

INTRODUCTION TO DATASTRUCTURE

Datastructure - It is not only the collection of data but also it includes the following things.

- Organisation of data
- Relationship among the data
- Operations performed on the data
- Access methods to access the data

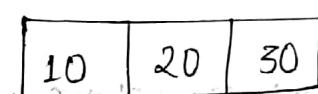
Types Of Datastructure -

- Linear
 - Static
- Non Linear
 - Non- static
 - Dynamic

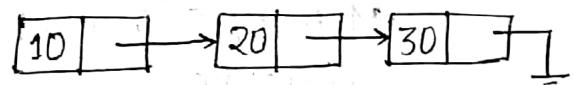
Linear Datastructure

Elements are organised in a linear fashion i.e sequential manner.

Ex - Stack



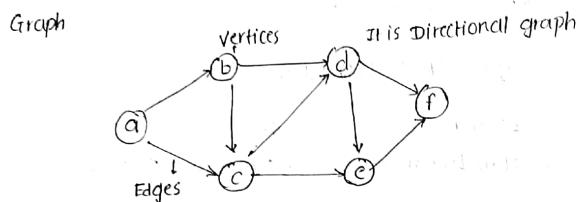
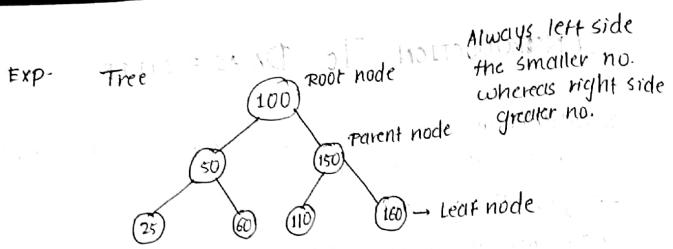
Linked List -



Non-Linear Datastructure

Elements are organised in non-linear fashion i.e they are not organised in sequential manner.

- Tree
- Graph
- Network
- Database
- Database
- Database
- Database



Static Datastructure -

Memory allocation at the time of compilation. And once the memory is allocated it will not change.

Ex- Array, stack using array, Queue using array

Dynamic Datastructure-

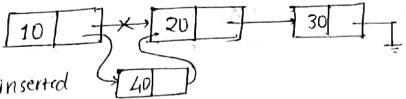
Memory allocation can be done at the time of run time.

And once the memory is allocated it can be changed.

Ex- Linked list, stack using linked list, Queue using linked list

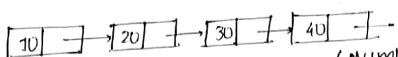
New element can be inserted in between	New element is inserted before i.e. elements get added at left side (before)	New element is inserted after i.e. elements get added at right (after)
--	--	--

Linked List



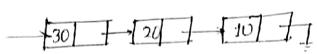
Element can be inserted in between

Queue



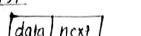
(Number/Elements are arranged in ascending order)

Stack

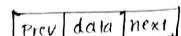


Elements are added these side (i.e. arranged in descending here)

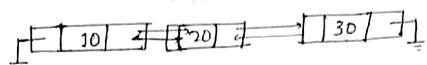
Linked List node



single LL
(directional)



double LL
(bidirectional)



Stack-

It is a Linear datastructure in which elements are pushed and popped through one end called as top of stack (tos).

It follows LIFO policy i.e. Last In First Out.

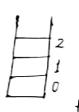
If tos = -1, stack is empty
If tos = max - 1, stack is full

#define Max 50

Macro

tos = -1; stack is empty

Stack operation -
 $\#define MAX 50$
 MACRO or Macro definition



tos = -1

→ struct stack

{

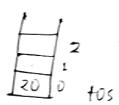
```
int tos;
int elems[MAX];
}s;
```

→ int isEmpty()

{

```
if (s.tos == -1)
  return 1;
else
  return 0;
}
```

(adding new elem)



Push(20)

→ int Push()
 {
 enter data i.e x
 check stack is full
 i.e if (isFull())
 display stack is full;
 else
 s.elems[+ + s.tos] = x;
 }

→ int isFull()

{

```
if (s.tos == MAX - 1)
  return 1;
else
  return 0;
}
```

(Deleting elem)
 POP() POP()



→ int POP()

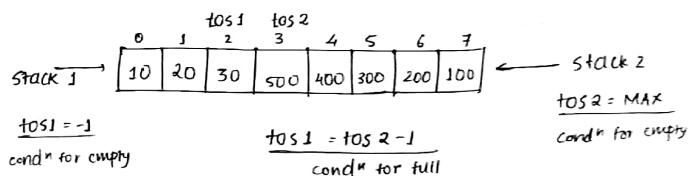
{
 check stack empty
 i.e.
 if (s.tos == -1)
 display stack is empty;
 else
 return s.elems[s.tos--];
}

→ int show()

{

check stack empty
 i.e.
 if (s.tos == -1)
 display stack is empty;
 else
 for (i=0; i < s.tos; i++)
 display elems[i];
 s.elems[i];
 }

Two stacks in a single array-



```

→ struct stack
{
    int tos1, tos2;
    int elems[MAX];
} s;

→ int isEmpty1()
{
    if (s.tos1 == -1)
        return 1;
    else
        return 0;
}

→ int isFull1()
{
    if (s.tos1 == s.tos2 - 1)
        return 1;
    else
        return 0;
}

```

```

→ int isEmpty2()
{
    if (s.tos2 == MAX)
        return 1;
    else
        return 0;
}

```

```

→ int push1()
{
    Enter data i.e x;
    check stack is full
    i.e. IF (isFull1())
        display stack is full;
    else
        s.elems[++s.tos1] = x;
}

```

```

→ int pop1()
{
    check stack is empty
    i.e. if (isEmpty1())
        display stack is empty;
    else
        s.elems[s.tos1--];
}

```

```

→ int show1()
{
    check stack 1 is empty
    i.e.
        if (isEmpty1())
            display stack 1 is empty;
        else
            for (i=0; i < s.tos1; i++)
                display elems at
                    s.elems[i];
}

```

```

→ int push2()
{
    Enter data i.e x;
    check stack is full
    i.e.
        if (isFull2())
            display stack is full;
        else
            s.elems[--s.tos2] = x;
}

```

```

→ int pop2()
{
    check stack 2 is empty
    i.e.
        if (isEmpty2())
            display stack 2 is empty;
        else
            s.elems[s.tos2++];
}

```

```

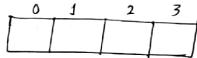
→ int show2()
{
    check stack 2 is empty
    i.e.
        if (isEmpty2())
            display stack 2 is empty;
        else
            for (i=MAX-1; i >= s.tos2; i--)
                display elems at
                    s.elems[i];
}

```

Queue -

It is a linear datastructure in which elements are inserted through one end called as rear and removed through one end called as front.

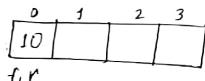
Initially



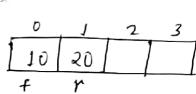
(front) f = 0

(rear) r = -1

insert (10)



insert (20)



→ Struct Queue

define MAX 50
Macro

```
{  
    int r, f;  
    int elems[MAX];  
} q;
```

→ int isEmpty()

```
{  
    if (q.r < q.f)  
        return 1;  
    else  
        return 0;  
}
```

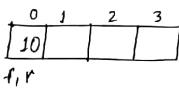
→ int isFull()

```
{  
    if (q.r == MAX-1)  
        return 1;  
    else  
        return 0;  
}
```

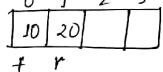
→ int insert()

```
{  
    Enter data x;  
    check queue is full;  
    i.e if (isFull())  
        display queue is full;  
    else  
        q.elems[+ + q.r] = x;  
}
```

insert (10)



insert (20)



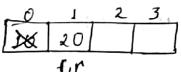
→ show()

```
{  
    check queue is Empty;  
    i.e if (isEmpty())  
        display queue is empty;  
    else  
        for (i = q.f; i <= q.r; i++)  
            display elems at  
            q.elems[i];  
}
```

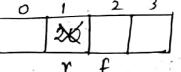
→ remove()

```
{  
    check queue is empty;  
    i.e if (isEmpty())  
        display queue is empty;  
    else  
        q.elems[q.f++];  
}
```

remove()



remove()



Program for stack - (Menu driven program)

```

Void main ()
{
    int ch, x;
    S::tos = -1;

    do
    {
        printf ("Enter 1. push 2. pop 3. show 4. Exit");
        printf ("Enter the choice");
        scanf ("%d", &ch);

        switch (ch)
        {
            Case 1 : push (x);
            break;

            Case 2 : x = pop ();
            If (x == -1)
                Stack is empty
            else
                print f ("%d is removed", x);
            break;

            Case 3 : show();
            break;

            Case 4 : Exit (0);
            break;
        }
    } while (ch != 4);
    getch();
}

```

Applications of Stack - Prefix ($+21$)

- Infix to postfix conversion
 - ↓
 - operator comes ($(R+)$) between two operands ($(R+)$)
- Postfix expression evaluation
- Removal of recursion
- Decimal to binary conversion
- Reverse of a string
 - 1) Infix to Postfix conversion-
 - In infix expression, operator lies in between operands, e.g - $A+B$
 - In postfix expression, operator lies after the operands, e.g - $AB+$

Operator	Priority	Associativity
(Raise to)	1 (highest) ↑	L to R
*	2	R to L
/	3	L to R
+, -	4	L to R

$\text{Exp} \rightarrow a + b * c$

current symbol	stack	postfix exp
a	Empty	a
+	+	a
b	+	ab
*	+*	ab
c	+*	abc
EOS End of symbol	empty	<u>abc * +</u>

$\rightarrow (A+B)*C - D/E * (F/G)$

current symbol	stack	postfix exp
((empty
A	(A
+	(+	A
B	(+	AB
)	empty	AB +
*	*	AB +
C	*	AB + C
-	-	AB + C *
D	-	AB + C * D
/	- /	AB + C * D

current symbol	stack	postfix exp
E	- /	AB + C * DE
*	- *	AB + C * DE /
C	- * C	AB + C * DE /
F	- * C	AB + C * DE / F
/	- * C /	AB + C * DE / F
G	- * C /	AB + C * DE / FG
)	- *	AB + C * DE / FG /
EOS	empty	<u>AB + C * DE / FG / * -</u>

→ Don't consider priority of brackets for this case.

