# In-class Exercise:

## General – simple probability / combination / relative frequency

1. The Board of Directors of a company wishes to form a quality management committee to monitor the quality of their products. The company has 5 Scientists, 4 Engineers and 6 Accountants. Find the probability that the committee will contain 2 Scientists, 1 Engineer and 2 Accountants.

Solution:

Probability to pick 2 scientists among 5 is 2/5 = 0.4

Probability to pick 1 engineer among 4 is 1/4 = 0.25

Probability to pick 2 accountants from 6 is 2/6= 0.33

Probability that committee will have 2 scientists, 1 engineer and 2 accountants is 2/5 \* 1/4 \* 2/6

**Addition rule: (Mutually exclusive and not mutually exclusive events)**

**2**. A number is selected at random from the numbers 1 to 30. What is the probability that

1. It is divisible by either 3 or 7

Solution:

10 numbers divisible by 3 between 1 to 30

4 numbers divisible by 7 between 1 to 10

But 21 is divisible by both 3 and 7 hence removing that count from one of the occurrences. Hence probability of one number either divisible by 3 or 7 is 1/(10)+1/(4-1) = 0.43

1. It is divisible by 5 or 13?

Solution:

6 numbers divisible by 5 between 1 to 30 hence 6/30

3 numbers divisible by 13 between 1 to 10 hence 2/30

Hence probability of one number is 6/30 + 2/30 = 0.26

## Multiplication theorem (independent events)

**3**. The odds favoring the event of a person hitting a target are 3 to 5. The odds against the event of another person hitting the target are 3 to 2. If each of them fire once at the target, find the probability that:

1. both of them hit it

Solution:

Person1: Hits 3/5 times

Person2: Hits 2/5 times

1. at least one of them hit the target.

## BAYES Theorem

**4.** A bin contains 3 different types of lamps. The probability that a type 1 lamp will give over 100 hours of use is 0.7, with the corresponding probabilities for type 2 and 3 lamps being 0.4 and 0.3 respectively. Suppose that 20 per cent of the lamps in the bin are of type 1, 30 per cent are of type 2 and 50 per cent are of type 3.

1. What is the probability that a randomly selected lamp will last more than 100 hours?

**Solution**: Lamp1 lasting is 0.7 probable, Lamp2 lasting. Is 0.4 possible and Lamp3 lasting is 0.3

Bin 1 has 20% L1 lamps leading to 0.14 of L1 lasting, 30% of L2 lamps leading to 0.12 lasting and L3 lamps of 50% leading to 0.15 lasting.

Summing all probabilities = 0.14 + 0.12 + 0.15 gives 0.41 probability

1. Given that a selected lamp lasted more than 100 hours, what are the conditional probabilities that are of type 1, type 2, and type 3?

**Solution**: L1 = 0.14 / 0.41 = 0.34

L2 = 0.12/ 0.41 = 0.29

L3 = 0.15 / 0.41 = 0.36

**Contingency table / Bivariate table**

**5.** In one of the cities of India out of 1,00,000 people 51,500 are male and 48,500 are female. Among the males, 9,000 use cosmetics. Among the women, 30,200 use cosmetics. If a person is selected at random, what is the probability that:

1. He or she uses cosmetics.

Solution: (9000 + 30200)/100000 = 0.392

1. A **male** or a **person using cosmetics** is chosen. P(A or B)

Solution: (9000/100000)+((9000+30200)/100000) = 0.482

1. A male not using cosmetics or a female using cosmetics, is chosen.

Solution: ((51500 – 9000)/100000)+(30200/100000) = 0.727

1. A male not using cosmetics is chosen.

Solution: ((51500 – 9000)/100000 = 0.425

6. The probabilities that drivers A, B and C will drive home safely after dining at party are 2/5, 3/7 and 3/4, respectively. What is the probability that they will all drive home safely after the late night party?

