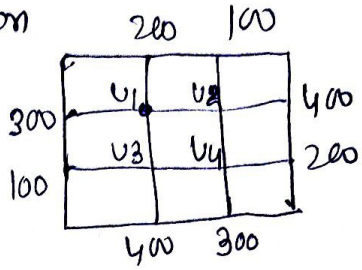
# Solution of PDE Continued.

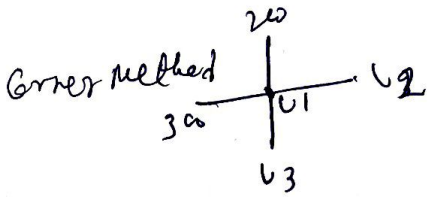
Q.2. Solve the elliptic equation $u_{xx} + u_{yy} = 0$



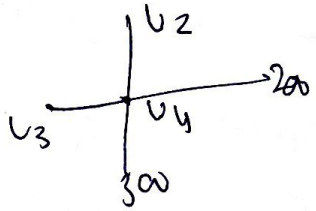

Sol$^n$:- Let $u_1, u_2, u_3, u_4$ are interior.

Here since we cannot calculate interior points either by cross averaging or diagonal method.

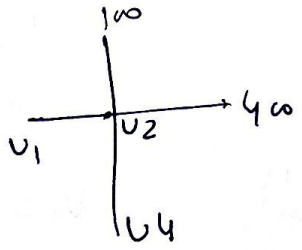
1st iteration :- Let $u_2 = u_3 = 0$



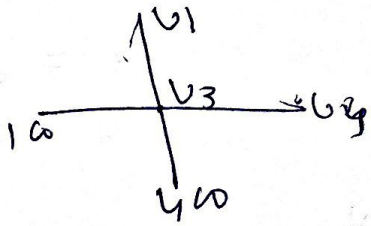

$$u_1 = \frac{1}{4}[300 + 200 + u_2 + u_3]$$
$$= 125$$




$$u_4 = \frac{1}{4}[u_3 + u_2 + 300 + 200]$$
$$= \frac{1}{4}[0 + 0 + 300 + 200] = 125$$




$$u_2 = \frac{1}{4}[u_1 + 100 + u_4 + 400]$$
$$= \frac{1}{4}[125 + 100 + 125 + 400] = 187.5$$




$$u_3 = \frac{1}{4}[100 + u_1 + u_4 + 400]$$
$$= \frac{1}{4}[100 + 125 + 125 + 400]$$
$$= 187.5$$

IInd iteration $u_1 = \frac{1}{4}[300 + 200 + u_2 + u_3]$

$$= \frac{1}{4}[300 + 200 + 187.5 + 187.5] = 218.75$$

$$v_2 = \frac{1}{4}\left[ v_1 + 1\omega + v_4 + 4\omega \right]$$

$$= \frac{1}{4}\left[ 218.75 + 1\omega + 125 + 4\omega \right]$$

$$= 210.937$$

$$v_3 = \frac{1}{4}\left[ 1\omega + v_1 + v_4 + 4\omega \right]$$

$$= \frac{1}{4}\left[ 100 + 218.75 + 125 + 4\omega \right]$$

$$= 210.937$$

$$v_4 = \frac{1}{4}\left[ v_3 + v_2 + 3\omega + 200 \right]$$

$$= \frac{1}{4}\left[ 210.937 + 210.937 + 300 + 200 \right]$$

$$= 230.468$$

III iteration

$v_1 = 230.468$

$v_2 = 240.234$

$v_3 = 240.234$

$v_4 = 295.117$

IV

$v_1 = 245.117$

$v_2 = 260.058$

$v_3 = 260.058$

$v_4 = 305.029$

V

$v_1 = 255.029$

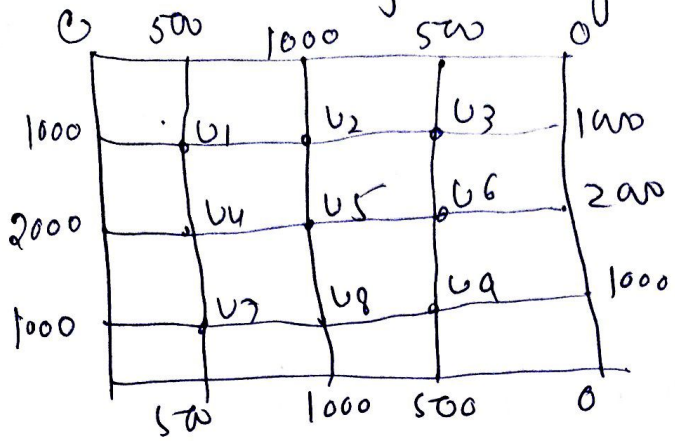
$v_2 = 265.014$

$v_3 = 265.014$

$v_4 = 307.507$ //

Qus 3.

