Module-4-Test of Significance

Testing of Hypothesis

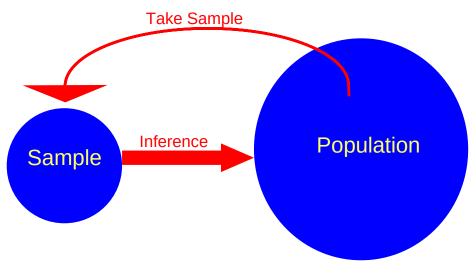
Important Definitions and Results:

Population:

The group of individuals, under study is called population. The population may be finite or infinite.

Sample:

A finite subset of statistical individuals in a population is called sample.



Sample size:

The number of individuals in a sample is called the sample size. It is denoted by n when n<30, then the sample is called small sample when n≥30, then the sample is called large sample.

Sampling error:

On examining a sample of a particular stuff we arrive at a decision of purchasing or rejecting the stuff. The error involved in such approximation is known as sampling error.

Parameters & statistics:

Parameters : The statistical constants of population(namely mean μ &,varianceσ^2) parameters.

statistics: The Statistical constants computed from sample observations alone(namely mean $\bar{x}$&,variances^2)

Notation:

	Population	sample
Size	N	n
Mean	μ	$\bar{x}$
Standard Deviation	σ	s
Proportion	P	p

Sampling distribution:

If we draw a sample of size 'n' from a given finite population of size N then total number of possible samples is ${}^{N}c_n$ .From each of the ${}^{N}c_n$ samples if we compute a statistic (mean, variance)and then we form a frequency distribution for these ${}^{N}c_n$ values of a statistic. Such a distribution is called sampling distribution of that statistic.

Standard error:

The Standard deviation of the sampling distribution is called standard error.

Testing of Hypothesis:

Hypothesis:

A Statistical hypothesis is a statement about a population parameter.

Hypothesis testing(or) Test of significance:

The procedure which enable us to decide on the basis of sample result whether a hypothesis is true or not is called test of hypothesis (or) test of significance there are two type

1.Null Hypothesis

2.Alternative hypothesis

Null hypothesis: Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true . Null hypothesis is denoted by H_0

Alternative hypothesis:

Any hypothesis which is complementary to null hypothesis is called alternative hypothesis and is denoted by H_1 .

For example: To test the null hypothesis that the population has a specified mean μ_0

Null Hypothsis $H_0: \mu = \mu_0$

Alternative Hypothesis $(i) H_1: \mu \neq \mu_0 (Two\ tailed)$

$(ii)\ H_1: \mu < \mu_0 (left\ tailed)$

$(ii) H_1: \mu > \mu_0 (Right\ tailed)$

Errors in Sampling:

1) **Type-I error ($\boldsymbol{\alpha}$):** Reject H_0When H_0 is true

2) **Type-II error ($\boldsymbol{\beta}$):** Accept H_0When H_0 is false

P(rejecting a good lot)= α

P(Accepting a bad lot)= β

Critical region:

A region in which null hypothesis H_0 is rejected is called the critical region (or) region of rejection .

Level of significance (l.o.s):

The maximum probability of making type-I error is called level of significance and is denoted by α . Where α = P (making Type-I error)

= P (H_0 is rejected when it is true)

This can be measured in terms of percentage i.e. 5%, 1%, 10% etc…….

Power of the test:

The probability of rejecting a false hypothesis is called power of the test and is denoted by $1-\beta$

Power of the test =P (H_0 is rejected when it is true)

= 1- P (H_0 is accepted when it is false)

Test statistic:

The test statistic is defined as the difference between the sample statistic value and the hypothetical value, divided by the standard error of the statistic.

i.e. test statistic $Z = \dfrac{t - E(t)}{S.E(t)}$

= 1- P (Committing Type-II error) = 1- β

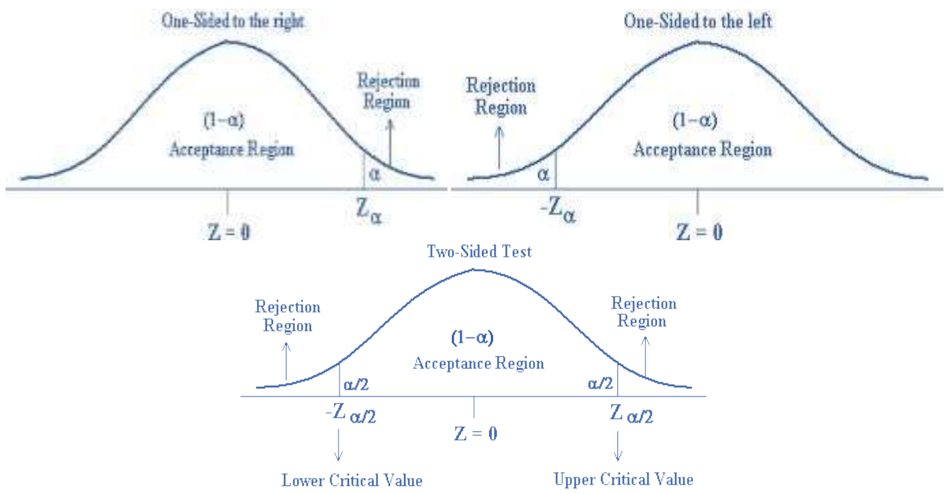
One tailed and two tailed tests:

A test with the null hypothesis $H_0 : \theta = \theta_0$ against the alternative hypothesis $H_1 : \theta \neq \theta_0$, it is called a **two tailed test**. In this case the critical region is located on both the tails of the distribution.

A test with the null hypothesis $H_0 : \theta = \theta_0$ against the alternative hypothesis $H_1 : \theta > \theta_0$ (right tailed alternative) or $H_1 : \theta < \theta_0$ (left tailed alternative) is called **one tailed test**. In this case the critical region is located on one tail of the distribution.

$H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ ------- right tailed test

$H_0 : \theta = \theta_0$ against $H_1 : \theta < \theta_0$ ------- left tailed test



Utility of standard error:

1. It is a useful instrument in the testing of hypothesis. If we are testing a hypothesis at 5% l.o.s and if the test statistic i.e. $|Z| = \left|\frac{t - E(t)}{S.E(t)}\right| > 1.96$ then the null hypothesis is rejected at 5% l.o.s otherwise it is accepted.
2. With the help of the S.E we can determine the limits with in which the parameter value expected to lie.
3. S.E provides an idea about the precision of the sample. If S.E increases the precision decreases and vice-versa. The reciprocal of the S.E i.e. $\frac{1}{S.E}$ is a measure of precision of a sample.
4. It is used to determine the size of the sample.

Procedure for testing of hypothesis:

1. Set up a null hypothesis i.e. $H_0 : \theta = \theta_0$.
2. Set up a alternative hypothesis i.e. $H_1 : \theta \neq \theta_0$ or $H_1 : \theta > \theta_0$ or $H_1 : \theta < \theta_0$
3. Choose the level of significance i.e. α .
4. Select appropriate test statistic Z.
5. Select a random sample and compute the test statistic.
6. Calculate the tabulated value of Z at α % l.o.s i.e. Z_α .
7. Compare the test statistic value with the tabulated value at α % l.o.s. and make a decision whether to accept or to reject the null hypothesis.

Test of significance of small samples:

When the size of the sample n<30, then the sample is a small sample.

The following are some important tests for small samples.

i. Student's 't' test
ii. F-test
iii. χ^2-test

Small sample tests:

Student's 't' test(For single mean):

The student's 't' is defined by the statistic,

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

Where $\bar{x} = \frac{1}{n}\sum x_i = sample\ mean$ $\qquad$ $\mu = population\ mean$

$s^2 = \frac{1}{n-1}\sum_i (x_i - \bar{x})^2$ $\qquad$ $n = sample\ size$

$n-1 = degrees\ of\ freedom$

Note:

1. If the standard deviation of a sample is **given directly** then $\boldsymbol{t} = \frac{\bar{\boldsymbol{x}}-\boldsymbol{\mu}}{\frac{\boldsymbol{S.D}}{\sqrt{\boldsymbol{n-1}}}}$
2. If the standard deviation of a sample is **not given directly (i.e., if we have to find S.D)** then $\boldsymbol{t} = \frac{\bar{\boldsymbol{x}}-\boldsymbol{\mu}}{\frac{\boldsymbol{S.D}}{\sqrt{\boldsymbol{n}}}}$
3. If calculated value of $|t|$ > tabulated value of $|t|$ at 5% or 1% l.o.s then the null hypothesis H_0 is rejected.
4. If calculated value of $|t|$ < tabulated value of $|t|$ at 5% or 1% l.o.s then the null hypothesis H_0 is accepted.
5. If $t_{0.05}$ is the table value of $|t|$ for n-1 degrees of freedom at 5% l.o.s then 95% confidence limit for μ is given by $\bar{\boldsymbol{x}} \pm \boldsymbol{t_{0.05}} \frac{\boldsymbol{S}}{\sqrt{\boldsymbol{n-1}}}$

Applications (uses) of t-distribution:

1. To test if the sample mean $(\bar{x})$ differs significantly from the hypothetical value μ of population mean.
2. To test the significance of the difference between two sample means.
3. To test the significance of an observed sample correlation coefficient and sample regression coefficient.

Assumptions for t-distribution:

1. The parent population from which sample is drawn is normal.
2. The population standard deviation σ is unknown.
3. Sample size n<30.

Student's 't'-test for single mean:

Problems:

Type-I When S.D is given directly:

1. **A machinist is making engine parts with axle diameters of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a S.D of 0.40 inch. Compute the statistic you would use to test whether the work is meeting specifications.**

Solution: Given n = 10<30 (small sample)

Sample mean $\bar{x}$ = 0.742 inches.

Population mean μ = 0.700 inches.

Sample S.D = 0.040 inches.

Null Hypothesis: H_0 : The product is meeting the specifications. i.e., μ = 0.700

Alternative Hypothesis: H_1 : The product is not meeting the specifications. i.e., $\mu \neq$ 0.700(Two tailed)

Test Statistic:

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = \frac{0.742-0.700}{\frac{0.040}{\sqrt{10-1}}} = 3.15$$

Calculated $|t|$ = 3.15

Tabulated $|t|$ at 5% level with 10-1 = 9 degrees of freedom is 2.26.

∵ Calculated $|t|$ > Tabulated $|t|$(i.e., 3.15 > 2.26)

∴ H_0 is rejected.

2. **The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increases to 153.7 and showed a S.D of 172, was advertising campaign successful.**

Solution: Given n = 22<30 (small sample)

Sample mean $\bar{x}$ = 153.7

Population mean μ = 146.3.

Sample S.D = 17.2

Null Hypothesis: H_0 : The advertising campaign was not successful. i.e., μ = 146.3

Alternative Hypothesis: H_1 : The advertising campaign was successful. i.e., μ > 146.3(Right tailed)

Test Statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \frac{153.7 - 146.3}{\frac{17.2}{\sqrt{22-1}}} = 1.97$$

Calculated $|t|$ = 1.97

Tabulated $|t|$ at 5% level with 22-1 = 21 degrees of freedom is 1.72.

∵ Calculated $|t|$ > Tabulated $|t|$(i.e., 1.97 > 1.72)

∴ H_0 is rejected.

i.e., The advertising campaign was successful.

3. **A sample of 26 bulbs gives a mean life of 990 hours with a S.D of 20 hours. The manufacture claims that the mean life of bulbs is 1000 hours. Is the sample not up to the standard?**

Solution: Given n = 26<30 (small sample)

Sample mean $\bar{x}$ = 990

Population mean μ = 1000

Sample S.D = 20

Null Hypothesis: H_0 : The sample is upto the standard. i.e., μ = 1000.

Alternative Hypothesis: H_1 : The advertising campaign was successful. i.e., μ < 146.3(Left tailed)

Test Statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \frac{990 - 1000}{\frac{20}{\sqrt{26-1}}} = -2.5$$

Calculated $|t|$ = 2.5

Tabulated $|t|$ at 5% level with 26-1 = 25 degrees of freedom is 1.708.

