Mathemania 2025
Round 2

Round and round
Let's go around in circles and dance
We will clap our hands and sing
La-la-la-la, let's have fun dancing
Ring-a, ring-a, ring-a
Ring-a, ring-a-ring
Ring-a, ring-a, ring-a
Ring-a, ring-a-ring
We will go hand in hand
And have fun jumping around

Gi-Hun, the determined and resilient main character of ***Squid Game***, returns to face the dangerous and gritty games, this time with **455 others**. But this time, the stakes are higher. To his shock, a new masked man emerges—someone from his past, none other than his college math professor. The professor, a lover of complex mathematical puzzles, has convinced the VIPs to transform the deadly games into a new, twisted challenge—**a world where mathematics and strategy collide, with a dash of drama.**

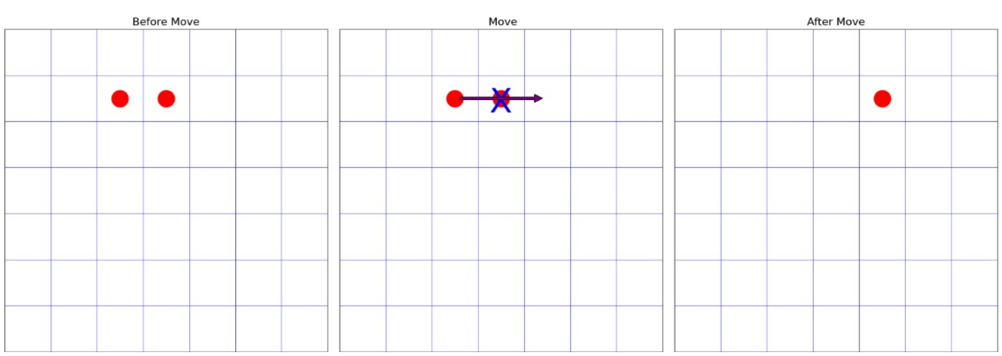
Now, **Gi-Hun** and his friends must rely on their wits to solve intricate problems, navigating through challenges where survival isn't just about strength but intellect. The fate of **Gi-Hun** and his companions rests in your hands. Will you guide them to victory, or will the games claim yet another victim?

Problem 1 [200 Points]

Gi-Hun stands before the Salesman, who is known for recruiting players into the deadly games. The Salesman's face is unmistakably visible, and he smiles cryptically as he presents the next challenge.

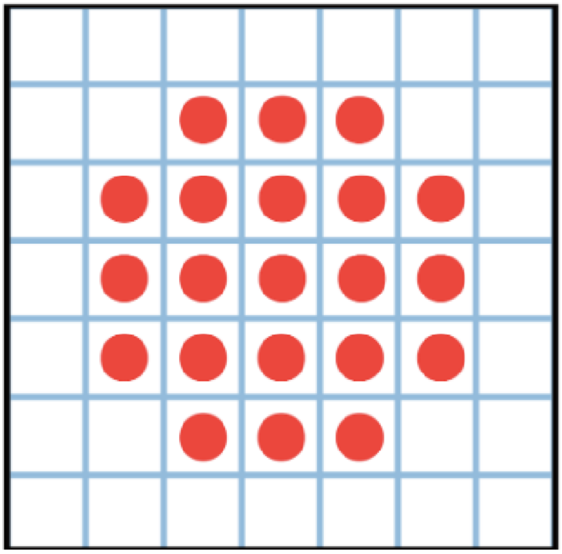
"To prove your worth," says the Salesman, "you must solve this problem. It is inspired by the Ddakji game, which you are all too familiar with."

You are given a 7×7 grid, with pointers in some of the squares. The only move allowed is as follows: you can move in any horizontal or vertical direction.