Solve following boundary value problem (BVP)




Sol$^{n}$ Let interior are $u_1\ u_2\ u_3\ u_4\ u_5\ u_6\ u_7\ u_8\ u_9$

By symmetry

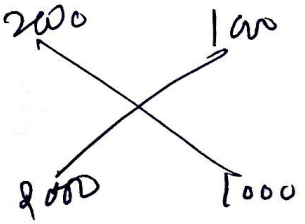
$u_1 = u_3 = u_7 = u_9$

$u_2 = u_8$

$u_4 = u_6$

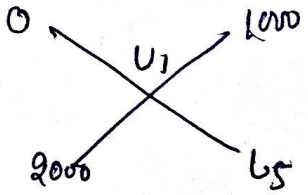
∴ we have to calculate $u_1, u_2, u_4$ and $u_5$ only

By mean averaging formula



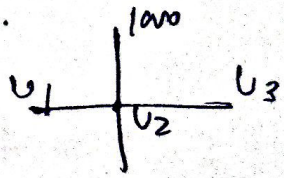

$$u_5 = \frac{1}{4}[2000 + 1000 + 1000 + 2000]$$

$$= 1500$$




$$u_1 = \frac{1}{4}[0 + u_5 + 2000 + 1000]$$

$$= \frac{1}{4}[1500 + 2000 + 1000 = 1125$$



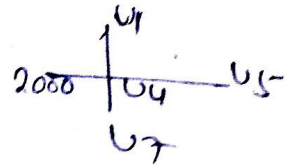

$$u_2 = \frac{1}{4}[u_1 + 1000 + u_3 + u_5]$$

$$= \frac{1}{4}[1125 + 1000 + 1125 + 1500]$$

$$= 1167.5$$

Using averaging formula,

$$U_4 = \frac{1}{4}[2000 + U_1 + U_5 + U_7]$$




$$= \frac{1}{4}[2000 + 1125 + 1500 + 1125]$$

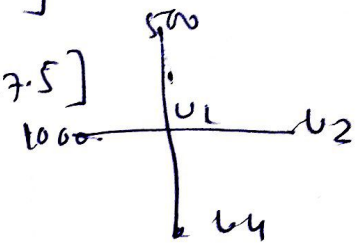
as $U_7 = U_1$

$$= 1437.5$$

1) Iteration

Use averaging formula :-

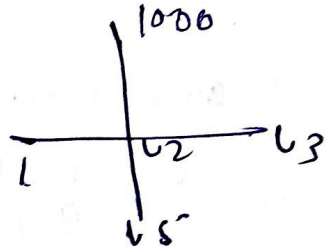
$$U_1 = \frac{1}{4}[1000 + 500 + U_2 + U_4]$$




$$= \frac{1}{4}[1000 + 500 + 1187.5 + 1437.5]$$

$$= 1031.25$$

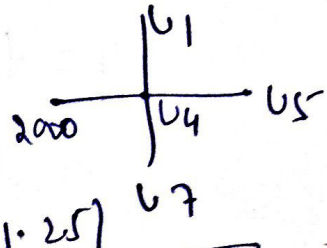
$$U_2 = \frac{1}{4}[U_1 + 1000 + U_3 + U_5]$$