$\rightarrow a \uparrow b * c - d + e / f / (g + h)$

current symbol	stack	postfix exp
cs	empty	a
a	empty	a
↑	↑	a
b	↑	ab
*	*	ab ↑
c	*	ab ↑ c
-	-	ab ↑ c *
d	-	ab ↑ c * d
+	+	ab ↑ c * d -
e	+	ab ↑ c * d - e
/	+/	ab ↑ c * d - e

cs	stack	Postfix exp ⁿ
f	+ /	ab↑c * d - ef
/	+ /	ab↑c * d - ef /
c	+ / c	ab↑c * d - ef /
g	+ / c	ab↑c * d - ef / g
+	+ / c +	ab↑c * d - ef / g
h	+ / (+	ab↑c * d - ef / gh
)	+ /	ab↑c * d - ef / gh +
EOS	empty	ab↑c * d - ef / gh + / +

Algorithm

Step 1 - Start
 Step 2 - Let s be the empty stack i.e $tos = -1$
 Step 3 - Read the infix expression as an array of characters
 Step 4 - Analyse each character of infix expression and perform the following steps till end of input.
 i) Let cs be the next input character.
 ii) If (cs is an operand)
 copy cs to postfix expression
 else
 {
 if (stack is empty)
 push (s, cs)
 else
 {

check priority of stack top and cs
 if (priority of stack top \geq priority of cs)
 {
 $ts = \text{pop}(s)$
 add ts to postfix expression
 }
 else
 push (s, cs)
 }
 step 5 - if ($cs == '('$)
 push (s, cs)
 if ($cs == ')'$)
 pop operators between (&)
 and add to the postfix expression
 step 6 - while (stack is not empty) $\Rightarrow pos // \text{input exp}^n$ finish
 {
 $ts = \text{pop}(s)$
 add ts to the postfix expression
 }
 step 7 - display postfix expression
 step 8 - stop

$$(A + B + C) + C$$

2) Evaluation of postfix expression

In postfix expression, operator lies after the operands e.g. $A B +$

EXP - $\rightarrow 9, 2, 1, 3, -$

9	2	1
9	2	1

$$OP2 = \text{pop}(S) \text{ i.e. } 2$$

$$OP1 = \text{pop}(S) \text{ i.e. } 9$$

$$\begin{aligned} \text{value} &= OP1 \text{ CS } OP2, \\ &= 9/2 \end{aligned}$$

(9/2) is not a valid operand

$$= 4.5$$

push(S, value)

3	4	5
3	4	5

$$OP2 = 3$$

$$OP1 = 4.5$$

$$\text{value} = 4.5 - 3$$

$$= 1.5$$

push(S, value)

1.5
1.5

End of i/p

$$\text{value} = \text{pop}(S)$$

value of postfix expression

$$= 1.5$$

$(9 + 2) / (3 - 1.5)$

$\rightarrow 5, 6, 2, +, *, 12, 7, 1, -, 1$

5	6	2	12	7	1
5	6	2	12	7	1

$$OP2 = \text{pop}(S) \text{ i.e. } 2$$

$$OP1 = \text{pop}(S) \text{ i.e. } 6$$

$$\begin{aligned} \text{value} &= OP1 \text{ CS } OP2 \\ &= 6 + 2 \\ &= 8 \end{aligned}$$

push(S, value)

8	*
8	*

$$OP2 = 8$$

$$OP1 = 5$$

$$\text{value} = 5 * 8$$

$$= 40$$

push(S, value)

12	7	1
12	7	1

$$OP2 = 7$$

$$OP1 = 12$$

$$\text{value} = 12 / 7$$

$$= 1.714$$

push(S, value)

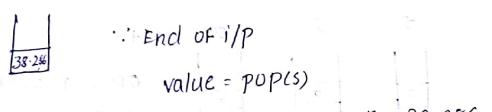
1.714
1.714

$$OP2 = 1.714$$

$$OP1 = 40$$

$$\text{value} = 40 - 1.714$$

$$= 38.286$$

\rightarrow Given input $4, 2, +, 3, *, 3, -, 8, 4, /, 1, 1, 1, +, 1, +$

 value of postfix exprn = 38.286
= 38.3

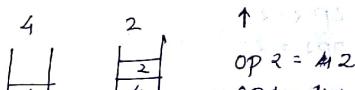
Algorithm

Step 1 - start
 Step 2 - Let s be the empty stack i.e $tos = -1$
 Step 3 - Read the postfix expression as an array of characters
 Step 4 - Analyse each character and perform the following steps till end of input
 i) Let cs be the next input character
 ii) IF (cs is an operand)
 push (s, cs)
 else
 {
 $OP_2 = pop(s)$
 $OP_1 = pop(s)$
 value = $OP_1 \text{ } CS \text{ } OP_2$
 push ($s, value$)
 }

Step 5 - pop the value from stack and display it.

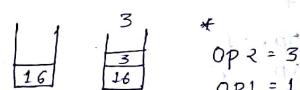
Step 6 - stop

$\rightarrow 4, 2, +, 3, *, 3, -, 8, 4, /, 1, 1, 1, +, 1, +$



$OP_2 = 4$
 $OP_1 = 2$

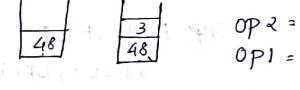
$value = 2 * 4$



$OP_2 = 3$
 $OP_1 = 16$

$value = 16 * 3$

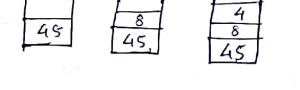
$= 48$



$OP_2 = 3$
 $OP_1 = 48$

$value = 48 - 3$

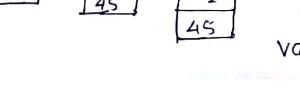
$= 45$



$OP_2 = 4$
 $OP_1 = 8$

$value = 8 / 4$

$= 2$



$OP_2 = 1$
 $OP_1 = 1$

$value = 1 + 1$

$= 2$



$push(s, value)$

$\begin{array}{|c|} \hline 2 \\ \hline 2 \\ \hline 45 \\ \hline \end{array}$
 OP2 = 2
 OP1 = 2
 value = 2/2
 = 1
 push(s, value)

$\begin{array}{|c|} \hline 1 \\ \hline 45 \\ \hline \end{array}$
 +

 OP2 = 1
 OP1 = 45
 value = 45 + 1
 = 46
 push(s, value)

$\begin{array}{|c|} \hline 46 \\ \hline \end{array}$
 ∵ BNF End. of I/P
 value = pop(s)
 Value of postfix exp
 = 46

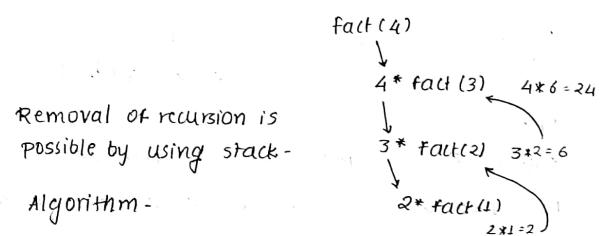
3) Removal of Recursion -

→ A function that calls itself until base condition is satisfied is called as recursion.

→ Let us consider to find factorial of a number.

→ int fact (int x)

```
{
  if (x == 0 || x == 1)
    return 1;
  else
    return (x * fact(x-1));
}
```



Removal of recursion is possible by using stack -

Algorithm -

Step 1 : start

Step 2 : Let S be the empty stack i.e tos=-1

Step 3 : Enter number n

Step 4 : push the number onto the stack till n > 1

```

→ while (n>1)
{
  Push(n)
  n--
}
  
```

Step 5: Initialize fact=1

perform pop operation till stack empty
→ while (! stack is empty)
fact = fact * POP()

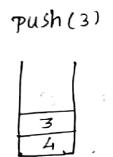
Step 6: Display factorial of a number &

Step 7: Stop

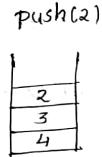
Exp - n=4



$$n = n-1 \\ = 3$$



$$n = 2$$



$$n = 1 \\ \therefore \text{stop push op"}$$

$$\text{fact} = 1$$

$$\text{pop i.e. } 2$$

$$\text{POP i.e. } 3$$

$$\text{POP i.e. } 4$$

$$\therefore \text{fact} = 1 * 2 = 2$$

$$\therefore \text{fact} = 2 * 3 = 6$$

$$\therefore \text{fact} = 6 * 4 = 24$$

∴ stack is empty

∴ stop pop op"

∴ factorial is 24

4) Decimal to Binary conversion-

Algorithm-

Step 1: Start

Step 2: Let S be the empty stack i.e. TOS = -1

Step 3: Enter number n

Step 4: push r onto the stack till q=0

while ($n! = 0$)
{
 $r = n \% 2$
 push(r)
 $n = n / 2$

Step 5: perform POP
operation till stack is empty

while (! stack is empty)
 POP()