∵ Calculated $|t|$ > Tabulated $|t|$(i.e., 2.5 > 1.708)

∴ H_0 is rejected.

i.e., The sample is not up to the standard.

4. **The mean life time of a sample of 25 fluorescent light bulbs produced by a company is computed to be 1570 hrs with a S.D 120 hrs. The company claims that the average life of the bulbs produced by the company is 1600 hrs using the l.o.s of 0.05. Is the claim complete.**

Solution: Given n = 25<30 (small sample)

Sample mean $\bar{x}$ = 1570

Population mean μ = 1600

Sample S.D = 120

Null Hypothesis: H_0 : The claim is acceptable. i.e., μ = 1600 hrs.

Alternative Hypothesis: H_1 : $\mu \neq$ 1600 hrs (Two tailed).

Test Statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \frac{1570 - 1600}{\frac{120}{\sqrt{25-1}}} = -1.22$$

Calculated $|t|$ = 1.22

Tabulated $|t|$ at 5% level with 25-1 = 24 degrees of freedom is 2.06.

∵ Calculated $|t|$ > Tabulated $|t|$(i.e., 1.22 < 2.06)

∴ H_0 is accepted.

i.e., The claim that the average life of the bulbs produced by the company is 1600 hrs is acceptable.

5. **A soap manufacturing company was distributing a particular brand of soap through a large number of retail shops. Before a heavy advertisement campaign, the man sales per week per shop was 140 dozens. After the campaign, a sample of 26 shops was taken and the mean sales was found to be 147 dozens with standard deviation 16. What conclusion do you draw on the impact of advertisement on sales? Use 5% significance level.**

Solution: It is given S.D = 16, n = 26, $\bar{x}$ = 147, μ = 140

Null Hypothesis: H_0: The advertisement was not successful. i.e., μ = 140 against

Alternative Hypothesis: $H_1 = \mu \neq 140$

Test Statistic:

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = \frac{147-140}{\frac{16}{\sqrt{25}}} = \frac{7}{\frac{16}{5}} = \frac{7}{3.2} = 2.19$$

From the table, for (26-1) = 25 d.o.f., t_{005} = 2.06. Since Computed value of t > $t_{0.05}$, we reject the null hypothesis. That is, the advertisement may be considered to have changed the average sales volume.

6. **A random sample of 25 cups from a certain coffee dispensing machine yields a mean 6.9ounces per cup. Use α = 0.05 level of significance to test, on the average, the machine dispense μ = 7.0 ounces. Assume that the distribution of ounces per cup is normal, and that the variance is the known quantity σ^2 = 0.01ounces.**

Solution:

Given n = 25, $\bar{x} = 6.9, \mu = 7.0, \sigma^2 = 0.01$

$S.D = 0.01 = \sigma$

Null Hypothesis H_0: μ =7 (The sample mean $\bar{x}$ does not differ significantly from the population mean μ .

Alternative Hypothesis H_1: $\mu \neq 7$.

Test statistic:

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} = \frac{6.9-7}{0.1/\sqrt{24}} = -4.89$$

Calculated Value of $|t| = 4.89$ and table value = 1.645

Since the calculated value of t > the tabulated value of t, we reject the null hypothesis H_0.
i.e., the population mean μ = 7 is unacceptable.

Home Work:

1. A machine is designed to produce insulating washers for electrical devices of average thickness of 0.025 cm. A random sample of 10 washers was found to have a thickness of 0.024 cm with a S.D of 0.002 cm. Test the significance of the deviation. Table value for t for 9 degrees of freedom at 5% l.o.s is 2.262.
   ($t_{0.05}$=2.262, D.f = 9, $|t| = 1.5$)
2. The average breaking strength of the steel rods is specified to be 18.5 thousand pounds. To test this a sample of 14 rods was tested. The mean and standard deviations obtained were 17.85 and 1.955 respectively. Is the result of the experiment significant?
   ($t_{0.05}$=2.16, D.f =13, $|t| = 1.199$)

Student's t-test:

Type-II When S.D is not given directly:

1. **A random sample of 10 boys had the following I.Q's 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Does the data support the assumption of a population mean I.Q of 100? Find a reasonable range in which most of the mean I.Q values of samples of 10 boys lie.**

Solution: Given n = 10<30 (small sample)

Since the standard deviation (S.D) is not given, we have to find out S.D and sample mean.

$$\bar{x} = \frac{\sum x}{n}, s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$$

x	x-$\bar{x}$ x-97.2	(x-$\bar{x}$)^2
70	70-97.2 = -27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.84
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84
$\sum x = 972$		1833.60

Sample mean $\bar{x} = \frac{\sum x}{n} = \frac{972}{10} = 97.2$

$s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2 = \frac{1833.60}{9} = 203.73$

S.D = $\sqrt{203.73} = 14.27$

Null Hypothesis: H_0 : The data support the assumption of a population mean I.Q of 100 i.e., μ = 100

Alternative Hypothesis: H_1 : $\mu \neq 100$ (two tailed)

Test Statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{97.2 - 100}{\frac{14.72}{\sqrt{10}}} = -0.61$$

Calculated $|t|$ = 0.61

Tabulated $|t|$ at 5% level with 10-1 = 9 degrees of freedom is 2.26.

∵ Calculated $|t|$ > Tabulated $|t|$(i.e., 0.61 < 2.26)

∴ H_0 is accepted.

i.e., The data support the assumption of a population mean I.Q of 100.

Next to find the reasonable range in which most of the mean I.Q values of samples of 10 boys lie.

$$\bar{x} \pm t_{0.05}\frac{s}{\sqrt{n-1}} = 97.2 \pm 2.26 \times 4.514 = 97.2 \pm 10.20 = 107.41, 86.99$$

The 95% confidence limits with in which the mean I.Q values of sample of 10 boys will lie in (range), $[86.99, 107.41]$

2. **The height of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches?**

Solution: Given n = 10<30 (small sample)

Since the standard deviation (S.D) is not given, we have to find out S.D and sample mean.

$$\bar{x} = \frac{\sum x}{n}, S^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$$

x	x-$\bar{x}$	(x-$\bar{x}$)^2

	x-66	
70	70-66 = 4	16
67	1	1
62	-4	16
68	2	4
61	-5	25
68	2	4
70	4	16
64	-2	4
64	-2	4
66	0	0
$\sum x = 660$		90

Sample mean $\bar{x} = \frac{\sum x}{n} = \frac{660}{10} = 66$

$s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2 = \frac{90}{9} = 10$

S.D = $\sqrt{10} = 3.16$

Null Hypothesis: H_0 : The average height is not greater than 64 inches

i.e., $\mu = 64$

Alternative Hypothesis: H_1 : $\mu > 64$ (Right tailed)

Test Statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{66 - 64}{\frac{3.16}{\sqrt{10}}} = 2$$

Calculated $|t|$ = **2**

Tabulated $|t|$ at 5% level with 10-1 = 9 degrees of freedom is 1.833.

∵ Calculated $|t|$ > Tabulated $|t|$(i.e., 2 > 1.833)

∴ H_0 is rejected.

i.e., The average height is greater than 64 inches

3. **A certain pesticide is packed into bags by a machine. A random sample of 10 bags is drawn and their contents are found to weight (in Kg) as follows: 50, 49, 52, 44, 45, 48, 46, 45, 49, 45. Test if the average packing can be taken to be 50 kg. ($t_{0.05}$ = 2.262 at 9 d.f)**

Solution: Given n = 10<30 (small sample)

Since the standard deviation (S.D) is not given, we have to find out S.D and sample mean.

$$\bar{x} = \frac{\sum x}{n}, s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$$

x	x-$\bar{x}$ x-47.3	(x-$\bar{x}$)^2
50	50-47.3 = 2.7	7.29
49	1.7	2.89
52	4.7	22.09
44	-3.3	10.89
45	-2.3	5.29
48	0.7	0.49
46	-1.3	1.69
45	-2.3	5.29
49	-1.7	2.89
45	-2.3	5.29
$\sum x = 473$		64.1

Sample mean $\bar{x} = \frac{\sum x}{n} = \frac{473}{10} = 47.3$

$s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2 = \frac{64.1}{9} = 7.12$

S.D = $\sqrt{7.12} = 2.67$

Null Hypothesis: H_0 : The average packing is 50 kgs.

i.e., μ = 50

Alternative Hypothesis: H_1 : $\mu \neq 50$ (two tailed)

Test Statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{47.3 - 50}{\frac{2.67}{\sqrt{10}}} = -3.19$$

Calculated $|t|$ = $\mathbf{3.19}$

Tabulated $|t|$ at 5% level with 10-1 = 9 degrees of freedom is 2.262.

∵ Calculated $|t|$ > Tabulated $|t|$(i.e., $\mathbf{3.19}$> 2.262)

∴ H_0 is rejected.

i.e., The average packing is not 50 kgs.

4. **The individuals are chosen at random from a population and their highest are found to be in inches 63, 63, 66, 67, 68, 69, 70, 70, 71 and 71. In the light of these data, decreases the suggestion that the mean height in the population is 66. Use 5% significance level.**

Solution: Given n = 10<30 (small sample)

Since the standard deviation (S.D) is not given, we have to find out S.D and sample mean.

$$\bar{x} = \frac{\sum x}{n}, s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
63	-4.8	23.04
63	-4.8	23.04
66	-1.8	3.24
67	-0.8	0.64
68	0.2	0.04
69	1.2	1.44
70	2.2	4.84
70	2.2	4.84
71	3.2	10.24
71	3.2	10.24
$\sum x = 678$		$\sum(x - \bar{x})^2 = 81.60$

$\bar{x} = \frac{678}{10} = 67.8$

$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{81.60}{9} = 9.066$

$\therefore s = \sqrt{9.066} = 3.011$

Null Hypothesis: H_0 : μ = 66” against

Alternative Hypothesis: H_1 : μ ≠ 66”

Test Statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{67.8 - 66}{\frac{3.01}{\sqrt{10}}} = \frac{1.80}{\frac{3.01}{3.16}} = 1.89$$

Calculated $|t|$ = 1.89

Two tailed table value of t with 9d. D. f at 5% level is found to be 2.26.

As computed value as t < tabulated 't'. So computed t does not lie in the critical region. We accept H_0 and conclude that the experiment provides no ground for doubt that the mean height is 66".

5. **A certain medicine administered to each 10 patients resulted in the following increase in the blood pressure 8, 8, 7, 5, 4, 1, 0, 0,-1,-1. Can it be concluded that the medicine was responsible for the increase in blood pressure.**

Solution: Given $\mu = 0$, n = 10 (Small sample n < 30) .
Since the standard deviation (S.D) is not given, we have to find out S.D and sample mean.

$$\bar{x} = \frac{\sum x}{n}, s^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
8	4.9	24.01
8	4.9	24.01
7	3.9	15.21
5	1.9	3.61
4	0.9	0.81
1	-2.1	4.41
0	-3.1	9.61
0	-3.1	9.61
-1	-4.1	16.81
-1	-4.1	16.81
$\sum x = 31$		$\sum (x - \bar{x})^2 = 124.9$

$$\bar{x} = \frac{\sum x}{n} = \frac{31}{10} = 3.1$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{124.9}{9} = 13.88$$

$$\therefore s = \sqrt{13.88} = 3.72$$

Null Hypothesis: $H_0 : \mu = 0$

Alternative Hypothesis: $H_1 : \mu \neq 0$ LOS $\alpha = 0.05$, d.f = 10 – 1 = 9 = 2.26

Test statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \frac{3.1 - 0}{\frac{3.5341}{\sqrt{3}}} = 2.6314\,\mu$$

Calculated Value of $|t| = 2.6314$ and table value = 2.26

Calculated Value of $|t| = 2.6314$ > table value = 2.26

We reject the null Hypothesis and we concluded that there is significant difference; the medicine is responsible for the increase of blood pressure.

