Solution of equations.

Algebraic and Transcendental equations.

Direct Method

1. All factor Method
2. Quadratic Method
3. Synthetic Division

Iterative Method

1. Newton Raphson Method
2. Secant Method.
3. Fixed point Method

Solution of System of Linear equations.

Direct Method.

1. Gauss elimination Method
2. Gauss-Jordon Method

Iterative Method

1. Gauss Jacobi Method
2. Gauss Seidal Method
3. Diagonal Dominance
4. Thomas Algorithm.

Algebraic and Transcendental equations

① **Algebraic equation :-**

Let $f(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_n$ — ①

in which n is positive integer and $a_0, a_1, a_2, \ldots a_n$ are constants also $a_n \neq 0$ then $f(x)$ is known as polynomial of n degree.

But if $f(x) = 0$ then

$a_0 x^n + a_1 x^{n-1} + \cdots a_n = 0$ is called algebraic equation of degree n, which is a purely polynomial

Ex| - $$\left.\begin{array}{l} x^3 - 3x + 1 = 0 \\ x^4 + 2x^3 - 3x^2 + 2x + 1 = 0 \end{array}\right\}$$ Algebraic equation

② **Transcendental Equations :-**

Those equations which contains polynomial, Trigonometric terms, logarithm terms, exponenential terms etc are called Transcendental equation.

Ex| - $$\left.\begin{array}{l} x^3 - 3x + 1 - \cos x - 1 = 0 \\ x \log_{10} x - 1 = 0 \\ 3x + \sin x - e^x = 0 \end{array}\right\}$$ Transcendental equation

Solution of an equation

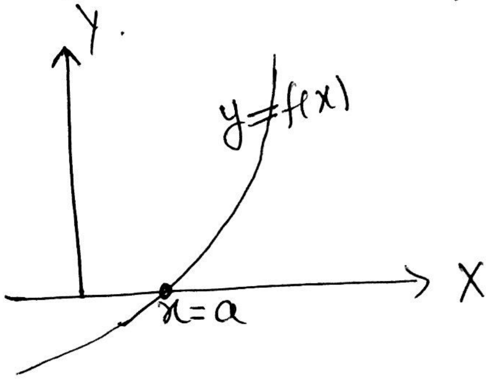
(1) The solution of an equation means finding roots or zeros of the equation. $f(x)=0$

Root or zeros of $f(x)=0$:-

If the value $x=a$ satisfies $f(x)=0$ i.e $f(a)=0$ then we say that, a is a root of $f(x)=0$.

Geometrical Interpretation of root of $f(x)$:-

A root of $f(x)=0$ is that value where the curve $y=f(x)$ crosses the x-axis. If the curve $y=f(x)$ crosses at $x=a$ the X-axis then $x=a$ is the root of $f(x)=0$



The process of finding the roots of the equation is known as solution of the equation

* The problem of solution is great importance in Mathematics

* If $f(x)$ is quadratic, cubic or a biquadratic expression then solutions of $f(x)=0$ are available.

* Need arises when the equations $f(x)=0$ are of higher degree and transcendental for which no direct method available in such case equations can be solved by Numerical Methods (Approximation Methods) only.

Ex:-

1. $x^2+5x+6=0$ can be solved easily.

$(x+2)(x+3)=0$ gives two root

$x=-2$ and $x=-3$

2. $2x^3+x^2-13x+6=0$ can be solved.

by trial method put $x=2$ which satisfies the given equation

$\therefore x=2$ is a root of $f(x)$. Dividing polynomial $2x^3+x^2-13x+6=0$ by $(x-2)$ gives quotient $2x^2+5x-3$ and remainder 0 solving quadratic equation

$$2x^2+5x-3=0$$

$$x=\frac{-5\pm\sqrt{5^2-4\times2\times(-3)}}{2\cdot2}=-3,\ 1/2$$

Hence the roots are

$x = 2, -3, 1/2$

Numerical Method (Approximation Method)

When equations can not be solved by any direct method then we have to use Numerical method only. Before doing this let us know some important points of equations.

(1) If $f(x) = 0$ is Algebraic or transcendental equation. If we find $f(a)$ and $f(b)$ such that both are of opposite sign that is

if $\boxed{f(a) = +ve, \ f(b) = -ve}$

OR

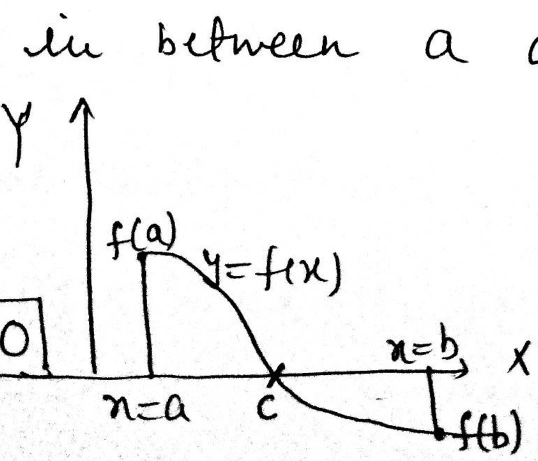
$\boxed{f(a) = -ve \quad f(b) = +ve}$

we can say $\boxed{f(a) \times f(b) < 0 \text{ i.e. } -ve \text{ value}}$

Hence roots lies in between a and b

Geometrically

Here c is a root such that $\boxed{f(a)\, f(b) < 0}$



(2) Every equation of nth degree has only n roots (real or imaginary)

