5. $p(x) = {}^{18}C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{18-x}$, $x = 0, 1, 2, \ldots, 18$

10. $80\left(\frac{1}{4}+\frac{3}{4}\right)^{10}$ or $80\left(\frac{3}{4}+\frac{1}{4}\right)^{10}$.

11. $\sum_{x=5}^{7} {}^7C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{7-r}$.

14. The statement is not correct.

18. (i) cdot59049 = ·6065 (ii) ·32805 = ·3032 (iii) ·08146 = ·9855.

19. 1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1.

20. Binomial, 1, 10, 44, 117, 205, 246, 205, 117, 44, 10, 1.

21. $128\left(\frac{1}{2}+\frac{1}{2}\right)^7$.

22. Expected frequencies are 1, 12, 66, 220, 495, 792, 924, 792, 495, 220, 66, 12, 1. Expected mean = 6, actual mean = 6·139, expected σ = 1·732, actual σ = 1·712.

5.7. Poisson Distribution*

(Bhopal 1994, 96; Jabalpur 2014; Jiwaji 84; Sagar 2014)

The Poisson Distribution is a particular limiting form of the Binomial distribution when p (or q) is very small and n is large enough so that np (or nq) is a finite constant say m.

Under the conditions, $p(r)$, the probability of r successes in the Binomial distribution,

$$p(r) = P(X = r) = {}^nC_r p^r q^{n-r}$$

can be written as

$$\frac{n!}{r!(n-r)!}\left(\frac{m}{n}\right)^r\left(1-\frac{m}{n}\right)^{n-r} = \frac{m^r}{r!}\left(1-\frac{m}{n}\right)^n \frac{n!}{(n-r)!\,n^r\left(1-\frac{m}{n}\right)^r}$$

When n tends to infinity, we have

$$\lim_{r\to\infty}\left(1-\frac{m}{n}\right)^n = e^{-m},\ \lim_{n\to\infty}\left(1-\frac{m}{n}\right)^r = 1.$$

Hence by making n tend to infinity, the probability of r successes is

$$= \frac{m^r}{r!}e^{-m}\lim_{n\to\infty}\frac{n!}{(n-r)!}\cdot\frac{1}{n^r}.$$

Using Stirling's formula for $n!$, viz.,

$$n! = \sqrt{2\pi}\ n^{n+1/2}e^{-n}$$

we see that the probability of r successes is

$$= \frac{m^r e^{-m}}{r!}\lim_{n\to\infty}\frac{\sqrt{2\pi}n^{n+1/2}e^{-n}}{n^r\sqrt{2\pi}(n-r)^{n-r+1/2}e^{-n+r}}$$

*...thematician S.D. Poisson in 1837.

$$= \frac{m^r e^{-m}}{r! e^r} \lim_{n \to \infty} \left[\frac{1}{\left(1-\frac{r}{n}\right)^n \left(1-\frac{r}{n}\right)^{-r+1/2}} \right]$$

$$= \frac{m^r e^{-m}}{r! e^r} \cdot \frac{1}{e^{-r}.1} = \frac{m^r e^{-m}}{r!}$$

Hence, $$\lim_{n \to \infty, np = m, p \to 0} P(X = r) = \frac{m^r e^{-m}}{r!}.$$

Hence the successive terms of the Binomial distribution become

$$e^{-m}, me^{-m}, \frac{m^2 e^{-m}}{2!}, \ldots, \frac{m^r e^{-m}}{r!}, \ldots$$

Thus the limit of $(p+q)^n$ is

$$e^{-m}\left(1 + \frac{m}{1!} + \frac{m^2}{2!} + \ldots + \frac{m^r}{r!} + \ldots\right).$$

Alternative Method :

$$p(r) = P(X = r) = {}^nC_r \, p^r (1-p)^{n-r}$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} p^r \left(1 - \frac{np}{n}\right)^{n-r}$$

$$= \frac{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{r-1}{n}\right)}{r!} \frac{(np)^r}{\left(1-\frac{np}{n}\right)^r} \left(1-\frac{np}{n}\right)^n$$

$\therefore$ $p(r)$ = the probability of r successes in Poisson distribution

$$= \lim_{p \to 0, n \to \infty, np = m} P(X = r)$$

$$= \lim_{n \to \infty} \frac{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{r-1}{n}\right)}{r!} \frac{m^r}{\left(1-\frac{m}{n}\right)^r} \left(1-\frac{m}{n}\right)^n$$

$$= \frac{m^r . e^{-m}}{r!} \qquad \left[\because \lim_{n\to\infty}\left(1-\frac{m}{n}\right)^n = e^{-m}\right.$$

and $$\lim_{n\to\infty}\left(1-\frac{m}{n}\right)^n = (1-0)^r = 1.$$

It is called *Poisson distribution* with parameter m.