Step 6: Display

Step 7: Stop

Exp - n=13

$$r = n \% 2$$

$$r = 13 \% 2 = 1$$

$$\text{push}(1)$$

$$r = 6 \% 2$$

$$r = 0$$

$$\text{push}(0)$$

$$r = 3 \% 2$$

$$r = 1$$

$$\text{push}(1)$$



$$n = n/2$$

$$\therefore n = 13/2 = 6$$



$$n = n/2$$

$$\therefore n = 6/2 = 3$$



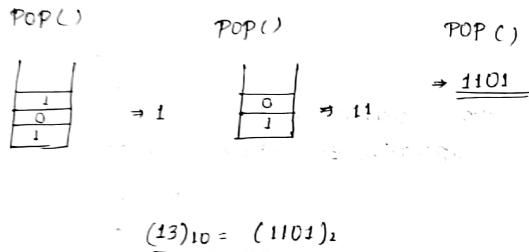
$$n = n/2$$

$$\Rightarrow n = 3/2 = 1$$

$r = n/2$
 $= 1/2$
 $\underline{r=1}$
 push(1)


Now $q=0$

stop push op"



5) Reverse of a String

* Algorithm -

Step 1: Start

Step 2: Let s be the empty stack i.e. $\text{tos} = -1$

Step 3: Enter the string and read the characters of string

Step 4: Analyse each character and perform the steps till the end of input

i) Let cs be the input character

ii) while ($\text{string}[i] \neq \text{lo}$)

{

$cs = \text{string}[i]$

$\text{push}(cs)$

$i++$

}

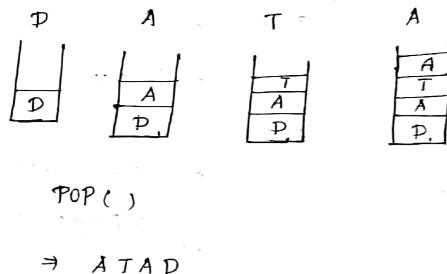
Step 5: pop the value from stack and display it if

\rightarrow while (!stack is empty)

Step : stop

$\text{POP}()$

EXP - DATA



6) well-formedness of parenthesis

Algorithm-

Step 1: Start

Step 2: Let s be the empty stack i.e $TOS = -1$

Step 3: Enter the parenthesis
Read

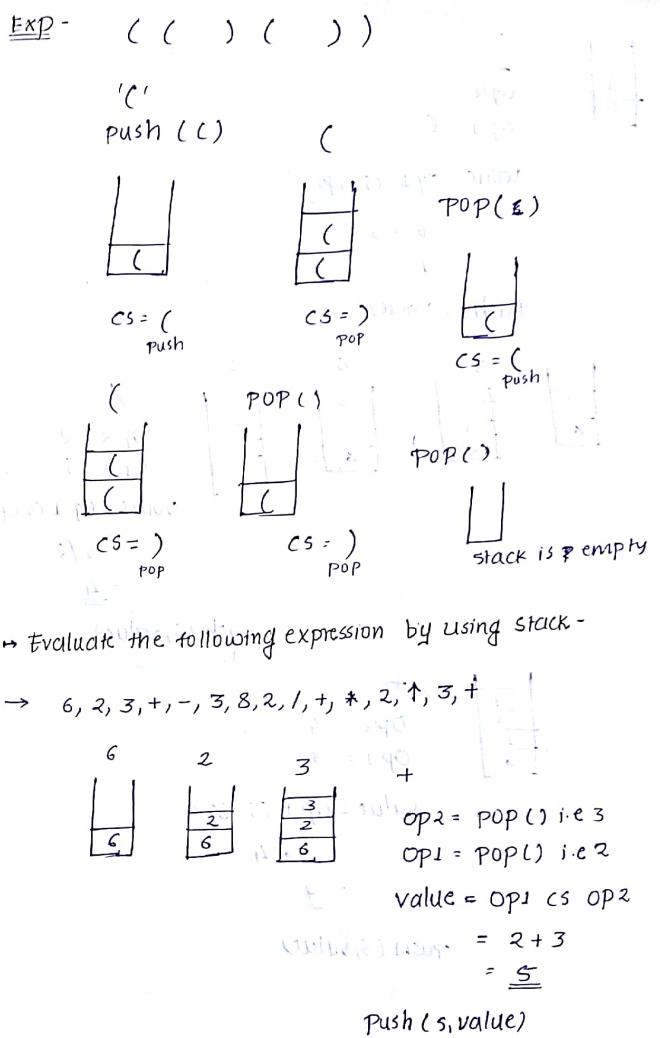
Step 4: Analyse each parenthesis, and perform the following steps -

- Let cs be the input character
- while (~~string~~ $cs \neq \text{EOF}$)
 - if ($cs == '('$) push (cs)
 - else if (stack is empty) display error
 - else pop()

if ($EOS \& ! \text{stack empty}$) display error

else Accept

Step 5: Stop





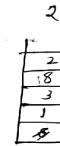
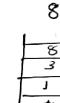
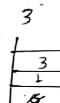
$$\begin{aligned}OP2 &= 5 \\OP1 &= 6\end{aligned}$$

$\text{value} = \text{OP1 CS OP2}$

$$= 6 - 5$$

$$= 1$$

$\text{push}(s, \text{value})$



$$\begin{aligned}OP2 &= 2 \\OP1 &= 8\end{aligned}$$

$\text{value} = \text{OP1 CS OP2}$

$$= 8 / 2$$

$$= \underline{\underline{4}}$$

$\text{push}(s, \text{value})$



$$\begin{aligned}OP2 &= 4 \\OP1 &= 3\end{aligned}$$

$\text{value} = \text{OP1 CS OP2}$

$$= 3 + 4$$

$$= \underline{\underline{7}}$$

$\text{push}(s, \text{value})$



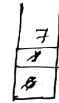
$$\begin{aligned}OP2 &= 7 \\OP1 &= 1\end{aligned}$$

$\text{value} = \text{OP1 CS OP2}$

$$= 1 * 7$$

$$= \underline{\underline{7}}$$

$\text{push}(s, \text{value})$



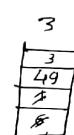
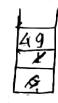
$$\begin{aligned}OP2 &= 2 \\OP1 &= 7\end{aligned}$$

$\text{value} = \text{OP1 CS OP2}$

$$= 7 * 2$$

$$= \underline{\underline{14}}$$

$\text{push}(s, \text{value})$



$$\begin{aligned}OP2 &= 3 \\OP1 &= 49\end{aligned}$$

$\text{value} = \text{OP1 CS OP2}$

$$= 49 + 3$$

$$= \underline{\underline{52}}$$

$\text{push}(s, \text{value})$

$\text{pop}()$

$$\text{value} = \underline{\underline{52}}$$

Transfer the following expression to postfix and then evaluate by assuming $a=1; b=2; c=3; d=4;$
 $e=6; f=6; g=1; i=3; j=3$

$$= a + b - c * d / e + f \frac{g}{i+j}$$

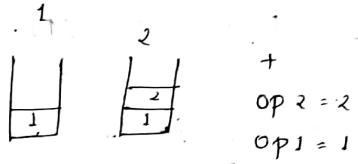
Current symbol	Stack	Postfix exp ⁿ
a	Empty	a
+	+	a
b	+	ab
-	-	ab+
c	-*	ab+c
*	-*	ab+c*
d	-*	ab+cd
/	-/	ab+cd*
e	-/	ab+cd*
+	+*	ab+cd*
f	+	ab+cd*
*	+*	ab+cd*
g	+*	ab+cd*
/	+*/	ab+cd*
(+/(ab+cd*
i	+/(ab+cd*

current symbol	stack	postfix exp ⁿ
+	+/(+	ab+cd*
)	+/(+	ab+cd*
EOS	Empty	ab+cd*

postfix expression is $- ab+cd*$ $\frac{e}{f} g \uparrow ij + / +$

1, 2, 3, 4, 6, 1, - , 6, 1, 1, 3, 3, +, /, +

$\Rightarrow 1, 2, +, 3, 4, *, 6, /, -, 6, 1, 1, 3, 3, +, /, +$

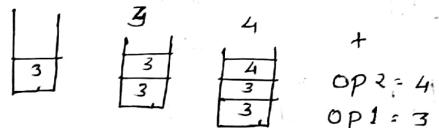


Value = op1 + op2

= 1 + 2

= 3

push(s, value)

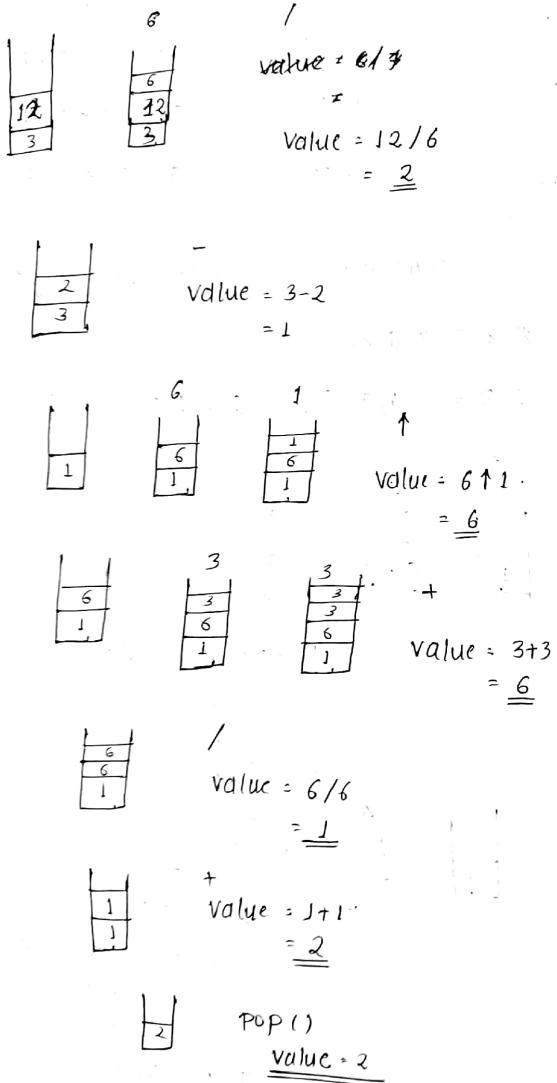


value = op1 + op2

= 3 + 4

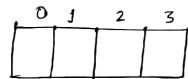
= 12

push(s, value)



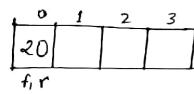
Circular Queue -

Let us consider Linear queue with max = 4

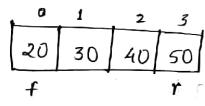


f=0
r=-1

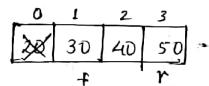
insert (20)



insert (30), insert (40), insert (50)



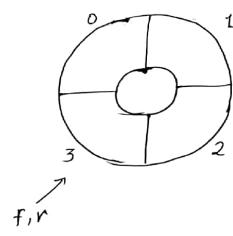
remove ()



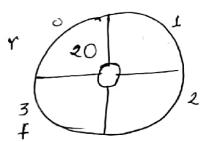
we cannot insert new element into the queue even if there is empty place for a new element because full condition of Queue is satisfied i.e [r = max - 1]

To overcome these circular queue can be used in which two ends of a queue are joined together to form a ring.

