JEE-MAIN EXAMINATION – JANUARY 2026

(22ND JANUARY 2026) **TIME: 9:00 A.M. TO 12:00 NOON**

MATHEMATICS TEST PAPER WITH SOLUTION

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer :

1. Let f and g be functions satisfying $f(x+y)=f(x)f(y)$, $f(1)=7$ and $g(x+y)=g(xy), g(1)=1$, for all $x, y \in \mathbb{N}$. If $\sum_{x=1}^{n}\left(\frac{f(x)}{g(x)}\right)=19607$, then n is equal to:

(1) 7
(2) 6
(3) 4
(4) 5

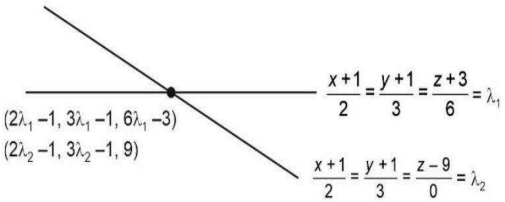
Answer (4)

Sol. $\because f(x+y)=f(x)f(y)$
$\Rightarrow f(x)=a^x$
$\because f(1)=7 \Rightarrow a=7 \Rightarrow f(x)=7^x, \forall x \in N$
$\because g(x+y)=g(xy), g(1)=1$
Put $x=y=1 \Rightarrow g(2)=g(1)=1$
Put $x=1, y=2 \Rightarrow g(3)=g(2)=1$
Put $x=1, y=3 \Rightarrow g(4)=g(3)=1$
$\Rightarrow g(x)=1, \forall x \in N$
$\Rightarrow \sum_{x=1}^{n}\left(\frac{f(x)}{g(x)}\right)=\sum_{x=1}^{n}7^x=\frac{7(7^n-1)}{7-1}$
$\Rightarrow \frac{7}{6}(7^n-1)=19607$
$\Rightarrow 7^n=16807=7^5 \Rightarrow n=5$
$\Rightarrow$ Option (4) is Correct.

2. Let L be the line $\frac{x+1}{2}=\frac{y+1}{3}=\frac{z+3}{6}$ and let S be the set of all points (a, b, c) on L, whose distance from the line $\frac{x+1}{2}=\frac{y+1}{3}=\frac{z-9}{0}$ along the line L is 7. Then $\sum_{(a,b,c)\in S}(a+b+c)$ is equal to:

(1) 40
(2) 34
(3) 28
(4) 6

Answer (2)
Sol. Let $P(a, b, c)=(2t-1, 3t-1, 6t-3)$



$\Rightarrow \lambda_1 = \lambda_2 = 2$
$\Rightarrow$ Intersection point is $Q(3,5,9)$
$PQ = 7$
$\Rightarrow (2t-4)^2 + (3t-6)^2 + (6t-12)^2 = 49$
$\Rightarrow t^2(4+9+36) + t(-16-36-144) + 16$

$$+36 + 144 = 49$$

$t^2 - 4t + 4 = 1$
$t^2 - 4t + 3 = 0$
$t = 1,3$
$\Rightarrow P(1,2,3)$ or $P(5,8,15)$
$\Rightarrow \sum_{(a,b,c)\in S}(a+b+c) = 6 + 28 = 34$
$\Rightarrow$ Option (2) is Correct.

3. Let n be the number obtained on rolling a fair die. If the probability that the system
$x - ny + z = 6$
$x + (n-2)y + (n+1)z = 8$
$(n-1)y + z = 1$

has a unique solution is $\frac{k}{6}$, then the sum of k and all possible values of n is:
(1) 21
(2) 22
(3) 20
(4) 24

Answer (2)
Sol. $\Delta = -n^2 + 3n - 2$

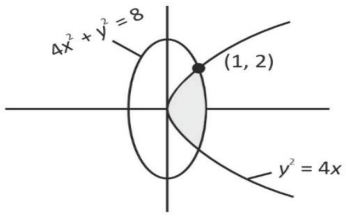
$$\Delta_1 = -7n^2 + 20n - 12$$
$$\Delta_2 = -n + 2$$
$$\Delta_3 = 0$$
$$\Rightarrow x = \frac{7n-6}{n-1}, y = \frac{1}{n-1}, z = 0$$

for unique solution $\Delta \neq 0, n^2 - 3n + 2 \neq 0 \Rightarrow n \neq 1,2$ possible values of n are 3,4,5,6
$\Rightarrow P(E) = \frac{k}{6} = \frac{4}{6} \Rightarrow k = 4$
$\Rightarrow$ Answer is $4 + (3+4+5+6) = 22$
Option (2) is Correct.

4. The area of the region $A = \{(x,y): 4x^2 + y^2 \leq 8 \text{ and } y^2 \leq 4x\}$ is:
(1) $\frac{\pi}{2} + 2$
(2) $\pi + 4$
(3) $\frac{\pi}{2} + \frac{1}{3}$
(4) $\pi + \frac{2}{3}$

Answer (4)

Sol.



Area $= 2\int_0^2 \sqrt{\frac{8-y^2}{4}} - \frac{y^2}{4} dx$

$$= \int_0^2 \left(\sqrt{8-y^2} - \frac{y^2}{2}\right) dy$$

$$= 4\sin^{-1}\left(\frac{y}{2^{3/2}}\right) + \frac{y\sqrt{8-y^2}}{2} - \frac{y^3}{6}\Bigg]_0^1$$

$$= 4\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \frac{2\times 2}{2} - \frac{8}{6}$$

$$= \frac{4\pi}{4} + 2 - \frac{4}{3}$$

$$= \left(\pi + \frac{2}{3}\right) \text{ sq. unit}$$

5. If $y = y(x)$ satisfies the differential equation $16(\sqrt{x+9\sqrt{x}})(4+\sqrt{9+\sqrt{x}})\cos y dy = (1+2\sin y)dx, x > 0$ and $y(256) = \frac{\pi}{2}, y(49) = \alpha$, then $2\sin\alpha$ is equal to:
(1) $3(\sqrt{2}-1)$
(2) $\sqrt{2}-1$
(3) $2(\sqrt{2}-1)$
(4) $2\sqrt{2}-1$

Answer (Bonus)

6. If $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is a solution of the system of equations $AX = B$, where adj $A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$, then $|x+y+z|$ is equal to:
(1) 1
(2) 3
(3) $\frac{3}{2}$
(4) 2

Answer (4)

Sol. $\text{adj}(A) = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$$\text{adj}(A) = |A| \cdot A^{-1}$$
$$AX = B$$
$$\Rightarrow X = A^{-1}B$$

$\frac{\text{adj}(A)}{|A|} \cdot B$

$\text{adj}(A) = |A|^2 = 4(10) - 2(-20) + 2(10)$

$= 100$

$|A| = \pm 10$

$X = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \cdot \frac{1}{10}$

$= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

$x = 2, y = -1, z = 1$

$|x + y + z| = 2$

7. Let the locus of the mid-point of the chord through the origin O of the parabola $y^2 = 4x$ be the curve S . Let P be any point on S . Then the locus of the point, which internally divides OP in the ratio $3:1$, is:

(1) $3x^2 = 2y$

(2) $2x^2 = 3y$

(3) $3y^2 = 2x$

(4) $2y^2 = 3x$

Answer (4)

Sol. $O(0,0), A(t^2, 2t)$

$M(h,k) \equiv \left(\frac{t^2}{2}, t\right)$

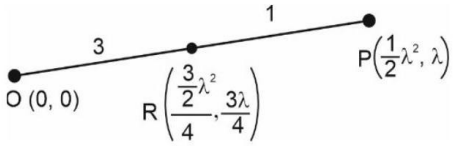
$h = \frac{t^2}{2}, k = t$

$h = \frac{k^2}{2}$

$\Rightarrow$ Locus of $M(h,k)$ is $y^2 = 2x$

$S: y^2 - 2x = 0$

$P\left(\frac{1}{2}\lambda^2, \lambda\right)$



Let $R \equiv (h,k)$

$\Rightarrow h = \frac{3}{8}\lambda^2, k = \frac{3\lambda}{4}$

$\Rightarrow h = \frac{3}{8}\left(\frac{4k}{3}\right)^2$

locus is

$x = \frac{3}{8} \times \frac{16}{9} y^2$

$3x = 2y^2$

8. Let $S = \{z \in \mathbb{C}: 4z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} |z|^2$ is equal to:

(1) $\frac{5}{64}$

(2) $\frac{7}{64}$

(3) $\frac{1}{16}$

(4) $\frac{3}{16}$

Answer (4)

Sol. $(4z^2) = -\bar{z}$

$= |4z^2| = |-\bar{z}|$

$= 4|z^2| = |z|$

$= |z| = 0, |z| = \frac{1}{4}$

$\bar{z} = -4z^2$

$z \cdot \bar{z} = -4z^3$

$-4 \cdot z^3 = \frac{1}{16}$

$z^3 = -\frac{1}{64}$

Total three roots

$\sum_{z \in S} |z|^2 = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + 0 = \frac{3}{16}$

9. Let C_r denote the coefficient of x^r in the binomial expansion of $(1+x)^n, n \in N, 0 \le r \le n$. If $P_n = C_0 - C_1 + \frac{2^2}{3}C_2 - \frac{2^3}{4}C_3 + \cdots + \frac{(-2)^n}{n+1}C_n$, then the value of $\sum_{n=1}^{25} \frac{1}{P_{2n}}$ equals.

