

Assignment 3

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AI21MTECH02003

I. PROBLEM 2.6

Problem: A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Solution :

Let

A : Event that item produced by operator A
B : Event that item produced by operator B
C : Event that item produced by operator C
D : Event that item produced is defective

We need to find out the probability that the item is produced by the operator A is defective i.e. $P(A|D)$

$$\begin{aligned}
 P(A|D) &= \frac{P(A).P(D|A)}{P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)} = \frac{0.005}{0.034} \\
 &= \frac{5}{34} \\
 &= 0.147
 \end{aligned}$$

$$\begin{aligned}
 P(A) &= \text{Probability of item is produced by operator A} \\
 &= 50\% = \frac{50}{100} = 0.5
 \end{aligned}$$

$$\begin{aligned}
 P(B) &= \text{Probability of item is produced by operator B} \\
 &= 30\% = \frac{30}{100} = 0.3
 \end{aligned}$$

$$\begin{aligned}
 P(C) &= \text{Probability of item is produced by operator C} \\
 &= 20\% = \frac{20}{100} = 0.2
 \end{aligned}$$

$$\begin{aligned}
 P(D|A) &= \text{Probability of a defective item is produced by operator A} \\
 &= 1\% = \frac{1}{100} = 0.01
 \end{aligned}$$

$$\begin{aligned}
 P(D|B) &= \text{Probability of a defective item is produced by operator B} \\
 &= 5\% = \frac{5}{100} = 0.05
 \end{aligned}$$

$$\begin{aligned}
 P(D|C) &= \text{Probability of a defective item is produced by operator C} \\
 &= 7\% = \frac{7}{100} = 0.07
 \end{aligned}$$

Putting the values in the formula :

$$\begin{aligned}
 P(A|D) &= \frac{P(A).P(D|A)}{P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)} \\
 &= \frac{0.5 \times 0.01}{0.5 \times 0.01 + 0.3 \times 0.05 + 0.2 \times 0.07} \\
 &= \frac{0.005}{0.005 + 0.015 + 0.014}
 \end{aligned}$$

Therefore, required probability = 0.147

Code source: https://github.com/harshal9876/AI5002/blob/main/Assignment_3/Codes/Asssignment_3.py