

Assignment 6

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AI21MTECH02003

I. PROB 5.9

Number of ways to get 1 aces

Problem: Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of $E(X)$ is .

- (a) $\frac{37}{221}$
- (b) $\frac{5}{13}$
- (c) $\frac{1}{13}$
- (d) $\frac{2}{13}$

$$\begin{aligned}
 &= \text{Number of ways to select 1 ace out of 4 ace cards} \times \text{Number of ways to select 1 card out of 48 non-ace cards} \\
 &= {}^4C_1 \times {}^{48}C_1 \\
 &= 4 \times 48 \\
 &= 192
 \end{aligned}$$

Solution:

Let X : be the number of aces obtained
we can get 0,1 or 2 aces
So the value of x is 0,1 or 2

$$\begin{aligned}
 P(X = 1) &= \frac{\text{Number of ways to get 1 aces}}{\text{Total number of ways}} \\
 &= \frac{192}{1326}
 \end{aligned}$$

Total number of ways to draw 2 cards out of 52 is :

$$\begin{aligned}
 \text{Total ways} &= {}^{52}C_2 \\
 &= 1326
 \end{aligned}$$

$P(X = 2)$
i.e probability of getting 2 aces
Number of ways to get 2 aces

$P(X=0)$: Probability of getting zero aces
Number of ways of getting zero aces
= Number of ways to select 2 cards out of non ace card
= Number of ways to select 2 cards out of (52-4) , 48 cards
= ${}^{48}C_2$
= 1128

$$\begin{aligned}
 &= \text{Number of ways to select 2 ace out of 4 ace cards} \\
 &= {}^4C_2 \\
 &= 6
 \end{aligned}$$

$$\begin{aligned}
 P(X = 0) &= \frac{\text{Number of ways to get zero aces}}{\text{Total number of ways}} \\
 &= \frac{1128}{1326}
 \end{aligned}$$

$$\begin{aligned}
 P(X = 2) &= \frac{\text{Number of ways to get 2 aces}}{\text{Total number of ways}} \\
 &= \frac{6}{1326}
 \end{aligned}$$

$P(X = 1)$
i.e probability of getting 1 ace

The probability distribution is :

X	0	1	2
P(X)	$\frac{1128}{1326}$	$\frac{192}{1326}$	$\frac{6}{1326}$

The expected value $E(X)$ is given by

$$\begin{aligned}
 \mu = E(X) &= \sum_{i=1}^n x_i p_i \\
 &= 0 \times \frac{1128}{1326} + 1 \times \frac{192}{1326} + 2 \times \frac{6}{1326} \\
 &= 0 + \frac{192 + 12}{1326} \\
 &= \frac{204}{1326} \\
 &= \frac{2}{13}
 \end{aligned}$$

Thus option D stating the estimation of getting aces when drawing 2 cards from a deck of cards is $\frac{2}{13}$

Code source: https://github.com/harshal9876/AI5002/blob/main/Assignment_6/Codes/Assignment_6.py