(1) 525

(2) 650

(3) 580

(4) 675

Answer (4)

Sol. $P_n = \sum_{r=0}^{n} \left(\frac{2^r}{r+1}\right)(-1)^r \left(n_{c_r}\right)$

$= \frac{1}{2(n+1)} \sum_{r=0}^{n} {}^{n+1}C_{r+1} \cdot 2^{(r+1)}(-1)^r$

$= \frac{-1}{2(n+1)} \sum_{r=0}^{n} {}^{n+1}C_{r+1} \cdot (-2)^{(r+1)}$

$= \frac{-1}{2(n+1)} [(1-2)^{n+1} - {}^{n+1}C_0(-2)^0]$

$= \frac{-1}{2(n+1)} [(-1)^{n+1} - 1]$

$P_{2n} = \frac{-1}{2(2n+1)}[-2] = \frac{1}{(2n+1)}$

$\Rightarrow \frac{1}{P_{2n}} = (2n+1) \Rightarrow \sum_{n=1}^{25}(2n+1) = 675$

10. Let S and S' be the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $P(\alpha, \beta)$ be a point on the ellipse in the first quadrant. If $(SP)^2 + (S'P)^2 - SP \cdot S'P = 37$, then $\alpha^2 + \beta^2$ is equal to:

(1) 11

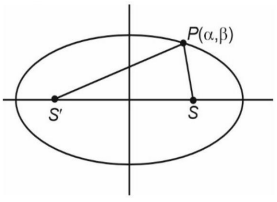
(2) 17

(3) 15

(4) 13

Answer (4)

Sol.



$PS + PS' = 2(S) = 10$

$(PS + PS')^2 = (37 + SP \cdot SP') + 2SP \cdot SP' = 100$

$\Rightarrow 3SP \cdot SP' = 63 \Rightarrow SP \cdot SP' = 21$

$\Rightarrow SP, SP' \in \{3,7\}$

$e = \sqrt{1 - \frac{9}{25}} = \frac{4}{5} \Rightarrow ae = 4$

$\sqrt{(\alpha+4)^2 + \beta^2} = 7$

$\sqrt{(\alpha-4)^2 + \beta^2} = 3$

$\Rightarrow (\alpha+4)^2-(\alpha-4)^2=49-9=40$

$\Rightarrow (2\alpha)(8)=40 \Rightarrow \alpha=\frac{40}{16}=\frac{5}{2}$

$\Rightarrow \beta^2+\frac{169}{4}=49 \Rightarrow \beta^2=\frac{27}{4}$

$\Rightarrow \alpha^2=\frac{25}{4}$

$\Rightarrow \alpha^2+\beta^2=\frac{52}{4}=13$

11. If $\lim_{x\to 0}\frac{e^{(a-1)x}+2\cos bx+(c-2)e^{-x}}{x\cos x-\log_e(1+x)}=2$ then $a^2+b^2+c^2$ is equal to:

(1) 5
(2) 9
(3) 3
(4) 7

Answer (4)

Sol. $\lim_{x\to 0}\frac{+\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots\right)(c-2)}{x\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots\right)-\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots\right)}$

$$\left(1+x(a-1)+\frac{(x(a-1)^2}{2!}+\cdots\right)+2\left(1-\frac{b^2x^2}{2!}+\frac{b^4x^4}{4!}+\cdots\right)$$

$$1+2+(c-2)+x(a-1-c+2)x^2$$

$$\Rightarrow\frac{\left(\frac{(a-1)^2}{2!}-b^2+\frac{(c-2)}{2}\right)+\cdots}{\frac{x^2}{2}-\frac{5}{6}x^3}$$

$3+c-2=0$

$\Rightarrow c=-1$

$a-c+1=0$

$\Rightarrow a=-2$

$\frac{\frac{9}{2}-b^2-\frac{3}{2}}{\frac{1}{2}}=2$

$\Rightarrow b^2=2$

$a^2+b^2+c^2=4+2+1=7$

12. Let $[\cdot$ denote the greatest integer function, and let $f(x)=\min\{\sqrt{2}x,x^2\}$. Let $S=\{x\in(-2,2)$: the function $g(x)=|x|[x^2]$ is discontinuous at $x\}$. Then $\sum_{x\in S}f(x)$ equals

(1) $1-\sqrt{2}$
(2) $\sqrt{6}-2\sqrt{2}$
(3) $2-\sqrt{2}$
(4) $2\sqrt{6}-3\sqrt{2}$

Answer (1)

Sol. $f(x)=\min\{\sqrt{2}x,x^2\}$

$g(x)=|x|[x^2].x\in(-2,2)$

$g(x)$ is discontinuous at $x=-1,1,-\sqrt{2},\sqrt{2},\sqrt{3},-\sqrt{3}$

$\sum_{x\in S}f(x)=-\sqrt{6}-2-\sqrt{2}+1+2+\sqrt{6}$

$=1-\sqrt{2}$

13. Let $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}\ \vec{b} = \lambda\hat{\jmath} + 2\hat{k}\ \lambda \in Z$ be two vectors. Let $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d}$ be a vector of magnitude 2 in yz-plane. If $|\vec{c}| = \sqrt{53}$, then the maximum possible value of $(\vec{c} \cdot \vec{d})^2$ is equal to:
(1) 208
(2) 52
(3) 26
(4) 104

Answer (1)

Sol. $\vec{c} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 0 & \lambda & 2 \end{vmatrix} = -(2+\lambda)\hat{\imath} - 4\hat{\jmath} + 2\lambda\hat{k}$

$|\vec{c}| = 53$

$(2+\lambda)^2 + 16 + 4\lambda^2 = 53$

$\Rightarrow 5\lambda^2 + 4\lambda - 33 = 0$

$\Rightarrow 5\lambda^2 + 15\lambda - 11\lambda - 33 = 0$

$\Rightarrow (5\lambda - 11)(\lambda + 3) = 0$

$\lambda = -3$ ($\because \lambda$ is an integer)

let $\vec{d} = b\hat{\jmath} + c\hat{k}$

$|\vec{d}| = 2 \Rightarrow b^2 + c^2 = 4$

$\vec{c} \cdot \vec{d} = -4b - 6c$

$(\vec{c} \cdot \vec{d})^2 = (4b + 6c)^2$

Using Cauchy-Schwartz inequality

$$(ax + by)^2 \leq (a^2 + b^2)(x^2 + y^2)$$
$$(4b + 6c)^2 \leq (4^2 + b^2)(b^2 + c^2)$$
$$\leq 52 \times 4 \leq 208$$

14. Let $P(10, 2\sqrt{15})$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose foci are S and $S\hat{I}$. If the length of its latus rectum is 8 , then the square of the area of $\triangle PSS'$ is equal to:
(1) 2700
(2) 4200
(3) 1462
(4) 900

Answer (1)

Sol. $\frac{100}{a^2} - \frac{60}{b^2} = 1$

Also $\frac{2b^2}{a} = 8$

$\Rightarrow b^2 = 4a$

$\frac{100}{a^2} - \frac{60}{4a} = 1$

$100 - 15a = a^2$

$a^2 + 15a - 100 = 0$

$(a + 20)(a - 5) = 0$

$a = -20, 5$

$\because b^2 = 4a \Rightarrow a$ cannot be negative

$\Rightarrow a = 5$

$$b^2 = 20$$

Now $H : \frac{x^2}{25} - \frac{y^2}{20} = 1$

$e^2 = 1 + \frac{20}{25} \Rightarrow e = \frac{3}{\sqrt{5}}$

$F:(\pm 3\sqrt{5},0)$ i.e S_1 & S_2

Area $PS_1S_2 = \frac{1}{2}\begin{vmatrix} 10 & 2\sqrt{15} \\ 3\sqrt{5} & 0 \\ -3\sqrt{5} & 0 \\ 10 & 2\sqrt{15} \end{vmatrix} = \frac{1}{2}[-6\sqrt{75} - 6\sqrt{75}]$

$$= 6\sqrt{75}$$

Area $^2 = 2700$

15. Let α, β be the roots of the quadratic equation $12x^2 - 20x + 3\lambda = 0, \lambda \in Z$. If $\frac{1}{2} \leqslant |\beta - \alpha| \leqslant \frac{3}{2}$, then the sum of all possible values of λ is

(1) 4
(2) 6
(3) 1
(4) 3

Answer (4)

