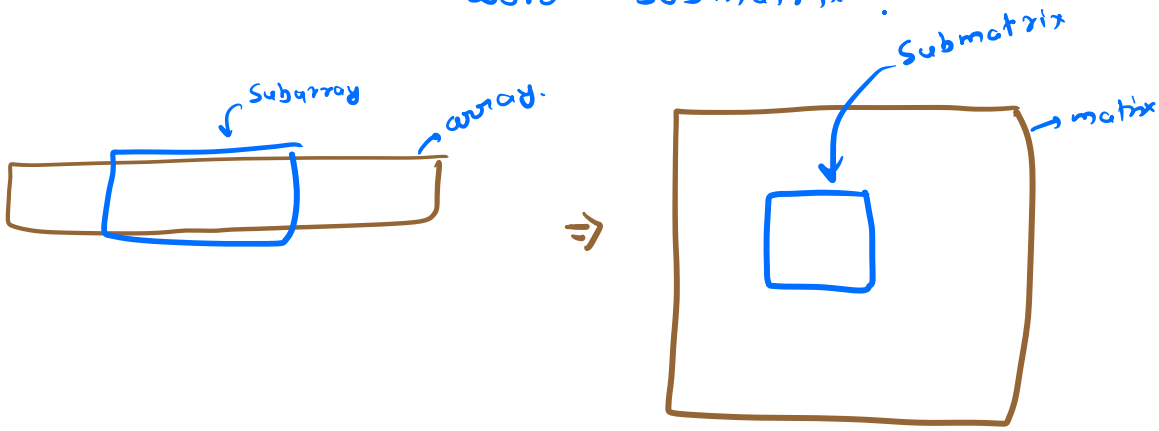
2D Arrays (Matrices)

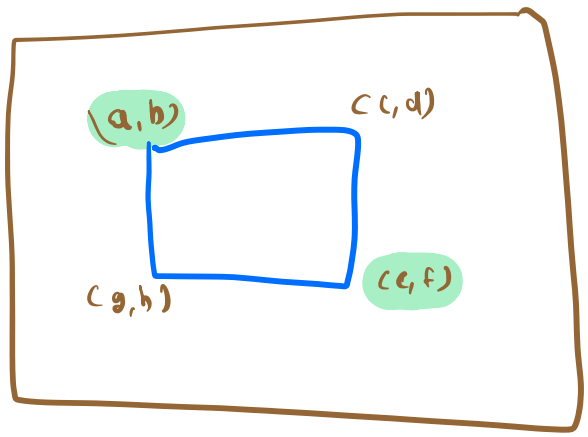
Given an 2D array, what do you understand by word "submatrix".



2	-1	3	2	1
9	3	5	3	4
12	13	3	8	6
0	1	1	7	1
-1	5	6	2	3
7	8	9	10	11

Submatrix → should be rectangle in size (side parallel to x-y axis)

How many min points (elemint) needed to represent a submatrix.



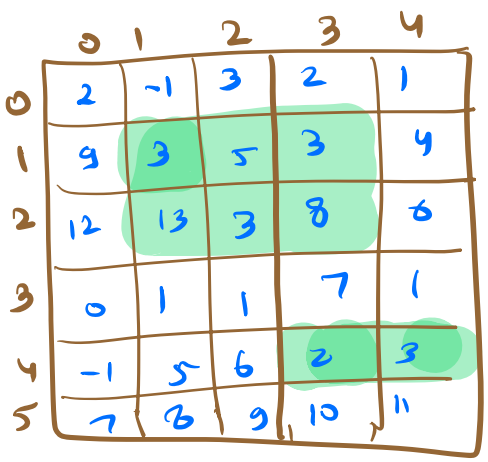
TL & BR ✓

2	-1	3	2	1
9	3	5	3	4
12	13	3	8	6
0	1	1	7	1
-1	5	6	2	3
7	8	9	10	11

Q. Given a matrix N×M. Q queries.

Each query → TL, BR

Find sum of the given sub-matrix of each query.



Q = 2

	(T, L)	(B, R)	
Q1	(1,1)	(2,3)	= 35
Q2	(4,3)	(4,4)	= 5

BF: For each query:

Iterate over submatrix & get the sum

(p, q)

(y, z)

```
for( i=p; i<=y; i++)
{   for( j=q; j<=z; j++)
    {   sum = sum + a[i][j]
```

