

MATHEMATICS TEST PAPER WITH SOLUTION

1. The positive integer n , for which the solutions of the equation $x(x+2) + (x+2)(x+4) + \dots + \{x + 2n - 2\}(x+2n) - \frac{8n}{3}$ are two consecutive even integers, is :

- (1) 3 (2) 6
(3) 12 (4) 9

Ans. (1)

Sol. $nx^2 + 2n^2 + \frac{4}{3}n(n+1)(n-1) = \frac{8n}{3}$

$$x^2 + 2nx + \frac{4}{3}(n^2 - 1) - \frac{8}{3} = 0$$

$$4n^2 - 16\left(\frac{n^2 - 1 - 2}{3}\right) = 4$$

$$3n^2 - 4n^2 + 12 = 3$$

$$n^2 = 9 \quad n = 3$$

2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbb{R}$ and $f'(a-1) = 0$, where a is a real number. Let $g(x) = f(\tan^2 x - 2\tan x + a)$, $0 < x < \frac{\pi}{2}$.

Consider the following two statements:

(I) g is increasing in $\left(0, \frac{\pi}{4}\right)$

(II) g is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Then ,

- (1) Neither (I) nor (II) is True
(2) Only (II) is True
(3) Only (I) is True
(4) Both (I) and (II) are True

Ans. (1)

Sol. $g'(x) = f'(\tan^2 x - 2\tan x + a) \cdot (2\tan x - 2) \sec^2 x$
 $= f'(a - 1 + (\tan + 1)^2) \cdot 2(\tan x - 1) \sec^2 x$

$$g \text{ is increasing in } \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$g \text{ is decreasing in } \left(0, \frac{\pi}{4}\right)$$

3. Let $f(x) = x^3 + x^2 f'(1) + 2x f''(2) + f'''(3)$, $x \in \mathbb{R}$. Then the value of $f'(5)$ is :

- (1) $\frac{62}{5}$ (2) $\frac{657}{5}$ (3) $\frac{2}{5}$ (4) $\frac{117}{5}$

Ans. (4)

Sol. $f'(x) = 3x^2 + 2x f'(1) + 2f''(2)$

$$f''(x) = 6x + 2 f'(1)$$

$$f'''(x) = 6$$

$$f''(2) = 12 + 2f'(1)$$

$$2f''(2) = -3 - f'(1)$$

$$5f''(2) = 6$$

$$f''(2) = \frac{6}{5}, f'(1) = -\frac{27}{5}$$

$$f''(5) = 75 + 10 \left(-\frac{27}{5} \right) + 2 \left(\frac{6}{5} \right)$$

$$= \frac{117}{5}$$

4. If the line $ax + 4y = \sqrt{7}$, where $a \in \mathbb{R}$, touches the ellipse $3x^2 + 4y^2 = 1$ at the point P in first quadrant, then one of the focal distance of P is

- (1) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{11}}$ (2) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{5}}$
 (3) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{5}}$ (4) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$

Ans. (4)

Sol. $\frac{x^2}{\frac{1}{3}} + \frac{y^2}{\frac{1}{4}} = 1, y = \frac{-a}{4}x + \frac{\sqrt{7}}{4}$

$$\frac{7}{16} = \frac{1}{3} \left(\frac{a^2}{16} \right) + \frac{1}{4}$$

$$21 + a^2 = 12 \Rightarrow a = \pm 3$$

$$\text{Tangent } 3x + 4y = \sqrt{7}$$

$$3xx_1 + 44y_1 = 1$$

$$x_1 = y_1 = \frac{1}{\sqrt{7}}$$

$$P \left(\frac{1}{\sqrt{7}}, \frac{1}{\sqrt{7}} \right)$$

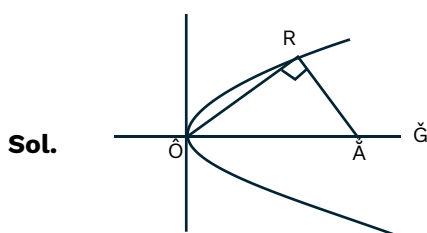
$$\text{Focal distance} = a \pm \frac{e}{\sqrt{7}}$$

$$\frac{1}{\sqrt{3}} \pm \frac{1}{2\sqrt{7}}$$

5. Let $y^2 = 12x$ be the parabola with its vertex at O. Let P be a point on the parabola and A be a point on the x-axis such that $\angle OPA = 90^\circ$. Then the locus of the centroid of such triangles OPA is

- (1) $y^2 - 6x + 4 = 0$
 (2) $y^2 - 9x + 6 = 0$
 (3) $y^2 - 2x + 8 = 0$
 (4) $y^2 - 4x + 8 = 0$

Ans. (3)



$$P(3t^2, 6t), O(0, 0)$$

$$PA : tx + 2y = 3t^3 + 12t$$

$$A(3t^2 + 12, 0)$$

$$h = \frac{3t^2 + 12 + 3t^2}{3}, k = \frac{6t}{3}$$

$$h = 2\left(\frac{k}{2}\right)^2 + 4$$

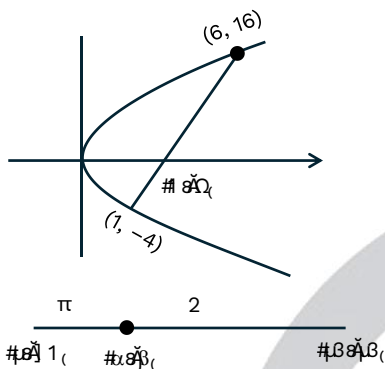
$$y^2 = 2x - 8$$

6. Let one end of a focal chord of the parabola $y^2 = 16x$ be $(16, 16)$. If $P(\alpha, \beta)$ divides this focal chord internally in the ratio 5 : 2, then the minimum value of $\alpha + \beta$ is equal to :

- (1) 22 (2) 7
(3) 5 (4) 16

Ans. (2)

Sol.



$$\alpha = \frac{37}{7}, \beta = \frac{12}{7}$$

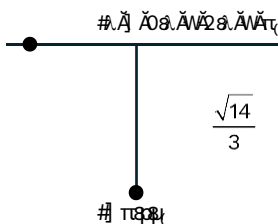
$$\alpha + \beta = 7$$

7. Let the line L pass through the point $(-3, 5, 2)$ and make equal angles with the positive coordinate axes. If the distance of L from the point $(-2, r, 1)$ is $\sqrt{\frac{14}{3}}$, then the sum of all possible values of r is

- (1) 12 (2) 16
(3) 6 (4) 10

Ans. (4)

Sol. $L : x + 3 = y - 5 = z - 2$



$$(\lambda - 1) + (\lambda + 5 - r) + (\lambda + 1) = 0$$

$$r = 3\lambda + 5$$

$$(\lambda + 1)^2 + 4\lambda^2 + (\lambda + 1)^2 = \frac{14}{3}$$

$$6\lambda^2 = \frac{14}{3} \Rightarrow \lambda = \pm \frac{\sqrt{7}}{3}$$

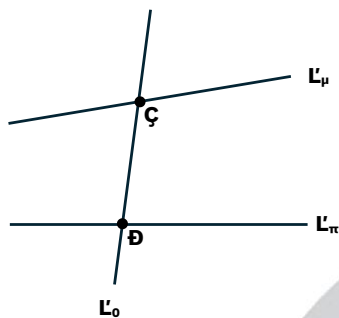
Sum of all possible values of $r = 10$

8. Let the line L_1 be parallel to the vector $-3\hat{i} + 2\hat{j} + 4\hat{k}$ and pass through the point $(2, 6, 7)$, and the line L_2 be parallel to the vector $2\hat{i} + \hat{j} + 3\hat{k}$ and pass through the point $(4, 3, 5)$. If the line L_3 is parallel to the vector $-3\hat{i} + 5\hat{j} + 16\hat{k}$ and intersects the lines L_1 and L_2 at the points C and D, respectively, then $|\overrightarrow{CD}|^2$ is equal to :

- (1) 171 (2) 290
(3) 312 (4) 89

Ans. (2)

Sol. C $(2 - 3\lambda, 6 + 2\lambda, 7 + 4\lambda)$
D $(4 + 2\mu, 3 + \mu, 5 + 3\mu)$



$$\frac{2\mu + 3\lambda + 2}{-3} = \frac{\mu - 2\lambda - 3}{5} = \frac{3\mu - 4\lambda - 2}{16}$$

$$= \frac{7\lambda + 8}{-13} = \frac{-2\lambda - 7}{-1}$$

$$7\lambda + 8 = -26\lambda - 91 \Rightarrow \lambda = -3$$

$$\mu = 2$$

$$C(11, 0, -5), D(8, 5, 11)$$

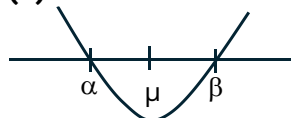
$$|\overrightarrow{CD}|^2 = 9 + 25 + 256 = 290$$

9. Let α and β be the roots of the equation $x^2 + 2ax + (3a + 10) = 0$ such that $\alpha < 1 < \beta$. Then the set of all possible values of a is :

- (1) $\left(-\infty, \frac{-11}{5}\right) \cup (5, \infty)$
(2) $(-\infty, -2) \cup (5, \infty)$
(3) $(-\infty, -3)$
(4) $\left(-\infty, \frac{-11}{5}\right)$

Ans. (4)

Sol.



