x	y
x_0	$f(x_0) = y_0$
$x_1 = x_0 + h$	$f(x_1) = y_1$
$x_2 = x_1 + h$	$f(x_2) = y_2$
$x_3 = x_2 + h$	$f(x_3) = y_3$
$x_4 = x_3 + h$	$f(x_4) = y_4$
$\vdots$	$\vdots$

Qus. based on Interpolation

$\downarrow$

Newton's forward difference formula

$\downarrow$

$$\boxed{y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 y_0 + \cdots \quad (1)}$$

Where $p = \frac{x - x_0}{h}$

x = The value for which we have to find y.

x_0 = starting value of x

h = class interval

Note :- When x is near to x_0 use forward otherwise Backward.

Qus 1. following table gives the values of x & y. Find $\Delta^4 y_0$. Also express y as a function of x and hence obtain $y(2.5)$

x	0	1	2	3	4
y	7	10	13	22	43

Soln:- Since we have to obtain $\Delta^4 y_0$. Use forward table

In this Qus. $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$

$y_0 = 7$ $y_1 = 10$ $y_2 = 13$, $y_3 = 22$, $y_4 = 43$

To find $\Delta^4 y_0$, $y(x)$ and then $y(2.5)$

x		y		Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	x_0	7	y_0				
				$3 = \Delta y_0$			
1	x_1	10	y_1		$0 = \Delta^2 y_0$		
				$3 = \Delta y_1$		$6 = \Delta^3 y_0$	
2	x_2	13	y_2		$6 = \Delta^2 y_1$		$0 = \Delta^4 y_0$
				$9 = \Delta y_2$		$6 = \Delta^3 y_1$	
3	x_3	22	y_3		$12 = \Delta^2 y_2$		
				$21 = \Delta y_3$			
4	x_4	43	y_4				

Hence $\boxed{\text{Ans } \Delta^4 y_0 = 0}$

2nd method to find $\Delta^4 y_0$

we can also find $\Delta^4 y_0$ by

$$(E-1)^4 y_0 = E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0$$

$$= y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$$

$$= 43 - 4\times 22 + 6\times 13 - 4\times 10 + 7$$

$$= \cancel{43} - \cancel{88} + \cancel{78} - \cancel{40} + \cancel{7}$$

$$\boxed{\Delta^4 y_0 = (E-1)^4 y_0 = 0}$$

To find polynomial :- put in Newton's forward difference formula. (1)

$$y_n(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{\underline{|2}}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{\underline{|3}}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{\underline{|4}}\Delta^4 y_0$$

Find $\boxed{p = \dfrac{x - x_0}{h} = \dfrac{x-0}{1}}$

OR $p = \dfrac{2.5-0}{1} = 2.5$

Since we have to find polynomial $f(x)$ we write $p = \frac{x-0}{1} = x$. (here)

$$y(x) = 7 + 3x + \frac{x(x-1)}{\lfloor 2}(0) + \frac{x(x-1)(x-2)}{\lfloor 3}(6)$$

$$+ \frac{x(x-1)(x-2)(x-3)}{\lfloor 4}(0)$$

$$\boxed{\begin{aligned} y(x) &= 7 + 3x + x^3 - 2x^2 + 2x \\ &= 7 + 5x - 2x^2 + x^3 \end{aligned}} \longrightarrow \text{Ans.}$$

To find $y(2.5)$

$$y(2.5) = 7 + 5(2.5) - 2(2.5)^2 + (2.5)^3$$

$$\boxed{y(2.5) = 16.375}$$

Ques 2:-

Find a polynomial of degree two for the following data by Newton's forward difference formula

x	0	1	2	3	4	5	6	7
y	1	2	4	7	11	16	22	29

Soln :- The difference table (forward) is :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0 (x_0)	1 (y_0)			
		$1 = \Delta y_0$		
1 (x_1)	2 (y_1)		$1\ (\Delta^2 y_0)$	
		$2\ (\Delta y_1)$		$0\ (\Delta^3 y_0)$
2 (x_2)	4 (y_2)		$1\ (\Delta^2 y_1)$	
		$3\ (\Delta y_2)$		$0\ (\Delta^3 y_1)$
3 (x_3)	7 (y_3)		$1\ (\Delta^2 y_2)$	
		$4\ (\Delta y_3)$		$0\ (\Delta^3 y_2)$
4 (x_4)	11 (y_4)		$1\ (\Delta^2 y_3)$	
		$5\ (\Delta y_4)$		$0\ (\Delta^3 y_3)$
5 (x_5)	16 (y_5)		$1\ (\Delta^2 y_4)$	
		$6\ (\Delta y_5)$		$0\ (\Delta^3 y_4)$
6 (x_6)	22 (y_6)		$1\ (\Delta^2 y_5)$	
		$7\ (\Delta y_6)$		
7 (x_7)	29 (y_7)			

Here $x_0 = 0$, $h = 1$ $\quad P = \frac{x - x_0}{h} = \frac{x-0}{1} = x$

Newton's forward difference formula

$$y(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{\lfloor 2}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{\lfloor 3}\Delta^3 y_0$$

$$y(x) = 1 + x\cdot(1) + \frac{x(x-1)}{\lfloor 2}(1) + \frac{x(x-1)(x-2)}{\lfloor 3}(0)$$

$$= 1 + x + \frac{(x^2 - x)}{2}$$

$$\boxed{y(x) = \frac{1}{2}(x^2 + x + 2)}$$

is the required polynomial of degree 2.

Another type Ques. (Missing term)

Ex! -

x:	1	2	3	4	5	6	7
	x_0	x_1	x_2	x_3	x_4	x_5	x_6
y:	2	4	8	–	32	64	128
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Solution! - There are 6 given values of y ∴ we use 6th degree polynomial formula which satisfy

$$\Delta^6 y_0 = 0. \qquad \text{—} \; (1)$$

$$\Delta = (E-1)$$

As $\Delta f(x) = f(x+h) - f(x)$

$$\Delta f(x) = E f(x) - f(x)$$

$$\Rightarrow \Delta f(x) = (E-1) f(x)$$

Hence $\boxed{\Delta = E - 1}$

1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

write in (1)

$$(E-1)^6 y_0 = 0$$

$$(E^6 - 6E^5 + 15E^4 - 20E^3 + 15E^2 - 6E + 1) y_0 = 0$$

$$E^6 y_0 - 6E^5 y_0 + 15E^4 y_0 - 20E^3 y_0 + 15E^2 y_0 - 6E y_0 + y_0 = 0$$

$$y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0 = 0$$

$$128 - 6(64) + 15(32) - 20y_3 + 15(8) - 6(4) + 2 = 0$$

$$\Rightarrow 20y_3 = 322$$

$$y_3 = 16.1 \Rightarrow \text{Missing value}$$