CS 559 — HW2

Q1) $y(x, w) = w_0 + w_1 x + w_2 x^2 + \dots + w_m x^m = \sum_{j=0}^{m} w_j x^j$ — ①

$E(w) = \frac{1}{2} \sum_{n=1}^{N} [y(x_n, w) - t_n]^2$ — ②

Substituting ① in ②, we get.

$$E(w) = \frac{1}{2} \sum_{n=1}^{N} \left[\sum_{j=0}^{M} w_j x_n^j - t_n \right]^2$$

Differentiating wrt. w_i then,

$$0 = \frac{2}{2} \sum_{n=1}^{N} \left[\sum_{j=0}^{n} w_j x_n^j - t_n \right] x_n^i$$

$$\Rightarrow \sum_{n=1}^{N} \left[\sum_{j=0}^{M} w_j x_n^j - t_n \right] x_n^i = 0$$

$$\Rightarrow \sum_{n=1}^{N} \sum_{j=0}^{m} w_j x_n^{i+j} = \sum_{n=1}^{N} t_n x_n^i$$

$$\Rightarrow \sum_{j=0}^{M} \left[\sum_{n=1}^{N} x_n^{i+j} \right] = \sum_{n=1}^{N} t_n x_n^i$$

$$\Rightarrow \sum_{j=0}^{M} A_{ij} w_j = T_i$$

$$A_{ij} = \sum_{n=1}^{N} x_n^{i+j}, \quad T_i = \sum_{n=1}^{N} t_n x_n^i$$

The sol^n of the set of linear eq. is

$$\Rightarrow \sum_{j=0}^{M} A_{ij} w_j = T_i \quad \text{— (3)}$$

For the regularized sum of squares of error function. The corresponding linear eq. are obtained by differentiation

Replace A_{ij} by $\tilde{A}_{ij} = A_{ij} + \lambda I_{ij}$ in eq. (3),

$$\therefore \sum_{j=0}^{M} \tilde{A}_{ij} w_j = T_i$$

$$\sum_{j=0}^{M} (A_{ij} + \lambda I_{ij}) w_j = T_i$$

Q2)

$$S_B = (m_2 - m_1)(m_2 - m_1)^T$$

$$S_W = \sum_{n \in C_1} (x_n - m_1)(x_n - m_1)^T + \sum_{n \in C_2} (x_n - m_2)(x_n - m_2)^T$$

$$w_0 = -w^T m \quad , \quad \sum_{n=1}^{N} (w^T x_n + w_0 - t_n) x_n = 0$$

$$\therefore \left(S_W + \frac{N_1 N_2}{N} S_B \right) w = N (m_1 - m_2).$$

$$S_W^2 = \sum_{x \in C_1} w^T (x - m_1)(x - m_1)^T w + \sum_{x \in C_2} w^T (x - m_2)(x - m_2)^T w$$

$$= w^T S_1 w + w^T S_2 w$$

$\therefore$ S_w comes out to be

$$S_w = \underline{\underline{w^T S_w w}}$$

$|||^{ly}$ $(m_2 - m_1)^2 = \underline{\underline{w^T S_B w}}$

where $S_B = (m_2 - m_1)(m_2 - m_1)^T$

$J(w) = \dfrac{w^T S_B w}{w^T S_w w}$ and here,

$$(w^T S_B w) S_w w = (w^T S_w w) S_B w$$

$\therefore$ $S_B w = (m_1 - m_2)(m_1 - m_2)^T w$ where $(m_1 - m_2)^T w$ is a scalar and always in the dir^n of $(m_1 - m_2)$

$$\Rightarrow \underline{\underline{w \propto S_w^{-1}(m_2 - m_1)}}$$