Therefore, the chances of 0, 1, 2, ..., r successes are respectively

$$e^{-m}, \frac{me^{-m}}{1!}, \frac{m^2 e^{-m}}{2!}, \ldots, \frac{m^r e^{-m}}{r!}.$$

so tha

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$[\because p+q=$

∴ The limiting form of Binomial distribution

$(q + p)^n$; where $p \to 0$, $n \to \infty$

so that $np = m$ is called Poisson's distribution.

Definition : The probability distribution of a random variable x is called Poisson distribution if x can assume non-negative integral values only and its distribution is given by

$$p(r) = P(X = r) = \begin{cases} \dfrac{e^{-m} m^r}{r!}, & r = 0, 1, 2, \ldots \\ 0, & r \neq 0, 1, 2, \ldots \end{cases}$$

5.8. Examples of Poisson Variates

(Jabalpur 1991)

The following are some examples of Poisson variates :

(Sagar 1997)

(1) The number of cars passing through a certain street in time t.

(2) The number of defective screws per box of 100 screws.

(3) The number of deaths in a district in one year by a rare disease.

(4) The number of suicides or deaths by heart attack in time t.

(5) The number of pieces of a certain merchandise sold by a store in time t.

(6) The number of printing mistakes at each page of the book.

(7) The number of accidents in some kind of time say 1 minute.

5.9. Constants of the Poisson Distribution

(Bhopal 1994)

(i) Mean or First moment about origin :

$$u_1' = E(X) = \sum_0^{\infty} \frac{e^{-m} m^r}{r!} \cdot r = \sum_0^{\infty} \frac{e^{-m} m^r}{(r-1)!}$$

$$= e^{-m} \left(m + \frac{m^2}{1!} + \frac{m^3}{2!} + \ldots \right)$$

$$= me^{-m} \left(1 + \frac{m}{1!} + \frac{m^2}{2!} + \ldots \right) = me^{-m} e^m = m.$$

(ii) Second moment about origin :

$$u_2' = E(X^2) = \sum_{r=0}^{\infty} \frac{r^2 m^r e^{-m}}{r!} = \sum_{r=0}^{\infty} \left[\{r + r(r-1)\} \frac{m^r e^{-m}}{r!} \right]$$

$$= \sum_0^{\infty} \frac{m^r e^{-m}}{(r-1)!} + \sum_0^{\infty} \frac{e^{-m} m^r}{(r-2)!}$$

$$= e^{-m} m \sum_{r=1}^{\infty} \frac{m^{r-1}}{(r-1)!} + e^{-m} m^2 \sum_{r=2}^{\infty} \frac{m^{r-2}}{(r-2)!}$$

$$= e^{-m} me^m + e^{-m} m^2 e^m$$

$$= m + m^2.$$

(iii) Third moment about origin :

$$u_3' = E(X^3) = \Sigma e^{-m} \frac{m^r}{r!} r^3$$
$$= e^{-m} \Sigma m^r \frac{[r(r-1)(r-2)+3r(r-1)+r]}{r!}$$
$$= e^{-m} m^3 \Sigma \frac{m^{r-3}}{(r-3)} + e^{-m} 3m^2 \Sigma \frac{m^{r-2}}{(r-2)!} + e^{-m} . m \Sigma \frac{m^{r-1}}{(r-1)!}$$
$$= m^3 + 3m^2 + m.$$

