Worked Example 9(B)

Example 1

Tests made on the breaking strength of 10 pieces of a metal wire gave the results: 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 kg. Test if the mean breaking strength of the wire can be assumed as 577 kg.

Let us first compute sample mean $\bar{x}$ and sample S.D.'s' and then test if $\bar{x}$ differs significantly from the population mean $\mu = 577$.

We take the assumed mean $A = \dfrac{568 + 596}{2} = 582$

$$di = x_i - A$$

$$\therefore \quad x_i = d_i + A$$

$$\therefore \quad \bar{x} = \frac{1}{n}\sum x_i = \frac{1}{n}\sum d_i + A$$

$$= \frac{1}{10} \times (-68) + 582 = 575.2 \text{ (see Table 9.7 given below)}$$

Table 9.7

x_i	$d_i = x_i - A$	d_i^2
578	– 4	16
572	– 10	100
570	– 12	144
568	– 14	196
572	– 10	100
570	– 12	144
570	– 12	144
572	– 10	100
596	14	196
584	2	4
Total	– 68	1144

$$s^2 = \frac{1}{n}\sum d_i^2 - \left(\frac{1}{n}\sum d_i\right)^2$$

$$= \frac{1}{10} \times 1144 - \left(\frac{1}{10} \times -68\right)^2 = 68.16$$

$$\therefore \quad s = 8.26$$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n-1}} = \frac{575 \cdot 2 - 577}{8 \cdot 26 / \sqrt{9}}$$
$$= -0.65$$

$v = n - 1 = 9.$

$H_0 : \bar{x} = \mu$ and $H_1 : x \neq \mu.$

Let LOS be 5%. Two tailed test is to be used.

From the t-table, for $v = 9, t_{5\%} = 2.26$. Since $|t| < t_{5\%}$, the difference between $\bar{x}$ and μ is not significant or H_0 is accepted. ∴ The mean breaking strength of the wire can be assumed as 577 kg at 5% LOS

∴ cal $|t| <$ tab t 5% not significant accepted

Example 2

A machinist is expected to make engine parts with axle diameter of 1.75 cm. A random sample of 10 parts shows a mean diameter 1.85 cm. with a S.D. of 0.1 cm. On the basis of this sample, would you say that the work of the machinist is inferior?

$\bar{x} = 1.85, \quad s = 0.1, \quad n = 10$ and $\mu = 1.75.$

$H_0 : \bar{x} = \mu ; \quad H_1 : \bar{x} \neq \mu$

Two tailed test is to be used. Let L.O.S. be 5%

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n-1}} = \frac{0 \cdot 10}{0 \cdot 1 / \sqrt{9}} = 3 \quad \text{and} \quad v = n - 1 = 9.$$

From the t-table, for $v = 9, \quad t_{0.05} = 2.26$ and $t_{0.01} = 3.25.$

∴ $|t| > t_{0.05}$ and $|t| < t_{0.01}$

∴ H_0 is rejected and H_1 is accepted at 5% level, but H_0 is accepted and H_1 is rejected at 1% level. i.e. At 5% LOS, the work of the machinist can be assumed to be inferior, but at 1% LOS, the work cannot be assumed to be inferior.

Example 3

A certain injection administered to each of 12 patients resulted in the following increases of blood pressure:

5, 2, 8, – 1, 3, 0, 6, – 2, 1, 5, 0, 4.

Can it be concluded that the injection will be, in general, accompanied by an increase in B.P.? The mean of the sample is given by $\bar{x} = \frac{1}{n} \sum x = \frac{31}{12} = 2.58$

The S.D. 's' of the sample is given by

$$s^2 = \frac{1}{n} \sum x^2 - \left(\frac{1}{n} \sum x \right)^2 = \frac{1}{12} \times 185 - (2.58)^2 = 8.76$$

∴ $s = 2.96$

$H_0 : \bar{x} = \mu,$ where $\mu = 0$, i.e. the injection will not result in increase in B.P.

$H_1 : \bar{x} > \mu$

Right-tailed test is to be used. Let L.O.S. be 5%. Now $t_{5\%}$ for one-tailed test for $(\nu = 11) = t_{10\%}$ for two-tailed test for $(\nu = 11) = 1.80$ (from t-table)

Now $$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{2.58 - 0}{2.96/\sqrt{11}} = 2.89$$

We see that $|t| > t_{10\%}\ (\nu = 11)$

$\therefore$ H_0 is rejected and H_1 is accepted.

i.e. we may conclude that the injection is accompanied by an increase in B.P.

