Adams Bashforth Method :-

This is also predictor corrector method.

The formula is without proof.

predictor formula

$$y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right] \quad (1)$$

Corrector formula

$$y_{n+1} = y_n + \frac{h}{24}\left[9y_{n+1}' + 19y_n' - 5y_{n-1}' + y_{n-2}'\right] \quad (2)$$

Application of Adam Bashforth method is same as Milen's method. To apply predictor Adam Bashforth method four values of y must be given in problem. To find y_4 put $n=3$ and to find y_5 put $n=4$.

predictor formula put $n=3$ in (1)

$$\boxed{y_4 = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]} \quad (3)$$

Corrector formula put $n=3$ in (2)

$$\boxed{y_4 = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]} \quad (4)$$

Q.1 Using Adam's method determine $y(0.4)$ and $y(0.5)$ correct to 3 decimals given that $\frac{dy}{dx} = 0.5xy$ and $y(0)$, $y(0.1)$, $y(0.2)$, $y(0.3)$ have values 1.0, 1.0025, 1.0101, 1.0228 respectively.

Sol^n :- $y' = \frac{dy}{dx} = 0.5xy$

$x_0 = 0, \quad x_1 = 0.1 \quad x_2 = 0.2 \quad x_3 = 0.3$

$y_0 = 1 \quad y_1 = 1.0025 \quad y_2 = 1.0101 \quad y_3 = 1.0228$

$h = 0.1$

predictor formula :-

$$y_4 = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$y_0' = 0.5x_0y_0 = 0$

$y_1' = 0.5x_1y_1 = 0.0501$

$y_2' = 0.1010$

$y_3' = 0.5x_3y_3 = 0.1534$

$\therefore y_4 = 1.0228 + \frac{0.1}{24}\left[55(0.1534) - 59(0.1010) + 37(0.0501) - 9(0)\right]$

$$\boxed{y_4 = 1.0410}$$

Corrector formula

$$y_4 = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

$y_4' = 0.5x_4y_4 = 0.2082$

$\therefore y_4 = 1.0228 + \frac{0.1}{24}\left[9(0.2082) + 19(0.1534) - 5(0.1010) + 0.0501\right] = 1.0410$

To find $y(0.5)$

put $n=4$.

$$y_5 = y_4 + \frac{h}{24}\left[55 y_4' - 59 y_3' + 37 y_2' - 9 y_1'\right]$$

$$y_4' = 0.5\, x_4 y_4 = 0.2082$$

$$\therefore y_5 = 1.0410 + \frac{0.1}{24}\left[55(0.2082) - 59(0.1534) + 37(0.1016) - 9(0.0501)\right]$$

$\boxed{y_5 = 1.0649}$ predicted

Corrected y_5

$$y_5 = y_4 + \frac{h}{24}\left[9 y_5' + 19 y_4' - 5 y_3' + y_2'\right]$$

$$y_5' = 0.5\, x_5 y_5$$

$$= 0.5(0.5)(1.0649) = 0.2662$$

$$\therefore y_5 = 1.0410 + \frac{0.1}{24}\left[9(0.2662) + 19(0.2082) - 5(0.1534) + 0.1010\right]$$

$\boxed{y_5 = 1.0649.}$ Ans

Q2 practice.