HOME WORK:

1. The average breaking strength of steel rods is specified to be 17.5. To test this, sample of 14 rods tested & gave the following results 15, 18, 16, 21, 19, 17, 17, 15, 17, 20, 19, 17, 18. Is the result of the experiment significant? Also obtain the 95% confidence limits.
($t_{0.05}$=2.16, D.f =13, $|t| = 0.7090$)

2. A random sample of 16 values from a normal population showed a mean of 53 and a sum of squares of deviations from the mean equals to 150. Can this sample be regarded as taken from the population having 56 as mean? obtain the 95% confidence limits of the mean of the population.

Hint: Given $\sum(x_i - \bar{x})^2 = 150, find\ S^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$

($t_{0.05}$=2.13, D.f =15, $|t| = 3.79$)

Student's t-test:

Difference of means (independent samples):

To test the significant difference between two means $\bar{x}_1$ & $\bar{x}_2$ of sample sizes n_1 and n_2,

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$s^2 = \frac{\sum(x_1 - \bar{x}_1)^2 + \sum(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

(or)

$$s^2 = \frac{1}{n_1 + n_2 - 2}[n_1 s_1^2 + n_2 s_2^2]$$

s_1 *and* s_2 are sample standard deviation.

$n_1 + n_2 - 2 = degrees\ of\ freedom$

Problems:

6. **Samples of two types of electric light bulbs were tested for length of life and following data were obtained**

	Type-I	Type-II
Sample number	$n_1 = 8$	$n_2 = 7$
Sample means	$\bar{x}_1 = 1234\ hrs$	$\bar{x}_2 = 1036\ hrs$
Sample S.D	$s_1 = 36\ hrs$	$s_2 = 40\ hrs$

(A.U NOV/DEC 2010)

Is there a difference in the means sufficient to warrant that type I is superior to type II regarding length of life.

Solution: Given that $n_1 = 8, n_2 = 7, \bar{x}_1 = 1234, \bar{x}_2 = 1036, s_1 = 36, s_2 = 40$

Null hypothesis H_0: The two types I and II of electric bulbs are identical.

i.e., H_0: $\mu_1 = \mu_2$

Alternate Hypothesis H_1: $\mu_1 > \mu_2$ (right tailed).

To find *S*:

$$s^2 = \frac{1}{n_1 + n_2 - 2}[n_1 s_1^2 + n_2 s_2^2] = \frac{1}{8 + 7 - 2}[8(36)^2 + 7(40)^2] = 1659.08$$

$$s = \sqrt{1659.08} = 40.73$$

Test Statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{1234 - 1036}{40.73\sqrt{\frac{1}{8} + \frac{1}{7}}} = 9.39$$

Calculated $|t|$ = $\mathbf{9.39}$

Degrees of freedom (D.f): 8+7-2 = 13

Tabulated value of t for 13 D.f at 5% l.o.s is 1.77(one-tailed)

∵ Calculated $|t|$ > Tabulated $|t|$(i.e., $\mathbf{9.39}$> 1.77)

∴ H_0 is rejected.

7. **Two types of batteries are tested for their length of life and the following data are obtained:**

Battery	No.of samples	Mean life in Hrs	Variance
type A	9	600	121
type B	8	640	144

Is there is a significant difference in the two means? (Value of t for 15 degrees of freedom at 5% level is 2.131)

Solution: Given $n_1 = 9, n_2 = 8, \bar{x}_1 = 600, \bar{x}_2 = 640, {s_1}^2 = 121, {s_2}^2 = 144$

We set up **Null hypothesis H_0:** There is no significant difference in the two means

i.e., $H_0 : \mu_1 = \mu_2$

Against **Alternate Hypothesis** $H_1 : \mu_1 \neq \mu_2$

The appropriate test statistic is Fisher's t which under H_0 follows t distribution with (n_1+n_2-2) d.o.f.

To find *S*:

$$s^2 = \frac{1}{n_1+n_2-2}[n_1{s_1}^2 + n_2{s_2}^2] \Rightarrow s = \sqrt{\frac{9\times 121 + 8\times 144}{9+8-2}} = \sqrt{149.4} = 12.2$$

Test Statistic:

We compute t $\dfrac{600-640}{12.2\sqrt{\dfrac{1}{9}+\dfrac{1}{8}}} = \dfrac{-40}{12.2\times 0.486} = -6.7$

d.o.f. = $(n_1 + n_2 - 2) = 9 + 8 - 2 = 15$

Computed $|t|$ = 6.7 which is larger than tabulated t (=2.13 for 15 d.o.f. of and $\alpha = 0.05$)

So we reject H_0 and accept H_1.

Our conclusion is there is significant difference in the two means.

8. **Two random samples gave the following results.**

Sample	Size	Sample Mean	Sum of squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population using t-test

Solution: : Given $n_1 = 10, n_2 = 12, \bar{x}_1 = 15, \bar{x}_2 = 14, {s_1}^2 = 90, {s_2}^2 = 108$

Null hypothesis H_0: The two samples have been drawn from the same normal population.

i.e. H_0 = $\mu_1 = \mu_2 \ and \ \sigma_1^2 = \sigma_2^2$

Against **Alternate Hypothesis** $H_1 : \mu_1 \neq \mu_2 and \ \sigma_1^2 \neq \sigma_2^2$

The appropriate test statistic is Fisher's t which under H_0 follows t distribution with (n_1+n_2-2) d.o.f.

To find *S*:

$$s^2 = \frac{1}{n_1+n_2-2}\left[\sum(x_1-\overline{x_1})^2 + \sum(x_2-\overline{x_2})^2\right] = \frac{1}{20}(90+108) = 9.9$$

Test Statistic:

The test statistic t = $\dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}} = \dfrac{15-14}{\sqrt{9.9\left(\dfrac{1}{10}+\dfrac{1}{12}\right)}}$ = 0.742.

d.f = $(n_1 + n_2 - 2) = 20$.

Tabulated value of t for 20 d.f at 5% level of significance is 2.086.

Since calculated value of t = 0.74 < the tabulated value of t = 2.086, we accept the null hypothesis H_0 = $\mu_1 = \mu_2 \ and \ \sigma_1^2 = \sigma_2^2$

Hence the given samples come from the same normal population

9. **Two independent samples of sizes 8 and 7 contained the following values:**

Sample-I: 19 17 15 21 16 18 16 14

Sample-II: 15 14 15 19 15 18 16

Is the difference between the sample means significant? (A.U APR/MAY 2010)

Solution: Null hypothesis H_0: There is no significant difference between the means.

i.e., $\mu_1 = \mu_2$

Alternate Hypothesis H_1: $\mu_1 \neq \mu_2$

To find S:

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
19	19 – 17 = 2	4	15	15 – 16 = -1	1
17	0	0	14	-2	4
15	-2	4	15	-1	1
21	4	16	19	3	9
16	-1	1	15	-1	1
18	1	1	18	-2	4
16	-1	1	16	0	0
14	-3	9			
$\sum x = 136$		36	$\sum y = 112$		20

$\bar{x} = \frac{136}{8} = 17, \bar{y} = \frac{112}{7} = 16, n_1 = 8, n_2 = 7$

$$s^2 = \frac{\sum(x_1-\bar{x}_1)^2 + \sum(x_2-\bar{x}_2)^2}{n_1+n_2-2} = \frac{1}{8+7-2}[36+20] = 4.30$$

$$s = \sqrt{4.30} = 2.07$$

Test Statistic:

$$t = \frac{\bar{x}-\bar{y}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{17-16}{2.07\sqrt{\frac{1}{8}+\frac{1}{7}}} = 0.93$$

Calculated $|t|$ = 0.93

Degrees of freedom (D.f): 8+7-2 = 13

Tabulated value of t for 13 D.f at 5% l.o.s is 2.16(two-tailed)

∵ Calculated $|t|$ < Tabulated $|t|$(i.e., 1.39< 2.16)

∴ H_0 is accepted.

10. **Below are given the gain of weight (in lbs) of pigs fed on two diets A & B.**

Diet A	**25**	**32**	**30**	**34**	**24**	**14**	**32**	**24**	**30**	**31**	**35**	**25**	**-**	**-**	**-**
Diet B	**44**	**34**	**22**	**10**	**47**	**31**	**40**	**30**	**32**	**35**	**18**	**21**	**35**	**29**	**22**

Test if the two diets differ significantly as regards to their effect on increase in weight.

Solution: Null hypothesis H_0: There is no significant difference between the mean increase in weight due to diets A & B. i.e., $\mu_1 = \mu_2$

Alternate Hypothesis H_1: $\mu_1 \neq \mu_2$

To find *S*:

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
25	25 – 28 = - 3	9	44	44 – 30 = 14	196
32	4	16	34	4	16

30	2	4	22	-8	64
34	6	36	10	-20	400
24	-4	16	47	17	289
14	-14	196	31	1	1
32	4	16	40	10	100
24	-4	16	30	0	0
30	2	4	32	2	4
31	3	9	35	5	25
35	7	49	18	-12	144
25	3	9	21	-9	81
-	-	-	35	5	25
-	-	-	29	-1	1
-	-	-	22	-8	64
$\sum \boldsymbol{x} = \mathbf{336}$		380	$\sum \boldsymbol{y} = \mathbf{450}$		1410

$\bar{x} = \frac{336}{12} = 28, \bar{y} = \frac{450}{15} = 30, n_1 = 12, n_2 = 15$

$$S^2 = \frac{\sum(x_1 - \bar{x}_1)^2 + \sum(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2} = \frac{1}{12 + 15 - 2}[380 + 1410] = 71.6$$

$$s = \sqrt{71.6} = 8.46$$

Test Statistic:

$$t = \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{28 - 30}{8.46\sqrt{\frac{1}{12} + \frac{1}{15}}} = -0.609$$

Calculated $|t|$ = 0.609

Degrees of freedom (D.f): 12+15-2 = 25

Tabulated value of t for 25 at 5% l.o.s is 2.06(two-tailed)

∵ Calculated $|\boldsymbol{t}|$ < Tabulated $|\boldsymbol{t}|$(i.e., $\mathbf{0.609}$< 2.06)

∴ H_0 is accepted.

HOME WORK:

11. The average number of articles produced by two machines per day are 200 & 250 with S.D 20 & 25 respectively on the basis of records of 25 days production. Can you regard both the machines equally efficient at 1% l.o.s
    (s = 23.10, $|\boldsymbol{t}| = \mathbf{7.65}$, $t_{0.01}$ = 2.58, D.f = 48)
12. The means of two random samples of size 9 and 7 are 196.42 & 198.82 respectively. The sum of the squares of deviation from mean are 26.94 & 18.73 respectively. Can the sample be considered to have been drawn from the same normal population.
    (s = 1.81, $|\boldsymbol{t}| = \mathbf{2.63}$, $t_{0.05}$ = 2.15, D.f = 14)
13. The height of six randomly chosen sailors are in inches 63, 65, 68, 69, 71 & 72. Those of 10 randomly chosen soldiers are 61,62, 65, 66, 69, 69, 70, 71, 72 & 73. Discuss the light that these data throw on the suggestion that sailors are on the average taller than soldiers.
    (s = 15.257, $|\boldsymbol{t}| = \mathbf{0.099}$, $t_{0.05}$ =1.76, D.f = 14)

Test for equality of two means – paired t test

(Testing of significance of difference of two means – Dependent samples)

This test used when for the same sample values which indicate an increase or decrease inthe variable is given. Before and after values of the variable are normally given or the differences may be given directly.

Here **d = (X_{after} – X_{before}) or (X_{before} – X_{after})**

We take **Null Hypothesis** $H_0 : \underset{d}{\mu} = 0$ against $H_1 : \underset{d}{\mu} \neq 0$

Or **Alternative Hypothesis** $H_1 : \underset{d}{\mu} > 0$ or $H_1 : \underset{d}{\mu} < 0$

Test statistic:

$$t = \frac{\bar{d}}{s / \sqrt{n}}$$

Where,

$$\bar{d} = \sum d / n \quad \text{and} \quad s = \sqrt{\frac{\sum (d - \bar{d})^2}{n-1}} = \sqrt{\frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right]}$$

and **d. o. f** is **n – 1.**

Problems:

1. **To verify whether a course in accounting improved performance, a similar test was given to 12 participant both before and after the course. The marks are**

Before course	**44**	**40**	**61**	**52**	**32**	**44**	**70**	**41**	**67**	**72**	**53**	**72**
After course	**53**	**38**	**69**	**57**	**46**	**39**	**73**	**48**	**73**	**74**	**60**	**78**

Whether the course is useful?

