Machine Learning for Engineering and Science Applications

Linear combinations

Span

Linear Independence

Linear combination

- Linear combination : A linear combination of the set of vectors $\{\boldsymbol{v}^{(1)}, \dots, \boldsymbol{v}^{(n)}\}$ is given by multiplying each vector by a corresponding scalar coefficient and adding the results

$$\sum_i \alpha_i \boldsymbol{v}^{(i)}$$

Example : $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \ \boldsymbol{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$

$$\boldsymbol{v}_3 = v_1 + 2v_2 = \begin{bmatrix} 5 \\ 2 \\ 9 \end{bmatrix}$$

Note: $\boldsymbol{v}_3 = [\boldsymbol{v}_1 \ \boldsymbol{v}_2] \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Matrix multiplications can be interpreted in terms of linear combinations of columns

```
>> V = [1 2; 2 0; 3 3]

V =

     1     2
     2     0
     3     3
```

```
>> A = [1 3;2 4]

A =

     1     3
     2     4
```

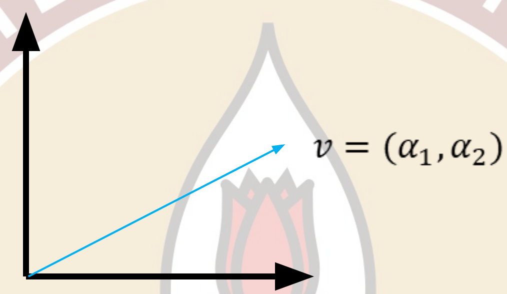
```
>> V*A

ans =

     5    11
     2     6
     9    21
```

Span

- Span : The span of a set of vectors is the set of all vectors obtainable by a linear combination of the original vectors.



Example : The span of the coordinate vectors $v_1 = (1,0),\ v_2 = (0,1)$ is?

Ans :

The span of all the columns of a matrix is called the column space

Note: The equation $A\boldsymbol{x} = \boldsymbol{b}$ has a solution only if $\boldsymbol{b}$ lies in the column space of A

Linear independence

- Linear independence : A set of vectors is linearly independent if none of these vectors can be written as a linear combination of the other vectors.

Example: $\{v_1 = (1,0), v_2 = (0,1)\}$ linearly independent

$\{v_1 = (1,0), v_2 = (0,1), v_3 = (3,4)\}$ linearly dependent

Mathematically, $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ is linearly independent if and only if the linear combination $\alpha_1 \boldsymbol{v}_1 + \alpha_2 \boldsymbol{v}_2 + \cdots \alpha_k \boldsymbol{v}_k = 0$ means that all the $\alpha_i = 0$