

7.1.4)

$$G: \begin{cases} S \Rightarrow AAA \mid B \\ A \Rightarrow aA \mid B \\ B \Rightarrow \epsilon \end{cases}$$

Simplifying CFGs:

$$G \Rightarrow G' \Rightarrow G'' \Rightarrow G''' \Rightarrow G''''$$

i) Eliminate  $\epsilon$ -prod.

Nullable variables are:  $\{S, A, B\}$

$$G': \begin{cases} S \Rightarrow AAA \mid AA \mid A \mid B \\ A \Rightarrow aA \mid a \mid B \end{cases}$$

ii) Eliminate unit productions:

Unit Pairs | non unit production grammar.

|          |  |
|----------|--|
| $(S, S)$ | $S \Rightarrow AAA \mid AA$                        |
| $(S, A)$ | $S \Rightarrow aA \mid a$                          |
| $(S, B)$ | $-$ (empty since no productions are defined for B) |
| $(A, A)$ | $A \Rightarrow aA \mid a$                          |
| $(A, B)$ | $-$  |
| $(B, B)$ | $-$  |

$$G'' : \begin{cases} S \Rightarrow AAA \mid AA \mid aA \mid a \\ A \Rightarrow aA \mid a \end{cases}$$

iii) Useless symbols: both S and A are generating  
both S and A are reachable  
 $\Rightarrow$  nothing to remove. ( $G''' = G''$ )

iv) Chomsky Normal form:

Answer:  $\Rightarrow$

$$G''' : \begin{cases} S \Rightarrow AA_1 \mid AA \mid X_a A \mid a \\ A_1 \Rightarrow AA \\ A \Rightarrow X_a A \mid a \\ X_a \Rightarrow a \\ S \Rightarrow \epsilon \end{cases}$$



Ex: 2  $G'$ :

$S \Rightarrow aAaB \mid baB \mid aC \mid D$   
 $A \Rightarrow aA \mid B \mid C \mid \epsilon$   
 $B \Rightarrow bB$   
 $C \Rightarrow cC \mid Dab \mid \epsilon$   
 $D \Rightarrow dbB \mid a \mid b \mid \epsilon$

i) Eliminate  $\epsilon$ -prod:

Nullable variables:  $\{S, A, C, D\}$

$G'$   
 $\rightarrow S \Rightarrow aAaB \mid aaB \mid baB \mid aC \mid a \mid D$   
 $A \Rightarrow aA \mid a \mid B \mid C$   
 $B \Rightarrow bB$   
 $C \Rightarrow cC \mid c \mid Dab \mid ab$   
 $D \Rightarrow dbB \mid a \mid b$

ii) Eliminate Unit Prod:

Unit Pairs:

$(S, S)$   
 $(S, D)$   
 $(A, A)$   
 $(A, B)$   
 $(A, C)$   
 $(B, B)$   
 $(C, C)$   
 $(D, D)$

non-unit prods:  $G''$

$S \Rightarrow aAaB \mid aaB \mid baB \mid aC \mid a$   
 $S \Rightarrow dbB \mid a \mid b$   
 $A \Rightarrow aA \mid a$   
 $A \Rightarrow bB$   
 $A \Rightarrow cC \mid c \mid Dab \mid ab$   
 $B \Rightarrow bB$   
 $C \Rightarrow cC \mid c \mid Dab \mid ab$   
 $D \Rightarrow dbB \mid a \mid b$



Ex:2 contd....)

iii) Remove Useless Symbols:

a) Generating symbols:  $\{S, A, C, D\}$

non-Generating symbols:  $\{B\}$

Eliminating B:

$\rightarrow S \Rightarrow aC/a/a/b$

$A \Rightarrow aA/a/cC/c/Dab/ab$

$C \Rightarrow cC/c/Dab/ab$

$D \Rightarrow a/b$

b) Unreachable symbols:  $\{A\}$

Eliminating A:

$\rightarrow S \Rightarrow aC/a/b$

$C \Rightarrow cC/c/Dab/ab$

$D \Rightarrow a/b$

iv) Convert to CNF:

$\rightarrow S \Rightarrow X_a C/a/b$

$C \Rightarrow X_c C/c/DX_{ab}/X_a X_b$

$D \Rightarrow a/b$

$X_a \Rightarrow a$

$X_b \Rightarrow b$

$X_c \Rightarrow c$

$X_{ab} \Rightarrow X_a X_b$

final  
answer.

$+ S \Rightarrow \epsilon$