Bit Manipulation 1

Agenda

- Number System Basics
- Binary to Decimal
- Decimal to Binary
- Adding 2 binary numbers
- Bitwise Operators
    - Basic Properties
    - Basic Problems

Number System Basics

Hundreds Tens Units/ones

$$\begin{array}{ccc} 10^2 & 10^1 & 10^0 \\ 7 & 3 & 4 \end{array} : \quad 700 + 30 + 4 = 7\times10^2 + 3\times10^1 + 4\times10^0$$

$$6\,5\,9\,4 : \quad 6000 + 500 + 90 + 4 = 6\times10^3 + 5\times10^2 + 9\times10^1 + 4\times10^0$$

$\left.\begin{array}{l} \text{Digits} = 0 \text{ to } 9 \rightarrow 10 \text{ digits} \\ \text{Base power} = 10 \end{array}\right\}$ Decimal number system

Other Number Systems

→ Binary - 2

→ Octal - 8

→ Hexadecimal - 16

Binary → Digits 0 and 1

Binary → Power = 2

1) $$\left(\overset{2^4}{1}\ \overset{2^3}{0}\ \overset{2^2}{1}\ \overset{2^1}{0}\ \overset{2^0}{0}\right)_2 = 1\times 2^4 + \cancel{0\times 2^3} + 1\times 2^2 + \cancel{0\times 2^1} + \cancel{0\times 2^0}$$

$$= 16 + 4$$

$$= 20$$

2) $$\left(\overset{2^4}{1}\ \overset{2^3}{0}\ \overset{2^2}{1}\ \overset{2^1}{1}\ \overset{2^0}{0}\right)_2 = 1\times 2^4 + \cancel{0\times 2^3} + 1\times 2^2 + 1\times 2^1 + \cancel{0\times 2^0}$$

Quiz 1

$$= 16 + 4 + 2$$

$$= 22$$

3) $$\left(\overset{2^2}{1}\ \overset{2^1}{2}\ \overset{2^0}{0}\right)_2 \rightarrow \text{Invalid input}$$

Digits in binary – 0 and 1

Decimal to Binary

1. Repeatedly divide the number by 2, till you get 0.
2. Note down the remainders.
3. Take the remainders in reverse.

2	37
2	18
2	9
2	4
2	2
2	1
	0

1
0
1
0
0
1

$(100101)_2$

Quiz 2

2	25
2	12
2	6
2	3
2	1
	0

1
0
0
1
1

$(11001)_2$

Adding 2 decimal numbers

	1	1	
	7	8	9
	+	+	+
+	1	4	2
	9	3	1

$s = 9$
$d = 9 \% 10$
$c = 9 / 10$

	7	8	3	9
+	3	9	4	8

Adding 2 binary numbers

			0	1	1	0	
	22 ←	1	0	1	1	0	
		+	+	+	+	+	
+	7 ←	0	0	1	1	1	
	29 ←	1	1	1	0	1	

$s = 1$
$d = 1 \% 2$
$c = 1 / 2$

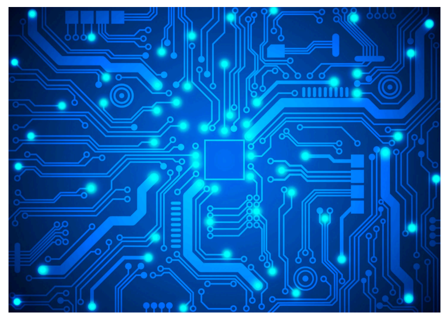
Quiz 3

		0	0	1	1	
	19 ←	1	0	0	1	1
		+	+	+	+	+
+	9 ←	0	1	0	0	1
	28 ←	1	1	1	0	0

Why Binary ?

We humans use a decimal, or base-10, numbering system, presumably because people have 10 fingers

Early computers were designed around the decimal numbering system. This approach made the creation of computer logic capabilities unnecessarily complex and did not make efficient use of resources. (For example, 10 vacuum tubes were needed to represent one decimal digit.)



