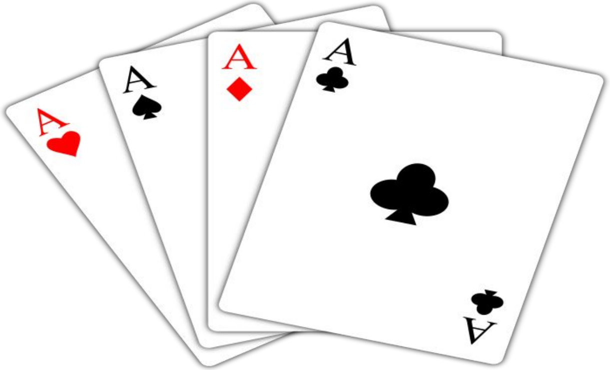


PROBABILITY

Consider the following statement

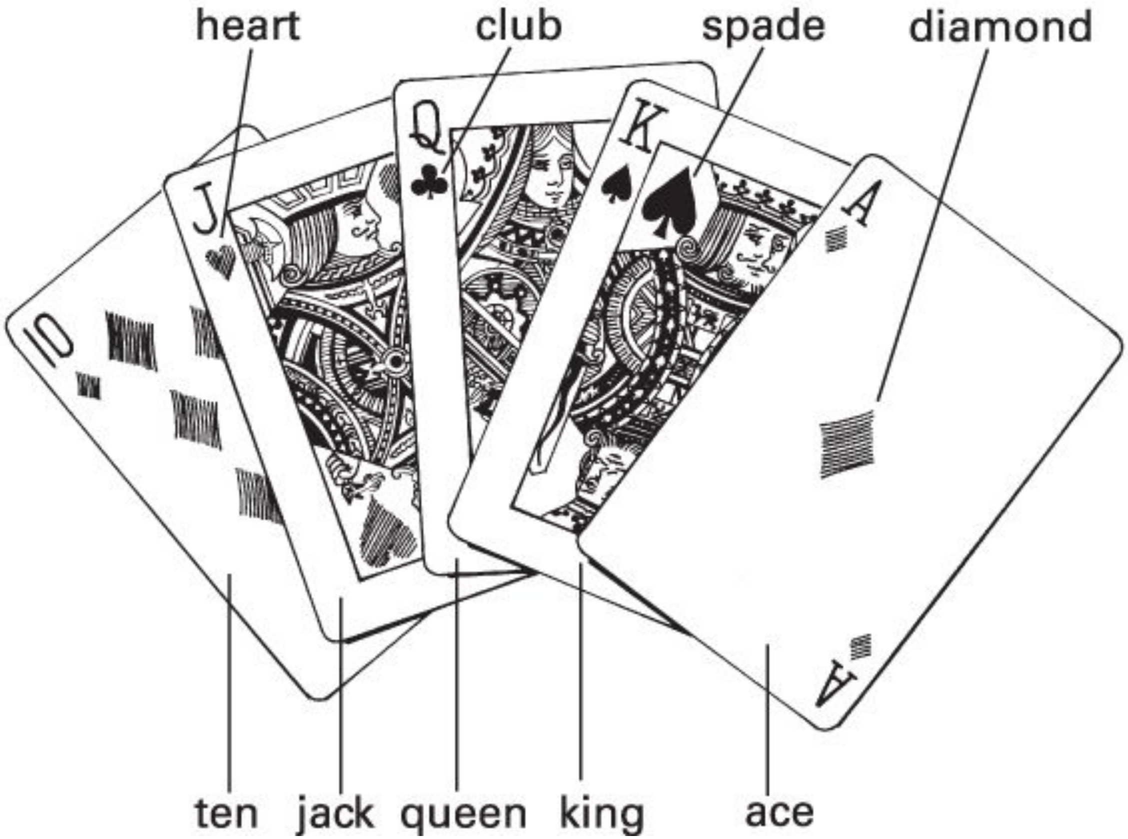
- The chance of India winning a cricket match against Pakistan is good
- The chance that it will rain on 15th August is pretty high
- The chance that price of share issued by Tata group would go up in the next two years is very high