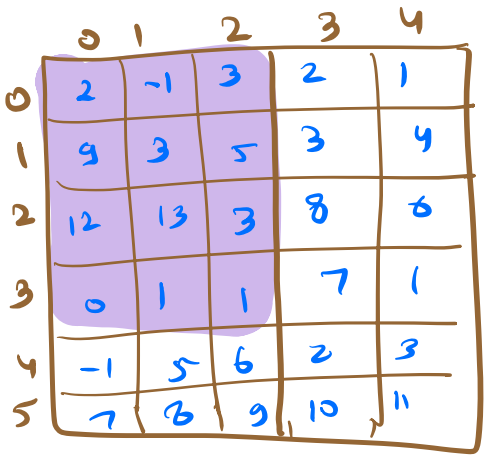
xdim sum

TC : $(Q \times N \times M)$ — each query

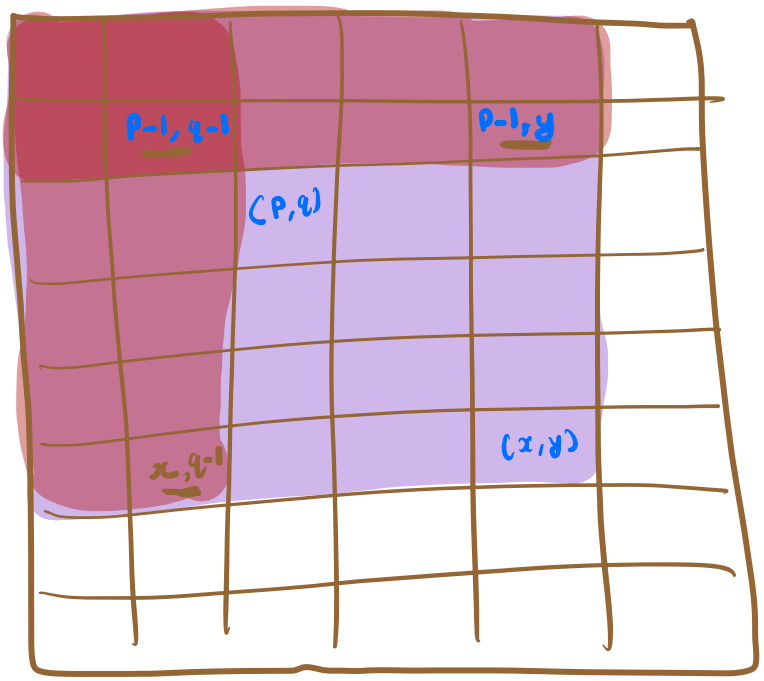
SC : O(1)

1 D array PF[i] → sum of all elements from 0 to i

2 D array PF[i][j] → sum of all submatrix from (0,0) to (i,j)



PF[3][2]