→ circular queue

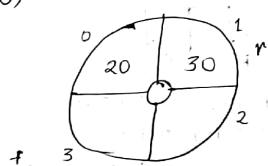


insert (20)

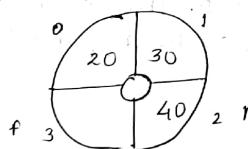


No element should be added where there is a f(front)

insert (30)



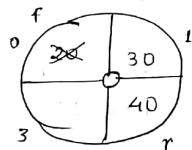
insert (40)



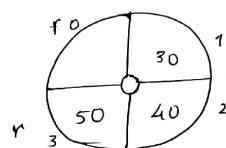
rear = (rear + 1) % Max

remove ()

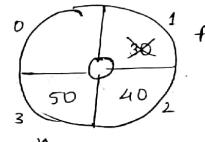
remove ()



Insert (50)



remove ()



→ only the drawback is just the one place remains empty
otherwise complete queue is filled.

Algorithm-
→ # define max 50
→ struct cqueue
{

int r, f;
int elements[MAX];
} cq;

(macro is processed at time of
compile)
(function is processed at time of
run)

```

→ int isEmpty()
{
    if (cq.r == cq.f)
        return 1;
    else
        return 0;
}

```

```

→ void insert (int x)
{

```

③ update rear
i.e.

$$cq.\text{rear} = (cq.r + 1) \% \text{MAX}$$

④ if (isFull())
 display "queue is full"; //queue overflow
else
 cq.items [cq.r] = x
}

 cq.r = cq.r + 1
 cq.front = cq.front + 1
 cq.items[cq.r] = 10
}

```

→ int isFull()
{
    if (cq.r == cq.f)
        return 1;
    else
        return 0;
}

```

```

→ int removes()
{

```

① if (isEmpty())
{
 return -1; //queue underflow
}
else
 update front
② i.e.
 cq.f = (cq.f + 1) \% MAX
③ return (cq.items [cq.f]);
}

```

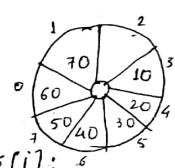
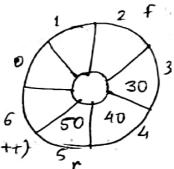
→ void display()
{

```

if (isEmpty())
{

display "queue is empty";

else
{
 if (cq.front < cq.r)
 {
 for (i = cq.f + 1; i <= cq.r; i++)
 display elements at cq.items [i];
 }
 else
 for (i = cq.f + 1; i <= MAX - 1; i++)
 display ele. at cq.items [i];
}
}



```

for (i=0; i<=(q-1); i++)
    display elems at cq.elems[i];
}
}

```

C programming -

→ Sum of digits -

```

#include <stdio.h>
#include <conio.h>

void main ()
{
    int n, r, sum=0; clrscr();
    printf("Enter the number");
    scanf("%d", &n);
    while (n>0)
    {
        r = n%10;
        sum = sum+r;
        n = n/10;
    }
    printf("sum of digit is %d", sum);
    getch();
}

```

→ Fibonacci series -

```

0 1 1 2 3 5 8
#include <stdio.h>
#include <conio.h>

void main ()
{
    int a, n, a+b, c; a=0, b=1, c;
    clrscr();
    printf("Enter the number");
    scanf("%d", &n);
    while (n>0)
    {
        a = b;
        b = c;
        c = a+b;
    }
    printf("Fibonacci series is %d", c);
    getch();
}

```

→ Prime number -

```

#include <stdio.h>
#include <conio.h>

void main ()
{
    int n;
    clrscr();
    printf("Enter the number");
    scanf("%d", &n);
}
```

```

while
while &
{
for(j=2; i<=n-1; i++)
{
    if (n-j, i!=0)
        flag = 1;
    break;
}
else
if (flag == 1)
    It is prime no. ;
else
    It is not a prime no. ;
}

```

For 2 marks -

- Q.1) Explain stack with its operation.
 - 2) Explain queue with its operation.
 - 3) Why circular queue is used.
 - 4) Difference between stack and queue.
 - 5) Types of Datastructure.
 - 6) Explain Recursion.
 - 7) Explain ADT (Abstract Datatype) with example.
 - 8) What is priority queue and its types.
 - 9) Explain time complexity.
 - 10) What are the applications of stack.
 - 11) What are the characteristics of algorithm.
- For 5 marks -
- 1) stack using array. (Algorithm)
 - 2) Queue using array.
 - 3) circular queue using array.
 - 4) Infix to postfix conversion.
 - 5) Postfix expression evaluation.
 - 6) Asymptotic notations.

ADT (Abstract Datatype)

- Abstract means to hide the details from the user.
- In context of datastructures there are various types of data which are stored into datastructure and purpose of each data structure is different.
- For exp - Queue can be used for job scheduling.
Graph can be used to find shortest distance between two cities.
Stack can be used to find expression value and so on.
- Not all users are interested about how things are happen rather they expected final result.
- ADT means what operations datastructures provide rather how these operations give final result.

Stack ADT

```
int isEmpty(s) // check whether stack is empty  
int isFull(s) // check whether stack is full  
void push(s,x) // insert x into the stack  
int pop(s) // removes the top element from the stack  
void show(s) // display elements from the stack  
int stacktop(s) // returns the top element from stack
```

Queue ADT

```
int isEmpty(q) // check whether queue is empty  
int isFull(q) // check whether queue is full  
void insert(q,x) // insert x into the queue  
int removes(q) // removes the element from the queue  
void display(q) // display elements from the queue
```

Characteristics of Algorithm-

- Algorithm is a step-by-step procedure to solve a given problem.
- It is a blueprint of a program that map inputs to output
- Characteristics -
 - Input - It represents input data given by user externally.
 - Output - It represents the result given by program after successful completion.
 - Finiteness - An algorithm must be terminated after finite number of steps.
 - Definiteness - An algorithm must contain clear, precise, and unambiguous statement.
 - Efficiency - It represents time complexity and space complexity of an algorithm must be as low as possible

Exp - Let us consider addition of two numbers

Step1: start

Step2: Accept two numbers a and b

Step3: $c = a + b$

Step4: print c

Step5: stop

Characteristics -

- There are two inputs a and b
- Addition of a and b is c
- Algorithm terminates in 5 steps
- Above algorithm contains clear, precise and unambiguous statement
- Time complexity is $O(1)$ and space complexity is 6 bytes (Big-oh).

Priority Queue -

- In linear queue elements are inserted through rear end removed through front end and it follows first in first out (FIFO) policy.
- In priority queue elements are inserted through rear end but elements are removed based on priority.
There are two types -
 - ↳ Ascending priority queue
 - ↳ Descending priority queue

1) Ascending priority Queue -

→ Elements are inserted through rear end but removed in ascending order.

For exp -

0	1	2
10	15	12

removes()

10 removed

0	1	2
15	12	

removes()

12 removed

0	1	2
15		

2) Descending priority Queue -

→ Elements are inserted through rear end but removed in descending order.