Probability

- Chance of happening and not happening of an event

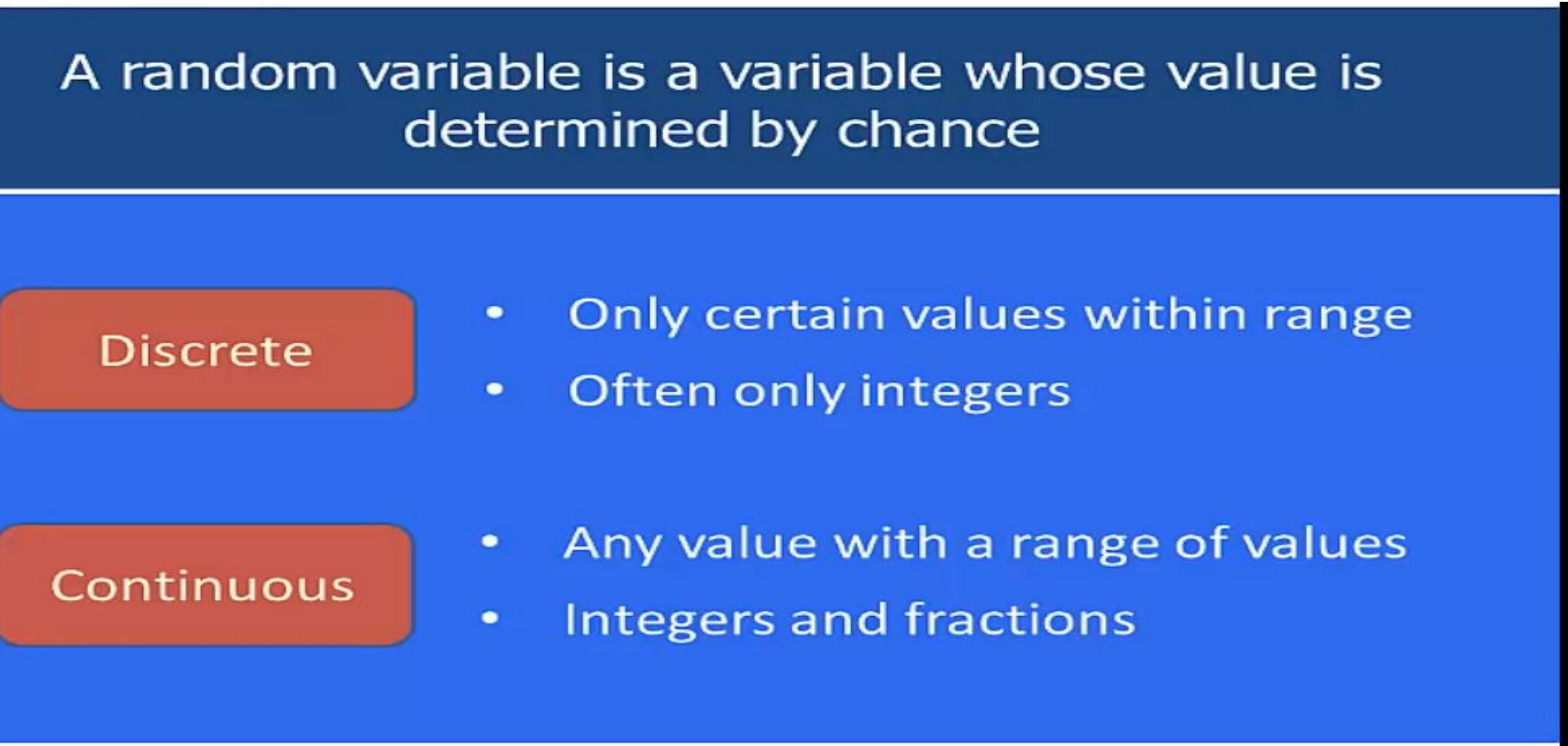
$$\text{Probability} = \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ cases}$$

playing cards



Some Important Terms and Concepts

Random Variable



Random variable

A variable whose value is determined by chance

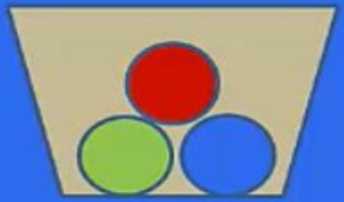
Ordinary variable	Random variable
• People, places, things • Values can vary	• People, places, things • Values can vary • Value based on chance

Ordinary variable

What color is the ball?

- Color is a variable
- Not a random variable

Random variable



What color is the ball?