$$1 + 2a + 3a + 10 < 0$$

$$a < -\frac{11}{5}$$

10. A random variable X takes values $0, 1, 2, 3$ with probabilities $\frac{2a+1}{30}, \frac{8a-1}{30}, \frac{4a+1}{30}, b$ respectively, where $a, b \in \mathbb{R}$. Let μ and σ respectively be the mean and standard deviation of X such that $\sigma^2 + \mu^2 = 2$. Then $\frac{a}{b}$ is equal to :

- (1) 30 (2) 2
(3) 60 (4) 12

Ans. (3)

Sol. $\sum p_i = 1$

$$14a + 30b = 29$$

X	0	1	2	3
P				

$$\sigma^2 = 0 + 1 \left(\frac{8a-1}{30} \right) + 4 \left[\frac{4a+1}{30} \right] + 9b - \mu^2$$

$$2 = \frac{24a+3}{30} + 9b \Rightarrow 24a + 270b = 57$$

$$a = 2, b = \frac{1}{30}$$

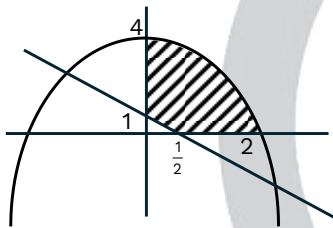
11. If the area of the region $\{(x, y) : 1 - 2x \leq y \leq 4 - x^2, x \geq 0, y \geq 0\}$ is $\frac{\alpha}{\beta}$, $\alpha, \beta \in \mathbb{N}$, $\gcd(\alpha, \beta) = 1$, then

the value of $(\alpha + \beta)$ is :

- (1) 73 (2) 85
(3) 91 (4) 67

Ans. (1)

Sol.



$$\frac{2}{3} \times 8 - \frac{1}{4} = \frac{61}{12}$$

12. Let $a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \dots, \frac{a_{10}}{2^9}$ be a G.P. of common ratio $\frac{1}{\sqrt{2}}$. $a_1 + a_2 + \dots + a_{10} = 62$, then a_1 is equal to :

- (1) $2(\sqrt{2} - 1)$ (2) $2 - \sqrt{2}$
(3) $\sqrt{2} - 1$ (4) $2(2 - \sqrt{2})$

Ans. (1)

Sol. $\frac{a_2}{2a_1} = \frac{1}{\sqrt{2}} = \frac{a_3}{2a_2} = \frac{a_4}{2a_3} = \dots$

a_1, a_2, a_3, \dots G.P. with C.R. $\sqrt{2}$

$$\frac{a_1 \left((\sqrt{2})^{10} - 1 \right)}{\sqrt{2} - 1} = 62$$

$$a_1 = \frac{62(\sqrt{2} - 1)}{31} = 2(\sqrt{2} - 1)$$

13. Let $A = \{x : |x^2 - 10| \leq 6\}$ and $B = \{x : |x - 2| > 1\}$. Then

- (1) $A \cup B = (-\infty, 1] \cup [2, \infty)$

- (2) $A - B = [2, 3)$
 (3) $A \cap B = [-4, -2] \cup [3, 4]$
 (4) $B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$

Ans. (4)

Sol. $A - 6 \leq x^2 - 10 \leq 6$

$$4 \leq x^2 \leq 16$$

$$x \in [-4, -2] \cup [2, 4]$$

$$B : x - 2 > 1 \text{ or } x - 2 < -1$$

$$x \in (-\infty, 1) \cup (3, \infty)$$

- 14.** For the matrices $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -29 & 49 \\ -13 & 18 \end{bmatrix}$, if $(A^{15} + B) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then among the following

which one is true ?

- (1) $x = 5, y = 7$ (2) $x = 18, y = 11$
 (3) $x = 11, y = 2$ (4) $x = 16, y = 3$

Ans. (3)

Sol. $A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

$$A^3 = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix} \dots\dots\dots A^n = \begin{bmatrix} 2n+1 & -4n \\ n & -(2n-1) \end{bmatrix}$$

$$A^{15} = \begin{bmatrix} 31 & -60 \\ 15 & -29 \end{bmatrix}, A^{15} + B = \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix}$$

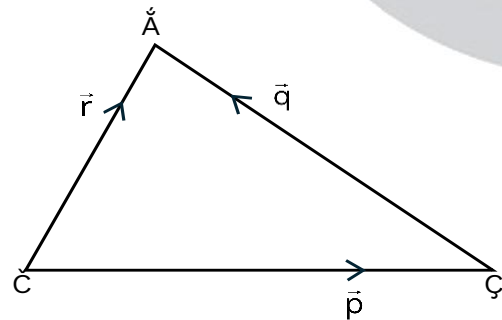
$$2x - 11y = 0$$

- 15.** For a triangle ABC, let $\vec{p} = \vec{BC}$, $\vec{q} = \vec{CA}$ and $\vec{r} = \vec{BA}$. If $|\vec{p}| = 2\sqrt{3}$, $|\vec{q}| = 2$ and $\cos \theta = \frac{1}{\sqrt{3}}$ where θ is the

angle between \vec{p} and \vec{q} , then $|\vec{p} \times (\vec{q} - 3\vec{r})|^2 + |\vec{r}|^2$ is equal to :

- (1) 340 (2) 220
 (3) 410 (4) 200

Ans. (4)



Sol.

$$\vec{p} + \vec{q} = \vec{r}$$

$$12 + 4 + 2 \times \frac{2\sqrt{3} \times 2}{\sqrt{3}} = |\vec{r}|^2 \Rightarrow |\vec{r}|^2 = 24$$

$$|\vec{p} \times \vec{q} - 3\vec{p} \times (\vec{p} + \vec{q})|^2$$

$$= 4|\vec{p} \times \vec{q}|^2 = 4 \times 12 \times 4 \times \frac{2}{3} = 128$$

$$128 + 3 \times 24 = 200$$

16. Let $y = y(x)$ be the solution of the differential equation

$$\sec x \frac{dy}{dx} - 2y = 2 + 3 \sin x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), y(0) = -\frac{7}{4}. \text{ Then } y\left(\frac{\pi}{6}\right) \text{ is equal to :}$$

- (1) $-\frac{5}{2}$ (2) $-\frac{5}{4}$
 (3) $-3\sqrt{3} - 7$ (4) $-3\sqrt{2} - 7$

Ans. (1)

Sol. $\frac{dy}{dx} - 2y \cos x = 2 \cos x + 3 \sin x \cos x$

IF = $e^{-2 \sin x}$

$$y e^{-2 \sin x} = \int (2 + 3 \sin x) \cos x e^{-2 \sin x} dx$$

$$= \int e^{-2t} (2 + 3t) dt$$

$$= (2 + 3t) \left(\frac{-e^{-2t}}{2} \right) + \frac{3}{2} \int e^{-2t} dt = (2 + 3t) \left(\frac{-e^{-2t}}{2} \right) - \frac{3}{4} e^{-2t} + C$$

$$y = \frac{-(2 + 3 \sin x)}{2} - \frac{3}{4} + C e^{2 \sin x}$$

$$-\frac{7}{4} = -\frac{7}{4} + C e^{2 \sin x} \Rightarrow C = 0$$

$$y\left(\frac{\pi}{6}\right) = -\frac{5}{2}$$

17. Let $A = \{2, 3, 5, 7, 9\}$. Let R be the relation on A defined by xRy if and only if $2x \leq 3y$. Let ℓ be the number of element in R , and m be the minimum number of element required to be added in R to make it a symmetric relation. Then $\ell + m$ is equal to:

- (1) 23 (2) 25
 (3) 21 (4) 27

Ans. (2)

Sol. $R = \left\{ \begin{array}{l} (2, 2), (2, 3), (2, 5), (2, 7), (2, 9) \\ (3, 2), (3, 3), (3, 5), (3, 7), (3, 9) \\ (5, 3), (5, 5), (5, 7), (5, 9) \\ (7, 5), (7, 9), (7, 7), (9, 7), (9, 9) \end{array} \right\}$

$$\ell = 19$$

$$m = 6$$

$$\ell + m = 25$$

18. If the system of equations

$$3x + y + 4z = 3$$

$$2x + \alpha y - z = -3$$

$$x + 2y + z = 4$$

has no solution, then the value of α is equal to :

- (1) 19 (2) 4
 (3) 13 (4) 23

Ans. (1)

Sol. $D = \begin{vmatrix} 3 & 1 & 4 \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$$\Rightarrow 3(\alpha + 2) - (3) + 4(4 - \alpha) = 0$$

$$\alpha = 19$$

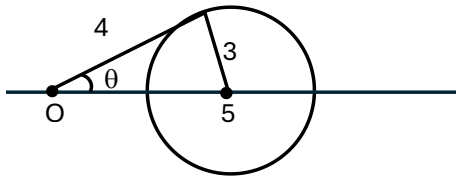
19. Let z be the complex number satisfying $|z - 5| \leq 3$ and having maximum positive principal argument.