Sol. $|\beta - \alpha|^2 \in \left[\frac{1}{4}, \frac{9}{4}\right]$

$$(\alpha^2 + \beta^2 - 2\alpha\beta) = (\alpha + \beta)^2 - 4\alpha\beta \in \left[\frac{1}{4}, \frac{9}{4}\right]$$
$$= \left(\frac{20}{12}\right)^2 - 4\left(\frac{3\lambda}{12}\right) \in \left[\frac{1}{4}, \frac{9}{4}\right]$$
$$\Rightarrow \frac{25}{9} - \lambda \in \left[\frac{1}{4}, \frac{9}{4}\right]$$
$$\Rightarrow 100 - 36\lambda \in [9,81]$$
$$36\lambda - 100 \in [-81, -9]$$
$$36\lambda \in [19,91]$$
$$\lambda \in \left[\frac{19}{36}, \frac{91}{36}\right]$$

then integral values of λ is 1,2

$\Rightarrow$ Sum of integral values of λ is $1 + 2 = 3$

16. Among the statements

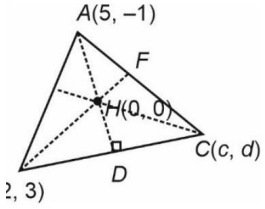
(S1) : If $A(5,-1)$ and $B(-2,3)$ are two vertices of a triangle, whose orthocentre is $(0,0)$, then its third vertex is $(-4,-7)$ and

(S2) : If positive numbers $2a, b, c$ are three consecutive terms of an A.P., then the lines $ax + by + c = 0$ are concurrent at $(2,-2)$,

(1) both are incorrect
(2) both are correct
(3) only (S1) is correct
(4) only (S2) is correct

Answer (2)

Sol. (S1)



$B(-2,3)$

$$\left.\begin{matrix} M_{CH}: \frac{d}{c} \\ M_{AB}: -\frac{4}{7} \end{matrix}\right\} \Rightarrow \frac{4d}{72} = 1 \Rightarrow 4d = 7c \quad (1)$$

$M_{BH} \cdot M_{AC} = \frac{d+1}{c-5} \times \left(\frac{-3}{2}\right) = -1$

$3d + 3 = 2c - 10$ solving (1) & (2)

$$2c - 3d = 13 \ \ldots(2)\ (c,d) \equiv (-4,-7) \qquad (2)$$

S 1 is true.

(S2) : 2a, $b, c \rightarrow$ AP

$\Rightarrow 2b = 2a + c$

$ax + by + c = 0$

$ax + by + 2b - 2a = 0$

$a(x-2) + b(y+2) = 0$

$(x-2) + \lambda(y+2) = 0$

$L_1 + \lambda L_2 = 0$

$\Rightarrow (x, y) = (2, -2)$ concurrent: $\therefore$ (S2 is true)

17. If the mean deviation about the median of the numbers $k, 2k, 3k, \ldots\ldots, 1000k$ is 500 , then k^2 is equal to

(1) 16

(2) 4

(3) 1

(4) 9

Answer (2)

Sol. data set

$k, 2k, 3k, \ldots\ldots, 1000k.$

Median $= \frac{500k+501k}{2} = 500.5k$

Mean-deviation about median

$$500 = \frac{|500.5k - k| + |500.5k - 2k| + \cdots + |1000k - 500.5k|}{1000}$$

$$= \frac{2[499.5k + 498.5k + \cdots + 0.5k]}{1000}$$

$$250000 = \frac{500}{2}[0.5 + 499.5]k$$

$\Rightarrow\ k = 2$

$k^2 = 4$

18. Let $f(x) = [x]^2 - [x+3] - 3, x \in \mathbf{R}$, where $[\cdot]$ is the greatest integer function. Then

(1) $f(x) = 0$ for finitely many values of x

(2) $f(x) > 0$ only for $x \in [4, \infty)$

(3) $f(x) < 0$ only for $x \in [-1,3)$

(4) $\int_0^2 f(x)dx = -6$

Answer (3)

Sol. $f(x) = [x]^2 - [x+3] - 3$

$f(x) = [x]^2 - [x] - 6$

$= ([x] - 3)([x] + 2)$

$f(x) = 0$ for $x \in [-2,1) \cup [3,4)$

$f(x) > 0$

$\Rightarrow ([x] - 3)([x] + 2) > 0$

$[x] \in (-\infty, -2) \cup (3, \infty)$

$x \in (-\infty, -2) \cup [4, \infty)$

$f(x) < 0$

$x \in [-1,3)$

19. Let the domain of the function $f(x) = \log_3 \log_5 \left(7 - \log_2 (x^2 - 10x + 85)\right) + \sin^{-1}\left(\left|\frac{3x-7}{17-x}\right|\right)$ be $(\alpha, \beta]$. Then $\alpha + \beta$ is equal to:

(1) 8
(2) 12
(3) 10
(4) 9

Answer (4)

Sol. $\log_5 \left(7 - \log_2 (x^2 - 10x + 85)\right) > 0$

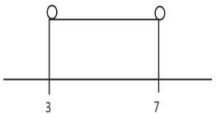
$\Rightarrow 7 - \log_2 (x^2 - 10x + 85) > 1$

$\Rightarrow \log_2 (x^2 - 10x + 85) < 6$

$\Rightarrow x^2 - 10x + 85 < 64$

$\Rightarrow x^2 - 10x + 21 < 0$

$(x - 7)(x - 3) < 0$



$\Rightarrow x < (3,7)$

$\left|\frac{3x-7}{17-x}\right| \leq 1$

$\Rightarrow |3x - 7| \leq |17 - x|$

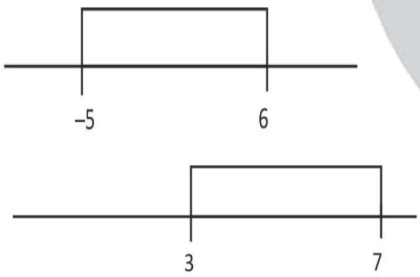
$\Rightarrow 9x^2 + 49 - 42x \leq 289 + x^2 - 34x$

$\Rightarrow 8x^2 - 8x - 240 \leq 0$

$\Rightarrow x^2 - x - 30 \leq 0$

$(x - 6)(x + 5) \leq 0$

$x \in [-5,6]$



$x \in (3,6]$

20. The number of elements in the relation $R = \{(x, y): 4x^2 + y^2 < 52, x, y \in \mathbf{Z}\}$ is

(1) 77
(2) 89
(3) 67
(4) 86

Answer (1)

Sol. $4x^2 + y^2 < 52, x, y \in I$

$\Rightarrow 4x^2 < 52 \Rightarrow x^2 < 13 \Rightarrow x^2 \in \{0,1,4,9\}$

If $x^2 = 0 \Rightarrow y^2 < 52 \Rightarrow y^2 \in \{0,1,4,9,16,25,36,49\}$

$1 \times (1 + 2 \times 7) = 15$

If $x^2 = 1 \Rightarrow y^2 < 48 \Rightarrow y^2 \in \{0,1,4,9,16,25,36\}$

$2 \times (1 + 2 \times 6) = 26$

If $x^2 = 4 \Rightarrow y^2 < 36 \Rightarrow y^2 \in \{0,1,4,9,16,25\}$

$2 \times (1 + 2 \times 5) = 22$

If $x^2 = 9 \Rightarrow y^2 < 16 \Rightarrow y^2 \in \{0,1,4,9\}$
$2 \times (1 + 2 \times 3) = 14 \Rightarrow 77$ pairs

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let a vector $\vec{a} = \sqrt{2}\hat{\imath} - \hat{\jmath} + \lambda\hat{k}, \lambda > 0$, make an obtuse angle with the vector $\vec{b} = -\lambda^2\hat{\imath} + 4\sqrt{2}\hat{\jmath} + 4\sqrt{2}\hat{k}$ and an angle $\theta, \frac{\pi}{6} < \theta < \frac{\pi}{2}$, with the positive z-axis. If the set of all possible values of λ is $(\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to ___.

Answer (05.00)

Sol. $\vec{a} \cdot \vec{b} < 0$
$\Rightarrow -\sqrt{2}\lambda^2 - 4\sqrt{2} + 4\sqrt{2}\lambda < 0$
$\lambda^2 - 4\lambda + 4 > 0 \Rightarrow (\lambda - 2)^2 > 0$
$\Rightarrow \lambda \in R - \{2\}$
also, $\frac{\vec{a}\cdot\hat{k}}{|\vec{a}||\hat{k}|} = \cos\theta, \theta \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$
$\Rightarrow \frac{\lambda}{\sqrt{\lambda^2+3}} = \cos\theta \in \left(\frac{\sqrt{3}}{2}, 1\right)$
$\Rightarrow \frac{\lambda^2}{\lambda^2+3} \in \left(\frac{3}{4}, 1\right)$
$\left(1 - \frac{3}{\lambda^2+3}\right) \in \left(\frac{3}{4}, 1\right)$
$\Rightarrow \frac{-3}{\lambda^2+3} \Rightarrow \left(\frac{-1}{4}, 0\right)$
$\frac{1}{\lambda^2+3} \in \left(0, \frac{1}{12}\right)$
$\Rightarrow \lambda^2 + 3 \in (0,12)$
$\Rightarrow \lambda^2 + 3 < 12$
$\Rightarrow \lambda^2 < 9 \Rightarrow \lambda \in (-3,3)$
but $\lambda > 0 \Rightarrow \lambda \in (0,3)$
$\Rightarrow (\alpha - \beta) - \gamma = (0,3) - \{2\}$
$\Rightarrow \alpha + \beta + \gamma = 0 + 2 + 3 = 5$

22. Let S be the set of the first 11 natural numbers.

Then the number of elements in $A = \{B \subseteq S : n(B) \geqslant 2$ and the product of all elements of B is even$\}$ is ___.