Example 4

The mean lifetime of a sample of 25 bulbs is found as 1550 hours with a S.D. of 120 hours. The company manufacturing the bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptable at 5% level of significance?

$\bar{x} = 1550, \quad s = 120, \quad n = 25 \quad$ and $\quad \mu = 1600.$

$H_0 : \bar{x} = \mu$ and $H_1 : \bar{x} < \mu.$

Left-tailed test is to be used. LOS = 5%

$$\text{Now } t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{-50\sqrt{24}}{120} = -2.04 \text{ and } \quad \nu = 24$$

$t_{5\%}$ for one-tailed test for $(\nu = 24) = t_{10\%}$ for two-tailed test for $(\nu = 24) = 1.71.$

We see that $|t| > |t_{10\%}|$

$\therefore$ H_0 is rejected and H_1 is accepted at 5% LOS

i.e. The claim of the company cannot be accepted at 5% LOS

Example 5

The heights of ten males of a given locality are found to be 175, 168, 155, 170, 152, 170, 175, 160, 160 and 165 cms. Based on this sample, find the 95% confidence limits for the height of males in that locality.

We shall first find the mean $\bar{x}$ and S.D. 's' of the sample, by taking the assumed mean $A = 165$ (Table 9.8).

$$d_i = x_i - A$$

$$\therefore \quad \bar{x} = A + \bar{d}$$

$$= 165 + \frac{1}{10} \times 0 = 165.$$

$$s^2 = \frac{1}{n}\sum d_i^2 - \left(\frac{1}{n}\sum d_i\right)^2$$

$$= \frac{1}{10} \times 578 = 57.8$$

$$\therefore \quad s = 7.6$$

From the t-table,

$$t_{5\%}\ (\nu = 9) = 2.26.$$

The 95% confidence limits for μ are

$$\left(\bar{x} - 2.26\frac{s}{\sqrt{n-1}},\ \bar{x} + 2.26\frac{s}{\sqrt{n-1}}\right)$$

$$\left(165 - \frac{2.26 \times 7.6}{\sqrt{9}},\ 165 + \frac{2.26 \times 7.6}{\sqrt{9}}\right)$$

i.e. (159.3, 170.7)

i.e. the heights of males in the locality are likely to lie within 159.3 cm and 170.7 cm.

Table 9.8

x_i	$d_i = x_i - A$	d_i^2
175	10	100
168	3	9
155	− 10	100
170	5	25
152	− 13	169
170	5	25
175	10	100
160	− 5	25
160	− 5	25
165	0	0
Total	0	578

Example 6

Two independent samples of sizes 8 and 7 contained the following values:

Sample I : 19, 17, 15, 21, 16, 18, 16, 14

Sample II : 15, 14, 15, 19, 15, 18, 16

Is the difference between the sample means significant?

Table 9.9

Sample I			Sample II		
x_1	$d_1 = x_1 - 18$	d_1^2	x_2	$d_2 = x_2 - 16$	d_2^2
19	1	1	15	− 1	1
17	−1	1	14	− 2	4
15	− 3	9	15	− 1	1
21	3	9	19	3	9
16	− 2	4	15	− 1	1
18	0	0	18	2	4
16	− 2	4	16	0	0
14	− 4	16			
Total	− 8	44	Total	0	20

For sample I. $\bar{x}_1 = 18 + \bar{d}_1 = 18 + \frac{1}{8}\sum d_1$

$$= 18 + \frac{1}{8} \times (-8) = 17.$$

$$s_1^2 = \frac{1}{n_1}\sum d_1^2 - \left(\frac{1}{n_1}\sum d_1\right)^2$$

$$= \frac{1}{8} \times 44 - \left(\frac{1}{8} \times -8\right)^2 = 4.5$$

$\therefore$ $s_1 = 2.12.$

For sample II. $\bar{x}_2 = 16 + \bar{d}_2 = 16 + \frac{1}{7}\sum d_2 = 16.$

$$s_2^2 = \frac{1}{n_2}\sum d_2^2 - \left(\frac{1}{n_2}\sum d_2\right)^2$$

$$= \frac{1}{7} \times 20 - \left(\frac{1}{7} \times 0\right)^2 = 2.857$$

$\therefore$ $s_2 = 1.69$

$H_0 : \bar{x}_1 = \bar{x}_2$ and $H_1 : \bar{x}_1 \neq \bar{x}_2$

Two-tailed test is to be used. Let LOS be 5 %

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{17 - 16}{\sqrt{\left(\frac{8 \times 4{\cdot}5 + 7 \times 2{\cdot}857}{13}\right)\left(\frac{1}{8} + \frac{1}{7}\right)}}$$

$$= 0.93$$

Also $\nu = n_1 + n_2 - 2 = 13.$

From the t-table, $t_{5\%}$ $(\nu = 13) = 2.16$

Since $|t| < t_{5\%}$, H_0 is accepted and H_1 is rejected.

i.e. the two sample means do not differ significantly at 5% LOS

Example 7

Table 9.10 give's the biological values of protein from cow's milk and buffalo's milk at a certain level. Examine if the average values of protein in the two samples significantly differ.