(iv) Fourth moment about origin :

$$u_4' = \Sigma e^{-m} \frac{m^r}{r!} r^4$$
$$= e^{-m} \Sigma m^r \frac{r(r-1)(r-2)(r-3)+6r(r-10)(r-2)+7r(4-1)+r]}{r!}$$
$$= e^{-m} m^4 \Sigma \frac{m^{r-4}}{(r-4)!} + e^{-m} 6m^3 \Sigma \frac{m^{r-3}}{(r-3)!}$$
$$+ e^{-m} . 7m^2 \Sigma \frac{m^{r-2}}{(r-2)!} + e^{-m} m \Sigma \frac{m^{r-1}}{(r-1)!}$$
$$= m^4 + 6m^3 + 7m^2 + m.$$

(v) First moment about mean :

$u_1 = 0$ always

(vi) Variance or Second moment about mean : (Jiwaji 1980)

$$V(x) = u_2 = E(X-EX)^2 = E(X^2) - (EX)^2$$
$$= u_2' - (u_1')^2 = m^2 + m - m^2 = m$$

(vii) Standard deviation :

$$\sigma = \sqrt{\mu_2} = \sqrt{m}.$$

Remark : Note that Mean = Variance = m.

(viii) Third moment about mean :

$$u_3 = u_3' - {}^3C_1 u_2' . m + {}^3C_2 u_1' m^3 - {}^3C_3 u_0' m^3$$
$$= m^3 + 3m^2 + m - 3(m^3 + m) . m + 3m^3 - m^3 = m$$

Remark : Note that

Mean = Variance = Third moment about mean = m.

(ix) Fourth moment about mean :

$$u_4 = u_4' - {}^4C_1 u_3' m + {}^4C_3 u_2' m^2 - {}^4C_3 u_1' m^3 + {}^4C_4 u_1' m^4$$
$$= m^4 + 6m^3 + 7m^2 + m - 4(m^3 + 3m^2 + m) m$$
$$+ 6(m^2 + m) m^2 - 4m^4 + m^4$$
$$= 3m^2 + m.$$

(x) Pearson's coefficients :

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{m^2}{m^3} = \frac{1}{m}; \beta_2 = \frac{\mu_4^2}{\mu_2^2} = \frac{3m^2 + m}{m^2} = 3 + \frac{1}{m}$$

$$\gamma_1 = \sqrt{\beta_1} = \frac{1}{\sqrt{m}}, \quad \gamma_2 = \beta_2 - 3 = \frac{1}{m}.$$ (Jabalpur 1990; Jiwaji 89)

(b) We know that for a Poisson distribution

$$\text{mean} = \text{variance} = \mu_3 = m > 0 \text{ always}$$

Hence Poisson distribution is always positively skew for $m > 0$.

Example 4. *For Poisson distribution*

$$P(X = x) = e^{-m} \frac{m^x}{x!}, \quad x = 0, 1, 2, \ldots, \infty.$$

Prove that as m tends to infinity γ_1 and γ_2 both approach to zero.

Solution : We know that

$$\gamma_1 = \frac{1}{\sqrt{m}}, \gamma_2 = \frac{1}{m}$$

$$\Rightarrow \quad \gamma_1 \to 0 \text{ as } m \to \infty$$

and $\quad \gamma_2 \to 0$ as $m \to \infty$.

Example 5. *Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience shose that 2 percent of such fuses are defective.*

(Bhopal 1992, 95; Jiwaji 94, 98; Sagar 92; Vikram 89)

Solution : Here $m = np = 200 \times \frac{2}{100} = 4.$

$$e^{-4} = 1 - 4 + \frac{1}{2} \cdot 4^2 - \frac{1}{6} 4^3 + \frac{1}{24} \cdot 4^4 + \ldots = 0 \cdot 0183$$

Hence $\quad P(r \leq 5) = \sum_{r=0}^{5} e^{-4} \frac{4^4}{r!}$

$$= e^{-4} \left(1 + 4 + \frac{4^2}{2} + \frac{4^3}{6} + \frac{4^4}{24} + \frac{4^5}{120}\right) = 0{\cdot}7845.$$

Example 6. *In a Poisson distribution probability for x = 0 is 10%. Find the mean, given that* $\log_3 10 = 2{\cdot}3026$.

