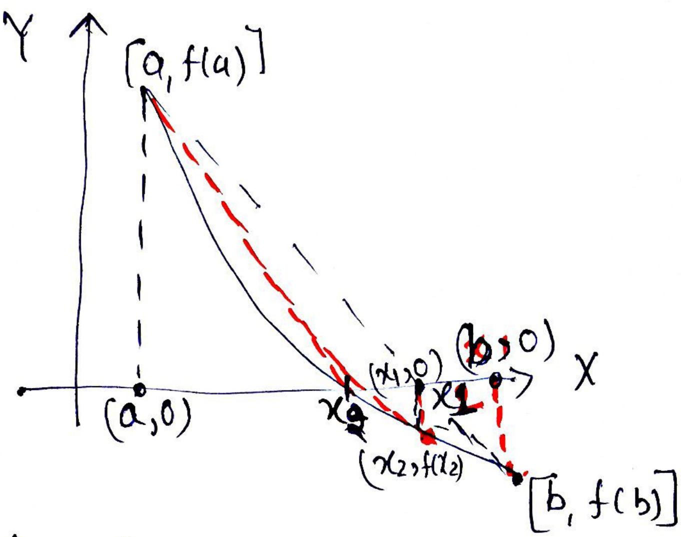
Regular falsi method closely resembles the Bisection Method.

Let $f(x)=0$ be an equation choose a and b such that $f(a)\cdot f(b)<0$

The curve $y=f(x)$ cuts at x-axis between these two point a and b. Hence root must lie between a and b



Draw the chord ab. The equation of the chord joining two points a and b is

$$y-f(a)=\frac{f(b)-f(a)}{b-a}\cdot(x-a) \quad \text{—} ①$$

$$\boxed{y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)}$$

The chord ① cuts on x-axis at x_1 where $y=0$

Hence from ①

$$0-f(a)=\frac{f(b)-f(a)}{b-a}\cdot(x_1-a)$$

$$\Rightarrow -f(a)\cdot b+\cancel{af(a)}=x_1f(b)-x_1f(a)-af(b)+\cancel{af(a)}$$

$$\therefore \boxed{x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}} \rightarrow \text{first approximation}$$

$a<x_1<b$

Now check weather

(i) $f(x_1) \cdot f(a) < 0$ OR

(ii) $f(x_1)\, f(b) < 0$

If $f(x_1) \cdot f(a) < 0$ then root (x_2) lies in between a and x_1 such that $a < x_2 < x_1$ then

$$x_2 = \frac{a\, f(x_1) - x_1\, f(a)}{f(x_1) - f(a)}.$$

In this way repeat the process till the root is obtained.

= x =

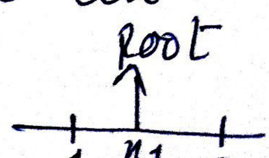
Ques 1 Compute the root of $x^3 - 5x + 3 = 0$ in interval $(1, 2)$ by using Regular falsi method.

Sol:- Let $f(x) = x^3 - 5x + 3$.

Here $a = 1$ $\quad f(a) = f(1) = 1^3 - 5(1) + 3 = -1$

$b = 2$ $\quad f(b) = f(2) = 2^3 - 5(2) + 3 = 1$

$\because$ $f(a) < 0$ & $f(b) > 0$ root lies in between $a = 1$ and $b = 2$



$$\therefore\ x_1 = \frac{a\, f(b) - b\, f(a)}{f(b) - f(a)} = \frac{1 \times f(2) - 2\, f(1)}{f(2) - f(1)}$$

$$x_1 = \frac{1\times(1) - 2(-1)}{1-(-1)} = \frac{3}{2} = \boxed{1.5}$$

find $f(x_1) = f(1.5) = (1.5)^3 - 5(1.5) + 3 = -1.125$

Now $f(a) = f(1) = -1, -ve$

$f(b) = f(2) = 1 + ve$

$f(x_1) = f(1.5) = -1.125\ (-ve)$

$\therefore$ Root (x_2) lies in between $(\overset{x_1}{1.5}, \overset{b}{2})$ as $f(1.5)\, f(2) < 0.$ & $1.5 < x_2 < 2$

1.5 x_2 2

$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)} = \frac{1.5\times(f(2)) - 2f(1.5)}{f(2) - f(1.5)}$$

$$= \frac{1.5\times 1 - 2\times(-1.125)}{1-(-1.125)} = \boxed{1.7647}$$

$f(x_2) = f(1.7647) = -0.3279.$

Now $f(b) = f(2) = 1\ +ve$

$f(x_1) = f(1.5) = -1.125$

$f(x_2) = f(1.7647) = -0.3279$

$\therefore$ root (x_3) lies in between 2 and (1.7647) as $f(2)\cdot f(1.7647) < 0$ Also, $x_2 < x_3 < b$

$$\therefore x_3 = \frac{x_2 \cdot f(b) - b\, f(x_2)}{f(b) - f(x_2)}$$

1.7647 x_3 2

$$= \frac{1.7647\cdot(f(2)) - 2\times f(1.7647)}{f(2) - f(1.7647)} =$$

$$= \frac{1.7647\times(1) - 2\times(-0.3279)}{1-(-0.3279)} = \boxed{1.823}$$

Now $f(x_3) = f(1.823) = -0.05756$

Again $f(b) = f(2) = 1 \quad +ve$

$f(x_1) = f(1.5) = -1.125$

$f(x_2) = f(1.7647) = -0.3279$

$f(x_3) = f(1.823) = -0.05756$

$\because f(2) = +ve$ & $f(x_3) = -ve$ $\therefore$

Root x_4 lies in between b and x_3.

$\therefore x_3 < x_4 < b$.

Now

$$\therefore x_4 = \frac{x_3 f(b) - b f(x_3)}{f(b) - f(x_3)}$$

$$x_4 = \frac{1.823 \times f(2) - 2 \times (f(1.823))}{f(2) - f(1.823)}$$

$$= \frac{1.823 \times 1 - 2 \times (-0.05756)}{1 - (-0.05756)}$$

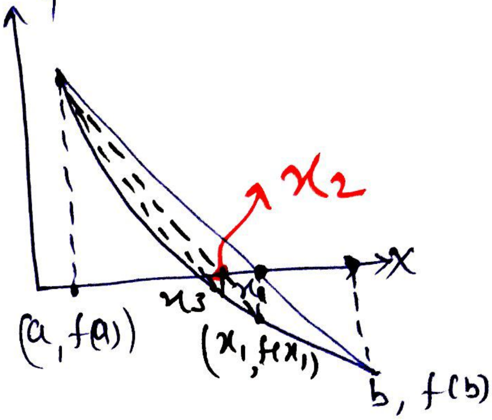
$$\boxed{x_4 = 1.8324}$$

$\because x_3 = x_4 \therefore$ And Root $= 1.8324$

Secant p/250. Secant Method

This method is same as Regular Falsi method. except the condition $f(a)f(b)<0$

In this intersection of the chord with X-axis is the next approximation (x_1)



$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$ → Same as regular falsi

∵ x_1 is nearer to b ∴ replace a by b and b by x_1

$$\therefore x_2 = \frac{b f(x_1) - x_1 f(b)}{f(x_1) - f(b)}$$

∵ x_3 is close to x_2 and x_1 replace b by x_1 and x_1 by x_2

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

Continue in this way.

The method fails if $f(x_{k-1}) = f(x_{k-2})$
So this method does not converge surely. But regular method surely converge, this is the Draw Back of Secant Method