H.W of lecture No. 2

Qus 1. Evaluate $\sqrt{12}$ to four decimal places by N-R (Newton Raphson Method)

Qus 2. Compute real root of $x \log_{10} x = 1.2$ Correct to 3 decimal places by N-R Method.

Qu

Find **by** N-R method, the root of the equation $e^x = 4x$, which is approximately 2, correct to three decimals.

Sol:- Let $f(x) = e^x - 4x$

   & $f'(x) = e^x - 4$

Now root is approximately 2 ∴, start with 2

$$f(2) = e^2 - 8 = 7.389056 - 8$$
$$= -0.61094 \quad (-ve)$$

$$f(3) = e^3 - 12 = 20.0855 - 12$$
$$= 8.08554 \quad (+ve)$$

Hence roots lies in between 2 & 3.

Now: $|f(2)| < |f(3)|$

∴ root is nearer to $x_0 = 2$. (Let $x_0 = 2.1$)

N-R formula $\boxed{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}}$

$$x_{n+1} = x_n - \left(\frac{e^{x_n} - 4x_n}{e^{x_n} - 4}\right)$$

1st approximation :- put $n = 0$

$$x_1 = x_0 - \left(\frac{e^{x_0} - 4x_0}{e^{x_0} - 4}\right) = 2.1 - \frac{e^{2.1} - 4(2.1)}{e^{2.1} - 4}$$

$$= 2.1561$$

2nd approximation :- put n = 1 & $x_1 = 2.1561$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.1561 - \frac{e^{2.1561} - (4 \times 2.1561)}{e^{2.1561} - 4}$$

$$\boxed{x_2 = 2.1533}$$

3rd approximation :- put n = 2 & $x_2 = 2.1533$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.1533 - \frac{(e^{2.1533} - 4 \times 2.153}{(e^{2.1533} - 4)}$$

$$\boxed{x_3 = 2.1533}$$

$\therefore$ $x_2 = x_3$ $\therefore$ root of f(x) is

$$\boxed{Ans :: x_3 = 2.1533}$$

Table

Ituation	x_{n+1}	$f(x_{n+1})$
1	2.1561	0.01299
2	2.1533	0.000035
3	2.1533	0.000035

↓ same

$\therefore$ root is $\underline{2.1533}$ Ans.

P|1.31

Q:– find the root of $\cos x = x e^x$ by N-R Method by taking $x_0 = 0.5$