Solution: Given $n = 12$ and the samples are dependent, so we use paired t-test.

Null hypothesis H_0: The course is not useful.

i.e., $\bar{d} = 0$

Alternate Hypothesis H_1: $\bar{d} \neq 0$. We take **d = ($X_{after} - X_{before}$).**

Before course	After course	$d = A.C - B.C$	d^2
44	53	9	81
40	38	-2	4
61	69	8	64
52	57	5	25
32	46	14	196
44	39	-5	25
70	73	3	9
41	48	7	49
67	73	6	36
72	74	2	4
53	60	7	49
72	78	6	36
		$\sum d = 60$	$\sum d^2$ =578

$\bar{d} = \frac{\sum d}{n} = \frac{60}{12} = 5,$

$$s^2 = \frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1} = \frac{578 - \frac{3600}{12}}{12-1} = \frac{578 - 300}{11} = 25.273$$

$$s = \sqrt{25.273} = 5.03$$

Test Statistic:

$$t = \frac{\bar{d}}{\frac{s}{\sqrt{n}}} = \frac{5}{\frac{5.03}{\sqrt{12}}} = 5 \times \frac{\sqrt{12}}{5.03} = 3.44$$

Calculated $|t|$ = 3.44

Degrees of freedom (D.f): n-1 = 12 – 1 = 11.

Tabulated value of t for 11 at 5% l.o.s is 2.201(two-tailed)

∵ Calculated $|\boldsymbol{t}|$ >Tabulated $|\boldsymbol{t}|$

∴ H_0 is rejected. i.e., The course is useful.

2. An IQ test was administered to 5 persons before and after they were trained. The results are given below:

	I	II	III	IV	V
IQ before training	**110**	**120**	**123**	**132**	**125**
IQ after training	**120**	**118**	**125**	**136**	**121**

Test whether there is any change in IQ after the training programme. (Given $t_{0.01,4}$=4.6)

Solution: This problem is a case for paired t test as the scores before training (x_{before}) and after training (x_{after}) are not independent, but the latter to be affected by the former i.e. they are correlated (dependent).

We take **d = (X_{after} – X_{before}).**

And set up null hypothesis $H_0 : \frac{\mu}{d} = 0$

i.e, H_0: There is no change in the I.Q after the training programmed.

against $H_1 : \frac{\mu}{d} \neq 0$

I.Q = x_{before}	I.Q = x_{after}	d	d^2
110	120	10	100
120	118	-2	4
123	125	2	4
132	136	4	16
125	121	-4	16
		$\sum d = 10$	$\sum d^2$ =140

$\bar{d} = \frac{\sum d}{n} = \frac{10}{5} = 2,$

$$s^2 = \frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1} = \frac{140 - \frac{100}{5}}{5-1} = \frac{140-20}{4} = 30$$

$$s = \sqrt{30} = 5.48$$

Test Statistic:

$$t = \frac{\bar{d}}{\frac{s}{\sqrt{n}}} = \frac{2}{\frac{5.48}{\sqrt{5}}} = 2 \times \frac{\sqrt{5}}{5.48} = 0.82$$

Calculated $|t|$ = 0.82

Degrees of freedom (D.f): n-1 = 5 – 1 = 4.

Tabulated value of t for 4 at 1% l.o.s is 4.6(two-tailed)

∵ Calculated $|\boldsymbol{t}|$ <Tabulated $|\boldsymbol{t}|$

∴ H_0 is rejected. i.e., The course is useful.

3. **A manufacturer of shock absorbers is comparing the durability of one of his models with those of his competitors. To conduct the test the manufacturer installed one of his shock absorbers, also one of his competitor's shock absorbers on each of ten pairs of cars selected at random. Each of the cars was driven 20,000 miles. Then, each shock absorber was measured for strength. The results are shown below:**

Car	Manufacturer's Shock	Competitor's Shock
1	10.0	9.6
2	11.7	11.9
3	13.7	13.1
4	9.9	9.4
5	9.8	10.0
6	14.4	14.0
7	15.1	14.6
8	10.6	10.8
9	9.8	9.4
10	12.1	12.3

At the 0.1 level of significance, is there any evidence that the manufacturer's shock absorbers last longer?

Solution:

Null Hypothesis:

The manufacturer's shock absorbers do not last significantly longer than the competitor's shock absorbers.

Alternative Hypothesis:

The manufacturer's shock absorbers last significantly longer than the competitor's shock absorbers.

Car	Manufacturer's Shock	Competitor's Shock	Difference d
1	10.0	9.6	0.4
2	11.7	11.9	-0.2
3	13.7	13.1	0.6
4	9.9	9.4	0.5
5	9.8	10.0	-0.2
6	14.4	14.0	0.4
7	15.1	14.6	0.5
8	10.6	10.8	-0.2
9	9.8	9.4	0.4
10	12.1	12.3	-0.2

$$\bar{d} = \frac{\sum d}{n} = \frac{2}{10} = 0.2$$

$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{1.5 - \frac{2^2}{10}}{9}} = 0.35$$

$$t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} = t = \frac{0.2}{\frac{0.35}{\sqrt{10}}} = 1.807$$

Critical region at 1% level of significance $t < 1.383$

Since the computed $|t| = 0.82$ is less than 1.383, H_0 cannot be rejected and we conclude that The manufacturer's shock absorbers do not last significantly longer than the competitor's shock absorbers.

F-Test:

To test whether if there is any significant difference between two estimates of population variance (or) To test if the two samples have come from the same population we use F-test.

In this case we set up null hypothesis $H_0 = \boldsymbol{\sigma_1}^2 = \boldsymbol{\sigma_2}^2$(i.e., The population variances are same).

Under H_0 the test statistic is,

$$F = \frac{S_1^2}{S_2^2}\left(\frac{gretaer\ variance}{smaller\ variance}\right)$$

Where

$$s_1^2 = \frac{\sum(x-\bar{x})^2}{n_1 - 1}; n_1 = First\ sample\ size$$
$$s_2^2 = \frac{\sum(y-\bar{y})^2}{n_2 - 1}; n_2 = Second\ sample\ size$$

Note:

1. Always $s_1^2 > s_2^2$
2. Degrees of freedom are $\gamma_1 = n_1 - 1; \gamma_2 = n_2 - 1$
3. If sample variance S^2 is given we can obtain population variance S^2 by using the relation $n S^2 = (n-1) S^2$

Problems:

1. **In one sample of 8 observations the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in another sample of 10 observations it was 102.6. Test whether this difference is significant at 5% level.**

Solution: Given $n_1 = 8$; $n_2 = 10$

$\sum(x-\bar{x})^2 = 84.4$, $\sum(y-\bar{y})^2 = 102.6$

$$s_1^2 = \frac{\sum(x-\bar{x})^2}{n_1 - 1} = \frac{84.4}{7} = 12.057$$
$$s_2^2 = \frac{\sum(y-\bar{y})^2}{n_2 - 1} = \frac{102.6}{9} = 11.4$$

Null hypothesis H_0: There is no significant difference in the sample variance.

i.e., $s_1^2 = s_2^2$

$$F = \frac{S_1^2}{S_2^2} = \frac{12.05}{11.4} = 1.057$$

Calculated F = 1.057

Degrees of freedom (D.f): $\gamma_1 = n_1 - 1 = 8 - 1 = 7; \gamma_2 = n_2 - 1 = 10 - 1 = 9$

Tabulated value of F for (7, 9) at 5% l.o.s is 3.29.

∵ Calculated F < Tabulated F i.e., 1.057< 3.29

∴ H_0 is accepted.

2. **Two independent samples of eight and seven items respectively had the following values of the variables.**

Sample-1: 9 11 13 11 15 9 12 14

Sample-2: 10 12 10 14 9 8 10

Do the two estimates of population variance differ significantly at 5% level of significance.

(A.U APR/MAY 2010)

Solution:

Given $n_1 = 8, n_2 = 7$

Null Hypothesis: H_0: The two estimates of population variance does not differ significantly.

i.e., H_0 : $\sigma_1^2 = \sigma_2^2$

Alternative Hypothesis: H_1: $\sigma_1^2 \neq \sigma_2^2$

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
9	9 – 11.75	7.56	10	10 – 10.43	0.18

	= -2.75			= -0.43	
11	-0.75	0.56	12	1.57	2.46
13	1.25	1.56	10	-0.43	0.18
11	-0.75	0.56	14	3.57	12.74
15	3.25	10.56	9	-1.43	2.04
9	-2.75	7.56	8	-2.43	5.90
12	0.25	0.06	10	-0.43	0.18
14	2.25	5.06	-		
$\sum x = 94$		33.48	$\sum y = 73$		23.68

$$\bar{x} = \frac{\sum x}{n_1} = \frac{94}{8} = 11.75 \qquad s_1^2 = \frac{\sum(x-\bar{x})^2}{n_1 - 1} = \frac{33.48}{8-1} = 4.78$$

$$\bar{y} = \frac{\sum y}{n_2} = \frac{73}{7} = 10.43 \qquad s_2^2 = \frac{\sum(y-\bar{y})^2}{n_2 - 1} = \frac{23.68}{7-1} = 3.94$$

Test statistic:

$$F = \frac{s_1^2}{s_2^2} = \frac{4.78}{3.94} = 1.21$$

Calculated F= $\mathbf{1.21}$

Tabulated value of F at 5% l.o.s is 4.21(two-tailed)

∵ Calculated F < Tabulated F(i.e., $\mathbf{1.21}$< 4.21)

∴ H_0 is accepted.

3. The nicotine content in milligrams of two samples of tobacco were found to be as follows:

Sample A	**24**	**27**	**26**	**21**	**25**	**-**
Sample B	**27**	**30**	**28**	**31**	**22**	**36**

(i) **Can it be said that two samples come from normal populations having the same mean.**

(ii) **Can it be said that two samples come from normal populations having the same Variance.**

Solution:

Given $n_1 = 5, n_2 = 6$

Equality of means will be tested by t-test and the equality of variances will be tested by F-test. Since F-test assumes equality of variances, we shall apply t-test first.

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
24	24 - 24.6 = -0.6	0.36	27	-2	4
27	2.4	5.76	30	1	1
26	1.4	1.96	28	-1	1
21	-3.6	12.96	31	2	4
25	0.4	0.16	22	-7	49
-	-	-	36	7	49
123	0	21.2	174	0	108

$$\bar{x} = \frac{\sum x}{n_1} = \frac{123}{5} = 24.6 \qquad \bar{y} = \frac{\sum y}{n_2} = \frac{174}{6} = 29$$

$$\sum(x-\bar{x})^2 = 21.2$$

$$\sum (y-\bar{y})^2 = 108$$

For t-test:

$$s^2 = \frac{1}{n_1+n_2-2}\left[\sum (x-\bar{x})^2 + \sum (y-\bar{y})^2\right] = \frac{1}{5+6-2}[21.2+108] = 14.35$$

$$s = \sqrt{14.35} = 3.78$$

For F-test:

$$s_1{}^2 = \frac{\Sigma(x-\bar{x})^2}{n_1-1} = \frac{21.2}{4} = 5.3$$

$$s_2{}^2 = \frac{\Sigma(y-\bar{y})^2}{n_2-1} = \frac{108}{5} = 21.6$$

Here $s_2{}^2 > s_1{}^2$

(i) Using students t-test(mean):

Null Hypothesis: H_0 : The two samples have been drawn from the normal populations with the same mean. i.e., $\mu_1 = \mu_2$.

Alternative Hypothesis: $H_1 : \mu_1 \neq \mu_2$

Test Statistic:

$$t = \frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{24.6-29}{3.78\sqrt{\frac{1}{5}+\frac{1}{6}}} = -1.92$$

Calculated value of $|t|$ = 1.92

Degrees of freedom (D.f): 5+6-2 = 9

Tabulated value of t for 9 D.f at 5% l.o.s is 2.262(two-tailed)

∵ Calculated $|t|$ < Tabulated $|t|$(i.e., 2.262 < 1.92)

∴ H_0 is accepted.