Solution : Let m be the mean. Then

$$P(X = x) = e^{-m} \frac{m^x}{x!} \qquad \Rightarrow P(0) = e^{-m}.$$

$$\therefore \quad e^{-m} = 10\% = \frac{10}{100} = 0{\cdot}1 \text{ or } \quad e^m = 10$$

$$\therefore \quad m = \log_e 10 = 2{\cdot}3026.$$

Example 7. *A car–hire–firm has two cars, which it hires, out day by day. The number of demands for a car on each day is distributed as Poisson distribution with mean 1·5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused.* $[e^{-1.5} = 0{\cdot}2231]$. **(Jiwaji 1971)**

Solution : The proportion of days when no car will be required is

$$e^{-m} \frac{m^0}{m!} = 0{\cdot}2231.$$

The probability when no car, one car, two cars will be required is

$$= e^{-m}\frac{m^0}{m!} + e^{-m}\frac{m}{1!} + e^{-m}\frac{m^2}{2!}$$

$$= e^{-m}\left(1+m+\frac{1}{2}m^2\right)$$

$$= 0{\cdot}2231\,(1 + 1{\cdot}5 + 1{\cdot}125) = 0{\cdot}8087375.$$

$\therefore$ The proportion of days on which some demand is refused

$$= 1 - 0{\cdot}8087375 = 0{\cdot}1912625.$$

Example 8. *A telephone switch board handles 600 calls on the average during a rush hour. The board can make a maximum of 20 connections per minute. Use Poisson distribution to estimate the probability that the board will be over taxed during any given minute.* [$e^{-1} = {\cdot}00004539$].

Solution : Mean (per minute) $= \frac{600}{60} = 10.$

Hence, the probability for using 0 to 20 calls per minute

$$= \sum_{0}^{20} e^{-m}\frac{m^r}{r!} = e^{-10}\sum_{0}^{20}\frac{10^4}{r!}$$

$$= {\cdot}0004539 \sum_{0}^{20}\frac{10^4}{r!}.$$

Hence the probability that the board will be over taxed during any given minute *i.e.* when the calls will be more than 20 connections per minute.

$$= 1 - {\cdot}00004539 \sum_{0}^{20}\frac{10^4}{r!}.$$

Example 9. *Six coins are tossed 6400 times. Using Poisson distribution, find the approximate probability of getting six heads x times and 2 times.* **(Vikram 1992)**

Solution : Let the coins be unbiassed i.e. the probability of getting a head = the probability of getting a tail for each coin.

$\therefore$ The probability of getting 6 heads with 6 coins

$$= \left(\frac{1}{2}\right)^6 = \frac{1}{64} = p, \text{ say}$$

$\therefore$

$$np = 6400 \times \frac{1}{64} = 100 = m, \text{ say.}$$

Let

X = No. of tosses with 6 heads.

Then according to Poisson. Law

$$P(X = x) = e^{-m}\frac{m^x}{x!} = \frac{e^{-100}(100)^x}{x!}$$

and

$$P(X = 2) = e^{-m}\cdot\frac{m^2}{2!} = e^{-100}\frac{(100)^2}{2!} = 5000\,e^{-100}.$$

Example 10. *If $P(X=0) = P(X=1) = a$ in a Poisson distribution, show that $a = \frac{1}{e}$.*

Solution : Let $\quad P(X=x) = e^{-m}\dfrac{m^x}{x!}, \; x = 0, 1, 2, \ldots, \infty$

then $\quad P(X=0) = e^{-m}$ and $P(X=1) = e^{-m}\cdot m$

$\because \quad P(X=0) = P(X=1)$

$\Rightarrow \quad e^{-m} = e^{-m}\, m \Rightarrow m = 1$

But $\quad P(X=0) = a$

$\Rightarrow \quad e^{-1} = a$

$\therefore \quad a = \dfrac{1}{e}. \qquad [\because m = 1]$

Example 11. *If $P(X=2) = 9P(X=4) + 90\,P(X=6)$, in the Poisson distribution then find $E(X)$*

Solution : Let $\quad P(X=x) = e^{-m}\dfrac{m^x}{x!}, \; x = 0, 1, \ldots, \infty$

Given $\quad P(X=2) = 9P(X=4) + 90\,P(X=6)$

$$\Rightarrow \quad \frac{e^{-m}m^2}{2!} = 9e^{-m}\frac{m^4}{4!} + 90\,e^{-m}\frac{m^6}{6!}$$

$$\Rightarrow \quad m^4 + 3m^2 - 4 = 0$$

$$\Rightarrow \quad (m^2+4)(m^2-1) = 0$$

$$\Rightarrow \quad m = 1 \qquad [\because \text{ other values are not admissible}]$$

