INVERSE OF A MATRIX

Gauss - Jordan Method

Let us consider a 3×3 non singular matrix A. If the matrix X is the inverse of A, then AX = I

i.e., $$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This matrix equation gives nine equations which are equivalent to the three system of equations.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{21} \\ x_{31} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{12} \\ x_{22} \\ x_{32} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{13} \\ x_{23} \\ x_{33} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

By using Gauss Jordan elimination method we can find the unknowns $x_{11}, x_{21}, x_{31}, \ldots$, etc., which gives the inverse of the given matrix. The method is explained in the following example.

Example 1

Find the inverse of the matrix $\begin{pmatrix} 5 & -2 \\ 3 & 4 \end{pmatrix}$ *by Gauss – Jordan method.* *[A.U. Nov. '96]*

Solution

Let $$A = \begin{pmatrix} 5 & -2 \\ 3 & 4 \end{pmatrix}$$

and $$X = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$$ be the inverse of A.

so that, $$AX = I$$

Step 1: **Write the augmented system**

$$\left[\begin{array}{cccc} 5 & -2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$$

Step 2 : $R_1 \to R_1 \div 5$

$$\begin{bmatrix} 1 & \frac{-2}{5} & \frac{1}{5} & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix}$$

Step 3 : $R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 1 & \frac{-2}{5} & \frac{1}{5} & 0 \\ 0 & \frac{26}{5} & \frac{-3}{5} & 1 \end{bmatrix}$$

Step 4 : $R_2 \to R_2 \div \frac{26}{5}$

$$\begin{bmatrix} 1 & \frac{-2}{5} & \frac{1}{5} & 0 \\ 0 & 1 & \frac{-3}{26} & \frac{5}{26} \end{bmatrix}$$

Step 5 : $R_1 \to R_1 + \left(\frac{2}{5}\right) R_2$

$$\begin{bmatrix} 1 & 0 & \frac{2}{13} & \frac{1}{13} \\ 0 & 1 & \frac{-3}{26} & \frac{5}{26} \end{bmatrix}$$

Hence the inverse of the given matrix is

$$\begin{bmatrix} \frac{2}{13} & \frac{1}{13} \\ \frac{-3}{26} & \frac{5}{26} \end{bmatrix} = \frac{1}{26}\begin{bmatrix} 4 & 2 \\ -3 & 5 \end{bmatrix}$$

Example 2

Using Gauss – Jordan method find the inverse of the matrix.

$$\begin{pmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{pmatrix}$$

[A.U. Apr./May '04]

Solution

Let $A = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$

and $$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$$ be the inverse of A.

so that, $AX = I$

Step 1 : **Write the augmented system.**

$$\left[\begin{array}{cccccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 3 & 5 & 0 & 0 & 1 \end{array}\right]$$

Step 2 : $R_2 \to R_2 - R_1, \quad R_3 \to 2R_3 - R_1$

$$\left[\begin{array}{cccccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 1 & -1 & -2 & -1 & 1 & 0 \\ 0 & 4 & 7 & -1 & 0 & 2 \end{array}\right]$$

Step 3 : $R_3 \to R_3 + 4R_2$

$$\left[\begin{array}{cccccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 1 & 0 \\ 0 & 0 & -1 & -5 & 4 & 2 \end{array}\right]$$

Step 4 : $R_2 \to R_2 - 2R_3$

$$\left[\begin{array}{cccccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 0 & 9 & -7 & -4 \\ 0 & 0 & -1 & -5 & 4 & 2 \end{array}\right]$$

Step 5 : $R_1 \to R_1 + 2R_2$

$$\left[\begin{array}{cccccc} 2 & 0 & 3 & 19 & -14 & -8 \\ 0 & -1 & 0 & 9 & -7 & -4 \\ 0 & 0 & -1 & -5 & 4 & 2 \end{array}\right]$$

Step 6 : $R_1 \to R_1 + 3R_3$

$$\left[\begin{array}{cccccc} 2 & 0 & 0 & 4 & -2 & -2 \\ 0 & -1 & 0 & 9 & -7 & -4 \\ 0 & 0 & -1 & -5 & 4 & 2 \end{array}\right]$$

Step 7 : $R_1 \to R_1 \div 2, \; R_2 \to R_2 \div (-1), \; R_3 \to R_3 \div (-1)$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -9 & 7 & 4 \\ 0 & 0 & 1 & 5 & -4 & -2 \end{array}\right]$$

Hence the inverse of the given matrix is

$$\begin{bmatrix} 2 & -1 & -1 \\ -9 & 7 & 4 \\ 5 & -4 & -2 \end{bmatrix}$$

Example 3

Find the inverse of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ *using Gauss – Jordan method.* *[A.U. Nov. '04, Apr '00, Oct. '96]*

Solution

Let $$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

and $$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$$ be the inverse of A.

