Numerical Solution of ODE.

Euler's Modified Method (EMM)

Let $\frac{dy}{dx} = f(x, y)$

Method! –

Steps: ① first apply Euler's method to get an approximate value of y_1

Euler's formula:

$$y_1 = y_0 + h f(x_0, y_0)$$

② This value of Euler's method y_1 is used in Euler's modified method to find better approximation of y_1

$$y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1)]$$

(y_1 from step ①.)

This value of y_1 is again used in Euler's modified method to find next ~~Second~~ approximation of y_1

$$y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1)]$$

(y_1 from step ②)

The process is continued until the values obtained are equal.

To calculate y_2

$$y_2 = y_1 + h\, f(x_1, y_1) \quad \text{——} (8)$$

↑ from first iteration y_1 (final)

· Euler's Modified Method

I iteration: $y_2 = y_1 + \frac{h}{2}\, f(x_1, y_1) + f(x_2, y_2)$

↑ from (13)

II iteration

$$y_2 = y_1 + \frac{h}{2}\, f(x_1, y_1) + f(x_2, y_2)$$

↑ from I iteration

Continue till two equal values of y_2

—— ✻ ——

Qus 1 $y' = -2xy^2$

$y(0) = 1 \quad x = 0.25 \; (0.25) \; 0.5$

Soln -

Clearly $h = 0.25$

$x_0 = 0 \quad y_0 = 1$

$f(x, y) = -2xy^2$

Step 1 Euler's Method

$y_1 = y_0 + h f(x_0, y_0)$

$= 1 + (0.25) \; f(0, 1)$

$= 1$

$\therefore y_1 = 1$ at $x_1 = x_0 + h = 0.25$

Step 2 Euler's Modified Method

$y_1 = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1)]$

Iteration

$= 1 + \frac{0.25}{2} [f(0, 1) + f(0.25, 1)]$

$= 1 + 0.1250 \; [0 - 0.5]$

$= 0.9375$

IInd $y_1 = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1)]$

$= 1 + \frac{0.25}{2} [f(0, 1) + f(0.25, 0.9375)]$

$= 1 + 0.1250 [0 - 0.4395]$

$= 0.9451$

III rd $y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1)]$

$= 1 + \frac{0.25}{2}[f(0,1) + f(0.25, 0.9451)]$

$= 1 + 0.1250 [0 - 0.4466]$

$\boxed{y_1 = 0.9442}$

IV th $y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1)]$

$= 1 + \frac{0.25}{2}[f(0,1) + f(0.25, 0.9442)]$

$= 1 + 0.1250 [0 - 0.4457]$

$\boxed{y_1 = 0.9443}$ ~ III rd situation y_1

$\therefore$ $y_1 = .9443.$

Find y_2

step (1) find y_2 by Euler's method

$y_2 = y_1 + h f(x_1, y_1)$

$= 0.9443 + 0.25 f(0.25, 0.9443)$

$x_1 = x_0 + h$.

$= 0.9443 + 0.25[-0.4459]$

$y_2 = 0.8328$

Step 2 Calculate Modified Method

I Iteration

$$y_2 = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2)]$$

$$= 0.9443 + \frac{0.25}{2}[f(0.25, 0.9443) + f(0.5, 0.8328)]$$

$$= 0.9443 + 0.1250[-0.4459 - 0.6936]$$

$y_2 = 0.8019$

II $y_2 = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2)]$

$$= 0.9443 + \frac{0.25}{2}[f(0.25, 0.9443) + f(0.5, 0.8019)]$$

$$= 0.9443 + 0.1250[-0.4459 - 0.6430]$$

$= 0.8082$

III $y_2 = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2)]$

$= 0.9443 + \frac{0.25}{2}\left[f(0.25, 0.9443) + f(0.5, 0.8082)\right]$

$= 0.9443 + 0.1250\left[-0.4459 - 0.6532\right]$

$= 0.8069$

IV $y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2)\right]$

$= 0.9443 + \frac{0.25}{2}\left[f(0.25, 0.9443) + f(0.5, 0.8069)\right]$

$= 0.9443 + 0.1250\left[-0.4459 - 0.6511\right]$

$= 0.8072$

V $y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2)\right]$

$= 0.9443 + \frac{0.25}{2}\left[f(0.25, 0.9443) + f(0.5, 0.8072)\right]$

$= 0.9443 + 0.1250\left[-0.4459 - 0.6516\right]$

$= 0.8071$

$\therefore$ IV $=$ V $= 0.8071$ Ans

$y(0.5) = 0.8071$

Practice Ques! -

4.19.

Q $\frac{dy}{dx} = y - \frac{2x}{y}$ take $h = 0.1$

given $y(0) = 1$ at $x = 0.1$