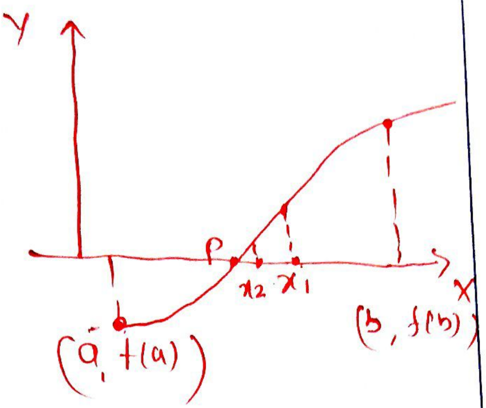
Geometrical Concept

Method | **Curve** | **Interpretation**

① Bisection Method



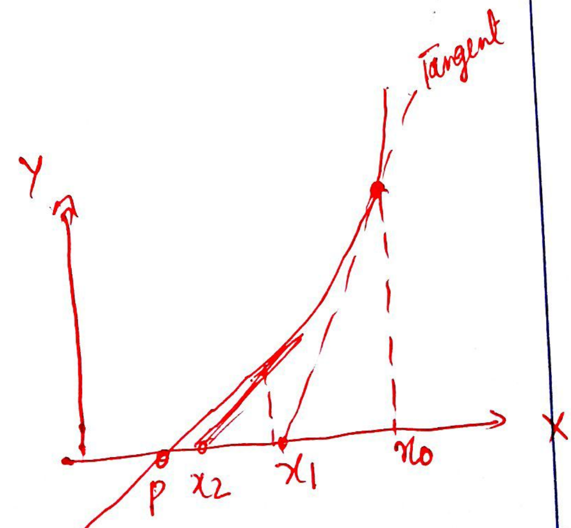
$x_1 = \frac{a+b}{2}$ as $f(a)\,f(b) < 0$

$x_2 = \frac{a+x_1}{2}$ as $f(a)\cdot f(x_1) < 0$

Continue till Convergent solution.

1. In each step we find the mid value on the curve so that $f(a)f(b) < 0$.
2. $f(x)$ should be Continous.
3. The Convergence is slow & linear
4. The error decreases in step 0.5
5. To overcome this problem we see another Method

② N-R Method

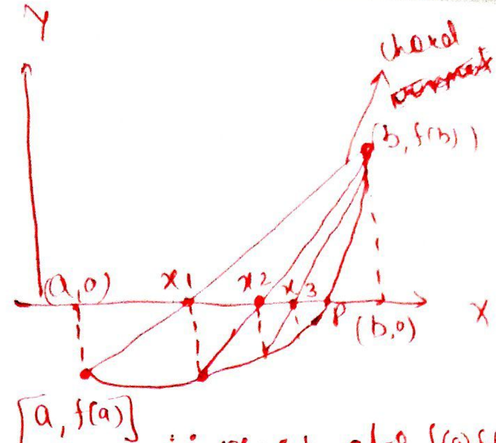


Find x_0 approximate root such that $a < x_0 < b$ where $f(a)\cdot f(b) < 0$ and if $|f(a)| < |f(b)|$ then $x_0 = a$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1. In each step we draw the tangent to the curve starting from a point x_0 as first approximate Root of $f(x)$.
2. The point of intersection with X-axis gives x_1 with the tangent
3. It is faster order of Convergence Quadratic (double) 2.

3. Regular Falsi Method (R.F.)



$\because a < x_1 < b$ also $f(a)f(b) < 0$

Step ①

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

Now if $x_1 < x_2 < b$ also $f(x_1) \cdot f(b) < 0$

Step ②

$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)}$$

Now if $x_2 < x_3 < b$ also $f(x_2) \cdot f(b) < 0$

Step ③

$$x_3 = \frac{x_2 \cdot f(b) - b f(x_2)}{f(b) - f(x_2)}$$

Continue till we get root

Interpretation

① In each step we draw the chord joining two points which cuts on X-axis so that we get first approximate root x_1

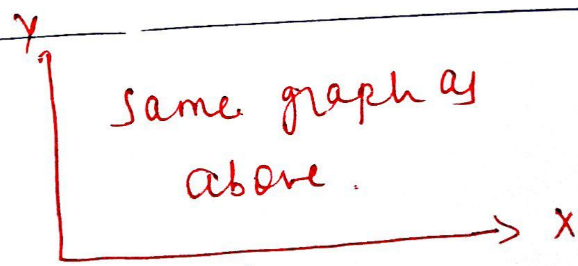
Also
① $f(x_1) f(a) < 0$
OR
$a < x_2 < x_1$
② $f(x_1) \cdot f(b) < 0$
and
$x_1 < x_2 < b$
Continue This

