

1 Problem Manual 6.2.1

Question :

Let:

$$X1 \sim \mathcal{N}(0, 1) \text{ and } X2 \sim \mathcal{N}(0, 1)$$

Evaluate the joint PDF of X1, X2, given by

$$p_{X1, X2}(x1, x2) = p_{X1}(x1)p_{X2}(x2)$$

Solution :

Normal distribution is defined as :

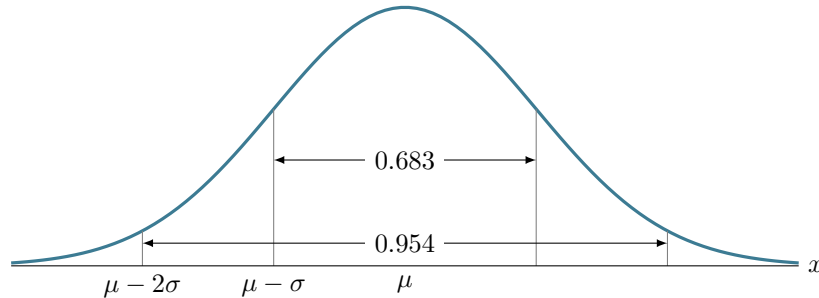
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \dots\dots\dots(1)$$

Where :

$\mu = \text{Mean}$

$\sigma = \text{Standard deviation}$

Given by the curve :



Given in the question that $\mu = 0$ and $\sigma = 1$ equation(1) transforms to

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Similarly the individual PDF of X1 and X2 would be defined as :

$$p_{X1}(x1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x1^2}{2}} \dots\dots\dots(2)$$

$$p_{X2}(x2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x2^2}{2}} \dots\dots\dots(3)$$

The joint probability distribution given by $p_{X_1, X_2}(x_1, x_2)$ is obtained by multiplying (1) and (2) is :

$$p_{X_1, X_2}(x_1, x_2) = \frac{1}{2\pi} e^{-\frac{(x_1+x_2)^2}{2}}$$

2 Problem Manual 6.2.2

Question :

Let

$$X1 = \sqrt{V} \cos \theta$$

$$X2 = \sqrt{V} \sin \theta$$

Evaluate the Jacobian

$$J = \begin{vmatrix} \frac{\partial X1}{\partial V} & \frac{\partial X2}{\partial V} \\ \frac{\partial X1}{\partial \theta} & \frac{\partial X2}{\partial \theta} \end{vmatrix}$$

Solution:

Using partial derivative

$$\frac{\partial X1}{\partial V} = \frac{1}{2\sqrt{V}} \cos \theta$$

$$\frac{\partial X2}{\partial V} = \frac{1}{2\sqrt{V}} \sin \theta$$

$$\frac{\partial X1}{\partial \theta} = -\sqrt{V} \sin \theta$$

$$\frac{\partial X2}{\partial \theta} = \sqrt{V} \cos \theta$$

Substituting the resultanat Jacobian would be

$$J = \begin{vmatrix} \frac{1}{2\sqrt{V}} \cos \theta & \frac{1}{2\sqrt{V}} \sin \theta \\ -\sqrt{V} \sin \theta & \sqrt{V} \cos \theta \end{vmatrix}$$