

Assignment 2

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I. PROBLEM 1.8

Problem: A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

- (a) At least once
- (b) Exactly once
- (c) At least twice?

Solution : Let X represent the number of winning prizes in 50 lotteries. The trials are Bernoulli trials. Clearly, X has a binomial distribution with $n = 50$ and $p = \frac{1}{100}$

$$q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$$

$$P(X = x) = {}^nC_x q^{n-x} p^x = {}^{50}C_x \left(\frac{99}{100}\right)^{50-x} \left(\frac{1}{100}\right)^x$$

$$(a) P(\text{At least once}) = P(X \geq 1)$$

$$\begin{aligned} &= 1 - P(X < 1) \\ &= 1 - P(X = 0) \\ &= 1 - {}^{50}C_0 \left(\frac{99}{100}\right)^{50-x} \left(\frac{1}{100}\right)^x \\ &= 1 - {}^{50}C_0 \left(\frac{99}{100}\right)^{50} \left(\frac{1}{100}\right)^0 \\ &= 1 - \left(\frac{99}{100}\right)^{50} (1) \\ &= 1 - \left(\frac{99}{100}\right)^{50} \\ &= 0.3950 \end{aligned}$$

$$(b) P(\text{Winning exactly once}) = P(X = 1)$$

$$\begin{aligned} &= {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)^1 \\ &= 50 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right) \\ &= \frac{1}{2} \left(\frac{99}{100}\right)^{49} \\ &= 0.3055 \end{aligned}$$

$$(c) P(\text{At least twice}) = P(X \geq 2)$$

$$\begin{aligned} &= 1 - P(X < 2) \\ &= 1 - P(X \leq 1) \\ &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2} \left(\frac{99}{100}\right)^{49} \\ &= 1 - \left(\frac{99}{100}\right)^{49} \left[\frac{99}{100} + \frac{1}{2}\right] \\ &= 1 - \left(\frac{99}{100}\right)^{49} \left(\frac{149}{100}\right) \\ &= 1 - \left(\frac{149}{100}\right) \left(\frac{99}{100}\right)^{49} \\ &= 0.0894 \end{aligned}$$