Then $34 \left| \frac{5z - 12}{5iz + 16} \right|^2$ is equal to :

- (1) 16 (2) 12
(3) 26 (4) 20

Ans. (4)

Sol.



$$Z = \frac{16}{5} + \frac{12i}{5}$$

$$34 \left| \frac{16 + 12i - 12}{16i - 12 + 16} \right|^2$$

$$= 34 \times \left| \frac{4(1 + 3i)}{4(1 - 4i)} \right|^2 = \frac{34 \times 10}{17} = 20$$

20. The largest $n \in \mathbb{N}$, for which 7^n divides $101!$, is :

- (1) 16 (2) 18
(3) 15 (4) 19

Ans. (1)

Sol. $\left[\frac{101}{7} \right] + \left[\frac{101}{7^2} \right] = 14 + 2 = 16$

21. Let $[\cdot]$ denote the greatest integer function and $f(x) = \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n \left[\frac{k^2}{3^x} \right]$. Then $12 \sum_{j=1}^{\infty} f(j)$ is equal to

Ans. (2)

Sol. $\lim_{n \rightarrow \infty} \frac{1}{n^3} \left[\frac{1^2}{3^x} + \frac{2^2}{3^x} + \dots + \frac{n^2}{3^x} - \left\{ \frac{1}{3^x} \right\} - \left\{ \frac{2^2}{3^x} \right\} - \dots \right] = \frac{1}{3 \cdot 3^x}$

$$f(x) = \frac{1}{3^{x+1}}$$

$$12 \left[\frac{\frac{1}{9}}{1 - \frac{1}{3}} \right] = 12 \times \frac{1}{9} \times \frac{3}{2} = 2$$

22. If $\int_0^1 4 \cot^{-1}(1 - 2x + 4x^2) dx = a \tan^{-1}(2) - b \log_e(5)$, where $a, b \in \mathbb{N}$, then $(2a + b)$ is equal to

Ans. (9)

Sol. $\int_0^1 4 \tan^{-1} \left[\frac{2x - (2x - 1)}{1 + 2x(2x - 1)} \right] dx =$

$$4 \int_0^1 [\tan^{-1} 2x - \tan^{-1}(2x - 1)] dx = 4(I_1 + I_2)$$

$$I_2 = \int_0^1 \tan^{-1}(2x-1) dx$$

$$I_2 = \int_0^1 \tan^{-1}(1-2x) dx$$

$$2I_2 = 0$$

$$I_1 = \left(x \tan^{-1} 2x \right)_0^1 - \int_0^1 \frac{2x}{1+4x^2} dx = \tan^{-1} 2 - \left(\frac{1}{4} \ell n(1+4x^2) \right)_0^1 = \tan^{-1} 2 - \frac{1}{4} \ell n 5$$

$$4(I_1 + I_2) = 4 \tan^{-1} 2 - \ell n 5$$

$$a = 4, b = 1$$

23. Let the maximum value of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ for $x \in \left[-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}} \right]$ be $\frac{m}{n} \pi^2$, where $\gcd(m, n)$. Then

$m + n$ is equal to _____

Ans. (65)

Sol. $(\sin^{-1}x)^2 + \left(\frac{\pi}{2} - \sin^{-1}x \right)^2$

$$2(\sin^{-1}x)^2 - \pi \sin^{-1}x + \frac{\pi^2}{4}$$

Let $\sin^{-1}x = t$, $t \in \left[-\frac{\pi}{3}, \frac{\pi}{4} \right]$

$$2t^2 - \pi t + \frac{\pi^2}{4}$$

Maximum value at $t = \frac{-\pi}{3}$

$$\frac{2\pi^2}{9} + \frac{\pi^2}{3} + \frac{\pi^2}{4} = \frac{29\pi^2}{36}$$

$$= 65$$

24. $\left(\frac{1}{{}^{15}C_0} + \frac{1}{{}^{15}C_1} \right) \left(\frac{1}{{}^{15}C_1} + \frac{1}{{}^{15}C_2} \right) \dots \left(\frac{1}{{}^{15}C_{12}} + \frac{1}{{}^{15}C_{13}} \right) = \frac{\alpha^{13}}{{}^{14}C_0 {}^{14}C_1 \dots {}^{14}C_{12}}$, then 30α is equal to

Ans. (32)

Sol. $\frac{1}{{}^{15}C_r} + \frac{1}{{}^{15}C_{r+1}} = \frac{{}^{16}C_{r+1}}{{}^{15}C_{r+1} \cdot {}^{15}C_r} = \frac{16! r!(r+1)!(15-r)!(14-r)!}{(r+1)!(15-r)!15!15!}$

$$= \frac{16 r!(14-r)!}{15!}$$

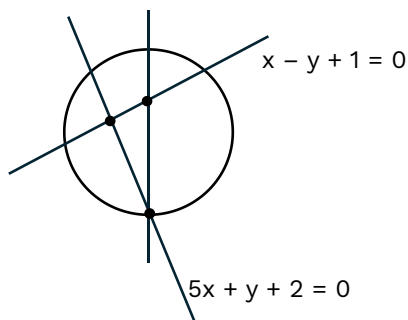
$$= \left(\frac{16}{15} \right) \frac{1}{{}^{14}C_r}$$

Now, $\left(\frac{16}{15} \right)^{13} \frac{1}{{}^{14}C_0 {}^{14}C_1 \dots {}^{14}C_{12}}$, $\alpha = \frac{16}{15}$

$$30\alpha = 32$$

25. If P is a point on the circle $x^2 + y^2 = 4$, Q is a point on the straight line $5x + y + 2 = 0$ and $x - y + 1 = 0$ is the perpendicular bisector of PQ, then 13 times the sum of abscissa of all such points P is _

Ans. (2)

**Sol.**

$$P(2\cos\theta, 2\sin\theta)$$

$$Q(\alpha, -5\alpha - 2)$$

$$\frac{2\cos\theta + \alpha}{2} - \frac{2\sin\theta - 2 - 5\alpha}{2} + 1 = 0$$

$$\cos\theta - \sin\theta + 3\alpha + 2 = 0$$

$$\frac{2\sin\theta + 5\alpha + 2}{2\cos\theta - \alpha} = -1$$

$$2\sin\theta + 5\alpha + 2 = -2\cos\theta + \alpha$$

$$\cos\theta + \sin\theta + 2\alpha + 1 = 0$$

$$(3\alpha + 2)^2 = (2\alpha + 1)^2 = 2$$

$$13\alpha^2 + 16\alpha + 3 = 0$$

$$(\alpha + 1)(13\alpha + 3) = 0 \Rightarrow \alpha = -1, \alpha = -\frac{3}{13}$$

$$\cos\theta - \sin\theta = 1 \quad \text{or} \quad \cos\theta - \sin\theta = -\frac{17}{13}$$

$$\cos\theta + \sin\theta = 1 \quad \text{or} \quad \cos\theta + \sin\theta = \frac{-7}{13}$$

$$\cos\theta = 1 \quad \text{or} \quad \cos\theta = \frac{-12}{13}$$

$$2(1) = +2\left(-\frac{12}{13}\right) = 2\left[1 - \frac{12}{13}\right] = \frac{2}{13}$$

$$= 2$$

PHYSICS TEST PAPER WITH SOLUTION

- 26.** Consider two identical metallic spheres of radius R each having charge Q and mass m . Their centers have an initial separation of $4R$. Both the spheres are given an initial speed of u towards each other. The minimum value of u , so that they can just touch each other is: **[Electrostatics]**

(Take $k = \frac{1}{4\pi\epsilon_0}$ and assume $kQ^2 > Gm^2$ where G is the Gravitational constant)

$$(1) \sqrt{\frac{KQ^2}{4mR} \left(1 - \frac{Gm^2}{KQ^2}\right)} \quad (2) \sqrt{\frac{KQ^2}{4mR} \left(1 + \frac{Gm^2}{KQ^2}\right)}$$

$$(3) \sqrt{\frac{KQ^2}{2mR} \left(1 - \frac{Gm^2}{KQ^2}\right)} \quad (4) \sqrt{\frac{KQ^2}{2mR} \left(1 - \frac{Gm^2}{2KQ^2}\right)}$$

Ans. (1)**Sol.** $KE_i + PE_i = KE_f + PE_f$

$$\left(\frac{1}{2}mu^2 + \frac{1}{2}mu^2\right) + \left(\frac{KQ^2}{4R} - \frac{GMm}{4R}\right) = (0)$$

$$+ \left(\frac{KQ^2}{2R} - \frac{Gm^2}{2R}\right)$$

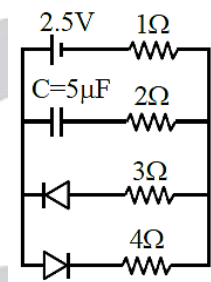
$$mu^2 = \left(\frac{KQ^2}{2R} - \frac{KQ^2}{4R}\right) - \left(\frac{Gm^2}{2R} - \frac{Gm^2}{4R}\right)$$

$$mu^2 = \frac{KQ^2 - Gm^2}{4R}$$

$$u = \sqrt{\frac{KQ^2}{4mR} \left(1 - \frac{Gm^2}{KQ^2}\right)}$$

27. The charge stored by the capacitor C in the given circuit in the steady state is μC .

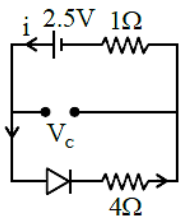
[semiconductor]



- (1) 12.5
(2) 10
(3) 7.5
(4) 5

Ans. (2)

Sol.



In steady state

$$i = \frac{2.5}{5} = 0.5 \text{ A}$$

$$V_c = 4 \times 0.5$$

$$V_c = 2 \text{ V}$$

Charge

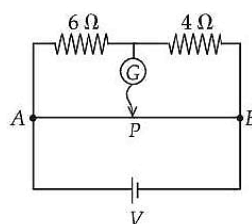
$$Q = CV_2$$

$$= 5 \times 2$$

$$= 10 \mu\text{C}$$

28. The total length of potentiometer wire AB is 50 cm in the arrangement as shown in figure. If P is the point where the galvanometer shows zero reading then the length AP is cm.

