Assignment 3

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I. Problem 2.6

Problem: A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Solution :

Let
A : Event that item produced by operator A
B : Event that item produced by operator B
C : Event that item produced by operator C
D : Event that item produced is defective

We need to find out the probability that the item is produced by the operator A is defective
i.e.$P(A|D)$

$$P(A|D) = \frac{P(A).P(D|A)}{P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)}$$

P(A) = Probability of item is produced by operator A
=$50\% = \frac{50}{100} = 0.5$
P(B) = Probability of item is produced by operator B
=$30\% = \frac{30}{100} = 0.3$
P(C) = Probability of item is produced by operator C
=$20\% = \frac{20}{100} = 0.2$
$P(D|A)$ = Probability of a defective item is produced by operator A
=$1\% = \frac{1}{100} = 0.01$
$P(D|B)$ = Probability of a defective item is produced by operator B
=$5\% = \frac{5}{100} = 0.05$
$P(D|C)$ = Probability of a defective item is produced by operator C
=$7\% = \frac{7}{100} = 0.07$
Putting the values in the formula :

$$P(A|D) = \frac{P(A).P(D|A)}{P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)}$$

$$\begin{aligned}
&= \frac{0.5 \times 0.01}{0.5 \times 0.01 + 0.3 \times 0.05 + 0.2 \times 0.07} \\
&= \frac{0.005}{0.005 + 0.015 + 0.014} \\
&= \frac{0.005}{0.034} \\
&= \frac{5}{34} \\
&= 0.147
\end{aligned}$$

Therefore, required probability = 0.147

Code source: https://github.com/harshal9876/AI5002/blob/main/Assignment_3/Codes/Asssignment_3.py