Module-2

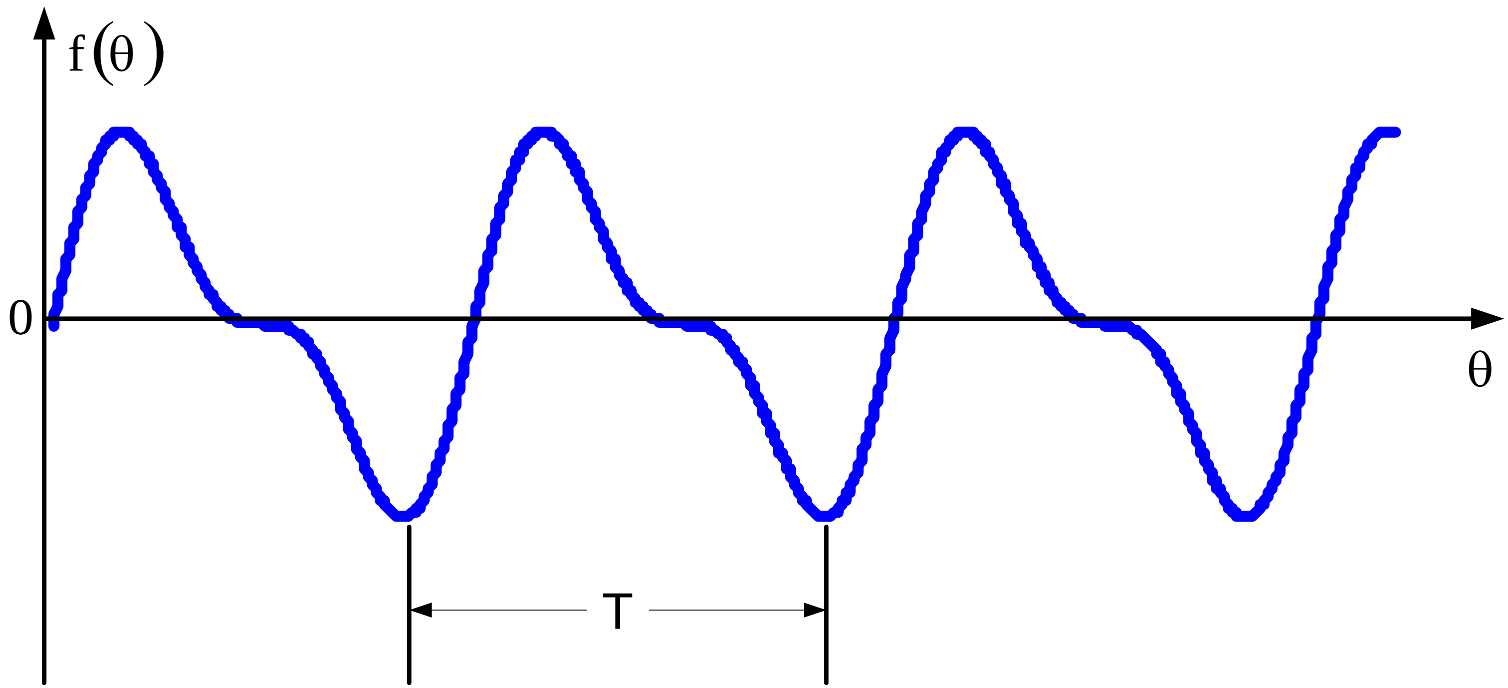
Fourier Series

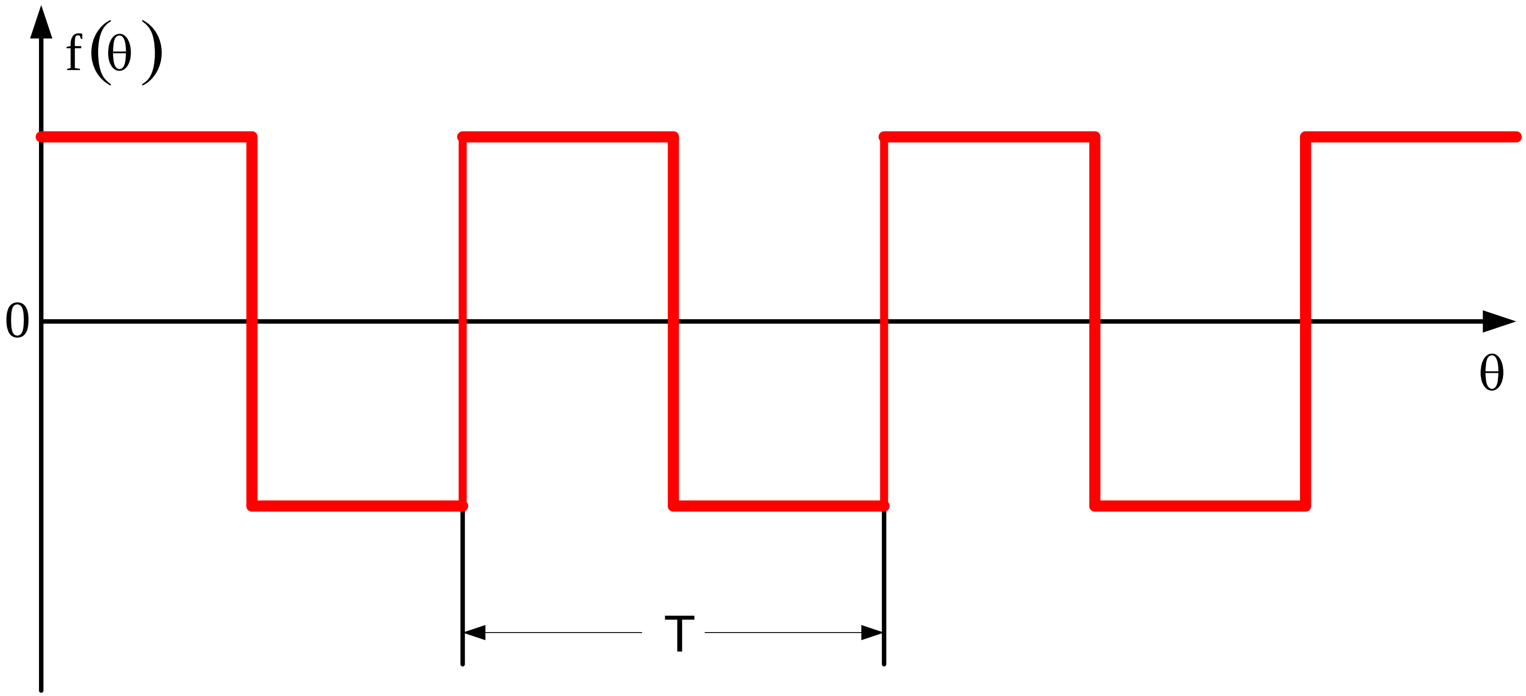
The Mathematic Formulation

- Any function that satisfies

$$f(t) = f(t + T)$$

where *T* is a constant and is called the *period* of the function.


f(θ)
0
θ
T


f(θ)
0
θ
T

Result

Let m and n be integers, $m \neq 0$, $n \neq 0$ for $m \neq$

1. $$\int_{\alpha}^{\alpha+2\pi} \cos mx \cdot \cos nx \, dx = 0$$

2. $$\int_{\alpha}^{\alpha+2\pi} \sin mx \cdot \cos nx \, dx = 0$$

3. $$\int_{\alpha}^{\alpha+2\pi} \sin mx \cdot \cos nx \, dx = 0$$

4. $$\int_{\alpha}^{\alpha+2\pi} \cos mx \, dx = 0$$

5. $$\int^{\alpha+2\pi} \sin mx \, dx = 0$$

for $m = n$

1. $$\int_{\alpha}^{\alpha+2\pi} \cos mx \cdot \cos nx\,dx = \int_{\alpha}^{\alpha+2\pi} \cos^2 mx\,dx = \pi$$

2. $$\int_{\alpha}^{\alpha+2\pi} \sin^2 mx\,dx = \pi$$

3. $$\int_{\alpha}^{\alpha+2\pi} \cos mx \cdot \sin mx\,dx = 0$$

9.3 FOURIER SERIES

The Fourier series is an infinite series which is represented in terms of the trigonometric sine an[d cosine]
functions of the form

$$f(x) = \frac{a_0}{2} + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + a_3 \cos 3x + b_3 \sin 3x$$

or $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos nx + b_n \sin n,x \right]$$

where the constants a_0, a_n, and b_n are called Fourier coefficients.

