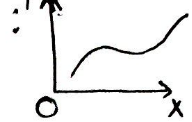
Interpolation with Unequal intervals :



① Lagrange form :-

② Newton's divided difference form :-

The various interpolation formula derived so far has the disadvantage of being applicable only to equally spaced values (x). Hence it is desirable to study the formula to develop interpolation formula for unequal spaced values of x. which are ① Lagrange form ② Newton's Divided difference form.

① Lagrange form :-

Let x takes the values $x_0, x_1, x_2 \ldots x_n$ and y takes the values $y_0, y_1, y_2 \ldots y_n$ corresponding to the values of x. then

$$f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)\ldots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\ldots(x_2-x_n)} y_2 + \ldots \quad (1)$$

is called Lagrange's interpolation formula

Proof :- of Lagrange formula

Let $y = f(x)$ be a function which takes the values $(x_0, y_0), (x_1, y_1), (x_2, y_2) --- (x_n, y_n)$

Let the polynomial of the form

$$y = f(x) = a_0(x-x_1)(x-x_2)--(x-x_n) +$$
$$a_1(x-x_0)(x-x_2)--(x-x_n) +$$
$$a_2(x-x_0)(x-x_1)\cdot(x-x_3)--(x-x_n) +$$
$$= = = =$$
$$a_n(x-x_0)(x-x_1) \cdot --- (x-x_{n-1}) \quad — (1)$$

put $x = x_0,\ y = y_0$ in (1) we get

$$y_0 = a_0(x_0-x_1)(x_0-x_2)--(x_0-x_n) + 0 + 0 + --$$

$$\Rightarrow a_0 = \frac{y_0}{(x_0-x_1)(x_0-x_2)--(x_0-x_n)}$$

Now put $x = x_1,\ y = y_1$ in (1) we get

$$y_1 = 0 + a_1(x_1-x_0)(x_1-x_2)--(x_1-x_n) + 0 + 0 --$$

$$\text{or} \quad a_1 = \frac{y_1}{(x_1-x_0)(x_1-x_2)--(x_1-x_n)}$$

= = = Similary

$a_2, a_3 -- a_n$. put $a_0, a_1, a_2 --$ in (1).

we get

$$y = \frac{y_0(x-x_1)(x-x_2)--(x-x_n)}{(x_0-x_1)(x_0-x_2)--(x_0-x_n)} + \frac{y_1(x-x_0)(x-x_2)--(x-x_n)}{(x_1-x_0)(x_1-x_2)--(x_1-x_n)}$$
$$- - -$$

Which is known as Lagrange's interpolation formula. This can also be written as.

$$y = L_0 y_0 + L_1 y_1 + L_2 y_2 -- L_n y_n \quad (2)$$

where,

$$L_0 = \frac{(x-x_1)(x-x_2)\cancel{(x-x_3)} -- (x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3) -- (x_0-x_n)}$$

$$L_1 = \frac{(x-x_0)(x-x_2)(x-x_3) -- (x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3) -- (x_1-x_n)}$$

$$L_2 = \frac{(x-x_0)(x-x_1)(x-x_3) ---- (x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3) --- (x_2-x_n)}$$

$$L_n = \frac{(x-x_0)(x-x_1)(x-x_3) --- (x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)(x_n-x_3) -- (x_n-x_{n-1})}$$

To find Lagrange's formula/ polyinomial find L_0, L_1, L_2 --- are put in (2) to get polynomial.

Important
Note

(1) Lagrange's formula can be applied whether the values x_i are equally spaced or not.

(2) It is easy to remember but lots of calculations.

(3) It is also used to split the given function into partial fractions.

Example:- based on Lagrange's interpolation formula:

(1) Given the values

x:	5 (x_0)	7 (x_1)	11 (x_2)	13 (x_3)	17 (x_4)
y:	150 (y_0)	392 (y_1)	1452 (y_2)	2366 (y_3)	5202 (y_4)

find $f(9)$ using Lagrange's formula.

Solution/- L. formula

$$y(x) = y_0 L_0 + y_1 L_1 + y_2 L_2 + y_3 L_3 + y_4 L_4 \quad (1)$$

Here x is not equally spaced.

$$L_0 = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}$$

$$= \frac{(x-7)(x-11)(x-13)(x-17)}{(5-7)(5-11)(5-13)(5-17)}$$

at $x = 9$.

$$L_0 = \frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)}$$ (Calculate)

$$L_1 = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$= \frac{(x-5)(x-11)(x-13)(x-17)}{(7-5)(7-11)(7-13)(7-17)}$$

at $x = 9$

$$L_1 = \frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)}$$ (Calculate)

$$L_2 = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$= \frac{(x-5)(x-7)(x-13)(x-17)}{(11-5)(11-7)(11-13)(11-17)}$$

at $x = 9$

$$L_2 = \frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)}$$ (Calculate)

$$L_3 = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}$$

$$= \frac{(x-5)(x-7)(x-11)(x-17)}{(13-5)(13-7)(13-11)(13-17)}$$

at $x = 9$

$$L_3 = \frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)} \quad \text{(Calculate)}$$

$$L_4 = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

$$= \frac{(x-5)(x-7)(x-11)(x-13)}{(17-5)(17-7)(17-11)(17-13)}$$

at $x = 9$

$$L_4 = \frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)} \quad \text{(Calculate)}$$

Calculate L_0, L_1, L_2, L_3, L_4 and put in ①.

$$y(9) = y_0L_0 + y_1L_1 + y_2L_2 + y_3L_3 + y_4L_4$$

$$= 150L_0 + 392L_1 + 1452L_2 + 2366L_3 + 5202L_4$$

$$= -\frac{50}{3} + \frac{3136}{15} + \frac{3872}{3} - \frac{2366}{3} + \frac{578}{5} = 810$$

Ans.

Assignments

Ques 1 Use Lagrange formula to find $y(10)$ from following table. Also write polynomial $y(x)$.

x:	5	6	9	11
y:	12	13	14	16.

Ques 2 Find the parabola passing through the points (0,0) (1,1) and (2,20), using Lagrange's formula hence find $f(3)$.

Note :- Submit it before 1:10 for getting attendence.