- Color is based on chance
- Color is random variable

Random Experiment

- A random experiment is a process whose outcome is uncertain.
- The result may be any one of the various possible outcomes but may not be same every time.
- Eg: if an unbaised dice is thrown it will not always fall with any particular number up. Any of the six numbers on the dice can come up.

Sample Space

- A set of all possible outcomes from an experiment is called sample space.
- Eg: **Rolling a die**
- There are six possible outcomes and the sample space consists of six elements:
- {1, 2, 3, 4, 5, 6}.

Trial and Event

- The performance of a random experiment is called a ***trial*** and the outcome an ***event***.
- Eg: Throwing of a dice would be called a trials and the result falling of any one of six numbers 1,2, 3, 4, 5, 6 an event.

Simple event

- An event consisting of only one possible outcome is called simple event.
- Eg: in tossing a dice the chance of getting 3 is a single event (because 3 occurs in the dice only once)

Compound event

- An event consisting of more than one possible outcomes is called compound event.
- Eg: in a dice the chance of getting an odd number is a compound event (because odd numbers are more than one i.e. 1, 3 and 5)

Exhaustive Cases or Events

- It is the total number of all the possible outcomes of an experiment.
- For eg: in the throw of a single dice the exhaustive cases are 6. however if 2 dice are thrown simultaneously the exhaustive number of cases would be (6x6=36)

Favourable cases

- The number of outcomes which result in the happening of a desired event.
- Eg: in a single throw of a dice the number of favourable cases for getting an odd number are three , i.e 1, 3 and 5

Mutually Exclusive Events

- If in an experiment the occurrence of an event prevents or rules out the happening of all other events in the same experiment, then these events are said to be mutually exclusive events.
- Eg: in tossing a coin, the events head and tail are mutually exclusive , because if the outcome is head, then the possibility of getting a tail in the same trial is ruled out.

Equally Likely Events

- Events are said to be equally likely if the chance of their happening is equal.
- eg: in a throw of an unbiased dice, the coming up of 1, 2, 3, 4, 5, 6 is equally likely.

Independent and Dependent Events

- Two or more events are said to be independent if the happening of any one does not depend on the happening of the other.
- Events which are ***not independent*** are called dependent events
- Eg: if we draw a card from a pack of well shuffled cards and again draw a card from the rest of pack of cards (consisting 51 cards), then the second draw is dependent on the first. But if on the other hand we draw a second card from the pack by replacing the card drawn the second draw is known as independent of the first.

1. What is the probability of getting an even number in a single throw with a dice?
2. What is the probability of getting tail in a throw of a coin?
3. A bag contains 6 white balls, 9 black balls. What is the probability of drawing a black ball?

An unbiased cubic dice is thrown. What is the probability of getting-

- 6
- An even number
- An odd number
- A multiple of 2
- A multiple of 3

One card is drawn from a well shuffled pack of 52 playing cards. What is the probability that it is a –

- King
- King of red color
- King of heart
- Numeric card
- Numeric card bearing a multiple of 2
- Red numeric card bearing a multiple of 2
- Black odd numeric card

Find the probability of getting a red ace when a card is drawn at random from an ordinary deck of cards.

Solution

Since there are 52 cards and there are 2 red aces, namely, the ace of hearts and the ace of diamonds, $P(\text{red ace}) = \frac{2}{52} = \frac{1}{26}$.

A card is drawn from an ordinary deck. Find these probabilities.

a. Of getting a jack

b. Of getting the 6 of clubs (i.e., a 6 and a club)

c. Of getting a 3 or a diamond

d. Of getting a 3 or a 6

Solution

a. Refer to the sample space in Figure 4–2. There are 4 jacks so there are 4 outcomes in event E and 52 possible outcomes in the sample space. Hence,

$$P(\text{jack}) = \tfrac{4}{52} = \tfrac{1}{13}$$

b. Since there is only one 6 of clubs in event E, the probability of getting a 6 of clubs is

$$P(6 \text{ of clubs}) = \tfrac{1}{52}$$

c. There are four 3s and 13 diamonds, but the 3 of diamonds is counted twice in this listing. Hence, there are 16 possibilities of drawing a 3 or a diamond, so

$$P(3 \text{ or diamond}) = \tfrac{16}{52} = \tfrac{4}{13}$$

This is an example of the inclusive or.

d. Since there are four 3s and four 6s,

$$P(3 \text{ or } 6) = \tfrac{8}{52} = \tfrac{2}{13}$$

This is an example of the exclusive or.