(ii) Using F-test (variance):

Null Hypothesis: H_0 : The two samples have been drawn from the normal populations with the same variance. i.e., $H_0 : \sigma_1{}^2 = \sigma_2{}^2$.

Alternative Hypothesis: $H_1 : \sigma_1{}^2 \neq \sigma_2{}^2$

Test Statistic:

$$F = \frac{S_2{}^2}{S_1{}^2} = \frac{21.6}{5.3} = 4.07$$

Calculated value of F = 4.07

Degrees of freedom (D.f): $\boldsymbol{\gamma_1 = n_1 - 1 = 6 - 1 = 5; \gamma_2 = n_2 - 1 = 5 - 1 = 4}$

Tabulated value of F for (5, 4) at 5% l.o.s is 6.26.

∵ Calculated F < Tabulated F i.e., 4.07< 6.26

∴ H_0 is accepted.

i. e., The two samples have been drawn from the normal populations with the same variances.

4. Two random samples gave the following results.

Sample	Size	Sample Mean	Sum of squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population.

Solution: Given $\boldsymbol{n_1} = 10, \boldsymbol{n_2} = 12$

Null Hypothesis: H_0: The two samples have been drawn from the same normal population.

i.e. H_0: $\mu_1 = \mu_2$ *and* $\sigma_1^2 = \sigma_2^2$

Alternative Hypothesis: $H_1 : \mu_1 \neq \mu_2$ and $\sigma_1{}^2 \neq \sigma_2{}^2$

Equality of means will be tested by t-test and the equality of variances will be tested by F-test. **Since t-test assumes equality of variances, we shall apply F-test first**.

(i) Using F-test (variance):

Null Hypothesis: H_0: $\sigma_1^2 = \sigma_2^2$

Alternative Hypothesis: $H_1 : \sigma_1^2 \neq \sigma_2^2$

Given $n_1 = 10$, $n_2 = 12$, $\bar{x}_1 = 15, \bar{x}_2 = 14$

$\sum (x_1 - \bar{x}_1)^2 = 90, \sum (x_2 - \bar{x}_2)^2 = 108$

$$S_1^2 = \frac{1}{n_1 - 1}\sum (x_1 - \overline{x_1})^2 = \frac{90}{9} = 10,\ S_2^2 = \frac{1}{n_2 - 1}\sum (x_2 - \overline{x_2})^2 = \frac{108}{11} = 9.82$$

Test statistic:

$$F = \frac{S_1^2}{S_2^2} = 1.018$$

Calculated F = 1.018.

$F_{0.05}$ (9, 11) = 2.90

Since calculated F < tabulated F, we accept the null hypothesis H_0.

i.e., $\sigma_1^2 = \sigma_2^2$

Since $\sigma_1^2 = \sigma_2^2$, we can apply t-test now.

(ii) Using students t-test(mean):

Null Hypothesis: H_0: $\mu_1 = \mu_2$

Alternative Hypothesis: $H_1 : \mu_1 \neq \mu_2$

$$s^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum (x_1 - \overline{x_1})^2 + \sum (x_2 - \overline{x_2})^2\right] = \frac{1}{20}(90 + 108) = 9.9$$

Test statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{15 - 14}{\sqrt{9.9\,(\frac{1}{10} + \frac{1}{12})}} = 0.742.$$

d.f = ($n_1 + n_2 - 2$) = 20.

Tabulated value of t for 20 d.f at 5% level of significance is 2.086.

Since calculated value of t = 0.74 < the tabulated value of t = 2.086, we accept the null hypothesis H_0: $\mu_1 = \mu_2$

Combining (i) and (ii), we conclude that the samples have come from the same normal population.

5. **In comparing the variability of family income in two areas, a survey fielded the following data. $n_1 = 100$, $n_2 = 100$, $s_1^2 = 25$, $s_2^2 = 10$ where n_1 and n_2 are the sample sizes and s_1^2, s_2^2 are the sample variance of incomes for the two areas respectively. Assuming that the populations are normal test the hypothesis H_0: $\sigma_1^2 = \sigma_2^2$ against H_1: $\sigma_1^2 > \sigma_2^2$ at 5% level of significance.**

   Solution:

   Given $n_1 = 100$, $n_2 = 100$, $s_1^2 = 25$, $s_2^2 = 10$

   H_0: There is no significant difference between the variances.

   The test statistic is $F = \frac{s_1^2}{s_2^2} = \frac{25}{10} = 2.5$

   Tabulated value of F for (99, 99) d.f is 1.35.

   Since calculated F > tabulated F, we reject the null hypothesis H_0.

   i.e., the variances differ significantly.

Chi Square Distribution (χ^2 distribution) – Definition:

If O_i (i = 1, 2, 3, …n) are set of observed (experimental) frequencies & E_i (i = 1, 2, 3, …n) are the corresponding set of expected (theoretical or hypothetical) frequencies then the statistic χ^2 is defined by,

$$(Chisquare)\ \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

and the degrees of freedom of this statistic is v = n-1.

Note:

1. The number of ***degrees of freedom*** is the total number of observations less the number of independent constraints imposed on the observations.
2. Chi-Square Distribution has only one parameter called the degrees of freedom. (d.o.f).
3. For fitting binomial distribution v = n – 1, Poisson distribution v = n – 2 and Normal distribution v = n – 3.
4. The shape of chi-square distribution depends on the number of degrees of freedom.

The important uses (applications) of χ^2 test are:

(a) As a test for goodness of fit
(b) As a test for independence of attributes
(c) To test the homogeneity of independent estimates of the population variance.

Conditions:

(a) The sample observations should be independent.
(b) The total frequency should be large, say greater than 50.
(c) Constraints of the cell frequencies must be linear.
(d) No theoretical cell frequency should be less than 5.

Method -1:

χ2 **test The Goodness of Fit Test:**

It is a very powerful test for testing the significance of the discrepancy between theory and experiment. It helps us to find if the deviation of the experiment from theory is just by chance or it is really due to the inadequacy of the theory to fit the observed data.

Pearsonian Chi-Square is

$$(Chisquare)\ \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Where E_i = Expected frequency

O_i = Observed frequency

K = Number of Categories

Problems:

1. **A survey of 320 families with five children each revealed the following distribution:**

No. of Boys	**0**	**1**	**2**	**3**	**4**	**5**
No. of Girls	**5**	**4**	**3**	**2**	**1**	**0**
No. of Families	**12**	**40**	**88**	**110**	**56**	**14**

Is this result consistent with the hypothesis that male and female births are equally probable? (A.U APR/MAY 2010)

Solution: Given n = 6 (Small sample n < 30).

Null hypothesis H_0: Male and female births are equally probable.

p(male birth) = $\frac{1}{2}$; q =1 - $\frac{1}{2} = \frac{1}{2}$ $(\because \boldsymbol{p + q = 1\ in\ probability\ theory})$

Based on H_0, the probability that a family of 5 children has r male children = $\boldsymbol{5_{c_r}\left(\frac{1}{2}\right)^5}$

(by binomial distribution req. probability = $\boldsymbol{n_{c_r}(p)^r(q)^{n-r} = 5_{c_r}\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{5-r} = 5_{c_r}\left(\frac{1}{2}\right)^{r+5-r} = 5_{c_r}\left(\frac{1}{2}\right)^5}$)

Alternate Hypothesis H_1: Male and female births are not equally probable.

∴ Expected number of families having r male children = 320 × $\boldsymbol{5_{c_r}\left(\frac{1}{2}\right)^5 = 320 \times 5_{c_r}\frac{1}{2^5}}$

$\boldsymbol{= 320 \times 5_{c_r}\frac{1}{32} = 10 \times 5_{c_r}}$(320 = total no. of families).

Test statistic $= \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

O_i	E_i	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
12	$10 \times 5_{c_0} = 10$	12 – 10 = 2	4	$\frac{4}{10} = 0.4$
40	$10 \times 5_{c_1} = 50$	-10	100	2
88	$10 \times 5_{c_2} = 100$	-12	144	1.44
110	$10 \times 5_{c_3} = 100$	10	100	1
56	$10 \times 5_{c_4} = 50$	6	36	.72
14	$10 \times 5_{c_5} = 10$	4	16	1.6
				$\sum \frac{(O_i - E_i)^2}{E_i} = 7.16$

$\boldsymbol{\gamma = n - 1 = 6 - 1 = 5}$

At 5% l.o.s table value for 5 D.f chi-square distribution is $\chi_{0.05}{}^2 = \mathbf{11.07}$

Since calculated value $\chi^2 = \mathbf{7.16 < 11.07}$, H_0 is accepted.

2. **A Personal Manager is interested in trying to determine whether absenteeism is greater on one day of the week than on another. His records for the past year show the following sample distribution.**

Day of Week	**:Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
No. of absentees	**:66**	**56**	**54**	**48**	**75**

Test whether the absentee is uniformly distributed over the week.

Solution:

Given n = 5 (Small sample n < 30)

We set up $\mathbf{H_0}$: Number of absence is uniformly distributed over the week against

$\mathbf{H_1}$: Number of absence is not uniformly distributed over the week.

The number of absentees during a week are 300 and if absentism is equally probable on all days, then we should expect $\frac{300}{5} = 60$ absentees on each day of the data as follows:

Test statistic $= \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Category	O_i	E_i	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
Monday	66	60	6	36	0.60
Tuesday	57	60	-3	9	0.15
Wednesday	54	60	-6	36	0.60
Thursday	48	60	-12	144	2.40
Friday	75	60	-15	225	3.75
Total					$\sum \frac{(O_i - E_i)^2}{E_i} = 7.50$

The critical value of χ^2 = 9.49 at $\alpha = 0.05$ and d.o.f. = (5-1) =4

As calculated value of χ^2 = 7.5 is less than its critical value, the null hypothesis is accepted.

3. **A die tossed 264 times with the following results.**

a)

Face	**1**	**2**	**3**	**4**	**5**	**6**
Frequency	**40**	**32**	**28**	**58**	**54**	**52**

Show that the die is biased.

Solution:

Given n = 6 (Small sample n < 30)

We set up $\mathbf{H_0}$: The die is unbiased.

$\mathbf{H_1}$: The die is biased.

The expected frequency of each of the numbers $\frac{264}{6} = 44$

Test statistic $= \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Face	O_i	E_i	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1	40	44	-4	16	0.36
2	32	44	-12	144	3.27
3	28	44	-16	256	5.82
4	58	44	14	196	4.45
5	54	44	10	100	2.27
6	52	44	8	64	1.45
Total					$\sum \frac{(O_i - E_i)^2}{E_i} = 17.62$

The critical value of χ^2 = 11.07 $\alpha = 0.05$ and d.o.f. = (6-1) =5

As calculated value of χ^2 = 17.62 is > critical value, the null hypothesis is rejected.
Hence the die is biased.

4. The theory predicts the proportion of beans in the 4 groups A, B, C and D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the 4 groups were 882, 313, 287 and 118. Does the experimental result support the theory? (A.U MAY/JUN 2012)

Solution:

H_0: The experimental result support the theory.
i.e., The four categories are in the ratio 9:3:3:1

H_1: The experimental result does not support the theory.

The expected frequencies of the four classes are,

$$\frac{9}{16} \times 1600 = 900, \ \frac{3}{16} \times 1600 = 300, \ \frac{3}{16} \times 1600 = 300, \ \frac{1}{16} \times 1600 = 100$$

Test statistic $= \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Group	O_i	E_i	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
A	882	900	18	324	0.36
B	313	300	13	169	0.56
C	287	300	13	169	0.56
D	118	100	18	324	3.24
		1600			$\sum \frac{(O_i - E_i)^2}{E_i}$ =4.72

d.o.f. = (4-1) =3
i.e, Calculated value of χ^2 = 4.72. Table value = 7.81 at 5% level.
Calculated value χ^2 = 4.72 < table value, so we accept H_0 at 5% level of significance.
The four categories are in the ratio **9:3:3:1.**

4. A sample analysis of examination results of 500 students was made. It was found that 220 students have failed, 170 have secured a third class, and 90 have secured a second class and the rest, a first class. Do these figures support the general belief that the above categories are in the ratio 4:3:2:1 respectively?