Solution: 2/5 \*. 3/7 \*. ¾ = 0.12

7. A factory has 65% male workers. 70% of the total workers are married. 47% of the workers are married males. Find the probability that a worker chosen randomly is:

|  |  |  |
| --- | --- | --- |
| Male workers | 65% of x | 35% of x is female |
| Married workers | 70% of x | 30% of x is unmarried |
| Married Male workers | 47% of married workers | 53% of married workers are female |
|  |  |  |
| Let x = 100 |  |  |
| Male workers | 65 |  |
| Female workers | 35 |  |
| Married workers | 70 |  |
| Unmarried workers | 30 |  |
| Married male workers | 32.9 |  |
| Married female workers | 37.1 |  |

1. i. Married female
2. Solution: 37.1%
3. ii. A male and married

Solution: 32.9%

8. A research group says, illiquidity of scripts is occurring only one-third as often as quoting below par. The probability of both illiquidity and below par is 0.05. If 80% of scripts have none of these problems, how low must the illiquidity problem probability be?

Solution: p\_no\_issues\_in\_script = 0.8

p\_issues\_in\_script = 1-p\_no\_issues\_in\_script

p\_below\_par = 3/4\*(p\_issues\_in\_script+0.05) = 0.062

9. The probabilities that “A” and “B” will tell the truth are 2/3 and 4/5 respectively. What is the probability that

P\_A = 2/3

P\_A1 = 1-P\_A

P\_B = 4/5

P\_B1 = 1-P\_B

i) they agree with each other

Solution: (P\_A\*P\_B)+(P\_A1\*P\_B1) =0.6

ii) they contradict each other while giving a witness in the court.

Solution: (P\_A\*P\_B1)+(P\_B\*P\_A1) = .4

10. Five Managers A, B C, D and E of a Bank are considered for a 3 member Trade delegation to represent the bank in an international trade conference. Construct the sample space and find the probability that

sample space = {ABC,ABD,BCD,BCE,CDE,ACD,ACE,ADE,BDE}

a.) A is selected

Solution: 6/10 = 0.6

b.) A is not selected

Solution: 1 – (6/10) = 0.4

c.) Either A or D (not both) is selected.

Solution: 6/10 = 0.6

(Assume Natural assignment of probability).

11. A recently developed car has two important components A and B. The probability of failure of A and B are 0.2 and 0.1. What is the probability that the car will fail?

Solution: p(a) + p(b) - p(a) \* p(b) = 0.28

12. A manufacturing firm is engaged in the production of steel pipes in its three plants with a daily production of 1000, 1500 and 2500 units respectively. Based on the past experience, it is known that the fractions of defective pipes produced by the three plants are respectively 0.04, 0.09 and 0.07. If a pipe is selected from a day’s total production and found to be defective, find out:

P\_A = 1000/5000

P\_B = 1500/5000

P\_C = 2500/5000

# probability of defective pipes

P\_A\_given\_E = 0.04

P\_B\_given\_E = 0.09

P\_C\_given\_E = 0.07

* 1. What is the probability of the defective pipes?

Solution: (P\_A \* P\_A\_given\_E) + (P\_B\* P\_B\_given\_E) + (P\_C\* P\_C\_given\_E) = 0.07

* 1. What is the probability that it has come from the second plant?

Solution: (P\_B\*P\_B\_given\_E)/(P\_A\*P\_A\_given\_E + P\_B\*P\_B\_given\_E + P\_C\* P\_C\_given\_E) = 0.38

13. What is the probability of getting 53 Mondays in a leap year?

Solution: (366 – (52\*7)) / 7 = 0.28

14. Box I contains 5 Red and 6 Blue balls. Box II contains 6 Red and 4 Blue balls. A ball is drawn at random from box I and is transferred to box II. Now from Box II, a ball is drawn at random. What is the probability that it is red?

box1 = 5R+6B

box2 = 6R+4B

Solution: prob\_red\_box1 = 5/11

prob\_blue\_box1 = 6/11

prob\_red\_box2 = ((5/11)\*(7/11)+(6/11\*6/11)) = 0.58

**15.** The probability of Mr. Sunil solving a problem is. The probability of Mr. Anish solving is. What is the probability that a given problem will be solved?