Example 12. *For a Poisson distribution with mean m, show that*

$$u_{r+1} = mr\,.\,u_{r-1} + m\frac{d\mu_r}{dm}.$$

where

$$u_r = \sum_{x=0}^{\infty}(x-m)^r\frac{e^{-m}m^r}{x!}.$$

(Indore 1998; Jabalpur 91; Vikram 90, 93)

Solution : $\dfrac{d\mu_r}{dm} = \dfrac{d}{dm}\left[\sum_{x=0}^{\infty}(x-m)^r\dfrac{e^{-m}m^x}{x!}\right]$

$$= -r\sum_{x=0}^{\infty}(x-m)^{r-1}\frac{e^{-m}m^x}{x!} + \sum_{x=0}^{\infty}\frac{(x-m)^r}{x!}\left[xm^{x-1}e^{-m} - m^x e^{-m}\right]$$

$$= -ru_{r-1} + \sum_{x=0}^{\infty}\frac{(x-m)^r e^{-m}m^{x-1}(x-m)}{x!}$$

Multiplying both sides by m, we

Illustrative Examples

Example 1. *Fit Poisson's distribution to the following and calculate theoretical frequencies :*

Deaths	0	1	2	3	4
Frequencies	122	60	15	2	1

(Bhopal 1974; Indore 90; Vikram 91, 93)

Solution : Here,

Mean, $$m = \frac{122\times0+60\times1+15\times2+2\times3+1\times4}{122+60+15+2+1}$$

$$= \frac{60+30+6+4}{200} = \frac{1}{2} = 0{\cdot}5.$$

Now $$e^{-m} = e^{-0\cdot5} = 1-(0{\cdot}5)+\frac{1}{2}(0{\cdot}5)^2-\frac{1}{6}(0{\cdot}5)^3+\ldots$$

$$= 1-0{\cdot}5+0{\cdot}125-0{\cdot}0208+\ldots = 0{\cdot}61 \text{ nearly.}$$

The theoretical frequency of *r* deaths is given by

$$N\,e^{-m}\,\frac{m^r}{r!} = 200\times(\cdot61)\,\frac{(0{\cdot}5)^r}{r!}.$$

Computation :

x	$P(X=x)$	$NP(X=x)$, *Expected Frequency*
0	$e^{-m}=0{\cdot}61$	$200\times0{\cdot}61=122$
1	$me^{-m}=0{\cdot}305$	$200\times0{\cdot}305=61$
2	$\frac{m^2}{2!}e^{-m}=0{\cdot}0762$	$200\times0{\cdot}0762=15$
3	$\frac{m^3}{3!}e^{-m}=0{\cdot}0127$	$200\times0{\cdot}0127=2$
4	$\frac{m^4}{4!}e^{-m}=0{\cdot}0016$	$200\times0{\cdot}0016=0$
Total	$1{\cdot}0055 \cong 1$	200

Example 2. *Find the mean and standard deviation for the table of deaths of women over 85 year old recorded in a three year period.*

No. of deaths recorded in a day	0	1	2	3	4	5	6	7
No. of days	364	376	218	89	33	13	2	1

(Bhopal 1997; Sagar 89)

Find the expected number of days with one death recorded for the Poisson series fitted to the data.

Solution : Here $N = 1096$ (total frequencies)

x	f	fx	fx^2
0	364	0	0
1	376	376	376
2	218	436	872
3	89	267	801
4	33	132	528
5	13	65	325
6	2	12	72
7	1	7	49
	1096	1295	3023

Hence,

$$m = \frac{1295}{1096} = 1 \cdot 18.$$

$$\sigma^2 = \frac{3023}{1096} - (1 \cdot 18)^2$$
$$= 2 \cdot 758 - 1 \cdot 392$$
$$= 1 \cdot 366.$$

$$\therefore \quad \sigma = 1 \cdot 17 \text{ nearly.}$$

$$e^{-1 \cdot 17} = 1 - 1 \cdot 17 + \frac{(1 \cdot 17)^2}{2!} - \frac{(1 \cdot 17)^3}{3!} + \frac{(1 \cdot 17)^4}{4!} - \frac{(1 \cdot 17)^5}{5!} + \ldots$$
$$= 1 - 1 \cdot 17 + 0 \cdot 68445 - 0 \cdot 26693 + 0 \cdot 67808 - 0 \cdot 018 \cong 0 \cdot 31.$$