Solution:

Given n = 4 (Small sample n < 30)

H_0: the results in the four categories are in the ratio 4:3:2:1
H_1: the results in the four categories are not in the ratio 4:3:2:1
The expected frequencies of the four classes are,

$$\frac{4}{10}\times 500 = 200,\ \frac{3}{10}\times 500 = 150,\ \frac{2}{10}\times 500 = 100,\ \frac{1}{10}\times 500 = 50$$

Test statistic $= \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Class	O_i	E_i	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
Failures	220	200	20	400	2.00
III	170	150	20	400	2.67
II	90	100	- 10	100	1.00
I	20	50	-30	900	18.00
		500			$\sum \frac{(O_i - E_i)^2}{E_i}$ =23.67

i.e, Calculated value of χ^2 = 23.67. Table value = 7.815 at 5% level.
Calculated value χ^2 = 23.67 > table value, so we reject H_0 at 5% level of significance. The four categories are not in the ratio 4:3:2:1

Method -2:

χ^2 test - for independence of attributes:

Frequencies related to two attributes may be tested to find whether an association between the attributes exist or not for the two –way table between the attributes the expected frequencies are calculated for each cell of the observed frequencies. The Chi-square value is less than evaluated using the formula,

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Where $E_i = \frac{RowTotal \times ColumnTotal}{Totalfrequency}$

The corresponding null hypothesis is H_0 (attributes are independent) against alternative hypothesis H_1 (attributes are not independent)

Rejection Rule: If the calculated value of Chi-Square is greater than or equal to the tabulated value for (r-1)(c-1) d.o.f. where r = number of rows, and c=number of columns, then H_0 rejected otherwise accepted.

Note: for 2 x2 tables, the Chi-square formula for test of independence of attributes becomes simplified and can be used directly as $\chi^2 = \frac{N(ad-bc)^2}{R_1 R_2 C_1 C_2}$

Where given table explains the notation,

2 X 2 table		Row table
a	b	R_1
c	d	R_2

Column $C_1 C_2 N$ = Total Frequency Total

Rejection Rule: The null hypothesis is rejected when the Chi-square value, is equal to or exceeds the tabulated value for (2-1)(2-1) =1 d.o.f.

Yates' Correction: For (2 x 2) table, there is always only one degree of freedom. To get better result of χ^2 it is necessary to make a correction to the above formula and corrected formula is given by

$$\chi^2 = \frac{N\left\{|ad - bc| - \frac{N}{2}\right\}^2}{R_1 R_2 C_1 C_2}$$

Problems:

1. **Find the value of χ^2 for the following 2×2 contingency table:**

6	**2**
3	**5**

Solution: Here a = 6, b = 2, c = 3, d = 5

$$\chi^2 = \frac{N\left[|ad - bc| - \frac{N}{2}\right]^2}{(a+b)(a+c)(b+d)(c+d)}$$

and N= a + b + c + d = 16

χ^2 = 1.0159

2. **Find the coefficient of attributes for the 2×2 contingency table**

7	**6**
10	**8**

Solution:

Coefficient of attributes = $\frac{ad - bc}{ad + bc}$

= $\frac{56 - 60}{56 + 60}$ = - 0.0345

3. **Given the following contingency table for hair color and eye color, find the value of Chi square. Is there good association between the two?**

(A.U NOV/DEC 2010)

		Hair color			
		Fair	Brown	Black	Total
	Blue	15	5	20	40
Eye	Grey	20	10	20	50
color	Brown	25	15	20	60
	Total	60	30	60	150

Solution: Null hypothesis H_0: The given attributes are independent.

Alternate Hypothesis H_1: The given attributes are not independent.

Test statistic:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Given,

		Hair color			
		Fair	Brown	Black	Total
Eye color	Blue	15	5	20	40
	Grey	20	10	20	50
	Brown	25	15	20	60
	Total	60	30	60	150

O_i	$E_i = \frac{\textbf{Row total}\times\textbf{column total}}{\textbf{Grand total}}$	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
15	$\frac{40\times60}{150} = 16$	15 – 16 = -1	1	$\frac{1}{16} = 0.06$
5	$\frac{40\times30}{150} = 8$	-3	9	1.13

20	$\frac{40\times60}{150}=16$	4	16	1.00
20	$\frac{50\times60}{150}=20$	0	0	0.00
10	$\frac{50\times30}{150}=10$	0	0	0.00
20	$\frac{50\times60}{150}=20$	0	0	0.00
25	$\frac{60\times60}{150}=24$	1	1	0.04
15	$\frac{60\times30}{150}=12$	3	9	0.75
20	$\frac{60\times60}{150}=24$	-4	16	0.67
				$\sum\frac{(O-E)^2}{E}=3.65$

$$D.f=(r-1)(s-1)=(3-1)(3-1)=4$$

At 5% l.o.s table value for 4 D.f chi-square distribution is ${\chi_{0.05}}^2=9.48$

Since calculated value $\chi^2=3.65<9.48$, H_0 is accepted.

4. The following data are collected on the two characters

	Smokers	**Non Smokers**
literates	**83**	**57**
illiterates	**45**	**68**

Based on this, can you say that there is no relationship between smoking and literacy?

Solution:

Null hypothesis H_0: literacy and smoking habit are independent

Alternate Hypothesis H_1: literacy and smoking habit are not independent

Test statistic:

$$\chi^2=\sum\frac{(O_i-E_i)^2}{E_i}$$

O_i	$E_i=\frac{\text{Row total}\times\text{column total}}{\text{Grand total}}$	$\frac{(O_i-E_i)^2}{E_i}$
83	$\frac{128\times140}{253}=70.83$	2.09
57	$\frac{125\times140}{253}=69.17$	2.14
45	$\frac{128\times113}{253}=57.17$	2.59
68	$\frac{125\times113}{253}=55.83$	2.65
		9.48

$\alpha=0.05$ d.f (r-1)(c-1) = (2-1)(2-1) = 1 x 1 = 1

χ^2 Calculated value = 9.48 and χ^2 1d.f at 5% table value = 3.84

χ^2 Calculated value = 9.48 > χ^2 table value.

∴We reject the null hypothesis.

There is some relationship between literacy and smoking habit.

Home Work:

1. The following table gives the number of aircraft accidents that occur during various days of the week. Find whether the accidents are uniformly distributed over the weekdays.

Days :	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
No of accident:	14	16	8	12	11	9	14

[Ans. χ^2 = 4.16 < 12.59, Yes]

2. Three Hundred digits were chosen at random from a set of tables. The frequencies of the digits were as follows:

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	28	29	33	31	26	35	32	30	31	25

Using χ^2 test assess the hypothesis that the digits were distributed in equal number in the table. **[Ans. Yes, χ^2 = 2.864 <16.92]**

3. A die tossed 120 times with the following results.

b)

Face	1	2	3	4	5	6	Total
Frequency	30	25	18	10	22	15	120

Test the hypothesis that the dice is unbiased. **[Ans. No, χ 2 = 12.90 > 11.07]**

c)

Face	: 1	2	3	4	5	6
Observed freq	: 15	22	20	14	18	31

Can you regard the die as honest. (Givenx^2= 13.6 at 5% level of significance for 5 d.o.f.

4. From the table given below, test whether the colour of the son's eyes is associated with that of father's eyes.

		Eye colour of sons	
		Not light	Light
Eye colour of fathers	Not light	230	148
	Light	151	471

(Given χ^2 = 3.84 at 1df &5% significance)

[Ans. 133.39 > 3.84, there is relation]

5. The following are collected on two characters.

	Cinema goers	Non cinema goers
Literate	92	48
Illiterate	208	52

Based on this can you conclude that there is a relation between habit of cinema going and literacy (Given 5% points of χ^2 distribution are 3.841, 5.991 and 9.488 for degrees of freedom 1, 2 and 4 respectively) **[Ans. χ^2 = 9.9 > 3.84, Yes there is relation]**

6. Calculate the expected frequencies for the following data presuming the two attributes viz. , conditions of home and condition of child are independent.

Condition	Condition of home	
of child	Clean	Dirty
Clean	70	50
Fairly clean	80	20
Dirty	35	45

[Ans. χ^2 = 25.633 > 5.99, there is an association between the two]

7. In an examination on immunization of cattle from tuberculosis the following result were obtained.

	Affected	Unaffected
Inoculated	12	28
Not inoculated	13	7

Examine the effect of vaccine in controlling the incidence of the disease.

[Ans. χ^2 6.729 > 3.84 , Yes.]

8. Two sample polls of votes for two candidates A and B for public office are taken, one from among residents of rural areas and the other from urban areas. The results are given in the table. Examine whether the nature of the area is related to voting preference in this election.

	Vote for		
Area	A	B	Total
Rural	620	380	1000
Urban	550	450	1000
Total	1170	830	2000

[Ans. χ^2 =10.089 > 3.84, Nature of area is related to voting]

9. Two groups of 100 people each were taken for testing the use of a vaccine.15 persons contracted the disease out of the inoculated persons while 25 contracted the disease in the other group. Test the efficiency of vaccine using χ^2 – test.

[Ans. Not effective, χ^2 =3.125 < 3.84]

10. In a survey of 200 boys, of which 75 were intelligent. 40 had skilled fathers, while 85 of the unintelligent boys had unskilled fathers. Do these data support the hypothesis that skilled fathers have intelligent boys.

[Ans. Yes, χ^2 = 8.89 > 3.84]

11. Out of 800 persons 25% were literates and 300 had enjoyed T.V. programmes. 30% of those who had not enjoyed T.V. Programmes were literate. Test at 5% level of significance whether T.V. Programmes influence literacy.

Degree of freedom :	1	2	3	4
Chi – Square at 5% level:	3.841	5.999	7.815	9.448

[Ans. χ^2 = 17.78 (H_o is rejected T.V. program significantly influence literacy]

12. Out of 8000 graduates in a town 800 are females. Out of 1600 graduates employees 120 are females.Test whether there is any distinction made in the appointment on the basis of female or male criterion.

[Ans. χ^2 = 13.89. Yes distinction is made]

Large sample tests:

The sample size which is greater than or equal to 30 is called as large sample and the test depending on large sample is called large sample test.

The assumption made while dealing with the problems relating to large samples are

Assumption-1: The random sampling distribution of the statistic is approximately normal.

Assumption-2: Values given by the sample are sufficiently closed to the population value and can be used on its place for calculating the standard error of the statistic.

Assumptions: z-test

1. The underlying distribution is normal or the Central Limit Theorem can be assumed to hold
2. The sample has been randomly selected and the sample size is ≥ 30.
3. The population standard deviation is known or the sample size is at least 25.

Critical values of Z for selected levels of significance:

Level of significance (α)	0.10	0.05	0.01	0.005
Two Tail Critical values of |Z|	1.645	1.96	2.58	2.81
Right tail Critical value of Z	1.28	1.645	2.33	2.58
Let tail Critical value of Z	-1.28	-1.645	-2.33	-2.58

Large sample test for single mean (or) test for significance of single mean:

Test statistic:

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

Where σ is the S.D of the population.

Now calculate $|Z|$

Find out the tabulated value of Z at α % l.o.s i.e. Z_α

If $|Z| > Z_\alpha$, reject the null hypothesis H_0

If $|Z| < Z_\alpha$, accept the null hypothesis H_0

Note:

1. If the population standard deviation is unknown then we can use Test statistic, with sample S.D s,

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}} \sim N(0,1)$$

2. At 5% level 95% confidence limits are $\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$
3. At 1% level 99% confidence limits are $\bar{x} - 2.58\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 2.58\frac{\sigma}{\sqrt{n}}$

Problems:

(1) In order to test whether average weekly maintenance cost of a fleet of buses is more than Rs. 500, a random sample of 50 buses was taken. The mean and the standard deviation were found to be Rs. 508 and Rs. 40 (Use $\alpha = 0.05$)

Solution:

Given n = 50 > 30 (large sample), $\bar{x} = 508, s = 40$

Null hypothesis $H_0 : \mu = 500$

Alternate Hypothesis H_1: μ > 500.

