$$= \frac{\frac{3}{4} \times \frac{1}{4}}{1-\left(\frac{3}{4}\right)^4} = \frac{48}{175}.$$

Similarly, C's chance is the sum of the infinite series

$$= \left(\frac{3}{4}\right)^2 \times \frac{1}{4} + \left(\frac{3}{4}\right)^6 \times \frac{1}{4} + \ldots = \frac{36}{175}$$

Similarly, D's chance is the sum of the infinite series,

$$= \left(\frac{3}{4}\right)^3 \times \frac{1}{4} + \left(\frac{3}{4}\right)^7 \times \frac{1}{4} + \ldots.$$

$$= \frac{\left(\frac{3}{4}\right)^3 \times \frac{1}{4}}{1-\left(\frac{3}{4}\right)^4} = \frac{27}{175}.$$

CW

Example 18. *One bag contains 5 white balls and 7 black balls, another contain white balls and 5 black balls. If a bag is chosen at random and a ball is drawn from it, wh is the chance that it is white ?*

Solution : Here there are two bags. The chance of selecting the first bag = $\frac{1}{2}$.

Now the chance that a white ball is drawn from the first bag

$$= \frac{{}^5C_1}{{}^{12}C_1} = \frac{5}{12}.$$

Hence if first bag is chosen and a white ball is drawn from it, the probability of event E_1 is

$$P(E_1) = \frac{1}{2} \times \frac{5}{12} = \frac{5}{24}.$$

Similarly, second bag is chose and a white ball is drawn from it, the probability of event E_2 is

$$(E_2) = \frac{1}{2} \times \frac{3}{8} = \frac{3}{16}.$$

Since both above events are mutually exclusive, the required probability is

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) = \left(\frac{5}{24}\right) + \left(\frac{3}{16}\right) = \frac{19}{48}.$$

Example 19. *Urn contains 10 white and 3 black balls. Another urn contains 3 wh and 5 black balls. Two balls are transferred from the first urn and placed in the second the one ball is taken from the later. What is the probability that it is a white ball ?*

Solution :

Urn	Old positions		New position					
			A_1		A_2		A_3	
	I	II	I	II	I	II	I	II
White balls	0	3	8	59	4	10	3	
Black balls	3	5	3	5	2	6	1	7
Total	3	8	11	10	11	10	11	10

The event A of drawing two balls from the first urn can happen in the form of following three mutually exclusive events :

A_1 : both are white

A_2 : one is white and one is black

A_3 : both are black.

Let B = A ball I drawn from the II urn is white when the two balls have been placed in it from the first urn in any one of the three mutually exclusive forms A_1, A_2, A_3.

$$\therefore \quad B = (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B)$$

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$$

$$\Rightarrow \quad P(B) = P(A_1)\, P(B/A_1) + P(A_2)\, P(B/A_2) + P(A_3)\, P(B/A_3)$$

or
$$P(B) = \frac{^{10}C_2}{^{10}C_2} \times \frac{5}{10} + \frac{^{10}C_1 \times {^3C_1}}{^{13}C_2} \times \frac{4}{10} + \frac{^3C_2}{^{13}C_2} \times \frac{3}{10}$$

$$= \frac{10\times 9}{10\times 12} \times \frac{5}{10} + \frac{10\times 3\times 2}{13\times 12} \times \frac{4}{10} + \frac{3\times 2}{13\times 12} \times \frac{3}{10}$$

$$= \frac{10}{12} + \frac{2}{13} + \frac{3}{260} = \frac{75+40+3}{260} = \frac{118}{260} = \frac{59}{130}.$$

Example 20. *Each of the three bags contains 5 white and 7 red balls. One ball is drawn from the first bag and is placed in second bag. Then one ball is drawn from the second bag and placed in the third bag. Now, what is the probability of drawing a white ball from the third bag ?*

Solution : Let A = The ball from III bag is white

A can happen as the union of the following mutually exclusive events :

A_1 = White from I, white from II and white from III (*www*)

A_2 = White from I, black from II and white from III (*wbw*)

A_3 = Black from I, white from II and white from III (*bww*)

A_4 = Black from I, black from II and white from III (*bbw*)

where
$$P(A_1) = \frac{5}{12} \times \frac{6}{13} \times \frac{6}{13} = \frac{180}{12\times 13\times 13}$$

$$P(A_2) = \frac{5}{12} \times \frac{7}{13} \times \frac{5}{13} = \frac{175}{12\times 13\times 13}$$

$$P(A_3) = \frac{7}{12} \times \frac{5}{13} \times \frac{6}{13} = \frac{210}{12\times 13\times 13}$$

$$P(A_4) = \frac{7}{12} \times \frac{8}{13} \times \frac{5}{13} = \frac{280}{12\times13\times13}$$

$$\therefore \qquad P(A) = \frac{180+175+210+280}{12\times13\times13} = \frac{845}{2028}.$$

Example 21. *A is one of 6 horses entered for a race, and is to be ridden by one of two Jockeys B and C. It is 2 to 1 that B rides A in which case all the horses are equally likely to win; if C rides A his chance is trebled. What are the odds against his winning ?*

(Vikram 1993)

Solution : Given P (Jockey B rides horse A)

$$= \frac{2}{2+1} = \frac{2}{3}$$

and in this case all six horses are equally likely to win.

$$\therefore P(A \text{ wins when } B \text{ rides}) = \frac{2}{3} \times \frac{1}{6} = \frac{1}{9} \qquad ...(1)$$

Again, P (Jockey C rides the horse A)

$$= \frac{1}{2+1} = \frac{1}{3}$$

and in this case the chance of A's winning being trebled *i.e.*

$$3 \times \frac{1}{6} = \frac{3}{6}.$$

$$\therefore P(A \text{ wins when } C \text{ rides}) = \frac{1}{3} \times \frac{3}{6} = \frac{1}{6} \qquad ...(2)$$

From (1) and (2), the two events are mutually exclusive.

Hence $\qquad P(A \text{ wins}) = \frac{1}{6} + \frac{1}{9} = \frac{5}{18} = \frac{5}{5+13}.$

$\therefore$ Odds against A's winning are 13 to 5.

Example 22. *In each of a set of games it is 2 to 1 in favour the winner of the previous game. What is the chance that the player who wins the first game shall win, at least 3 games out of the next four games ?*

(Jiwaji 1994, 9...)

Solution : In each of a set of games it is 2 to 1 in favour of the winning of the previous game

$\Rightarrow P$ (winning a game/he has won the previous game)

$$= \frac{2}{2+1} = \frac{2}{3} = P(W/W), \text{ say}$$

$$\therefore \qquad P(W^C/W) = 1 - \frac{2}{3} = \frac{1}{3}, P(W/W^C) = \frac{1}{3}$$

and $\qquad (W^C/W^C) = \frac{2}{3}, \quad$ where $\quad W =$ he wins,

$W^C =$ he does not win