Machine Learning for Engineering and Science Applications

Basic Operations

Tensor operations covered in this video

- Addition, Broadcasting
- Multiplication
  - Matrix Product, Dot Product, Hadamard Product (Elementwise multiply)
- Transpose
- Inverse

Skip this video if you know this basic material

Addition

- Normal Matrix addition: $A_{ij} + B_{ij} = C_{ij}$

```
>> A = [ 1 2 3; 4 5 6]

A =

     1     2     3
     4     5     6

>> B = [ 1 5 6; 1 7 8]

B =

     1     5     6
     1     7     8

>> C = A + B

C =

     2     7     9
     5    12    14
```

- Broadcasting: $A_{ij} + b_j = C_{ij}$
  - Adding a vector to a matrix by repeating the vector
  - Done automatically in MATLAB and Numpy

```
>> A

A =

     1     2     3
     4     5     6

>> b = [1 1 1];
>> A + b

ans =

     2     3     4
     5     6     7
```

Multiplication

- Matrix Product : $C = AB$
  - $C_{ij} = \sum_k A_{ik} B_{kj}$
  - Sizes must match

```
>> A

A =

     1     2     3
     4     5     6

>> b

b =

     3
     4
     5

>> C = A*b

C =

    26
    62
```

- Hadamard Product: $C = A \odot B$
  - Elementwise multiplication
  - A, B and C must be of the same size

```
>> A

A =

     1     2     3
     4     5     6

>> B

B =

     1     5     6
     1     7     8

>> A.*B

ans =

     1    10    18
     4    35    48
```

Multiplication (contd)

- Dot Product : $\vec{a} \cdot \vec{b} = \alpha$
  - $\alpha = \sum_i a_i b_i$
  - Can also be written as $\alpha = a^T b = b^T a$

```
>> a = [ 1 2 3]'

a =

     1
     2
     3

>> b = [4 5 6]'

b =

     4
     5
     6

>> alpha = dot(a,b)

alpha =

    32
```

```
>> alpha1 = a'*b

alpha1 =

    32
```

```
>> alpha2 = b'*a

alpha2 =

    32
```

Transpose and Inverse

- Transpose : $B = A^T$
  - $B_{ij} = A_{ji}$

```
>> A

A =

     1     2     3
     4     5     6

>> B = A'

B =

     1     4
     2     5
     3     6
```

- Inverse: $I = A^{-1}A = AA^{-1}$
  - Above definition is for a square matrix
  - Not all square matrices have an inverse
  - For non-square cases we can define something called the pseudoinverse

```
>> A = rand(3)

A =

    0.7814    0.5567    0.7802
    0.2880    0.3965    0.3376
    0.6925    0.0616    0.6079

>> inv(A)

ans =

   50.2178  -66.1993  -27.6885
   13.3927  -14.8948   -8.9171
  -58.5694   76.9290   34.0937

>> A*inv(A)

ans =

    1.0000    0.0000   -0.0000
    0.0000    1.0000   -0.0000
    0.0000    0.0000    1.0000
```