When a single die is rolled, what is the probability of getting a number less than 7?

Solution

Since all outcomes—1, 2, 3, 4, 5, and 6—are less than 7, the probability is

$$P(\text{number less than 7}) = \tfrac{6}{6} = 1$$

The event of getting a number less than 7 is certain.

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

a. A person has type O blood.

b. A person has type A or type B blood.

c. A person has neither type A nor type O blood.

d. A person does not have type AB blood.

Solution

Type	Frequency
A	22
B	5
AB	2
O	21
Total	50

a. $P(\text{O}) = \dfrac{f}{n} = \dfrac{21}{50}$

b. $P(\text{A or B}) = \dfrac{22}{50} + \dfrac{5}{50} = \dfrac{27}{50}$

(Add the frequencies of the two classes.)

c. $P(\text{neither A nor O}) = \dfrac{5}{50} + \dfrac{2}{50} = \dfrac{7}{50}$

(Neither A nor O means that a person has either type B or type AB blood.)

d. $P(\text{not AB}) = 1 - P(\text{AB}) = 1 - \dfrac{2}{50} = \dfrac{48}{50} = \dfrac{24}{25}$

(Find the probability of not AB by subtracting the probability of type AB from 1.)

Theorems of Probability

- Addition Theorem
- Multiplication Theorem

THE ADDITION THEOREM

The Addition Theorem of Probability is studied under two headings:

- ***Addition Theorem for Mutually Exclusive Events***
- ***Addition Theorem for Not Mutually Exclusive Events***

THE ADDITION THEOREM FOR MUTUALLY EXCLUSIVE EVENTS

The Addition Theorem states that if A and B are two mutually exclusive events, then the probability of occurrence of either A or B is the sum of the individual probabilities of A and B.

Symbolically,

$$P\ (A \text{ or } B) = P\ (A) + P\ (B)$$

OR

$$P\ (A + B) = P\ (A) + P\ (B)$$

It is also known as the **'Theorem of Total Probability.'**

EXAMPLES ILLUSTRATING THE APPLICATION OF THE ADDITION THEOREM

Question: A card is drawn from a pack of 52 cards. What is the probability of getting either a king or a queen?

Solution: There are 4kings and 4 queens in a pack of 52 cards.

The probability of drawing a king card is P (K) = $\frac{4}{52}$

and the probability of drawing a queen card is P (Q) = $\frac{4}{52}$

Since, both the events are mutually exclusive, the probability that the card drawn is either a king or a queen is

P (K or Q) = P (K) + P (Q)

$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

Question : A perfect die is tossed. What is the probability of throwing 3 or 5?

Solution : The probability of throwing 3 is P (A) = $\frac{1}{6}$

The probability of throwing 5 is P (B) = $\frac{1}{6}$

P (A or B) = P (A) + P (B)

$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

Question :A card is drawn at random from a pack of cards. Find the probability that the drawn card is either a club or an ace of diamond.

Solution :Probability of drawing a club P (A) = $\frac{13}{52}$

Probability of drawing an ace of diamond P (B) = $\frac{1}{52}$

P (A or B) = P (A) + P (B)

$$\frac{13}{52} + \frac{1}{52} = \frac{14}{52} = \frac{7}{26}$$

ADDITION THEOREM FOR NOT MUTUALLY EXCLUSIVE EVENTS

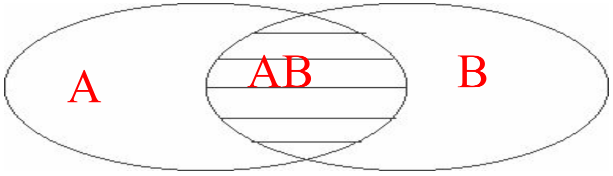
Two or more events are known as partially overlapping if part of one event and part of another event occur together. Thus, when the events are not mutually exclusive the addition theorem has to be modified.

