CS 583 - Assignment 1

Q1) $x : [5, -3, -1, 2]^T$

1. l_2 norm $: ||x||_2 = (\Sigma_i x_i^2)^{1/2}$

$\therefore \quad ||x||_2^2 = (\Sigma_i x_i^2)$
$= 5^2 + (-3)^2 + (-1)^2 + 2^2$
$= 25 + 9 + 1 + 4$
$= \underline{\underline{39}}$

2. l_1 norm $: ||x||_1 = \Sigma_i |x_i|$
$= |5| + |-3| + |-1| + |2|$
$= 5 + 3 + 1 + 2$
$= \underline{\underline{11}}$

3. $x^T a = \begin{bmatrix} 5 \\ -3 \\ -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ 6 \\ -1 \end{bmatrix} = (5 \times 4) + (-3 \times -2) + (-1 \times 6) + (2 \times -1)$
$= 20 + 6 - 6 - 2$
$= \underline{\underline{18}}$

Q2) $A = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \quad B = \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix}$

1. $Ab = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 6(-4) + (1)(5) + (-2)(2) \\ -5(-4) + 5(7) + 2(9) \end{bmatrix}$

$= \begin{bmatrix} -23 \\ 73 \end{bmatrix}$

2. $AA^T = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \begin{bmatrix} 6 & -5 \\ 1 & 7 \\ -2 & 9 \end{bmatrix}$

$= \begin{bmatrix} 41 & -41 \\ -41 & 155 \end{bmatrix}$

(Q3) $x = [x_1, x_2, x_3)$, $y = \frac{x_1^2}{2} + \log_e x_2 - \frac{x_1}{x_3}$

$\frac{\partial y}{\partial x_1} = \frac{2x_1}{2} - \frac{1}{x_3} = \frac{\cancel{2} \times 9}{\cancel{2}} - 2 = 7$

$\frac{\partial y}{\partial x_2} = \frac{1}{x_2} = \frac{1}{1} = 1$

$\frac{\partial y}{\partial x_3} = \frac{x_1}{x_3^2} = \frac{9}{(1/2)^2} = 36$

Using $x = [9, 1, 1/2]$

$\therefore \frac{dy}{dx} = [7, 1, 36]$

Q4 $f(\omega) = \|X\omega - y\|_2^2 + \lambda\|\omega\|_2^2$

~~$\frac{\partial f(\omega)}{\partial \omega} = 2\|X\omega - y\| + 2\lambda\|\omega\|$~~

~~$\frac{\partial f(\omega)}{\partial \omega} = 2X^T(X\omega - y) + 2\lambda\omega$~~

$X = (x_{ij})_{n\times d}$ $\quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}_{n\times 1}$ $\quad \omega = \begin{pmatrix} \omega_1 \\ \vdots \\ \omega_d \end{pmatrix}_{d\times 1}$

$$X\omega = \left(\sum_{j=1}^{d} x_{ij}\,\omega_j\right)_{i=1}^{n} = \begin{pmatrix} \sum_{j=1}^{d} x_{1j}\omega_j \\ \sum_{j=1}^{d} x_{2j}\omega_j \\ \vdots \\ \sum_{j=1}^{d} x_{nj}\omega_j \end{pmatrix}$$

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$$\|X\omega - y\|_2^2 = \left(\sum_{j=1}^{d} x_{1j}\omega_j - y_1\right)^2 + \left(\sum_{j=1}^{d} x_{2j}\omega_j - y_2\right)^2 + \dots + \left(\sum_{j=1}^{d} x_{nj}\omega_j - y_n\right)^2$$

$$= \sum_{i=1}^{n}\left(\sum_{j=1}^{d} x_{ij}\,\omega_j - y_i\right)^2$$

~~$\|X\omega - y\|_2^2 =$~~

$F: \mathbb{R}^d \to \mathbb{R}$

$$F(\omega) = \sum_{i=1}^{n}\left(\sum_{j=1}^{d} x_{ij}\,\omega_j - y_i\right)^2 + \lambda\left(\sum_{l=1}^{d} \omega_l^2\right)$$

$$\frac{\partial f(\omega)}{\partial \omega_k} = \sum_{i=1}^{n} 2\left[\left(\sum_{j=1}^{d} x_{ij}\omega_j - y_i\right) x_{ik} + \lambda \omega_k\right]_{k=1}^{d}$$

$$\frac{\partial f(\omega)}{\partial \omega} = 2\left(\sum_{i=1}^{n}\left(\sum_{j=1}^{d} x_{ij}\, \omega_j - y_i\right) x_{ik}\right)_{k=1}^{d} + 2\lambda(\omega_k)_{k=1}^{d}$$

$X^T(X\omega - y)$ where $X^T \in M_{d\times n}(\mathbb{R})$
$X\omega - y \in M_{n\times 1}(\mathbb{R})$.

$$\therefore \frac{\partial f(\omega)}{\partial \omega} = 2\left(\sum_{i=1}^{n}\left(\sum_{j=1}^{d} x_{ij}\, \omega_j - y_i\right) x_{ik}\right)_{k=1}^{d} + 2\lambda(\omega_k)_{k=1}^{d}$$

$$= 2X^T(X\omega - y) + 2\lambda\omega$$

$$\therefore \frac{\partial f(\omega)}{\partial \omega} = 2X^T(X\omega - y) + 2\lambda\omega$$