Table 9.10

Cow's milk (x_1):	1.82,	2.02,	1.88,	1.61,	1.81,	1.54
Buffalo's milk (x_2):	2.00,	1.83,	1.86,	2.03,	2.19,	1.88

$n = 6$

$\bar{x}_1 = \frac{1}{6} \times 10.68 = 1.78$

$s_1^2 = \frac{1}{6} \times \Sigma x_1^2 - (\bar{x}_1)^2 = \frac{1}{6} \times 19.167 - (1.78)^2 = 0.0261$

$\bar{x}_2 = \frac{1}{6} \times 11.79 = 1.965$

$s_2^2 = \frac{1}{6} \times \Sigma x_2^2 - (\bar{x}_2)^2 = \frac{1}{6} \times 23.2599 - (1.965)^2 = 0.0154$

As the two samples are independent, the test statistic is given by t

$$= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2 + s_2^2}{n-1}}}$$

with $\nu = 2n - 2$ [Refer to note (2) under Test (2)]

$$t = \frac{1{\cdot}78 - 1{\cdot}965}{\sqrt{\frac{0{\cdot}0261 + 0{\cdot}0154}{5}}} = \frac{-0{\cdot}185}{\sqrt{\cdot 0083}} = -2.03 \text{ and } \nu = 10.$$

$H_0 : \bar{x}_1 = \bar{x}_2$ and $H_1 : \bar{x}_1 \neq \bar{x}_2$.

Two tailed test is to be used. Let LOS be 5%

From t-table, $t_{5\%}\ (\nu = 10) = 2.23$.

Since $|t| < t_{5\%}\ (\nu = 10)$, H_0 is accepted.

i.e. the difference between the mean protein values of the two varieties of milk is not significant at 5% level.

Example 8

Samples of two types of electric bulbs were tested for length of life and the following data were obtained.

	Size	Mean	S.D.
Sample I	8	1234 hours	36 hours
Sample II	7	1036 hours	40 hours

Is the difference in the means sufficient to warrant that type I bulbs are superior to type II bulbs?

$\bar{x}_1 = 1234, \quad s_1 = 36, \quad n_1 = 8; \ \bar{x}_2 = 1036, \quad s_2 = 40, \quad n_2 = 7$

$H_0 : \bar{x}_1 = \bar{x}_2 ; \quad H_1 : \bar{x}_1 > \bar{x}_2.$

Right-tailed test is to be used. Let LOS be 5%

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{198}{\sqrt{\left(\frac{21568}{13}\right)\left(\frac{1}{8} + \frac{1}{7}\right)}} = \frac{198}{21{\cdot}0807}$$

$= 9.39$

$\nu = n_1 + n_2 - 2 = 13$

$t_{5\%}$ ($\nu = 13$) for one-tailed test $= t_{10\%}$ ($\nu = 13$) for two tailed test $= 1.77$ (from t-table)

Now $t > t_{10\%}$ ($\nu = 13$)

$\therefore$ H_0 is rejected and H_1 is accepted

i.e. Type I bulbs may be regarded superior to type II bulbs at 5% LOS

Example 9

The mean height and the S.D. height of eight randomly chosen soliders are 166.9 cm. and 8.29 cm. respectively. The corresponding values of six randomly chosen sailors are 170.3 cm and 8.50 cm. respectively. Based on this data, can we conclude that soldiers are, in general, shorter than sailors?

$\bar{x}_1 = 166.9,\ s_1 = 8.29,\ n_1 = 8;\ \bar{x}_2 = 170.3,\ s_2 = 8.50,\ n_2 = 6.$

$H_0 : \bar{x}_1 = \bar{x}_2;\ H_1 : \bar{x}_1 < \bar{x}_2.$

Left-tailed test is to be used. Let LOS be 5%.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{-3.4}{\sqrt{\left(\frac{983.29}{12}\right)\left(\frac{1}{8} + \frac{1}{6}\right)}}$$

$$= -0.695$$

$$\nu = n_1 + n_2 - 2 = 12$$

$t_{5\%}$ ($\nu = 12$) for one-tailed test $= t_{10\%}$ ($\nu = 12$) for two tailed test $= 1.78$ (from t-table)

Now $|t| < t_{10\%}$ ($\nu = 12$)

$\therefore$ H_0 is accepted and H_1 is rejected.

i.e. based on the given data, we cannot conclude that soldiers are in general shorter than sailors.