Modified Addition Theorem states that if A and B are not mutually exclusive events, the probability of occurrence of either A or B or both is equal to the probability of that event A occurs, plus the probability that event B occurs minus the probability that events common to both A and B occur simultaneously. Symbolically,

$$\mathbf{P\ (A\ or\ B\ or\ Both\ ) = P\ (A) + P\ (B) - P\ (AB)}$$

The following figure illustrates this point:

NOT MUTUALLY EXCLUSIVE EVENTS



Overlapping Events

EXAMPLES ILLUSTRATING THE APPLICATION OF THE MODIFIED ADDITION THEOREM

Question: A card is drawn at random from a well shuffled pack of cards. What is the probability that it is either a spade or a king?

Solution: The Probability of drawing a spade P (A) $=\frac{13}{52}$

The Probability of drawing a King P (B) $= \frac{4}{52}$

The Probability of drawing a King of Spade P (AB) $= \frac{1}{52}$

P (A or B or Both) = P (A) + P (B) – P (AB)

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}$$

MULTIPLICATION THEOREM OF PROBABILITY

If two events A and B are independent then the probability that both of them will occur is equal to the product of their individual probabilities.

P (AB) = P (A∩B) = P (A and B) = P (A). P (B)

From a pack of 52 cards, two cards are drawn at random one after the other with replacement. What is the probability that both cards are kings?

The probability of drawing a king P (A) = 4/52

The probability of drawing again the king after replacement P (B) = 4/52

Since the two events are independent, the probability of drawing two kings is:

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

A bag contains 4 red balls, 3 white balls and 5 black balls. Two balls are drawn one after the other with replacement. Find the probability that first is red and the second is black.

Probability of red ball in the first draw = 4/12

The probability of a black ball in the second draw = 5/12

Since the events are independent, the probability that first is red and the second are black will be:

$$P(1R).P(1B) = \frac{4}{12} \times \frac{5}{12} = \frac{20}{144} = \frac{10}{72} = \frac{5}{36}$$

Permutations and Combinations

Factorial Formula for Permutations

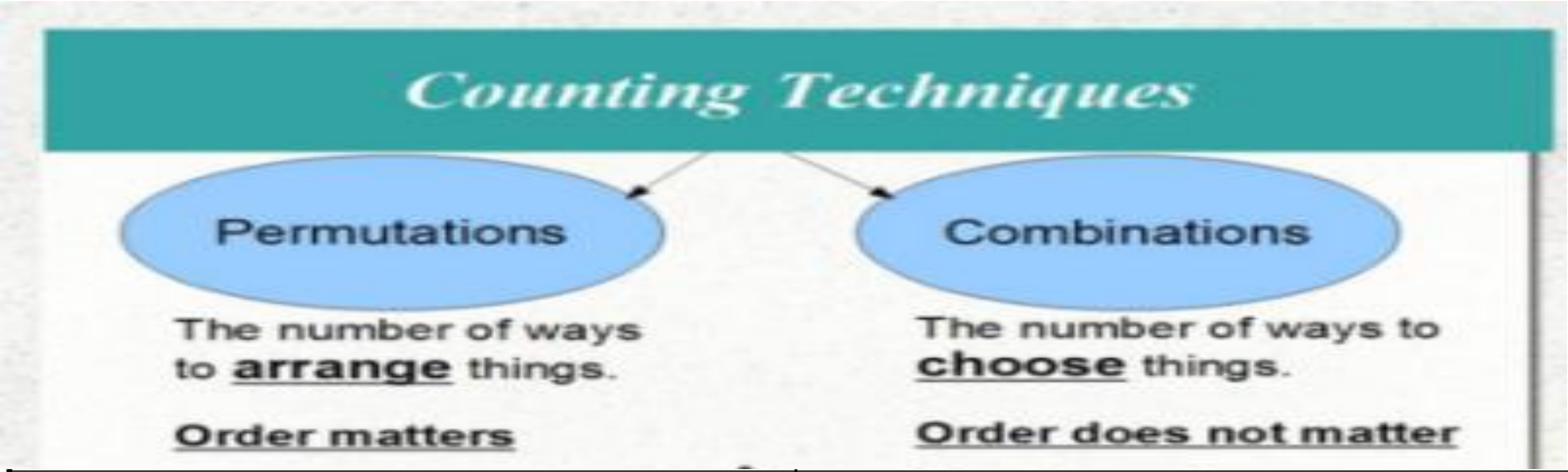
The number of **permutations**, or *arrangements*, of n distinct things taken r at a time, where $r \leq n$, can be calculated as

$$_nP_r = \frac{n!}{(n-r)!}.$$

Factorial Formula for Combinations

The number of **combinations**, or *subsets*, of n distinct things taken r at a time, where $r \leq n$, can be calculated as

$$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{r!(n-r)!}.$$



${}_nP_r = n!/(n-r)!$	${}_nC_r = n!/[\ r!(n-r)!]$
Clue words: arrangement, schedule, order	Clue words: group, sample, selection