The significance level is 0.05 and H_1 is a right tailed.

The test criterion is the Z test.

Test statistic:

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{508 - 500}{40/\sqrt{50}} = \frac{8}{5.657} = 1.414$$

$|\boldsymbol{z}| = \mathbf{1.414}$

At α = 0.05, the critical value of Z = 1.645 (i.e. right tailed) and

The computed value of Z (1.414) < critical value of Z (1.645)

∴ we accept null hypothesis H_0 that μ = 500. i.e., The average maintenance cost is not more than Rs. 500.

(2) The quality control department as processing company specifies that the mean net weight per pack as its produce must be 20 gms. Experience has shown that the weight are approximately normally distributed with a S-d, of 1.5 gms. A random sample of 50 packs yield a mean weight if 19.5 gms as this sufficient evidence to indicate that the true mean weight of the packs has decreased (use 5% significance levels).

Solution:

Given n = 50 > 30 (large sample), $\bar{x} = 19.5, \sigma = 1.5$

Null hypothesis $H_0 : \mu = 20$

Alternate Hypothesis H_1: μ < 20.

The significance level is 0.05 and H_1 is a left tailed.

The test criterion is the Z test.

Test statistic:

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{19.5 - 20}{1.5/\sqrt{50}} = \frac{-0.5}{0.212} = -2.358$$

$|\boldsymbol{z}| = \mathbf{2.358}$

At $\alpha = 0.05$ level of significance the critical value of Z (Left tailed) is -1.645.

The computed value of Z (2.358) > critical value of Z (-1.645)

∴ we reject null hypothesis H_0 that μ = 20.

(3) A company is engaged in the packaging of a suppressing quantity tea in jar of 500 gm each. The company is of the view that as long as jars contains 500 gms of tea, the process is in control. The standard deviation is 50 gm. A sample of 225 jars is taken at random and the sample average is found to be 510 gm. Has the process gone out of control at 5% l.o.s.

Solution

Given n = 225 > 30 (large sample), $\bar{x} = 510, \sigma = 50$

Null hypothesis $H_0 : \mu = 500$ gms

Alternate Hypothesis H_1: $\mu \neq 500$ gms

The significance level is 0.05 and H_1 is a two tailed.

The test criterion is the Z test.

Test statistic:

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{510-500}{50/\sqrt{225}} = \frac{10}{3.33} = 3$$

$|z| = 3$

At $\alpha = 0.05$ level of significance the critical value of Z (two tailed) is 1.96.

The computed value of Z (3) > critical value of Z (1.96)

∴ we reject null hypothesis H_0 that $\mu = 500$ gms.

This means process is not in control.

4. The average (mean) live weight of a farmer's steers prior to slaughter was 390 pounds in past years. This year his 50 steers were fed on a new diet. Suppose we consider these 50 steers on the new diet as a random sample taken from a population of all possible steers that may be fed the diet now or in the future and it S.D is give by 35.2. Use the sample data given below and α =.01 to test the research hypothesis that the mean live weight for steers on the new diet is greater than 380.

Solution: Given $\bar{x}$ =390, n = 50(large sample), s = 35.2.

Null hypothesis H_0: $\mu = 380$

Alternate Hypothesis H_1: $\mu > 380$

Test Statistic:

$$z = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{390-380}{35.2/\sqrt{50}} = 2.01$$

$|z| = 2.01$

At $\alpha = 0.01$ level of significance the critical value of Z (right tailed) is 2.33.

The computed value of Z (2.01) > critical value of Z (2.33)

∴ we accept null hypothesis H_0 that $\mu = 380$

There is not sufficient evidence to conclude that the mean live weight for steers on the new diet is greater than 380.

5. Chennai Municipality uses thousands of fluorescent light bulbs each year. The brand of bulb it currently uses has a mean life of 900 hours. A manufacturer claims that its new brand of bulbs, which cost the same as the brand the university currently uses, has a mean life of more than 900 hours. The university has decided to purchase the new brand if, when tested, the test evidence supports the manufacturer's claim at α = .05. Suppose sixty-four bulbs were tested with the following results:

$\bar{x}$ = 930 hours s = 80 hours

Will Chennai Municipality purchase the new brand of fluorescent bulbs? Conduct hypothesis test.

Solution: Given $\bar{x}$ =930 hours, n = 64(large sample), s = 80 hours

Null hypothesis H_0: $\mu = 900$
Alternate Hypothesis H_1: $\mu > 900$ (the mean life for the new brand of bulbs is higher than the mean life for the old brand)
Test Statistic:

z = $\frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{930-900}{80/\sqrt{64}} = \frac{30}{10} = 3.00$

$|z| = \mathbf{3.00}$

At $\alpha = 0.05$ level of significance the critical value of Z (right tailed) is 1.645.

The computed value of Z (3.00) > critical value of Z (1.645)

∴ we reject null hypothesis H_0 that $\mu = 900$.

There is sufficient evidence to conclude that the mean life for the new brand of bulbs is greater than 900.

Large sample test for difference between two means:

Test statistic

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

Now calculate $|Z|$

Find out the tabulated value of Z at α % l.o.s i.e. Z_α

If $|Z| > Z_\alpha$, reject the null hypothesis H_0

If $|Z| < Z_\alpha$, accept the null hypothesis H_0

Note:

1. If σ_1^2 and σ_2^2 are unknown then we can consider S_1^2 and S_2^2 as the estimate value of σ_1^2 and σ_2^2 respectively.

2. Under H_0: $\mu_1 = \mu_2$, if the samples are drawn from the same population where $\sigma_1 = \sigma_2 = \sigma$, then $Z = \frac{\overline{x_1} - \overline{x_2}}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

4. If σ_1, σ_2 are not known and $\sigma_1 \neq \sigma_2$, then $Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$

Problem:

1. **The means of 2 large samples 1000 & 2000 members are 67.5 inches & 68.0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2.5 inches?**

(A.U NOV/DEC 2010, A.U MAY/JUNE 2012)

Solution:

Given $n_1 = 1000$; $n_2 = 2000$ (large sample test)

$\overline{x_1} = 67.5\ inches; \overline{x_2} = 68\ inches$

σ = 2.5 $inches$

Null hypothesis H_0: The samples have been drawn from the same population of S.D 2.5 inches.

i.e., H_0: $\mu_1 = \mu_2$ and σ = 2.5 $inches$

Alternate Hypothesis H_1: $\mu_1 \neq \mu_2$

Test Statistic:

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}} = \frac{67.5 - 68}{\sqrt{\frac{(2.5)^2}{1000} + \frac{(2.5)^2}{2000}}} = \frac{-0.5}{0.0968} = -5.16$$

Calculated $|Z|$ = 5.16

Tabulated value of Z at 5% l.o.s is 1.96.

$\because$ Calculated $|Z|$ > Tabulated $|Z|$(i.e., 5.16> 1.96.)
$\therefore$ H_0 is rejected.
$i.e.$, The samples are not drawn from the same population of S.D 2.5 inches.

2. **The mean yield of two sets of plots and their variability are as given below.**

	Set of 40 plots	**Set of 60 plots**
Mean yield per plot	**1258 Kg**	**1243 Kg**
Standard deviation per plot	**34**	**28**

Examine at 5% level whether the difference in mean yields of the two sets of plots is significant.

(A.U MAY/JUNE-2010)

Solution:
Given $n_1 = 40$; $n_2 = 60$ (large sample test)
$\overline{x_1} = 1258\ kgs; \overline{x_2} = 1243\ kgs$
$s_1 = 34, s_2 = 28$
Null hypothesis H_0: There is no significant difference in mean yields of two sets of plots.
i.e., H_0: $\overline{x_1} = \overline{x_2}$
Alternate Hypothesis H_1: $\overline{x_1} \neq \overline{x_2}$ (two tailed)
Test Statistic:

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{S_1^{\ 2}}{n_1} + \frac{S_2^{\ 2}}{n_2}}} = \frac{1258 - 1243}{\sqrt{\frac{(34)^2}{40} + \frac{(28)^2}{60}}} = \frac{15}{\sqrt{28.9 + 13.067}} = \frac{15}{\sqrt{41.967}} = \frac{15}{6.48} = 2.31$$

Calculated $|Z|$ = 2.31
Tabulated value of Z at 5% l.o.s is 1.96.
$\because$ Calculated $|Z|$ > Tabulated $|Z|$(i.e., 2.31> 1.96.)
$\therefore$ H_0 is rejected.

Large sample test for single proportion (or) test for significance of proportion:
Test statistic $z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$
Now calculate $|Z|$
Find out the tabulated value of Z at α % l.o.s i.e. Z_α
If $|Z| > Z_\alpha$, reject the null hypothesis H_0
If $|Z| < Z_\alpha$, accept the null hypothesis H_0
Large samples
Test of significance for single proportion:

1. **In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers.**

(A.U MAY/JUNE 2011,NOV/DEC 2010)

Solution: Given n = 600 (large sample).
$\boldsymbol{x}$ = Number of smokers = 325
p = Proportion of smokers in the **sample.**
$= \frac{\boldsymbol{x}}{\boldsymbol{n}} = \frac{\mathbf{325}}{\mathbf{600}} = 0.5417$
$\boldsymbol{P}$ = Proportion of smokers in the **Population.**
$= \frac{\mathbf{1}}{\mathbf{2}} = 0.5$
$\boldsymbol{Q} = 1 - P = 1 - 0.5 = 0.5$
Null Hypothesis: H_0: The number of smokers and non-smokers are equal in the city.
Alternative Hypothesis: H_1: P > $\mathbf{0.5}$ (Right tailed)
Test statistic:

$$z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.5417 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{600}}} = 2.04$$

Calculated $|Z|$ = 2.04

Tabulated value of Z at 5% l.o.s is 1.64(Right-tailed)

∵ Calculated $|Z|$ > Tabulated $|Z|$(i.e., 2.04> 1.64)

∴ H_0 is rejected.

$i.e.$, The majority of men in this city are smokers.

2. **Experience has shown that 20% of a manufactured product is of top quality. In one day's production of 400 articles, only 50 are of top quality. Show that either the production of the day chosen was not a representative sample (or) the hypothesis of 20% was wrong.**

(A.U APR/MAY 2010)

Solution: Given n = 400 (large sample).

p = Proportion of top quality products in the sample.

$= \frac{\mathbf{50}}{\mathbf{400}} = \frac{\mathbf{1}}{\mathbf{8}}$

$\boldsymbol{P}$ = Proportion of top quality products in the Population.

$= \frac{\mathbf{20}}{\mathbf{100}} = \frac{\mathbf{1}}{\mathbf{5}}$

$\boldsymbol{Q} = 1 - P = 1 - \frac{1}{5} = \frac{4}{5}$

Null Hypothesis: H_0: Proportion of top quality products in the Population P = $\frac{1}{5}$

Alternative Hypothesis: H_1: P ≠ $\frac{1}{5}$

Test statistic:

$$z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{\frac{1}{8} - \frac{1}{5}}{\sqrt{\frac{\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)}{400}}} = \frac{-\frac{3}{40}}{\sqrt{\frac{4}{25 \times 400}}} = -3.75$$

Calculated $|Z|$ = 3.75

Tabulated value of Z at 5% l.o.s is 1.96(two-tailed)

∵ Calculated $|Z|$ > Tabulated $|Z|$(i.e., 3.75 > 1.96)

∴ H_0 is rejected.

3. **The product manager wises to determine whether or not to change the package design for her product. She feels that it will be worth considering only if more than 60% of the non-users prefer the new box to the old one. She selects or random sample of 100 persons who are non-users and finds that 73 persons prefer the new box. Should she change the design? (Significance level α = 0.05)**

Solution: Given n = 100 (large sample).