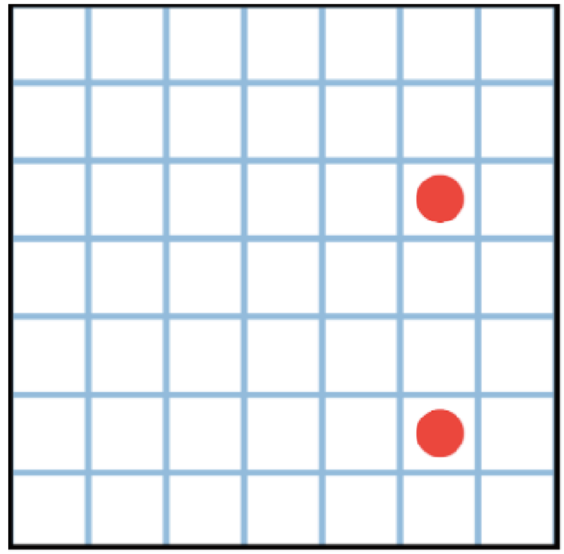


The Salesman watches closely, observing Gi-Hun's every move. The challenge is clear, but the outcome remains uncertain. Will Gi-Hun succeed or fall short?

Your Task: Is it possible to go from (1) to (2) by playing this game? Help Gi-Hun solve the puzzle and determine if success is within reach.



(1)



(2)

Problem 2

[100 Points]

The Front Man begins the first game by recounting a legend of loyalty among the circles. His voice echoes ominously:

> "Once upon a time in the world of geometry, there existed a grand and mighty circle, C_0, with a noble radius of 4 units. This magnificent circle was not alone , accompanying it were four smaller but equally devoted circles, denoted C_1, C_2, C_3, and C_4, each with a radius of 2 units. These smaller circles, though smaller in size, shared a bond of loyalty with their greater counterpart, C_0, forever remaining in touch with it from the inside.
>
> At the beginning of time, $t = 0$, all four smaller circles were perfectly aligned, tangent to C_0 at the same point, say A, marking the beginning of their synchronized dance.
>
> The smaller circles were not static, but dynamic beings. They began to move, revolving at a constant rate around the center of C_0 in a clockwise direction, but always maintaining their loyalty to C_0 by staying tangent to it from within.
>
> As time passed, each of these smaller circles completed its own revolution. By the time $t = t_0$, the point of tangency of smaller circles with bigger one had returned to point A once again. At this moment, each C_i, where $i = 1, 2, 3, 4$, had completed i full revolutions."

"Now," the Front Man continues, his tone commanding,"to better understand this journey, we define the following regions:"

$$S_i = \{P \mid P \textbf{ lies inside exactly } i \textbf{ circles}\} \quad \textbf{for} \quad 1 \leq i \leq 5$$

"The function $F_{S_i}(t)$ computes the area of the region S_i at any time t, where t belongs to $\mathbb{R}^+ \cup \{0\}$."

—

A tense silence fills the room, before the bold and menacing voice of the Front Man commands once more:

—

"Let us now turn our attention to the critical problems:

$G(t) = F_{S_1}(t) - F_{S_3}(t)$ attains its minimum value $\boldsymbol{\alpha}$ times in the interval $[0, t_0]$,
$H(t) = F_{S_4}(t) + F_{S_5}(t)$ attains its minimum value $\boldsymbol{\beta}$ times in the interval $[0, t_0]$,
The minimum value of $G(t)$ attained in the interval $[0, t_0]$ is $\boldsymbol{\gamma}$.

Tell me the value of $\boldsymbol{\gamma^2 + \alpha^2 + \beta^2}$."

Problem 3

[80 Points]

The Front Man's voice echoes through the arena, drawing every player's attention: "You have survived the test of loyalty and motion. But survival is fleeting without the sharpness of logic. Now, you must confront the **Bridge of Logic**, a challenge as perilous as it is precise."

The arena transforms, revealing a vast chasm. Suspended across it is a bridge made of glowing tiles, each engraved with cryptic symbols. The Front Man explains:

> "This bridge is no ordinary one—it is governed by the laws of mathematics. Each step you take requires solving a mystery of functions. To cross the bridge, you must solve the following :
>
> $$f(x + f(y)) = f(x) + y, \quad \forall x, y \in \mathbb{R}.$$
>
> Where $f : \mathbb{R} \to \mathbb{R}$ is a function mapping the real line to itself. The solution to this equation is the key to unlocking the correct sequence of steps. A single misstep will cause the tile beneath you to shatter, plunging you into the abyss below."

