6/10/20.

Ques. based on Case - II.

Evaluate

(1) $\lim_{\substack{x\to 0\\ y\to 0}} \frac{y^2-x^2}{y^2+x^2}$, $x \neq 0,\ y \neq 0$

Sol :- $\lim_{y\to 0}\left[\lim_{x\to 0}\frac{y^2-x^2}{y^2+x^2}\right] = \lim_{y\to 0}\left[\frac{y^2-0}{y^2+0}\right] = \lim_{y\to 0} 1 = L_1$

and $\lim_{\substack{x\to 0\\ y\to 0}} \frac{y^2-x^2}{y^2+x^2} = \lim_{x\to 0}\left[\lim_{y\to 0}\frac{y^2-x^2}{y^2+x^2}\right] = \lim_{x\to 0}\left[\frac{-x^2}{x^2}\right] = -1$
$= L_2$

$\because L_1 \neq L_2$ $\therefore$ Limit does not exist.

(2) $\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^3-y^3}{x^2+y^2}$, $x \neq 0,\ y \neq 0$.

Sol :- (1) $\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^3-y^3}{x^2+y^2} = \lim_{y\to 0}\left[\lim_{x\to 0}\frac{x^3-y^3}{x^2+y^2}\right] = \lim_{y\to 0}\left[\frac{0-y^3}{0+y^2}\right] = 0$
$= L_1$

(2) $\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^3-y^3}{x^2+y^2} = \lim_{x\to 0}\left[\lim_{y\to 0}\frac{x^3-y^3}{x^2+y^2}\right] = \lim_{x\to 0}\frac{x^3}{x^2} = 0 = L_2$

$\because L_1 = L_2$ $\therefore$ apply the path method

(3) Let $y = mx$ be a path

$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^3-y^3}{x^2+y^2} = \lim_{x\to 0}\left(\lim_{y\to 0}\frac{x^3-y^3}{x^2+y^2}\right) = \lim_{x\to 0}\left[\lim_{y\to mx}\frac{x^3-m^3x^3}{x^2+m^2x^2}\right]$

$$= \lim_{x\to 0} \frac{x^3 - m^3x^3}{x^2 + m^2x^2} = \lim_{x\to 0} \frac{x(1-m^3)}{(1+m^2)} = 0$$

$$= L_3$$

Again let $y = mx^2$. $\quad (y = mx^n)$

$$\lim_{x\to 0}\left[\lim_{y\to 0} \frac{x^3-y^3}{x^2+y^2}\right] = \lim_{x\to 0}\left[\lim_{y\to mx^2} \frac{x^3-y^3}{x^2+y^2}\right]$$

$$= \lim_{x\to 0}\left[\frac{x^3 - m^3x^6}{x^2+m^2x^4}\right] = \lim_{x\to 0} x\left[\frac{1-m^3x^3}{1+m^2x^2}\right]$$

$$= 0 = L_4$$

$\because L_1 = L_2 = L_3 = L_4 \therefore$ limit exists and its value $= 0$. Ans

Qus 2. Evaluate $\lim_{\substack{x\to 0\\ y\to 0}} (y^3 + x^3) \; ; x \neq 0, y \neq 0$

Sol^n|-

(1) $$\lim_{\substack{x\to 0\\ y\to 0}} (y^3+x^3) = \lim_{x\to 0}\left[\lim_{y\to 0} y^3+x^3\right]$$

$$= \lim_{x\to 0}\left[0 + x^3\right]$$

$$= 0 = L_1$$

(2) $$\lim_{\substack{x\to 0\\ y\to 0}} (y^3+x^3) = \lim_{y\to 0}\left[\lim_{x\to 0} y^3+x^3\right]$$

$$= \lim_{y\to 0}\left[y^3+0\right] = 0 = L_2$$

(iii) put $y = mx$

$$\lim_{\substack{x\to 0\\ y\to 0}} (y^3+x^3) = \lim_{x\to 0}\left[\lim_{y\to mx} y^3+x^3\right]$$

$$= \lim_{x\to 0}\left[m^3x^3+x^3\right]$$

$$= \lim_{x\to 0} x^3\left[m^3+1\right]$$

$$= 0 = L_3$$

(iv) put $y = mx^2$

$$\lim_{\substack{x\to 0\\ y\to 0}} (y^3+x^2) = \lim_{x\to 0}\left(\lim_{y\to mx^2} y^3+x^3\right]$$

$$= \lim_{x\to 0}\left[m^3x^6+x^3\right]$$

$$= \lim_{x\to 0} x^3\left[m^3x^3+1\right]$$

$$= 0 = L_4$$

$\because L_1 = L_2 = L_3 = L_4 = 0 \therefore$ limit exist and is equal to 0.

