Continuous opinion dynamics on small-world and scale-free network topology

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Abstract

Realistic networks are often modelled using graph networks. Properties of such model networks like the small world effect and heterogeneous distribution of connectivity degree influence the dynamics of the underlying real world network. One such important dynamics is opinion dynamics which is studied in the paper by Guo and Cai [1]. To study the opinion dynamic, a popular opinion model called the Deffuant model is improved further with heterogeneous convergence parameter in this paper. This new opinion model - Improved Deffuant Model (IDM) is then used to study the evolution of steady opinion s_* as a function of confidence parameter, ε. We reproduced some of the results of the paper which showed the dependence of opinion dynamics on confidence parameter and system topology. We are able to reproduce polarised as well as consensus state of steady state opinions. Moreover, we take their study forward and investigate the influence of different initial opinion distributions on the variation of steady state distribution with agent interaction parameter dependent on their opinion differences. We discover a new state other than polarization and consensus. We find the existence of communities of agents with strong, moderately strong and neutral opinions simultaneously.

I. INTRODUCTION

Many natural and man-made networks have been successfully studied as a framework of several celebrated opinion models. Some relevant results have been summarized in a recent review article by Toral and Tessone[2].

Social impact theory founded by Latan[3], Latan and Wolf[4], was developed as a meta theoretical framework for modelling situations where beliefs, attributes or behaviors of an individual are influenced by those of others around him/her. Based on the social impact theory, there are two celebrated opinion models proposed in recent years. One celebrated model is the binary opinion model that is proposed by Sznajd-Weron and Sznajd[5] (S model) to describe a simple mechanism of making up decisions in a closed community.The other is the continuous model proposed by Deffuant *et al.*[6].

In D model, the opinions of individual varies continuously between zero and one. Each agent selects randomly one of the other agents and checks first if an exchange of opinions makes sense. If the two opinions differ by less than $\varepsilon (0 < \varepsilon < 1)$, each opinion moves partly in the direction of the other, by amount $\mu \Delta s$, where Δs is the opinion difference and μ the convergence parameter $(0 < \mu \leq 0.5)$; otherwise, the two refuse to discuss and no opinion is changed. The parameter ε is called confidence bound or confidence parameter.

Many previous works about D model have considered the convergence parameters μ between pairwise agents are uniform on regular lattices and complex networks like Deffuant *et al.*[6], Porfiri, Bollt, and Stilwell[7], Weisbuch, Deffuant, and Amblard[8], Weisbuch[9]. On the other hand, our society is not the homogeneous one, i.e., each individual has his/her confidence parameter ε and convergence parameter *mu* (i.e., the influence of individual). Deffuant, Amblard, and Weisbuch[10] studied the opinion dynamics of D model with different confidence parameter and analyzed the role of the extremists and got many fruitful results. However, in our society, we often change our opinion as the one of individual who is a famous expert about the particular issue according to the celebrity effect. In this current paper, it is assumed that the larger the agent's connectivity is, the more famous expert the agent will be. Hence, the convergence parameters μ between pairwise agents are different, which is a function of the opposite's connectivity k.

II. THE MODEL

In this work we analyse the evolution of opinions of agents connected by different types of network structures such as small world network and scale free network. The agents are allowed to change their opinions continuously based on their interaction with each other. If two agents have close enough opinions and decide to alter opinions upon interaction (that is with some probability) then each of their opinions will shift slightly towards that of the other. With time, the state of opinions converges to a steady state. The steady state depends on the threshold ε for differences in opinions of the interacting agents. We see the steady state behavior for different threshold values of ε and find that a bifurcation occurs in the system. Our findings are in good agreement with the work described by Guo and Cai [1].

In our present work, we study the dynamics of continuous opinion of improved Deffuant model (IDM) with heterogeneous convergence parameters μ in complex networks.

