Gauss Jordan Method :-

* This Method is modified form of Gauss elimination
* In this Augmented Matrix is reduced into diagonal Matrix (instead upper triangular)

* Here we make the component entry 0 below the ~~pivot~~ a_{11} and below and above ~~pivot~~ a_{22} and so on.

* Here we get the solution without back substitution

Disadvantage :- Gauss elimination.

(1) Back substitution required.

(2) If any of a_{ii} ~~pivot~~ (a_{11}, a_{22}, a_{33}) becomes zero the method fails.

(3) In this case rearrangement of the equation is required.

(4) $a_{11} \neq 0$. If it is zero make the N/S. arrangements. by interchanging the Rows.

Advantage :- of Gauss Jordan :-

1. Back substitution not required.
2. — —

Method:-

Step 1 Write the equation in augmented form

Convert matrix into diagonal matrix :-

Step 2 Make a_{11} as ~~zero~~ one

Step 3 Below a_{11} make component as zero by elementary Row operation
$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Step 4 Now make a_{22} as 1

Step 5 Below and above a_{22} make a_{12} and a_{32} as zero by operation

Step 6 Now make a_{33} as 1.

Step 7 above a_{33} make a_{13} and a_{23} as zero again by elementary Row opn.

Step 8 finally get the answer without back Substitution process.

Disadvantage of Gauss Jordan:-

Lots of Computation are required than Gauss elimination method.

Ques Solve following by Gauss Jordan Method

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$

Sol:- Step 1 write in Augmented Matrix. $[A \,\vdots\, B]$

Where $A = \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix}$ & $B = \begin{bmatrix} 12 \\ 13 \\ 14 \end{bmatrix}$

$$\sim \left[\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 2 & 2 & 10 & 14 \end{array}\right]$$

Step 2 Convert Augmented Matrix into diagonal Matrix, Make a_{11} as 1. (Use $R_1 \to R_1 \div 10$)

$$\sim \left[\begin{array}{ccc|c} 1 & 1/10 & 1/10 & 12/10 \\ 2 & 10 & 1 & 13 \\ 2 & 2 & 10 & 14 \end{array}\right]$$

Step 3 Below ~~pivot~~ $a_{11} = 1$ Make $a_{21} = 0$ & $a_{31} = 0$

Apply $R_2 \to R_2 - 2R_1$ & $R_3 \to R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1/10 & 1/10 & 12/10 \\ 0 & 98/10 & 8/10 & 106/10 \\ 0 & 18/10 & 98/10 & 116/10 \end{array}\right]$$

Step 4 Make a_{22} as 1

$R_2 \to R_2 \div 98/10.$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{10} & \frac{1}{10} & \frac{12}{10} \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & \frac{18}{10} & \frac{98}{10} & \frac{116}{10} \end{array}\right] \sim \left[\begin{array}{cccc} \cancel{1} & \cancel{\frac{1}{10}} & \cancel{\frac{1}{10}} & \cancel{\frac{12}{10}} \\ \cancel{0} & \cancel{1} & \cancel{\frac{4}{49}} & \cancel{\frac{53}{49}} \\ \cancel{0} & \cancel{\frac{9}{10}} & \cancel{49} & \end{array}\right]$$

Step 5 Below a_{22} and above a_{22} Make entry 0

Apply

$R_3 \to R_3 - \frac{18}{10} R_2$ & $R_1 \to R_1 - \frac{1}{10} R_2$

$$\left[\begin{array}{ccc|c} \cancel{1} & \cancel{\frac{1}{10}} & \cancel{\frac{1}{10}} & \cancel{\frac{12}{10}} \\ & & & \\ & & & \\ & & & \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0.0918 & 1.0918 \\ 0 & 1 & 4/49 & 53/49 \\ 0 & 0 & 9.6530 & 9.6530 \end{array}\right]$$

Step 6 Make a_{33} as 1 Apply $R_3 \to R_3 \div (9.6530)$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0.0918 & 1.0918 \\ 0 & 1 & 4/49 & 53/49 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Step 7 above a_{33} Make a_{13} and a_{23} as zero

$R_1 \to R_1 - 0.0918(R_3)$

$R_2 \to R_2 - 4/49(R_3)$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Step 8 get Solution $z = 1,\ y = 1,\ x = 1.$

<u>Checking</u> :- $10x + y + z = 12 \Rightarrow 10 \times 1 + 1 + 1 = 12$ Verified.