④ Secant Method

Advantage over Regular Falsi:- No need to check in each step

$\boxed{f(a) \cdot f(b) < 0}$

$a \quad b \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad --$



Step ① is same as R.F

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$x_2 = \frac{b f(x_1) - x_1 f(b)}{f(x_1) - f(b)}$$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

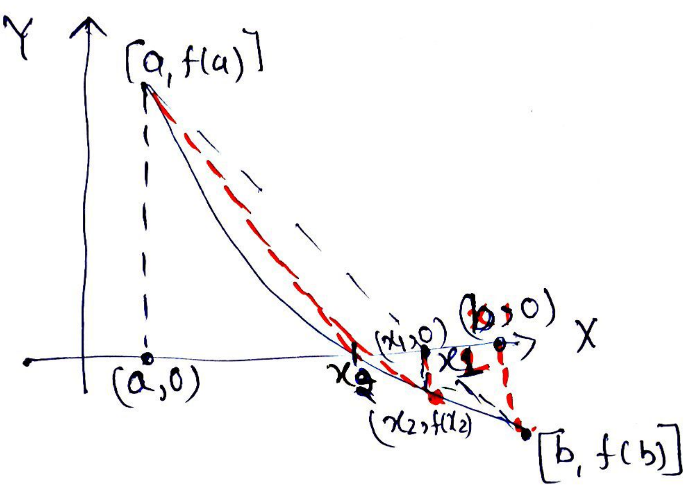
$$x_4 = x_2 f(x_3) - x_3 f(x_2) / f(x_3) - f(x_2)$$

① In each step we get eq of chord
② The method does not necessarily convergent
③ It fails if $f(x_{n-1}) = f(x_n)$
④ This is drawback over R.F.
⑤ R.F. surely converges

Regular falsi method closely resembles the Bisection method.

Let $f(x)=0$ be an equation choose a and b such that $f(a)\cdot f(b)<0$

The curve $y=f(x)$ cuts at X-axis between these two point a and b. Hence root must lie between a and b



Draw the chord ab.. The equation of the chord joining two points a and b is

$$y-f(a)=\frac{f(b)-f(a)}{b-a}\cdot(x-a) \quad \text{—(1)}$$

$$\boxed{y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)}$$

The chord (1) cuts on X-axis at x_1 where $y=0$

Hence from (1)

$$0-f(a)=\frac{f(b)-f(a)}{b-a}\cdot(x_1-a)$$

$$\Rightarrow -f(a)\cdot b+\cancel{af(a)}=x_1 f(b)-x_1 f(a)-af(b)+\cancel{af(a)}$$

$$\therefore \boxed{x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}} \longrightarrow \text{first approximation}$$

$a<x_1<b$

Q. Compute the real root of $x^3 - 5x + 3 = 0$ in the interval $(1,2)$ by Regular Falsi (R.F.)

Solution:- By R.F

Root lies in between 1 and 2 as

$$\boxed{\begin{array}{ll} f(1) = -1 = -ve & \\ f(2) = +1 = +ve & \text{Also } f(1)\cdot f(2) < 0 \end{array}}$$ *

Find First iteration $x_1 = \dfrac{a f(b) - b f(a)}{f(b) - f(a)}$ Here $a = 1$ $b = 2$

$f(a) = f(1) = -1$ $f(b) = f(2) = +1$

$$= \frac{1 \times (+1) - 2 \times (-1)}{+1 - (-1)} = \frac{+3}{+2} = 1.5.$$

$a = 1$	$b = 2$	$x_1 = 1.5$
$f(a)$	$f(b)$	$f(x_1)$
$f(1) = -1$	$f(2) = +1$	$f(1.5) = -1.125$

Second iteration $\because f(b)\cdot f(x_1) < 0$ *

$\therefore$ Root lies in between $b = 2$ & $x_1 = 1.5$

$x_2 = \dfrac{a f(b) - b f(a)}{f(b) - f(a)}$ Here $a = 2$, $b = 1.5$

$f(a) = f(2) = +1$ $f(b) = f(1.5) = -1.125$

$$= \frac{2 \times (-1.125) - (1.5)(+1)}{-1.125 - (+1)} = \frac{-2.25 - 1.5}{} = \frac{\cancel{-}3.75}{\cancel{-}2.13} = 1.7647$$

