Input

$$\Sigma = \{0,1\}$$

Let $L = \{ww^R \mid w \in \{0,1\}^* \}$,

$$\begin{cases} S(0) = \{a^n b^n \mid n \geq 1\} & \longrightarrow CFL \; \checkmark \\ S(1) = \{cc, dd\} & \longrightarrow CFL \; \checkmark \end{cases}$$

To show:  $S(L)$  is a CFL

Approach:  to build a grammar for

$$S(L)$$

$G :$ | $S \Rightarrow 0S0 \mid 1S1 \mid \varepsilon$ | $\Rightarrow$ CFG for $L$

$G_0 :$ | $S_0 \Rightarrow aS_0b \mid ab$ | $\Rightarrow$ CFG for $S(0)$

$G_1 :$ | $S_1 \Rightarrow cc \mid dd$ | $\Rightarrow$ CFG for $S(1)$

(G') for $S(L)$ :

$G' :$  $S \Rightarrow \underset{S_0 \quad S_0}{0S0} \mid \underset{S_1 \quad S_1}{1S1} \mid \varepsilon$

$G' \longrightarrow$ | $S \Rightarrow S_0 S S_0 \mid S_1 S S_1 \mid \varepsilon$
$S_0 \Rightarrow aS_0b \mid ab$
$S_1 \Rightarrow cc \mid dd$