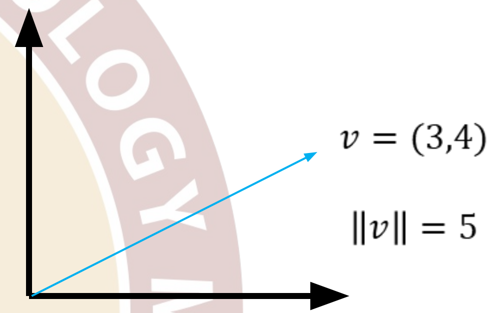
Machine Learning for Engineering and Science Applications

Norms

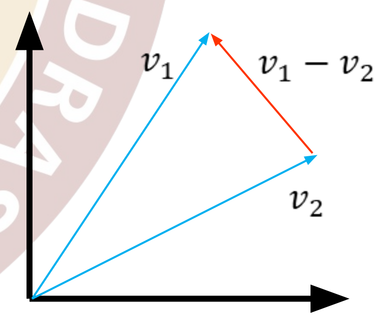
The reason to use norms

- Machine Learning uses tensors as the basic units of representation
  - Vectors, Matrices, etc..

- Two reasons to use norms
  1. To estimate how "big" a vector/tensor is



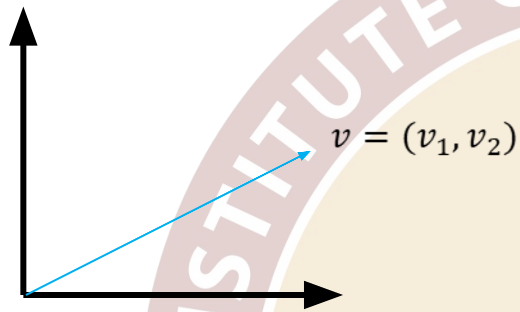
  2. To estimate "how close" one tensor is to another
     - That is how "big" the *difference between two tensors* is
     - Example : How close is one image to another?

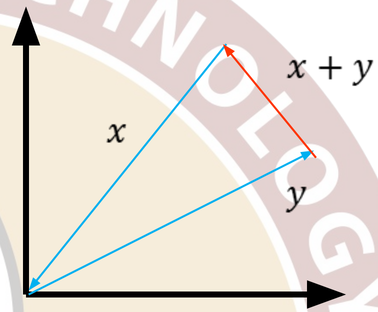


Norm is the generalization of the notion of "length" to vectors, matrices and tensors

Definition of a norm

Norms are a way of measuring the "length" of vectors, matrices, etc





Mathematically, a norm is any function ***f*** that satisfies

- $f(\boldsymbol{x}) = 0 \Rightarrow \boldsymbol{x} = \boldsymbol{0}$
- $f(\boldsymbol{x} + \boldsymbol{y}) \leq f(\boldsymbol{x}) + f(\boldsymbol{y})$ (Triangle Inequality)
- $\forall \alpha \in \mathbb{R},\ f(\alpha \boldsymbol{x}) = |\alpha| f(\boldsymbol{x})$ (Linearity)

Some standard norms

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- $f(\boldsymbol{x}+\boldsymbol{y}) \leq f(\boldsymbol{x}) + f(\boldsymbol{y})$ (Triangle Inequality)
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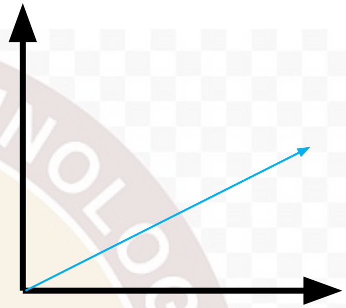

```
>> v = [-5,3,2]'

v =

    -5
     3
     2
```

Vector Norms

1. Euclidean Norm : $\|v\|_2 = (v_1^2 + v_2^2 + v_3^2 \ldots\ldots + v_n^2)^{\frac{1}{2}}$
   - Also called the 2-norm or the L^2 norm
   - Corresponds to our usual notion of distance
2. 1-norm : $\|v\|_1 = |v_1| + |v_1| + \cdots |v_n|$
3. p-Norm : $\|v\|_p = (|v_1|^p + |v_2|^p + \cdots + |v_n|^p)^{\frac{1}{p}}$
4. ∞-Norm : $\|v\|_\infty = \max(|v_1|, |v_2|, \ldots, |v_n|)$

```
>> norm(v,2)

ans =

    6.1644

>> norm(v,1)

ans =

    10

>> norm(v,inf)

ans =

     5
```

```
>> A = [1 2;2 0]

A =

     1     2
     2     0

>> norm(A,'fro')

ans =

     3
```