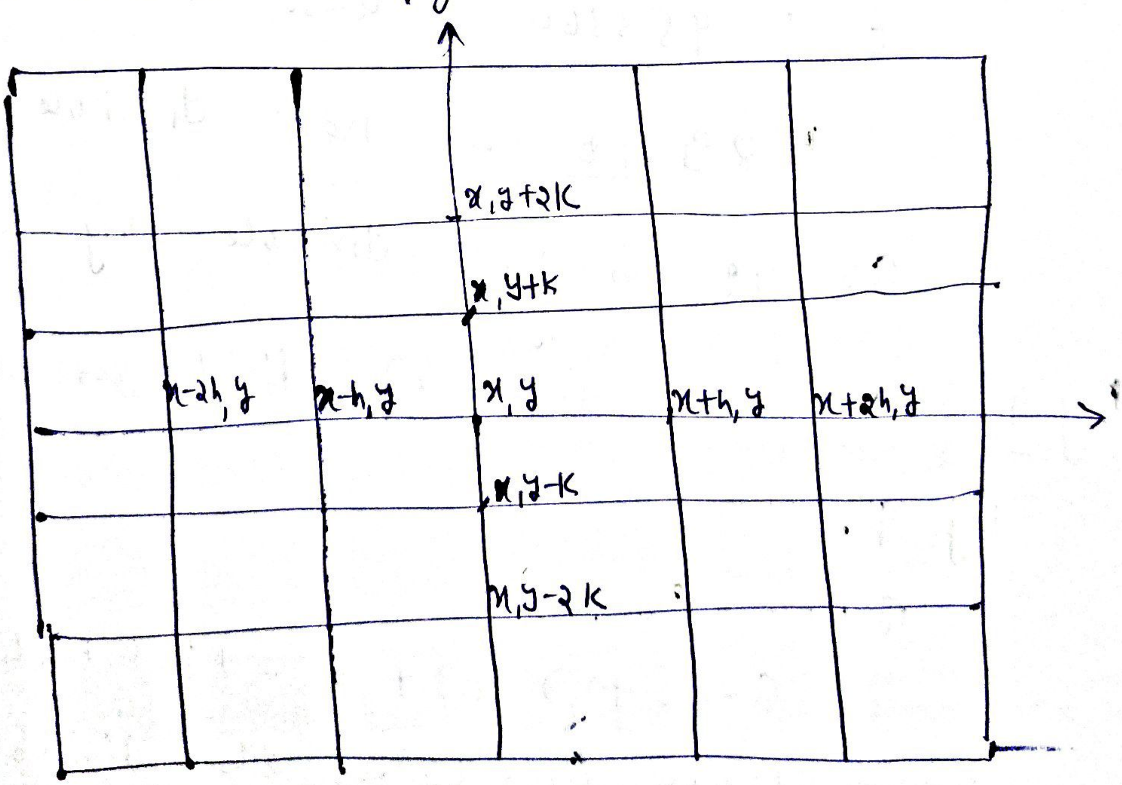
Using Adam's method determine $y(0.2)$ given that $y' = 2x^2 + 2y$ with $y(-0.6) = 0.1918$, $y(-0.4) = 0.4140$, $y(-0.2) = 0.6655$, $y(0) = 1$

Finite difference method for 2nd order Boundary value problems.

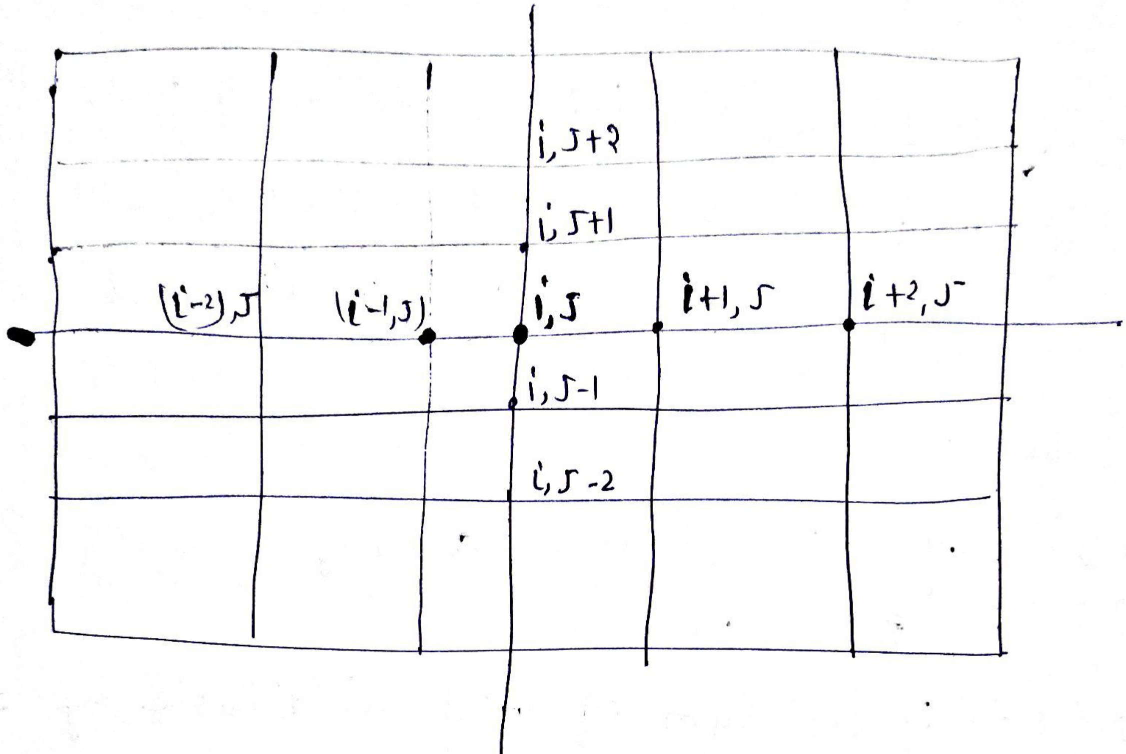
Geometrical representation of partial differential equation.