so that, $AX = I$

Step 1 : **Write the augmented system.**

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{bmatrix}$$

Step 2 : $R_2 \to R_2 - R_1, \quad R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{bmatrix}$$

Step 3 : $R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{bmatrix}$$

Step 4 : $R_2 \to R_2 \div 2,\ R_3 \to R_3 \div -4$

$$\left[\begin{array}{cccccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & \frac{-1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \end{array}\right]$$

Step 5 : $R_1 \to R_1 + R_2$

$$\left[\begin{array}{cccccc} 1 & 2 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -3 & \frac{-1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \end{array}\right]$$

Step 6 : $R_2 \to R_2 + 3R_3$

$$\left[\begin{array}{cccccc} 1 & 2 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{-5}{4} & \frac{-1}{4} & \frac{-3}{4} \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \end{array}\right]$$

Step 7 : $R_1 \to R_1 - 2R_2$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 3 & 1 & \frac{3}{2} \\ 0 & 1 & 0 & \frac{-5}{4} & \frac{-1}{4} & \frac{-3}{4} \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \end{array}\right]$$

Hence the inverse of the given matrix is

$$\left[\begin{array}{ccc} 3 & 1 & \frac{3}{2} \\ \frac{-5}{4} & \frac{-1}{4} & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \end{array}\right]$$

Example 4

Find the inverse of $\begin{pmatrix} 2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & -8 \end{pmatrix}$ *using Gauss – Jordan method.* *[A.U. Mar. '96]*

Solution

Let
$$A = \begin{pmatrix} 2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & -8 \end{pmatrix}$$

and
$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$$ be the inverse of A.

so that,
$$AX = I$$

Step 1 : **Write the augmented system.**

$$\left[\begin{array}{cccccc} 2 & 2 & 6 & 1 & 0 & 0 \\ 2 & 6 & -6 & 0 & 1 & 0 \\ 4 & -8 & -8 & 0 & 0 & 1 \end{array}\right]$$

Step 2 : $R_2 \to R_2 - R_1, \quad R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{cccccc} 2 & 2 & 6 & 1 & 0 & 0 \\ 0 & 4 & -12 & -1 & 1 & 0 \\ 0 & -12 & -20 & -2 & 0 & 1 \end{array}\right]$$

Step 3 : $R_3 \to R_3 + 3R_2$

$$\left[\begin{array}{cccccc} 2 & 2 & 6 & 1 & 0 & 0 \\ 0 & 4 & -12 & -1 & 1 & 0 \\ 0 & 0 & -56 & -5 & 3 & 1 \end{array}\right]$$

Step 4 : $R_3 \to R_3 \div (-56)\,,\ R_2 \to R_2 \div 4\,,\ R_1 \to R_1 \div 2$

$$\left[\begin{array}{cccccc} 1 & 1 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -3 & \frac{-1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{5}{56} & \frac{-3}{56} & \frac{-1}{56} \end{array}\right]$$

Step 5 : $R_1 \to R_1 - 3R_3$, $R_2 \to R_2 + 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & \frac{13}{56} & \frac{9}{56} & \frac{3}{56} \\ 0 & 1 & 0 & \frac{1}{56} & \frac{5}{56} & \frac{-3}{56} \\ 0 & 0 & 1 & \frac{5}{56} & \frac{-3}{56} & \frac{-1}{56} \end{array}\right]$$

Step 6 : $R_1 \to R_1 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{12}{56} & \frac{4}{56} & \frac{6}{56} \\ 0 & 1 & 0 & \frac{1}{56} & \frac{5}{56} & \frac{-3}{56} \\ 0 & 0 & 1 & \frac{5}{56} & \frac{-3}{56} & \frac{-1}{56} \end{array}\right]$$

Hence the inverse of the given matrix is

$$\begin{bmatrix} \frac{12}{56} & \frac{4}{56} & \frac{6}{56} \\ \frac{1}{56} & \frac{5}{56} & \frac{-3}{56} \\ \frac{5}{56} & \frac{-3}{56} & \frac{-1}{56} \end{bmatrix} = \frac{1}{56}\begin{pmatrix} 12 & 4 & 6 \\ 1 & 5 & -3 \\ 5 & -3 & -1 \end{pmatrix}$$

Example 5

Find the inverse of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$ *using Gauss – Jordan method.* *[A.U. Nov. '96]*

Solution

Let $$A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$$

and $$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$$ be the inverse of A

$$AX = I$$

so that,

Step 1: Write the augmented system

$$\left(\begin{array}{cccccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 0 & -6 & -7 & 0 & 0 & 1 \end{array}\right)$$

Step 2: $R_2 \rightarrow R_2 - 3R_1$

$$\left(\begin{array}{cccccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -3 & 1 & 0 \\ 0 & -6 & -7 & 0 & 0 & 1 \end{array}\right)$$

