# 1 Problem Manual 6.6.1

**Question :**
Let:

$$X1 \sim \mathcal{N}(0, 1) and X2 \sim \mathcal{N}(0, 1)$$

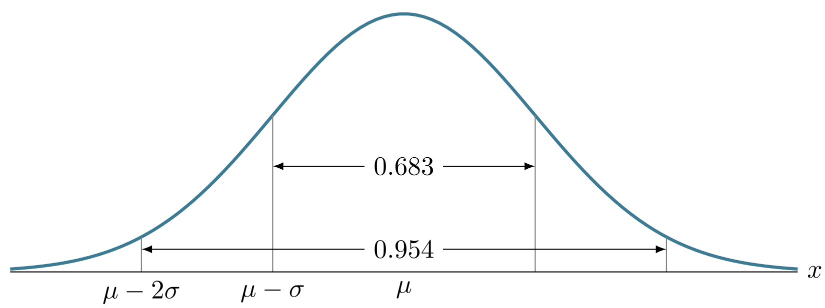
Evaluate the joint PDF of X1, X2, given by

$$p_{X1,X2}(x1, x2) = p_{X1}(x1)p_{X2}(x2)$$

**Solution :**
Normal distribution is defined as :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} ............(1)$$

Where :
$\mu = Mean$
$\sigma = standarddeviation$
Given by the curve :




Given in the question that $\mu = 0$ and $\sigma = 1$ equation(1) transforms to

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Similarly the individual PDF of $X1$ and $X2$ would be defined as :

$$p_{X1}(x1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x1^2}{2}} ................(2)$$

$$p_{X2}(x2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x2^2}{2}} .................(3)$$

The joint probability distribution given by $p_{X1,X2}(x1, x2)$ is obtained by multiplying (1) and (2) is :

$$p_{X1,X2}(x1, x2) = \frac{1}{2\pi} e^{-\frac{(x1+x2)^2}{2}}$$

## 2 Problem Manual 6.6.2

**Question :**
Let

$$X1 = \sqrt{V} cos\theta$$
$$X2 = \sqrt{V} sin\theta$$

Evaluate the Jacobian
$$J = \begin{vmatrix} \frac{\partial X1}{\partial V} & \frac{\partial X2}{\partial V} \\ \frac{\partial X1}{\partial \theta} & \frac{\partial X2}{\partial \theta} \end{vmatrix}$$

**Solution:**
Using partial derivative

$$\frac{\partial X1}{\partial V} = \frac{1}{2\sqrt{V}} cos\theta$$
$$\frac{\partial X2}{\partial V} = \frac{1}{2\sqrt{V}} sin\theta$$
$$\frac{\partial X1}{\partial \theta} = -\sqrt{V} sin\theta$$
$$\frac{\partial X2}{\partial \theta} = \sqrt{V} cos\theta$$

Substituting the resultanat Jacobian would be

$$J = \begin{vmatrix} \frac{1}{2\sqrt{V}} cos\theta & \frac{1}{2\sqrt{V}} sin\theta \\ -\sqrt{V} sin\theta & \sqrt{V} cos\theta \end{vmatrix}$$