For exp -

0	1	2
20	13	17

removes()

20 removed

0	1	2
13	17	

removes()

17 removed

0	1	2
13		

Asymptotic Notations

1) Big - Oh (O)

Let $f(n)$ and $g(n)$ be the two functions, we can say that $f(n)$ is big-oh (O) of $g(n)$

$$\text{i.e. } f(n) = O(g(n))$$

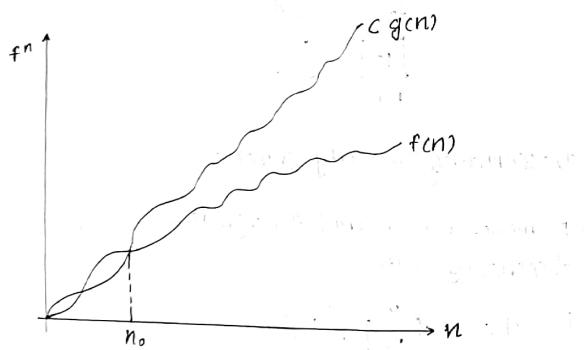
If it satisfies the following condition -

$$f(n) \leq c g(n)$$

where c is the constant and

n be number of inputs such that $n > n_0$

Graphical Representation -



Ex - Let

$$f(n) = 2n^2 + n + 100$$

$$\therefore 2n^2 + n + 100 < \frac{c}{3}n^2$$

$$\therefore f(n) = O(n^2)$$

2) Big - Omega (Ω)

Let $f(n)$ and $g(n)$ be the two functions, we can say that $f(n)$ is big-omega (Ω) of $g(n)$

$$\text{i.e. } f(n) = \Omega(g(n))$$

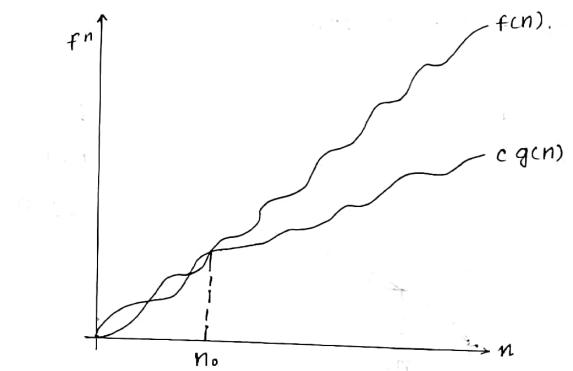
If it satisfies the following condition -

$$f(n) \geq c g(n)$$

where c is the constant and

n be the number of inputs such that $n > n_0$

Graphical Representation -



Ex -

$$\text{Let } f(n) = 2n^2 + n + 100$$

$$\therefore 2n^2 + n + 100 > \frac{c}{2}n^2$$

$$\therefore f(n) = \Omega(n^2)$$

3) Big-Theta (Θ)

→ It is a combination of both big-O and big-Omega notations.

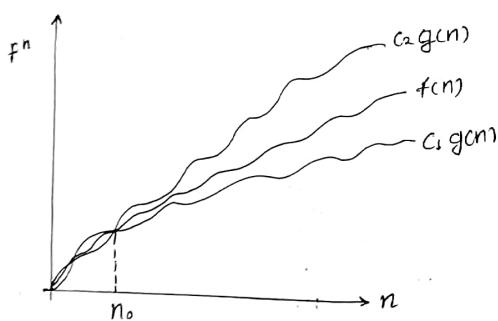
→ Let $f(n)$ and $g(n)$ be two functions, we can say that $f(n)$ is big-Theta (Θ) of $g(n)$ if $f(n) = \Theta(g(n))$

If it satisfies following condition -

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

where c_1 and c_2 are constants and n be number of inputs such that $n > n_0$

Graphical Representation-



Exp - Let

$$f(n) = 2n^2 + n + 100$$

$$\therefore 2n^2 < 2n^2 + n + 100 < 3n^2$$

$$\therefore f(n) = \Theta(n^2)$$

Time Complexity

→ It represents the time required by computer machine to execute a program,

accept the inputs from user and display output.

→ There are various notations used to calculate time complexity like -

→ Big-oh (O)

→ Big-Omega (Ω)

→ Big-Theta (Θ)

Types -

1) Worst case T_C -

→ It represents the upper bound on the running time on any input.

→ It gives us an assurance that the algorithm will never go beyond these limits.

2) Average T_C -

→ It is an estimation of running time for average inputs which are selected randomly.

→ This case assumes that inputs are equally likely.

→ Each input is given to the program and calculate its run time.

→ Finally, total time is divided by no. of inputs that gives average time complexity.

3) Best TC -

→ It is used to analyse an algorithm under optimal condition.
For ex - In linear search, if element found at first position then it is the base best case.

Examples -

4) Constant -

Algo	no. of times instructions executed
Add()	
{	
int a, b, sum;	1
sum = a+b;	1
display sum	1
}	
Total	<u>3</u>

As 3 is constant $\therefore TC = O(1)$

whether it is 3 or 30 then it is constant and can be written as 1

2) Linear -

Algo	no. of times instructions executed
Test()	
{	
for(i=1; i<=n; i++)	$n+1$
print i	n
}	
Total	<u>$2n+1$</u>

3) Quadratic -

Algo	no. of times instructions executed
Test()	
{	
for(j=1; j<=n; j++)	$n+1$
for(i=1; i<=n; i++)	$n(n+1)$
print i	$n \cdot n$
}	
Total	<u>$2n^2 + 2n + 1$</u>

$\therefore TC = O(n^2)$

4) Logarithmic -

for multiplication & division

Algo	no. of times instructions executed
Test()	
{	
i = 1	1
while (i <= n)	$\log_2 n + 1$
{	
print i;	$\log_2 n$
i = i * 2	$\log_2 n$
}	
Total	<u>$3\log_2 n + 2$</u>

Let $n = 1000$

$i = 1$	
$= 2$	
4	
8	
16	10 times
32	
64	
128	\log_2^{1024}
256	$\log_2^{2^{10}}$
512	\log_2^{1024}
1024	$10 \log_2^2$
	10

$\therefore TC = O(\log n)$

Searching And Sorting -

Linear Search / Sequential Search -

- Elements are search in sequential manner from first position till element found.
- Maximum number of passes required by these searching are n .
- Minimum it requires single pass.

Exp-

$$x[7] = \{ 10, 15, 12, 18, 19, 25, 13 \}$$

Let no. = 18

Step 1

Pass 1 : $i=0$

$$x[0] = 10 \neq 18$$

Not found, increment i

Pass 2 : $i=1$

$$x[1] = 15 \neq 18$$

Not found, increment i

Pass 3 : $i=2$

$$x[2] = 12 \neq 18$$

Not found, increment i

Pass 4 : $i=3$

$$x[3] = 18$$

Element found at location 3

\therefore Elements found at location 3

Algorithm -

- Start
- Enter the location of element to be search, i.e. no.
- Enter the number of elements i.e. n and location of array.
- Enter the elements and store into array x.
- int linear search(x, no, n)

```
for (i=0; i<n; i++) {  
    if (x[i] == no)  
        return i; // Element found  
    }  
return -1; // Element not found
```

- If function returns -1 then display 'element not found'

```
else  
    Element found at position i.e. if (i=-1)  
    { element not found  
    }
```

- stop

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

```
if (i=-1)  
    display "Element not found"  
else  
    display "Element found"
```

→ Binary search -

- In linear search, element will be search from 1st position to last position hence it is a time consuming method.
- To overcome these, binary search method can be used.
- To search element by using binary search, elements need to be in a sorted order

Exp - $x[1] = \{12, 25, 28, 31, 45, 51, 62, 68, 73\}$

i) Number to be found is 25

Let no = 25

Pass 1 :

$$\text{low} = 0$$

$$\text{high} = n-1 = 9$$

$$\text{mid} = (\text{low} + \text{high})/2 = 4$$

$$x[4] = 45 \neq \text{no}$$

$$\therefore \text{no} < x[4]$$

\therefore no may be present on left side of $x[4]$

Pass 2 :

$$\text{low} = 0$$

$$\text{high} = \text{mid} + 1 = 3$$

$$\text{mid} = (0+3)/2 = 1$$

$$x[1] = 25 = \text{no}$$

\therefore no. found at postn 1

2) Let no = 73

Pass 1 :

$$\text{low} = 0$$

$$\text{high} = n-1 = 9$$

$$\text{mid} = (\text{low} + \text{high})/2 = 4$$

$$x[4] = 45 \neq \text{no}$$

$$\therefore \text{no} > x[4]$$

\therefore no may be present on right side of $x[4]$

Pass 2 :

$$\text{low} = 4, \text{mid} + 1 = 4+1 = 5$$

$$\text{high} = \text{mid}$$

$$\text{high} = n-1 = 9$$

$$\text{mid} = (5+9)/2 = 7$$

$$x[7] = 68 \neq \text{no}$$

\therefore no. not found at position 7

Pass 3 :

$$\text{low} = \text{mid} + 1 = 7+1 = 8$$

$$\text{high} = 9$$

$$\text{mid} = (8+9)/2 = 8$$

$$x[8] = 73 = \text{no}$$

\therefore no. found at postn 8

∴ no. found at postn 8

3) Let no=50

Pass 1 : low = 0

$$high = n-1 = 9$$

$$mid = (0+9)/2 = 4$$

$$x[4] = 45 \neq no$$

$$\therefore no > x[4]$$

∴ no. may be present on right side of x[4]

Pass 2 : low = mid + 1 = 4 + 1 = 5

$$high = 9$$

$$mid = (5+9)/2 = 7$$

$$x[7] = 68 \neq no$$

$$\therefore no < x[7]$$

∴ no. not found at position 7

Pass 3 :

$$low = 5$$

$$high = mid - 1 = 6 \quad 7 - 1 = 6$$

$$mid = (5+6)/2 = 5$$

$$x[5] = 51 \neq no$$

$$\therefore no < x[5]$$

∴ no. not found at position 5

Pass 4 :

$$low = 5$$

$$high = mid - 1 = 5 - 1 = 4$$

since low > high

∴ Element not found

Algorithm -

1) start

2) Enter the number of elements i.e n

3) Enter the elements and store into array x

If elements are not sorted, then sort it first by using any sorting technique -

- 1) Radix sort
- 2) Binary tree sort
- 3) Bubble sort
- 4) shell sort
- 5) selection sort

4) ~~Program~~ Enter the elements to be search i.e no

5) bin_search (x, no, n)

{

$$low = 0$$

$$high = n-1$$

while (low <= high)

{

$$mid = (low+high)/2$$

if ($x[mid] == no$)

return mid // ele. found at mid

else

if ($x[mid] < no$)

low = mid + 1

else

high = mid - 1

```

if (no < x[mid])
    high = mid - 1
else
    low = mid + 1
}
return -1 //elements not found