Solution: P\_Sunil = 0.75

P\_Anish = 0.25

P\_Sunil\_and\_anish = P\_Sunil\*P\_Anish

P\_Sunil\_or\_Anish = P\_Sunil + P\_Anish - P\_Sunil\_and\_anish = 0.8125

16. A company has two plants to manufacture televisions. Plant 1 manufactures 80% of the televisions and Plant 2 manufactures 20%. At plant 1, 85 out of 100 televisions are rated standard quality or better. At plant 2, only 65 out of 100 televisions are rated standard quality or better.

P\_p1 = 0.8

P\_p2 = 0.2

P\_std\_given\_A = 0.85

P\_std\_given\_B = 0.65

P\_total\_std = (P\_p1\*P\_std\_given\_A) + (P\_p2 \* P\_std\_given\_B)

1. What is the probability that the television selected at random came from plant 1 if, it is known that the television is of standard quality?

Solution: P\_std\_p1 = (P\_p1\*P\_std\_given\_A)/P\_total\_std = 0.83

1. b. What is the probability that the television came from plant 2 if it is known that the television is of standard quality?

A - event of drawing a television from plant 1

B - event of drawing a television from plant 2

C be the event of std quality tv by either plant 1 or 2

Solution: P\_std\_p2 = (P\_p2 \* P\_std\_given\_B)/P\_total\_std = 0.16

17. An urn contains 100 marbles: 45 are green, and 20 of these green marbles are swirled. The rest of them are red, and 30 of the red ones are swirled. The marbles that are not swirled are clear. What is the probability of drawing:

P\_g = 45/100

P\_r = 55/100

P\_gs = 20/100

P\_rs = 30/100

P\_rc = 25/100

P\_gc = 25/100

* 1. A green marble from the urn

Solution: 45 / 100

* 1. A clear marble from the urn

Solution: P\_rc + P\_gc = 0.5

* 1. A green, swirled marble

Solution: 20 /100

* 1. A red, clear marble

Solution: 25/100

* 1. A swirled marble

Solution: P\_rs + P\_gs = 0.5

18. The probabilities that component A and component B of a machine will fail are 0.09 and 0.06 respectively. The machine will fail if any one of them fails. Find the probability that it will fail.

P\_A = 0.09

P\_B = 0.06

P\_A\_and\_B = P\_A \* P\_B

Solution: P\_A + P\_B - P\_A\_and\_B = 0.14

19. Data on the largest equity and debt funds provided 1-year and 3-year percentage returns for the period ending March 31, 2008. Suppose we consider a 1-year return in excess of 50% to be high and a 3-year return in excess of 100% to be high. 11 of the funds had 1-year returns in excess of 50% , 9 of the funds had 3-year in excess of 100% and five of the funds had both 1-year returns in excess of 50% and 3-year returns of 100%

n\_A = 11

n\_B = 9

n\_A\_and\_B = 5

1. What is the probability of a high 1-year return, and what is the probability of a high 3-year return?

Solution: n\_A/(n\_A + n\_B - n\_A\_and\_B) = 0.73 (1 year return)

n\_B/(n\_A + n\_B - n\_A\_and\_B) = 0.6 ( 3 year return)

1. What is the probability of both a high 1-year return and a high 3-year return?

Solution: n\_A\_and\_B/(n\_A + n\_B - n\_A\_and\_B) = 0.33

1. What is the probability of neither a high 1-year return nor a high 3-year return?

Solution: 1 - (P\_A + P\_B - P\_A\_and\_B) = 0.0

20. An investment analyst presents the following table giving probabilities of next year’s economic conditions normal or good or very good, in the country and probabilities of the movement increase or decline

Growth in GDP condition Movement in Stock Market

Increase Decline

Zero growth (same) 0.33 0.17

Positive (≥ +1%) 0.24 0.06

Negative (< -1%) 0.18 0.02

Find the probability that the market will increase, given that the GDP growth will be positive.

P\_A\_and\_B = 0.24

P\_B = 0.3

Solution: P\_A\_and\_B/P\_B = 0.8