Indian Institute of Technology Kharagpur

CS29003: Algorithms Laboratory, Spring 2022

Assignment 7: Heaps and Priority Queues

2PM – 5PM — 8th March, 2022

General Instructions (to be followed strictly)

Submit a single C/C++ source file.
Do not use global variables unless you are explicitly instructed so.
Do not use Standard Template Library (STL) of C++.
Use proper indentation in your code and include comments.
Name your file as <roll_no>_a7.<extn>
Write your name, roll number, and assignment number at the beginning of your program.

In this assignment, your task is to find positive integer solutions to the equation $a+b^2 = c^3+d^4$ in the range $[1, n]$ (i.e., a, b, c, d must be from the set $1, 2, \ldots, n$), where n is an integer entered by the user. For example, if $n = 5$, we have $1 + 1^2 = 1^3 + 1^4$, $5 + 2^2 = 2^3 + 1^4$, $1 + 4^2 = 1^3 + 2^4$ and $3 + 5^2 = 3^3 + 1^4$ and so there are 4 solutions. This can be done using $O(n^2)$ space – store all possible sums $a + b^2$ for $a, b \in [1, n]$ in one array (of size n^2) and the sums $c^3 + d^4$ for $[c, d] \in [1, n]$ in another; then sort the two arrays and look for matches. But we can do better, using only $O(n)$ space. The idea is to use two min heaps each of size $O(n)$.

Implement a min heap consisting of triples (x, y, z) with *z acting as the key*, with the following functions.

- *build-heap*: returns a new empty heap
- *get-min*: returns the item with smallest key
- *remove-min*: removes (and returns) the item with smallest key and restores the heap property
- *insert*: inserts a new triple maintaining the heap property

In order to find all positive integer solutions to $a+b^2 = c^3+d^4$, you use the above min heap implementation to define two minimum priority queues H_1 consisting of triples of the form $(a, b, a + b^2)$ and H_2 consisting of triples of the form $(c, d, c^3 + d^4)$ where the third component defines priority.

Let $\min(H_i)$ denote the minimum priority element in H_i for $i = 1, 2$. Initially, populate H_1 with triples $(a, 1, a + 1)$ for $a = 1, 2, \ldots, n$ and H_2 with triples $(c, 1, c^3 + 1)$ for $c = 1, 2, \ldots, n$. Until one of the priority queues becomes empty, do the following:

- if $\min(H_1) < \min(H_2)$, remove the minimum item $(a, b, a + b^2)$ of H_1 and then, if $b < n$, insert the item $(a, b + 1, a + (b + 1)^2)$ into H_1.
- if $\min(H_1) > \min(H_2)$, then remove the minimum item $(c, d, c^3 + d^4)$ of H_2 and if $d < n$, insert the item $(c, d + 1, c^3 + (d + 1)^4)$ into H_2.
- if $\min(H_1) = \min(H_2)$, extract minimum items $(a, b, a + b^2)$, $(c, d, c^3 + d^4)$ from H_1, H_2, print the four integers a, b, c, d in a new line and insert items $(a, b+1, a+(b+1)^2)$ in H_1 if $b < n$, $(c, d+1, c^3+(d+1)^4)$ in H_2 if $d < n$.

In the *main()* function,

- Read a positive integer n (≤ 500).
- Print all positive integer solutions to the equation $a+b^2 = c^3+d^4$ in the range $[1,n]$, each in a separate line. A solution a,b,c,d must be printed as $a\ \ b\ \ c\ \ d$.
- Print the total number of solutions.

Do not use any built-in library functions.

- **Sample Output 1**

```
n = 8

  1   1   1   1
  5   2   2   1
  1   4   1   2
  8   4   2   2
  3   5   3   1
  7   6   3   2
  1   8   4   1

Total number of solutions: 7
```

- **Sample Output 2**

```
n = 16

  1   1   1   1
  5   2   2   1
 13   2   1   2
  8   4   2   2
  3   5   3   1
  7   6   3   2
  1   8   4   1
 16   8   4   2
  1   9   1   3
  8   9   2   3
  8  10   3   3
  5  11   5   1
  1  12   4   3
 10  14   5   3
  7  15   6   2
  1  16   1   4
  8  16   2   4

Total number of solutions: 17
```

Policy on Plagiarism

Academic integrity is expected from all the students. Ideally, you should work on the assignment/exam consulting only the material we share with you. You are required to properly mention/cite anything else you look at. Any student submitting plagiarised code will be penalised heavily. Repeated violators of our policy will be deregistered from the course. Read this to know what is plagiarism.