sum of submatrix $[(p,q),(x,y)]$

$= PF[x][y] - PF[p-1][y]$

$- PF[x][q-1]$

$+ PF[p-1][q-1]$

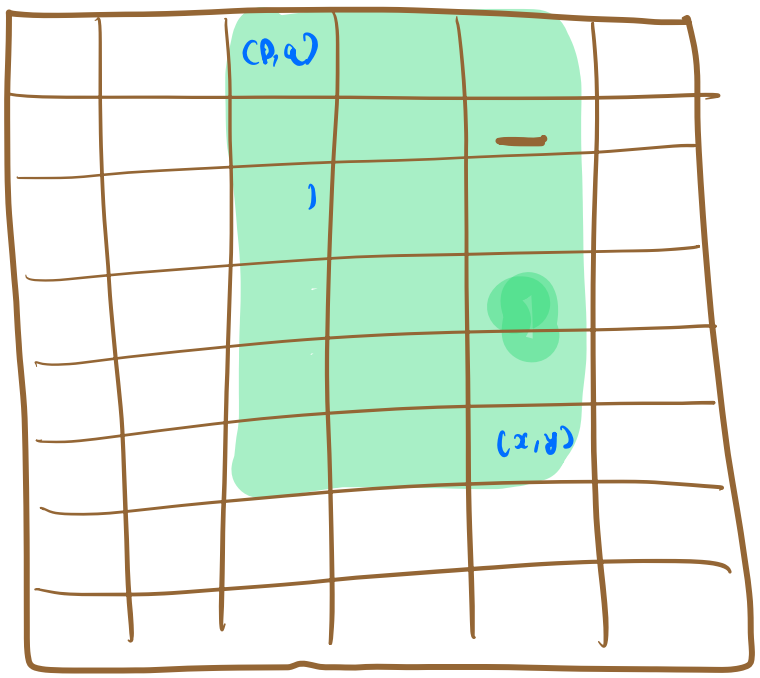
T_1 (a_1, b_1) TL

T_2 $(a_2 b_2)$ BR

$sum[T_1, T_2] = PF[a_2][b_2] - PF[a_1-1][b_2]$

$- PF[a_2][b_1-1]$

$+ PF[a_1-1][b_1-1]$



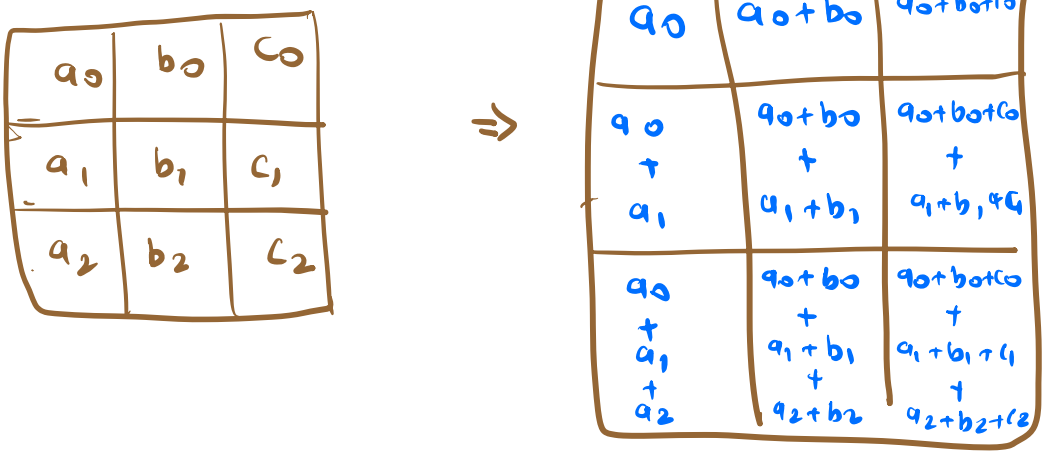
Query $a_1 b_1$ to $a_2 b_2$

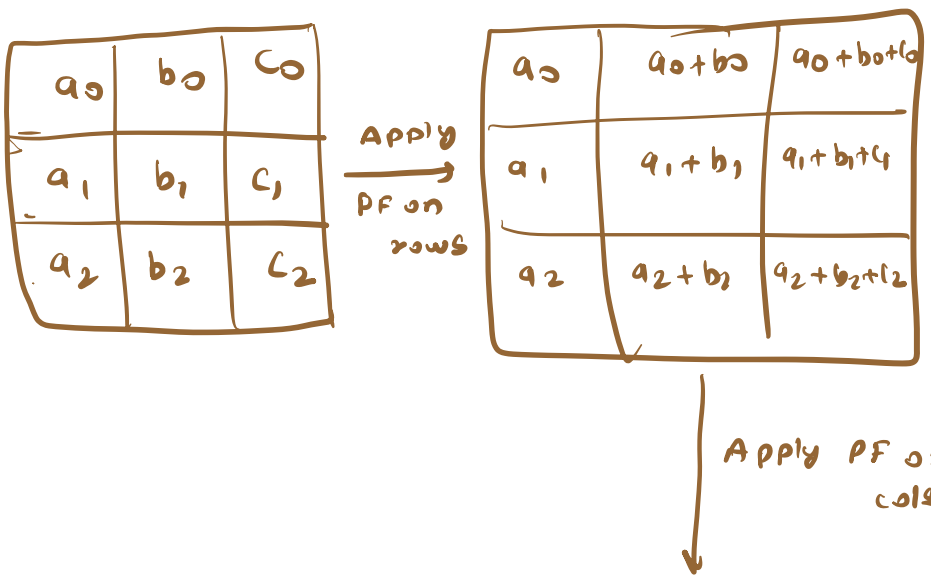
$ans = PF[a_2][b_2]$

If $(a_1 > 0)$ $ans = ans - PF[a_1-1][b_2]$

If $(b_1 > 0)$ $ans = ans - PF[a_2][b_1-1]$

If $(a_1>0$ & $b_1>0)$ $ans = ans + PF[a_1-1][b_1-1]$





Apply PF on cols

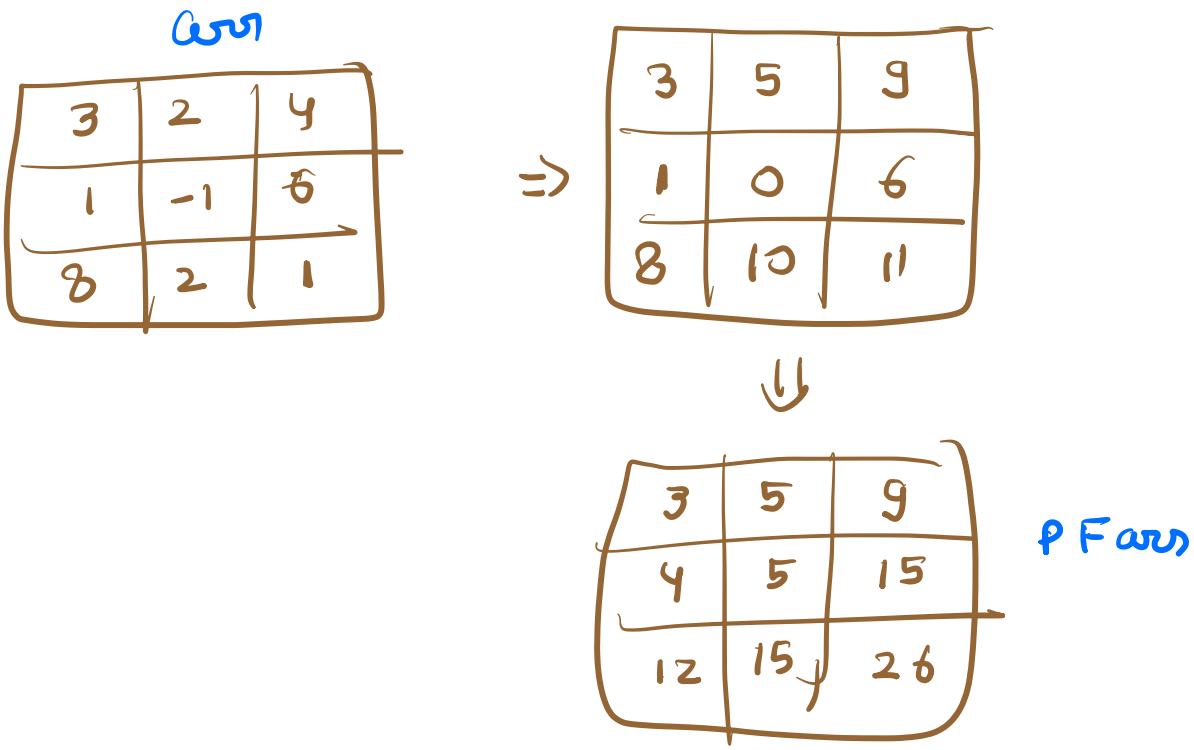
PF =

a_0	a_0+b_0	$a_0+b_0+c_0$
a_0 a_1	a_0+b_0 a_1+b_1	$a_0+b_0+c_0$ a_1 b_1 c_1