[Current Electricity]



- (1) 15 (2) 30
(3) 25 (4) 20

Ans. (2)

Sol. $\frac{R_1}{R_2} = \frac{R_{AP}}{R_{PB}}$

$$\frac{R_{AP}}{R_{PB}} = \frac{L_{AP}}{L_{PB}}$$

$$L_{AB} = 50, L_{PB} = 50 - L_{AP}$$

$$= \frac{6\Omega}{4\Omega} = \frac{L_{AP}}{50 - L_{AP}}$$

$$L_{AP} = \frac{150}{5} = 30\text{cm}$$

- 29.** A capacitor C is first charged fully with potential difference of V, and disconnected from the battery. The charged capacitor is connected across an inductor having inductance L. In 25% of the initial energy in the capacitor is transferred to the inductor. The value of t is S.

[Electromagnetic

Induction]

- (1) $\frac{\pi\sqrt{LC}}{3}$ (2) $\frac{\pi\sqrt{LC}}{6}$
(3) $\frac{\pi\sqrt{LC}}{2}$ (4) $\pi\sqrt{\frac{LC}{2}}$

Ans. (2)

Sol. Initial Energy $\rightarrow U_0 = \frac{1}{2}CV^2$

In LC circuit \rightarrow

$$U_C = U_0 \cos^2(\omega t)$$

Energy in inductor at time t

$$U_L = U_0 \sin^2(\omega t)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\sin^2(\omega t) = \frac{1}{4}$$

$$\sin(\omega t) = \frac{1}{2}$$

$$\omega t = \frac{\pi}{6}$$

$$t = \frac{\pi}{6\omega} = \frac{\pi}{6} \sqrt{LC}$$

- 30.** The r.m.s. speed of oxygen molecules at 47 °C is equal to that of the hydrogen molecules kept at °C. (Mass of oxygen molecule/mass of hydrogen molecule = 32/2)

[Heat and Thermodynamics]

- (1) -235 (2) -100
(3) -253 (4) -20

Ans. (3)

Sol. $(V_{rms})_{O_2} = (V_{rms})_{H_2}$

$$\sqrt{\frac{3RT_{O_2}}{M_{O_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$\frac{T_{O_2}}{M_{O_2}} = \frac{T_{H_2}}{M_{H_2}}$$

$$T_{H_2} = T_{O_2} \times \frac{M_{H_2}}{M_{O_2}} = 320K \times \frac{1}{16} = 20K$$

$$T_{H_2} (\text{in } ^\circ\text{C}) = 20K - 273 = -253^\circ\text{C}$$

31. Two cars A and B each of mass 10^3 kg are moving on parallel tracks separated by a distance of 10 m, in same direction with speeds 72 km/h and 36 km/h. The magnitude of angular momentum of car A with respect to car B is J.s.

[Rotational Motion]

- (1) 3.6×10^5 (2) 10^5
(3) 3×10^5 (4) 2×10^5

Ans. (2)

Sol. $V_A = 72 \text{ km/h} = 20 \text{ m/s } \hat{i}$

$$V_B = 36 \text{ km/h} = 10 \text{ m/s } \hat{i}$$

$$\vec{V}_{AB} = \vec{V}_{\text{rel}} = \vec{V}_A - \vec{V}_B = 20 - 10 = 10 \text{ m/s } \hat{i}$$

$$\vec{r}_{AB} = 10\text{m} \hat{j}$$

$$\vec{L} = \vec{r}_{AD} \times (m\vec{V}_{AB})$$

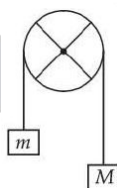
$$= r_{AB} m V_{AB} \sin \theta$$

$$\theta = 90^\circ$$

$$L = 10 \times 10^3 \times 10 \\ = 10^5 \text{ J.s}$$

32. The pulley shown in figure is made using a thin rim and two rods of length equal to diameter of the rim. The rim and each rod have a mass of M. Two blocks of mass of M and m are attached to two ends of a light string passing over the pulley, which is hinged to rotate freely in vertical plane about its center. The magnitudes of the acceleration experienced by the blocks is (assume no slipping of string on pulley).

[Rotational Motion]



(1) $\frac{(M-m)g}{\left[\left(\frac{13}{6}\right)M+m\right]}$ (2) $\frac{(M-m)g}{M+m}$

(3) $\frac{(M-m)g}{\left[\left(\frac{8}{3}\right)M+m\right]}$ (4) $\frac{(M-m)g}{2M+m}$

Ans. (3)

Sol. MOI of pulley

$$I = MR^2 + \frac{2}{3}MR^2 = \frac{5}{3}MR^2$$

For mass M (moving down)

$$Mg - T_2 = Ma$$

For mass m (moving up)

$$T_1 - mg = ma$$

For pulley

$$(T_2 - T_1) R = I \alpha$$

$$(T_2 - T_1) R = \frac{5}{2} MR^2 \times \frac{a}{R}$$

$$T_2 - T_1 = \frac{5}{3} Ma$$

$$[T_2 = Mg - Ma]$$

$$[T_1 = mg + ma]$$

$$a = \frac{(M - m)g}{\frac{8}{3}M + m}$$

- 33.** The kinetic energy of a simple harmonic oscillator is oscillating with angular frequency of 176 rad/s. The frequency of this simple harmonic oscillator is.....Hz

$$\left[\text{Take } \pi = \frac{22}{7} \right]$$

[Simple Harmonic Motion]

- (1) 14 (2) 88
(3) 28 (4) 176

Ans. (1)

Sol. $f_{KE} = \frac{\omega_{KE}}{2\pi}$

$$f_{KE} = \frac{176}{2 \times \frac{22}{7}} = 28 \text{ Hz}$$

$$f = \frac{f_{KE}}{2} = \frac{28}{2} = 14 \text{ Hz}$$

- 34.** Given below are two statements:

Statement I: In a Young's double slit experiment, the angular separation of fringes will increase as the screen is moved away from the plane of the slits

Statement II: In a Young's double slit experiment, the angular separation of fringes will increase when monochromatic source is replaced by another monochromatic source of higher wavelength

In the light of the above statements, choose the correct answer from the options given below:

[Wave Optics]

- (1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is false but Statement II is true
(4) Statement I is true but Statement II is false

Ans. (3)

Sol. Angular fringe width = $\frac{\lambda}{d}$

- 35.** A battery with EMF E and internal resistance r is connected across a resistance R . The power consumption in R will be maximum when:

[Current Electricity]

- (1) $R = 2r$ (2) $R = \frac{r}{2}$

(3) $R = \sqrt{2}r$ (4) $R = r$

Ans. (4)

Sol. $I = \frac{F}{R+r}$

$$P = \left(\frac{E}{R+r} \right)^2 R = \frac{E^2 R}{(R+r)^2}$$

$$\frac{dP}{dr} = E^2 \left[\frac{(R+r)^2(1) - R(2)(R+r)(1)}{(R+r)^4} \right] = 0$$

$$(R+r)^2 - 2R(R+r) = 0$$

$$(R+r) [(R+r) - 2R] = 0$$

$$(R+r)(r-R) = 0$$

$$R = r$$

36. Keeping the significant figures in view, the sum of the physical quantities 52.01 m, 153.2 m and 0.123 m is:

[Unit and Dimensions]

(1) 205 M (2) 205.333m

(3) 205.33m (4) 205.3 m

Ans. (4)

Sol. $52.01 + 153.2 + 0.123 = 205.333 \text{ m}$

Rounding the calculated sum of 205.333 m to one decimal place gives.
205.3 m

37. A spherical body of radius r and density σ falls freely through a viscous liquid having density ρ and viscosity η and attains a terminal velocity v_0 . Estimated maximum error in the quantity η is: (Ignore errors associated with σ , ρ and g , gravitational acceleration)

[Properties of matter]

(1) $2 \frac{\Delta r}{r} - \frac{\Delta v_0}{v_0}$ (2) $\frac{2\Delta r}{r} + \frac{\Delta v_0}{v_0}$

(3) $2 \left[\frac{\Delta r}{r} - \frac{\Delta v_0}{v_0} \right]$ (4) $2 \left[\frac{\Delta r}{r} + \frac{\Delta v_0}{v_0} \right]$

Ans. (2)

Sol. $v_0 = \frac{2r^2(T - \rho)}{9\eta}$

$$\eta = \frac{2r^2 g(T - \rho)}{9v_0}$$

$$\frac{\Delta \eta}{\eta} = \frac{2\Delta r}{r} + \frac{\Delta v_0}{v_0}$$

38. Surface tension of two liquids (having same densities), T_1 and T_2 , are measured using capillary rise method utilizing two tubes with inner radii of, r_1 and r_2 where $r_1 > r_2$. The measured liquid heights in these tubes are h_1 and h_2 respectively. [Ignore the weight of the liquid about the lowest point of meniscus]. The heights, h_1 and h_2 surfaces tensions T_1 and T_2 satisfy the relation:

[Surface Tension]

(1) $h_1 < h_2$ and $T_1 = T_2$

(2) $h_1 = h_2$ and $T_1 = T_2$

(3) $h_1 > h_2$ and $T_1 = T_2$

(4) $h_1 > h_2$ and $T_1 < T_2$

Ans. (1)