He steps back, his shadow looming as he concludes:

> **"Find all possible functions f that satisfy this equation. Solve this, and the bridge will guide you safely to the other side. Fail, and the Squid Game claims another victim."**

Problem 4

[160 Points]

As you step into the dimly lit room, you find yourself facing an ancient, massive vault door, inscribed with mysterious symbols. The vault is said to hold untold treasures of knowledge, accessible only to those who can prove their mastery over the intricate workings of numbers.

The Front Man's voice fills the air:

> **"Across the steps of numbers you must tread,**
> **A sum awaits, where floors are spread.**
> **Each term a fraction, halved and more,**
> **Until the integers they do restore.**
>
> **From zero to k, the sum must be,**
> **A balance of parts, as you will see.**
> **The right-hand side will bring the light,**
> **Prove it true, and cross the night."**

"Now," the Front Man intones, "show that this equation holds true, and the secrets of the vault shall be yours."

The equation appears:

$$\sum_{\mathbf{n=0}}^{\mathbf{k}} \left\lfloor \frac{\mathbf{1}}{\mathbf{2}} \left(\frac{\mathbf{x}}{\mathbf{2^{n-1}}} + \mathbf{1} \right) \right\rfloor = \left\lfloor \mathbf{x} + \frac{\mathbf{1}}{\mathbf{2}} \right\rfloor + \lfloor \mathbf{x} \rfloor - \left\lfloor \frac{\mathbf{x}}{\mathbf{2^k}} \right\rfloor$$

Problem 5

[200 Points]

You step into a vast arena, the polished ground gleaming. At the far end, VIPs sit behind golden masks, their gaze unblinking. A hush falls as a screen hums to life.

A fearing voice echoes:

"The VIPs have grown bored of the old games and have proposed a new challenge—one of intellect. Solve the riddle. A single misstep, and the consequences will be dire."

The screen displays the integral:

$$\mathbf{I} = \int_{\mathbf{0}}^{\infty} \frac{\sin \mathbf{x}}{\mathbf{x}\left(\mathbf{a}\sin^{\mathbf{2}}(\mathbf{x}) + \mathbf{b}\cos^{\mathbf{2}}(\mathbf{x})\right)^{\mathbf{5}}}\,\mathbf{dx}$$

The voice continues:

"Solve this to move forward. Every decision seals your fate."

Your Task: Solve the integral, showing every step of your work.

Problem 6

[120 Points]

Gi-Hun and his friends, having barely escaped their previous challenge, are now faced with a new, mysterious puzzle.

"In the world of numbers, truth hides in plain sight,
A pattern awaits, but beware the flight.
Just as Player 001 deceived with a smile,
This equation's twists will test you for a while."

says the Front Man and took the steps back, and the room grows tense as a screen illuminates the problem before you:

Find all $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x) = \pm\sqrt{\left(2024(x+1) + 2\lim_{n\to\infty}\sum_{k=1}^{n}\frac{x^2k}{n^2}f\left(\frac{(n-k)x}{n}\right)f'\left(\frac{(n-k)x}{n}\right)\right)}.$$

A pause lingers in the air, before the silence is broken by a voice from the shadows. It's Player 001, stepping forward with an unsettling calmness:

"A game of deception, a challenge to win,
With tricks in the numbers, so deep within.
But beware, for not all is as it seems,
Solve with precision, or be lost in your dreams."

Help Gi-Hun and his friends solve this problem and uncover the next piece of the puzzle that could reveal the true faces behind the game.

Problem 7

[80 Points]

Gi-Hun and his friends, having narrowly escaped the previous challenge, now retaliate with a risky plan to uncover the VIPs. Their success hinges on proving the following recurrence. The challenge appears on the screen:

Suppose $a_0 \in \mathbb{R}$ and $a_{n+1} = [a_n]\{a_n\}$ for all $n \geq 0$. Prove that $\exists N \in \mathbb{N}$ such that $\forall n \geq N$, $a_{n+2} = a_n$, where $\lfloor x \rfloor$ represents the greatest integer less than or equal to x, and $\{x\} denotes the fractional part of$x.

Your Task: Gi-Hun and his friends' success in uncovering the VIPs depends on proving this statement. Help them by proving the recurrence.