Answer (1979)

Sol. $n(B) \geq 2$ and product of all elements of B is even
$\Rightarrow$ at least one even element in B.
$\Rightarrow$ Let $n(B) = K, K \geq 2$,
then at least one even element
$\Rightarrow$ total ways - no even number picked
$({}^{11}C_K - {}^{6}C_K)$
$\Rightarrow$ number of such sets are covered in Set A
$\Rightarrow n(A) = \sum_{K=2}^{11}({}^{11}C_K - {}^{6}C_K)$
$= (2^{11} - 11 - 1) - (2^6 - 6 - 1)$
= 1979

23. Let $\cos(\alpha + \beta) = -\frac{1}{10}$ and $\sin(\alpha + \beta) = \frac{3}{8}$, where $0 < \alpha < \frac{\pi}{3}$ and $0 < \beta < \frac{\pi}{4}$. If $\tan 2\alpha = \frac{3(1-r\sqrt{5})}{\sqrt{11}(s+\sqrt{5})}$, $r, s \in$ N, then $r + s$ is equal to ___ .

Answer (20)

Sol. $\cos(\alpha+\beta)=\frac{-1}{10}\Rightarrow \alpha+\beta>\frac{\pi}{2}$

$\sin(\alpha-\beta)=\frac{3}{8}\Rightarrow \alpha-\beta>0$

$$\cos(\alpha+\beta)=-\frac{1}{10}\Rightarrow \sin(\alpha+\beta)=\frac{\sqrt{99}}{10}$$

$$\tan(2\alpha)=\tan((\alpha+\beta)+(\alpha-\beta))$$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha-\beta)\tan(\alpha+\beta)}$$

$$\tan(\alpha+\beta)=-\sqrt{99},\tan(\alpha-\beta)=\frac{3}{\sqrt{55}}$$

$$\Rightarrow \tan(2\alpha)=\frac{-\sqrt{99}+\frac{3}{\sqrt{55}}}{1+\frac{3\sqrt{99}}{\sqrt{55}}}=\frac{-3\sqrt{11}\cdot\sqrt{55}+3}{\sqrt{55}+9\sqrt{11}}$$

$$=\frac{3(1-\sqrt{11}\sqrt{55})}{\sqrt{11}(\sqrt{5}+9)}=\frac{3(1-11\sqrt{5})}{\sqrt{11}(9+\sqrt{5})}$$

Comparing, $r=11,s=9$

$\Rightarrow r+s=20$

24. Suppose a,b,c are in A.P and $a^2,2b^2,c^2$ are in G.P. If $a<b<c$ and $a+b+c=1$, then $9(a^2+b^2+c^2)$ is equal to ___

Answer (9)

Sol. $a,b,c\rightarrow$ A.P. and $a+b+c=1$

$$\Rightarrow \text{Let } b=\frac{1}{3},a=\frac{1}{3}-d,c=\frac{1}{3}+d$$

$a^2,2b^2,c^2\rightarrow$ G.P.

$$\Rightarrow 4b^4=a^2c^2$$

$$\Rightarrow 4\times\left(\frac{1}{3}\right)^4=\left(\frac{1}{9}-d^2\right)^2$$

$$\frac{1}{9}-d^2=\left(\frac{2}{9}\right) \text{ or } \frac{-2}{9}$$

$$d^2=\frac{-1}{9} \text{ absured, } d^2=\frac{3}{9}=\frac{1}{3}$$

Now, $9(a^2+b^2+c^2)$

$$=9\left(\left(d-\frac{1}{3}\right)^2+\left(d+\frac{1}{3}\right)^2+\frac{1}{9}\right)$$

$$=9\left(2d^2+\frac{2}{9}+\frac{1}{9}\right)=9\left(\frac{2}{3}+\frac{3}{9}\right)=6+3=9$$

25. Let [.] be the greatest integer function. If

$\alpha=\int_0^{64}\left(x^{1/3}-\left[x^{1/3}\right]\right)dx$, then

$\frac{1}{\pi}\int_0^{\alpha\pi}\left(\frac{\sin^2\theta}{\sin^6\theta+\cos^6\theta}\right)d\theta$ is equal to ___ .

Answer (36)

Sol. $\alpha=\int_0^{64}\left(x^{1/3}-\left[x^{1/3}\right]\right)dx$

$$=\int_0^1\left(x^{1/3}-0\right)dx+\int_1^8\left(x^{1/3}-1\right)dx$$

$$+\int_8^{27}\left(x^{1/3}-2\right)dx+\int_{27}^{64}\left(x^{1/3}-3\right)dx$$

$$= \frac{3x^{4/3}}{4}\Big|_0^1 + \left(\frac{3x^{4/3}}{4} - x\right)\Big|_1^8 + \left(\frac{3x^{4/3}}{4} - 2x\right)\Big|_8^{27}$$
$$+ \left(\frac{3x^{4/3}}{4} - 3x\right)\Big|_{27}^{64} = 36$$

Now, Let $I = \frac{1}{\pi}\int_0^{36} \frac{\sin^2\theta}{(\sin^6\theta + \cos^6\theta)} d\theta = 36$

PHYSICS TEST PAPER WITH SOLUTION

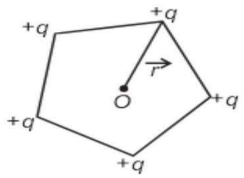
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer :

26. Five positive charges each having charge q are placed at the vertices of a pentagon as shown in the figure. The electric potential (V) and the electric field ($\vec{E}$) at the center O of the pentagon due to these five positive charges are :



(1) $V = \frac{5q}{4\pi\varepsilon_o r}$ and $\vec{E} = \frac{5q}{4\pi\varepsilon_o r^2}\hat{r}$

(2) $V = 0$ and $\vec{E} = 0$

(3) $V = \frac{5q}{4\pi\varepsilon_o r}$ and $\vec{E} = 0$

(4) $V = \frac{5q}{4\pi\varepsilon_0 r}$ and $\vec{E} = \frac{5\sqrt{3}q}{8\pi\varepsilon_0 r^2}\hat{r}$

Answer (3)

Sol. Due to geometrical symmetry

$$E = 0 \& V = \frac{5kq}{r}$$

27. Three small identical bubbles of water having same charge on each coalesce to form a bigger bubble. Then the ratio of the potentials on one initial bubble and that on the resultant bigger bubble is :
(1) $1:2^{2/3}$
(2) $3^{2/3}:1$
(3) $1:3^{1/3}$
(4) $1:3^{2/3}$

Answer (4)

Sol. $Q = 3q$ = change on big bubble

$$R^3 = 3r^3 \text{ or } R = 3^{1/3}r$$
$$V_Q = \frac{KQ}{R} = \frac{K3q}{3^{1/3}r} = \frac{kq}{r}3^{2/3}$$
$$V_q = \frac{kq}{r}$$
$$\frac{V_q}{V_Q} = \frac{1}{3^{2/3}}$$

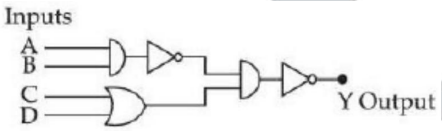
28. If ϵ, E and t represent the free space permittivity, electric field and time respectively, then the unit of $\frac{\epsilon E}{t}$ will be :
(1) $\mathrm{A/m}$
(2) Am
(3) $\mathrm{A/m^2}$
(4) $\mathrm{Am^2}$

Answer (3)

Sol. For $E = \frac{Q}{A\varepsilon_0} \Rightarrow \varepsilon_0 E = \frac{AT}{L^2}$

$$\frac{\varepsilon_0 E}{t} = \frac{A}{L^2} = (\mathrm{A/m^2})$$

29. The correct truth table for the given input data of the following logic gate is :



(1)

Inputs				Output
A	B	C	D	Y
1	1	0	1	1
0	0	1	1	0
1	0	1	0	0
1	1	1	1	1

(2)

Inputs				Output
A	B	C	D	Y
1	1	0	1	0
0	0	1	1	0
1	0	1	0	1
1	1	1	1	1

(3)

Inputs				Output
A	B	C	D	Y
1	1	0	1	0
0	0	1	1	0
1	0	1	0	1
1	1	1	1	1

(4)

Inputs				Output
A	B	C	D	Y
1	1	0	1	0
0	0	1	1	1
1	0	1	0	1
1	1	1	1	1

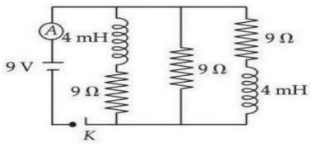
Inputs				Output
A	B	C	D	Y
1	1	0	1	1
0	0	1	1	0
1	0	1	0	1
1	1	1	1	0

Answer (1)

Sol. $Y = \overline{AB}(C + D) = AB + \overline{C + D}$

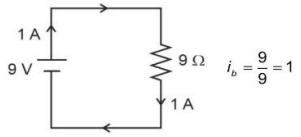
$$= AB + \overline{CD}$$

30. Figure shows the circuit that contains three resistances (9Ω each) and two inductors (4 mH each). The reading of ammeter at the moment switch K is turned ON , is ___ A.



