Assignment

Harshal Verma
AI21MTECH02003

I. Prob 4.6

Problem: Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Solution:
Let **X** : Throwing a die is a binomial distribution
So X has a binomial distribution given by
P(X = x) = ${}^{n}C_{x}q^{n-x}p^{x}$
Where :
n = Number of times the die is thrown
p = Probability of getting a six = $\frac{1}{6}$
q = 1-p = 1- $\frac{1}{6}$ = $\frac{5}{6}$
Hence
P(X = x) = ${}^{6}C_{x}(\frac{5}{6})^{6-x}(\frac{1}{6})^{x}$
We need to find the probability of throwing at most 2 sixes in 6 throws of a single die
i.e $P(X \leq 2)$:

$$\begin{aligned}
P(X \leq 2) &= P(X=0) + P(X=1) + P(X=2) \\
&=^{6} C_0(\frac{5}{6})^6(\frac{1}{6})^0 +^{6} C_1(\frac{5}{6})^5(\frac{1}{6})^1 +^{6} C_2(\frac{5}{6})^4(\frac{1}{6})^2 \\
&= 1 \times 1 \times (\frac{5}{6})^6 + 6 \times \frac{1}{6} \times (\frac{5}{6})^5 + 15 \times (\frac{1}{6})^2 \times (\frac{5}{6})^4 \\
&= (\frac{5}{6})^6 + (\frac{5}{6})^5 + 15 \times \frac{1}{36} \times (\frac{5}{6})^4 \\
&= (\frac{5}{6})^6 + (\frac{5}{6})^5 + \frac{5}{12} \times (\frac{5}{6})^4 \\
&= (\frac{5}{6})^4 \times ((\frac{5}{6})^2 + \frac{5}{6} + \frac{5}{12}) \\
&= (\frac{5}{6})^4 \times (\frac{25}{36} + \frac{5}{6} + \frac{5}{12}) \\
&= (\frac{5}{6})^4 \times (\frac{25+30+15}{36}) \\
&= (\frac{5}{6})^4 \times (\frac{70}{36}) \\
&= (\frac{5}{6})^4 \times (\frac{35}{18}) \\
&= 0.9377
\end{aligned}$$

Thus the required probability is 0.9377

Code source: https://github.com/harshal9876/AI5002/blob/main/Assignment_5/Codes/Assignment_5.py