To deal with the basic electronic states of on and off, Von Neumann suggested using the binary numbering system

ON OFF

1 and 0

Bitwise Operators

AND	OR	NOT (inverse)	XOR	Left shift	Right shift
&	|	~	^	<<	>>

Truth Table

if both are 1, then 1 else 0

if either is 1 then 1

Addition without carry

Same same, puppy shame

a	b	a & b	a | b	a ^ b	~a	~b
0	0	0	0	0	1	1
0	1	0	1	1	1	0
1	0	0	1	1	0	1
1	1	1	1	0	0	0

$$\begin{array}{r} 0 \\ +\,0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 0 \\ +\,1 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ +\,0 \\ \hline 1 \end{array} \qquad \begin{array}{r} \cancel{1} \\ 1 \\ +\,1 \\ \hline 0 \end{array}$$

Basic Problems on Bitwise Operators

$a = 29, \quad b = 19$

a :	1	1	1	0	1
b :	1	0	0	1	1
a & b:	1	0	0	0	1
a | b:	1	1	1	1	1
a ∧ b:	0	1	1	1	0

$a = 13, \quad b = 10$

	a :	1	1	0	1	
	b :	1	0	1	0	
Quiz 4	**a & b:**	1	0	0	0	→ 8
Quiz 5	**a | b:**	1	1	1	1	→ 15
Quiz 6	**a ∧ b:**	0	1	1	1	→ 7

Properties of Bitwise Operators

		3	2	1	0
a = 10	:	1	0	1	0
& 1	:	0	0	0	1

print(a & 1) : $\underbrace{0000}_{0}$

		3	2	1	0
a = 11	:	1	0	1	1
& 1	:	0	0	0	1

print(a & 1) : $\underbrace{0\ 0\ 0\ 1}_{1}$

		3	2	1	0
a = 14	:	1	1	1	0
& 1	:	0	0	0	1

print(a & 1): 0000

		3	2	1	0
a = 13	:	1	1	0	1
& 1	:	0	0	0	1

print(a & 1) : 0 0 0 1

Observations

10 & 1 = 0

11 & 1 = 1

14 & 1 = 0

13 & 1 = 1

Even → 0

Odd → 1

n % 2 == 0
↳ slow

Faster

if ((n & 1) == 1)

n is odd

else

n is even

Few more properties

Break till 10:15 PM

1) $a \& 0 = 0$

2) $a \& a = a$

$$\begin{array}{rr} a: & 1011 \\ \& \ a: & 1011 \\ \hline a: & 1011 \end{array}$$

3) $a \mid 0 = a$

$$\begin{array}{rr} a: & 1011 \\ \mid \ 0: & 0000 \\ \hline & 1011 \end{array}$$

4) $a \mid a = a$

5) $a \wedge 0 = a$

Important

$$\begin{array}{rr} a: & 1011 \\ \wedge \ 0: & 0000 \\ \hline & 1011 \end{array}$$

6) $a \wedge a = 0$

Important

$$\begin{array}{rr} a: & 1011 \\ \wedge \ a: & 1011 \\ \hline & 0000 \end{array}$$

7) $a \wedge 1$ → even $+1$ → odd -1

odd

$$\begin{array}{rrl} a: & 1011 & \rightarrow 11 \\ \wedge \ 1: & 0001 & -1 \\ \hline & 1010 & \rightarrow 10 \end{array}$$

even

$$\begin{array}{rrl} a: & 1010 & \rightarrow 10 \\ \wedge \ 1: & 0001 & +1 \\ \hline & 1011 & \rightarrow 11 \end{array}$$

8) $a \mid 1$ → TODO

Just a bit more ...

$a + b = b + a$
$a \times b = b \times a$

$$\left.\begin{aligned} a \,\&\, b &= b \,\&\, a \\ a \,|\, b &= b \,|\, a \\ a \wedge b &= b \wedge a \end{aligned}\right\}$$

Commutative property

Associative Property

$a \,\&\, b \,\&\, c = c \,\&\, b \,\&\, a$

$(a \,\&\, b) \,\&\, c \rightarrow c \,\&\, (a \,\&\, b)$

$a \,|\, b \,|\, c = c \,|\, b \,|\, a$

$a \wedge b \wedge c = c \wedge b \wedge a$

Q. What is the value of

$$a \wedge b \wedge a \wedge d \wedge b$$

$$= a \wedge a \wedge b \wedge b \wedge d$$

$$= 0 \wedge 0 \wedge d$$

$$= 0 \wedge d$$

$$= d$$

Q. What is the value of

$$c \wedge f \wedge a \wedge f \wedge c \wedge g \wedge a$$

$$= g$$

Quiz 7

$$1 \wedge 3 \wedge 5 \wedge 3 \wedge 2 \wedge 1 \wedge 5$$

$$= 1 \wedge 1 \wedge 3 \wedge 3 \wedge 5 \wedge 5 \wedge 2$$

$$= 0 \wedge 0 \wedge 0 \wedge 2$$

$$= 2$$

Single Number

Given N array elements, every element repeats twice except 1. Find the unique element.