```

$\rightarrow x[] = \{ 11 \ 5 \ 21 \ 3 \ 29 \ 17 \ 2 \ 43 \}$

\Rightarrow Let no = 29, 1

First sort the given elements

passes required for sorting n-1

Pass 1: 1) 11, 5

$11 > 5$

swap

$5 \ 11$

$\{ 5 \ 11 \ 21 \ 3 \ 29 \ 17 \ 2 \ 43 \}$

2) 11, 21

$11 < 21$

No swap

$\{ 5 \ 11 \ 21 \ 3 \ 29 \ 17 \ 2 \ 43 \}$

3) 21, 3

$21 > 3$

swap

$\{ 5 \ 11 \ 3 \ 21 \ 29 \ 17 \ 2 \ 43 \}$

4) 21, 29

$21 < 29$

No swap

$\{ 5 \ 11 \ 3 \ 21 \ 17 \ 29 \ 2 \ 43 \}$

5) 29, 17

$29 > 17$

swap

$\{ 5 \ 11 \ 3 \ 21 \ 17 \ 29 \ 2 \ 43 \}$

6) 29, 2

$29 > 2$

swap

$\{ 5 \ 11 \ 3 \ 21 \ 17 \ 2 \ 29 \ 43 \}$

7) 29, 43

$29 < 43$

No swap

$\{ 5 \ 11 \ 3 \ 21 \ 17 \ 2 \ 29 \ 43 \}$

Pass 2: $\{ 5 \ 11 \ 3 \ 21 \ 17 \ 2 \ 29 \ 43 \}$

1) 5, 11

$5 < 11$

No swap

$\{ 5 \ 11 \ 3 \ 21 \ 17 \ 2 \ 29 \ 43 \}$

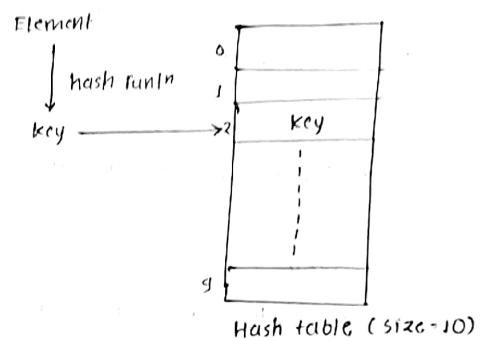
2) 11, 13

11 < 13 11 > 3
No swap

{ 5 3 11 21 17 8 29 43 }

Hashing -

→ It is one of the searching technique in which elements are stored into hash table by using various hash functions.



Hash Functions -

1) Modulo division Method -

→ Address of an element can be calculated by performing mod operation i.e. addr = element % table size

Exp -

$$\text{Addr of } 25 = 25 \% 10 = 5$$

2) Direct Method -

→ Element becomes the address.

For exp. Addr of 25 is 25

→ It requires large table size (with empty spaces in both)

3) Subtraction Method-

- A fixed number is subtracted from each element to find address.
- For exp - Addr of 125 = $125 - 100$ (Assuming 100 as fixed no)
 $= 25$
Element will be stored at 25

4) Digit Analysis Method-

- These method is generally used whenever element contains more number of digits.

For exp - Let no = 6225264
Digit 2 and 6 present multiple times hence address is 26

5) Length Dependant Method-

- Length of an ~~the~~ element becomes the address.

For exp -
265
Addr is 3

6) Length Independant Method-

- Address of an element can be calculated by using length and some part of element.

For exp -
Address of 125 = $3 + 12 = 15$?

7) Folding Method-

a) Fold shifting Method-

Elements are partitioned based on length of address, then these parts are added and ignore carry if generated.

Exp -

$$no = 12345678$$

Let address having 2 digits

∴ no. is partitioned into 12, 34, 56, 78

$$12 + 34 + 56 + 78$$

$$= \begin{array}{c} 180 \\ \uparrow \\ \text{address} \\ \text{ignore} \end{array}$$

b) Fold boundary Method-

This method is similar to fold shifting but elements present at the boundary, digits are interchanged.

For exp -

$$no = 12345678$$

Let address having 2 digits

∴ no. is partitioned into 12, 34, 56, 78

$$\Rightarrow 21 + 34 + 56 + 87$$

$$= \begin{array}{c} 198 \\ \uparrow \\ \text{Address} \\ \text{ignore} \end{array}$$

8) collision handling Method

whenever multiple elements hashes to the same address it is called as collision.

For exp -
Let us consider table size as 10 and hashing function as modulo division

$$\text{addr} = \text{element} \% \text{table size}$$

$$\therefore \text{addr of } 12 = 12 \% 10 = 2$$

$$\text{addr of } 32 = 32 \% 10 = 2 \rightarrow \underline{\text{collision}}$$

Collision handling techniques -

i) open addressing

i) Linear probing -

→ whenever collision occurs, start searching an empty slot by moving in downward direction till last position of hash table.

→ If no empty slot found then start searching from first position of hash table.

→ If there is no empty slot in entire hash table then it is advisable to increase the size of hash table.

→ Following eqn is used to find address of an element -

$$h(k, i) = (h(k) + i) \% m$$

where k is key element, i is prob number = $0, 1, 2, \dots$

$h(k)$ is $k \% m$

m is table size

Exp - 72, 27, 36, 24, 63, 81, 92, 101

Table size = $m = 10$

Table size = $10 = m$

Initially, all slots occupy -1 (By array)
0 (If by linked list)

0	-1
1	-1
2	-1
3	-1
4	-1
5	-1
6	-1
7	-1
8	-1
9	-1

$$\begin{aligned} h(72, 0) &= (h(72) + 0) \% 10 = 10 \\ &= (72 \% 10 + 0) \% 10 \\ &= 2 \end{aligned}$$

$$h(27, 0) = (27 \% 10 + 0) \% 10 = 7$$

$$h(36, 0) = (36 \% 10 + 0) \% 10 = 6$$

$$h(24, 0) = (24 \% 10 + 0) \% 10 = 4$$

$$h(63, 0) = (63 \% 10 + 0) \% 10 = 3$$

$$h(81, 0) = (81 \% 10 + 0) \% 10 = 1$$

$$h(92, 0) = (92 \% 10 + 0) \% 10 = 2$$

collision

$$h(92,1) = (92 \bmod 10 + 1) \bmod 10 = 3 \text{ collision}$$

$$h(92,2) = (92 \bmod 10 + 2) \bmod 10 = 4 \text{ collision}$$

$$h(92,3) = (92 \bmod 10 + 3) \bmod 10 = 5 \text{ collision}$$

$$h(101,0) = (101 \bmod 10 + 0) \bmod 10 = 1 \text{ collision}$$

$$h(101,1) = (101 \bmod 10 + 1) \bmod 10 = 2 \text{ collision}$$

$$h(101,2) = (101 \bmod 10 + 2) \bmod 10 = 3 \text{ collision}$$

$$h(101,3) = (101 \bmod 10 + 3) \bmod 10 = 4 \text{ collision}$$

$$h(101,4) = (101 \bmod 10 + 4) \bmod 10 = 5 \text{ collision}$$

$$h(101,5) = (101 \bmod 10 + 5) \bmod 10 = 6 \text{ collision}$$

$$h(101,6) = (101 \bmod 10 + 6) \bmod 10 = 7 \text{ collision}$$

$$h(101,7) = (101 \bmod 10 + 7) \bmod 10 = 8 \text{ collision}$$

→ Drawback of this technique is primary clustering where elements are stored at one particular area and rest of hash table is empty. To overcome this quadratic technique can be used.

0	
1	81
2	72
3	63
4	24
5	92
6	36
7	27
8	101
9	

ii) Quadratic probing -

→ Instead of searching for i th position in linear probing we search for i^2 position in Quadratic probing that overcomes primary clustering.

→ Following eqn is used to find address of an element -

$$h(k,i) = (h(k) + c_1 i + c_2 i^2) \bmod m$$

where k is key element, $i = \text{prob no.} = 0, 1, 2, \dots$

$c_1, c_2 = \text{constant but } c_2 \neq 0$.

$m = \text{table size}$

$$h(k) = k \bmod m$$

For exp -

$$72, 27, 36, 24, 92, 101, 81$$

Initially, the table size is $m = 10$

all slot occupy -1

Assume $c_1 = 0, c_2 = 3$

Initially,

$$h(72,0) = (72 \bmod 10 + 0 + 0) \bmod 10 = 2$$

$$h(27,0) = (27 \bmod 10 + 0 + 0) \bmod 10 = 7$$

$$h(36,0) = (36 \bmod 10 + 0 + 0) \bmod 10 = 6$$

$$h(24,0) = (24 \bmod 10 + 0 + 0) \bmod 10 = 4$$

$$h(92,0) = (92 \bmod 10 + 0 + 0) \bmod 10 = 2$$

Collision
C can be
assume 0 and c_2 betw 1 to 3

$$h(92, 0) = (92 \bmod 10 + 0 + 3) \bmod 10 = 5$$

$$h(101, 0) = (101 \bmod 10 + 0 + 0) \bmod 10 = 1$$

$$h(81, 0) = (81 \bmod 10 + 0 + 0) \bmod 10$$

0	
1	101
2	72
3	81
4	24
5	92
6	56
7	23
8	
9	

= 1
collision

$$h(81, 1) = (81 \bmod 10 + 0 + 3) \bmod 10$$

= 4
Collision

$$h(81, 2) = (81 \bmod 10 + 0 + 12) \bmod 10$$

= 3

(iii) Double hashing -

→ This technique uses two hashing functions h_1 and h_2 to find the address of key element.