We choose a pairwise agents i and j randomly at each time step. If the two opinions differ more than a fixed threshold parameter $\varepsilon(0<\varepsilon<1)$, which is called the confidence parameter, both opinions refuse to discuss and no opinion is changed. If, instead, $|s_i(t)-s_j(t)|<\varepsilon$, then each opinion moves partly in the direction of the other as:

$$s_i(t+1) = s_i(t) + \mu_j[s_j(t) - s_i(t)], \text{with prob.} p_i$$

$$s_j(t+1) = s_j(t) + \mu_i[s_i(t) - s_j(t)], \text{with prob.} p_j$$

where $\mu_j = \frac{k_j}{(2*k_{max})}$ is the convergence parameter $(0<\mu\leq 1/2)$ that agent j interacts other agents and k_{max} is the largest connectivity degree in social complex system. The probability $p_j(=1-\mu_j)$ is the probability that agent j is persuaded to change his/her opinion, since each agent has the ability to keep his/her opinion from changing.

III. RESULTS

In this section, we present the results obtained from the Improved Deffudant model. The number of agents in the system are N who are connected as per a small world network or a scale-free network. The model is allowed to evolve for T time steps until a steady state is achieved. We initialize the opinions of all agents at $t=0$ as uniformly distributed random numbers between 0 and 1. The process is repeated for different values of ε and we investigate the change in steady state opinion distribution of agents with variation in ε. As noted earlier, ε indicates the difference in opinion of two interacting agents under which they are probably ready to change their opinions. We observe a bifurcation in the system and we note that our results are in very good agreement with those shown by Guo and Cai [1].

We also extend the analysis of Guo and Cai [1] by changing the initial opinion distribution of agents at $t = 0$ from uniformly random to Gaussian and bimodal distributions. We find remarkable difference in the bifurcation diagrams for each case of initial opinion distributions, as discussed in this section.

A. Small World Network

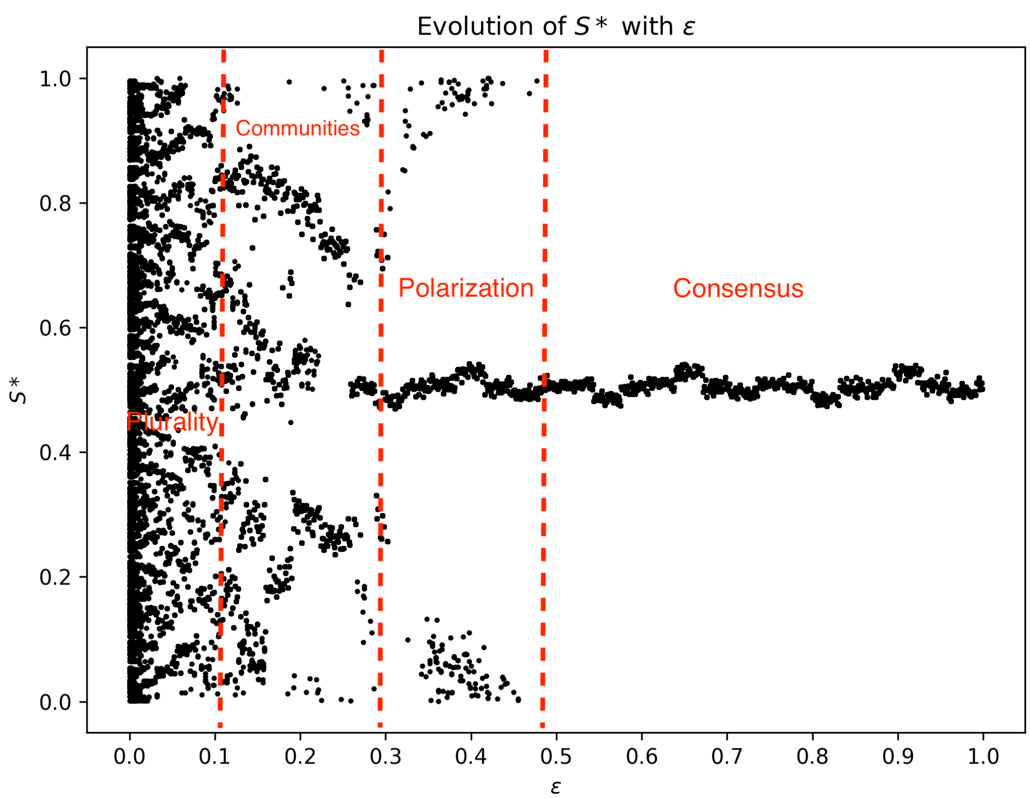

Fig. 1. Evolution of steady opinions of 500 agents in a small world network where the initial distribution of opinions is drawn from a uniform random distribution. Network parameters : $\phi = 0.05, k = 9$

