

Q1)

$$M = (A - BD^{-1}C)^{-1}$$

where A, B, C, D and M are square matrices of size n

$$M = (A - BD^{-1}C)^{-1} = \frac{1}{A - \frac{BC}{D}} = \frac{D}{AD - BC} \quad \text{--- (1)}$$

Inverse of $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is given by,

$$\begin{aligned} \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} &= \frac{1}{AD - BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \\ &= \begin{bmatrix} \frac{D}{AD - BC} & \frac{-B}{AD - BC} \\ \frac{-C}{AD - BC} & \frac{A}{AD - BC} \end{bmatrix} \end{aligned}$$

Now solving each element,

$$(1) \quad \frac{D}{AD - BC} = M \quad \{ \text{From (1)} \}$$

$$(2) \quad \frac{-B}{(AD - BC)} = \frac{-BD}{(AD - BC)} = -MBD^{-1}$$

$$(3) \quad \frac{-C}{(AD - BC)} = \frac{-CD}{(AD - BC)D} = -MCD^{-1}$$

$$\begin{aligned} (4) \quad \frac{A}{AD - BC} &= \frac{AD^2}{D^2(AD - BC)} \\ &= \frac{AD^2 - DBD + D(CB)}{D^2(AD - BC)} \\ &= \frac{D + CDB}{\frac{AD - BC}{D^2}} \\ &= \frac{D + CMB}{D^2} \end{aligned}$$

$$= \frac{1}{D} + \frac{CMB}{D^2} = \underline{\underline{D^{-1} + D^{-1}CMB D^{-1}}}$$

Hence, substituting all (1) in a matrix eqn, we get,

$$\begin{bmatrix} A & C \\ B & D \end{bmatrix}^{-1} = \begin{bmatrix} M & -MBD^{-1} \\ -D^{-1}CM & D^{-1} + D^{-1}MBCD^{-1} \end{bmatrix}$$

Q2) ① $P(a=0)$ is given by

$$0.192 + 0.144 + 0.048 + 0.216 = 0.6 \Rightarrow P(a=1) = 0.4$$

$$P(b=0) \text{ is given by } 0.592 \Rightarrow P(b=1) = 0.408$$

$\therefore P(a=0)$ and $P(b=0)$ is given by,

$$P(a=0)P(b=0) = 0.6 \times 0.592 = 0.3552$$

$$\text{Also, } P(a=0, b=0) = 0.192 + 0.144 = 0.336$$

$$\text{Hence } P(a=0)P(b=0) \neq P(a=0, b=0)$$

② Now, for $c=0$,

$$P(c=0) = 0.192 + 0.048 + 0.192 + 0.048 = 0.48$$

$$P(c=1) = 1 - P(c=0) = 0.52$$

$$③ P(a=0, c=0) = 0.192 + 0.048 = 0.24$$

$$\therefore P(a=0|c=0) = P(a=0, c=0) / P(c=0) = 0.24 / 0.48 = 0.5$$

$$\text{Similarly } P(b=0, c=0) = 0.384$$

$$P(b=0|c=0) = 0.384 / 0.48 = 0.8$$

$$\therefore P(b=0|c=0)P(a=0|c=0) = 0.5 \times 0.8 = 0.4$$

$$P(a=0, b=0, c=0) = 0.192$$

$$\therefore P(a=0, b=0|c=0) = P(a=0, b=0, c=0) / P(c=0) = 0.192 / 0.48 = 0.4$$

Hence proved that the values are equal.