Sol. $h = \frac{2T \cos \theta}{\rho g r}$

$$h \propto \frac{T}{r}$$

$$r_1 > r_2$$

$$h \propto \frac{1}{r}$$

$$h_1 < h_2$$

$$T_1 = T_2$$

- 39.** A river of width 200 m is flowing from west to east with a speed of 18 km/h. A boat, moving with speed of 36 km/h in still water, is made to travel one-round trip (bank to bank of the river). Minimum time taken by the boat for this journey and also the displacement along the river bank are and respectively. **[Kinematics]**

(1) 20 s and 100 m

(2) 40 s and 0 m

(3) 40 s and 200 m

(4) 40 s and 100 m

Ans. (3)

Sol. $d = 200$ m

$$V_r = 18 \text{ km/h} = 5 \text{ m/s}$$

$$V_b = 36 \text{ km/h} = 10 \text{ m/s}$$

$$t_1 = \frac{200}{10} = 20 \text{ sec}$$

$$t = 2t_1 = 40 \text{ sec}$$

$$x_1 = V_r \times t_1 = 5 \times 20 = 100 \text{ m}$$

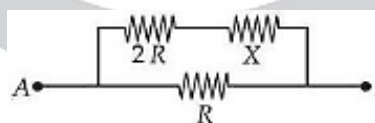
Round trip drift

$$x = 2x_1 = 200 \text{ m}$$

40 sec and 200 m

- 40.** Two known resistances of $R \Omega$ and $2R \Omega$ and one unknown resistance $X \Omega$ are connected in a circuit as shown in the figure. If the equivalent resistance between points A and B in the circuit is $X \Omega$, then the value of X is _____ Ω .

[Current Electricity]



(1) $(\sqrt{3} - 1)R$

(2) R

(3) $2(\sqrt{3} - 1)R$

(4) $(\sqrt{3} + 1)R$

Ans. (1)

Sol. $X = \frac{R(2R + x)}{R + (2R + x)}$

$$X^2 = 2RX - 2R^2 = 0$$

$$X = \frac{-2R \pm \sqrt{(R)^2 + 4R^2}}{2}$$

$$X = (-1 \pm \sqrt{3})R$$

$$X = (\sqrt{3} - 1)R$$

41. The energy of an electron in an orbit of the Bohr's atom is $-0.04E_0$ eV where E_0 is the ground state energy. If L is the angular momentum of the electron in this orbit and h is the Planck's constant, then $\frac{2\pi L}{h}$ is _____: **[Modern Physics]**

- (1) 2 (2) 4
(3) 5 (4) 6

Ans. (3)

Sol. $-0.04E_0 = \frac{-E_0}{n^2}$

$$\frac{1}{n^2} = 0.04$$

$$n^2 = \frac{100}{4}$$

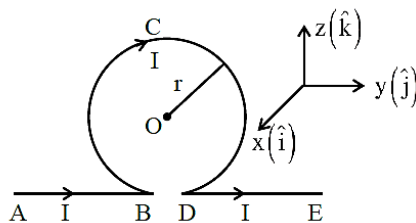
$$n^2 = 25$$

$$n = 5$$

$$\frac{2\pi L}{h} = n$$

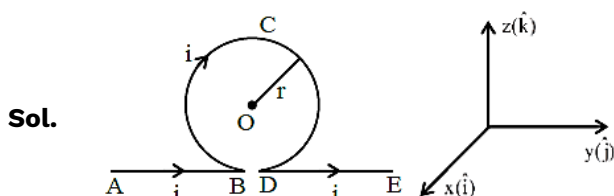
$$\frac{2\pi L}{h} = 5$$

42. An infinitely long straight wire carrying current I is bent in a planer shape as shown in the diagram. The radius of the circular part is r . The magnetic field at the centre O of the circular loop is: **[Magnetic Effect of current]**



- (1) $\frac{\mu_0}{2\pi} \frac{I}{r} (\pi + 1) \hat{i}$
(2) $-\frac{\mu_0}{2\pi} \frac{I}{r} (\pi - 1) \hat{i}$
(3) $\frac{\mu_0}{2\pi} \frac{I}{r} (\pi - 1) \hat{i}$
(4) $-\frac{\mu_0}{2\pi} \frac{I}{r} (\pi + 1) \hat{i}$

Ans. (2)



Sol.

$$\vec{B}_O = \vec{B}_{AB} + \vec{B}_{DE} + \vec{B}_{BCD}$$

$$= \frac{\mu_0 i}{4\pi r} \hat{i} + \frac{\mu_0 i}{4\pi r} \hat{i} - \frac{\mu_0 i}{2r} \hat{i}$$

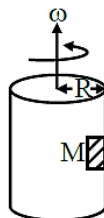
$$= \frac{\mu_0 i}{2\pi r} \hat{i} - \frac{\mu_0 i}{2r} \hat{i}$$

$$= \frac{\mu_0 i}{2\pi r} (1 - \pi) \hat{i}$$

$$= -\frac{\mu_0 i}{2\pi r} (\pi - 1) \hat{i}$$

43. A large drum having radius R is spinning around its axis with angular velocity ω , as shown in figure. The minimum value of ω so that a body of mass M remains stuck to the inner wall of the drum, taking the coefficient of friction between the drum surface and mass M as μ , is:

[Circular Motion]



- (1) $\sqrt{\frac{\mu g}{R}}$ (2) $\sqrt{\frac{2g}{\mu R}}$
 (3) $\sqrt{\frac{g}{2\mu R}}$ (4) $\sqrt{\frac{g}{\mu R}}$

Ans. (4)

Sol. $\mu\omega^2 R \geq g$

$$\omega^2 \geq \frac{g}{\mu R}$$

$$\omega_{\min} = \sqrt{\frac{g}{\mu R}}$$

44. A body of mass 2 kg is moving along x-direction such that its displacement as function of time is given by $x(t) = at^2 + \beta t + \gamma$ m, where $a = 1 \text{ m/s}^2$, $\beta = 1 \text{ m/s}$ and $\gamma = 1 \text{ m}$. The work done on the body during the $t = 2 \text{ s}$ to $t = 3 \text{ s}$, is [Work Power Energy]

- (1) 49 (2) 42
 (3) 24 (4) 12

Ans. (3)

Sol. $\omega = \Delta K = K_f - K_i = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$

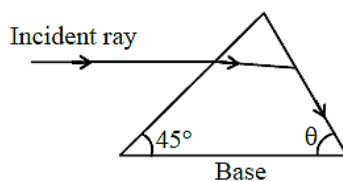
$$\omega = \frac{1}{2}(2\text{kg})(7\text{m/s})^2 - \frac{1}{2}(2\text{kg})(5\text{m/s})^2$$

$$\omega = 1(49) - 1(25)$$

$$\omega = 24 \text{ J}$$

45. As shown in the diagram, when the incident ray is parallel to base of the prism, the emergent ray grazes along the second surface. Incident ray

[Geometrical Optics]

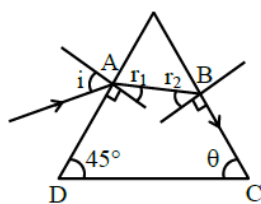


If refractive index of the material of prism is $\sqrt{2}$, the angle of prism is.

- (1) 60° (2) 75°
 (3) 90° (4) 45°

Ans. (1)

Sol.



For grazing emergence

$$\sin r_2 = \frac{1}{\mu}$$

By Snell's law at incident surface

$$1 \times \frac{1}{\sqrt{2}} = \sqrt{2} \sin r_1$$

$$r_1 = 30^\circ$$

$$r_1 + r_2 = A$$

$$A = 75^\circ$$

$$75^\circ + 45^\circ + \theta = 180^\circ$$

$$\theta = 60^\circ$$

46. An electromagnetic wave of frequency 100 MHz propagates through a medium of conductivity, $\sigma = 10$ mho/m. The ratio of maximum conduction current density to maximum displacement current density is _____.

[EM Waves]

$$\left[\text{Take } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 / \text{C}^2 \right]$$

Ans. 1800

Sol. $J_c = TE$

$$J_d = \omega \epsilon E$$

$$\frac{J_c}{J_d} = \frac{T}{\omega \epsilon} = \frac{10}{(2\pi \times 10^8) \times \frac{1}{(4\pi \times 9 \times 10^9)}}$$

$$\frac{J_c}{J_d} = \frac{40\pi \times 9 \times 10^9}{2\pi \times 10^8}$$

$$\frac{J_c}{J_d} = \frac{360 \times 10}{2} = 1800$$

47. The terminal velocity of a metallic ball of radius 6 mm in a viscous fluid is 20 cm/s. The terminal velocity of another ball of same material and having radius 3 mm in the same fluid will be cm/s.