Problem 8

[80 Points]

Gi-Hun and his friends stand under the watchful eyes of the Pink Guards. The cold, steel walls reflect harsh light as the next challenge appears on the glowing screen. The tension is palpable as they prepare for what lies ahead.

With a swift motion, the Front Man gestures to the screen. The equation begins to form:

$$f(x) = \int_x^{x+1} \sin(e^t)\, dt.$$

"Now, your mission is clear: show that"

$$e^x |f(x)| < 2$$

"and prove that"

$$e^{-x} f(x) = \cos(e^x) - e^{-1} \cos(e^{x+1}) + r(x),$$

"where the remainder term $r(x)$ satisfies the bound $|r(x)| < Ce^{-x}$ for some constant C. Only through careful understanding and attention to detail can you unlock the next phase of this twisted game."

The Pink Guards stand silently, their eyes scanning every movement. The room is filled with tension, and Gi-Hun knows that every step from here on must be precise.

Problem 9

[100 Points]

The stage is set in a cold, dimly lit arena. A single, infinite line stretches across the floor, marked with

$$\frac{1}{2}\left(3^{100} + 1\right)$$

evenly spaced points glowing faintly. Seong Gi-hun, bruised and determined, faces the enigmatic **Front Man**, who stands cloaked in his signature black uniform.

A robotic voice echoes:

> *"This is the game before final game. The rules are simple. You must color exactly 2^{100} of these points red, ensuring that no red point is equidistant from two other red points. Fail, and you lose everything. Succeed, and you win freedom."*

The Front Man steps forward, his voice calm yet menacing:

> *"Gi-Hun, this is no game of luck or brawn. It's a game of reason. If you think you can outsmart me, prove it. Begin."*

Problem 10

[80 Points]

The room is silent except for the faint hum of the spotlight that now shines on the prize—the **45.6 billion won**, suspended high above in a transparent vault. The Front Man steps forward, his gaze cold and unyielding, and hands Gi-Hun a sheet with the following details:

- You start at the **orange city**.
- From any city, you may move to a directly connected city.
- The probability of traveling along a road is proportional to its weight. For example:
  - Starting at the orange city, the probability of moving along a road with weight 3 is $\frac{3}{7}$, if the total weights of outgoing roads are 7.

– If you then arrive at a city connected to green with a road weight 5, and the total weights of outgoing roads are 9, the probability of moving to green is $\frac{5}{9}$.

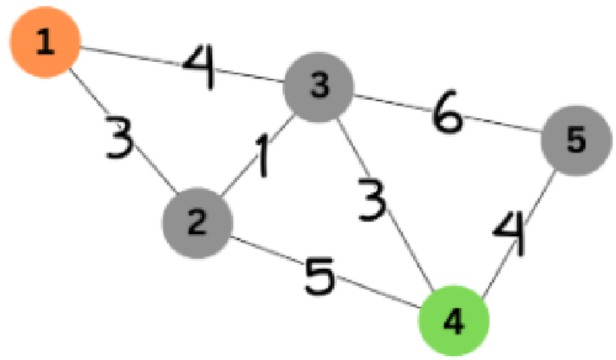
– Therefore, the probability of reaching green in **two steps** (via this route) is $\frac{3}{7} \cdot \frac{5}{9}$.

The Front Man continues:

"Now, your task is to determine: ***What is the probability of reaching the green city after exactly 8 steps, starting from orange?*** *Success will mean claiming the fortune. Failure..."* He pauses, letting the implication hang in the air. "Good luck."

The lights dim, and all eyes are on Gi-Hun as the timer begins its relentless countdown.

Can you help Gi-Hun to make it out?



In a world of danger, games unfold,
Where the Pink Guards in masks are bold.
Gi-Hun steps forward with a heart of grace,
While Player 001 leaves a cryptic trace.

The VIPs from their lavish chairs,
Watch the chaos unfold with no cares.
The robotic voice, so cold and clear,
Echoes through the players' growing fear.

The Salesman smiles, with motives dark,
Leading them all to a deadly mark.
The game is over, the players are few,
But Squid Game's lessons remain with you.