Example 10

The following data relate to the marks obtained by 11 students in two tests, one held at the beginning of a year and the other at the end of the year after intensive coaching. Do the data indicate that the students have benefited by coaching?

Test 1:	19,	23,	16,	24,	17,	18,	20,	18,	21,	19,	20
Test 2:	17,	24,	20,	24,	20,	22,	20,	20,	18,	22,	19

The given data relate to the marks obtained in two tests by the same set of students. Hence the marks in the two tests can be regarded as correlated and so the t-test for paired values should be used.

Let $d = x_1 - x_2$,

where x_1, x_2 denote the marks in the two tests.

Thus the values of d are $2, -1, -4, 0, -3, -4, 0, -2, 3, -3, 1$.

$$\Sigma d = -11 \quad \text{and} \quad \Sigma d^2 = 69$$

$$\bar{d} = \frac{1}{n}\Sigma d = \frac{1}{11} \times -11 = -1$$

$$s^2 = s_d^2 = \frac{1}{n}\Sigma d^2 - (\bar{d})^2 = \frac{1}{11} \times 69 - (-1)^2 = 5.27$$

$$s = 2.296$$

$H_0 : \bar{d} = 0$, i.e. the students have not benefited by coaching; $H_1 : \bar{d} < 0$ (i.e. $\ldots$).

One-tailed test is to be used. Let LOS be 5%

$$t = \frac{\bar{d}}{s/\sqrt{n-1}} = \frac{-1}{2{\cdot}296/\sqrt{10}} = -1.38 \quad \text{and} \quad \nu = 10$$

$t_{5\%}$ $(\nu = 10)$ for one-tailed test $= t_{10\%}$ $(\nu = 10)$ for two-tailed test $= 1.81$ from t-table).

Now $|t| < t_{10\%}$ $(\nu = 10)$

$\therefore H_0$ is accepted and H_1 is rejected.

i.e. there is no significant difference between the two sets of marks.

i.e. the students have not benefitted by coaching.

Example 11

F dist

1 A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. Could both samples be from populations with the same variance?

$$n_1 = 13, \quad \hat{\sigma}_1^2 = 3.0 \quad \text{and} \quad \nu_1 = 12$$

$$n_2 = 15, \quad \hat{\sigma}_2^2 = 2.5 \quad \text{and} \quad \nu_2 = 14.$$

$H_0 : \hat{\sigma}_1^2 = \hat{\sigma}_2^2$, i.e. The two samples have been drawn from populations with the same variance.

$H_1 : \hat{\sigma}_1^2 \neq \hat{\sigma}_2^2$. Let L.O.S. be 5%

$$F = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} = \frac{3{\cdot}0}{2{\cdot}5} = 1.2$$

$$\nu_1 = 12 \quad \text{and} \quad \nu_2 = 14.$$

$F_{5\%}$ $(\nu_1 = 12, \quad \nu_2 = 14) = 2.53$, from the F-table.

$F < F_{5\%}$. $\therefore H_0$ is accepted

i.e. the two samples could have come from two normal populations with the same variance.

s^2 variance of sample
σ = S.D of population
s = S.D of sample
$S = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$

Example 12

2 Two samples of sizes nine and eight gave the sums of squares of deviations from their respective means equal to 160 and 91 respectively. Can they be regarded as drawn from the same normal population?

$$n_1 = 9, \quad \Sigma(x_i - \bar{x})^2 = 160, \quad \text{i.e. } n_1 s_1^2 = 160$$

$$n_2 = 8, \quad \Sigma(y_i - \bar{y})^2 = 91, \quad \text{i.e. } n_2 s_2^2 = 91$$

$$\hat{\sigma}_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{1}{8} \times 160 = 20; \quad \hat{\sigma}_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{1}{7} \times 91 = 13$$

Since $\hat{\sigma}_1^2 > \hat{\sigma}_2^2$, $v_1 = n_1 - 1 = 8$ and $v_2 = n_2 - 1 = 7$

$$H_0 : \hat{\sigma}_1^2 = \hat{\sigma}_2^2 \text{ and } H_1 : \hat{\sigma}_1^2 \neq \hat{\sigma}_2^2.$$

Let the LOS be 5%

$$F = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} = \frac{20}{13} = 1.54$$

$F_{5\%}(v_1 = 8, v_2 = 7) = 3.73$, from the F-table.

