1 Problem Manual 6.2.1

Question :
Let:

$$X1 \sim \mathcal{N}(0,\,1) and X2 \sim \mathcal{N}(0,\,1)$$

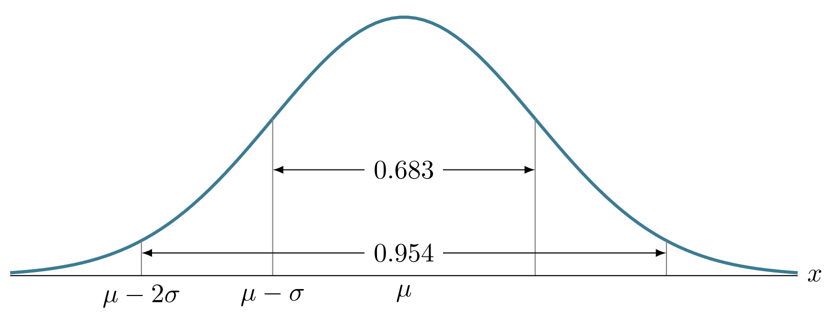
Evaluate the joint PDF of X1, X2, given by

$$p_{X1,X2}(x1,x2) = p_{X1}(x1)p_{X2}(x2)$$

Solution :
Normal distribution is defined as :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} ....(1)$$

Where :
$\mu = Mean$
$\sigma =$ Standard deviation
Given by the curve :



Given in the question that $\mu = 0$ and $\sigma = 1$ equation(1) transforms to

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Similarly the individual PDF of $X1$ and $X2$ would be defined as :

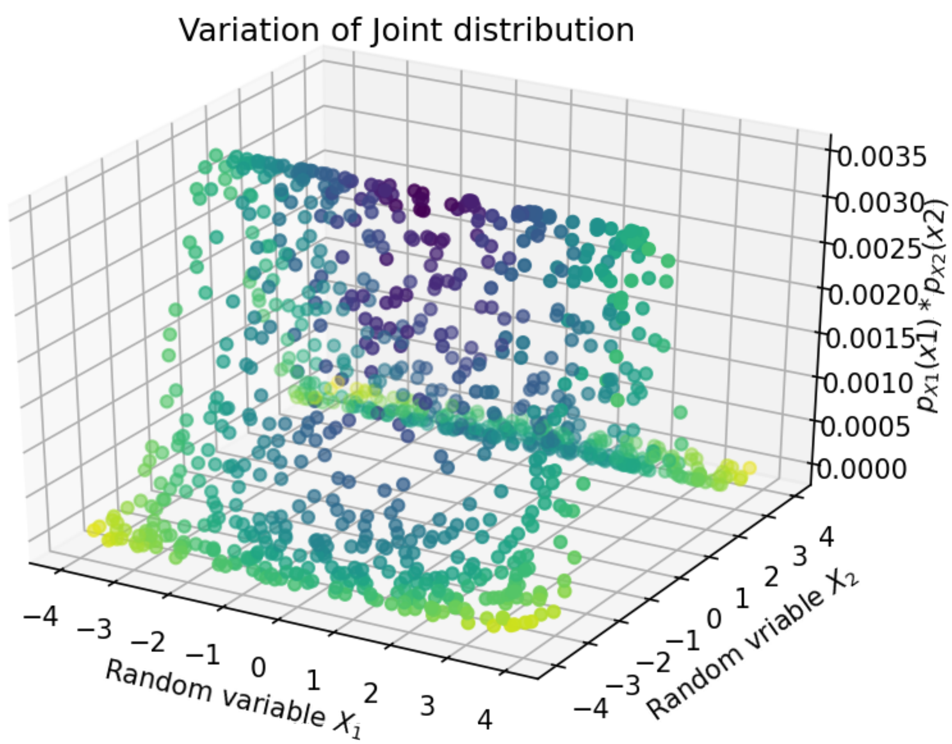
$$p_{X1}(x1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x1^2}{2}} ....(2)$$

$$p_{X2}(x2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x2^2}{2}} ....(3)$$

The joint probability distribution given by $p_{X1,X2}(x1, x2)$ is obtained by multiplying (1) and (2) is :

$$p_{X1,X2}(x1, x2) = \frac{1}{2\pi} e^{-\frac{(x1+x2)^2}{2}} ....(4)$$

Graphically equivalent to :



https://github.com/harshal9876/AI5002/blob/main/Joint$_{d}istribution_{a}ssignment_{1}(1).png$

$JointBi-variableGaussiandistributionisgivenas:$

$p_{X1,X2}(x1,x2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp -\frac{z}{2(1-\rho^2)}$..........(5)
$Where$

z = $(\mathrm{x}_1 - \mu_1)^2 \frac{}{\sigma_1^2 + \frac{(x_2-\mu_2)^2}{\sigma_2^2} - \frac{2\rho(x_1-\mu_1)((x_2-\mu_2))}{\sigma_1\sigma_2}}$
and

