MAT2003-APPLIED NUMERICAL METHODS

Presented by
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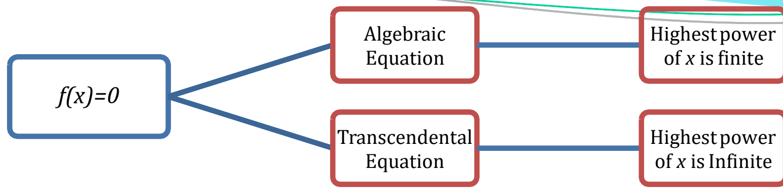
OUTLINE:

- Syllabus
- Module-1
- References

SYLLABUS

MODULE-1:
Algebraic, Transcendental and Linear system of equations

- **Algebraic and Transcendental Equations**
- **Methods to solve the equations**
- **Direct and Iterative methods**



- To find the solution of the equation $f(x) = 0$.

ALGEBRAIC EQUATION:

- An Equation which contains algebraic terms is called as an algebraic equation.

- Consider the equation of the form f(x)=0.

If f(x) is a quadratic, cubic or biquadratic expression then algebraic formulae are available for expressing the roots.
But when f(x) is a polynomial of higher degree or an expression

Example: $x^3 - x^2 - 1 = 0$

Here Highest power of x is finite.

TRANSCENDENTAL EQUATION:

- An equation which contains trigonometric, exponential and logarithmic functions is called as a transcendental equation.

1 +cos x - 5x, x tan x - cosh x, e - sin x etc., algebraic methods are not available.

Example: $e^{x}-x-1=0, sinx=1, log_{10}(x-1)=0$

Quiz 1

- This equation $x^3 - x = 1$ is a

- [ ] Algebraic equation.
- [ ] Transcendental equation
- [ ] quadratic equation
- [ ] Cubic equation

METHODS

In order to solve these type of equations following methods exist:

- DIRECTIVE METHODS:

  The methods which are used to find solutions of given equations in the direct process is called as directive methods.

  Example: remainder theorem, Factorization method etc

  Note:

  By using Directive Methods, it is possible to find exact solutions of the given equation.

- ITERATIVE METHODS (INDIRECT METHODS):

  The methods which are used to find solutions of the given equation in some indirect process is called as Iterative Methods

  Note:

  By using Iterative methods, it is possible to find approximate solution of the given equation and also it is possible to find single solution of the given equation at the same time.

ITERATIVE/INDIRECT METHODS:

- Bisection method
- Newton-Raphson method
- False-position or Regula-falsi method
- Secant Method
- Fixed point iteration method

BISECTION METHOD

- This method is based on the repeated application of intermediate value property.

- ❑Locate the interval (a, b) in which root lies.
- ❑Bisect the interval (a, b).
- ❑Choose the half interval in which the root lies.
- ❑Bisect the half interval.
- ❑Repeat the process until the root converges.

BISECTION METHOD ALGORITHM

STEP-1: Consider $f(x) = 0$ be the given equation.

STEP-2: Choose a and b such that f(a).f(b)<0.

Theorem-1.1: If f(x) is continuous in [a, b] and f(a) and f(b) are of opposite signs, then f(p)=0 for atleast one p in (a, b).

STEP-3: $x_n = \frac{a+b}{2}$

STEP-4: **Stop till the last solution repeats. It will be the required approximate solution.**

Ex. 01 Find a real root of the equation $f(x) = x^3 - x - 1 = 0$ correct upto 4-decimal places.

Solution Here $f(x) = x^3 - x - 1 = 0$

$\Rightarrow f(0) = -1 < 0$

$\Rightarrow f(1) = 1^3 - 1 - 1 = -1 < 0$

$\Rightarrow f(2) = 2^3 - 2 - 1 = 8 - 2 - 1 = 5 > 0$

$\Rightarrow$ Root lies in $(1, 2)$.

(*) Initial root, $x_0 = \frac{1+2}{2} = \frac{3}{2} = 1.5$

$\Rightarrow f(x_0) = f(1.5) = (1.5)^3 - 1.5 - 1$

$= 0.8750 > 0$

$\Rightarrow$ Root lies in $(1, 1.5)$.

(*) 1^{st} iteration, $x_1 = \frac{1+1.5}{2} = 1.25$

$\Rightarrow f(x_1) = f(1.25) = (1.25)^3 - 1.25 - 1$

$= -0.2969 < 0$

$\Rightarrow$ Root lies in $(1.25, 1.5)$.

* 2nd iteration, $x_2 = \frac{1.25+1.5}{2} = 1.3750$

$$\Rightarrow f(x_2) = f(1.3750)$$

$$= 0.2246 > 0$$

$\Rightarrow$ Root lies in $(1.25, 1.3750)$.

* 3rd iteration, $x_3 = \frac{1.25+1.3750}{2} = 1.3125$

$$\Rightarrow f(x_3) = f(1.3125) = -0.0515 < 0$$

$\Rightarrow$ Root lies in $(1.3125, 1.3750)$.

$\vdots$

n	a	b	$x_n = \frac{a+b}{2}$	$f(x_n)$
0	1	2	1.5	$0.8750 > 0$
1	1	1.5	1.25	$-0.2969 < 0$
2	1.25	1.5	1.3750	$0.2246 > 0$
3	1.25	1.3750	1.3125	$-0.0515 < 0$
4	1.3125	1.3750	1.3438	$0.0826 > 0$
5	1.3125	1.3438	1.3282	$0.0147 > 0$
6	1.3125	1.3282	1.3204	$-0.0186 < 0$
7	1.3204	1.3282	1.3243	$-0.0018 < 0$
8	1.3243	1.3282	1.3263	$0.0065 > 0$
9	1.3243	1.3263	$\boxed{1.3253}$	$0.0025 > 0$
10	1.3243	1.3253	$\boxed{1.3248}$	$0.0003 > 0$
11	1.3243	1.3248	1.3246	$-0.0007 < 0$

→ Hence the required solution is $\boxed{1.325}$. Ans.

!!!THE END!!!

THANK YOU FOR YOUR

ATTENTION