I with List – II according to shape.

List - I	List – II
(A) XeO_3	(I) BrF_5
(B) XeF_2	(II) NH_3
(C) XeO_2F_2	(III) $[I_3]^-$
(D) $XeOF_4$	(IV) SF_4

Choose the correct answer from the options given below:

(1) A-II, B-III, C-IV, D-I (2) A-III, B-II, C-IV, D-I

(3) A-II, B-I, C-III, D-IV (4) A-II, B-III, C-I, D-IV

Ans : (1)

Sol : XeF_2 & I_3^- : 2 bond pair 3 lone pair

$XeOF_4$ & BrF_5 : 5 bond pair 1 lone pair

XeO_2F_2 & SF_4 : 4 bond pair 1 lone pair

XeO_3 & NH_3 : 3 bond pair 1 lone pair

66. Total number of alkali insoluble solid sulphonamides obtained by reaction of given amines with Hinsberg's reagent is _____.

Aniline, N-Methylaniline, Methanamine, N, N – Dimethylmethamine, N – Methyl methanamine, Phenylmethanamine, N-Propylaniline, N-Phenylaniline, N, N-Dimethylaniline, Allyl amine, Isopropyl amine

(1) 2 (2) 4 (3) 5 (4) 8

Ans : (3)

Sol : Secondary amine are insoluble alkali after reaction with Hinsberg reagent.

67. The plot of $\log_{10} K$ vs $\frac{1}{T}$ gives a straight line. The intercept and slope respectively are (where K is equilibrium constant).

(Where K is the equilibrium constant).

(1) $-\frac{\Delta S^\circ R}{2.303}, \frac{\Delta H^\circ R}{2.303}$ (2) $-\frac{\Delta H^\circ}{2.303R}, \frac{\Delta S^\circ}{2.303R}$

(3) $\frac{2.303R}{\Delta H^\circ}, \frac{2.303R}{\Delta S^\circ}$ (4) $\frac{\Delta S^\circ}{2.303R}, \frac{-\Delta H^\circ}{2.303R}$

Ans : (4)

Sol : $\log K = -\frac{\Delta H^\circ}{2.303RT} + \frac{\Delta S^\circ}{2.303R}$

Slope $= -\frac{\Delta H^\circ}{2.303R}$,

y-intercept $= \frac{\Delta S^\circ}{2.303R}$

68. The reactions which produce alcohol as the product are:

A. $CH_4 + O_2 \xrightarrow[\Delta]{Mo_2O_3}$

B. $2CH_3CH_3 + 3O_2 \xrightarrow[\Delta]{(CH_3COO)_2Mn}$

C. $(CH_3)_3CH \xrightarrow{KMnO_4}$

D. $2CH_4 + O_2 \xrightarrow{Cu/523\ K/100\ atm.}$

E. $CH_3-CH=CH-CH_3 \xrightarrow{KMnO_4/H^+}$

Choose the correct answer from the options given below:

(1) C and D only (2) A, C and E only

(3) A and D only (4) B, D and E only

Ans : (1)

Sol : $KMnO_4$ is a good oxidizing agent and it oxidises $NO_2^-, C_2O_4^{2-}$ and I^- ions.

69. Consider the following aqueous solutions.

I. 2.2 g Glucose in 125 mL of solution.

II. 1.9 g Calcium chloride in 250 mL of solution.
III. 9.0 g Urea in 500 mL of solution.
IV. 20.5 g Aluminium sulphate in 750 mL of solution.
The correct increasing order of boiling point of these solutions will be:
[Given: Molar mass in g mol^{-1}: H = 1, C = 12, N = 14, O = 16, Cl = 35.5, Ca = 40, Al = 27 and S = 32]
(1) III < I < II < IV (2) I < II < III < IV (3) II < III < IV < I (4) II < III < I < IV

Ans : (2)

Sol : $\Delta T_b = i.k_b.m$

For dilute solution (M = m)

Molarity	i × m
$M_{glucose} = \frac{0.025}{180} \times \frac{1000}{125} = 0.09$	0.09 × 1
$M_{urea} = \frac{9}{60} \times \frac{1000}{500} = 0.3$	0.3 × 1
$M_{CaCl_2} = \frac{1.9}{111} \times \frac{1000}{250} = 0.068$	0.068 × 3
$M_{Al_2(SO_4)_2} = \frac{20.5}{342} \times \frac{1000}{750} \simeq 0.08$	0.08 × 5

Order of $\Delta T_b = Al_2(SO_4)_3$ > Urea > $CaCl_2$ > Glucose

70. Consider the following reactions

$$Na_2B_4O_7 \xrightarrow{\Delta} 2X + Y$$

$$CuSO_4 + Y \xrightarrow{\text{Non-Luminous flame}} Z + SO_3$$

$$2Z + 2X + \text{Carbon} \xrightarrow{\text{Luminous flame}} 2Q + Na_2B_4O_7 + CO$$