a_0 a_1 a_2	a_0+b_0 a_1+b_1 a_2+b_2	a_0 b_0 c_0 a_1 b_1 c_1 a_2 b_2 c_2

Build PF array

1. Apply Psum on rows
2. Apply Psum on cols

✓



TC: O(2×NM + Q.1) = TC(NM+Q)

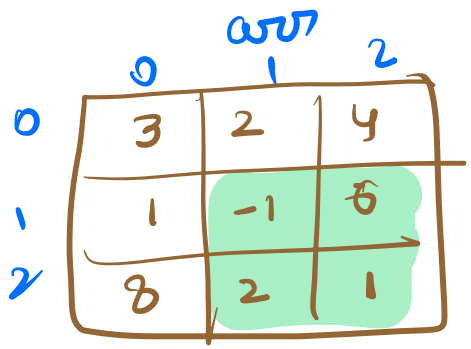
2×NM → PFArray

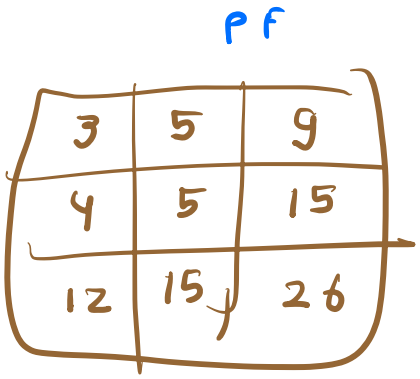
Q.1 → Queries

SC: O(NM + 1) = SC: O(NM)

NM → PFarray

1 → O(1)





sum (1,1 to 2,2)

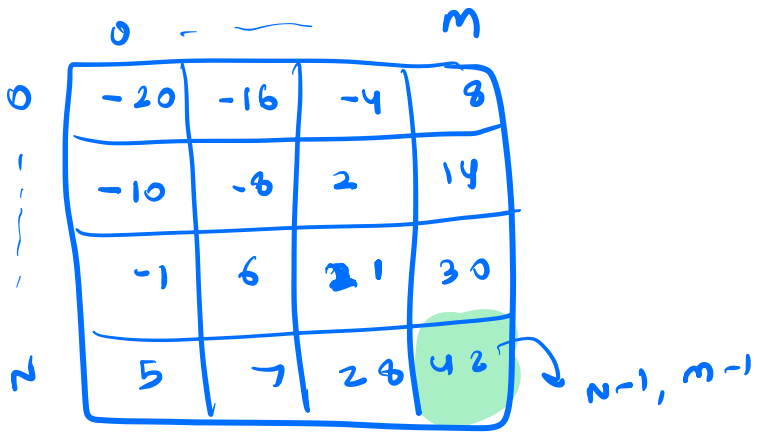
$$= PF[2][2] - PF[0][2] - PF[2][0] + PF[0][0]$$

$$= 26 - 9 - 12 + 3$$

$$= 8$$

Q. Given an array (2D). (Sorted row-wise & col-wise)

Find max submatrix-sum.



For submatrix BR is fixed

= (N-1, m-1)

-13	-10	-7
-12	-9	-5
-9	-5	-3

TL is left

Explore all options & get the sum.

N x m options.