(1) 1
(2) 3
(3) zero
(4) 2

Answer (1)
Sol. As the switch is turned ON
Inductors act as open circuit
Thus,



31. Given below are two statements :

Statement I: For a mechanical system of many particles total kinetic energy is the sum of kinetic energies of all the particles.
Statement II: The total kinetic energy can be the sum of kinetic energy of the center of mass w.r.t. to the origin and the kinetic energy of all the particles w.r.t. the center of mass as the reference.

In the light of the above statements, choose the correct answer from the options given below :
(1) Both Statement I and Statement II are false
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Statement I is false but Statement II is true

Answer (2)

Sol. Both statements are true as

$$KE = \frac{1}{2}\mu v_{rev}^2 + \frac{1}{2}(m_1 + m_2)v_{cm}^2$$
$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

32. Which of the following are true for a single slit diffraction?
A. Width of central maxima increases with increase in wavelength keeping slit width constant.
B. Width of central maxima increases with decrease in wavelength keeping slit width constant.
C. Width of central maxima increases with decrease in slit width at constant wavelength.
D. Width of central maxima increases with increase in slit width at constant wavelength.
E. Brightness of central maxima increases for decrease in wavelength at constant slit width.
(1) A, D only
(2) B, C only
(3) B, D only
(4) A, D, E only

Answer (4)

Sol. $W = \frac{2\lambda a}{D}$

$W \propto \lambda \propto \mathbf{a}$

and $W \propto \frac{1}{D}$

Since decreasing λ decreases width of central maximum causing small region to receive more intensity their brightness also increases.

33. In parallax method for the determination of focal length of a concave mirror, the object should always be placed:
(1) at any point beyond the focus (F) of the mirror
(2) beyond the centre of the curvature (C) of the mirror ONLY
(3) between the pole (P) and the focus (F) of the concave mirror ONLY
(4) between the focus (F) and the centre of curvature (C) of the mirror ONLY

Answer (1)

Sol. Only real images are analyzed in parallel method.

34. The smallest wavelength of Lyman series is 91 nm . The difference between the largest wavelengths of Paschen and Balmer series is nearly ___ nm .
(1) 1217
(2) 1550
(3) 1784
(4) 1875

Answer (1)

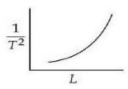
Sol. $\frac{1}{\lambda_0} = RZ^2\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = RZ^2$

$\frac{1}{\lambda_\rho} = RZ^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7RZ^2}{16 \times 9}$

$\frac{1}{\lambda_B} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5RZ^2}{4 \times 9}$

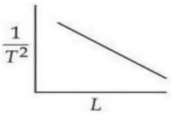
$\lambda_\rho - \lambda_B = \frac{9}{RZ^2}\frac{(80-28)}{35} = 1217 \text{ nm}$

35. Using a simple pendulum experiment g is determined by measuring its time period T. Which of the following plots represent the correct relation between the pendulum length L and time period T ?

(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. $T^2 = (2\pi)^2 \frac{L}{g}$

$$g = (2\pi)^2 \frac{L}{T^2}$$

Since g is constant and

$L \to x; y \to \frac{1}{T^2}$

xy = constant

Thus is rectangular hyperbola.

36. In an open organ pipe v_3 and v_6 are 3^{rd} and 6^{th} harmonic frequencies, respectively. If $v_6 - v_3 = 2200$ Hz then length of the pipe is ___ mm. (Take velocity of sound in air is 330 m/s)

(1) 200

(2) 275

(3) 250

(4) 225

Answer (4)

Sol. $v_3 = \frac{3C}{2l}; v_6 = \frac{6C}{2l}$

$$v_6 - v_3 = \frac{3C}{2l} = v_3 = 2200 \text{ Hz}$$

$$l = \frac{3 \times 330 \times 10^3}{4400} \text{ mm}$$

$$l = \frac{9}{4} \times 10^2 = 225 \text{ mm}$$

37. Given below are two statements :

Statement I: A satellite is moving around earth in the orbit very close to the earth surface. The time period of revolution of satellite depends upon the density of earth.

Statement II: The time period of revolution of the satellite is $T = 2\pi\sqrt{\frac{R_e}{g}}$ (for satellite very close to the earth surface), where R_e radius of earth and g acceleration due to gravity.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

Answer (3)

Sol. $m\omega^2 R = \frac{GMm}{R^2}$

$\omega = \sqrt{\frac{g}{R}} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$

$\omega^2 R = \frac{G\rho\frac{4}{3}\pi R^3}{R^2}$

$\omega \propto \sqrt{\rho}$ only

38. Given below are two statements :

Statement-I: An object moves from position r_1 to position r_2 under a conservative force field $\vec{F}$. The work done by the force is $W = -\int_{r_1}^{r_2} \vec{F} \cdot \overrightarrow{dr}$.

Statement-II: Any object moving from one location to another location can follow infinite number of paths. Therefore, the amount of work done by the object changes with the path it follows for a conservative force.

In the light of the above statements, choose the correct answer from the options given below :
(1) Both Statement-I and statement-II are true
(2) Statement-I is false but statement-II is true
(3) Statement-I is true but statement-II is false
(4) Both Statement-I and statement-II are false

Answer (4)
Sol. Work done by force

$$W = \int_{r_1}^{r_2} \vec{F} \cdot \overrightarrow{dr}$$

for conservation force W is independent of path & only depends on initial & final position.
39. Light is incident on a metallic plate having work function 110×10^{-2} J. If the produced photoelectrons have zero kinetic energy then the angular frequency of the incident light is ___ rad/s. ($h = 6.63 \times 10^{-34}$ J. s).
(1) 1.04×10^{13}
(2) 1.66×10^{15}
(3) 1.66×10^{16}
(4) 1.04×10^{16}

Answer (4)

Sol. $\phi = \frac{h\omega}{2\pi}$

$$\omega = \frac{110 \times 10^{-20} \times 2\pi}{6.63 \times 10^{-34}} = 1.04 \times 10^{-18+}$$
$$\omega = 1.04 \times 10^{16}$$

40. An electric power line having total resistance of 2Ω, delivers 1 kW of power at 250 V . The percentage efficiency of transmission line is
___ .
(1) 92.5
(2) 100
(3) 96.9
(4) 86.5

Answer (3)

Sol. $P_{(\text{in})} = 1\ \text{kW}$

$$I = \frac{1000}{250} = 4\ \text{A}$$
$$P_{(\text{loss})} = IRR = 32\text{watt}$$

So efficiency

$$\eta = \frac{968}{1000} \times 100 \approx 96.8\%$$

41. When a part of a straight capillary tube is placed vertically in a liquid, the liquid raises upto certain height h. If the inner radius of the capillary tube, density of the liquid and surface tension of the liquid decrease by 1% each, then the height of the liquid in the tube will change by ___ %.
(1) -3
(2) +3
(3) -1
(4) +1

Answer (4)

Sol. $h = \frac{2T\cos\theta}{\rho g r}$

$$\frac{\Delta h}{h} = \frac{\Delta T}{T} - \frac{\Delta r}{r} - \frac{\Delta p}{p}$$
So $\left(\frac{\Delta h}{h}\right) \times 100 = (-1 + 1 + 1)$
$= +1\%$

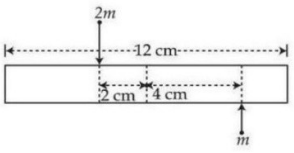
42. The wavelength of light, while it is passing through water is 540 nm . The refractive index of water is $\frac{4}{3}$. The wavelength of the same light when it is passing through a transparent medium having refractive index of $\frac{3}{2}$ is ___ nm .
(1) 540
(2) 380
(3) 480
(4) 840

Answer (3)

Sol. $\mu\lambda$ = constant

$$\Rightarrow 540 \times \frac{4}{3} = \lambda \times \frac{3}{2}$$
$$\Rightarrow \lambda = 480\ \text{nm}$$

43. A uniform bar of length 12 cm and mass 20 m lies on a smooth horizontal table. Two point masses m and $2m$ are moving in opposite directions with same speed of v and in the same plane as the bar, as shown in figure. These masses strike the bar simultaneously and get stuck to it. After collision the entire system is rotating with angular frequency ω. The ratio of v and ω is :



(1) 32
(2) 66
(3) 33
(4) $2\sqrt{88}$

Answer (3)

Sol. $v_0 = \frac{2mv-m}{23m} = \frac{v}{23}$

$l\omega = mv \times \frac{4}{100} + \frac{4mv}{100} = 8mv \times 10^{-2}$

$\Rightarrow I = \frac{20ml^2}{12} + \frac{ml^2}{9} + \frac{2ml^2}{36} = \frac{66}{36}ml^2$

$\Rightarrow \omega = \frac{8m \times 10^{-2} \cdot v}{66ml^2} \times 36$

$\Rightarrow \frac{v}{\omega} = \frac{66I^2}{8 \times 36 \times 10^{-2}}$

$\Rightarrow \frac{v}{\omega} = \frac{33}{100}$

As the lengths are given in cm that's why 100 appearing in denominator.

44. Consider two boxes containing ideal gases A and B such that their temperatures, pressures and number densities are same. The molecular size of A is half of that of B and mass of molecule A is four times that of B. If the collision frequency in gas B is $32 \times 10^{18}/s$ then collision frequency in gas A is ___ /s.