$p = \frac{73}{100} = 0.73$

$\boldsymbol{P} = \frac{\mathbf{60}}{\mathbf{100}} = 0.6$

$\boldsymbol{Q} = 1 - P = 1 - 0.6 = 0.4$

Null Hypothesis: H_0: P = 0.6.

Alternative Hypothesis: H_1 : P > 0.6 (right-tailed)

Test statistic:

$$z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.73 - 0.6}{\sqrt{\frac{0.6 \times 0.4}{100}}} = \frac{0.13}{\sqrt{\frac{0.24}{100}}} = 2.65$$

Calculated $|Z|$ = 2.65

At α = 0.05, $Z_{0.05}$ = 1.645 as the test is right-tailed.

As computed $|Z|$ > Tabulated value as Z (= 1.645), Null hypothesis is rejected

∴ we can conclude that the sample gives sufficient evidence that more than 60% of the non-users prefer the new box design and the product manger should change the package design of her product at the specified significance level.

Large sample test for Difference of proportions (or) test for significance of proportion:

Test statistic $Z = \dfrac{p_1 - p_2}{\sqrt{PQ(\frac{1}{n_1} + \frac{1}{n_2})}} \sim N(0,1)$

When P is not known P can be calculated by $P = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ and $Q = 1 - P$

Now calculate $|Z|$

Find out the tabulated value of Z at α % l.o.s i.e. Z_α

If $|Z| > Z_\alpha$, reject the null hypothesis H_0

If $|Z| < Z_\alpha$, accept the null hypothesis H_0

Problem:

1. **A machine puts out 10 defective units in a sample of 200 units. After the overhauling the machine puts out 4 defective units in a sample of 100 units. Has the machine been improved. (use $\alpha = 0.05$)**

Solution:

Given $n_1 = 200,\ p_1 = \dfrac{10}{200} = 0.05,\ n_2 = 100,\ p_2 = \dfrac{4}{100} = 0.04$

Null Hypothesis: $H_0 : p_1 = p_2$

i.e., The proportions of defective before and after overhauling are equal**.**

Alternative Hypothesis: $H_1 : p_1 > p_2$

i.e., The proportion of defectives has decreased after overhauling.

Proportion $P = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \dfrac{10 + 4}{200 + 100} = 0.047$

$Q = 1 - p = 0.953$

Test statistic:

$$Z = \frac{p_1 - p_2}{\sqrt{PQ(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.05 - 0.04}{\sqrt{0.047 \times 0.953\left(\frac{1}{200} + \frac{1}{100}\right)}} = 0.385$$

Calculated $|Z|$ = 0.385

At α = 0.05, $Z_{0.05}$ = 1.645 as the test is right-tailed.

Calculated Value of | z | = 0.385 < table value = 1.645

Since | z | < 1.645, we accept the null hypothesis H_0 at 5% level of significance.

i.e., The proportions of defective before and after overhauling are equal**.**

3. **In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant?**

Solution:

Given n_1 = 900, n_2 = 1600, p_1 = 0.2, p_2 = 0.185

Null Hypothesis: $H_0 : p_1 = p_2$

i.e., The differences between the two proportions are not significant

Alternative Hypothesis: $H_1 : p_1 > p_2$

i.e., The differences between the two proportions are significant.

Proportion P = $\dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ = 0.1904.

Q = 1 – P = 0.8906.

Test statistic:

$$Z = \frac{p_1 - p_2}{\sqrt{PQ(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.2 - 0.185}{\sqrt{0.1904 \times 0.8906\left(\frac{1}{900} + \frac{1}{1600}\right)}} = 0.9375$$

Calculated $|Z|$ = 0.9375

At α = 0.05, $Z_{0.05}$ = 1.645 as the test is right-tailed.

Calculated Value of | z | = 0.9375 < table value = 1.645

Since | z | < 1.645, we accept the null hypothesis H_0 at 5% level of significance.

i.e., the differences between the two proportions are not significant.

Solved Two Mark Questions:

TESTING OF HYPOTHESIS

PART-A

1. **Define Type I and Type II errors.**

   Solution:

   Type I error: Reject H_0 when it is true

   Type II error: Accept H_0 when it is false

2. **If we want to estimate the true proportion of defectives in a very large shipment of adobe bricks, and that we want to be at least 95% confident that the error is at most 0.04. How large a sample will we need if we know that the true proportion does not exceed 0.12?**

   Solution:

   $Z_{0.05}$ = 1.96, E = 0.04 and P = 0.12

   $$n = z_{\alpha}^2 \frac{(PQ)}{E^2}$$

   =253.55

   $\cong$ 254

3. **In a sample of 100 ceramic pistons made for an experimental diesel engine, 18 were cracked. Construct a 95% confidence interval for the true proportion of cracked pistons.**

   Solution:

   95% confidence interval for the true proportion is

   $$\left(p - z_{0.05}\sqrt{\frac{pq}{n}}, p + z_{0.05}\sqrt{\frac{pq}{n}}\right)$$

   $$\left(0.18 - 1.96 \times \sqrt{\frac{0.18 \times 0.82}{100}}, 0.18 + 1.96 \times \sqrt{\frac{0.18 \times 0.82}{100}}\right)$$

   i.e., (0.1047, 0.2553)

4. **The length of certain machine parts looked upon as a random variable having a normal distribution with a mean of 2cm and a standard deviation of 0.05cm. We want to test the null hypothesis μ = 2 against the alternative hypothesis $\mu \neq 2$ on the basis of a random sample of size n = 30 and mean 2.01. If the probability of a Type I error is to be α = 0.05, find whether the sample mean differ significantly from the population mean.**

   Solution:

   $$z = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}} = \frac{2.01 - 2.00}{0.05 / \sqrt{30}} = 1.095$$

   Since $\alpha = 0.05, z_{\alpha} = 1.96$.

   $z < z_{\alpha}$. Therefore the null hypothesis $\mu = 2$ is accepted. There is no significant difference between the sample mean and population mean.

5. **In a random sample of 1000 people in Maharashtra, 540 are rice eaters and the rest are wheat eaters. If both rice and wheat are equally popular in the state, find the standard error of the proportion of wheat eaters.**

   Solution:

   n = 1000, P = ½ and Q = ½

   $$S.E. = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{0.5 \times 0.5}{1000}} = 0.0138$$

6. **Write short notes on critical region.**

   Solution:

   A region in the sample space which amounts to the rejection of H_0 is known as the critical region or region of rejection.

7. **Distinguish between parameters and statistics.**

   Solution: Statistical constants of the population namely mean(μ), variance(σ^2), etc., are usually referred as parameters. Statistical measures computed from the sample observations alone are known as Statistic.(eg.)Mean(x), variance(s^2), etc.,

8. **Define degrees of freedom**

   Solution:

   Number of independent variables which make up the statistic is known as degrees of freedom.

9. **Explain level of significance**

Solution:

The probability 'α' that a random value of the statistic t belongs to the critical region is known as the level of significance. ie., level of significance is the size of the Type I error or maximum producer's risk. Usually 5% and 1% levels of significance are employed in testing of hypothesis.

If E(X) be the expected value of X and $Z = \frac{X - E(X)}{S.E(X)}$

If $P(|Z| > 1.96) = 0.05$, we say that H_0 is rejected at 5% level of significance.

10. **Define critical value**

Solution:

The value of the test statistic which separates the critical region and the acceptance region is called the critical value or significant value. This value is dependent on (i) the level of significance and (ii) the alternative hypothesis, whether it is one –tailed or two-tailed.

11. **What do you mean by Confidence level and confidence limits**

Solution:

If α is the probability level and if the estimated value of a statistic is θ and if $P(c_1<\theta<c_2) = 1-\alpha$ then c_1 and c_2 are called the confidence limits and the interval (c_1, c_2) is known as confidence interval at the probability level α.

12. **A normal population has a mean 0.1 and s.d. 2.1. find the probability that mean of a sample of size 900 will be negative.**

Solution:

$$P(\bar{x} < 0) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{o-\mu}{\sigma/\sqrt{n}}\right)$$

$$= P\left(z < \frac{-\mu}{\sigma/\sqrt{n}}\right)$$

Given $\mu = 0.1$, $\sigma = 2.1$, n = 900

$$P(\bar{x} < 0) = P\left(z < -\frac{0.1}{2.1/\sqrt{900}}\right) = P(z < -1.43) = 0.0764$$

13. **In a recent study, 69 of 120 meteorites were observed to enter the earth's atmosphere with a velocity of less than 26 miles per second. If we estimate the corresponding true proportion as o.575, what can we say with 95% confidence about the maximum error?**

Solution:

$$E = z_\alpha \sqrt{\frac{PQ}{n}}$$

$$= 1.96 \times \sqrt{\frac{0.575 \times 0.425}{120}} = 0.0884$$

14. **A random sample of 500 toys was taken from a large consignment and 65 were found to be defective. Find the percentage of defective toys in the consignment.**

Solution:

n= 500 p = 65/500 = 0.13 and q = 1- p = 0.87

Confidence interval for proportion of defective toys is

$$\left(p - 3\sqrt{\frac{pq}{n}}, p + 3\sqrt{\frac{pq}{n}}\right)$$

(0.0849, 0.1751)

Percentage of defective toys in the consignment lies between 8.5 and 17.51.

15. **Test the hypothesis that σ = 10 given that s = 15 for a random sample of size 50 from a normal population.**

Solution:

H_0: $\sigma = 10$

Given that n = 50 and s = 15

$$z = \frac{s-\sigma}{\sigma/\sqrt{2n}} = \frac{15-10}{10/\sqrt{100}} = 5.05$$

$z_{0.05} = 1.96$

$|z| > z_{0.05}$. Hence, H_0 is rejected.

16. What are the uses of 'F' – test?

Solution:

(i) F - test is used to test whether two independent samples have been drawn from the normal populations with the same variance

(ii) F – test is used to test whether the two independent estimates of the population variance are homogeneous or not.

21. **Write the applications of 'χ^2' test.**

Solution:

χ^2 - test is used

(i) to test the goodness of fit.

(ii) to test the independence of attributes.

(iii) to test the homogeneity of independent estimates of the population variance.

17. Define errors in sampling and critical region.

Solution:

Type I error: Reject H_0 when it is true

Type II error: Accept H_0 when it is false

A region in the sample space which amounts to the rejection of H_0 is known as the critical region or region of rejection.

18. Define null hypothesis and alternative hypothesis.

Solution:

Null hypothesis:

It is a definite statement about the parameter, that there is no difference. It is denoted by H_0

Alternative hypothesis:

Complementary hypothesis to null hypothesis is called the alternative hypothesis & is denoted by H_1.

19. What are the conditions under which chi-square test is valid?

Solution:

(i). The sample observations should be independent.

(ii). Constraints of the cell frequencies must be linear.

(iii) No theoretical cell frequency should be less than 5.

PART-B

1. **A cubical die is thrown 9000 times and a throw of three or four is observed 3240 times. Show that the die cannot be regarded as an unbiased one and find the extreme limits between which the probability of a throw of three or four lies.**

Solution:

H_0: The die is unbiased. i.e., P = $\frac{1}{3}$ (= the probability of getting 3 or 4)

H_1: P $\neq \frac{1}{3}$ Two tailed test is used. Level of significance α = 5 % $Z_\alpha = 1.96$

$$Z = \frac{X - np}{\sqrt{nPQ}} = \frac{3240 - \left(9000 \text{x} \frac{1}{3}\right)}{\sqrt{9000 \text{x} \frac{1}{3} \text{x} \frac{2}{3}}} = 5.37$$

$|Z| = 5.36 > Z_\propto = 1.96$. We reject H_0

We concluded that the dice is almost certainly biased. P $\neq \frac{1}{3}$

P = $\frac{3240}{9000} = 0.36$ and $Q = 1 - P = 1 - 0.36 = 0.64$

Hence the probable limits for the population proportions of successes may be taken by

$\widehat{P} \pm \sqrt{PQ/n} = 0.36 \pm \sqrt{\frac{0.36\text{x}0.64}{9000}} = 0.360 \pm 0.015$ = 0.345 and 0.375

Hence the probability of getting 3 or 4 almost certainly lies between 0.345 and 0. 375.