[Viscosity]

Ans. 5

Sol. $\frac{V_2}{V_1} = \left(\frac{r_2}{r_1} \right)^2$

$$\frac{V_2}{20} = \left(\frac{3}{6} \right)^2 = \frac{1}{4}$$

$$V_2 = 5 \text{ cm/sec}$$

48. A particle having electric charge 3×10^{-19} C and mass 6×10^{-27} kg is accelerated by applying an electric potential of 1.21 V. Wavelength of the matter wave associated with the particle is $a \times 10^{-12}$ m. The value of a is (Take Planck's constant = 6.6×10^{-34} J.s)

[Modern Physics]

Ans. 10

Sol. $KE = (3 \times 10^{-19} \text{ C}) \times (1.21 \text{ V})$
 $= 3.63 \times 10^{-19} \text{ J}$

$$KE = \frac{p^2}{2m}, \quad p = \sqrt{2m \times KE}$$

$$p = \sqrt{2 \times (6 \times 10^{-27} \text{ kg}) \times (3.63 \times 10^{-19} \text{ J})}$$

$$p = 6.6 \times 10^{-23} \text{ Kg m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34} \text{ J.s}}{6.6 \times 10^{-23} \text{ kg.m / s}}$$

$$\lambda = 1 \times 10^{-11} \text{ m} = 10 \times 10^{-12} \text{ m}$$

$$a = 10$$

- 49.** In a Young's double slit experiment set up, the two slits are kept 0.4. mm apart and screen is placed at 1 m from slits. If a thin transparent sheet of thickness $20 \mu\text{m}$ is introduced in front of one of the slits then center bright fringe shifts by 20 mm on the screen. The refractive index of transparent sheet α is given by $\frac{\alpha}{10}$ where α is

[Wave Optics]**Ans.** 14

Sol. $x = \frac{D}{d}(\mu - 1)t$

$$2 \times 10^{-2} = \frac{1}{4 \times 10^{-4}}(\mu - 1)(2 \times 10^{-5})$$

$$\mu - 1 = 0.4$$

$$\mu = 1.4$$

$$\mu = \frac{\alpha}{10} = 1.4$$

$$\alpha = 14$$

- 50.** A diatomic gas ($\gamma = 1.4$) does 100 J of work when it is expanded isobarically. Then the heat given to the gas _____ J.

[Thermodynamics]**Ans.** 350

Sol. $\frac{Q}{\omega} = \frac{nC_p \Delta T}{nR \Delta T} = \frac{C_p}{R}$

$$\frac{Q}{\omega} = \frac{C_p}{C_p - C_v}$$

$$\frac{Q}{\omega} = \frac{\frac{C_p}{C_v}}{\left(\frac{C_p}{C_v}\right) - 1}$$

$$= \frac{\gamma}{\gamma - 1}$$

$$Q = \omega \times \frac{\gamma}{\gamma - 1}$$

$$= 100 \times \frac{1.4}{1.4 - 1}$$

$$= 350 \text{ J}$$

CHEMISTRY TEST PAPER WITH SOLUTION

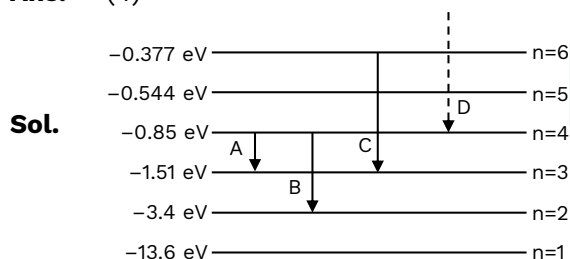
51. Consider the following spectral lines for atomic hydrogen: **[Atomic Structure]**

- A. First line of Paschen series
 B. Second line of Balmer series
 C. Third line of Paschen series
 D. Fourth line of Bracket series

The correct arrangement of the above lines in ascending order of energy is:

- (1) $D < C < A < B$ (2) $A < B < C < D$
 (3) $C < D < B < A$ (4) $D < A < C < B$

Ans. (4)



ΔE order is: $B > C > A > D$

52. Match List-I with List-II.

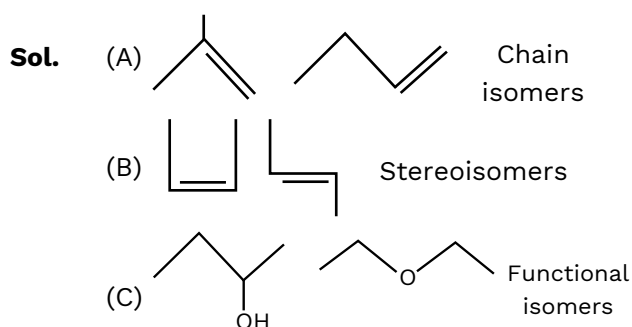
List-I Pair of Compounds		List-II Type of Isomers	
A.	2-Methylpropene and but-1-ene	I.	Stereoisomers
B.	Cis-but-2-ene and trans-but-2-ene	II.	Position isomers
C.	2-Butanol and diethyl ether	III.	chain isomers
D.	But-1-ene and but-2-ene	IV.	Functional group isomers

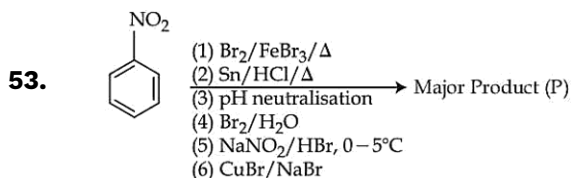
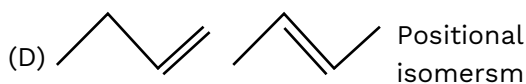
[Isomerism]

Choose the correct answer from the options given below:

- (1) A-III, B-I C-IV, D-II
 (2) A-III, B-I C-II, D-IV
 (3) A-I, B-IV, C-III, D-II
 (4) A-II, B-I, C-IV, D-III

Ans. (1)



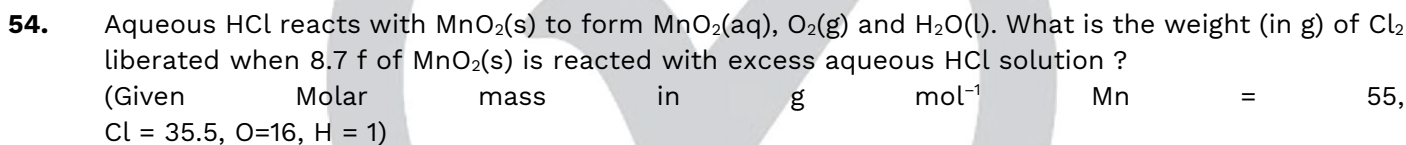
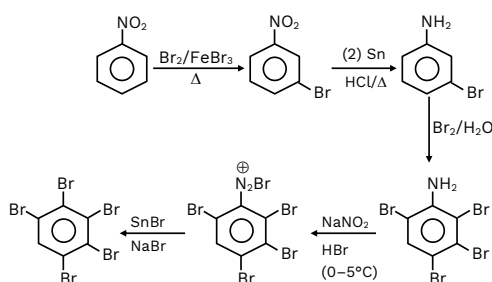


Consider the above sequence of reactions. The number of bromine atom(s) in the final product (P) will be:

[Aromatic Compounds]

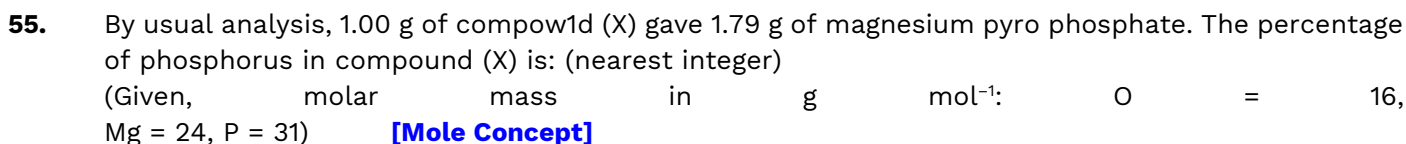
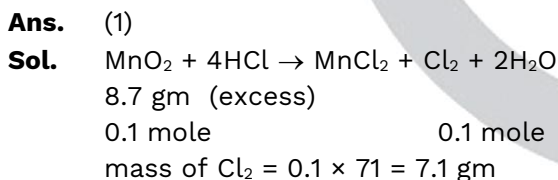
- (1) 1 (2) 6
(3) 5 (4) 3

Ans. (3)
Sol.

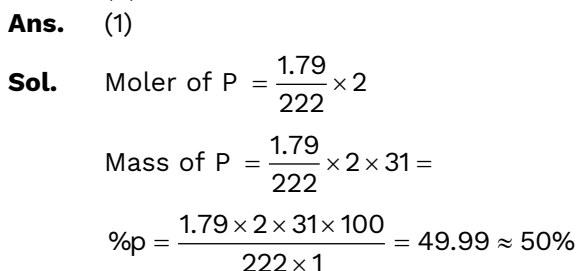


[Mole Concept]

- (1) 7.1 (2) 71
(3) 21.3 (4) 14.2



- (1) 50
(2) 30
(3) 20
(4) 40

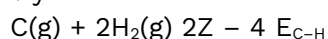


56. Consider the following data:
 $\Delta_f H^\circ$ (methane, g) = $-X \text{ kJ mol}^{-1}$
 Enthalpy of sublimation of graphite = $Y \text{ kJ mol}^{-1}$
 Dissociation enthalpy of H_2 = $Z \text{ kJ mol}^{-1}$
 The bond enthalpy of C-H bond is given by:

[Thermodynamics & Thermochemistry]

- (1) $\frac{X + Y + 2Z}{4}$ (2) $\frac{X + Y + 4Z}{2}$
 (3) $X + Y + Z$ (4) $\frac{-X + Y + Z}{4}$

Ans. (1)

Sol. $\text{C(s)} + 2\text{H}_2(\text{g}) \rightarrow \text{CH}_4(\text{g}) \quad \Delta_f H^\circ = -X$ $\downarrow y$ 

$$-X = Y + 2Z - 4 E_{\text{C-H}}$$

$$E_{\text{C-H}} = \frac{(x + y + 2z)}{4}$$

57. Match List-I with List-II.

List-I Reagents		List-II Reaction Name (Involving aldehydes)	
A.	$\text{H}_2, \text{Pd-BaSO}_4$	I.	Etard Reaction
B.	$\text{SnCl}_2, \text{HCl}$	II.	Rosenmund Reduction
C.	$\text{CrO}_2\text{Cl}_2, \text{CS}_2$	III.	Gatterman – Koch Reaction
D.	$\text{CO, HCl, Anhyd. AlCl}_3$	IV.	Stephen Reaction