(1) 8×10^8

(2) 2×10^8

(3) 4×10^8

(4) 32×10^8

Answer (3)

Sol. $f \propto \eta\sigma\bar{v}$

So $\frac{f_A}{f_B} = \frac{\sigma_A\bar{v}_A}{\sigma_B\bar{v}_B} = \frac{1}{4\times2} = \frac{1}{8}$

$\Rightarrow f_A = 4 \times 10^{\frac{18}{3}}/\text{sec}$

45. A laser beam has intensity of 4.0×10^{14} W/m^2. The amplitude of magnetic field associated with ___ T. (Take $\varepsilon_0, = 8.85 \times 10^{-12}\text{C}^2/\text{Nm}^2$ and $c = 3 \times 10^8$ m/s) beam is

(1) 1.83

(2) 5.5

(3) 2.0

(4) 18.3

Answer (1)

Sol. $I = \frac{1}{2}\varepsilon_0 E_0^2 \cdot c$

$= \frac{1}{2}\varepsilon_0 C^3 B_0^2$

$\Rightarrow \beta = \sqrt{\frac{2I}{\varepsilon_0 c^3}} = 1.83$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A capacitor P with capacitance 10×10^{-6} F is fully charged with a potential difference of 6.0 V and disconnected from the battery. The charged capacitor P is connected across another capacitor Q with capacitance 20×10^{-6} F. The charge on capacitor Q when equilibrium is established will be $\alpha \times 10^{-5}$C (assume capacitor Q does not have any charge initially), the value of is ___ .

Answer (4)

Sol. $v_f = \frac{6C}{C+C_1} = \frac{6\times10\times10^{-6}}{30\times10^{-6}} = 2$ volt

So, q1=$2 \times 20 \times 10^{-6}$

$= 4 \times 10^{-5}$

47. A conducting circular loop is rotated about its diameter at a constant angular speed of 100rad/s in a magnetic field of 0.5 T perpendicular to the axis of rotation. When the loop is rotated by 30° from the horizontal position, the induced EMF is 15.4 mV . The radius of the loop is ___ mm .

(Take $\pi = \frac{22}{7}$)

Answer (14)

Sol. ϕ = NAB constant

$$E = 15.4 \times 10^{-3} = NA\omega B\sin\,\omega t$$
$$R^2 = \frac{15.4 \times 10^{-3}}{100 \times 22} \times 2 \times 2 \times 7$$
$$R = 14 \text{ mm}$$

48. A cylindrical conductor of length 2 m and area of cross-section $0.2\ \text{mm}^2$ carries an electric current of 1.6 A when its ends are connected to a 2 V battery. Mobility of electrons in the conductor is $\alpha \times 10^{-3}\text{m}^2/\text{V.s}$. The value of α is
(electron concentration $= 5 \times 10^{28}/\text{m}^3$ and electron charge $= 1.6 \times 10^{-19}\text{C}$)

Answer (1)

Sol. $\mu = \frac{v}{E} = \frac{i}{neAE} = \frac{id}{neAV_b}$

$$\frac{1.6 \times 2}{5 \times 10^{28} \times 1.6 \times 10^{-19} \times 0.2 \times 10^{-6} \times 2}$$
$$\mu = 1 \times 10^{-3}$$

49. An insulated cylinder of volume $60\ \text{cm}^3$ is filled with a gas at 27°C and 2 atmospheric pressure. Then the gas is compressed making the final volume as $20\ \text{cm}^3$ while allowing the temperature to rise to 77°C. The final pressure is ___ atmospheric Pressure.

Answer (7)

Sol. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{350}{300} \times \frac{60}{20} = P_2 = 7$ atm

50. Two masses m and $2m$ are connected by a light string going over a pulley (disc) of mass $30m$ with radius $r = 0.1$ m. The pulley is mounted in a vertical plane and it is free to rotate about its axis. The $2m$ mass is released from rest and its speed when it has descended through a height of 3.6 m is ___ m/s. (Assume string does not slip and $g = 10\ \text{m/s}^2$)

Answer (2)

Sol. $a = \frac{(2m-m)}{2m+m+\frac{I}{r^2}}g = \frac{5}{9}\ \text{m/s}^2$

$V = \sqrt{2as} = 2\ \text{m/s}$

CHEMISTRY TEST PAPER WITH SOLUTION

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

51. Match List-I with List-II.

	List-I Reaction of Glucose with		List-II Product formed
A.	Hydroxylamine	I.	Gluconic acid
B.	Br_2 water	II.	Glucose pentacetate
C.	Excess acetic anhydride	III.	Saccharic acid
D.	Concentrated HNO_3	IV.	Glucoxime

Choose the correct answer from the options given below :
(1) A-III, B-I, C-IV, D-II
(2) A-IV, B-III, C-II, D-I
(3) A-I, B-III, C-IV, D-II
(4) A-IV, B-I, C-II, D-III

Answer (4)
Sol. Glucose $\xrightarrow{NH_2OH}$ Glucoxime $(A \rightarrow IV)$
Glucose $\xrightarrow[\text{excess}]{Br_2 \text{ water}}$ Gluconic acid $(B \rightarrow I)$
Glucose $\xrightarrow[\text{acetic anhydride}]{}$ Glucose pentaacetate
$(C \rightarrow II)$
Glucose $\xrightarrow{\text{Conc. } HNO_3}$ Saccharic acid $(D \rightarrow III)$

52. Among H_2 S, H_2O, NF_3, NH_3 and $CHCl_3$, identify the molecule (X) with lowest dipole moment value.

The number of lone pairs of electrons present on the central atom of the molecule (X) is :
(1) 3
(2) 0
(3) 2
(4) 1

Answer (4)
Sol. Lowest dipole moment is for NF_3
LP = 1 on nitrogen atom

53. Given below are two statements :

Statement 1 : The first ionization enthalpy of Cr is lower than that of Mn .

Statement II: The second and third ionization enthalpies of Cr are higher than those of Mn .

In the light of the above statements, choose the correct answer from the options given below :
(1) Both Statement I and Statement II are false
(2) Statement I is false but Statement II is true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are true

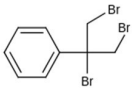
Answer (3)
Sol. IE_1 of $Cr = 653\ kJ\ mole^{-1}$
IE_1 of $Mn = 717\ kJ\ mole^{-1}$
Statement I is correct.
(IE_2 and IE_3) of $Cr <$ (IE_2 and IE_3) of Mn
Statement II is incorrect.
54. $[Ni(PPh_3)_2Cl_2]$ is a paramagnetic complex. Identify the INCORRECT statements about this complex.
A. The complex exhibits geometrical isomerism.
B. The complex is white in colour.
C. The calculated spin-only magnetic moment of the complex is 2.84 BM .
D. The calculated CFSE (Crystal Field Stabilization Energy) of Ni in this complex is $-0.8\Delta 0$.
E. The geometrical arrangement of ligands in this complex is similar to that in $Ni(CO)_4$.

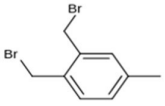
Choose the correct answer from the options given below :
(1) A and B Only
(2) A, B and D Only
(3) C, D and E Only
(4) C and D Only

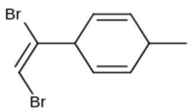
Answer (2)

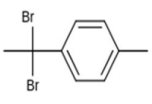
Sol. $Ni^{2+}: 4s^0 3d^8$
$\Rightarrow$ Tetrahedral
(A) $\Rightarrow$ Tetrahedral complexes doesn't exhibit GI

Compound is not white
CFSE $= \frac{1l-1}{1l-1}$
CFSE $= -0.6\Delta_t(4) + 0.4\Delta_t(4)$
$\Rightarrow\ -0.2\Delta_t \times 4 = -0.8\Delta_t(\text{not} - 0.8\Delta_0)$
D is incorrect
A, B and D are incorrect.
55. The dibromo compound [P] (molecular formula : $C_9H_{10}Br_2$ when heated with excess sodamide followed by treatment with dilute HCl gives [Q]. On warming [Q] with mercuric sulphate and dilute sulphuric acid yield [R] which gives positive lodoform test but negative Tollen's test. The compound [P] is :

(1) 

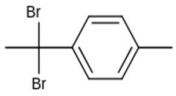
(2) 

(3) 

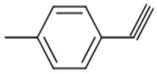
(4) 

Answer (4)

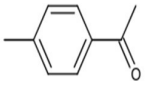
Sol. P :



Q :



R :



R gives lodoform test and negative Tollen's test.

56. Which of the following mixture gives a buffer solution with $\mathrm{pH} = 9.25$?

Given : $\mathrm{pK_b(NH_4OH)} = 4.75$

(1) $0.2\mathrm{MNH_4OH(0.5\ L)} + 0.1\mathrm{MHCl(0.5\ L)}$

(2) $0.4\mathrm{MNH_4OH(1\ L)} + 0.1\mathrm{MHCl(1\ L)}$

(3) $0.2\mathrm{MNH_4OH(0.4\ L)} + 0.1\mathrm{MHCl(1\ L)}$

(4) $0.5\mathrm{MNH_4OH(0.2\ L)} + 0.2\mathrm{MHCl(0.5\ L)}$

Answer (1)

Sol. pH = 9.25

$\mathrm{pOH} = 4.75$

$4.75 = 4.75 + \log \dfrac{\text{Salt}}{\text{Base}}$

$\mathrm{pH} = \mathrm{pK_b}$ at half equivalence point

$\Rightarrow$ Option 1 is correct.