Since $F < F_{5\%}$, H_0 is accepted.

i.e. the two samples could have come from two normal populations with the same variance.

We cannot say that the samples have come from the same population, as we are unable to test if the means of the samples differ significantly or not.

Example 13

3 Two independent samples of eight and seven items respectively had the following values of the variable.

Sample 1 :	9,	11,	13,	11,	15,	9,	12,	14
Sample 2 :	10,	12,	10,	14,	9,	8,	10	

Do the two estimates of population variance differ significantly at 5% level of significance?

For the first sample, $\Sigma x_1 = 94$ and $\Sigma x_1^2 = 1138$

$$\therefore \quad s_1^2 = \frac{1}{n_1} \sum x_1^2 - \left(\frac{1}{n_1} \sum x_1 \right)^2$$

$$= \frac{1}{8} \times 1138 - \left(\frac{1}{8} \times 94 \right)^2 = 4.19$$

For the second sample, $\Sigma x_2 = 73$ and $\Sigma x_2^2 = 785$

$$\therefore \quad s_2^2 = \frac{1}{n_2} \sum x_2^2 - \left(\frac{1}{n_2} \sum x_2 \right)^2$$

$$= \frac{1}{7} \times 785 - \left(\frac{1}{7} \times 73\right)^2 = 3.39$$

$$\hat{\sigma}_1^2 = \frac{n_1}{n_1 - 1} s_1^2 = 4.79 \quad \text{and} \quad \hat{\sigma}_2^2 = \frac{n_2}{n_2 - 1} s_2^2 = 3.96$$

Since $\hat{\sigma}_1^2 > \hat{\sigma}_2^2$, $\nu_1 = 7$ and $\nu_2 = 6$

$$H_0 : \hat{\sigma}_1^2 = \hat{\sigma}_2^2 \quad \text{and} \quad H_1 : \hat{\sigma}_1^2 \neq \hat{\sigma}_2^2$$

$$F = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} = \frac{4.79}{3.96} = 1.21$$

$F_{5\%}(\nu_1 = 7, \nu_2 = 6) = 4.21$, from the F-table. Since $F < F_{5\%}$, H_0 is accepted.
i.e. $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ do not differ significantly at 5% level of significance.

Example 14

Two random samples gave the following data:

	Size	Mean	Variance
Sample I	8	9.6	1.2
Sample II	11	16.5	2.5

Can we conclude that the two samples have been drawn from the same normal population?

Refer to Note (2) under F-test. To conclude that the two samples have been drawn from the same population, we have to check first that the variances of the populations do not differ significantly and then check that the sample means (and hence the population means) do not differ significantly.

$$\hat{\sigma}_1^2 = \frac{8 \times 1.2}{7} = 1.37 \; ; \quad \hat{\sigma}_2^2 = \frac{11 \times 2.5}{10} = 2.75$$

$$F = \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2} = \frac{2.75}{1.37} = 2.007 \text{ with degrees of freedom 10 and 7.}$$

From the F-table, $F_{5\%}(10, 7) = 3.64$

If $\quad H_0 : \hat{\sigma}_1^2 = \hat{\sigma}_2^2 \quad \text{and} \quad H_1 : \hat{\sigma}_1^2 \neq \hat{\sigma}_2^2,$

H_0 is accepted, since $F < F_{5\%}$
i.e. the variances of the populations from which samples are drawn may be regarded as equal.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{-6.9}{\sqrt{\left(\frac{9.6 + 27.5}{17}\right)\left(\frac{1}{8} + \frac{1}{11}\right)}}$$

$$= -\frac{6.9}{0.6864} = -10.05$$

and $\quad v = n_1 + n_2 - 2 = 17.$

$t_{5\%}\ (v = 17) = 2.11$, from the t table.

If $H_0 : \bar{x}_1 = \bar{x}_2$ and $H_1 : \bar{x}_1 \neq \bar{x}_2$, H_0 is rejected, since $|t| > t_{5\%}$,

i.e. the means of two samples (and so the populations) differ significantly.

$\therefore$ The two samples could not have been drawn from the same normal population.

Example 15

The nicotine contents in two random samples of tobacco are given below.

Sample I :	21	24	25	26	27	
Sample II :	22	27	28	30	31	36.

