Analysis Of Variance (ANOVA)

Hypothesis Testing In ANOVA

Null Hypothesis H_0 : $\mu_1 = \mu_2 = \mu_3 \cdots \mu_K$

Alternate Hypothesis H_1 : Atleast one of the mean is not equal

X $\boxed{\mu_1 \neq \mu_2 \neq \mu_3 \cdots \neq \mu_K}$

Test Statistics

$$F = \frac{\text{Variation between samples}}{\text{Variation within samples.}}$$

F test

σ^2 — Variance between Sample

	X_1	X_2	X_3
Variation within Sample	1	6	5
	2	7	6
	4	3	3
	5	2	2
	3	1	4
	$\Sigma X_1 = 15$	$\Sigma X_2 = 19$	$\Sigma X_3 = 20$
	$\bar{X}_1 = 3$	$\bar{X}_2 = 19/5$	$\bar{X}_3 = 4$

H_0 : $\bar{X}_1 = \bar{X}_2 = \bar{X}_3$

H_1 : Atleast one Sample mean is not equal

One Way ANOVA

One factor with atleast 2 levels, levels are independent

① Doctors want to test a new medication which reduces headache. They split the participant into 3 condition [15mg, 30mg, 45mg]. Later on the doctor ask the patient to rate the headache between [1-10]. Are there any differences between the 3 conditions using alpha = 0.05?

Ans)

15 mg	30mg	45mg
9	7	4
8	6	3
7	6	2
8	7	3
8	8	4
9	7	3
8	6	2

① Define Null and Alternate hypothesis?

H_0 $\mu_{15} = \mu_{30} = \mu_{45}$

H_1 : not all μ's are equal

② State Significance value

$\alpha = 0.05$ => C.I = 0.95

③ Calculate Degree of freedom

$N = 21$ $a = 3$ $n = 7$

$df_{between} = a - 1 = 3 - 1 = 2$

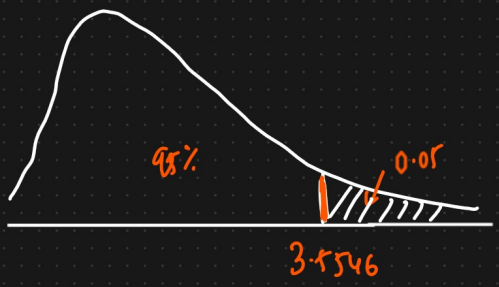
$df_{within} = N - a = 21 - 3 = 18$

$df_{total} = N - 1 = 21 - 1 = 20$

(2, 18)

⇓

F table



Critical value = 3.5546.

④ State Decision Rule

If F is greater than 3.5546, reject the null hypothesis.

⑤ Calculate Test Statistics

	SS	df	MS	F
Between				
Within				
Total				

$SS_{between}$ SS_{within} SS_{total}

15 mg	30mg	45mg
9	7	4
8	6	3
7	6	2
8	7	3
8	8	4
9	7	3
8	6	2

① $SS_{between} = \frac{\Sigma(\Sigma a_i)^2}{n} - \frac{T^2}{N}$

15mg : $9+8+7+8+8+9+8 = 57$

30mg : $7+6+6+7+8+7+6 = 47$

45mg : $4+3+2+3+4+3+2 = 21$

$= \frac{57^2+47^2+21^2}{7} - \frac{[57+47+21]}{21}$

$= \boxed{98.67}$

② $SS_{within} = \Sigma y^2 - \frac{\Sigma(\Sigma a_i)^2}{n}$

$= \Sigma y^2 - \left[\frac{57^2+47^2+21^2}{7}\right]$

$\Sigma y^2 = 9^2+8^2+7^2+8^2+8^2+9^2+8^2+7^2+6^2$
$+ \cdots\cdots$

$= 853.$

$= 853 - \left[\frac{57^2+47^2+21^2}{7}\right]$

$= \boxed{10.29}$

③ $SS_{Total} = \Sigma y^2 - \frac{T^2}{N}$

$= 853 - \frac{125^2}{21} = \boxed{108.95}$

	SS	df	MS	F
Between	98.67	2	49.34	
Within	10.29	18	0.54	
Total	108.95	20		

$$F = \frac{\text{Variation between samples}}{\text{Variation within samples.}}$$

$$F = \frac{MS_{between}}{MS_{within}}$$

$$F = \frac{49.34}{0.54} = 86.56$$

If F is greater than 3.5546, reject the null hypothesis.

$86.56 > 3.5546$, Reject the Null hypothesis