Step 3: $R_3 \rightarrow R_3 \div -6$

$$\left(\begin{array}{cccccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -3 & 1 & 0 \\ 0 & 1 & \frac{7}{6} & 0 & 0 & \frac{-1}{6} \end{array}\right)$$

Step 4: $R_2 \sim R_3$ (Interchange R_2 and R_3.)

$$\left(\begin{array}{cccccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & \frac{7}{6} & 0 & 0 & \frac{-1}{6} \\ 0 & 0 & 8 & -3 & 1 & 0 \end{array}\right)$$

Step 5: $R_1 \rightarrow \frac{R_3}{8}$

$$\left(\begin{array}{cccccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & \frac{7}{6} & 0 & 0 & \frac{-1}{6} \\ 0 & 0 & 1 & \frac{-3}{8} & \frac{1}{8} & 0 \end{array}\right)$$

Step 6: $R_2 \rightarrow R_2 - \frac{7}{6} R_3$

$$\left(\begin{array}{cccccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{7}{16} & \frac{-7}{48} & \frac{-1}{6} \\ 0 & 0 & 1 & \frac{-3}{8} & \frac{1}{8} & 0 \end{array}\right)$$

Step 7 : $R_1 \rightarrow R_1 + R_3$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{8} & \frac{1}{8} & 0 \\ 0 & 1 & 0 & \frac{7}{16} & \frac{-7}{48} & \frac{-1}{6} \\ 0 & 0 & 1 & \frac{-3}{8} & \frac{1}{8} & 0 \end{pmatrix}$$

Hence the inverse of the given matrix is

$$\begin{pmatrix} \frac{5}{8} & \frac{1}{8} & 0 \\ \frac{7}{16} & \frac{-7}{48} & \frac{-1}{6} \\ \frac{-3}{8} & \frac{1}{8} & 0 \end{pmatrix}$$

Example 6

Find the inverse of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ *using Gauss Jordan method.*

Solution

Let $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$

and $X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$ be the inverse of A

so that, $AX = I$

Step 1 : Write the augmented system

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{pmatrix}$$

Step 2 : Add multiples of first row to the other rows so that the elements in the second row first column and the third row first column become zero.

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{-3}{2} & 1 & 0 \\ 0 & \frac{7}{2} & \frac{17}{2} & \frac{-1}{2} & 0 & 1 \end{pmatrix} \quad \begin{array}{l} R_2 \to R_2 - \frac{3}{2} R_1 \\ R_3 \to R_3 - \frac{1}{2} R_1 \end{array}$$

Step 3 : Make the element in the third row and the second column as zero by using the element in the second row and the second column.

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{-3}{2} & 1 & 0 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{pmatrix} \quad R_3 \to R_3 - 7 R_2$$

Step 4 : Make the element in the second row and the third column zero by using the element in the third row and the third column.

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 6 & \frac{-17}{4} & \frac{3}{4} \\ 0 & 0 & -2 & 10 & -7 & 1 \end{pmatrix} \quad R_2 \to R_2 + \frac{3}{4} R_3$$

Step 5 : Make the element in the first row and third column to zero by using the element in the third row and third column.

$$\begin{pmatrix} 2 & 1 & 0 & 6 & \frac{-7}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 6 & \frac{-17}{4} & \frac{3}{4} \\ 0 & 0 & -2 & 10 & -7 & 1 \end{pmatrix} \quad R_1 \to R_1 + \frac{1}{2} R_3$$

Step 6 : Make the element in the first row and second column to zero by using the element in the second row and second column.

$$\left(\begin{array}{cccccc} 2 & 0 & 0 & -6 & 5 & -1 \\ 0 & \frac{1}{2} & 0 & 6 & \frac{-17}{4} & \frac{3}{4} \\ 0 & 0 & -2 & 10 & -7 & 1 \end{array}\right) \quad R_1 \to R_1 - 2R_2$$

Step 7 :

$$\left(\begin{array}{cccccc} 1 & 0 & 0 & -3 & \frac{5}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & 12 & \frac{-17}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -5 & \frac{7}{2} & \frac{-1}{2} \end{array}\right) \quad \begin{array}{l} R_1 \to \frac{R_1}{2} \\ R_2 \to R_2 \times 2 \\ R_3 \to R_3\left(\frac{-1}{2}\right) \end{array}$$

Hence the inverse of the given matrix is

$$\left(\begin{array}{ccc} -3 & \frac{5}{2} & \frac{-1}{2} \\ 12 & \frac{-17}{2} & \frac{3}{2} \\ -5 & \frac{7}{2} & \frac{-1}{2} \end{array}\right)$$

Example 7

Find the inverse of the matrix $\left(\begin{array}{ccc} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{array}\right)$ ***using Gauss Jordan method.***