Can you say that the two samples came from the same population?

$$\bar{x}_1 = \text{Mean of sample I} = \frac{123}{5} = 24.6$$

$$\bar{x}_2 = \text{Mean of sample II} = \frac{174}{6} = 29.0$$

$$s_1^2 = \text{Variance of sample I} = \frac{1}{5}\,\Sigma(x_i - 24.6)^2 = 4.24$$

$$s_2^2 = \text{Variance of sample II} = \frac{1}{6}\,\Sigma(x_i - 29.0)^2 = 18.0$$

$$\hat{\sigma}_1^2 = \frac{5}{4} \times 4.24 = 5.30 \text{ and } v = 4;\ \hat{\sigma}_2^2 = \frac{6}{5} \times 18.0 = 21.60 \text{ and } v = 5$$

$$H_0 : \hat{\sigma}_1^2 = \hat{\sigma}_2^2;\quad H_1 : \hat{\sigma}_1^2 \neq \hat{\sigma}_2^2$$

$$F = \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2} = \frac{21\cdot60}{5\cdot30} = 4.07$$

$F_{5\%}\ (5, 4) = 6.26.$

Since $\quad F < F_{5\%}$, H_0 is accepted.

$\therefore$ The variances of the two populations can be regarded as equal.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \frac{-4\cdot4}{\sqrt{\left(\dfrac{21\cdot2 + 108\cdot0}{9}\right)\left(\dfrac{1}{5} + \dfrac{1}{6}\right)}}$$

$$= \frac{-4\cdot4}{2\cdot2943} = -\,1.92$$

$n_1 + n_2 - 2$

$11 - 2 = 9$

and $\quad v = 9.$

From t-table, $F_{5\%}\ (v = 9) = 2.26.$

If $\quad H_0 : \bar{x}_1 = \bar{x}_2$ and $H_1 : \bar{x}_1 \neq \bar{x}_2$, H_0 is accepted

since $|t| < F_{5\%}$.

Appendix A

Statistical Tables

(Normal Table, t-table, χ^2-table and F-table)

z	*Ordinate*	*Area* = $\int_0^z$	z	*Ordinate*	*Area* = $\int_0^z$
0.00	0.3989	0.0000	1.55	0.1200	0.4394
0.05	0.3984	0.0199	1.60	0.1109	0.4452
0.10	0.3970	0.0398	1.65	0.1023	0.4505
0.15	0.3945	0.0596	1.70	0.941	0.4554
0.20	0.3910	0.0793	1.75	0.0863	0.4099
0.25	0.3867	0.0987	1.80	0.0790	0.4641
0.30	0.3814	0.1179	1.85	0.0721	0.4678
0.35	0.3752	0.1368	1.90	0.0656	0.4719
0.40	0.3683	0.1554	1.95	0.0596	0.4744
0.45	0.3605	0.1736	2.00	0.0540	0.4773
0.50	0.3521	0.1910	2.05	0.0488	0.4798
0.55	0.3429	0.2088	2.10	0.0440	0.4821
0.60	0.3332	0.2258	2.15	0.0396	0.4842
0.65	0.3230	0.2422	2.20	0.0355	0.4861
0.70	0.3123	0.2080	2.25	0.0317	0.4878
0.75	0.3011	0.2734	2.30	0.0283	0.4893
0.80	0.2897	0.2882	2.35	0.0252	0.4906
0.85	0.2780	0.3023	2.40	0.0224	0.4918
0.90	0.2661	0.3159	2.45	0.0224	0.4929
0.95	0.2541	0.3289	2.50	0.0198	0.4938
1.00	0.2420	0.3413	2.55	0.0175	0.4946
1.05	0.2299	0.3531	2.60	0.0155	0.4953
1.10	0.2179	0.3643	2.65	0.0136	0.4960

(Contd.)

z	Ordinate	Area = $\int_0^z$	z	Ordinate	Area = $\int_0^z$
1.15	0.2059	0.3749	2.70	0.0119	0.4965
1.20	0.1942	0.3849	2.75	0.0104	0.4970
1.25	0.1827	0.3944	2.80	0.0091	0.4974
1.30	0.1714	0.4032	2.85	0.0079	0.4978
1.35	0.1604	0.4115	2.90	0.0060	0.4981
1.40	0.1497	0.4192	2.95	0.0051	0.4984
1.45	0.1394	0.4265	3.00	0.0044	0.4987
1.50	0.1295	0.4332	3.05	0.0038	0.4989

