Assignment 2

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I. Problem 1.8

Problem: A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$ .What is the probability that he will win a prize

(a) At least once
(b) Exactly once
(c) At least twice?

Solution : Let **X** represent the number of winning prizes in 50 lotteries. The trials are Bernoulli trials. Clearly, **X** has a binomial distribution with $n = 50$ and $p = \frac{1}{100}$

q = 1 - p = 1 -$\frac{1}{100}$ = $\frac{99}{100}$

P(X = x) = ${}^nC_x q^{n-x} p^x$ = ${}^{50}C_x (\frac{99}{100})^{50-x} (\frac{1}{100})^x$

(a) P(At least once) = P(X $\geq$ 1)

$$
\begin{aligned}
&= 1 - P(X < 1) \\
&= 1 - P(X = 0) \\
&= 1 - {}^{50}C_x (\frac{99}{100})^{50} \\
&= 1 - {}^{50}C_0 (\frac{99}{100})^{50} (\frac{1}{100})^0 \\
&= 1 - (\frac{99}{100})^{50} (1) \\
&= 1 - (\frac{99}{100})^{50} \\
&= 0.3950
\end{aligned}
$$

(b) P(Winning exactly once) = P(X = 1)

$$
\begin{aligned}
&= {}^{50}C_1 (\frac{99}{100})^{49} (\frac{1}{100})^1 \\
&= 50 (\frac{99}{100})^{49} (\frac{1}{100}) \\
&= \frac{1}{2} (\frac{99}{100})^{49} \\
&= 0.3055
\end{aligned}
$$

(c) P(At least twice) = P(X $\geq$ 2)

$$
\begin{aligned}
&= 1 - P(X < 2) \\
&= 1 - P(X \leq 1) \\
&= 1 - [P(X = 0) + P(X = 1)] \\
&= 1 - P(X = 0) - P(X = 1) \\
&= 1 - (\frac{99}{100})^{50} - \frac{1}{2} (\frac{99}{100})^{49} \\
&= 1 - (\frac{99}{100})^{49} [\frac{99}{100} + \frac{1}{2}] \\
&= 1 - (\frac{99}{100})^{49} (\frac{149}{100}) \\
&= 1 - (\frac{149}{100}) (\frac{99}{100})^{49} \\
&= 0.0894
\end{aligned}
$$