(i) To Find the Coefficient a_0

Integrate both sides of (1) with respect to x in the interval α to $\alpha+2\pi$. Then,

$$\int_{\alpha}^{\alpha+2\pi} f(x)dx = \int_{\alpha}^{\alpha+2\pi} \frac{a_0}{2}dx + \int_{\alpha}^{\alpha+2\pi}\left[\sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)\right]dx$$

$$= \frac{a_0}{2}[x]_{\alpha}^{\alpha+2\pi} + \sum_{n=1}^{\infty}\left[a_n \int_{\alpha}^{\alpha+2\pi} \cos nx\, dx + b_n \int_{\alpha}^{\alpha+2\pi} \sin nx\, dx\right]$$

$$\int_{\alpha}^{\alpha+2\pi} f(x)dx = \frac{a_0}{2}\cdot 2\pi + 0 = a_0 \cdot \pi$$

[Using results (4) and (5), the last two integrals for all n will be zerc

Hence,
$$a_0 = \frac{1}{\pi}\int_{\alpha}^{\alpha+2\pi} f(x)dx \qquad (\text{:}$$

(ii) To Find the Coefficient a_n for $n = 1, 2, 3, \ldots$

Multiplying both sides of (1) by $\cos nx$ and integrating w. r. t. 'x' in the interval $(\alpha, \alpha+2\pi)$, we get

$$\int_{\alpha}^{\alpha+2\pi} f(x)\cos nx = \int_{\alpha}^{\alpha+2\pi} \frac{a_0}{2}\cos nx\cdot dx + \int_{\alpha}^{\alpha+2\pi}\left[\sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right)\right]\cos nx\,dx$$

$$= \int_{\alpha}^{\alpha+2\pi} \frac{a_0}{2}\cos nx\,dx + \sum_{n=1}^{\infty}\left[\int_{\alpha}^{\alpha+2\pi} a_n\cos^2 nx\,dx + \int_{\alpha}^{\alpha+2\pi} b_n\sin nx.\cos nx\,dx\right]$$

$$= \frac{a_0}{2}\int_{\alpha}^{\alpha+2\pi}\cos nx\,dx + \sum_{n=1}^{\infty}\left[\int_{\alpha}^{\alpha+2\pi} a_n\cos^2 nx\,dx\right] + \sum_{n=1}^{\infty}\left[\int_{\alpha}^{\alpha+2\pi} b_n\cos nx\cdot\sin nx

Similarly, $$b_n = \frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} f(x) \sin nx \, dx$$

The formulae (2), (3), and (4) are known as *Euler formulae.*

9.5 DIRICHLET'S CONDITIONS FOR A FOURIER SERIES

The sufficient conditions for the uniform convergence of a Fourier series are called Dirichlet's condition (after Dirichlet, a German mathematician).

All the functions that normally arise in engineering problems satisfy these conditions and, hence they can be expressed as a Fourier series.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$$ where a_0, a_n, and b_n are Fourier coefficients,

Provided:

(i) Function $f(x)$ is periodic, single valued and finite.

(ii) Function $f(x)$ has a finite number of discontinuities in any one period.

(iii) Function $f(x)$ has at the most of finite number of maxima and minima.

When these conditions are satisfied, the Fourier series converges to $f(x)$ at every point of continuity.

At a point of discontinuity, the sum of the series is equal to the mean of the right- and left-hand limits, i.e., $\frac{1}{2}[f(x+0)+f(x-0)]$, where $f(x+0)$ and $f(x-0)$ denote the limit on the right and the limit on the left respectively.

9.6 FOURIER SERIES FOR DISCONTINUOUS FUNCTIONS

In Article 9.4, we derived Euler's formula for a_0, a_n, and b_n on the assumption that $f(x)$ in conti in $(\alpha, \alpha + 2\pi)$.

However, if $f(x)$ has finitely many points of finite discontinuity, even then it can be expr as a Fourier series. The integrals for a_0, a_n, and b_n are to be evaluated by breaking up the ran integration.

Let $f(x)$ be defined by $f(x) = f_1(x); \quad \alpha < x < x_0$

$$= f_2(x); x_0 < x < \alpha + 2\pi$$

where x_0 is the point of finite discontinuity in the interval $(\alpha, \alpha + 2\pi)$.

The values of a_0, a_n, and b_n are given by

$$a_0 = \frac{1}{\pi}\left[\int_{\alpha}^{x_0} f_1(x)\,dx + \int_{x_0}^{\alpha+2\pi} f_2(x)\,dx\right]$$

$$a_n = \frac{1}{\pi}\left[\int_{\alpha}^{x_0} f_1(x)\cdot\cos nx\,dx + \int_{x_0}^{\alpha+2\pi} f_2(x)\cos nx\,dx\right]$$

$$b_n = \frac{1}{\pi}\left[\int_{\alpha}^{x_0} f_1(x)\sin nx\,dx + \int_{x_0}^{\alpha+2\pi} f_2(x)\cdot\sin nx\,dx\right]$$

At $x = x_0$, there is a finite jump in the graph of the function. Both the limits $f(x_0 - 0)$ and $f(x_0 + 0)$ exist but are unequal. The sum of the Fourier series

$$= \frac{1}{2}\left[f(x_0 - 0) + f(x_0 + 0)\right]$$

Example 1 Find the Fourier series expansion for the periodic function $f(x) = x;\ 0 < x < 2\pi$

Solution Consider the Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \tag{1}$$

The Fourier coefficients a_0, a_n, b_n are obtained as follows:

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx = \frac{1}{\pi}\int_0^{2\pi} xdx = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} = 2\pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\, dx = \frac{1}{\pi}\int_0^{2\pi} x\cos nx\, dx = \frac{1}{\pi}\left[x\cdot\frac{\sin nx}{n} - \left(-\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos 2n\pi}{n^2} - \frac{1}{n^2}\right] = \frac{1}{n^2\pi}\left[(-1)^2 - 1\right] = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_0^{2\pi} x\sin nx\,dx = \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-\frac{2\pi\cos 2n\pi}{n}\right] = -\frac{2}{n}(-1)^{2n} \qquad [\because \cos 2n\pi = (-1)^{2}$$

$$= -\frac{2}{n}$$

Substituting the values of a_0, a_n and b_n in (1), we get

$$x = \frac{2\pi}{2} + \sum_{n=1}^{\infty}\left(-\frac{2}{n}\right)\sin nx = \pi - 2\left(\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right)$$

(i) Even Function

A function $f(x)$ is said to be an even function, if

$$f(-x) = f(x) \text{ for all } x$$

For example, x^4, $\cos x$, $\sec x$ are even functions.

Notes

(a) The graph of $f(x)$ is symmetric about the y-axis.

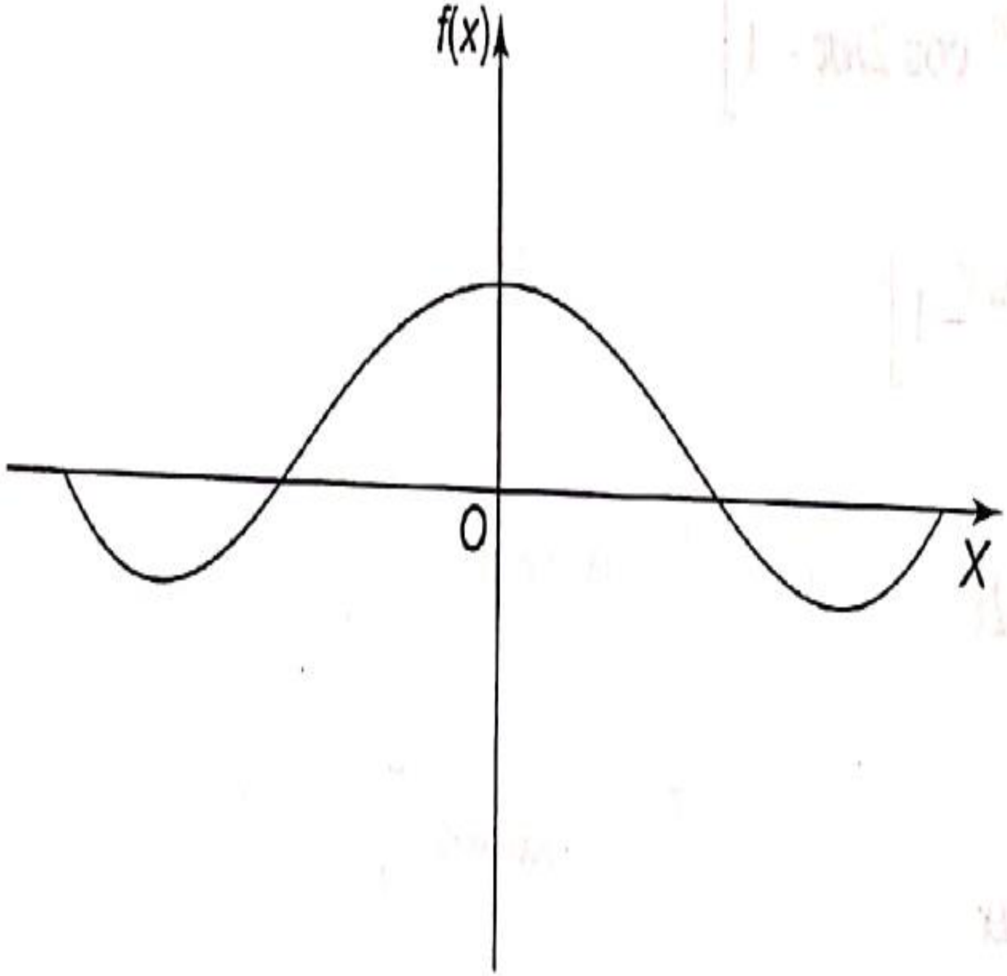
(b) $f(x)$ contains only even powers of x and may contain only $\cos x$, $\sec x$.