t-Table

12 D.of f

	Probability				
n	0.9	0.1	0.05	0.02	0.01
1	0.158	6.314	12.706	31.821	63.657
2	0.142	2.920	4.303	6.965	9.925
3	0.137	2.353	3.182	4.541	5.841
4	0.134	2.132	2.776	3.747	4.604
5	0.132	2.015	2.571	3.365	4.032
6	0.131	1.943	2.447	3.143	3.707
7	0.130	1.895	2.365	2.998	3.496
8	0.130	1.860	2.306	2.896	3.355
9	0.129	1.833	2.262	2.821	3.250
10	0.129	1.812	2.228	2.764	3.169
11	0.129	1.796	2.201	2.718	3.106
12	0.128	1.782	2.179	2.681	3.055
13	0.128	1.771	2.160	2.650	3.012
14	0.128	1.761	2.145	2.624	2.977
15	0.128	1.753	2.131	2.602	2.947
16	0.128	1.746	2.120	2.583	2.921
17	0.128	1.740	2.110	2.567	2.898
18	0.127	1.734	2.101	2.552	2.878
19	0.127	1.729	2.093	2.539	2.861
20	0.127	1.725	2.086	2.528	2.845
21	0.127	1.721	2.080	2.518	2.831
22	0.127	1.717	2.074	2.508	2.819
23	0.127	1.714	2.069	2.500	2.807
24	0.127	1.711	2.064	2.492	2.797
25	0.127	1.708	2.060	2.485	2.787
30	0.127	1.697	2.042	2.457	2.750
40	0.126	1.684	2.021	2.423	2.704
60	0.126	1.671	2.000	2.390	2.660
120	0.126	1.658	1.980	2.358	2.617
∞	0.126	1.645	1.960	2.326	2.576

χ^2-Table

n	Probability 0.99	0.95	0.10	0.05	0.02	0.01
1	0.000157	0.00393	2.706	3.841	5.412	6.635
2	0.0201	0.103	4.605	5.991	7.824	9.210
3	0.115	0.352	6.251	7.815	9.837	11.345
4	0.297	0.711	7.779	9.488	11.668	13.277
5	0.554	1.145	9.236	11.070	13.388	15.086
6	0.872	1.635	10.645	12.592	15.033	16.812
7	1.238	2.167	12.017	14.067	16.622	18.475
8	1.646	2.733	13.362	15.507	18.168	20.090
9	2.088	3.325	14.684	16.919	19.670	21.666
10	2.558	3.940	15.987	18.307	21.161	23.209
11	3.053	4.575	17.275	19.675	22.618	24.725
12	3.571	5.226	18.549	21.026	24.054	26.217
13	4.107	5.982	19.812	22.362	25.472	27.688
14	4.660	6.571	21.064	23.685	26.873	29.141
15	5.229	7.261	22.307	24.996	28.259	30.578
16	5.812	7.962	23.542	26.296	29.633	32.000
17	6.408	8.672	24.768	27.587	30.995	33.409
18	7.015	9.390	25.989	28.869	32.346	34.805
19	7.633	10.117	27.204	30.114	33.687	36.191
20	8.260	10.851	28.412	31.410	35.020	37.566
21	8.897	11.581	29.615	32.671	36.343	38.932
22	9.542	12.338	30.813	33.924	37.659	40.289
23	10.196	13.091	32.007	35.172	38.968	41.638
24	10.856	13.848	33.196	36.415	40.270	42.980
25	11.524	14.611	34.382	37.652	41.566	44.314
26	12.198	15.379	35.563	38.885	42.856	45.642
27	12.879	16.151	36.741	40.113	44.140	46.963
28	13.565	16.928	37.916	41.337	45.419	48.278
29	14.256	17.708	39.087	42.557	46.693	49.588
30	14.953	18.493	40.256	43.773	47.962	50.892

For larger values of n, the expression $\sqrt{2\chi^2} - \sqrt{2n-1}$ may be used as a normal variate with unit variance.