Ex/- $x^2+5x+6=0 \rightarrow$ Two roots (Real)

$x=-2,-3$

$2x^3+x^2-13x+6=0 \rightarrow$ Three roots (Real

$x=2,-3,1/2$

(3) Every equation of odd degree has atleast One real root (Not one Complex root)

(4) If equation belongs with Complex root, then roots occur in Conjugate pair.

Ex/- $3x^3-4x^2+x+88=0$ if $2+\sqrt{7}\,i$ is one Complex root then find others two.

By above (4) property if $2+\sqrt{7}\,i$ is one root then other will be $2-\sqrt{7}\,i$. Find the third root Now.

$$[x-(2+\sqrt{7}\,i)][x-(2-\sqrt{7}\,i)]=(x-2)^2-7i^2=0.$$

or $(x-2)^2+7=0$ gives $x^2-4x+11=0$. Divide $(3x^3-4x^2+x+88)$ By $(x^2-4x+11)$ gives $3x+8$ as quotient as remainder as 0. Hence roots are $(2\pm\sqrt{7}\,i)$, $-8/3$

(5) If $a+\sqrt{b}$ is an irrational root of $f(x)=0$ then $a-\sqrt{b}$ must also be its root always.

(6) Descarte's Rule!-

The equation $f(x)=0$ cannot have more positive roots than the changes of signs in $f(x)$ 2 and more negative roots than the changes of signs in $f(-x)$.

Ex|- $f(x) = 2x^7 - x^5 + 4x^3 - 5 = 0$

$+ \quad - \quad + \quad -$

1 2 3

$\boxed{\text{Deg of } f(x) - 7}$

$f(x)$ has 3 changes of signs $(+\,-\,+\,-)$. Thus $f(x)$ cannot have more than 3 positive roots.

Also $f(-x) = -2x^7 + x^5 - 4x^3 - 5 = 0$

$- \quad + \quad -$

1 2

$f(-x)$ has 2 changes of signs $(-\,+\,-)$. Thus $f(x)$ cannot have more than 2 negative roots.

If p is number of positive roots and q is number of negative roots then equation $f(x)$ has ~~at least~~ $n-(p+q)$ imaginary roots.

In above example $f(x)$ has $7-(3+2) = 2$ img root

Finding the root of Algebraic and Transcendental equation

① Bisection Method :-

Steps If $f(x) = 0$ is an equation.

(1) Find $f(a)$ and $f(b)$ such that $f(a) \cdot f(b) < 0$

(2) find $x_1 = \frac{a+b}{2}$

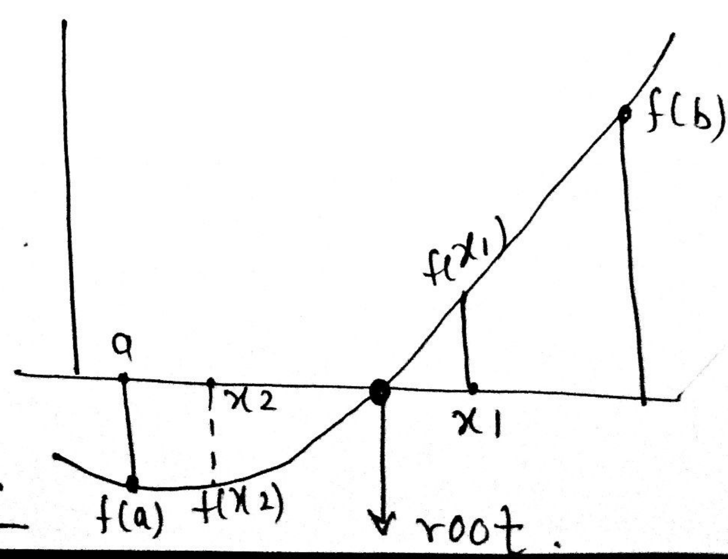
If $f(x_1) \cdot f(a) < 0$ then roots lies in between a and x_1 otherwise,

if $f(x_1) \cdot f(b) < 0$ then roots lies in between b and x_1

$\boxed{\text{Assume } f(x_1) \cdot f(a) < 0}$

③ find $x_2 = \frac{a+x_1}{2}$ and Repeat step (2) finding the root lying.

Geometrically :-



Note :-

(1) error decreases in each step by factor of 1/2

(2) Convergence is linear