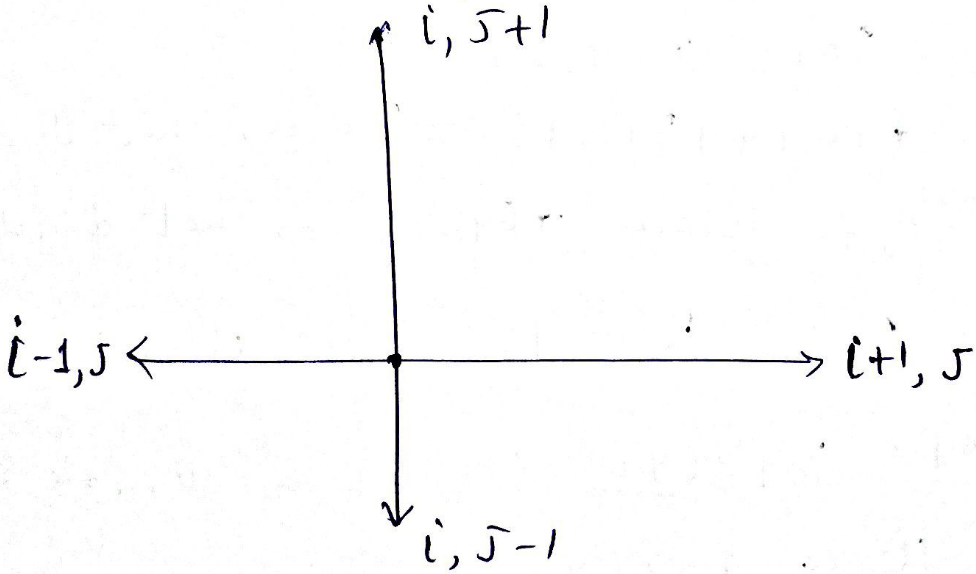
The XOY plane is divided into number of rectangles of sides $\Delta x = h$ and $\Delta y = K$ by drawing equidistant lines along x-axis and y-axis.

The point $x, x+h, x+2h$ ---- $x-h, x-2h$, ---- $y, y+K, y+2K$ ----, $y-K, y-2K$ --- are shown in figure.



In PDE U will be function of x & y. The value of $U(x, y)$ at (i, j) at $i, j = 0, 1, 2 \ldots$ is denoted by U_{ij}





Finite approximations (FDM). The numerical sol'n are obtained using boundary conditions.

$$\boxed{\frac{\partial U}{\partial x} = \frac{U_{i+1,j} - U_{i,j}}{h}} \; ; \; \boxed{\frac{\partial U}{\partial y} = \frac{U_{i,j+1} - U_{i,j}}{k}}$$

$$\boxed{\frac{\partial^2 U}{\partial x^2} = \frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{h^2}} \; ; \; \frac{\partial^2 U}{\partial y^2} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{K^2}$$