Table of F (Variance ratio)—1 Per Cent Points

ν_2 \ ν_1	1	2	3	4	5	6	8	12	24	∞
1	4052	4999	5403	5625	5764	5859	5981	6106	6234	6366
2	98.49	99.01	99.17	99.25	99.30	99.33	99.36	99.42	99.46	99.50
3	34.12	30.81	29.46	28.71	28.24	27.91	27.49	27.05	26.60	26.12
4	21.20	18.00	16.69	15.98	15.52	15.21	14.80	14.37	13.93	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.27	9.89	9.47	9.02

(Contd.)

v_2	v_1: 1	2	3	4	5	6	8	12	24	∞
6	13.74	10.92	9.78	9.15	8.75	8.47	8.10	7.72	7.31	
7	12.25	9.55	8.45	7.85	7.46	7.19	6.84	6.47	6.07	6.88
8	11.26	8.65	7.59	7.01	6.63	6.37	6.03	5.67	5.28	5.65
9	10.56	8.02	6.99	6.42	6.06	5.80	5.47	5.11	4.73	4.86
10	10.04	7.56	6.55	5.99	5.64	5.39	5.06	4.71	4.33	4.31
11	9.65	7.20	6.22	5.67	5.32	5.07	4.74	4.40	4.02	3.91
12	9.33	6.93	5.95	5.41	5.06	4.82	4.50	4.16	3.78	3.60
13	9.07	6.70	5.74	5.20	4.86	4.62	4.30	3.96	3.59	3.36
14	8.86	6.51	5.56	5.03	4.69	4.46	4.14	3.80	3.43	3.16
15	8.68	6.36	5.42	4.89	4.56	4.32	4.00	3.67	3.29	3.00
16	8.53	6.23	5.29	4.77	4.44	4.20	3.89	3.55	3.18	2.87
17	8.40	6.11	5.18	4.67	4.34	4.10	3.79	3.45	3.08	2.75
18	8.28	6.01	5.09	4.58	4.25	4.01	3.71	3.37	3.00	2.65
19	8.18	5.93	5.01	4.50	4.17	3.94	3.63	3.30	2.92	2.57
20	8.10	5.85	4.94	4.43	4.10	3.87	3.56	3.23	2.86	2.49
21	8.02	5.78	4.87	4.37	4.04	3.81	3.51	3.17	2.80	2.42
22	7.94	5.72	4.82	4.31	3.99	3.76	3.45	3.12	2.75	2.36
23	7.88	5.66	4.76	4.26	3.94	3.71	3.41	3.07	2.70	2.31
24	7.82	5.61	4.72	4.22	3.90	3.67	3.36	3.03	2.66	2.26
25	7.77	5.57	4.68	4.18	3.86	3.63	3.32	2.99	2.62	2.21
26	7.72	5.53	4.64	4.14	3.82	3.59	3.29	2.96	2.58	2.17
27	7.68	5.49	4.60	4.11	3.79	3.56	3.26	2.93	2.55	2.13
28	7.64	5.45	4.57	4.07	3.76	3.53	3.23	2.90	2.52	2.10
29	7.60	5.42	4.54	4.04	3.73	3.50	3.20	2.87	2.49	2.06
30	7.56	5.39	4.51	4.02	3.70	3.47	3.17	2.84	2.47	2.03
40	7.31	5.18	4.31	3.83	3.51	3.29	2.99	2.66	2.29	2.01
60	7.08	4.98	4.13	3.65	3.34	3.12	2.82	2.50	2.12	1.81
120	6.85	4.79	3.95	3.48	3.17	2.96	2.66	2.34	1.95	1.60
∞	6.64	4.60	3.78	3.32	3.02	2.80	2.51	2.18	1.79	1.38

Table of F—5 Per Cent Points

v_2	v_1: 1	2	3	4	5	6	8	12	24	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	238.9	243.9	249.0	253.4
2	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.41	19.45	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.84	8.74	8.64	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.91	5.77	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.68	4.53	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.91	2.74	2.54

(Contd.)

ν_2 \ ν_1	1	2	3	4	5	6	8	12	24	∞
11	4.84	3.98	3.59	3.36	3.20	3.09	2.95	2.79	2.61	2.40
12	4.75	3.88	3.49	3.26	3.11	3.00	2.85	2.69	2.50	2.30
13	4.67	3.80	3.41	3.18	3.02	2.92	2.77	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.64	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.55	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.51	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.48	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.42	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.40	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.38	2.20	2.00	1.76
24	4.26	3.40	3.10	2.78	2.62	2.51	2.36	2.18	1.98	1.73
25	4.24	3.38	2.99	2.76	2.60	2.49	2.34	2.16	1.96	1.71
26	4.22	3.37	2.98	2.74	2.59	2.47	2.32	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.30	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.44	2.29	2.12	1.91	1.65
29	4.18	3.33	2.93	2.70	2.54	2.43	2.28	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.09	1.89	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.00	1.79	1.51
60	4.00	3.15	2.76	2.52	2.37	2.25	2.10	1.92	1.70	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.02	1.83	1.61	1.25
∞	3.84	2.99	2.60	2.37	2.21	2.09	1.94	1.75	1.52	1.00