$\rho = cor(x1,x2) = \frac{V_{12}}{\sigma_1\sigma_2}$
where :

$x1, x2 = randomvariables$
μ_1 = Mean of random variable X1
μ_2 = Mean of random variable X2
σ_1 = Standard deviation of random variable X1
σ_2 = Standard deviation of random variable X2
$V_{12} = covariance(x1,x2)$

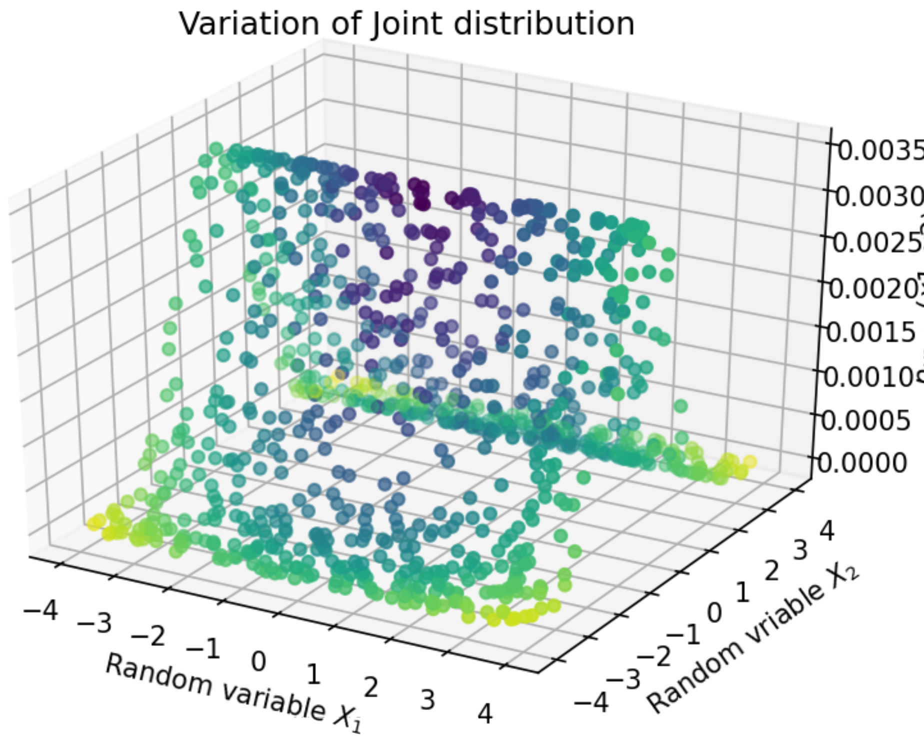
For a Bivariable Normal distribution having independent random variables
$\mu_1 = \mu_2 = 0$
$\sigma_1 = \sigma_2 = 1$
$V_{12} = 0$

The equation transforms to :
$z = x_1^2 + x_2^2$

Thus (4) transforms to

$$p_{X1,X2}(x1,x2) = \frac{1}{2\pi} e^{-\frac{(x1+x2)^2}{2}}$$

Graphically equivalent to



Source:https://github.com/harshal9876/AI5002/blob/main/Joint$_{d}istribution_{A}ssignment_{1}.png$

$Whichisthesameasofequation(4), thusprovingthatfortwonormalrandomdistribution$

$p_{X1,X2}(x1, x2) = p_{X1}(x1)p_{X2}(x2)$

2 Problem Manual 6.2.2

Question :
Let

$$X1 = \sqrt{V} cos\theta$$
$$X2 = \sqrt{V} sin\theta$$

Evaluate the Jacobian

$$J = \begin{vmatrix} \frac{\partial X1}{\partial V} & \frac{\partial X2}{\partial V} \\ \frac{\partial X1}{\partial \theta} & \frac{\partial X2}{\partial \theta} \end{vmatrix}$$

Solution:
Using partial derivative

$$\frac{\partial X1}{\partial V} = \frac{1}{2\sqrt{V}} cos\theta$$
$$\frac{\partial X2}{\partial V} = \frac{1}{2\sqrt{V}} sin\theta$$
$$\frac{\partial X1}{\partial \theta} = -\sqrt{V} sin\theta$$
$$\frac{\partial X2}{\partial \theta} = \sqrt{V} cos\theta$$

Substituting the resultanat Jacobian would be

$$J = \begin{vmatrix} \frac{1}{2\sqrt{V}} cos\theta & \frac{1}{2\sqrt{V}} sin\theta \\ -\sqrt{V} sin\theta & \sqrt{V} cos\theta \end{vmatrix}$$