(c) The product of two even functions is an even function.

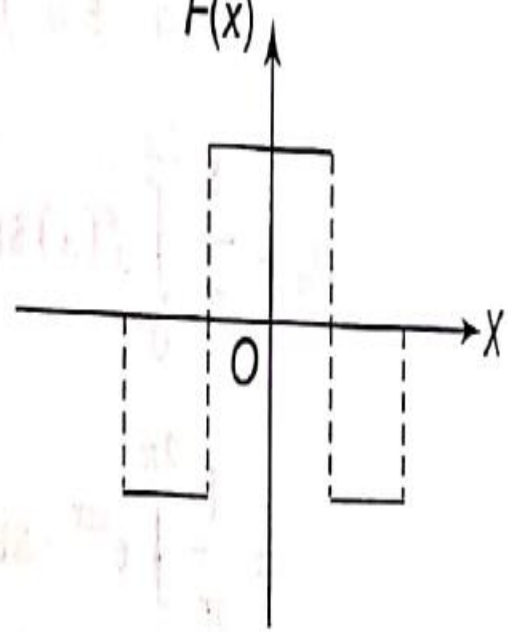
(d) The sum of two even functions is an even function.

(e) $\int_{-a}^{a} f(x)\, dx = 2\int_{0}^{a} f(x)\, dx$ if $f(-x) = f(x)$

Graphs of Even Functions



(a)



(b)

(ii) Odd Function

A function $f(x)$ is said to be an odd function if

$$f(-x) = -f(x) \text{ for all } x$$

For example, x, x^3, $\sin x$, $\tan x$ are odd functions.

Notes

(a) The graph of $f(x)$ is symmetric about the origin.

(b) $f(x)$ contains only odd powers of x and may contain only $\sin x$, $\operatorname{cosec} x$.

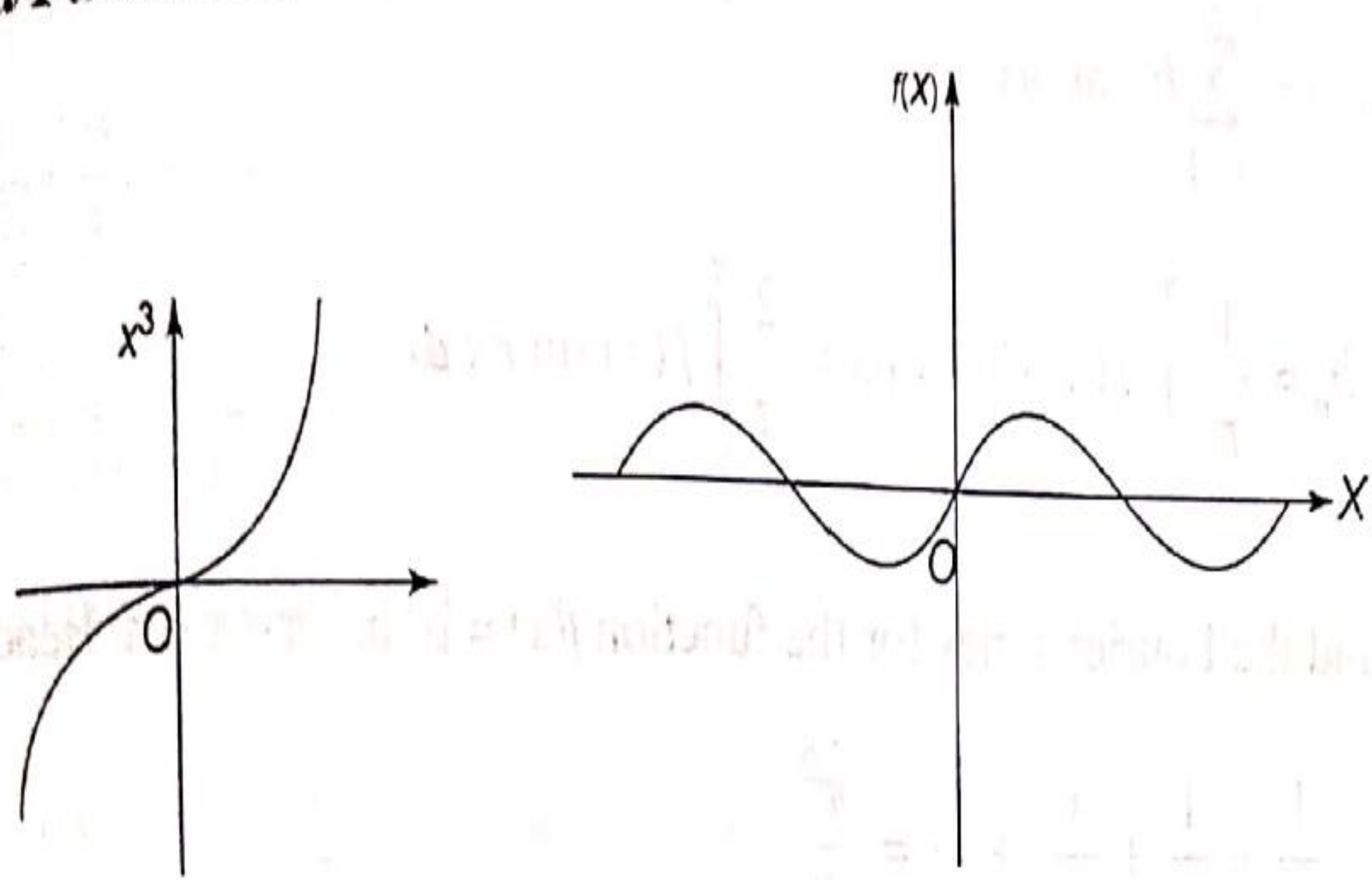
(c) The sum of two odd functions is odd.

(d) The product of an odd function and an even function is an odd function.

(e) Product of two odd functions is an even function.

(f) $\int_{-a}^{a} f(x)\,dx = 0$ if $f(-x) = -f(x)$

Graphs of Odd Functions



(a) (b)

(iii) Fourier Series for Even and Odd Functions

Let the Fourier series of $f(x)$ in $(-\pi, \pi)$ be

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos nx\, dx \quad \text{and}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin nx\, dx$$

Case I Consider $f(x)$ is an even function in $(-\pi, \pi)$. Then all b_n's will be zero. Thus, the Fourier series of an even function contains only cosine terms and is known as *Fourier cosine series* given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \tag{6}$$

where

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\,dx, \text{ and}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx\,dx$$

Case II

Case II Consider $f(x)$ is an odd function in $(-\pi, \pi)$, Then all a_n's will be zero. Also a_0 is zero since $f(x)$ is an odd function. Thus, the Fourier series of an odd function contains only sine terms and is known as *Fourier sine series* given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

where $$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx = \frac{2}{\pi}\int_{0}^{\pi} f(x) \sin nx \, dx$$

Example 7 Find the Fourier series for the function $f(x) = |x|$ in $-\pi < x < \pi$. Hence, show tha

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

Solution $f(x) = |x| = |-x| = f(-x)$

Therefore, $f(x)$ is an even function and, hence, $b_n = 0$,

Let the Fourier series

$$f(x) = |x| = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

Then, $$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} |x|\,dx = \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\left(\frac{x^2}{2}\right)^2$$

$$a_0 = \pi$$

and $$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} |x|\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$= \frac{2}{\pi}\left[x\left(\frac{\sin nx}{x}\right) - 1\cdot\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{1}{n^2}\right] = \frac{2}{\pi n^2}(\cos n\pi - 1)$$

$$= \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

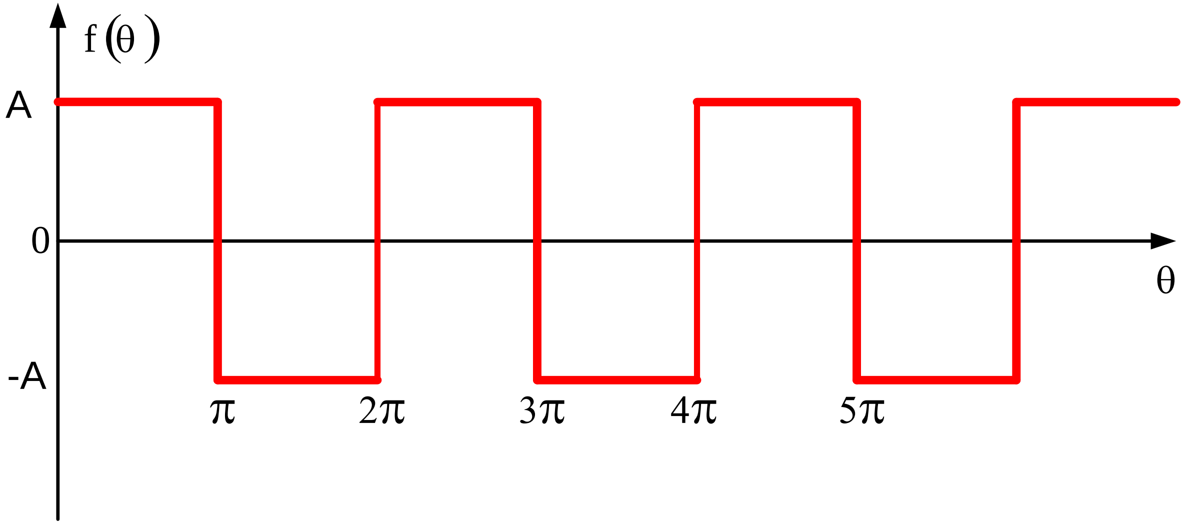
$$= \begin{cases} 0 & \text{if } n \text{ is even} \\ \dfrac{-4}{\pi n^2} & \text{if } n \text{ is odd} \end{cases}$$