[Carbonyl Compound]

Choose the correct answer from the options given below:

- (1) A-II, B-III, C-IV, D-I
 (2) A-IV, B-III, C-I, D-II
 (3) A-IV, B-I, C-II, D-III
 (4) A-II, B-IV, C-I, D-III

Ans. (4)

Sol. $\text{H}_2, \text{Pd-BaSO}_4 \rightarrow$ Rosenmund Reduction $\text{SnCl}_2/\text{HCl} \rightarrow$ Stephen Reaction $\text{CrO}_2\text{Cl}_2, \text{CS}_2 \rightarrow$ Etard Reaction $\text{CO, HCl, Anhydrous AlCl}_3 \rightarrow$ Gatterman Koch Reaction

58. Decomposition of A is a first order reaction at T(K) and is given by $\text{A(g)} \rightarrow \text{B(g)} + \text{C(g)}$.
 In a closed 1 L vessel, 1 bar A(g) is allowed to decompose at T(K). After 100 minutes, the total pressure was 1.5 bar. What is the rate constant (in min^{-1}) of the reaction ? ($\log 2 = 0.3$)

[Chemical Kinetics]

- (1) 6.9×10^{-1} (2) 6.9×10^{-3}
 (3) 6.9×10^{-2} (4) 6.9×10^{-4}

Ans. (2)

Sol. $\text{A} \rightarrow \text{B} + \text{C}$

t = 0 1 0 0

t = 100 min. 1-p p p

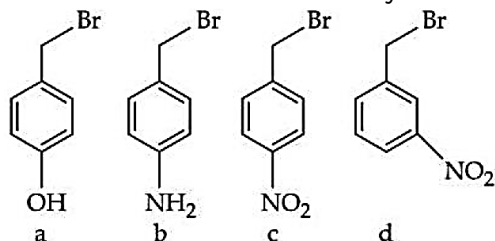
$$1.5 = 1 + p$$

$$p = 0.5$$

$$\Rightarrow t_{1/2} = 100 \text{ min.}$$

$$k = \frac{\ln 2}{100} = 6.9 \times 10^{-3}$$

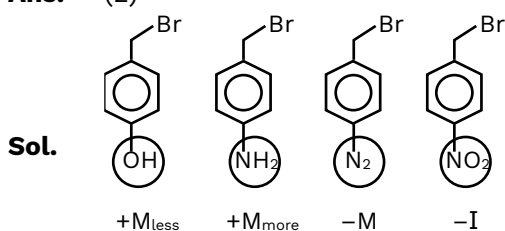
59. The correct order of reactivity of the following benzyl halides towards reaction with KCN is:



[Alkyl Halide (Halogen Derivatives)]

- (1) $a > b > c > d$
 (2) $b > a > d > c$
 (3) $b > a > c > d$
 (4) $a > b > d > c$

Ans. (2)



Sol.

Rate $\propto -M, -I$

$$\propto \frac{1}{+M}$$

60. Given below are two statements

Statement I: The correct order in terms of atomic/ ionic radii is $\text{Al} > \text{Mg} > \text{Mg}^{2+} > \text{Al}^{3+}$.

Statement II: The correct order in terms of the magnitude of electron gain enthalpy is $\text{Cl} > \text{Br} > \text{S} > \text{O}$.

In the light of the above statements, choose the correct answer from the options given below:

[Periodic Properties]

- (1) Both **Statement I** and **Statement II** are false
 (2) **Statement I** is false but **Statement II** is true
 (3) **Statement I** is true but **Statement II** is false
 (4) Both **Statement I** and **Statement II** are true

Ans. (2)

Sol.

Order of size: $\text{Mg} > \text{Al}$

Order of size: $\text{Mg}^{2+} > \text{Al}^{3+}$

Size of neutral atom > Size of cation

In 17th group order of $|\Delta_{\text{egH}}|$ $\text{Cl} > \text{F} > \text{Br} > \text{I}$

In 16th group order of $|\Delta_{\text{egH}}|$ $\text{S} > \text{Se} > \text{O}$

61. The correct statements are:

A. Activation energy for enzyme catalysed hydrolysis of sucrose is lower than that of acid catalysed hydrolysis.

B. During denaturation, secondary and tertiary structures of a protein are destroyed but primary structure remains intact.

C. Nucleotides are joined together by glycosidic linkage between C_1 and C_4 carbons of the pentose sugar.

D. Quaternary structure of proteins represents overall folding of the polypeptide chain.

Choose the correct answer from the options given below: [Biomolecules]

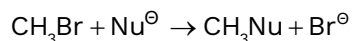
- (1) A, C and D Only (2) A, B and D Only
 (3) A and B Only (4) B and C Only

Ans. (3)

Sol. (A) Enzyme catalyst lowers activation energy more than acid catalysis \Rightarrow correct
 (B) Denaturation breaks sec/tertiary structure but primary remain intact \Rightarrow correct
 (C) In correct \Rightarrow Nucleotides are linked by phosphodiester bond ($3'-5'$) not glycosidic C_1-C_4 linkage
 (D) Incorrect overall folding of one chain is tertiary quaternary is arrangement of multiple subunits.

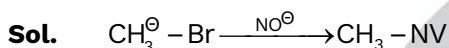
- 62.** The correct order of the rate of the reaction for the following reaction with respect to nucleophiles is:

[Alkyl Halide (Halogen Derivatives)]

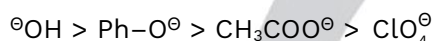


- (1) $PhO^\ominus > ^-\text{OH} > CH_3COO^\ominus > ClO_4^\ominus$
 (2) $ClO_4^\ominus > CH_3COO^\ominus > ^-\text{OH} > PhO^\ominus$
 (3) $CH_3COO^\ominus > PhO^\ominus > ^-\text{OH} > ClO_4^\ominus$
 (4) $^-\text{OH} > PhO^\ominus > CH_3COO^\ominus > ClO_4^\ominus$

Ans. (4)



Order of $\xrightarrow{\text{nucleophilicity}} +Br$



- 63.** Given below are two statements:

Statement I: Crystal Field Stabilization Energy (CFSE) of $[Cr(H_2O)_6]^{2+}$ is greater than that of $[Mn(H_2O)_6]^{2+}$.

Statement II: Potassium ferricyanide has a greater spin-only magnetic moment than sodium ferrocyanide.

In the light of the above statements, choose the correct answer from the options given below:

[Coordination Compound]

- (1) Both **Statement I** and **Statement II** are true
 (2) Both **Statement I** and **Statement II** are false
 (3) **Statement I** is true but **Statement II** is false
 (4) **Statement I** is false but **Statement II** is true

Ans. (1)

Sol. CFSE of $Cr(H_2O)_6^{2+}$ is $-0.6\Delta_0$

CFSE of $Mn(H_2O)_6^{2+}$ is zero.

Potential ferricyanide $K_3[Fe(CN)_6]$

$Fe^{3+}(3d^5)$ + strong field ligand.

Number of unpaired electron = 1

Sodium ferrocyanide $Na_4[Fe(CN)_6]$

$Fe^{2+}(3d^6)$ + Strong field ligand diamagnetic.

- 64.** The correct increasing order- of C-H(A), C-O(B), C=O(C) and C \equiv N(D) bonds in terms of covalent bond length is:

[Chemical Bonding]

- (1) $A < B < C < D$ (2) $A < D < C < B$
 (3) $D < C < B < A$ (4) $D < C < A < B$

Ans. (2)

Sol. C-H(A) C-O(B)

C=O(C) C \equiv N(D)

$C-O > C=O > C=N > C-H$

↑ ↑
State bond Smallest atom

65. Given below are four compounds:

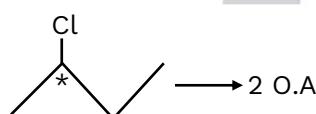
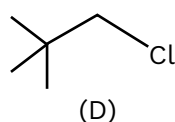
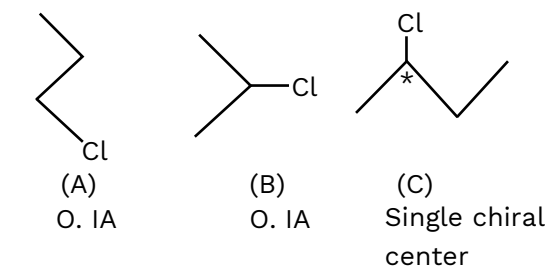
- (a) n-propyl chloride
- (b) iso-propyl chloride
- (c) sec-butyl chloride
- (d) neo-pentyl chloride

Percentage of carbon in the one which exhibits optical isomerism is:

[Isomerism]

- (1) 52 (2) 56
- (3) 46 (4) 40

Ans.
Sol.



$$\frac{12 \times 4}{92} \times 100 = 52.17$$

$$\approx 52\%$$

66. Given below are some of the statements about Mn and Mn_2O_7 . Identify the correct statements.

- A. Mn forms the oxide Mn_2O_7 , in which Mn is in its highest oxidation state.
- B. Oxygen stabilizes the Mn in higher oxidation states by forming multiple bonds with Mn.
- C. Mn_2O_7 is an ionic oxide.
- D. The structure of Mn_2O_7 consists of one bridged oxygen.

