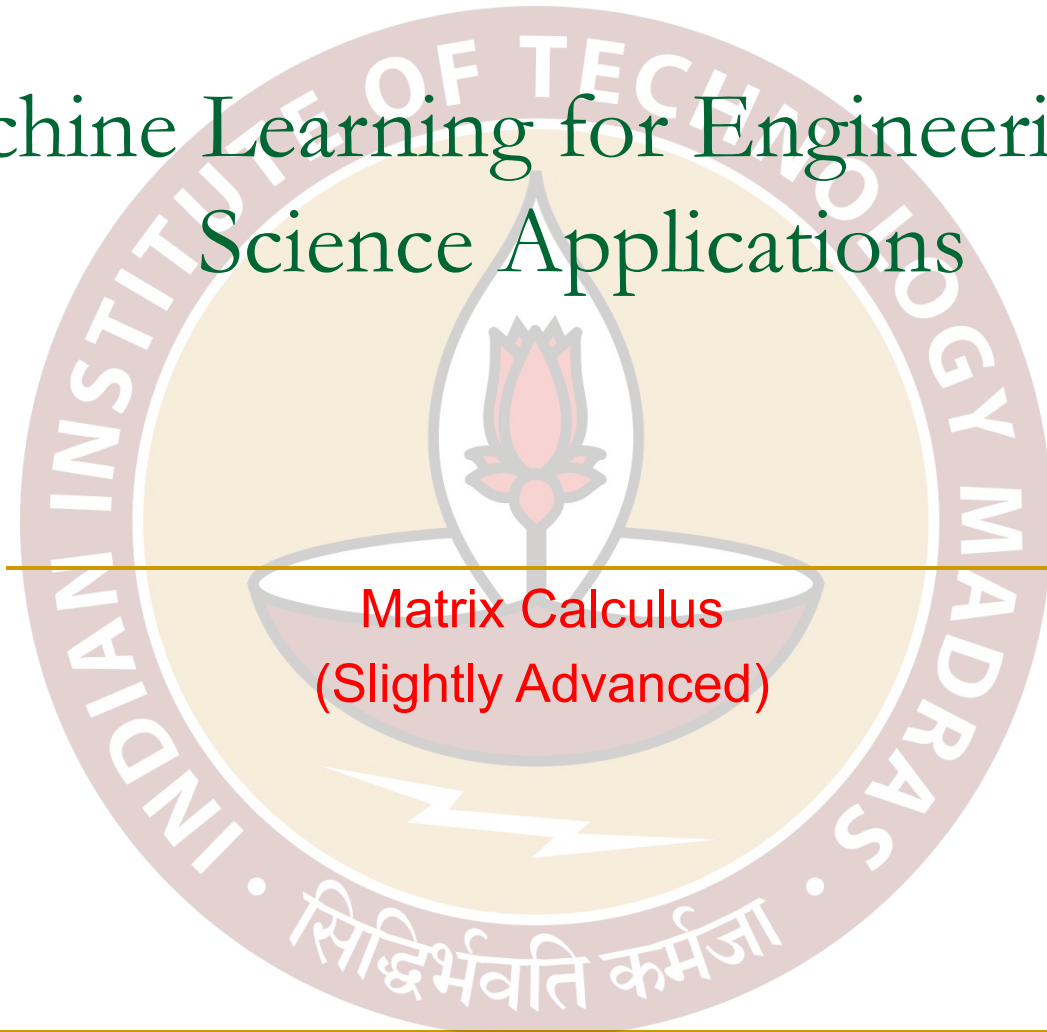


# Machine Learning for Engineering and Science Applications

Matrix Calculus  
(Slightly Advanced)