$\therefore$ $$|x| = \frac{\pi}{2} - \frac{4}{\pi}\left[\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right]$$

Putting $x = 0$ in (2), we get

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

Example 1. Find the Fourier series of the following periodic function.

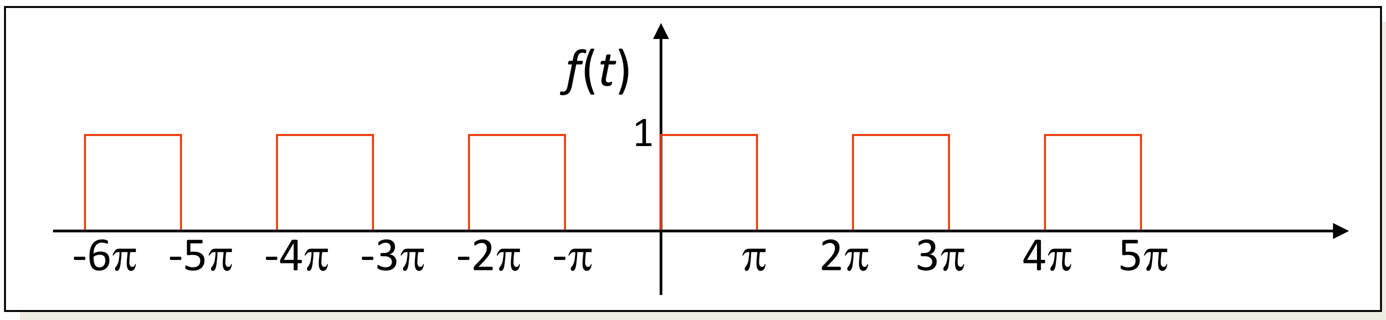


$$f(\theta) = A \quad \textbf{when} \quad 0 < \theta < \pi$$

$$= -A \quad \textbf{when} \quad \pi < \theta < 2\pi$$

$$f(\theta + 2\pi) = f(\theta)$$

Example (Square Wave)