TC: O (NM x NM) : $O(N^2 m^2)$

↓ PF sum

TC: O (NM x 1) : O(NM)

ans = $-\infty$

for (i=0 ; i<N ; i++)

{

for (j=0 ; j<m ; j++)

{

→ (i,j) to (N-1, m-1)

curr

ans = max (ans, curr)

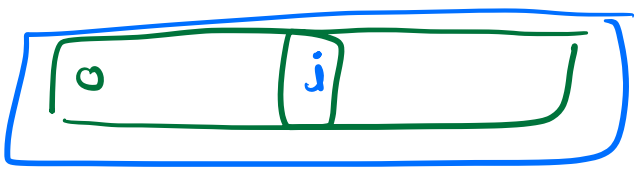
}

return ans

Q. Given array (1D). Find sum of all subarray-sum

Each subarray → add in ans

TC: $O(N^2)$



$$ans = \sum_{j=0}^{N-1} arr[j](j+1)*(N-j)$$

Sum of all subarrays-sum = arr[0] × no of subarray have arr[0] + arr[1] × no of subarray have arr[0] + ...

3	6	4	8

=

[3]	= 3
[6]	= 6
[4]	= 4
[8]	= 8
[3 6]	= 9
[6 4]	= 10
[4 8]	= 12

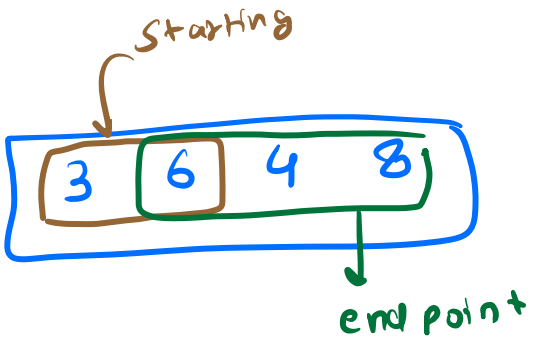
[364] = 13
[648] = 18
[3648] = 21

ans = 104

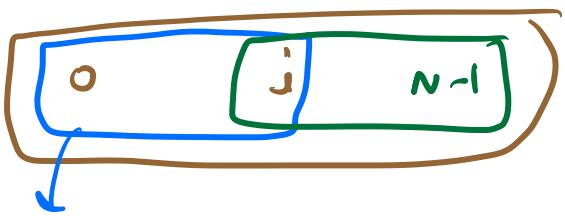
$$ans = 3 \times 4 + 6 \times 6 + 4 \times 6 + 8 \times 4$$

$$= 12 + 36 + 24 + 32$$

$$= 104$$



$= 2 \times 3 = 6$



$(i+1) \times (N-i)$

```
ans = 0
for( i = 0; i < N; i++)
{
    add contribution of ith element to the ans
    K = (i+1) (N-i)
    ans = ans + arr[i] x K
}
return ans;
```