→ Following eqn is used to find address of an element -

$$h(k, i) = (h_1(k) + i h_2(k)) \bmod m$$

where h_1 & h_2 = two hashing tables

For ex:-

Let us consider table size = $10 = M$

72, 27, 36, 24, 63, 81, 92, 101

$$\left. \begin{array}{l} h_1(k) = k \bmod 10 \\ h_2(k) = k \bmod 8 \end{array} \right\} \text{is not guaranteed}$$

h_1 can be taken

Initially, all slot occupy -1

0	-1
1	-1
2	-1
3	-1
4	-1
5	-1
6	-1
7	-1
8	-1
9	-1

$$h(72, 0) = (72 \bmod 10 + 0) \bmod 10 = 2$$

$$h(27, 0) = (27 \bmod 10 + 0) \bmod 10 = 7$$

$$h(36, 0) = (36 \bmod 10 + 0) \bmod 10 = 6$$

$$h(24, 0) = (24 \bmod 10 + 0) \bmod 10 = 4$$

$$h(63, 0) = (63 \bmod 10 + 0) \bmod 10 = 3$$

$$h(81, 0) = (81 \bmod 10 + 0) \bmod 10 = 1$$

$$h(92, 0) = (92 \bmod 10 + 0) \bmod 10 = 2$$

Collision

$$h(92, 1) = (92 \bmod 10 + 1 \cdot 92 \bmod 8) \bmod 10 = 6$$

Collision

$$h(92, 2) = (92 \bmod 10 + 2 \cdot 92 \bmod 8) \bmod 10 = 0$$

Collision

$$h(101, 0) = (101 \bmod 10 + 0) \bmod 10 = 1$$

Collision

$$h(101, 1) = (101 \bmod 10 + 1 \cdot 101 \bmod 8) \bmod 10 = 6$$

Collision

$$h(101, 2) = (101 \bmod 10 + 2 \cdot 101 \bmod 8) \bmod 10 = 1$$

Collision

$$h(101, 3) = (101 \bmod 10 + 3 \cdot 101 \bmod 8) \bmod 10 = 6$$

Collision

0	32
1	81
2	72
3	63
4	24
5	101
6	36
7	27
8	
9	

↑ address of 32

Table size is not fitting in my slot
so by taking previous 40 and changing them
to onwards 111 and occupy slot no 8

IV) Rehashing -

- when the hash table becomes nearly full the number of collision increases thereby degrading the performance of insertion and search operation.
- In such a case a better option is to create a new hash table with size double of the original hash table.
- All the entries of the original hash table are transferred to new hash table.

Let us consider hash table size 5 and hashing function as $\text{addr} = \text{element mod } 5$

∴ $\text{addr} = \text{element mod } 5$

For exp :- 31, 26, 17, 43

0	
1	31
2	26
3	17
4	43

$\text{addr of } 31 = 31 \text{ mod } 5 = 1$

$\text{addr of } 26 = 26 \text{ mod } 5 = 1$
Collision
∴ store in 1st slot

$\text{addr of } 17 = 17 \text{ mod } 5 = 2$
Collision
∴ store in 3rd slot

$\text{addr of } 43 = 43 \text{ mod } 5 = 3$
Collision
∴ store in 4th slot

∴ hash table is nearly full

∴ If we use rehashing then create another hash table having size 10

0	
1	31
2	
3	43
4	
5	
6	26
7	
8	17
9	

Elements are 31, 26, 17, 43

$\text{addr of } 31 = 31 \text{ mod } 10 = 1$

$\text{addr of } 26 = 26 \text{ mod } 10 = 6$

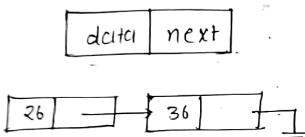
$\text{addr of } 17 = 17 \text{ mod } 10 = 7$

$\text{addr of } 43 = 43 \text{ mod } 10 = 3$

2) Chaining-

This technique uses the concept of linked list where each element is divided into two parts -
data and next pointer

For exp -



Let us consider table size 9 and hashing function
mod 10 division

Elements are 7, 24, 18, 52, 36, 54, 11, 23

Initially

0	
1	
2	
3	
4	
5	
6	
7	
8	

$$\text{addr of } 7 = 7 \bmod 9 = 7$$

$$\text{addr of } 24 = 24 \bmod 9 = 6$$

$$\text{addr of } 18 = 18 \bmod 9 = 0$$

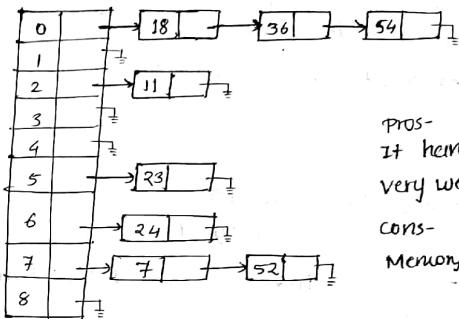
$$\text{addr of } 52 = 52 \bmod 9 = 7$$

$$\text{addr of } 36 = 36 \bmod 9 = 0$$

$$\text{addr of } 54 = 54 \bmod 9 = 0$$

$$\text{addr of } 11 = 11 \bmod 9 = 2$$

$$\text{addr of } 23 = 23 \bmod 9 = 5$$



Pros-
It handles collision very well
Cons-
Memory wastage in pointers

3) Bucket Hashing-

- This technique uses two dimensional array.
- If there is any collision then element will be stored in the same row but in the next column.

Exp -

Elements are 7, 24, 18, 52, 36, 54, 11, 23

Table size is 9x9 (no of rows should be equals to no of columns)

0	18	36	54					
1								
2	11							
3								
4								
5	23							
6	26							
7	7	52						
8								

$c_1 = 0, c_2 = 1$

→ Using linear probing and quadratic probing. Insert the following values in hash table of size 10.

28, 55, 71, 67, 11, 10, 90, 44

⇒ Linear probing

Initially, table size = 10

all slot occupy -1

0	-1
1	-1
2	-1
3	-1
4	-1
5	-1
6	-1
7	-1
8	-1
9	-1

$$h(28, 0) = (28 \bmod 10 + 0) \bmod 10 = 8$$

$$h(55, 0) = (55 \bmod 10 + 0) \bmod 10 = 5$$

$$h(71, 0) = (71 \bmod 10 + 0) \bmod 10 = 1$$

$$h(67, 0) = (67 \bmod 10 + 0) \bmod 10 = 7$$

$$h(11, 0) = (11 \bmod 10 + 0) \bmod 10 = 1$$

Collision

$$h(11, 1) = (11 \bmod 10 + 1) \bmod 10 = 2$$

$$h(10, 0) = (10 \bmod 10 + 0) \bmod 10 = 0$$

$$h(90, 0) = (90 \bmod 10 + 0) \bmod 10 = 0$$

Collision

$$h(90, 1) = (90 \bmod 10 + 1) \bmod 10 = 1$$

Collision

$$h(90, 2) = (90 \bmod 10 + 2) \bmod 10 = 2$$

Collision

$$h(90, 3) = (90 \bmod 10 + 3) \bmod 10 = 3$$

$$h(44, 0) = (44 \bmod 10 + 0) \bmod 10 = 4$$

0	10
1	71
2	11
3	90
4	44
5	55
6	67
7	28
8	
9	

Quadratic probing -

Initially table size = 10 = m

all slot occupy -1

0	-1
1	-1
2	-1
3	-1
4	-1
5	-1
6	-1
7	-1
8	-1
9	-1

$$h(28, 0) = (28 \bmod 10 + 0 + 0) \bmod 10 = 8$$

$$h(55, 0) = (55 \bmod 10 + 0 + 0) \bmod 10 = 5$$

$$h(71, 0) = (71 \bmod 10 + 0 + 0) \bmod 10 = 1$$

$$h(67, 0) = (67 \bmod 10 + 0 + 0) \bmod 10 = 7$$

$$h(11, 0) = (11 \bmod 10 + 0 + 0) \bmod 10 = 1$$

Collision

$$h(11, 1) = (11 \bmod 10 + 0 + 1) \bmod 10 = 2$$

$$h(10, 0) = (10 \bmod 10 + 0 + 0) \bmod 10 = 0$$

$$h(90, 0) = (90 \bmod 10 + 0 + 0) \bmod 10 = 0$$

Collision

$$h(90, 1) = (90 \bmod 10 + 0 + 1) \bmod 10 = 1$$

$$h(90, 2) = (90 \bmod 10 + 0 + 4) \bmod 10 = 4$$

Collision

$$h(90, 3) = (90 \bmod 10 + 0 + 9) \bmod 10 = 0$$

Collision

$$h(90, 4) = (90 \bmod 10 + 0 + 16) \bmod 10 = 6$$

Collision

Using linear probing

$$h(44, 0) = (44 \bmod 10 + 0 + 0) \bmod 10 = 4 \quad \text{collision}$$

$$h(44, 1) = (44 \bmod 10 + 0 + 1) \bmod 10 = 5 \quad \text{collision}$$

$$h(44, 2) = (44 \bmod 10 + 0 + 2) \bmod 10 = 8 \quad \text{collision}$$

$$h(44, 3) = (44 \bmod 10 + 0 + 3) \bmod 10 = 3$$

bit

Radix Sort-

→ This sorting technique requires 10 buckets (bucket 0 to bucket 9) and place the numbers into buckets based on digit.

→ Initially, unit place digit of all the numbers are considered and stored into buckets then 10th place, 100th place and so on.