$$= \frac{1}{4}[1031.25 + 1000 + 1031.25 + 500]$$

$$= \frac{1}{4}[4562.5] = 1140.5$$

$$U_4 = \frac{1}{4}[2000 + U_1 + U_5 + U_7]$$

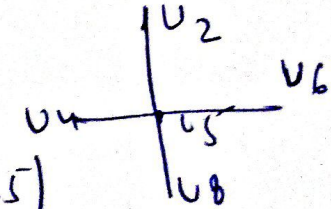



$$= \frac{1}{4}[2000 + 1031.25 + 1500 + 1031.25]$$

$U_7 = U_1$

$$= 1390.63$$

$$U_5 = \frac{1}{4}[U_4 + U_2 + U_6 + U_8]$$




$$= \frac{1}{4}[1390.63 + 1140.5 + 1390.63 + 1140.5]$$

$$= 1265.63$$

II iteration:-

$$u_1 = \frac{1}{4}\left[1000 + 500 + u_2 + u_4\right]$$
$$= \frac{1}{4}\left[1000 + 500 + 1140.5 + 1390.63\right]$$
$$= 1007.8$$
$$u_2 = \frac{1}{4}\left[u_1 + 1000 + u_3 + u_5\right]$$
$$= 1070.307$$
$$u_4 = 1320.807$$
$$u_5 = 1195.307$$

IV :- $u_1 = 972.1$
$u_2 = 1035.3$
$u_4 = 1285.2$
$u_5 = 1160.2$

V :- $u_1 = 955.1$
$u_2 = 1017.6$
$u_4 = 1267.63$
$u_5 = 1142.60$

---

Ques By Leibmann iteration method solve the elliptic equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$ satisfy the following

Boundaries

(a) $u(x, y) = 0$ for $0 \le y \le 4$
(b) $u(4, y) = 12 + y$ for $0 \le y \le 4$
(c) $u(x, 0) = 3x$ for $0 \le x \le 4$
(d) $u(x, 4) = x^2$ for $0 \le x \le 4$.

Soln:-



Here

$u(0,y) = 0$

Means $x = 0$ then $u = 0$

$u(4,y) = 12 + y \quad ; 0 \le y \le 4$

Means $x = 4$ & $u = 12 + y$

for 4 equal parts wl- $h = 1$, $k = 1$.

when $x = 1$, $y = 1$

$u(4,1) = 13$.

$x = 4 \quad y = 2 \quad u(4,2) = 14$

$x = 4, y = 3 \quad u(4,3) = 15$

$x = 4, \quad y = 4 \quad u(4,4) = 16$

when $y = 0$, $u(4,0) = 12$

Hence 12, 13, 14, 15, 16, are the Boundary values on the 4th Column.

next $u(x,0) = 3x$ for $0 \le x \le 4$

when $y = 0, x = 0$ $\quad u(0,0) = 0$

$y = 0 \quad x = 1$ $\quad u(1,0) = 3$

$u(2,0) = 6$

$u(3,0) = 9$

$u(4,0) = 12$

The Boundary values on first row are 0, 3, 6, 9, 12

Next - Boundary Condition.

$U(x,4) = x^2$ or $0 \le x \le 4$

When $y=4, x=0$ $\quad U(0,4) = 0$

$y=4 \quad x=1 \quad U(1,4) = 1$

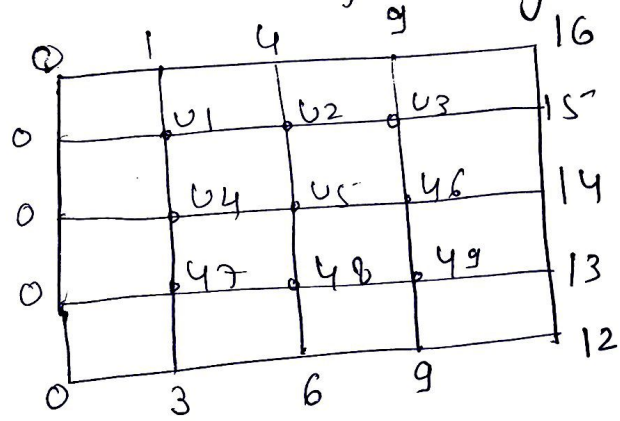
$y=4 \quad x=2 \quad U(2,4) = 4$

$y=4 \quad x=3 \quad U(3,4) = 9$

∴ The Boundary values on fourth row are

0, 1, 4, 9.

Hence we get following figure




Now start the iteration to find the solution upto (4 iteration)

After 4th iteration Ans! - $U_1 = 2.37$

$U_2 = 5.59$

$U_3 = 9.87$

$U_4 = 2.88$

$U_5 = 6.13$

$U_6 = 9.88$

$U_7 = 3.07$

$U_8 = 6.16$, $U_9 = 9.51$.