Q. Given 2D array. Find sum of all submatrix sum.

3	2
1	4

→ [3] = 3

↓ [2] = 2

↘ [1] = 1

↓ [4] = 4

→ [3 2] = 5

$\Rightarrow$ [1 4] = 5

$=$ [3 / 1] = 4

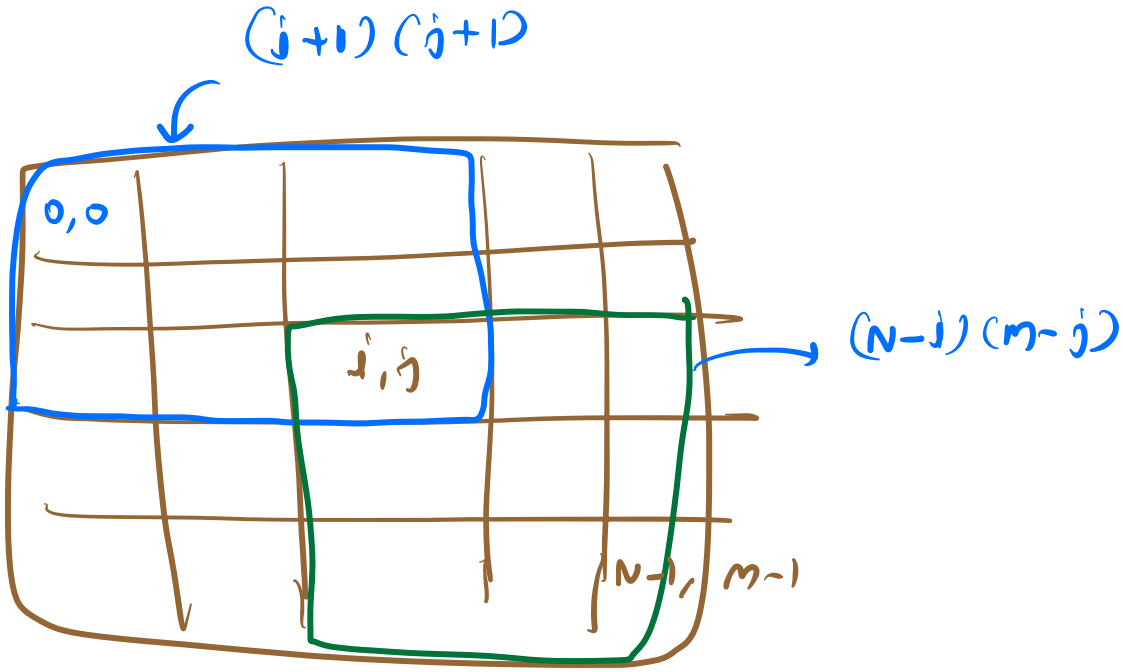
$=$ [2 / 4] = 6

$=$ [3 2 / 1 4] = 10

40

ans = $3 \times 4 + 2 \times 4 + 1 \times 4 + 4 \times 4$

$= 12 + 8 + 4 + 16 = 40$

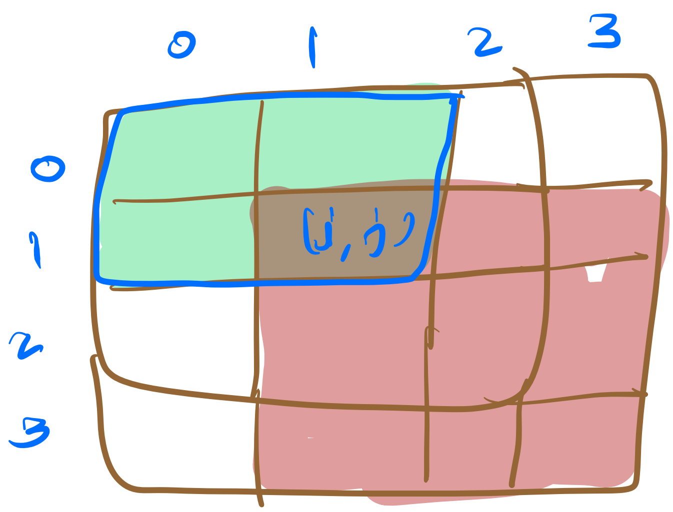


a[i][j] x K

$K = (i+1)(j+1)(N-i)(m-j)$

```
ans=0
for ( i=0; i<N; i++)
{
    for( j=0; j<m; j++)
    {
        K = (i+1)(j+1)(N-i)(m-j)
        ans= ans+ a[i][j]x K
    }
}
return ans
```

TC: O(N m)
SC: O(1)



TL

BR

(0,0)

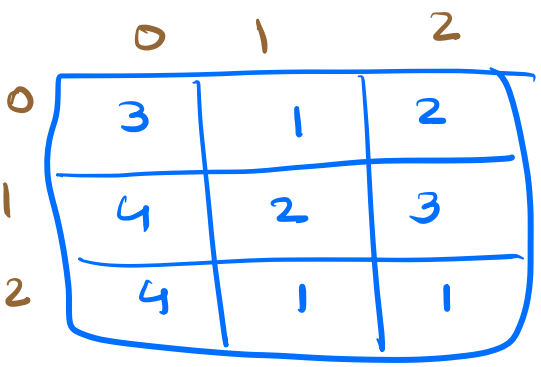
(0,1)

(1,0)

(1,1)

(0,1)
(1,2)
(1,3)
(2,1)
(2,2)
(2,3)
(3,1)
(3,2)
(3,3)

$4 \times 9 = 36$



$3 \times 3 = 9$

0,2

N=3
m=3

(i+1)(j+1)(N-i)(m-j)
(1)(3)(3)(1) = 9

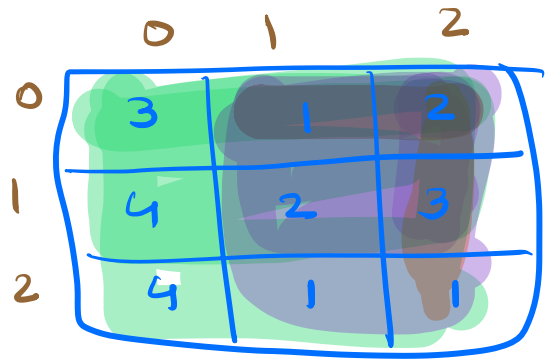
$2 \times 6 = 12$

ans = 0

ans = 0 + 3×9 = 27

ans = 27 + 1×12 = 39

ans = 39 +



→ Add contribution of each element

→ Contribution = a[i][j] × K ← no of submatrix

→ K = No of start × No of ending

⇒ No of starting = (i+1)(j+1)

No of ending = (N-i)(M-j)

Break Friday

10:45 - 9pm

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1. Submatrix
2. Sum of Submatrix
   → PF

(a1, b1) (a2, b2)

Sum = PF(a2)(b2) - PF(a1-1)(b2)

PF(a2)(b1-1) + PF(a1-1)(b1-1)

3. Array 2D [Sorted row-wise col-wise]

max submatrix sum

→ N-1, M-1 always BR.