We reproduce the findings on the Small World Network proposed by Watts and Strogatz Watts and Strogatz[11]. Initially N nodes are arranged in a circle, and each node is connected to k of it's nearest neighbours. Two edges are then picked at random and rewired with a probability ϕ. Rewiring the system introduces connections between distant nodes, creating "shortcuts" in the network. For all simulations using this network, we used a value of k = 9 and ϕ = 0.05.

Fig. 1 shows the evolution of the steady state opinion when the initial opinion was drawn from a uniform distribution between 0 and 1. Initially, for $\varepsilon \leq 0.1$ there is plurality of opinion. When ε is low, only agents who have similar opinions interact. As ε increases, agents with different opinions start interacting, which changes the steady state opinion in the society. For $0.1 \leq \varepsilon \leq 0.3$ we see the formation of communities, which evolves into polarization for $0.3 \leq \varepsilon \leq 0.5$. At this point, we can see the formation of two groups in the network. For ≥ 0.5 agents who have very different opinions also start interacting, and the agents reach a consensus.

B. Scale-free network

The codes used for the model on a scale-free network topology are attached in appendix **??**. We first obtain the results from scale-free network for the case where the initial opinions of the agents is a uniform random distribution as shown in Fig. 2. We observe a polarization state for $0.3 \leq \varepsilon \geq 0.5$. For $\varepsilon > 0.5$ we observe that all agents reach the same consensus. These results were also obtained by Guo and Cai[1] and confirm that our code for the improved Deffudant model is appropriate.

The polarization of opinions is observed since only those agents who have close opinions will interact and change their opinions. When the value of ε increases beyond a certain critical value, agents with even strong opinions can interact and as a result we obtain a steady state of consensus. Consensus occurs at the value of 0.5 indicating that all agents have neutral or unbiased opinions.

It is interesting to note that for Gaussian distribution, as we increase the threshold on opinion difference for interaction of agents, we see a diverging polarization of steady state opinions. In Gaussian distribution, the number of agents with extreme opinions (0 or 1) at the initial state is very less. At low thresholds of ε, the few agents with extreme opinions will drift towards the neutral state since their opinions are shifted by agents with very close by opinions only. But at larger thresholds of ε, agents with extreme opinions can interact with other agents with moderate opinions and change their opinions towards the extreme. As a result, communities of steady state opinions tend to diverge to more extreme values as we increase ε.

We further investigate the role of underlying initial opinions on the formation of consensus. First, we initialize a scale free network with opinions that follow a Gaussian distribution. The results for steady state opinion bifurcation for 500 agents is shown in Fig. 3. We observe that the opinions polarise at $\varepsilon = 0.15$. Clearly the polarization point in the bifurcation diagram is advanced as compared to that for uniform random distribution of opinions as seen in Fig. 2. We believe, the reason is that for a Gaussian distribution, to start with we have more agents with opinions closer to the neutral value of 0.5. Thus, agents tend to form community of closer opinions even for a small threshold of their opinion differences.

Finally, at a critical threshold $\varepsilon = 0.3$, all agents form a consensus resulting as a neutral opinion for all agents. Clearly, in the case of initial Gaussian distribution of opinions of agents, consensus is achieved earlier than in the case for uniformly random opinion initialization.

Next, we investigate the case where the initial opinions of agents is distributed according to a bimodal distribution such as that shown in Fig. 4(a). Note that such a bimodal distribution initiates an opinion distribution such that most agents are initially having extreme opinions and very few agents have neutral opinions. As a result, we expect that bifurcation to consensus will be delayed.