For one death, frequency $= Ne^{-m}\, m$

$$= 1096 \times (0 \cdot 311) \times (1 \cdot 17) = 397.5 \cong 398.$$

Example 3. *In* 1000, *consecutive issues of the 'Utopian Seven Daily Chronicle' the deaths of centenarians were recorded, the number x having frequency f according to the table.*

x	0	1	2	3	4	5	6	7	8
f	229	325	257	119	50	17	2	1	0

Show that the distribution is roughly Poissonian by calculating its mean, and then the frequencies in the Poisson distribution with the same mean and the same total frequency of 1000. *Also calculate the variance of the given distribution and compare it with the mean [given $e^{-1 \cdot 5} = \cdot 2231$ approximately].*

Solution :

x	f	fx	fx^2
0	229	0	0
1	325	325	325
2	257	514	1028
3	119	357	1071
4	50	200	800
5	17	85	425
6	2	12	72
7	1	7	49
8	0	0	0
Total	1000	1500	3770

$$\text{Mean of the series} = \frac{1500}{1000} = 1\cdot5 = m.$$

$$\text{Variance} = \sigma^2 = \frac{3770}{1600} - (1\cdot5)^2 = 3\cdot77 - 2\cdot25 = 1\cdot52.$$

Hence

$$\sigma = 1\cdot2 \text{ and } m = 1\cdot5.$$

But in Poisson distribution

$$\sigma = \sqrt{m} = \sqrt{1\cdot5} = 1\cdot2.$$

Hence the distribution is roughtly Poissonian.

Frequencies are given by $Ne^{-m} \dfrac{m^r}{r!}$

$$e^{-m} = e^{-1\cdot5} = 0\cdot2231.$$

Hence corresponding frequencies are as given below :

$$223\cdot1 \times \frac{m_0}{0!}, 223\cdot1 \times \frac{1\cdot5}{1!}, 223\cdot1 \times \frac{(1\cdot5)^2}{2!}, \ldots$$

x	$P(X=x)$	$NP(X=x)$
0	$e^{-m} = 0\cdot2231$	$1000 \times 0\cdot2231 = 223\cdot1$
1	$me^{-m} = 0\cdot3347$	$1000 \times 0\cdot3347 = 334\cdot7$
2	$\frac{m^2}{2!} e^{-m} = 0\cdot2510$	$1000 \times 0\cdot2510 = 251\cdot0$
3	$\frac{m^3}{3!} e^{-m} = 0\cdot1255$	$1000 \times 0\cdot1255 = 125\cdot5$
4	$\frac{m^4}{4!} e^{-m} = 0\cdot0471$	$1000 \times 0\cdot0471 = 47\cdot1$
5	$\frac{m^5}{5!} e^{-m} = 0\cdot0141$	$1000 \times 0\cdot0141 = 14\cdot1$
6	$\frac{m^6}{6!} e^{-m} = 0\cdot0035$	$1000 \times 0\cdot0035 = 3\cdot5$
7	$\frac{m^7}{7!} e^{-m} = 0\cdot0008$	$1000 \times 0\cdot0008 = 0\cdot8$
8	$\frac{m^8}{8!} e^{-m} = 0\cdot00002$	$1000 \times 0\cdot0008 = 0\cdot2$
Total	$1\cdot0000$	$1000\cdot0$

Example 4. *In 1,000 extensive sets of trials for an event of small probability the frequencies f of the number x of successes are found to be :*

x	0	1	2	3	4	5	6	7
f	305	365	210	80	28	9	2	1

Assuming it to be a Poissonian distribution calculate its mean, variance and expected frequencies for the Poissonian distribution with same mean.