=x=

Continuity :-

A function $f(x, y)$ is said to be Continous at a point (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b) = \text{Value of the function.}$$

Note

* A function is said to be Continous in a domain if it is Continous at every point of the domain.

Working Rule :- for a Continuity at (a, b)

Step 1 Find $f(a, b)$ which should be exist

Step 2 $\lim_{(x,y)\to(a,b)} f(x,y)$ should exist

Step 3 If $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$.

then function is Continous at (a, b)

Ques 1 Test the function $f(x,y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2} & x \neq 0, y \neq 0 \\ 0 & x = 0, y = 0 \end{cases}$

for the Continuity

Step 1

$f(0,0) = 0$ given

Step 2 find limit

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2}$$

$$= \lim_{x\to 0}\left[\lim_{y\to mx} \frac{x^3-y^3}{x^2+y^2}\right]$$

$$= \lim_{x\to 0} \frac{x^3-m^3x^3}{x^2+m^2x^2}$$

$$= 0$$

$\because$ limit of f(x) at origin = value of the function at (origin) (0,0)

Hence f is Continous at origin

Ques 2 Discuss the Continuity of $f(x,y)$

$$= \begin{cases} \dfrac{x}{\sqrt{x^2+y^2}} & ; x \neq 0, y \neq 0 \\ 2 & ; x = 0, y = 0 \end{cases}$$

at origin.

Soln - step 1 $f(0,0) = 2$

step 2 find limit

$$\lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}} = \lim_{x\to 0}\left[\lim_{y\to mx} \frac{x}{\sqrt{x^2+y^2}}\right]$$

$$= \lim_{x\to 0}\left[\frac{x}{\sqrt{x^2+m^2x^2}}\right]$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{1+m^2}}$$

Limit is not unique as depend upon m Hence limit does not exist.

Hence $f(x,y)$ is not continous.

Types of discontinuity :-

① first kind :- $f(x)$ is discontinous of first kind at $x = x_1$ if RHL $f(x_1+0)$ and LHL $f(x_1-0)$ exist but not equal

② Second kind :- $f(x)$ is discontinous of 2nd kind at $x = x_1$ if neither RHL $f(x_1+0)$ exists nor LHL $f(x_1-0)$ exists.

③ Removable discontinuity :- If RHL $f(x_1+0)$ is equal to LHL $f(x_1-0)$ is not equal to $f(x_1)$ then $f(x)$ is said to have removable discontinuity.

Ques Show that the given function are discontinous at all point $(2, 2)$

Sol! - $\cancel{\lim_{(x,y)}}$ $f(x,y) = \begin{cases} \dfrac{x^2+xy+x+y}{x+y} & ; (x,y) \neq (2,-2) \\ 4 & ; (x,y) = (2,-2) \end{cases}$

Soln/ -

$$\lim_{(x,y)\to(2,-2)} \frac{x^2+xy+x+y}{x+y}$$

$$= \lim_{(x,y)\to(2,-2)} \frac{\cancel{(x+y)}(x+1)}{\cancel{x+y}} = 2+1 = 3$$

$\because \lim_{(x,y)\to(2,-2)} \neq f(2,-2)$

$\therefore$ $f(x,y)$ is discontinous at $(2,-2)$

partial Derivatives

If $z = f(x,y)$ be function of two independent variables x and y.

The derivative of z with respect to x keeping y as a constant is called partial derivative of z 'w.r.t' 'x' and is denoted by $\frac{\partial z}{\partial x}$, $\frac{\partial f}{\partial x}$ or f_x

Similarly the partial derivative of z w r to y keeping x as a constant is denoted by $\frac{\partial z}{\partial y}$, $\frac{\partial f}{\partial y}$ or f_y

Notation $\frac{\partial z}{\partial x} = p$, $\frac{\partial z}{\partial y} = q$, $\frac{\partial^2 z}{\partial x^2} = r$, $\frac{\partial^2 z}{\partial y^2} = t$

$$\frac{\partial^2 z}{\partial x \partial y} = s.$$

Ques If $z(x+y) = x^2 + y^2$, show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

Sol^n:- Hint- find z first

$$z = \left(\frac{x^2+y^2}{x+y}\right)$$