Oxidation state of Cu in Z and Q, respectively are:

(1) +2 and +1 (2) +1 and +2 (3) +2 and +2 (4) +1 and +1

Ans : (1)

Sol : $Na_2B_4O_7.10H_2O \xrightarrow[-10H_2O]{\Delta} Na_2B_4O_7$

$$\xrightarrow{\Delta} 2NaBO_2 + B_2O_3$$

$$CuSO_4 + B_2O_3 \xrightarrow[\text{Non-Luminous flame}]{\Delta} \underset{\substack{\text{(Blue bead)}\\(Z)}}{\overset{+2}{Cu}}(BO_2)_2$$

$$\underset{(Z)}{Cu(BO_2)_2} \xrightarrow[\text{Luminous Flame}]{\Delta} \underset{\substack{\text{Colorless}\\(Q)}}{\overset{+1}{Cu}}BO_2$$

Oxidation states of Cu in Z and Q are +2 & +1.

SECTION – B

71. Fir strong electrolyte Λ_m increases slowly with dilution and can be represented by the equation

$\Lambda_m = \Lambda_m^0 - A^{1./2}$

Molar conductivity values of the solutions of strong electrolyte AB at 18°C are given below:

c[mol L^{-1}]	0.04	0.09	0.16	0.25
Λ_m [S cm^2 mol^{-1}]	96.1	95.7	95.3	94.9

The value of constant A based on the above data [in S cm^2 mol^{-1}/$(mol/L)^{1/2}$] unit is _____.

Ans : (4)

Sol : Using equation: $\Lambda_m = \Lambda_m^0 - A\sqrt{c}.$

$95.7 = \Lambda_m^0 - A\sqrt{0.04}$

$95.7 = \Lambda_m^0 - A \times 0.2$ ... (1)

$95.3 = \Lambda_m^0 - A \times \sqrt{0.09}$

$95.3 = \Lambda_m^0 - A \times 0.3$ ... (2)

From eq. (1) and eq. (2)

A = 4

72. A volume of x mL of 5 M $NaHCO_3$ solution was mixed with 10 mL of 2 M H_2CO_3 solution to make an electrolyte buffer. If the same buffer was used in the following electrochemical cell to record a cell potential of 235.3 mV, then the value of $x =$ _____ mL (nearest integer).

$Sn(s) \mid Sn(OH)_6^{2-}(0.5M) \mid HSnO_2^-(0.05M) \mid OH^- \mid Bi_2O_3(s) \mid Bi(s)$

Consider upto one place of decimal for intermediate calculations

$$\text{Given: } \begin{bmatrix} E^o_{HSnO_2^- \mid Sn(OH)_6^{2-}} = -0.9V \\ E^o_{Bi_2O_3 \mid Bi} = -0.44V \\ pKa_{(H_2CO_3)} = 6.11 \\ \frac{2.303RT}{F} = 0.059V \\ \text{Antilog } (1.29) = 19.5 \end{bmatrix}$$

Ans : (9819)

Sol : Calculate the standard cell potential $\left(E^o_{cell}\right)$

The overall reaction is:

Anode (Oxidation):

$Sn(OH)_2 + 2OH^- \rightarrow HSnO^- + 2H_2O + 2e^-$

Cathode (Reduction):

$$Bi_2O_3 + 3H_2O + 6e^- \rightarrow 2Bi + 6OH^-$$

The number of electrons transferred, n, is 6.

$$E^o_{\text{cell}} = E^o_{\text{cathode}} - E^o_{\text{anode}}$$

$$E^o_{\text{cell}} = -0.44V - (-0.9V) = 0.46V$$

Calculate the reaction quotient (Q) using the Nernst equation

The cell potential is given as

235.3 mV = 0.2353 V.

The Nernst equation is:

$$E_{\text{cell}} = E^o_{\text{cell}} - \frac{2.303\,RT}{n}\log Q$$

$$0.2353V = 0.46V - \frac{0.059V}{6}\log Q$$

$$\frac{0.059}{6}\log Q = 0.46V - 0.2353V = 0.2247V$$

$$\log Q = \frac{0.2247 \times 6}{0.059} \approx 22.85$$

$$Q = 10^{22.85} \approx 7.08 \times 10^{22}$$

Express the reaction quotient in terms of concentrations

The balanced overall reaction is:

$$3Sn(OH)_2 + 6OH^- + Bi_2O_3 \rightarrow 3HSnO^-$$

Note: Solids and water are not included in the expression for Q.

$$Q = \frac{\left[HSnO^-\right]^3}{\left[OH^-\right]^6}$$

$$7.08 \times 10^{22} = \frac{(0.05M)^3}{[OH^-]^6}$$

$$\left[OH^-\right]^6 = \frac{(0.05)^3}{7.08 \times 10^{22}} \approx 1.76 \times 10^{-27}$$

$$\left[OH^-\right] = \left(1.76 \times 10^{-27}\right)^{1/6} \approx 10^{-4.5}M$$

