Newton's Backward Interpolation formula

$$y_n(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$

Where $p = \dfrac{x - x_n}{h}$

$$\boxed{\begin{aligned}\nabla y_1 &= y_1 - y_0 \\ \nabla y_2 &= y_2 - y_1\end{aligned}}$$

Qus find a polynomial which takes the values.

x :	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
y :	1	2	4	7	11	16	22	29.

x		y		∇y	$\nabla^2 y$	$\nabla^3 y$
0	x_0	1	y_0			
				1 ∇y_1		
1	x_1	2	y_1		$\nabla^2 y_2 = 1$	
				2 ∇y_2		0 $\nabla^3 y_3$
2	x_2	4	y_2		$\nabla^2 y_3 = 1$	
				3 ∇y_3		0 $\nabla^3 y_4$
3	x_3	7	y_3		$\nabla^2 y_4 = 1$	
				4 ∇y_4		0 $\nabla^3 y_5$
4	x_4	11	y_4		$\nabla^2 y_5 = 1$	
				5 ∇y_5		0 $\nabla^3 y_6$
5	x_5	16	y_5		$\nabla^2 y_6 = 1$	
				6 ∇y_6		0 $\nabla^3 y_7$.
6	x_6	22	y_6		$\nabla^2 y_7 = 1$	
				7 ∇y_7		
7	x_7	29	y_7			

$$p = \frac{x - x_n}{h} = \frac{x-7}{1}$$

$$y = f_n(x) = y_7 + p \nabla y_7 + \frac{p(p+1)}{\lfloor 2} \nabla^2 y_7 + \frac{p(p+1)(p+2)}{\lfloor 3} \nabla^3 y_7 .$$

$$= 29 + (x-7)(7) + \frac{(x-6)(x-6)}{2}(1)$$

$$= \frac{1}{2}(x^2 + x + 2)$$

x

Some Ques based on interpolation operator:

Ques 1. Given $f_3 = 5$, $f_4 = -6$ $f_5 = 8$, $f_6 = 9$, $f_7 = 17$

Calculate $\Delta^4 f_3$.

Sol^n :-

x :	3	4	5	6	7
y :	5	-6	8	9	17

$$\Delta^4 f_3 = (E-1)^4 f_3$$

$$= (E^4 - 4E^3 + 6E^2 - 4E + 1) f_3$$

$$= E^4 f_3 - 4E^3 f_3 + 6E^2 f_3 - 4E f_3 + f_3$$

$$= f_7 - 4 f_6 + 6 f_5 - 4 f_4 + f_3$$

$$= 17 - 4(9) + 6(8) - 4(-6) + 5$$

$$= 17 - 36 + 48 + 24 + 5$$

$$=$$

Ques 2. find f_6 if $f_0 = 9$, $f_1 = 18$, $f_2 = 20$, $f_3 = 24$ given that the third differences are constant

Sol^n ! — Since third differences are constants

$\Delta^4 f_0 = 0$ $\Delta^5 f_0 = 0$ $\Delta^6 f_0 = 0$ and so on.

x:	0	1	2	3
	x_0	x_1	x_2	x_3
f:	9	18	20	24
	f_0	f_1	f_2	f_3

Find f_6

$$f_6 = E^6 f_0$$

$$= (1+\Delta)^6 f_0$$

$$= \left(1 + 6C_1\Delta + 6C_2\Delta^2 + 6C_3\Delta^3 + 6C_4\Delta^4 + 6C_5\Delta^5 + 6C_6\Delta^6\right) f_0$$

$$= (1 + 6\Delta + 15\Delta^2 + 20\Delta^3 + 0 + 0 + 0) f_0$$

$$= \left[1 + 6(E-1) + 15(E-1)^2 + 20(E-1)^3\right] f_0$$

$$= (1 + 6E - 6 + 15E^2 - 30E + 15 + 20E^3 - 60E^2 + 60E - 20) f_0$$

$$= (-10 + 36E - 45E^2 + 20E^3) f_0$$

$$= -10f_0 + 36Ef_0 - 45E^2 f_0 + 20E^3 f_0$$

$$= -10f_0 + 36f_1 - 45f_2 + 20f_3$$

$$= -10(9) + 36(18) - 45(20) + 20(24)$$

$$= 138$$

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Ques 3 From the following table find the missing value

x:	2 (x_0)	3 (x_1)	4 (x_2)	5 (x_3)	6 (x_4)
y:	45 (y_0)	49.02 (y_1)	54.1 (y_2)	– (y_3)	67.4 (y_4)

Soln :- Since only four values of y are given use

$$\Delta^4 y_0 = 0$$

$$\therefore (E-1)^4 y_0 = 0$$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1) y_0 = 0$$

Complete it ??