→ TL (NM) choty

↓

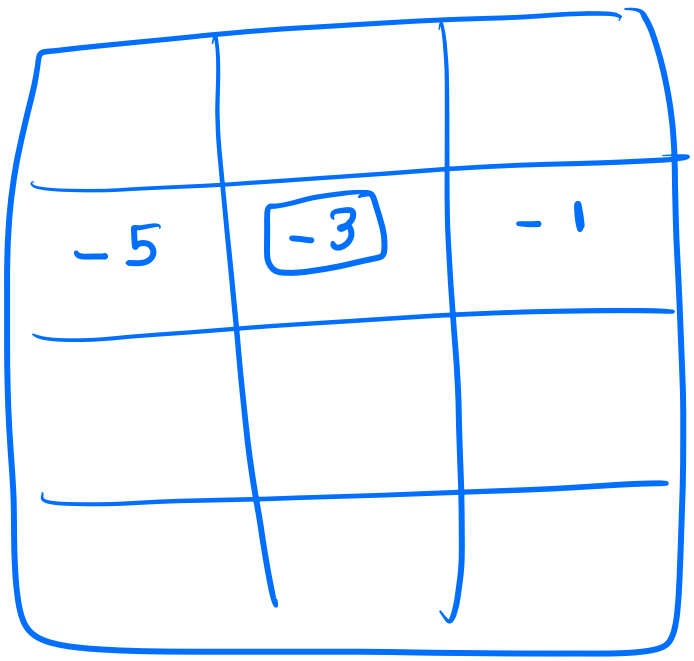
TC: O(NMx1)

4. Sum of Submatrix-sum

→ Add contribution of each element

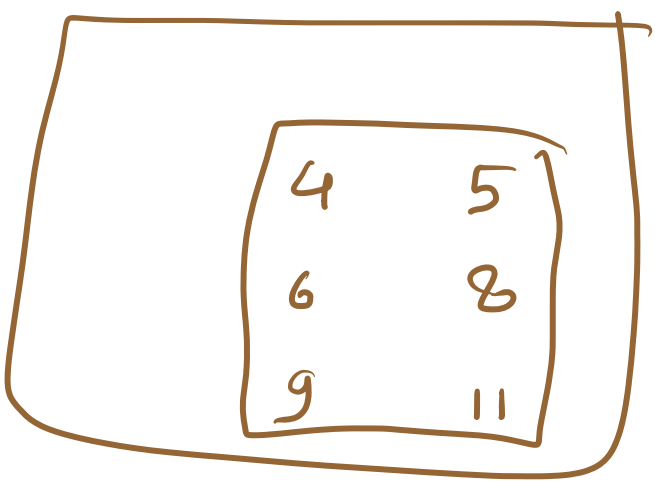
→ contri = a[i][j] x K

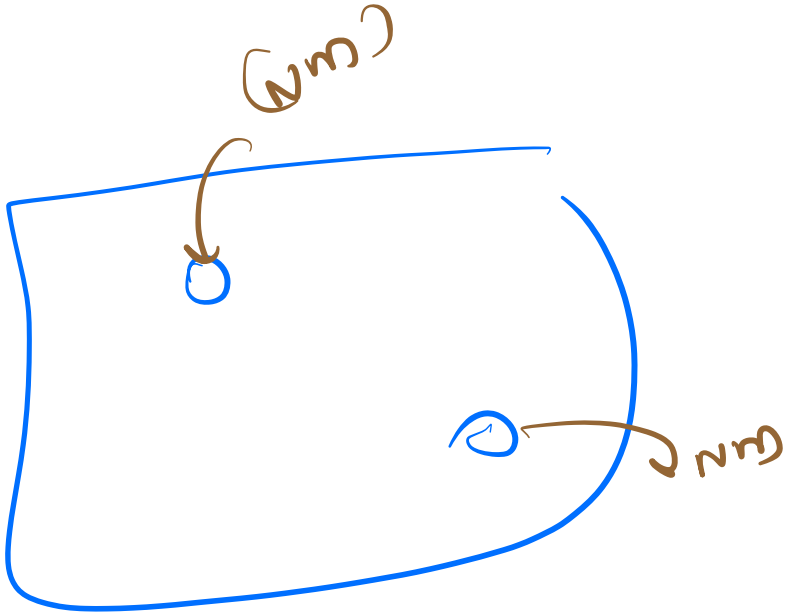
K = (i+1)(j+1)(N-i)(m-j)