Solution

Let $$A = \left(\begin{array}{ccc} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{array}\right)$$

and $$X = \left(\begin{array}{ccc} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{array}\right)$$ be the inverse of A.

so that, $$AX = I$$

Step 1 : Write the augmented system

$$\begin{pmatrix} 8 & -4 & 0 & 1 & 0 & 0 \\ -4 & 8 & -4 & 0 & 1 & 0 \\ 0 & -4 & 8 & 0 & 0 & 1 \end{pmatrix}$$

Step 2 : Performing $R_1 \rightarrow R_1 \div 8$, we get

$$\begin{pmatrix} 1 & -\frac{1}{2} & 0 & \frac{1}{8} & 0 & 0 \\ -4 & 8 & -4 & 0 & 1 & 0 \\ 0 & -4 & 8 & 0 & 0 & 1 \end{pmatrix}$$

Step 3 : Performing $R_2 \rightarrow R_2 + 4R_1$, we get

$$\begin{pmatrix} 1 & -\frac{1}{2} & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 6 & -4 & \frac{1}{2} & 1 & 0 \\ 0 & -4 & 8 & 0 & 0 & 1 \end{pmatrix}$$

Step 4 : Performing $R_2 \rightarrow R_2 \div 6$, we get

$$\begin{pmatrix} 1 & -\frac{1}{2} & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 1 & -\frac{2}{3} & \frac{1}{12} & \frac{1}{6} & 0 \\ 0 & -4 & 8 & 0 & 0 & 1 \end{pmatrix}$$

Step 5 : Performing $R_3 \rightarrow R_3 + 4R_2$, we get

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{1}{6} & \frac{1}{12} & 0 \\ 0 & 1 & -\frac{2}{3} & \frac{1}{12} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{16}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}$$

Step 6 : Performing $R_3 \to R_3 + \frac{16}{3}$, we get

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{1}{6} & \frac{1}{12} & 0 \\ 0 & 1 & -\frac{2}{3} & \frac{1}{12} & \frac{1}{6} & 0 \\ 0 & 0 & 1 & \frac{1}{16} & \frac{1}{8} & \frac{3}{16} \end{pmatrix}$$

Step 7 : Performing $R_1 \to R_1 + \frac{R_3}{3}$

and $R_2 \to R_2 + \frac{2}{3} R_3$, we get

$$\begin{pmatrix} 1 & 0 & 0 & \frac{3}{6} & \frac{1}{8} & \frac{1}{16} \\ 0 & 1 & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ 0 & 0 & 1 & \frac{1}{16} & \frac{1}{8} & \frac{3}{16} \end{pmatrix}$$

Hence the inverse of the given matrix is

$$\begin{pmatrix} \frac{3}{6} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{16} & \frac{1}{8} & \frac{3}{16} \end{pmatrix}$$

EXERCISES

Find the inverse of the following matrices using Gau: Jordan method.

1. $\begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$ *[Ans.* $\begin{pmatrix} \frac{1}{10} & \frac{3}{10} & \frac{1}{5} \\ \frac{21}{10} & \frac{-7}{20} & \frac{-2}{5} \\ \frac{-9}{10} & \frac{3}{10} & \frac{1}{5} \end{pmatrix}$ *]*

2. $\begin{pmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix}$ [*Ans.* $\begin{pmatrix} \frac{3}{4} & \frac{-10}{4} & \frac{-1}{4} \\ \frac{-1}{2} & 2 & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{-1}{2} \end{pmatrix}$]

3. $\begin{pmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{pmatrix}$ [*Ans.* $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{-1}{4} \\ \frac{-9}{8} & \frac{11}{8} & \frac{5}{8} \\ \frac{5}{8} & \frac{-7}{8} & \frac{-1}{8} \end{pmatrix}$]

4. $\begin{pmatrix} 1 & 3 & 4 \\ 3 & -1 & 6 \\ -1 & 5 & 1 \end{pmatrix}$ [*Ans.* $\begin{pmatrix} \frac{31}{2} & \frac{-17}{2} & -11 \\ \frac{9}{2} & \frac{-5}{2} & -3 \\ -7 & 4 & 5 \end{pmatrix}$]

5. $\begin{pmatrix} 3 & 1 & 2 \\ 2 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ [*Ans.* $\begin{pmatrix} \frac{1}{4} & \frac{-3}{4} & \frac{7}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{5}{4} \\ \frac{1}{4} & \frac{5}{4} & \frac{-13}{4} \end{pmatrix}$]

6. $\begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ [*Ans.* $\begin{pmatrix} \frac{-4}{35} & \frac{11}{35} & \frac{-1}{7} \\ \frac{-1}{35} & \frac{-6}{35} & \frac{5}{7} \\ \frac{6}{35} & \frac{1}{35} & \frac{-2}{7} \end{pmatrix}$]