Ques 4. Two missing term.

Estimate the production for 1964 and 1966 from following:

year	1961 (x_0)	1962 (x_1)	1963 (x_2)	1964 (x_3)	1965 (x_4)
production	200 (y_0)	220 (y_1)	260 (y_2)	– (y_3)	350 (y_4)

year	1966 (x_5)	1967 (x_6)
production	y_5	430 (y_6)

Solⁿ:- Since only 5 values of y are given we

$$\therefore \quad \Delta^5 y_K = 0$$

OR $(E-1)^5 y_K = 0$

OR $(E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1) y_K = 0$ — (1)

put $K=0$ in (1)

OR $(E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1) y_0 = 0$

OR $E^5 y_0 - 5E^4 y_0 + 10E^3 y_0 - 10E^2 y_0 + 5E y_0 - y_0 = 0$

OR $y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$

OR $y_5 - 5(350) + 10(y_3) - 10(260) + 5(220) - 206 = 0$

OR $y_5 + 10y_3 = 3450$ — (2)

put $K=1$ in (1)

$y_6 - 5y_5 + 10y_4 - 10y_3 + 5y_2 - y_1 = 0$

OR $430 - 5y_5 + 10(350) - 10y_3 + 5(260) - 220 = 0$

OR $5y_5 + 10y_3 = 5010$ — (3)

solve (2) & (3) we get $y_3 = 306$ & $y_5 = 390$

Ans.

Qus 5

let the sequence as follows:

2 9 28 65 126 217 _

Find 7th term
Also find general term

Solution! -

	x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	x_0	2 (y_0)	7 (Δy_0)	12 $(\Delta^2 y_0)$	6 $(\Delta^3 y_0)$	0 $(\Delta^4 y_0)$
1	x_1	9 (y_1)	19 (Δy_1)	18 $(\Delta^2 y_1)$	6 $(\Delta^3 y_1)$	0 $(\Delta^4 y_1)$
2	x_2	28 (y_2)	37 (Δy_2)	24 $(\Delta^2 y_2)$	6 $(\Delta^3 y_2)$	
3	x_3	65 (y_3)	61 (Δy_3)	30 $(\Delta^2 y_3)$		
4	x_4	126 (y_4)	91 (Δy_4)			
5	x_5	217 (y_5)				
6	x_6	??				

$$y(x) = y_0 + nc_1 \Delta y_0 + nc_2 \Delta^2 y_0 + \cdots nc_n \Delta^n y_0$$

$$= y_0 + nc_1 \Delta y_0 + nc_2 \Delta^2 y_0 + nc_3 \Delta^3 y_0 + nc_4 \Delta^4 y_0$$

$$= 2 + n\,7 + \frac{n(n-1)}{2}(12) + \frac{n(n-1)(n-2)}{6}(6) + 0$$

$$y(x) = n^3 + 3n^2 + 3n + 2$$

Hence 7th term is put $n = 6$

$$f_6 = 6^3 + 3(6^2) + 3(6) + 2 = \underline{344}.$$

Ques. 6 from the table given below, find $\sin 52^\circ$ by Newton's forward formula.

x	45°	50°	55°	60°
$y = \sin x$	0.7071	0.7660	0.8192	0.8660

Sol^n^ :- Forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
45°	0.7071			
		0.0589		
50°	0.7660		-0.0057	
		0.0532		-0.0007
55°	0.8192		-0.0064	
		0.0468		
60°	0.8660			

formula
$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 y_0$$

$$p = \frac{x - x_0}{h} = \frac{52-45}{5} = 1.4$$

$$y(52) = 0.7071 + (1.4)(0.0589) + \frac{(1.4)(0.4)}{\lfloor 2}(-0.0057)$$

$$+ \frac{(1.4)(0.4)(-0.6)}{\lfloor 3}(-0.0007)$$

$$\sin 52^\circ = \boxed{0.7880032} \text{ Ans.}$$