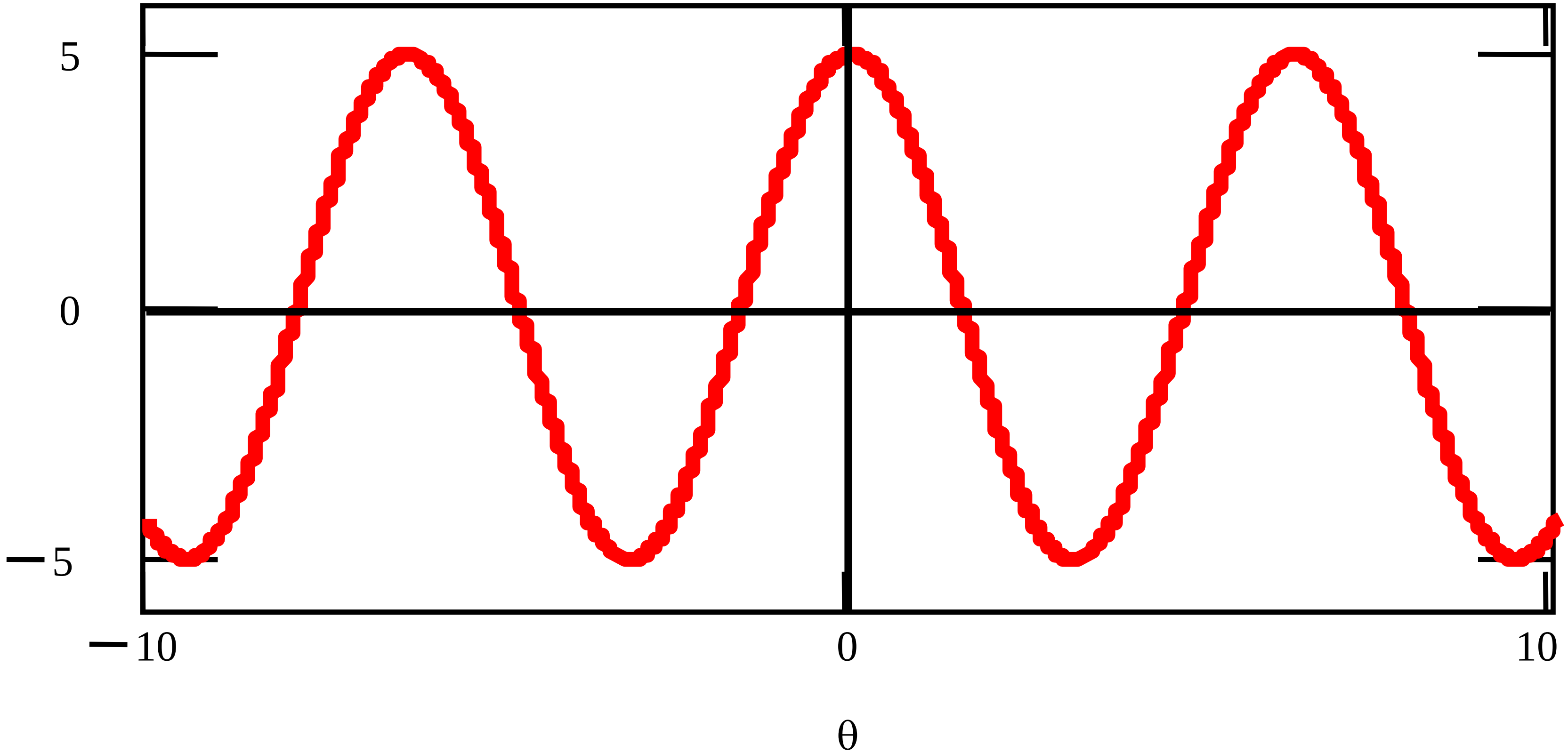


$$a_0 = \frac{2}{2\pi}\int_0^{\pi} 1 dt = 1$$

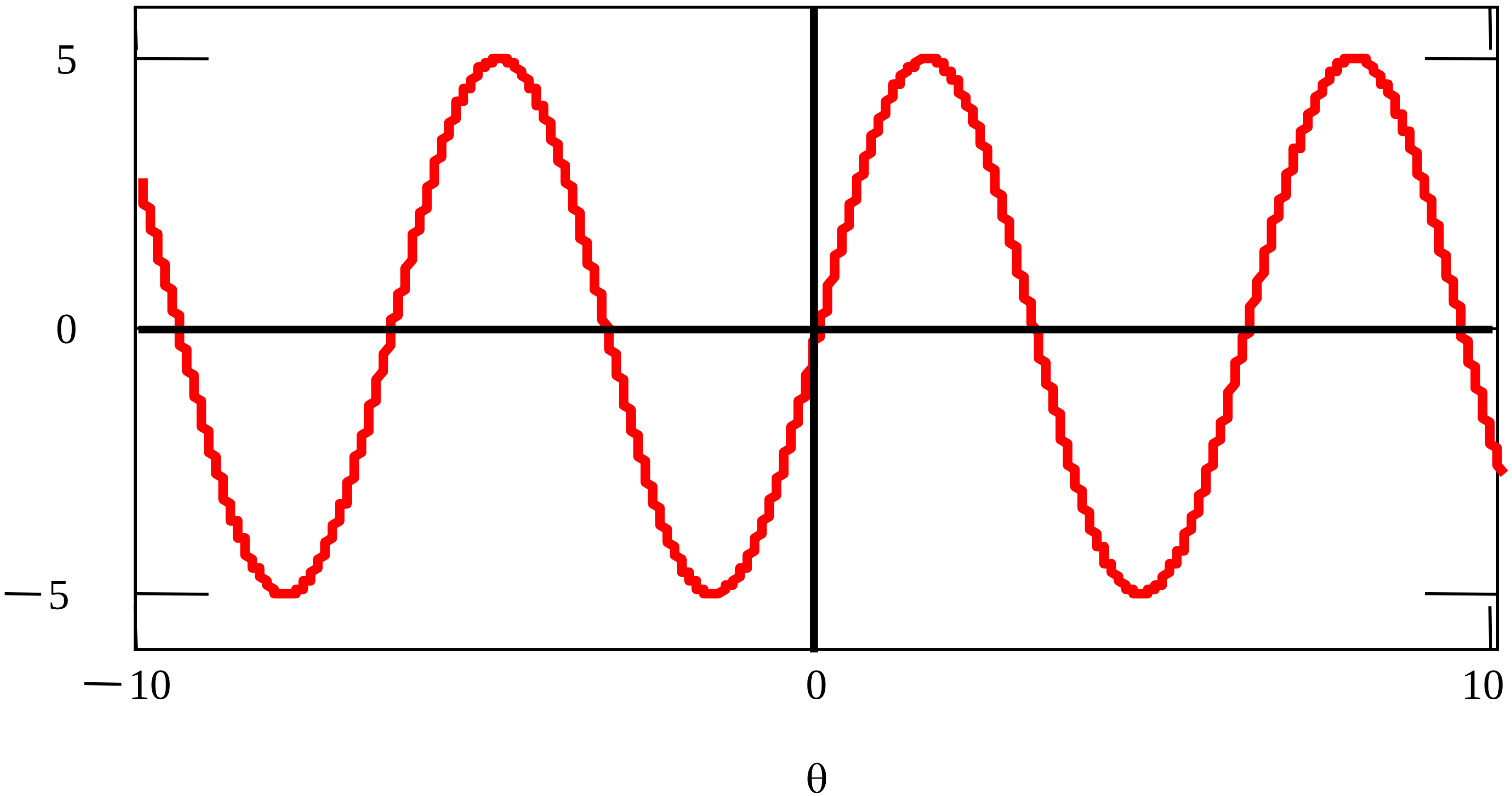
$$a_n = \frac{2}{2\pi}\int_0^{\pi} \cos nt dt = \frac{1}{n\pi}\sin nt\Big|_0^{\pi} = 0 \qquad n = 1,2,\cdots$$

$$b_n = \frac{2}{2\pi}\int_0^{\pi} \sin nt dt = -\frac{1}{n\pi}\cos nt\Big|_0^{\pi} = -\frac{1}{n\pi}(\cos n\pi - 1) = \begin{cases} 2/n\pi & n = 1,3,5,\cdots \\ 0 & n = 2,4,6,\cdots \end{cases}$$

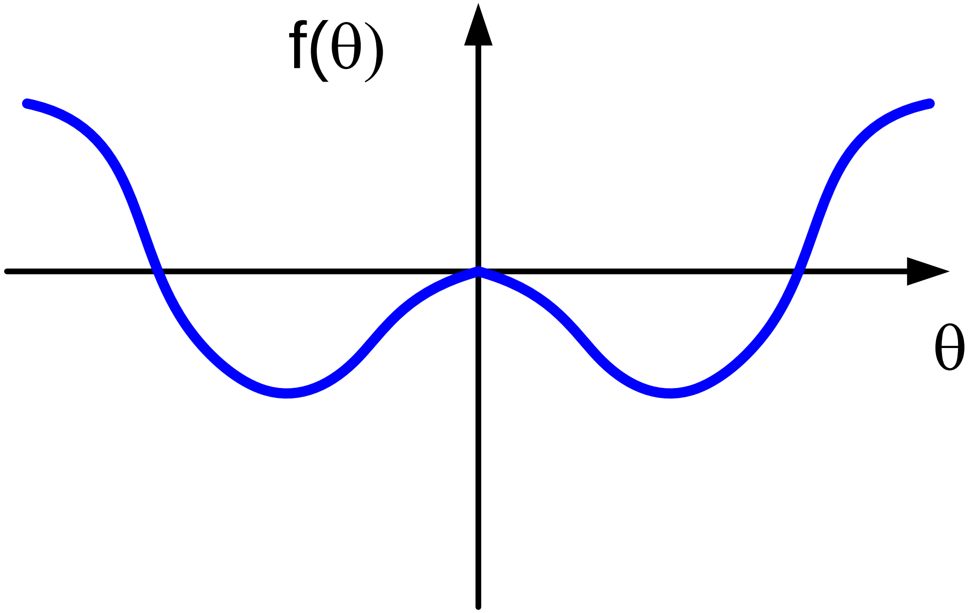
Even functions can solely be represented by cosine waves because, cosine waves are even functions. A sum of even functions is another even function.



Odd functions can solely be represented by sine waves because, sine waves are odd functions. A sum of odd functions is another odd function.



Even Functions

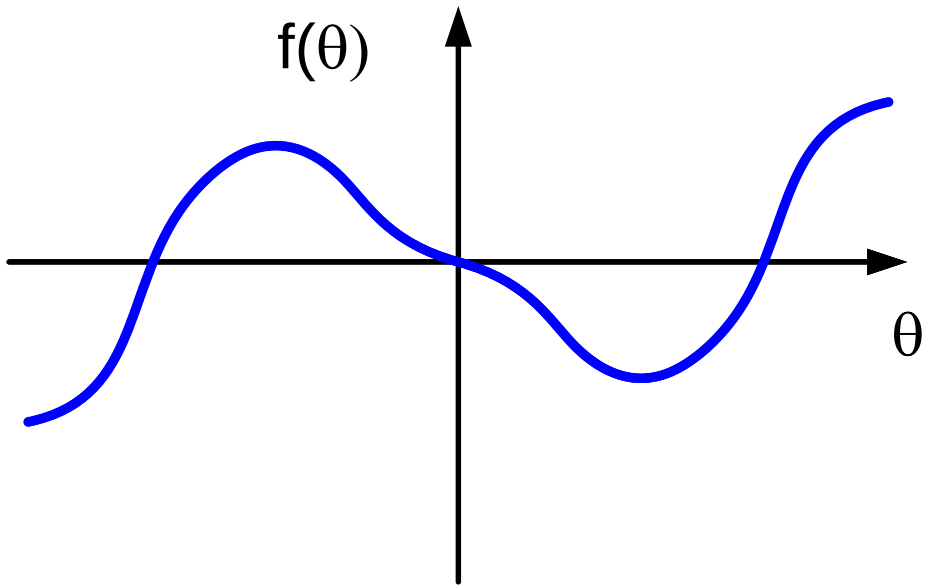


The value of the function would be the same when we walk equal distances along the X-axis in opposite directions.

Mathematically speaking -

$$f(-\theta) = f(\theta)$$

Odd Functions



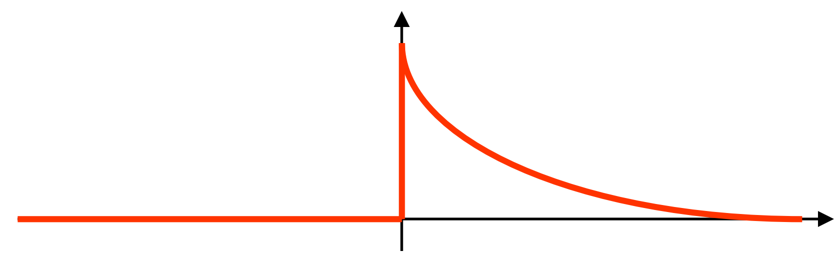
The value of the function would change its sign but with the same magnitude when we walk equal distances along the X-axis in opposite directions.

Mathematically speaking -

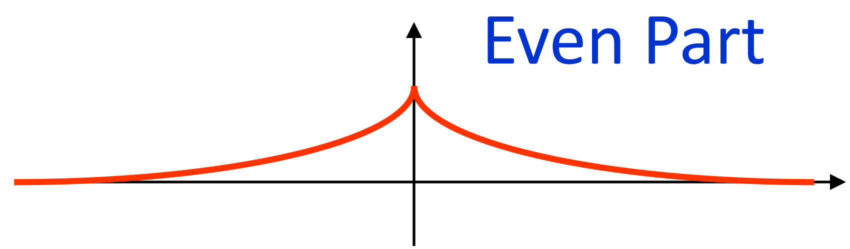
$$f(-\theta) = -f(\theta)$$

Example

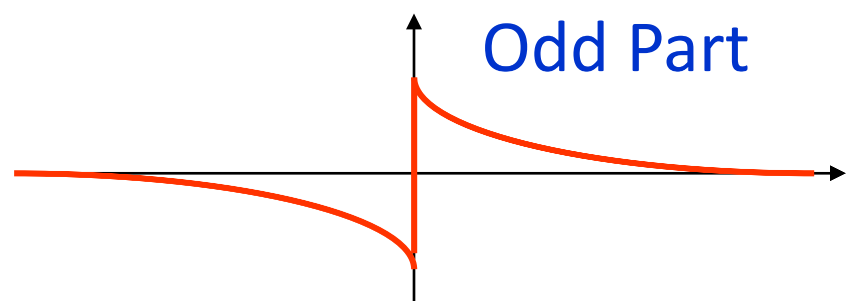
$$f(t)=\begin{cases} e^{-t} & t>0 \\ 0 & t<0 \end{cases}$$



