Predicter - Correcter formula:

This method is used to find fifth value namely y_4 for given first values y_0, y_1, y_2, y_3. The result is obtained by Newton's forward difference formula. Here the limits for n are 0 to 4.

Milne's predicter formula:

$$y_4 = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

where $y' = \frac{dy}{dx} = f(x, y)$

Milne's Correcter formula

$$y_4 = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

Ex Given $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 2$, $y(0.2) = 2.0933$, $y(0.4) = 2.1755$, $y(0.6) = 2.2493$. Find $y(0.8)$ using Milen's Method

Sol^n|- Here

$x_0 = 0$ $\quad y_0 = 2$

$x_1 = 0.2$ $\quad y_1 = 2.0933$

$x_2 = 0.4$ $\quad y_2 = 2.1755$

$x_3 = 0.6$ $\quad y_3 = 2.2493$

Now $\because y_4 = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$

& $y' = f(x,y) = \frac{1}{x+y}$.

$\therefore y_1' = f(x_1, y_1) = \frac{1}{x_1+y_1} = \frac{1}{0.2 + 2.0933} = 0.4360$

$y_2' = f(x_2, y_2) = \frac{1}{x_2+y_2} = \frac{1}{0.4 + 2.1755}$

$= 0.3882$

$y_3' = f(x_3, y_3) = \frac{1}{x_3+y_3} = \frac{1}{0.6 + 2.2493}$

$= 0.3509$

Now $y_4 = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$ gives

$y_4 = 2 + \frac{4(0.2)}{3}[2(0.436) - 0.3882 + 2(0.3509)]$

$\boxed{y_4 = 2.31616}$ at $\boxed{x_4 = 0.8}$

Corrector formula:

$y_4 = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$ — (A)

now $y_4' = f(x_4, y_4) = \frac{1}{x_4+y_4} = \frac{1}{0.8 + 2.31616}$

$= 0.3209$

from (A)

$\therefore y_4 = 2.1755 + \frac{0.2}{3}[0.3882 + 4(0.3509) + 0.3209]$

$\boxed{y_4 = 2.3163}$ Ans!.

Q (2) Using Milne's Method determine $y(0.4)$ given that $y' = x^2 + y^2 - 2$, $y(0) = 1$

Sol^n/- Here $x_0 = 0$, $y_0 = 1$ Let $h = (0.1)$

$$y' = \frac{dy}{dx} = x^2 + y^2 - 2 = f(x, y)$$

We have to find $x_1, x_2, x_3, y_1, y_2, y_3$ first.

From Taylor's series.

$$y_1 = y_0 + h y_0' + \frac{h^2}{\lfloor 2} y_0'' + \frac{h^3}{\lfloor 3} y_0''' + --- \quad (1)$$

$\because y' = x^2 + y^2 - 2.$

$\therefore y_0' = x_0^2 + y_0^2 - 2 = 0 + (1) - 2 = -1$

Also $y'' = 2x + 2yy'$

$$y_0'' = 2x_0 + 2y_0 y_0'$$
$$= 2(0) + 2(1)(-1) = -2$$

Now $y''' = 2 + 2yy'' + (y')^2$

$$y_0''' = 2 + 2y_0 y_0'' + (y_0')^2$$
$$= 2 + 2(1)(-2) + (-1)^2$$
$$= 2 - 4 + 2 = 0$$

Hence from (1)

$$y_1 = y_0 + \frac{h\, y_0'}{\lfloor 1} + \frac{h^2}{\lfloor 2} y_0'' + \frac{h^3}{\lfloor 3} y_0''' + \cdots$$

$$= 1 + \frac{(0.1)(-1)}{1} + \frac{(0.1)^2}{2}(-2) + \frac{(0.1)^3}{6}(0) + \cdots$$

$$= 1 - 0.1 - 0.01 + 0$$

$$y_1 = 0.89$$

y_2 from Taylor's series

$$y_2 = y_1 + h\, y_1' + \frac{h^2 y_1''}{\lfloor 2} + \frac{h^3}{\lfloor 3} y_1''' + \cdots$$

$x_1 = 0.1,\ y_1 = 0.89$

Now find $y_1',\ y_1'',\ y_1'''$

$$y_1' = f(x_1, y_1) = x_1^2 + y_1^2 - 2 = (0.1)^2 + (0.89)^2 - 2$$
$$= -1.1979$$

$$y_1'' = 2x_1 + 2y_1 y_1'$$
$$= 2(0.1) + 2(0.89)(-1.1979)$$
$$= -1.9323$$

$$y_1''' = 2 + 2y_1 y_1'' + 2(y_1')^2$$
$$= 2 + 2(0.89)(-1.9323) + 2(-1.1979)^2$$
$$= 1.4304$$

Hence

$$y_2 = y_1 + \frac{h}{\lfloor 1} y_1' + \frac{h^2}{\lfloor 2} y_1'' + \frac{h^3}{\lfloor 3} y_1''' + \cdots$$

$$= 0.89 + 0.1(-1.1979) + \frac{(0.1)^2}{\lfloor 2}(-1.19323)$$

$$+ \frac{(0.1)^3}{\lfloor 3}(1.4304)$$

$$= 0.89 - 0.1198 - 0.0097 + 0.0002$$

$$\boxed{y_2 = 0.7607} \text{ at } \boxed{x_2 = 0.2}$$

Now find y_3 at $x_3 = 0.3$.

$$y_2' = x_2^2 + y_2^2 - 2$$

$$= (0.2)^2 + (0.7607)^2 - 2$$

$$= -1.3813$$

$$y_2'' = 2x_2 + 2y_2 y_2'$$

$$= 2(0.2) + 2(0.7607)(-1.3813)$$

$$= -1.7016$$

$$y_2''' = 2 + 2y_2 y_2'' + 2(y_2')^2$$

$$= 2 + 2(0.7607)(-1.7016) + 2(-1.3813)^2$$

$$= 3.272$$

$$\therefore y_3 = y_2 + \frac{h}{\lfloor 1} y_2' + \frac{h^2}{\lfloor 2} y_2'' + \frac{h^3}{\lfloor 3} y_3''' + \cdots$$

$$\approx 0.7607 + \frac{(0.1)}{1}(-1.3813) + \frac{(0.1)^2}{2}(-1.7016) + \frac{(0.1)^3}{3}(3.2272) + \cdots$$

$y_3 = 0.6146$.

Now apply first predictor formula

$$y_4 = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right] \quad (B)$$

where

$x_0 = 0 \qquad y_0 = 1$

$x_1 = 0.1 \qquad y_1 = 0.89$

$x_2 = 0.2 \qquad y_2 = 0.7607$

$x_3 = 0.3 \qquad y_3 = 0.6146$

Also $y' = x^2 + y^2 - 2$

find y_1', y_2' y_3' first. put in (B)

then apply Corrector formula

$$y_4 = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right] \quad - (C)$$

find y_4' by predicted value y_4 & x_4 first put in (C). and find y_4.

$y_4 = 0.49$