In the bifurcation diagram shown in Fig. 4(b) we observe that the steady state opinion of

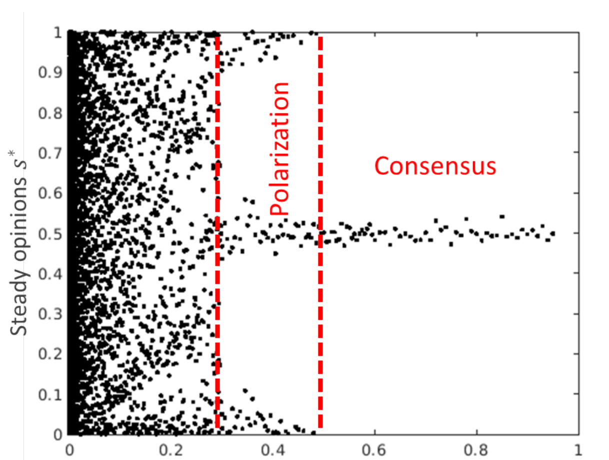

Fig. 2. Bifurcation diagram of steady opinions of 500 agents on a scale-free network obtained from the Improved Deffudant model run for 1000 time steps. The initial distribution of opinions varies according to a uniform random distribution.

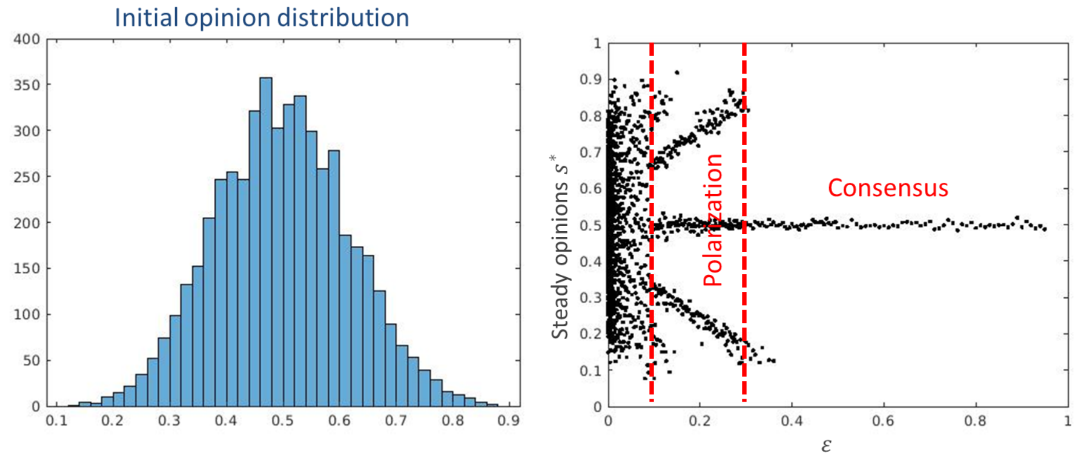

Fig. 3. (a) Distribution of initial opinions being Gaussian (b) Bifurcation diagram of steady opinions of 500 agents on a scale-free network obtained from the Improved Deffudant model run for 1000 time steps for such Gaussian distribution of initial opinions.