Calculate the pOH and pH of the buffer solution

$$pOH = -\log\left[OH^-\right] = 4.5$$

$$pH = 14 - pOH = 14 - 4.5 = 9.5$$

Use the Henderson-Hasselbalch equation to find the ratio of salt to acid

The buffer consist of $NaHCO_3$ (salt, HCO^-) and H_2CO_3 (acid).

$$pH = pK_a + \log\left(\frac{[\text{Salt}]}{[\text{Acid}]}\right)$$

$$9.5 = 6.11 + \log\left(\frac{[HCO^-]}{[H_2CO_3]}\right)$$

$$\log\left(\frac{[HCO^-]}{[H_2CO_3]}\right) = 9.5 - 6.11 = 3.39$$

$$\frac{[HCO^-]}{[H_2CO_3]} = 10^{3.39} \approx 2454.7$$

Calculate the value of x

The ratio of concentrations is equal to the ratio of moles since they are in the same solution after mixing.

$$\frac{\text{moles of } NaHCO_3}{\text{moles of } H_2CO_3} = 2454.7$$

Moles = Molarity × Volume (L)

$$\frac{5M \times x\ mL}{2m \times 10mL} = 2454.7$$

$$\frac{5x}{20} = 2454.7$$

$$x = \frac{2454.7 \times 20}{5} \approx 9818.8$$

The value of x is approximately 9819 mL (nearest integer).

73.

$$A \longrightarrow B \text{ (first reaction)}$$

$$C \longrightarrow D \text{ (second reaction)}$$

Consider the above two first-order reactions. The rate constant for first reaction at 500 K is double of the same at 300 K. At 500 K, 50% of the reaction becomes complete in 2 hour. The activation energy of the second reaction is half of that of first reaction. If the rate constant at 500 K of the second reaction becomes double of the rate constant of first reaction at the same temperature; then rate constant for the second reaction at 300 K is _____ 10^{-1} $hour^{-1}$ (nearest integer).

Ans : (5)

Sol :

For A → B

$$\ln(2)=\frac{E_{a_1}}{R}\left[\frac{1}{300}-\frac{1}{500}\right]$$

$$E_{a_1}=\frac{\ln 2\times R\times 1500}{2}$$

$$E_{a_2}=\frac{E_{a_1}}{2}=\frac{\ln 2\times R\times 1500}{4}$$

$$(K_1)_{\text{at 500 K}}=\frac{\ln 2}{2}$$

$$(K_1)_{\text{at 500 K}}=\ln 2$$

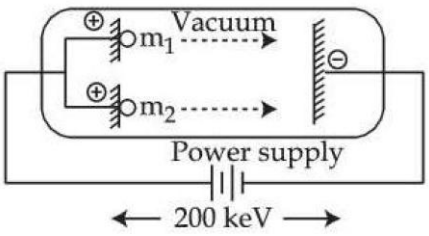
Now for C → D

$$\ln\left[\frac{(K_2)_{\text{at 500 K}}}{(K_2)_{\text{at 300 K}}}\right]=\left(\frac{\ln 2\times R\times 1500}{4}\right)\times\frac{1}{R}\times\left[\frac{1}{300}-\frac{1}{500}\right]$$

$$(K_2)_{\text{at 300 K}}=\frac{\ln 2}{\sqrt{2}}=0.49$$

$$(K_2)_{\text{at 300 K}}\times 10=4.9\simeq 5$$

74. Two positively charged particle m_1 and m_2 have been accelerated across the same potential difference of 200 keV as shown below.



[Given mass of $m_1=1$ amu and $m_2=4$ amu]

The deBroglie wavelength of m_1 will be x times of m_2. The value of x is _____. (nearest integer)

Ans : (2)

Sol : $\lambda_d=\frac{h}{\sqrt{2\text{m K.E.}}}$

Here KE is same i.e. 200 keV

So $\lambda_d \propto \frac{1}{\sqrt{m}}$

$$\frac{(\lambda_d)_{m_1}}{(\lambda_d)_{m_2}} = \sqrt{\frac{m_2}{m_1}} = \sqrt{4} = 2$$

$$(\lambda_d)_{m_1} = 2(\lambda_d)_{m_2}$$

So $x = 2$.

75. The number of isoelectronic species among $Sc^{3+}, Cr^{2+}, Mn^{3+}, Co^{3+}$ and Fe^{3+} is 'n'. If 'n' moles of AgCl is formed during the reaction of complex with formula $CoCl_3(en)_2NH_3$ with excess of $AgNO_3$ solution, then the number of electrons present in the t_{2g} orbital of the complex is _____.

Ans : (6)

Sol : Cr^{2+} and Mn^{3+} are isoelectronic

$n = 2$

Complex is: $[Co(en)_2NH_3Cl]Cl_2$

$\Rightarrow Co^{3+}\ 3d^6\ t_{2g}^{2.2.2}\ e_g^{0.0}$