Third iteration $\because$ $a = 1$ $b = 2$ $x_1 = 1.5$ $x_2 = 1.7647$

$f(1) = -1$ $f(2) = +1$ $f(1.5) = -1.125$ $f(x_2) = -0.3279$

$\therefore$ Root lies in between 2 and 1.7647 as

$f(2)\, f(1.7647) < 0$

Hence $a = 2, \quad b = 1.7647$

$f(a) = f(2) = +1 \quad f(b) = f(1.7647) = -0.3279$

$$\therefore x_3 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

$$= \frac{2 \times (-0.3279) - 1.7647\,(1)}{-0.3279 - 1}$$

$$= \frac{-(0.6558) - 1.7647}{-1.3279} = \frac{-2.4205}{-1.3279}$$

$$= 1.8228$$

$f(x_3) = f(1.8228) = -0.0576$

4th iteration :-

$\boxed{\begin{array}{lll} a = 2 & b = 1.7647 & x_3 = 1.8228 \\ f(a) = 1 & f(b) = -0.3279 & f(x_3) = \cancel{1.8228} \\ & & = -0.0576 \\ \because f(a) \cdot f(x_3) < 0 & & \end{array}}$

Root lies in between b & x_3.

Now $a = \cancel{1.7647}\ 2 \quad b = 1.8228$

$f(a) = \cancel{-0.3279}\ 1 \quad f(b) = -0.0576$

$$\therefore x_4 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

$$= \frac{2 \times (-0.0576) - (1.8228) \times (1)}{-0.0576 - 1} = \frac{-1.9380}{-1.0576}$$

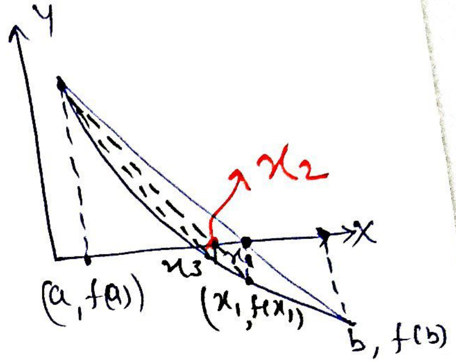
$$= \frac{(-0.1152) - (1.8228)}{-1.0576} = \frac{-1.9380}{-1.0576} = 1.8324$$

Ans.

Secant p/250. **Secant Method**

This method is same as Regular Falsi method. except the condition $f(a)f(b)<0$

In this intersection of the chord with X-axis is the next approximation (x_1)



$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$ → Same as regular falsi

∵ x_1 is nearer to b ∴ replace a by b and b by x_1

a b x_1 x_2 x_3 x_4

$$\therefore x_2 = \frac{b f(x_1) - x_1 f(b)}{f(x_1) - f(b)}$$

∵ x_3 is close to x_2 and x_1 replace b by x_1 and x_1 by x_2

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$ Continue in this way.

The method fails if $f(x_n) = f(x_{n-1})$

So this method does not converge surely. But Regular method surely converge, this is the Draw Back of Secant method

Q) Compute :- $x^3-5x+3=0$ in the interval $(1,2)$ by Secant, perform 4 iterations.

Solution :- Here, $f(x)=x^3-5x+3=0$, given $a=1$, $b=2$

$f(1)=1-5\times1+3=-1=-ve$

$f(2)=8-10+3=+1=+ve$

Surely root lies in between 1 & 2. Find first approximate root x_1

$$x_1=\frac{a f(b)-b f(a)}{f(b)-f(a)}$$ where $a=1$, $b=2$, $f(1)=-1$ $f(2)=1$

$$x_1=\frac{1\times1-2\times(-1)}{1-(-1)}=\frac{1+2}{2}=\frac{3}{2}=1.5$$

Now we have

1 a

2 b

1.5 x_1

1.7647 x_2

1.8735 x_3

x_4

$$x_2=\frac{a f(b)-b f(a)}{f(b)-f(a)}$$ Now $a=2$, $b=1.5$, $f(a)=f(2)=1$ $\underbrace{f(1.5)}_{f(b)}=-1.125$

$$x_2=\frac{2\times(-1.125)-1.5(1)}{(-1.125-1)}=1.7647$$

$$x_3=\frac{a f(b)-b f(a)}{f(b)-f(a)}$$ $a=1.5$, $b=1.7647$, $f(a)=f(1.5)=-1.125$, $f(b)=f(1.7647)=-0.3279$

$$x_3=\frac{1.5\times(-0.3279)-1.7647(-1.125)}{(-0.3279)-(-1.125)}=1.8735.$$

$$x_4=\frac{a f(b)-b f(a)}{f(b)-f(a)}$$ $a=1.7647$, $b=1.8735$, $f(a)=-0.3279$ $f(b)=0.2085$

$$=\frac{1.7647(0.2085)-(1.8735)(-0.3279)}{(0.2085-(-0.3279))}=\underline{\underline{1.8312}}$$ Ans

Practice Ques (H.W.)

Solve following by Both method

Ques 1

Find a root of the equation

$x - e^{-x} = 0$ Correct to 3 decimal places by both method (RF, secant)

Ans :- 0.567

Ques 2 By using Secant method find the root of equation $xe^{x} - \cos x = 0$ Correct to 4 decimal places