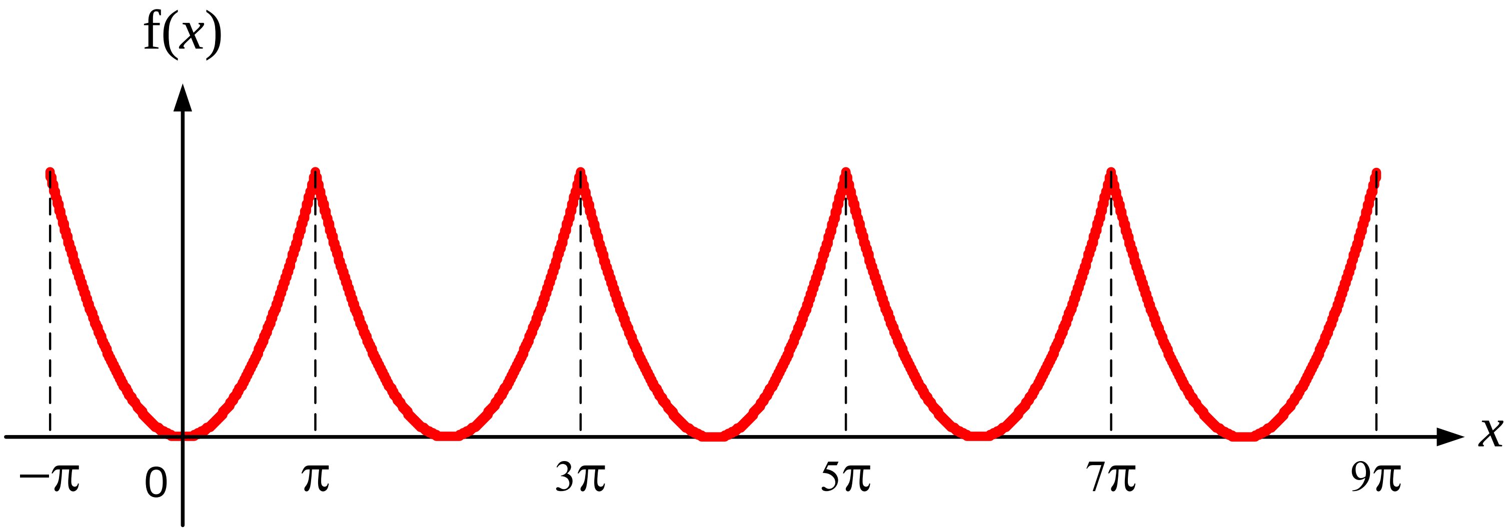
$$f_e(t)=\begin{cases} \frac{1}{2}e^{-t} & t>0 \\ \frac{1}{2}e^{t} & t<0 \end{cases}$$



$$f_o(t)=\begin{cases} \frac{1}{2}e^{-t} & t>0 \\ -\frac{1}{2}e^{t} & t<0 \end{cases}$$



Example 2. Find the Fourier series of the following periodic function.



$$f(x) = x^2 \quad when \quad -\pi \le x \le \pi$$

$$f(\theta + 2\pi) = f(\theta)$$

Functions Having Arbitrary Period

Assume that a function $f(t)$ has period, T. We can relate angle (θ) with time (t) in the following manner.

$$\theta = \omega t$$

ω is the angular velocity in radians per second.

$$\omega = 2\pi f$$

f is the frequency of the periodic function,

$$f(t)$$

$$\theta = 2\pi f t$$ where $$f = \frac{1}{T}$$

Therefore, $$\theta = \frac{2\pi}{T} t$$

$$\theta = \frac{2\pi}{T} t \qquad d\theta = \frac{2\pi}{T} dt$$

Now change the limits of integration.

$$\theta = -\pi \qquad -\pi = \frac{2\pi}{T} t \qquad t = -\frac{T}{2}$$

$$\theta = \pi \qquad \pi = \frac{2\pi}{T} t \qquad t = \frac{T}{2}$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)\,d\theta$$

$$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\cos n\theta \, d\theta \qquad n = 1, 2, \cdots$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos\left(\frac{2\pi n}{T}t\right)dt \qquad n = 1, 2, \cdots$$

12.3 Fourier Cosine and Sine Series

Fourier cosine Series

The Fourier series of an **even** function on the interval (-p, p) is the **cosine series**

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{p}x$$

$$a_0 = \frac{2}{p}\int_0^p f(x)dx$$

$$a_n = \frac{2}{p}\int_0^p f(x)\cos\frac{n\pi}{p}x\,dx$$

Fourier sine Series

The Fourier series of an **odd** function on the interval (-p, p) is the **sine series**

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{p}x$$

$$b_n = \frac{2}{p}\int_0^p f(x)\sin\frac{n\pi}{p}x\,dx$$

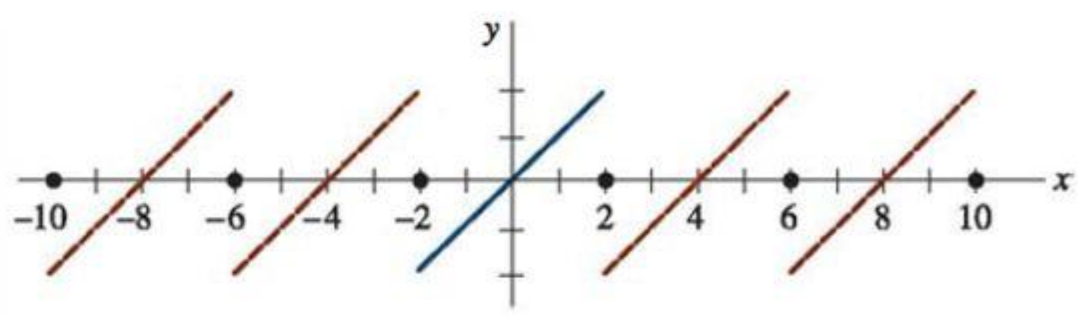
Example

Expand f(x) = x, -2 < x< 2, in a Fourier series.

$$b_n = \frac{4(-1)^{n+1}}{n\pi}$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{p}x$$

the series converges to the function on (-2, 2) and the periodic extension (of period 4)



Half-Range Expansions

Example: $f(x)=x^2,\ 0<x<L$

Expand
(a) in a cosine series,
(b) in a sine series,
(c) in a Fourier series

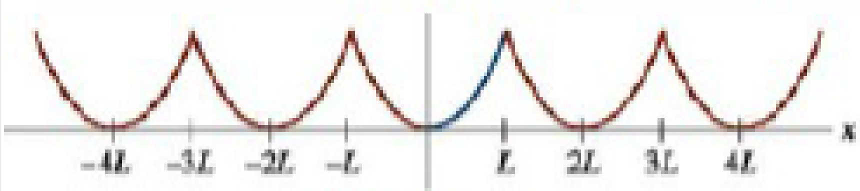
L-periodic extension

$$f(x)=\frac{L^2}{3}+\frac{L^2}{\pi}\sum_{n=1}^{\infty}\left\{\frac{1}{n^2\pi}\cos\frac{2n\pi}{L}x-\frac{1}{n}\sin\frac{2n\pi}{L}x\right\}.$$

(c) Fourier series

2L-periodic even extension

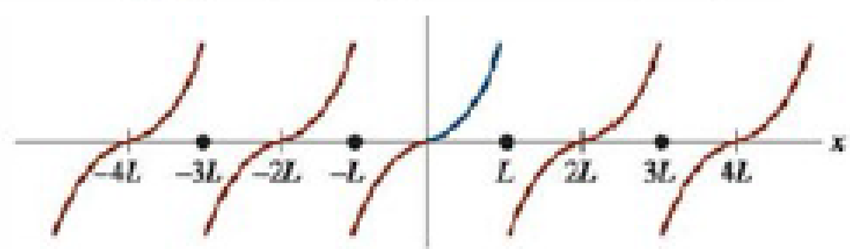
$$f(x)=\frac{L^2}{3}+\frac{4L^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\frac{n\pi}{L}x.$$