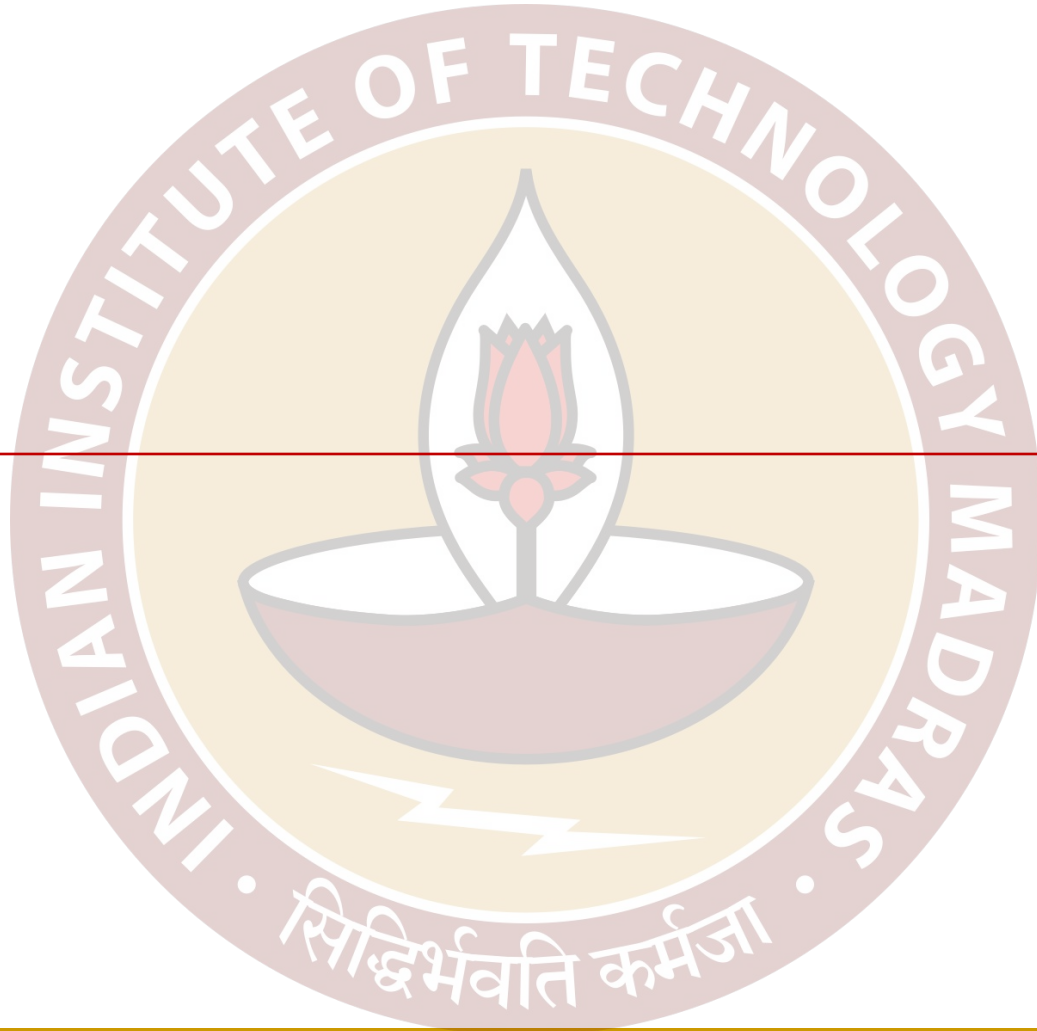
# Motivation

- Machine Learning training requires one to evaluate how one vector changes with respect to another
  - For example how output changes with respect to parameters
- This requires “matrix” calculus
- We will see some initial relations in this video
  - It is useful to understand these, but most of the course can be understood without this portion too.
- More advanced relations exist
  - Suggested resource : <https://explained.ai/matrix-calculus/index.html>

# Scalars and vectors

$$\left(\frac{\partial \mathbf{a}}{\partial x}\right)_i = \frac{\partial a_i}{\partial x}$$