57. Consider the following reduction processes :

$\mathrm{Al^{3+} + 3e^- \rightarrow Al(s), E^\circ = -1.66\ V}$

$\mathrm{Fe^{3+} + e^- \rightarrow Fe^{2+}, E^\circ = +0.77\ V}$

$\mathrm{Co^{3+} + e^- \rightarrow Co^{2+}, E^\circ = +1.81\ V}$

$\mathrm{Cr^{3+} + 3e^- \rightarrow Cr(s), E^\circ = -0.74\ V}$

The tendency to act as reducing agent decreases in the order:

(1) $\mathrm{Al > Cr > Fe^{2+} > Co^{2+}}$

(2) $\mathrm{Cr > Fe^{2+} > Al > Co^{2+}}$

(3) $\mathrm{Al > Fe^{2+} > Cr > Co^{2+}}$

(4) $\mathrm{Al > Cr > Co^{2+} > Fe^{2+}}$

Answer (1)

Sol. $\mathrm{RP} \propto \mathrm{E^\circ_{SRP}}$

$\Rightarrow \mathrm{Al > Cr > Fe^{2+} > Co^{2+}}$

58. The energy of first (lowest) Balmer line of H atom is x J. The energy (in J) of second Balmer line of H atom is

(1) $2x$

(2) $\frac{x}{1.35}$

(3) $1.35x$

(4) x^2

Answer (3)

Sol. $\Delta E_1 = R_H\left[\frac{1}{4}-\frac{1}{9}\right] = \frac{5R_H}{36} = x$

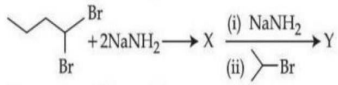
$$R_H = \frac{36x}{5}$$

$$\Delta E_2 = R_H\left[\frac{1}{4}-\frac{1}{16}\right]$$

$$= \frac{3R_H}{16} = \frac{3}{16}\times\frac{36x}{5}$$

$$= 1.35x$$

59. Consider the following reaction :



The product Y formed is

(1) 2-methylhex-3-yne

(2) Isopropylbut-1-yne

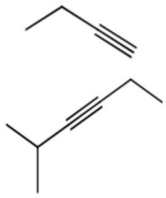
(3) 5-methylhex-2-yne

(4) 2-methylhex-2-yne

Answer (1)

Sol. X :

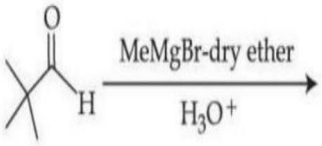
Y :



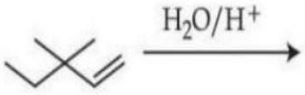
IUPAC Name : 2-methylhex-3-yne

60. 3,3-Dimethyl-2-butanol cannot be prepared by :

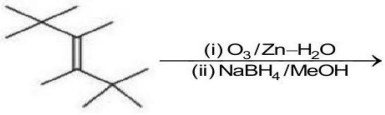
A.



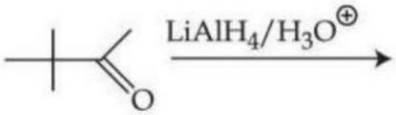
B.

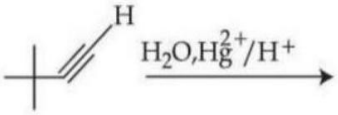


C.



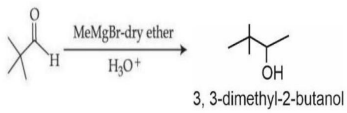
D.

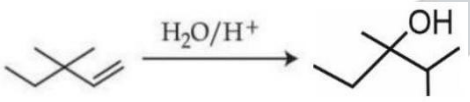


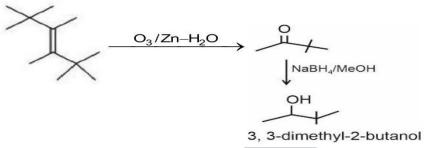
E. 

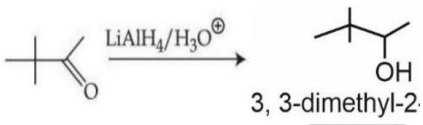
Choose the correct answer from the options given below
(1) B Only
(2) B and E Only
(3) B and C Only
(4) B, C and E Only

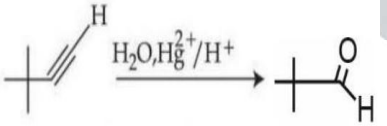
Answer (2)

Sol. A. 

B. 

C. 

D. 

E. 

61. Given below are two statements :

Statement I: Elements ' X ' and ' Y ' are the most and least electronegative elements, respectively among N, As, Sb and P . The nature of the oxides X_2O_3 and Y_2O_3 is acidic and amphoteric, respectively.

Statement II: BCl_3 is covalent in nature and gets hydrolysed in water. It produces $[B(OH)_4]^-$and $[B(H_2O)_6]^{3+}$ in aqueous medium.

In the light of the above statements, choose the correct answer from the options given below.
(1) Both Statement I and Statement II are false
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Statement I is false but Statement II is true

Answer (3)

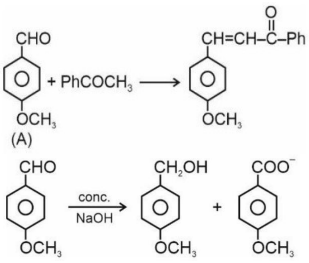
Sol.

	N	P	As	Sb
EN:	3	2.1	2.0	1.9

$X \rightarrow N$
$Y \rightarrow Sb$
$N_2O_3 \rightarrow$ acidic
$Sb_2O_3 \rightarrow$ amphoteric
BCl_3 on hydrolysis doesn't form $[B(H_2O)_6]^{3+}$ in aqueous medium.
62. The compound $A, C_8H_8O_2$ reacts with acetophenone to form a single product via crossAldol condensation. The compound A on reaction with conc. NaOH forms a substituted benzyl alcohol as one of the two products. The compound A is
(1) 4-methyl benzoic acid
(2) 4-methoxy benzaldehyde
(3) 4-hydroxy benzylaldehyde
(4) 2-hydroxy acetophenone

Answer (2)

Sol. $C_8H_8O_2 \Rightarrow DU = 5$



63. $A + 2B \rightarrow AB_2$
36.0 g of 'A' (Molar mass : 60 g mol^{-1}) and 56.0 g of ' B ' (Molar mass : 80 g mol^{-1}) are allowed to react. Which of the following statements are correct?
A. 'A' is the limiting reagent.
B. 77.0 g of AB_2 is formed.
C. Molar mass of AB_2 is 140 g mol^{-1}.
D. 15.0 g of A is left unreacted after the completion of reaction.

Choose the correct answer from the options given below:
(1) A and B only
(2) B and D only
(3) C and D only
(4) A and C only

Answer (2)
Sol. $A + 2B \rightarrow AB_2$

$n_A = \frac{36}{60} = 0.6$ mol

$n_B = \frac{56}{80} = 0.7$ mol

0.6 mol A require 0.12 mol$B$
$\rightarrow B$ is LR
$n_{AB_2} = 0.35$

Mass of $AB_2 = 0.35 \times 220$ g

$$= 77 \text{ g}$$

Molar mass of $AB_2 = 220$ g
0.7 mol of B reacts with 0.35 mol$A$
$\therefore$ Mass of A unreacted $= 0.25 \times 60$

$$= 15 \text{ g}$$

B, D are correct.
64. Correct statements regarding Arrhenius equation among the following are:
A. Factor $e^{-E/RT}$ corresponds to fraction of molecules having kinetic energy less than Ea.
B. At a given temperature, lower the Ea, faster is the reaction.
C. Increase in temperature by about $10^{\circ}C$ doubles the rate of reaction.
D. Plot of log k vs. $\frac{1}{T}$ gives a straight line with slope $= -\frac{Ea}{R}$.
Choose the correct answer from the options given below:
(1) B and D only
(2) B and C only
(3) A and B only
(4) A and C only

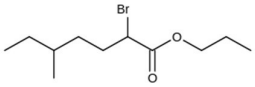
Answer (2)
Sol. $K = Ae^{-Ea/RT}$

$$\log K = \log A - \frac{Ea}{2.303RT}$$

Plot of log K vs $\frac{1}{T}$ has

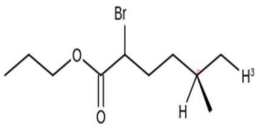
$\Rightarrow$ slope $= -\frac{Ea}{2.303R}$

Ea $\downarrow \Rightarrow -\frac{Ea}{RT} \uparrow \Rightarrow k \uparrow$ (rate increases)
B, C are correct.
65. The IUPAC name of the following compound is:



(1) n-propyl-2-bromo-5-methylheptanoate
(2) 2-bromo-5-methylhexylpropanoate
(3) n-propyl-1-bromo-4-methylehxanoate
(4) 2-bromo-5-methylpropanoate

Answer (1)

Sol. 