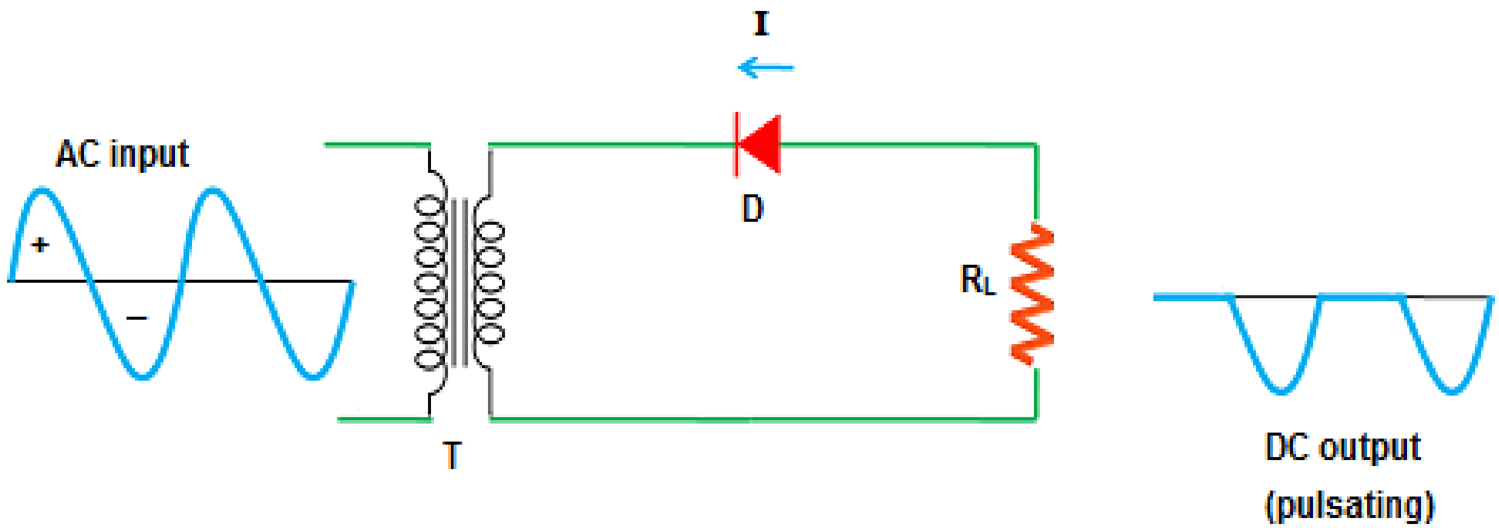
(a) Cosine series

2L-periodic odd extension

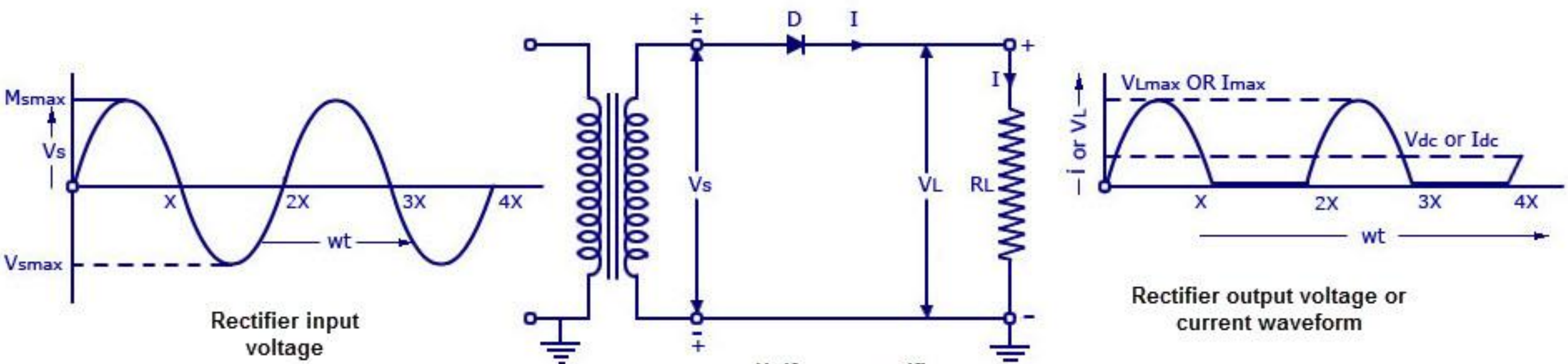
$$f(x)=\frac{2L^2}{\pi}\sum_{n=1}^{\infty}\left\{\frac{(-1)^{n+1}}{n}+\frac{2}{n^3\pi^2}\left[(-1)^n-1\right]\right\}\sin\frac{n\pi}{L}x.$$



(b) Sine series



Negative half wave rectifier



Rectifier input voltage

Half wave rectifier

Rectifier output voltage or current waveform

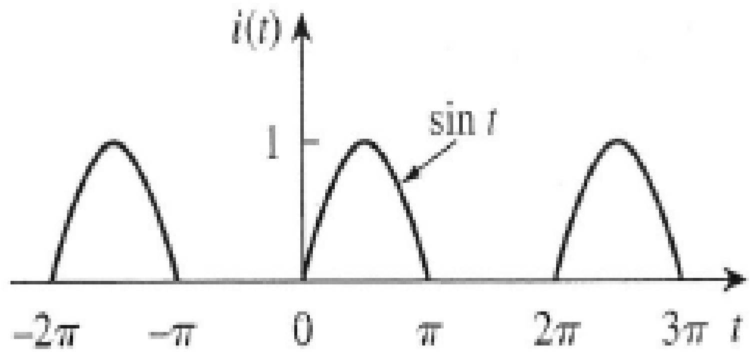
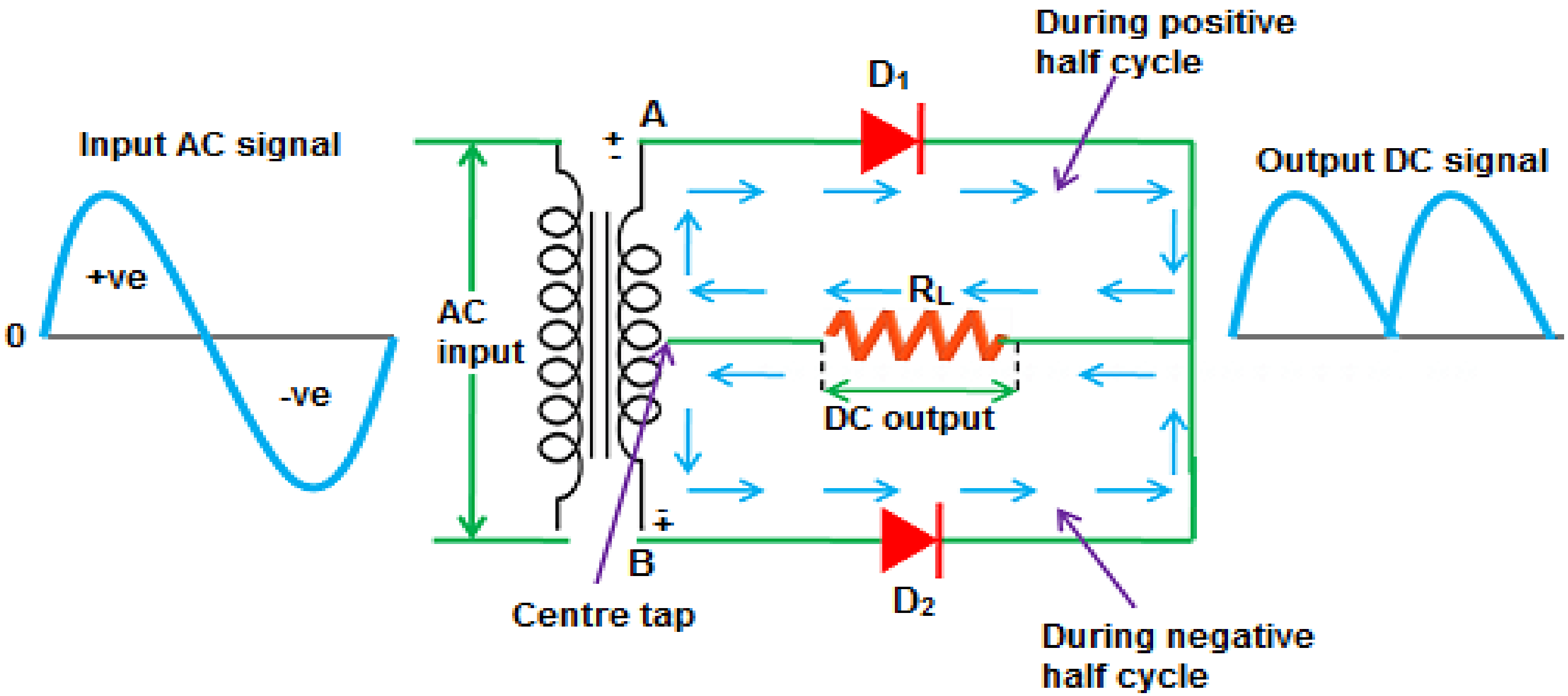
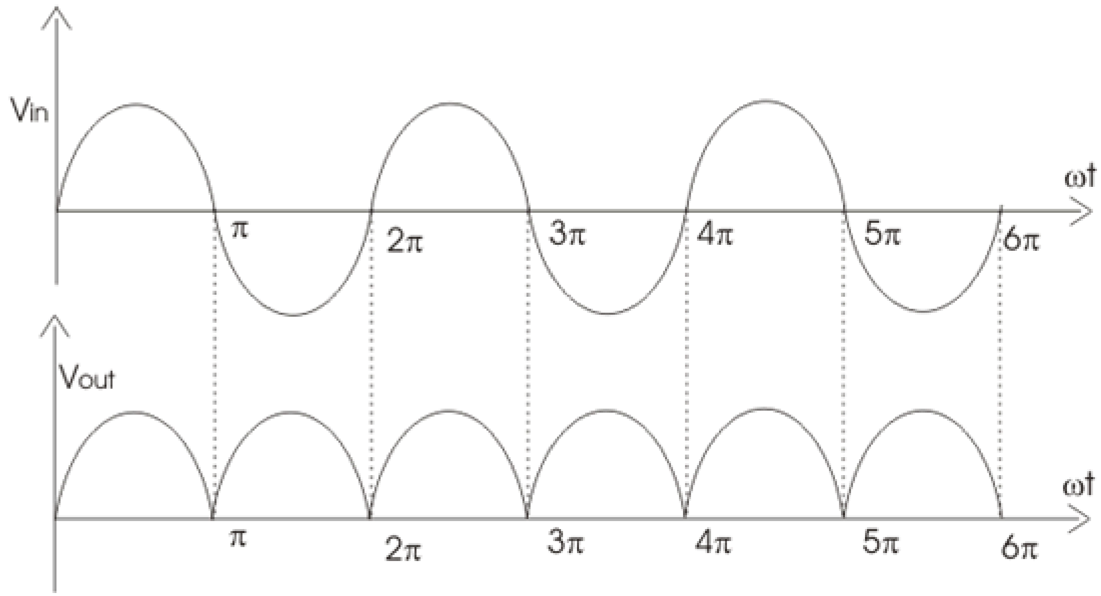