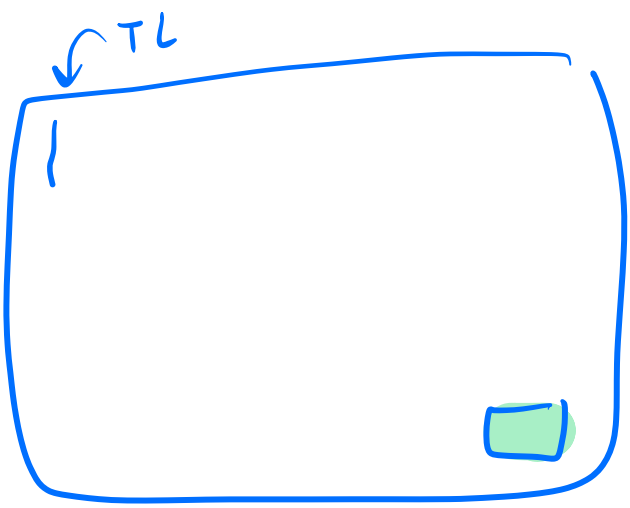
$-3-1 = -4$ ✓

$-3-5 = -8$ ✗





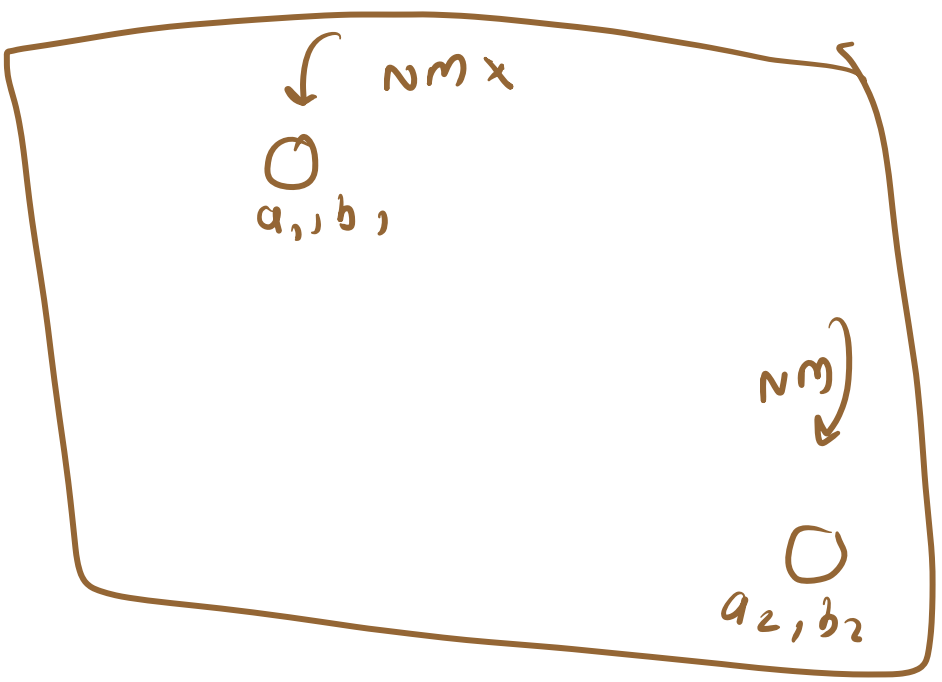
$= N^2 m^2$



$BR = N-1, m-1$

TL ≈ N × M

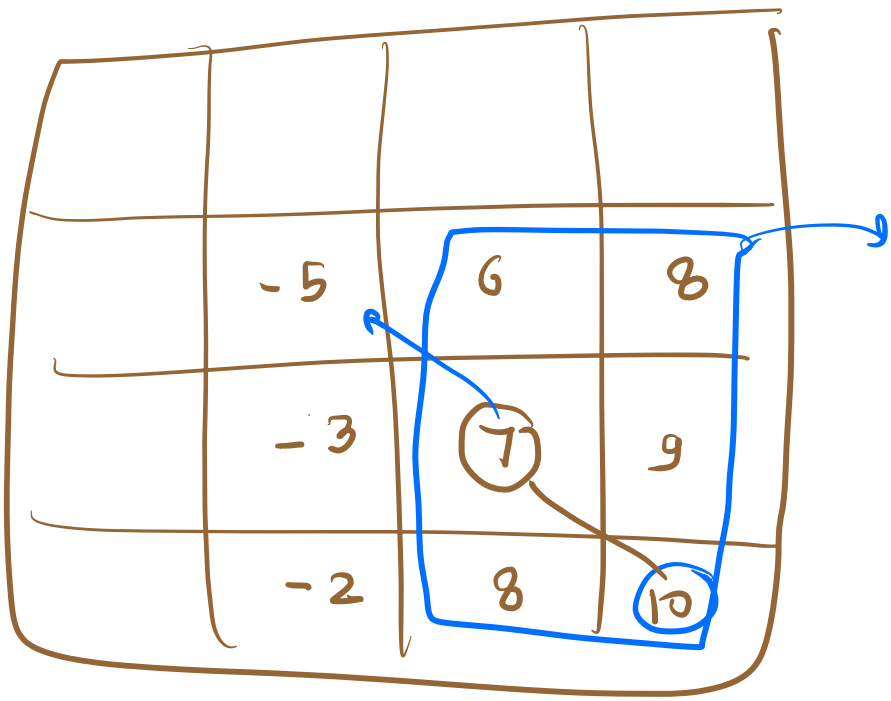
TC: O(N × M × 1)



NM × NM
$= N^2 M^2$

TC: $O(N^2 M^2 \times 1)$

1 OSA
2 OSA
MCQ


-5
6
8
-3
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9
-2
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10