Module - 2
Interpolation

Finite difference operators
- forward
- Backward
- Central
- Average, shift and differential operators (Relation)

Interpolation
- Newton forward Interpolation
- Newton Backward Interpolation
- Lagrange Interpolation (unequal intervals)
- Newton's divided difference Interpolation
- Cubic Spline Interpolation (with equally spaced)

Interpolation :-

It is a process of computing intermediate value of a function $f(x) = y$ from a given set of tabular values of a function

Let we have a set of corresponding values of x and y as follows

x :	x_0	x_1	x_2	--	x_n
y :	y_0	y_1	y_2	--	y_n

; $x_0 < x_i < x_n$

- If we find corresponding value of y_i to a value $x = x_i$ then This is Interpolation.

Ex.1 ! -

x !	0	1	2	3	4	5
y !	2	9	28	65	165	217

If we want to find out $f(2.5)$. This is interpolation.

Ex 2| -

x !	1	2	3	4	5	6	7
y !	2	4	8	—	32	64	128

If we want to find out $f(4)$. This is missing term.

Finite difference operators ! -

Let $y = f(x)$ be a given function let

x : $y = f(x)$

x_0 : $y_0 = f(x_0) = f_0$

x_1 : $y_1 = f(x_1) = f_1$

x_2 : $y_2 = f(x_2) = f_2$

$\vdots$

x_n : $y_n = f(x_n) = f_n$

where $x_0, x_1, x_2 \dots x_n$ are equally spaced such that $x_1 = x_0 + h$, $x_2 = x_0 + 2h$, $x_3 = x_0 + 3h$ --- $x_i = x_0 + ih$ where $(i = 0, 1, 2, \dots n)$

① Forward operator (Δ)

we write
$$\left\{\begin{array}{l} f(x_1) = f_1 \\ f(x_0) = f_0 \\ \vdots \\ f(x_n) = f_n \end{array}\right\}$$

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta f(x_0) = f(x_0+h) - f(x_0)$$

$$\boxed{\Delta f(x_0) = f(x_1) - f(x_0)} \quad \text{— (a)}$$

$$\Rightarrow \Delta f_0 = f_1 - f_0.$$

$$\Delta^2 f(x_0) = \Delta\left[\Delta f(x_0)\right]$$

$$= \Delta\left[f(x_1) - f(x_0)\right] \quad \text{By (a)}$$

$$=$$

$$= \Delta f(x_1) - \Delta f(x_0)$$

$$= f(x_1+h) - f(x_1) - f(x_1) + f(x_0)$$

$$\boxed{\begin{aligned} \Delta^2 f(x_0) &= f(x_1+h) - 2f(x_1) + f(x_0) \\ &= f(x_2) - 2f(x_1) + f(x_0) \end{aligned}} \quad \text{— (b)}$$

$$\boxed{\Delta^3 f(x_0) = f(x_3) - 3f(x_2) + 3f(x_1) - f(x_0)} \quad \text{— (c)}$$

$$\therefore \Delta^n f(x_0) = f(x_n) - {}^nC_1 f(x_{n-1}) + {}^nC_2 f(x_{n-2}) - \cdots (-1)^n f(x_0).$$

Tabular form.

x_K	f_K	Δf_K	$\Delta^2 f_K$	$\Delta^3 f_K$	....
x_0	f_0				
		$\Delta f_0 = f_1 - f_0$			
x_1	f_1		$\Delta^2 f_0 = \Delta f_1 - \Delta f_0$		
		$\Delta f_1 = f_2 - f_1$		$\Delta^3 f_0 = \Delta^2 f_1 - \Delta^2 f_0$	
x_2	f_2		$\Delta^2 f_1 = \Delta f_2 - \Delta f_1$	---	
		$\Delta f_2 = f_3 - f_2$			
x_3	f_3				
$\vdots$	$\vdots$				

Note :- $f_0 = y_0, \quad f_1 = y_1 \; - - -$

Ques 1 prepare the forward difference table for the following

x	$y=f(x)$	Δy first-difference	$\Delta^2 y$ 2nd diff	$\Delta^3 y$ 3rd diff	$\Delta^4 y$ 4th diff
x_0	$y_0=2$	$\Delta y_0 = 7$	$\Delta^2 y_0 = 12$	$\Delta^3 y_0 = 6$	$\Delta^4 y_0 = 0$
x_1	$y_1=9$	$\Delta y_1 = 19$	$\Delta^2 y_1 = 18$	$\Delta^3 y_1 = 6$	$\Delta^4 y_1 = 0$
x_2	$y_2=28$	$\Delta y_2 = 37$	$\Delta^2 y_2 = 24$	$\Delta^3 y_2 = 6$	
x_3	$y_3=65$	$\Delta y_3 = 61$	$\Delta^2 y_3 = 30$		
x_4	$y_4=126$	$\Delta y_4 = 91$			
x_5	$y_5=217$				

Ques 2 Evaluate

(i) $\Delta \tan^{-1} x$ (ii) $\Delta (e^x \log 2x)$

(iii) $\Delta (x^2 / \cos 2x)$ (iv) $\Delta^2 \cos 2x$.

Sol^n :- (i) $\Delta \tan^{-1} x = \tan^{-1}(x+h) - \tan^{-1}(x)$

$$= \tan^{-1}\left(\frac{x+h-x}{1+(x+h)\cdot x}\right) = \tan^{-1}\frac{h}{1+hx+x^2}$$

(ii) $\Delta (e^x \log 2x) = e^{x+h} \log 2(x+h) - e^x \log 2x$

$$= e^{x+h} \log 2(x+h) - \underbrace{e^{x+h} \log 2x} + \underbrace{e^{x+h} \log 2x} - e^x \log 2x$$

$$= e^{x+h} \log \frac{\cancel{2}(x+h)}{\cancel{2}\,x} + (e^{x+h} - e^x) \log 2x$$

$$= e^x \left[e^h \log\left(1+\frac{h}{x}\right) + (e^h - 1) \log 2x \right]$$

Solution