# Homework 4

Q1) Bayes' theoram describes updates to probabilities of the hypotheses, based on given data.

Let M, N denote two events. Now Baye's theoram provides a relⁿ b/w them.

$$P(M \cap N) = P(M)\, P(N/M) \quad - (1)$$
$$P(M \cap N) = P(N)\, P(M|N) \quad - (2)$$

Equating (1) and (2), we get

$$P(M/N) = \frac{P(N/M)\, P(M)}{P(N)}$$

In machine learning, various attributes and features are used for predictions and classifications.

Using the theoram, given probability distributions of various variables, we can calculate the probability of the response variable belonging to a particular value.

Thus our end goal is to find the best possible hypothesis, given initial probabilities. Thus prior prob. and best hypothesis can be found with the Bayes' theoram.

Q2) Wkt, from Bayes' rule,

$$P(cancer/+) = \frac{P(+/cancer) * P(cancer)}{P(+/cancer) * P(cancer) + P(+/\sim cancer) * P(\sim cancer)}$$

$$= \frac{0.98 \times 0.008}{0.98 \times 0.008 + 0.03 \times 0.992}$$

$$= \underline{\underline{0.21}}$$

Also,

$$P(cancer/++) = \frac{P(+/cancer) * P(cancer/+)}{P(+/cancer) * P(cancer/+) + P(+/\sim cancer) * P(\sim cancer/+)}$$

Where

$P(cancer/+) = 0.21$ and

$P(\sim cancer/+) = 0.79$

$$\therefore P(cancer/++) = \frac{0.98 \times 0.21}{0.9 \times 0.21 + 0.03 \times 0.79}$$

$$= \underline{\underline{0.91}}$$

$$\therefore P(\sim cancer/++) = 1 - P(cancer/++)$$

$$= \underline{\underline{0.09}}$$

Q3) Given, outlook = sun
Temp = cool
Humidity = high
wind = strong

Now, $P(Yes) = \frac{8}{12} = \underline{\underline{0.67}}$

$P(No) = \frac{4}{12} = \underline{\underline{0.33}}$

Now,

$P(strong / Yes) = 0.375$
$P(high / Yes) = 0.375$
$P(cool / Yes) = 0.375$
$P(sun / Yes) = 0.25$

$P(strong / No) = 0.5$
$P(high / No) = 0.75$
$P(cool / No) = 0.25$
$P(sun / No) = 0.75$

$\therefore P(Yes) = 0.67 \times 0.25 \times 0.375 \times 0.375 \times 0.375$
$= \underline{\underline{8.833 \times 10^{-3}}}$

$P(No) = 0.33 \times 0.75 \times 0.25 \times 0.75 \times 0.5$
$= \underline{\underline{0.0232}}$

Hence, Naive Bayes' classifier assigns the value of $\boxed{\text{NO}}$

Q4) For the first iteration,

$$net_c = w_{c0} + a \times w_{ca} + b \times w_{cb}$$
$$= 0.1 + 0.1 + 0$$
$$= \underline{\underline{0.2}}$$

$$\therefore O_c = \frac{1}{1+e^{-net_c}} = \frac{1}{1+e^{-0.2}} = \underline{\underline{0.55}}$$

$$\therefore net_d = w_{d0} + O_c * w_{dc}$$
$$= 0.1 + 0.55 \times 0.1$$
$$= \underline{\underline{0.155}}$$

Also,

$$O_d = \frac{1}{1+e^{-net_d}} = \frac{1}{1+e^{-0.155}}$$
$$= \underline{\underline{0.539}}$$

Hence using backpropagation,

$$\delta_d = O_d * (1-O_d) * (t_d - O_d)$$
$$= 0.539 \times (1-0.539) * (1-0.539)$$
$$= \underline{\underline{0.115}}$$

Now,

$$\Delta W_{dc} = \eta * \delta_d * O_c + \alpha \times 0$$
$$= 0.3 * 0.115 * 0.55 + 0$$
$$= \underline{0.019}$$

$\Delta W_{do} = 0.034$

$\therefore W_{dc} = W_{dc} + \Delta W_{dc}$
$= 0.1 + 0.019 = \underline{0.119}$

$W_{do} = W_{do} + \Delta W_{do}$
$= 0.1 + 0.034 = \underline{0.134}$

$$\delta_c = O_c * (1 - O_c) * (W_c + \delta_d)$$
$$= 0.55 * (1 - 0.55) * (0.1 \times 0.115)$$
$$= \underline{0.003}$$

$$\Delta W_{ca} = \eta * \delta_c * x_a + \alpha \times 0$$

$$= 0.3 * 0.003 * 1 = \underline{0.001}$$

and

$\Delta W_{co} = 0.001$

$\Delta W_{cb} = 0$

Now weights after $1^{st}$ iteration,

$w_{c0} = 0.101$ $\quad$ $w_{d0} = 0.134$

$w_{ca} = 0.101$ $\quad$ $w_{dc} = 0.119$

$w_{cb} = 0.100$

* Second interation

$net_c = w_{c0} + a\,w_{ca} + b\,w_{cb}$

$= 0.101 + 0 + 0.100 = 0.201$

$O_c = \frac{1}{1+e^{-0.201}} = 0.55$

$net_d = w_{d0} + O_c * w_{dc} = 0.134 + 0.55 * 0.119$

$= 0.1994$

Similarly, $O_d = \frac{1}{1+e^{-0.1994}} = 0.5496$

Now using backpropagation,

$\delta_d = O_d * (1 - O_d) * (t_d - O_d)$

$= 0.5496(1-0.5496)(0-0.5496)$

$= -0.136$

$$\Delta W_{dc} = \eta * \delta_d * O_c + \alpha \times \Delta W_{dc}(old)$$
$$= 0.3(-0.136)(0.55) + 0.9(0.019)$$
$$= -0.0053$$

and $\Delta W_{do} = -0.01$

$$\therefore W_{dc} = W_{dc} + \Delta W_{dc}$$
$$= 0.119 + (-0.003)$$
$$= 0.113$$

$$W_{do} = W_{do} + \Delta W_{do} = 0.134 + (-0.01)$$
$$= 0.124$$

$$\delta_c = O_c * (1 - O_c) * (W_{dc} * \delta_d)$$
$$= 0.55(1-0.55)(0.113)(-0.136)$$
$$= -0.0038$$

$$\therefore \Delta W_{ca} = \eta \times \delta_c * n_a + \alpha * W_{ca}(old)$$
$$= 0.3 * (-0.0038) * 0 + 0.9 * 0.101$$
$$= 0.009$$

and $\Delta W_{co} = 0$

$\Delta W_{cb} = -0.001$

Hence, final weights are given as

$w_{co} = 0.101$

$w_{ca} = 0.102$

$w_{cb} = 0.099$

$w_{do} = 0.124$

$w_{de} = 0.113$