Inverse Interpolation

So far, given a set of values of x and y, we have been finding the value of y corresponding to a certain value of x. While, the process of finding the value of x for a given value of y is called inverse interpolation.

To find inverse interpolation.

Two Methods:

(1) Lagrange's inverse interpolation (for unequal spaced data)

(2) Iterative Method (for equal spaced data).

① Lagrange's inverse interpolation ①

$$x = \frac{(y-y_1)(y-y_2)\cdots(y-y_n)\,x_0}{(y_0-y_1)(y_0-y_2)\cdots(y_0-y_n)} + \frac{(y-y_0)(y-y_2)\cdots(y-y_n)\,x_1}{(y_1-y_0)(y_1-y_2)\cdots(y_1-y_n)} + \frac{(y-y_0)(y-y_1)(y-y_3)-(y-y_n)\,x_2}{(y_2-y_0)(y_0-y_1)\,y_0-y_3)\cdots(y_2-y_n)} + \cdots$$

Ques. The following table gives the values of x and y:

x:	1.2	2.1	2.8	4.1	4.9	6.2
y:	4.2	6.8	9.8	13.4	15.5	19.6

find the value of x corresponding to $y = 12$ using Lagrange's method.

Soln - Here $x_0 = 1.2$, $x_1 = 2.1$, $x_2 = 2.8$ $x_3 = 4.1$ $x_4 = 4.9$ $x_5 = 6.2$ x find for $y = 12$

$y_0 = 4.2$, $y_1 = 6.8$ $y_2 = 9.8$ $y_3 = 13.4$ $y_4 = 15.5$ $y_5 = 19.6$.

using formula (1)

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)(y-y_4)(y-y_5)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)(y_0-y_4)(y_0-y_5)} x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)(y-y_4)(y-y_5)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)(y_1-y_4)(y_1-y_5)} x_1 + \frac{(y-y_0)(y-y_1)(y-y_3)(y-y_4)(y-y_5)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)(y_2-y_4)(y_2-y_5)} x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)(y-y_4)(y-y_5)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)(y_3-y_4)(y_3-y_5)} x_3 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)(y-y_3)(y-y_5)\,x_4}{(y_4-y_0)(y_4-y_1)(y_4-y_2)(y_4-y_3)(y_4-y_5)} +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)(y-y_3)(y-y_4)\,x_5}{(y_5-y_0)(y_5-y_1)(y_5-y_2)(y_5-y_3)(y_5-y_4)}$$

Substituting the given values and solving

$$x = \frac{(12-6.8)(12-9.8)(12-13.4)(12-15.5)(12-19.6)}{(4.2-6.8)(4.2-9.8)(4.2-13.4)(4.2-15.5)(4.2-19.6)} \times (1.2)$$

$$+ \frac{(12-4.2)(12-9.8)(12-13.4)(12-15.5)(12-19.6) \times (2.1)}{(6.8-4.2)(6.8-9.8)(6.8-13.4)(6.8-15.5)(6.8-19.6)}$$

$+ \quad - \quad - \quad - \quad + \quad - \quad - \quad - \quad - \quad + \quad - \quad - \quad - \quad - \quad + \quad - \quad -$

we get

$x = 3.55$

(2) Iterative method for equally spaced data.

Using Newton's forward interpolation formula

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 y_0$$
$$+ \cdots$$

from this

$$p = \frac{1}{\Delta y_0}\left[y - y_0 - \frac{p(p-1)}{\lfloor 2}\Delta^2 y_0 - \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 y_0 \right.$$
$$\left. - \cdots \right] \quad \text{— (1)}$$

neglect higher differences $\Delta^2 y_0, \Delta^3 y_0 \ldots$ we get first approximation

$$p_1 = \frac{(y - y_0)}{\Delta y_0}$$

find second approximation

$$p_2 = \frac{1}{\Delta y_0}\left[y - y_0 - \frac{p_1(p_1-1)}{\lfloor 2}\Delta^2 y_0\right]$$

p

find Third approximation

$$p_3 = \frac{1}{\Delta y_0}\left[y - y_0 - \frac{p_2(p_2-1)\Delta^2 y_0}{\lfloor 2} - \frac{p_2(p_2-1)(p_2-2)\Delta^3 y_0}{\lfloor 3}\right]$$

and so on. Continue the process till two approximations of p are same. Now use $x = x_0 + ph$ To find x.