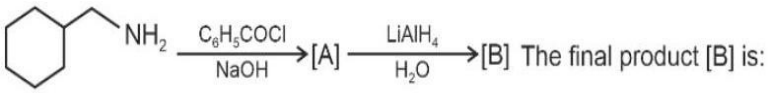
n-propyl-2-bromo-5-methylheptanoate
66. Identify the correct statements:
A. Hydrated salts can be used as primary standard.
B. Primary standard should not undergo any reaction with air.
C. Reactions of primary standard with another substance should be instantaneous and stoichiometric.
D. Primary standard should not be soluble in water.
E. Primary standard should have low relative molar mass.

Choose the correct answer from the options given below:
(1) A, B and C only
(2) A, B and E only
(3) D and E only
(4) A, B, C and E only

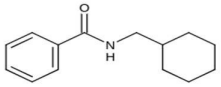
Answer (1)

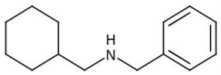
Sol. A, B and C are correct
A primary standard should be

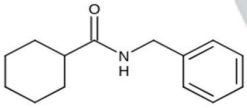
- Very pure
- Stable in Air (non-hygroscopic)
- Should not undergo any reaction with Air
- Reaction with another substance should be instantaneous and stoichiometric

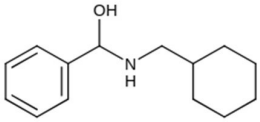
67. 

The final product [B] is:

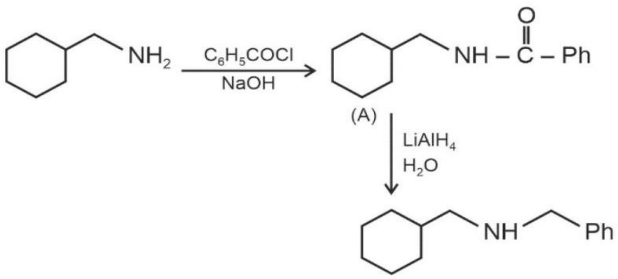
(1) 

(2) 

(3) 

(4) 

Answer (2)
Sol.



68. When 1 g of compound (X) is subjected to Kjeldahl's method of estimation of nitrogen, 15 mL $1M H_2SO_4$ was neutralized by ammonia evolved. The percentage of nitrogen in compound (X) is
(1) 42
(2) 21
(3) 0.42
(4) 0.21

Answer (1)
Sol. m . moles of $H_2SO_4 = 15$
m. moles of $NH_3 = 30$
m. moles of N = 30

$$\% \text{ by mass of N} = \frac{30 \times 10^{-3} \times 14}{1} \times 100$$
$$= 42\%$$

69. At $T(K)$,100 g of $98\% H_2SO_4(w/w)$ aqueous solution is mixed with 100 g of $49\% H_2SO_4(w/w)$ aqueous solution. What is the mole fraction of H_2SO_4 in the resultant solution?
(Given : Atomic mass $H = 1u; S = 32u; O = 16u$).
(Assume that temperature after mixing remains constant)
Options
(1) 0.337
(2) 0.1
(3) 0.9
(4) 0.663

Answer (1)

Sol. Total mass $= 200gm$
Moles of $H_2SO_4 = 1 + \frac{1}{2} = \frac{3}{2}$
Mass of $H_2SO_4 = 147gm$
Mass of $H_2O = 53gm$
Moles of $H_2O = 2.94$

$$X_{H_2SO_4} = \frac{1.5}{1.5 \times 2.94} = \frac{1.5}{4.44} = 0.337$$

70. Given below are two statements :

Statement I: $C < O < N < F$ is the correct order in terms of first ionization enthalpy values.

Statement II: $S > Se > Te > Po > O$ is the correct order in terms of the magnitude of electron gain enthalpy values.

In the light of the above statements, choose the correct answer from the options given below :
(1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is false but Statement II is true
(4) Statement I is true but Statement II is false

Answer (1)
Sol. $IE_1: C < O < N < F$
SI is correct
(EA) : $S > Se > Te > PO > O$
SII is correct

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. If the enthalpy of sublimation of Li is 155 kJ mol^{-1}, enthalpy of dissociation of F_2 is 150 kJ mol^{-1}, ionization enthalpy of Li is 520 kJ mol^{-1}, electron gain enthalpy of F is -313 kJ mol^{-1}, standard enthalpy of formation of LiF is -594 kJ mol^{-1}. The magnitude of lattice enthalpy of LiF is ___ $kJmol^{-1}$. (Nearest Integer)

Answer (1031)

Sol. $Li^+(g) + F^-(g) \rightarrow LiF(s)$

$$\begin{aligned}
&Li(s) \rightarrow Li(g) && \Delta H = 155 \text{ kJ mol}\\
&\frac{1}{2}F_2 \rightarrow F(g) && \Delta H = 75 \text{ kJ/mol}\\
&Li(g) \rightarrow Li^+(g) + e^- && \Delta H = 520\\
&e^- + F(g) \rightarrow F^-(g) && \Delta H = -313\\
&Li(s) + \frac{1}{2}F_2(g) \rightarrow LiF(s) && \Delta H = -594\\
& && -594 = 155 + 520 + 75 + (-313) + \Delta H_{lattice}\\
&\Delta H_{lattice} = -1031 &&
\end{aligned}$$

72. Consider $A \xrightarrow{k_1} B$ and $C \xrightarrow{k_2} D$ are two reactions. If the rate constant (k_1) of the $A \rightarrow B$ reaction can be expressed by the following equation $\log_{10} k = 14.34 - \frac{1.5\times10^4}{T/K}$ and activation energy of $C \rightarrow D$ reaction (E_{a_2}) is $\frac{1}{5}$ th of the $A \rightarrow B$ reaction (E_{a_1}), then the value of (E_{a_2}) is ___ $kJmol^{-1}$. (Nearest Integer)

Answer (57)

Sol. For reaction 1

$$\log k = 14.34 - \frac{1.5 \times 10^4}{T}$$

On comparing with

$$\log k = \log A - \frac{E_a}{2.303R}\frac{1}{T}$$

$$1.5 \times 10^4 = \frac{E_a}{2.303 \times K}$$

$$28.7 \times 10^4 = E_a$$

$$E_{a_2} = \frac{28.7 \times 10^4}{5} = 57.3 \text{ kJ}$$

73. Among the following oxides of 3 d elements, the number of mixed oxides are ___ .

Ti_2O_3, V_2O_4, Cr_2O_3, Mn_3O_4, Fe_3O_4, Fe_2O_3, Co_3O_4

Answer (3)

Sol. $Mn_3O_4 \rightarrow MnO \cdot Mn_2O_3$

$Fe_3O_4 \rightarrow FeO \cdot Fe_2O_3$

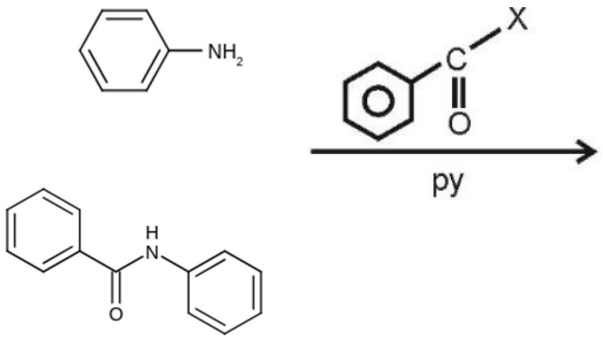
$Co_3O_4 \rightarrow CoO \cdot CO_2O_3$

74. The mass of benzanilide obtained from the benzoylation reaction of 5.8 g of aniline, if yield of product is 82%, is ___ g (nearest integer).

(Given molar mass in $gmol^{-1}$ H: 1, C: 12, N: 14, O : 16)

Answer (10)

Sol.



Moles of aniline $= \frac{5.8}{9.3} = 0.0623$

Moles of benzanilide $= 0.0623 \times 0.82 = 0.0511$

Mass of benzanilide $= 0.0511 \times 197 = 10$

75. Consider the following electrochemical cell :

$Pt|O_2(g)(1bar)|HCl(aq)||M^{2+}(aq, 1.0M)|M(s)$

The pH above which, oxygen gas would start to evolve at anode is ___ (nearest integer).

[Given $\left.\begin{matrix} E^{\circ}_{M^{2+}/M} = 0.994\ V \\ E^{\circ}_{O_2/H_2O} = 1.23\ V \end{matrix}\right\}$ standard reduction potential and $\frac{RT}{F}(2.303) = 0.059\ V$ at the given condition]

Answer (4)

Sol. At anode : $2H_2O \rightarrow 4H^+ + 4e^- + O_2$

At cathode : $M^{2+} + 2e^- \rightarrow M(s)$

Cell reaction $2H_2O + 2M^{2+} \rightarrow 4H^+ + O_2 + 2M(s)$

$$Q = \frac{[H^+]^4 p_{O_2}}{[M^{2+}]^2}$$

$$E = 0.994 - 1.23 - \frac{0.059}{4}\log\frac{[H^+]^4 \times 1}{1^2} > 0$$

$$-0.236 < 0.059\log[H^+]$$

$$4 < -\log[H^+]$$

$$4 < pH$$