

CS513 (HW-1)

Q1.1 $P(J) = 20\% = 0.2$
 $P(S) = 30\% = 0.3$
 $P(J \cap S) = 8\% = 0.08$

a- $P(J|S) = \frac{P(J \cap S)}{P(S)} = \frac{0.08}{0.3} = \frac{8}{30} \approx 0.267$

$= 26.7\%$

b- $P(J|\bar{S}) = \frac{P(J \cap \bar{S})}{P(\bar{S})} = \frac{P(J) - P(J \cap S)}{1 - 0.3}$

$= \frac{12}{70} = 0.171 \approx 17\%$

c $P(J \cap S | J \cup S) = \frac{P((J \cap S) \cap (J \cup S))}{P(J \cup S)} = \frac{P(J \cap S)}{P(J \cup S)} = \frac{0.08}{P(J) + P(S) - P(J \cap S)}$

$= \frac{0.08}{0.2 + 0.3 - 0.08} = \frac{0.08}{0.42} = 0.19$
 $= 19\%$

Q1.2 $P(H) = 80\% = 0.8$
 $P(S) = 90\% = 0.9$
 $P(H \cup S) = 91\% = 0.91$

$\therefore P(H \cap S) = P(H) + P(S) - P(H \cup S) = 0.79$

a- $P(\text{Only } H) = P(H) - P(H \cap S) = 0.8 - 0.79$

$= 0.01 \approx 1\%$

$$b - P(\text{Only } S) = P(S) - P(S \cap H) \\ = 0.9 - 0.79 = \boxed{0.11 = 11\%}$$

$$c \quad P(\text{Neither getting a B}) = P(\overline{H \cup S}) = 1 - 0.91 \\ = \boxed{0.09 = 9\%}$$

Q1.3 a Terry and Susan go together 8% of the days.

If the events were independent then individual parameter products are equal to the intersection.

In this case,

$$\boxed{20\% \times 30\% = 6\%} \text{ implying} \\ \boxed{\text{they are not independent}}$$

Q1.4 a. If these events are independent,

$$P(\text{Second} = 5 \text{ and Sum} = 6) = P(\text{Sum} = 6) * P(\text{Second} = 5)$$

$$\frac{5}{36} \times \frac{6}{36} \neq \frac{1}{36} \Rightarrow \boxed{\text{Not independent}}$$

b - If the events are independent

$$P(\text{first} = 5 \text{ and Sum} = 7) = P(\text{Sum} = 7) \text{ and } P(\text{first} = 5) \\ \frac{1}{36} \text{ and } \frac{6}{36} \times \frac{8}{36}$$

$$\frac{1}{36} = \frac{1}{36}$$

They are independent

Q1.5a.

$$P(\text{chose Tx}) = 60\%$$

$$P(\text{chose NJ}) = 10\%$$

$$P(\text{chose Ak}) = 30\%$$

$$P(\text{oil} | \text{Tx}) = 30\%$$

$$P(\text{oil} | \text{NJ}) = 10\%$$

$$P(\text{oil} | \text{Ak}) = 20\%$$

$$P(\text{oil}) = P(\text{oil} | \text{Tx}) \times P(\text{Tx}) + P(\text{oil} | \text{Ak}) \times P(\text{Ak}) + P(\text{oil} | \text{NJ}) \times P(\text{NJ})$$

$$= 0.30 \times 0.60 + 0.2 \times 0.3 + 0.1 \times 0.1$$

$$= 0.18 + 0.06 + 0.01$$

$$= 0.25$$

$$= 25\%$$

$$P(\text{Finding oil}) = 25\%$$

b-

$$P(\text{Tx} / \text{oil}) = \frac{P(\text{Tx} \cap \text{oil})}{P(\text{oil})}$$

$$= \frac{0.3 \times 0.6}{0.25} = \frac{18}{25}$$

$$= 72\%$$

Q1.6a-

$$P(\text{Not survive}) = \frac{\text{NS}}{T} = \frac{1490}{2201} \approx 0.677$$

$$\approx 67.7\%$$

b-

$$P(\text{First class}) = \frac{325}{2201} \approx 0.148 = 14.8\%$$

c-

$$P(\text{FC} / \text{survive}) = \frac{P(\text{FC} \cap \text{survive})}{P(\text{survive})} = \frac{203}{711}$$

$$= 0.285 \approx 28.5\%$$

$$d. P(\text{survived}) = 1 - 0.677 = \underline{\underline{32.3\%}} \quad - (1)$$

$$P(\text{first class}) = \underline{\underline{14.8\%}} \quad - (2)$$

$$\therefore (1) \times (2) \text{ gives us } \underline{\underline{4.78\%}}$$

$$P(\text{FC} \cap \text{S}) = \frac{203}{2201} = 9.22\%$$

Here they are not independent

$$e. P(\text{First child} / \text{survived}) = \frac{P(\text{First child} \cap \text{survived})}{P(\text{survived})}$$

$$= 6/711 \approx 0.0084$$

$$= \underline{\underline{0.84\%}}$$

$$f. P(\text{Adult} / \text{survived}) = \frac{P(\text{Adult} \cap \text{survived})}{P(\text{survived})} = \frac{654}{711} = \underline{\underline{0.9198 = 91.98\%}}$$

~~$$g. P(\text{age} / \text{survived}) = P(\text{C} / \text{survived}) + P(\text{A} / \text{survived}) = \frac{6}{711} + \frac{654}{711} = \frac{660}{711}$$~~

~~$$P(\text{First class} / \text{survived}) = \frac{203}{711} = \underline{\underline{28.55\%}}$$~~

~~clearly P(R) < P(S)~~

The events are not independent

$$g: P(\text{adult} \cap \text{Survived}) = \frac{197}{2201} = 0.089$$

$$P(\text{child} \cap \text{first}) = \frac{6}{2201} = 0.0027$$

$$P(\text{child}) = 57/2201, \quad P(\text{first}) = 203/2201$$

$$P(\text{adult}) = 654/2201$$

$$P(\text{child}) \times P(\text{first}) = 57 \times 203 / (2201)^2 = \underline{\underline{0.0023}}$$

$$P(\text{adult}) \times P(\text{first}) = 197 \times 203 / (2201)^2 = \underline{\underline{0.006}}$$

$$\rightarrow \text{Since } P(\text{adult} \cap \text{first}) \neq P(\text{adult}) \cdot P(\text{first})$$

\Rightarrow Adult and first class are not independent given that passenger survived

$$\rightarrow \text{Since } P(\text{child} \cap \text{first}) \neq P(\text{child}) \cdot P(\text{first})$$

\Rightarrow Child and first class are not independent given that passenger survived.