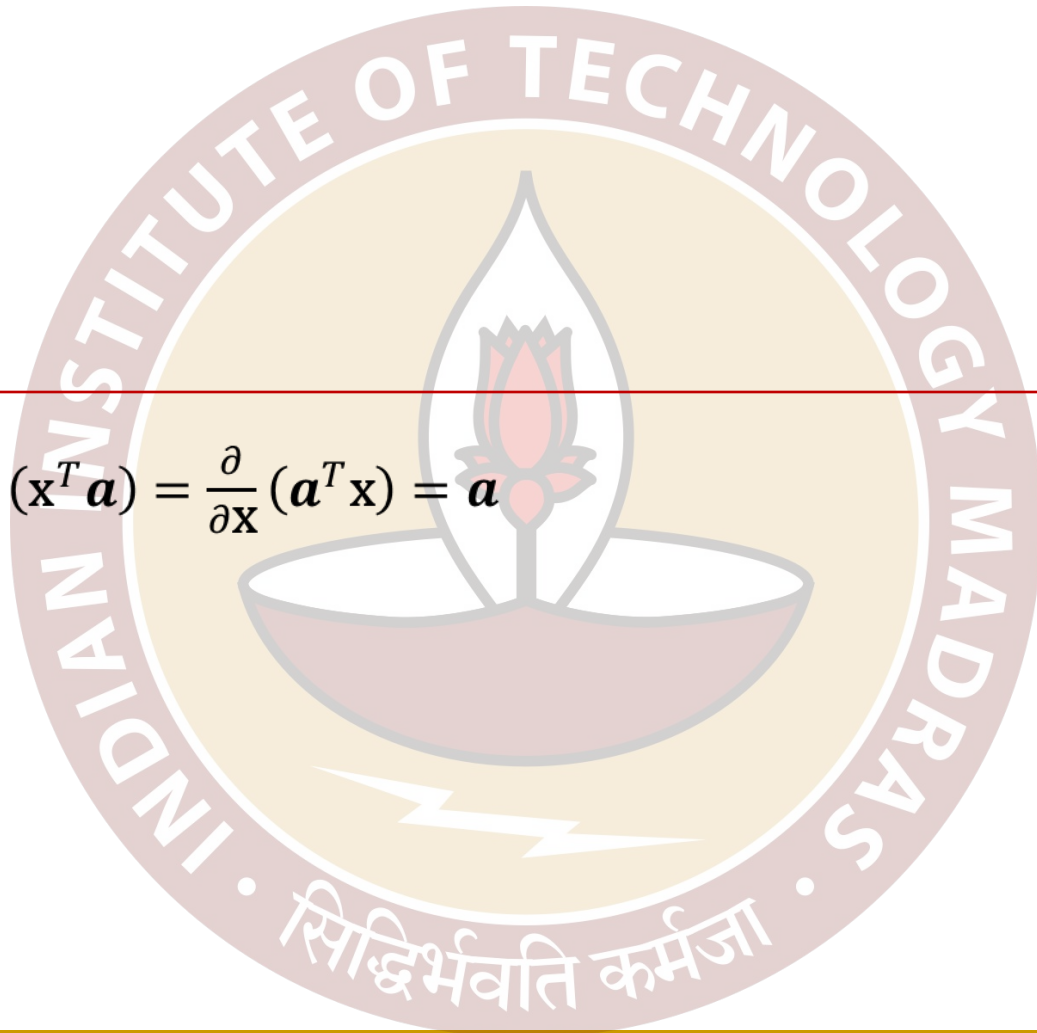
$$\left(\frac{\partial x}{\partial \mathbf{a}}\right)_i = \frac{\partial x}{\partial a_i}$$



# Vectors and vectors

$$\left(\frac{\partial \mathbf{a}}{\partial \mathbf{b}}\right)_{ij} = \frac{\partial a_i}{\partial b_j}$$

$$\frac{\partial}{\partial \mathbf{x}} (\mathbf{x} \cdot \mathbf{a}) = \frac{\partial}{\partial \mathbf{x}} (\mathbf{x}^T \mathbf{a}) = \frac{\partial}{\partial \mathbf{x}} (\mathbf{a}^T \mathbf{x}) = \mathbf{a}$$



# Matrices and vectors

$$\frac{\partial}{\partial \mathbf{x}} (\mathbf{A}\mathbf{B}) = \frac{\partial \mathbf{A}}{\partial \mathbf{x}} \mathbf{B} + \mathbf{A} \frac{\partial \mathbf{B}}{\partial \mathbf{x}}$$

$$\frac{\partial}{\partial \mathbf{x}} (\mathbf{x}^T \mathbf{A} \mathbf{x}) = (\mathbf{A} + \mathbf{A}^T) \mathbf{x}$$

Proof slightly involved (we will see a quick verification on the next slide)

**NOTE:** For symmetric matrices,  $\frac{\partial}{\partial \mathbf{x}} (\mathbf{x}^T \mathbf{A} \mathbf{x}) = 2\mathbf{A}\mathbf{x}$

# Derivative of the quadratic form

$$\frac{\partial}{\partial \mathbf{x}} (\mathbf{x}^T A \mathbf{x}) = (A + A^T) \mathbf{x}$$

Verification for  $2 \times 2$  case