To find x for y [using iterative method]

$$P_1 = \frac{(y - y_0)}{\Delta y_0}$$

$$P_2 = \frac{\left(y - y_0 - \frac{P_1(P_1 - 1)}{\lfloor 2} \Delta^2 y_0\right)}{\Delta y_0}$$

$$P_3 = \frac{1}{\Delta y_0}\left[y - y_0 - \frac{P_2(P_2-1)}{\lfloor 2} \Delta^2 y_0 - \frac{P_2(P_2-1)(P_2-2)\Delta^3 y_0}{\lfloor 3}\right]$$

$$P_4 = \frac{1}{\Delta y_0}\left[y - y_0 - \frac{P_3(P_3-1)}{\lfloor 2}\Delta^2 y_0 - \frac{P_3(P_3-1)(P_3-2)\Delta^3 y_0}{\lfloor 3} - \frac{P_3(P_3-1)(P_3-2)(P_3-3)\Delta^4 y_0}{\lfloor 4}\right]$$

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and so on

Continue till we get $P_i = P_{i+1} \doteq P$

Hence $\boxed{x = x_0 + Ph}$ gives value of x for y.

Note:-

This method (iterative method) is used to find inverse interpolation when x is equally distributed; otherwise use Lagrange Inverse interpolation.

Note :- Advantage of this method
The method is useful to find roots of an equation

Ques 1 find the values of x for $y = 3000$ by iterative method. where

x:	10	15	20
y:	1754	2648	3564

Here

	x	y	Δy	$\Delta^2 y$
x_0	10	y_0 1754	Δy_0 894	$\Delta^2 y_0$ 22
x_1	15	y_1 2648	Δy_1 916	
x_2	20	y_2 3564		

Also $y = 3000$ $\quad y_0 = 1754,\ y_1 = 2648,\ y_2 = 3564.$
$x_0 = 10,\ x_1 = 15,\ x_2 = 20$

find x for $y = 3000$, Also $h = 5$.

$$P_1 = \frac{1}{\Delta y_0}(y - y_0) = \frac{1}{894}(3000 - 1754) = 1.39$$

$$P_2 = \frac{1}{\Delta y_0}\left(y - y_0 - \frac{P_1(P_1 - 1)}{\lfloor 2}\Delta^2 y_0\right) =$$

$$= \frac{1}{894}\left[3000 - 1754 - \frac{1.39(1.39-1)}{\lfloor 2} \times 22\right] = 1.387$$

$$P_3 = \frac{1}{\Delta y_0}\left[(y - y_0 - \frac{P_2(P_2-1)}{\lfloor 2}\Delta^2 y_0 - \frac{P_2(P_2-1)(P_2-2)}{\lfloor 3}\Delta^3 y_0\right]$$

$$= \frac{1}{894}\left[3000 - 1754 - \frac{1.387(1.387-1)}{2} \times 22 + 0\right] = 1.387$$

$\because \quad p_2 = p_3 = 1.387$

$\therefore \; p = 1.387$

$\therefore \; x = x_0 + ph = 10 + 1.387 \times 5$

$= 16.935$

Hence ~~f(x)~~. $x = 16.935$ at $y = 3000$.

Ans.

Ques 2. using inverse interpolation find the real root of $x^3 + x - 3 = 0$ near to 1.2

x	$y = x^3 + x - 3$	Δy	$\Delta^2 y$	$\Delta^3 y$
1	-1	Δy_0 .431	$\Delta^2 y_0$ 0.066	$\Delta^3 y_0$ 0.006
1.1	-0.569	0.497	0.072	
1.2	-0.072	0.569		
1.3	0.497			

Apply the same above process as in Ques. 1