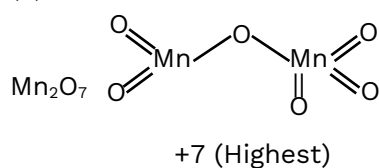
Choose the correct answer from the options given below:

[d-Block Elements]

- (1) A, B, C and D (2) A, B and D Only
- (3) A, C and D Only (4) A, B and C Only

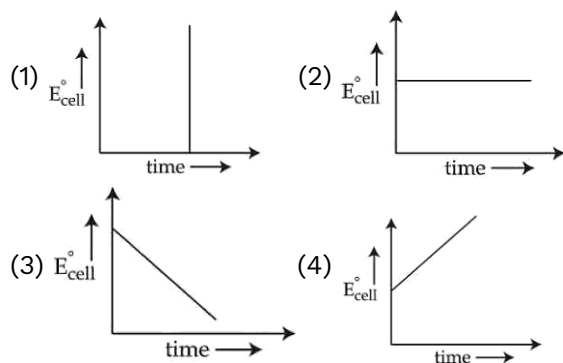
Ans. (2)

Sol.



67. For a closed circuit Daniell cell, which of the following plots is the accurate one at a given temperature?

[Electrochemistry]



Ans. (2)

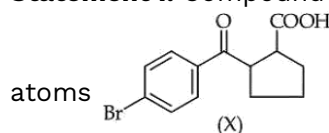
Sol. In Daniell cell:

$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{2.303RT}{2 \times F} \log \frac{[\text{Zn}^{+2}]}{[\text{Cu}^{+2}]}$$

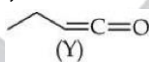
$$E_{\text{cell}}^{\circ} = \text{constant}$$

68. Given below are two statements:

Statement I: Compound (X), shown below, dissolves in NaHCO_3 solution and has two chiral carbon atoms



Statement II: Compound (Y), shown below, has two carbons with sp^3 hybridization, one carbon with sp^2 and one carbon with sp hybridization



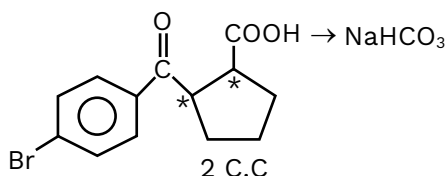
In the light of the above statements, choose the correct answer from the options given below:

[Isomerism]

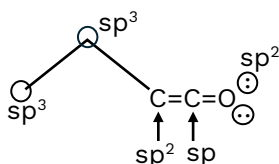
- (1) **Statement I** is true but **Statement II** is false
- (2) **Statement I** is false but **Statement II** is true
- (3) Both **Statement I** and **Statement II** are true
- (4) Both **Statement I** and **Statement II** are false

Ans. (3)

Sol. S-I

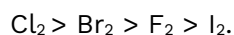


S-II

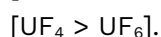


69. Given below are two statements:

Statement I: The correct order in terms of bond dissociation enthalpy is



Statement II: The correct trend in the covalent character of the metal halides is $[\text{SnCl}_4 > \text{SnCl}_2]$, $[\text{PbCl}_4 > \text{PbCl}_2]$ and



In the light of the above statements, choose the correct answer from the options given below:

[Chemical Bonding]

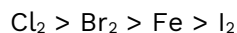
- (1) **Statement I** is true but **Statement II** is false
- (2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is false but **Statement II** is true

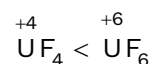
(4) Both **Statement I** and **Statement II** are false

Ans. (1)

Sol. Bond dissociation enthalpy



As polarising power of cation increase, covalent character increase.



70. On heating a mixture of common salt and $\text{K}_2\text{Cr}_2\text{O}_7$ in equal amount along with concentrated H_2SO_4 in a test tube, a gas is evolved. Formula of the gas evolved and oxidation state of the central metal atom in the gas respectively are:

[Salt Analysis]

(1) CrO_2Cl_2 and +5

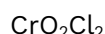
(2) CrO_2Cl_2 and +6

(3) $\text{Cr}_2\text{O}_2\text{Cl}_2$ and +6

(4) $\text{Cr}_2\text{O}_2\text{Cl}_2$ and +3

Ans. (2)

Sol. $\text{NaCl} + \text{K}_2\text{Cr}_2\text{O}_7 + \text{con. H}_2\text{SO}_4 \longrightarrow \text{CrO}_2\text{Cl}_2$
Red-brown vap.



Oxidation number of Cr = +6

71. The first and second ionization constants of H_2X are 2.5×10^{-8} and 1.0×10^{-13} respectively. The concentration of X^{2-} in 0.1 M H_2X solution is $____ \times 10^{-15}$ M.
(Nearest Integer)

Given 100

[Ionic Equilibrium]

Ans. (100)

Sol. $\text{H}_2\text{X} \rightleftharpoons \text{H}^+ + \text{HX}^- \quad K_{a1} = 2.5 \times 10^{-8}$
 $(0.1 - x) \quad (x+y) \quad (x-y)$
 $\text{HX}^- \rightleftharpoons \text{H}^+ + \text{X}^{2-} \quad K_{a2} = 1 \times 10^{-13}$
 $(x-y) \quad (x+y) \quad y$
Since, $K_{a1} \gg K_{a2}$, hence: $x \gg y$
 $(x-y) \approx x$ and $(x+y) \approx x \Rightarrow [\text{X}^{2-}] = K_{a2} = 1 \times 10^{-13} = 100 \times 10^{-15}$

72. The osmotic pressure of a living cell is 12 atm at 300 K. The strength of sodium chloride solution that is isotonic with the living cell at this temperature is $____ \text{g L}^{-1}$
(Nearest integer)

Given: $R = 0.08 \text{ Latrn K}^{-1} \text{ mol}^{-1}$

Assume complete dissociation of NaCl

(Given: Molar mass of Na and Cl are 23 and 35.5 g mol^{-1} respectively.)

[Liquid Solution]

Ans. (15)

Sol. $\pi = iCRT$

$$12 = 2 \times C \times 0.08 \times 300$$

$$c = \frac{1}{4} \frac{\text{mole}}{\text{litre}} = \frac{1}{4} \times 58.5$$

$$\frac{\text{gm}}{\text{litre}} = 14.625 \frac{\text{gm}}{\text{l}} = 15$$

73. A substance 'X' (1.5 g) dissolved in 150 g of a solvent 'Y' (molar mass = 300 g mol⁻¹) led to an elevation of the boiling point by 0.5 K. The relative lowering in the vapour pressure of the solvent 'Y' is _____ × 10⁻². (nearest integer)

[Given: K_b of the solvent = 5.0 K kg mol⁻¹]

Assume the solution to be dilute and no association or dissociation of X takes place in solution.

[Liquid Solution]

Ans. (3)

Sol. $\Delta T_b = K_b \times m$

$$0.5 = 5 \times \frac{n \times 1000}{150}$$

$$n = 0.015$$

$$\frac{p^\circ - p^s}{p^\circ} = \frac{n}{n+N} \approx \frac{n}{N} = \frac{1.5 \times 300}{100 \times 150}$$

$$= 0.03 = 3 \times 10^{-2}$$

74. Identify the metal ions among Co²⁺, Ni²⁺, Fe²⁺, V³⁺ and Ti²⁺ having a spin-only magnetic moment value more than 3.0 BM. The sum of unpaired electrons present in the high spin octahedral complexes formed by those metal ions is _____.

[Coordination Compounds]

Ans. (7)

Sol. Co²⁺(3d⁷) number of unpaired electron = 3

$$\mu_s = \sqrt{15} \text{ BM} > 3$$

Ni²⁺(3d⁸) number of unpaired electron = 2

$$\mu_s = \sqrt{8} \text{ BM} < 3$$

Fe²⁺(3d⁶) number of unpaired electron = 4

$$\mu_s = \sqrt{24} \text{ BM} > 3$$

V³⁺(3d²) number of unpaired electron = 2

$$\mu_s = \sqrt{8} \text{ BM} < 3$$

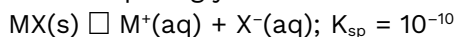
Ti²⁺(3d²) number of unpaired electron = 2

$$\mu_s = \sqrt{8} \text{ BM} < 3$$

In Fe(H₂O)₆²⁺ and Co(H₂O)₆²⁺

High spin High spin

75. MX is a sparingly soluble salt that follows the given solubility equilibrium at 298 K.



If the standard reduction potential for $\text{M}^+(\text{aq}) \xrightarrow{+e^-} \text{M(s)}$ is $(E_{\text{M}^+/\text{M}}^\ominus) = 0.79\text{V}$, then the value of the standard reduction potential for the metal/metal insoluble salt electrode $E_{\text{X}^-/\text{MX(s)}/\text{M}}^\ominus$ is _____ mV.

(nearest integer)

$$\left[\text{Given: } \frac{2.303RT}{F} = 0.059\text{V} \right]$$

[Electrochemistry]

Ans. (200)

$$\text{Sol. } E_{\text{X}^-/\text{MX}/\text{M}}^\ominus = E_{\text{M}^+/\text{M}}^\ominus - \frac{2.03RT}{F} \log \frac{1}{K_{\text{sp}}}$$

$$= 0.79 - 0.059 \times 10$$

= 0.02 V

= 200 mV