(Agra 1983; Delhi 64)

Solution :

x	f	fx	fx^2
0	305	0	0
1	365	365	365
2	210	420	840
3	80	240	720
4	28	112	448
5	9	45	225
6	2	12	72
7	1	7	49
	$N = 1000$	1201	2719

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{1201}{1000} \equiv 1{\cdot}2 \text{ approximately}$$

$$\sigma^2 = \frac{\Sigma fx^2}{N} = \frac{2719}{1000} - (1{\cdot}2)^2 = 2{\cdot}719 - 1{\cdot}144 = 1{\cdot}279$$

$$e^{-m} = e^{-1\cdot 2} = 1 - 1{\cdot}2 + \frac{(1{\cdot}2)^2}{2!} - \frac{(1{\cdot}2)^3}{3!} + \ldots = {\cdot}3012.$$

The expected frequencies for $x = 0, 1, 2, 3, 4, 5, 6, 7$ are respectively,

$$e_0 = Ne^{-m} = {\cdot}3012 \times 1000 = 301{\cdot}2,$$

$$e_1 = Ne^{-m}\, m = 301{\cdot}2 \times 1{\cdot}2 = 361{\cdot}4,$$

$$e_2 = Ne^{-m}\, \frac{m^2}{2!} = 301{\cdot}2 \times \frac{(1{\cdot}2)^2}{2!} = 216{\cdot}8,$$

$$e_3 = Ne^{-m} \cdot \frac{m^3}{3!} = 301{\cdot}2 \times \frac{(1{\cdot}2)^3}{3!} = 86{\cdot}7,$$

$$e_4 = Ne^{-m} \cdot \frac{m^4}{4!} = 301{\cdot}2 \times \frac{(1{\cdot}2)^4}{4!} = 26,$$

$$e_5 = Ne^{-m} \cdot \frac{m^5}{5!} = 301{\cdot}2 \times \frac{(1{\cdot}2)^5}{5!} = 6{\cdot}2,$$

$$e_6 = Ne^{-m} \cdot \frac{m^6}{6!} = 301{\cdot}2 \times \frac{(1{\cdot}2)^6}{6!} = 1{\cdot}2,$$

$$e_7 = Ne^{-m} \cdot \frac{m^7}{7!} = 301{\cdot}2 \times \frac{(1{\cdot}2)^7}{7!} = 0{\cdot}2.$$

Example 5. *In a book of 300 pages, a proofreader finds no error in 200 pages, in 75 pages one error on each page, in 20 pages two errors on each page and in 5 pages 3 errors on each page. Use Poisson distribution to these data and calculate theoretical frequency* [$e^{-0.43} = 0.6505$]

Solution : The observed frequency distribution is

No. of errors per page x	0	1	2	3	Total
No. of pages f	200	75	20	5	300

$$\therefore \quad \text{Mean } m = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{0\times 200+1\times 75+2\times 20+3\times 5}{300} = 0.43$$

Let Poisson's distribution be

$$P(X=x) = p(x) = e^{-m}(m^x/x!)$$

$$= e^{-0.43}\frac{(0.43)^x}{x!} = 0.6505\frac{(0.43)^x}{x!}$$

$\therefore$ The theoretical frequency for x errors

$$= Ne^{-m}\frac{m^x}{x!} = 300 \times 0.6505\frac{(0.43)^x}{x!} \quad ...(1)$$

Computation :

x	$P(X=x)$	$NP(X=x)$
0	$e^{-m} = 0.6505$	$300 \times 0.6505 = 195.15$
1	$me^{-m} = 0.2797$	$300 \times 0.2797 = 83.91$
2	$\frac{m^2}{2!}e^{-m} = 0.0601$	$300 \times 0.0601 = 18.03$
3	$\frac{m^3}{3!}e^{-m} = 0.0086$*	$300 \times 0.0086 = 2.58$*
Total	$0.9989 \cong 1.0000$	$299.67 \cong 300$

Remark : * However to have a total exactly equal to 1 we can find the last value by sub size 6 traction. Thus

$$P(3) = 1-[p(0)+p(1)+p(2)+...]$$
$$= 1-(.6505+.2797+.0601)$$
$$= 1-(.9903) = .0097$$

and the corresponding expected frequency $= 300 \times .0097 = 2.91$.

Exercise 5 (B)

1. Give some examples of the occurrence of Poisson distributon in different fields. Under the conditions to be stated derive Poisson distribution as a limiting form of a Binomial distribution. (Bhopal 1974; Jabalpur 88; Sagar 93, 94)