ar[5] = 6 9 6 10 9

↳ 10

ar[7] = 12 9 12 8 7 9 8

↳ 7

ar[5] = 2 9 7 2 7

↳ 9

Brute Force Idea

1) Run 2 loops → $O(n^2)$ time

2) Sort & check → $O(N \log N)$

Optimised Idea

Take XOR of all elements

```
singleNumber(int arr[]) {
        n = arr.len
        ans = 0
        for i → [0, n-1] :
              ans = ans ^ arr[i]
        return ans
}
```

Time - $O(N)$
Space - $O(1)$

Java

```java
int singleNumber(int[] nums) {
    int ans = 0;
    for (int x : nums) {
        ans = ans ^ x;
    }
    return ans;
}
```

Python

```python
def singleNumber(nums):
    ans = 0
    for x in nums:
        ans = ans ^ x
    return ans
```

ar[5] = 2 9 7 2 7

0 → 2 → 11 → 12 → 14 → 9

2 : 0010
9 : 1001
11 : 1011
7 : 0111
12 : 1100
2 : 0010
14 : 1110
7 : 0111
9 : 1001

Left shift operator <<

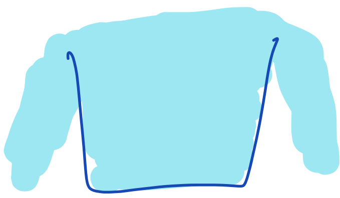
8 bits

	7	6	5	4	3	2	1	0	
$a = 10$	0	0	0	0	1	0	1	0	$= 10$
Discarded									
$a << 1$	0	0	0	1	0	1	0	0	$= 20 = 10 \times 2$
Discard									
$a << 2$	0	0	1	0	1	0	0	0	$= 40 = 10 \times 2^2$
Discard									
$a << 3$	0	1	0	1	0	0	0	0	$= 80 = 10 \times 2^3$
Discard									
$a << 4$	1	0	1	0	0	0	0	0	$= 160 = 10 \times 2^4$
Discard									
$a << 5$	0	1	0	0	0	0	0	0	$= \cancel{320}$

↳ 64

Largest 8 bit number = 255

Overflow



Exceeding your capacity

Generalisation (No overflow)

$a << 1 = a \times 2$

$a << 2 = a \times 2^2$

$a << 3 = a \times 2^3$

$a << 4 = a \times 2^4$

$a << N = a \times 2^N$

Quiz 8

$15 << 2$

$= 15 \times 2^2$

$= 15 \times 4$

$= 60$

Important Result

$1 << 1 \quad = \quad 1 \times 2$

$1 << 2 \quad = \quad 1 \times 2^2$

$1 << 3 \quad = \quad 1 \times 2^3$

$1 << N \quad = \quad 1 \times 2^N \quad = \quad 2^N$ ← O(1) time

Right Shift Operator >>

	7	6	5	4	3	2	1	0	
$a = 10$	0	0	0	0	1	0	1	0	$= 10$
$a >> 1$	0	0	0	0	0	1	0	1	$= 5 = \frac{10}{2}$
$a >> 2$	0	0	0	0	0	0	1	0	$= 2 = \frac{10}{4}$
$a >> 3$	0	0	0	0	0	0	0	1	$= 1 = \frac{10}{8}$
$a >> 4$	0	0	0	0	0	0	0	0	$= 0 = \frac{10}{16}$
$a >> 5$									

Generalisation

$$a >> 1 = \frac{a}{2}$$

$$a >> 2 = \frac{a}{2^2}$$

$$a >> 3 = \frac{a}{2^3}$$

$$a >> 4 = \frac{a}{2^4}$$

$$a >> i = \frac{a}{2^i}$$

Quiz 9

$a = 29$

$a >> 2$

$$29 >> 2 = \frac{29}{2^2} = \frac{29}{4} = 7$$

Doubts

Thank you

$2 ** 3 = 2^3$

(Python)

Single Number

3 5 3 6 3 3 5

$= \underline{3 \wedge 3} \wedge \underline{3 \wedge 3} \wedge \underline{5 \wedge 5} \wedge 6$

$= 0 \wedge 0 \wedge 0 \wedge 6$

$a >> i = \frac{a}{2^i}$

$a << i = a \times 2^i$

Contest 1 reattempt - 16^{th}

→ If you were not able to solve all questions in 1^{st} attempt

Then participate in this reattempt

Similar to the original set

2 9 ? 2 7

No of bits $\rightarrow$ Max value $-$ $2^N - 1$

N

Good Night

Thank You

Friday