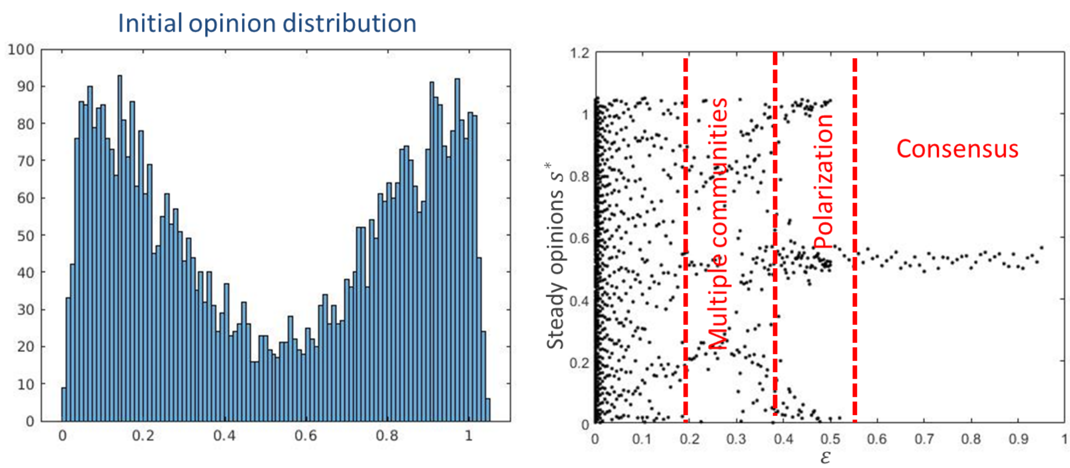

Fig. 4. (a) Distribution of initial opinions being bimodal and (b) Bifurcation diagram of steady opinions of 500 agents on a scale-free network obtained from the Improved Deffudant model run for 1000 time steps for such bimodal distribution of initial opinions.

agents is initially scattered throughout the range 0 to 1. For thresholds of opinion difference parameter $\varepsilon \approx 0.2$ we observe that steady state opinions tend to form multiple clusters. For the range $0.2 \leq \varepsilon \geq 0.4$ we observe multiple communities of agents with similar opinions. That is, some agents have strong opinions tending to 0 or 1 while some agents form moderately strong opinions tending to 0.8 or 0.2. Note that the formation of multiple communities was not found for the case where the initial opinion distribution was uniformly random.

As the threshold of opinion differences is increased, for the range $0.4 < \varepsilon < 0.55$ we observe complete polarization such that agents have either strong opinions (0 or 1) or neutral opinions. Finally, for the range $\varepsilon > 0.55$ all agents acquire neutral opinions in the steady state indicating that a consensus has been reached.

IV. DISCUSSION AND CONCLUSIONS

In this project, we study the evolution of opinions of agents connected according to a small-world or a scale-free network. The opinions of agents are allowed to vary continuously between the two extreme opinions 0 and 1. We follow the Improved Deffudant model proposed by Guo and Cai [1] and we successfully reproduce their results of bifurcation of steady state opinions of agents in a small-world and scale-free network. The initial opinions of agents is drawn from a uniformly random distribution. We find that as the parameter of agent interaction (ε) is allowed to increase, initially the widespread opinion distribution polarises into two strong opinions along with some neutral opinions, and finally achieve consensus. Our results are in agreement with those observed by Guo and Cai [1].

Polarization occurs since agents with opinions » 0.5 tend to interact and drift towards the extreme opinion of 1. Similarly agents with opinions « 0.5 tend to interact and form opinion close to 0. Where as agents with opinions close to 0.5 interact and form opinions closer to 0.5. Thus we obtain three dominant opinions in the steady state opinion distribution of agents. At larger thresholds of ε, agents with strong opinions can interact with agents having neutral opinions and drift towards the neutral opinion of 0.5 thus leading to the state of consensus.

We expand the study and investigate how the steady state of opinions changes if the initial opinion distribution of agents changes. For an initial Gaussian distribution, we observe polarization of opinions in steady state as well as consensus. However, such polarization and consensus states are reached for lower values of ε threshold. For bimodal distribution of initial opinions, with increase in ε we observe multiple communities of dominant opinions where agents have either very strong or moderately strong opinions or neutral opinions. Further, we also observe polarization and consensus being achieved in this system.

Thus, we conclude that not only the underlying network structure but also the social construct of initial opinions matter on how easily can a society form a consensus. A society which has strongly polarised views (like in the bimodal distribution) to start with will need a much larger acceptance threshold (similar to ε) in order to reach consensus. A society with a an open perspective (like in the case of Gaussian distribution) where most agents do not have very strong views to start with may achieve consensus for large acceptance thresholds, that is when the agents are ready to change opinions. But for moderate thresholds of ε, the society may form strongly polarized groups as we showed for the case of initial Gaussian distributed opinions.

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