Q5) $\mu_{f_s \to x}(x) = \sum_{x_1} \cdots \sum_{x_m} f_s(x_1, x_2, \ldots, x_M) \prod_{m=1}^{M} \mu_{x_m \to f_s}(x_m)$

and

$U_{x_m \to f_s}(x_m) = \prod_{f \in ne(x_m) \setminus f_s} \mu_{f \to x_m}(x_m)$,

$ne(x_m) \setminus f_s$ is the collection of all factors which are related to x_m, other than f_s.

If a leaf is a variable node x, its message to a variable node x is

$$\mu_{f \to x}(x) = f(x)$$

A node can send out a message if all the necessary messages that have arrived. Once all messages get sent out, the marginal probabilities can be computed as the following,

$$p(x) \propto \prod_{f \in ne(x)} \mu_{f \to x}(x)$$

Computational marginal of the set of variables x_s involved in a factor f_s.

$$p(x_s) \propto f_s(x_s) \prod_{x \in ne(f)} \mu_{x \to f}(x)$$

The joint probability of a single node x and all the observed nodes when we multiply the messages ~~coma~~ coming to x:

$$P(x, x_0) \propto \prod_{f \in ne(x)} \mu_{f \to x}(x)$$

The conditional is easily observed by normalization

$$P(x \mid x_0) = \frac{P(x, x_0)}{\sum_{x'} P(x', x_0)}$$

When the factor graph contains loops and not a tree, there is no guarantee that the ~~algo~~ algorithm will ever converge.