Permutations and Combinations

Evaluate each problem.

a) ${}_5P_3$

$$\frac{5!}{(5-3)!}$$

$$\frac{5!}{2!}$$

$$5 \cdot 4 \cdot 3$$

$$60$$

b) ${}_5C_3$

$$\frac{5!}{3!\,(5-3)!}$$

$$\frac{5!}{3!\,2!}$$

$$\frac{5 \cdot 4}{2}$$

$$10$$

c) ${}_6P_6$

$$\frac{6!}{(6-6)!}$$

$$\frac{6!}{0!}$$

$$\frac{720}{1}$$

$$720$$

d) ${}_6C_6$

$$\frac{6!}{6!\,(6-6)!}$$

$$\frac{6!}{6!\,0!}$$

$$\frac{6!}{6!\,1}$$

$$1$$

- Four students while entering a room found that seven chairs were lying vacant. In how many ways could the seats be occupied?

solution

$$7P_4 = \frac{7!}{(7-4)!} = 840$$

- If two coins are tossed. In how many ways can they fall?

Solution

First coin	Second coin
H	H
T	T
H	T
T	H

THE TWO COINS MAY FALL TOGETHER IN 4 WAYS

Theorem of permutation

- The number of permutations of n different objects taken all at a time when p objects are of one kind, q objects are of another kind, r objects are still of another kind and the rest are all different is equal to

$$\frac{n!}{p! \times q! \times r!}$$

Question

- In how many ways the letters of the word ACCOUNTANTS can be arranged?
- SOLUTION

- A = 2
- C=2
- O=2
- U=1
- N=2
- T=2
- S=1

the required permutation =

$$\frac{11!}{2!*2!*2!*2!} = 24,94,800$$

- In how many ways the letter of the word MISSISSIPPI can be arranged?
- Ans = 34,650

theorem

- If each object can be selected r times, then the number of permutations of n different objects taken r at a time is n^r

- There are four prizes- one for recitation, one for games, one for cleanliness and one for general knowledge. In how many ways can these prizes be distributed among eight students?
- Ans. $8^4=4096$
- in how many ways three letters can be placed in five boxes?
- Ans. $5^3=125$

- In how many ways a committee of 5 can be selected out of 8 persons?
- A man has 6 friends. In how many ways can he invite him?
- A student has to answer 5 questions out of 8 in an examination

  (i) how many choices has he?

  (ii) how many choices has he, if he must answer the first three questions?

answers

$$Ans\,1.\ \frac{8!}{5!\times(8-5)!}=56$$

$$Ans\,2.\ 6C_1+6C_2+6C_3+6C_4+6C_5+6C_6=63$$

$$Ans\,3.\ (i)\ 8C_5=56$$

$$(ii)\ 5C_3=10$$

Conditional Probability

- Two events A and B are said to be dependent when B can occur only when A is known to have occurred (or vice versa).

$$P(A \ \ and \ \ B) = P(A).\, P(B / A)$$

$$\frac{P(A \ \ and \ \ B)}{P(A)} = P(B / A)$$

A box contains black chips and white chips. A person selects two chips without replacement. If the probability of selecting a black chip *and* a white chip is $\frac{15}{56}$, and the probability of selecting a black chip on the first draw is $\frac{3}{8}$, find the probability of selecting the white chip on the second draw, *given* that the first chip selected was a black chip.

Let

$$B = \text{selecting a black chip} \qquad W = \text{selecting a white chip}$$

Then

$$\begin{aligned} P(W|B) &= \frac{P(B \text{ and } W)}{P(B)} = \frac{15/56}{3/8} \\ &= \frac{15}{56} \div \frac{3}{8} = \frac{15}{56} \cdot \frac{8}{3} = \frac{\overset{5}{\cancel{15}}}{\underset{7}{\cancel{56}}} \cdot \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{3}}} = \frac{5}{7} \end{aligned}$$

Hence, the probability of selecting a white chip on the second draw given that the first chip selected was black is $\frac{5}{7}$.