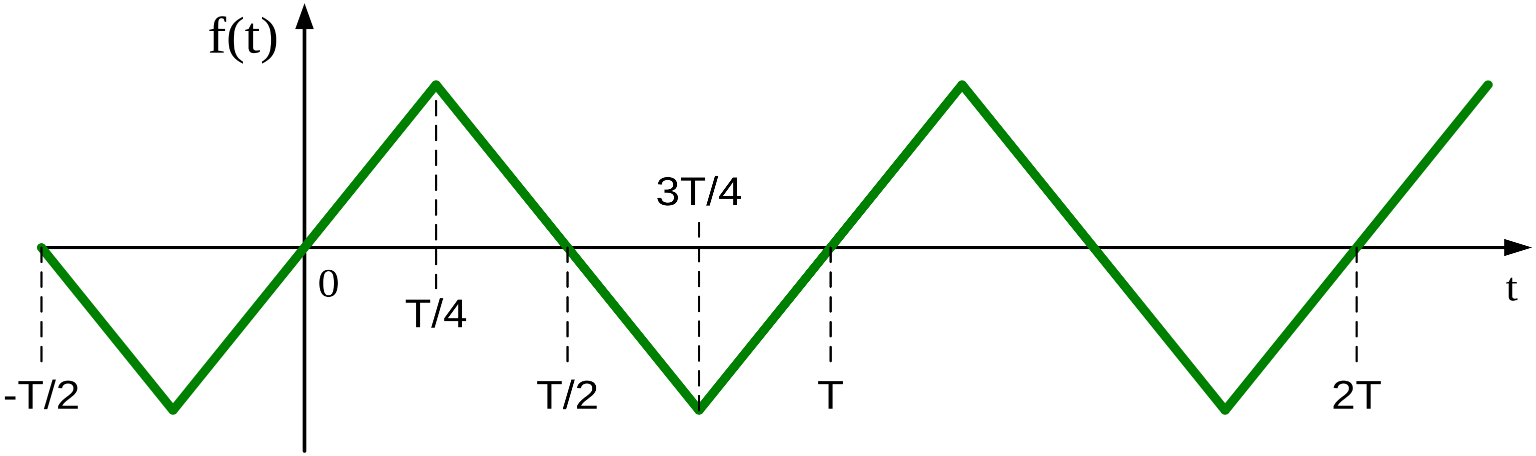
Figure 17.82
For Prob. 17.55.



Input AC signal
+ve
0
-ve
AC
input
A
B
Centre tap
D1
D2
RL
DC output
During positive
half cycle
During negative
half cycle
Output DC signal


Vin
ωt
π
2π
3π
4π
5π
6π
Vout
ωt
π
2π
3π
4π
5π
6π

Example 4. Find the Fourier series of the following periodic function.



$$f(t) = t \quad when -\frac{T}{4} \leq t \leq \frac{T}{4}$$

$$= -t + \frac{T}{2} \quad when \quad \frac{T}{4} \leq t \leq \frac{3T}{4}$$

Periodic Rectangular Wave (Fig. 260)

Find the Fourier coefficients of the periodic function $f(x)$ in Fig. 260. The formula is

$$(7) \qquad f(x) = \begin{cases} -k & \text{if} \quad -\pi < x < 0 \\ k & \text{if} \quad 0 < x < \pi \end{cases} \qquad \text{and} \qquad f(x + 2\pi) = f(x).$$

Functions of this kind occur as external forces acting on mechanical systems, electromotive forces in electric circuits, etc. (The value of $f(x)$ at a single point does not affect the integral; hence we can leave $f(x)$ undefined at $x = 0$ and $x = \pm\pi$.)

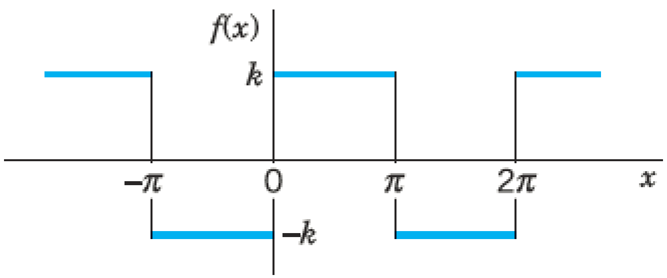


Fig. 260. Given function $f(x)$ (Periodic reactangular wave)

EXAMPLE 3 Half-Wave Rectifier

A sinusoidal voltage $E \sin \omega t$, where t is time, is passed through a half-wave rectifier that clips the nega portion of the wave (Fig. 265). Find the Fourier series of the resulting periodic function

$$u(t) = \begin{cases} 0 & \text{if} \quad -L < t < 0, \\ E \sin \omega t & \text{if} \quad 0 < t < L \end{cases} \qquad p = 2L = \frac{2\pi}{\omega}, \qquad L = \frac{\pi}{\omega}.$$

Solution. Since $u = 0$ when $-L < t < 0$, we obtain from (6.0), with t instead of x,

$$a_0 = \frac{\omega}{2\pi} \int_0^{\pi/\omega} E \sin \omega t \, dt = \frac{E}{\pi}$$

and from (6a), by using formula (11) in App. A3.1 with $x = \omega t$ and $y = n\omega t$,

$$a_n = \frac{\omega}{\pi} \int_0^{\pi/\omega} E \sin \omega t \cos n\omega t \, dt = \frac{\omega E}{2\pi} \int_0^{\pi/\omega} [\sin (1 + n) \omega t + \sin (1 - n) \omega t] \, dt.$$

If $n = 1$, the integral on the right is zero, and if $n = 2, 3, \cdots$, we readily obtain

$$a_n = \frac{\omega E}{2\pi} \left[-\frac{\cos (1 + n) \omega t}{(1 + n) \omega} - \frac{\cos (1 - n) \omega t}{(1 - n) \omega} \right]_0^{\pi/\omega}$$

$$= \frac{E}{2\pi} \left(\frac{-\cos (1 + n)\pi + 1}{1 + n} + \frac{-\cos (1 - n)\pi + 1}{1 - n} \right).$$

If n is odd, this is equal to zero, and for even n we have

$$a_n = \frac{E}{2\pi} \left(\frac{2}{1 + n} + \frac{2}{1 - n} \right) = -\frac{2E}{(n - 1)(n + 1)\pi} \qquad (n = 2, 4, \cdots).$$

In a similar fashion we find from (6b) that $b_1 = E/2$ and $b_n = 0$ for $n = 2, 3, \cdots$. Consequently,

$$u(t) = \frac{E}{\pi} + \frac{E}{2}\sin\omega t - \frac{2E}{\pi}\left(\frac{1}{1\cdot 3}\cos 2\omega t + \frac{1}{3\cdot 5}\cos 4\omega t + \cdots\right).$$

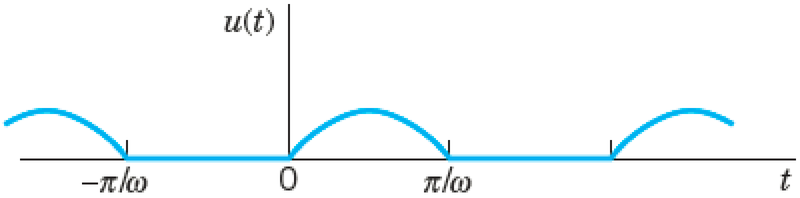


Fig. 265. Half-wave rectifier