Ex:- $x[] = 456, 330, 12, 9, 785, 892, 325, 53, 927$

Since largest number is 927 that has 3 digits

∴ These sorting requires 3 passes

pass 1-

buckets	0	1	2	3	4	5	6	7	8	9
observe the last digits and fill accordingly in buckets	330		012	053		785	456	927		009

$x[] = \{ 330, 012, 892, 053, 785, 325, 456, 927, 009 \}$

Pass 2

buckets	0	1	2	3	4	5	6	7	8	9
	009	012	325	330		053			785	892

$x[] = \{ 009, 012, 325, 927, 330, 053, 456, 785, 892 \}$

Pass 3

buckets	0	1	2	3	4	5	6	7	8	9
	009			325	456			785	892	927

$x[] = \{ 009, 012, 053, 325, 330, 456, 785, 892, 927 \}$

Algorithm-

Step 1 - start

Step 2 - Enter the number of elements i.e n

Step 3 - Enter the elements and store into array x

Step 4 - Find the largest number from array x and identify the number of digits

Step 5 - Find the number of passes i.e passes = no. of digits

Step 6 - Store the numbers into buckets i.e

```

div = 1
for (i=1; i <= passes; i++) // no of passes
{
    initialize all buckets to null i-1
    for (j=0; j <= 9; j++)
        bucket [j] = null
    for (k=0; k <= n-1; k++)
    {
        d = (x[k] / div) % 10
        copy x[k] to bucket [d]
    }
    copy elements from bucket to array x []
    div = div * 10
}

```

Step 7 - Display sorted elements from array x.

Step 8 - Stop

Time Complexity for radix sort -

It requires two loops - 1st loop requires no. of passes
i.e m^a
2nd loop is used to store elements into buckets i.e n

∴ Time complexity is $O(mn)$.

Heap sort -

→ This sorting technique consists of two steps - build heap & heapify

Build Heap -

→ Binary tree is created.

→ There are two ways - i) max heap
parent element is greater than its child elements
if not then perform swapping.

ii) min heap

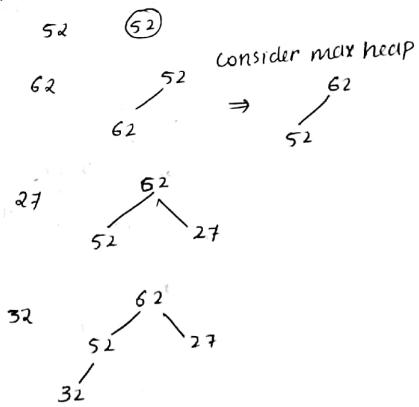
parent element is less than its child elements
if not then perform swapping.

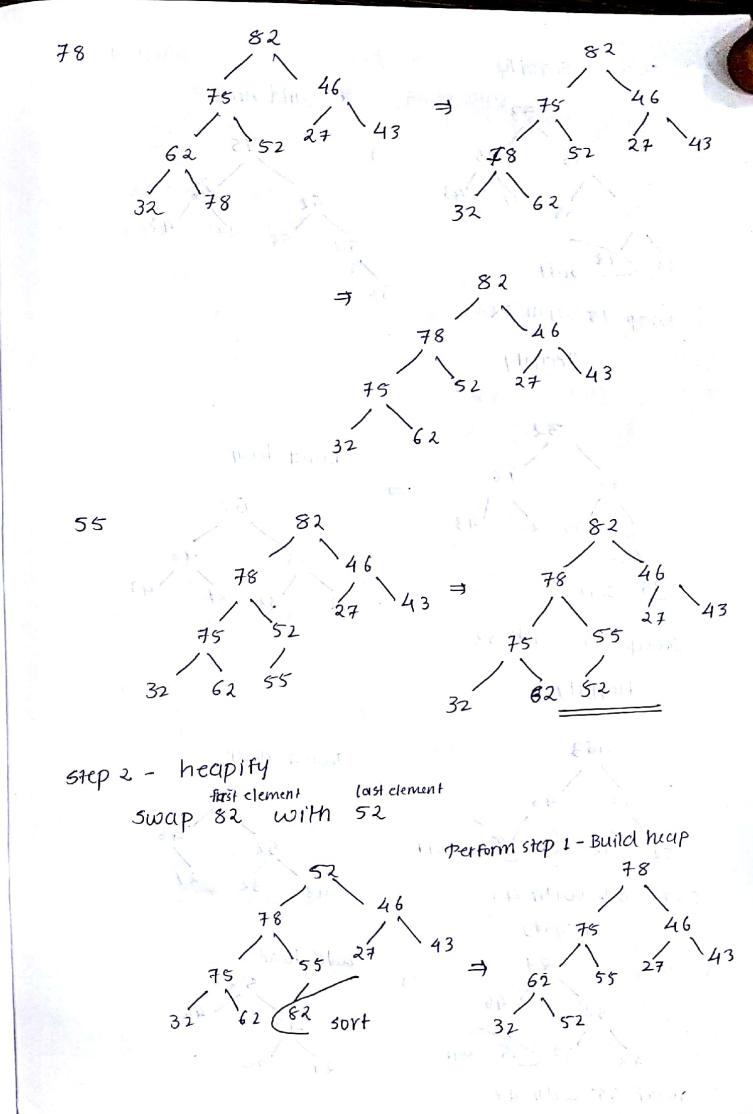
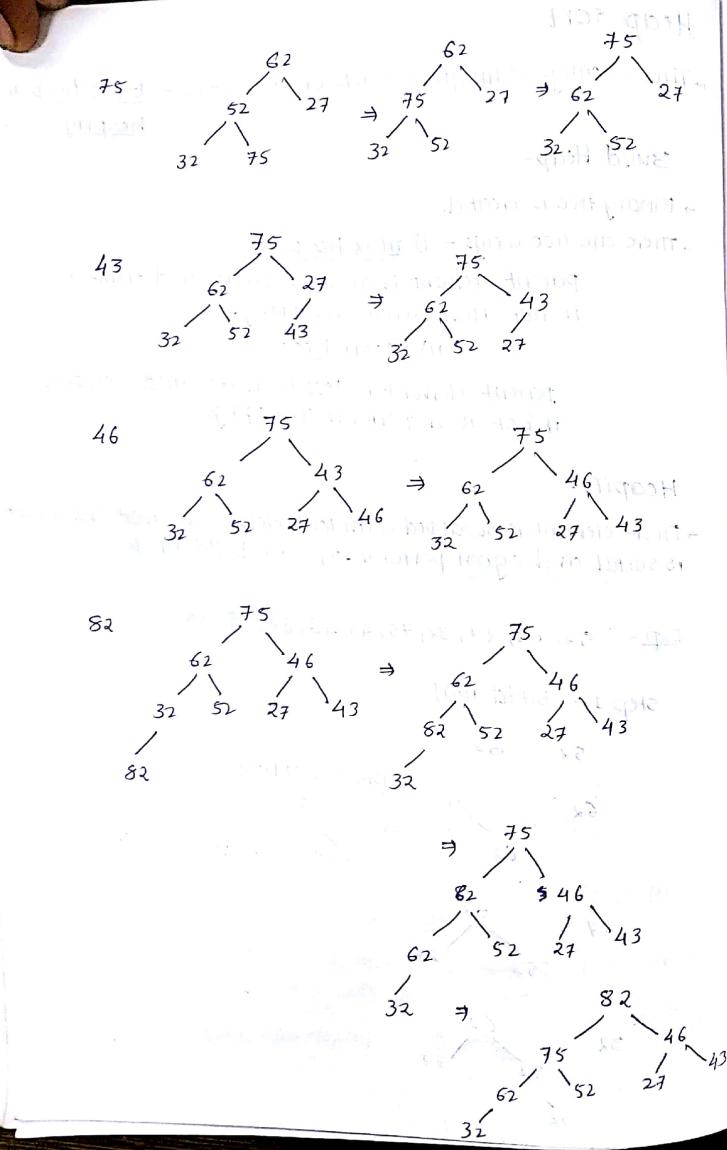
Heapify -

→ First element is swapped with last element so that one element is sorted and again perform step 1 i.e build heap.

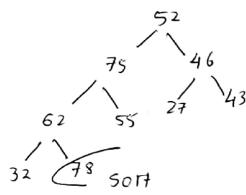
Ex - 52, 62, 27, 32, 75, 43, 46, 82, 78, 55

Step 1 - Build heap



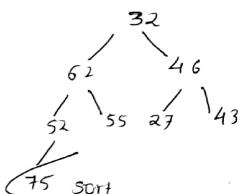


Step 2 - heapify



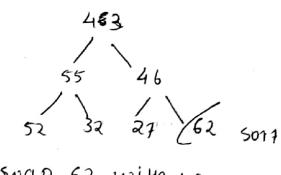
swap 78 with 52

Heapify



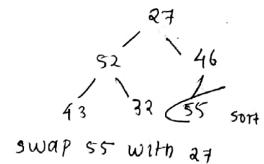
swap 75 with 32

Heapify



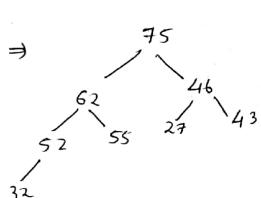
swap 62 with 43

Heapify

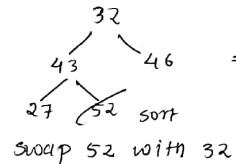


swap 55 with 27

Build heap

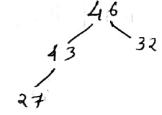


Heapify

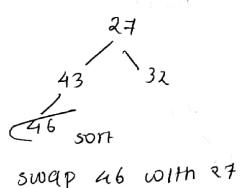


swap 52 with 32

Build heap

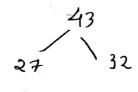


Heapify

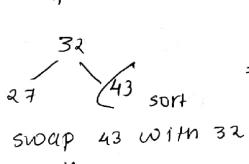


swap 46 with 27

Build heap



Heapify

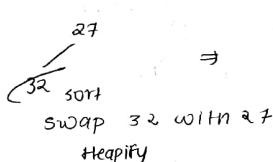


swap 43 with 32

Build heap



Heapify



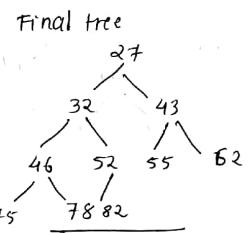
swap 32 with 27

Build heap



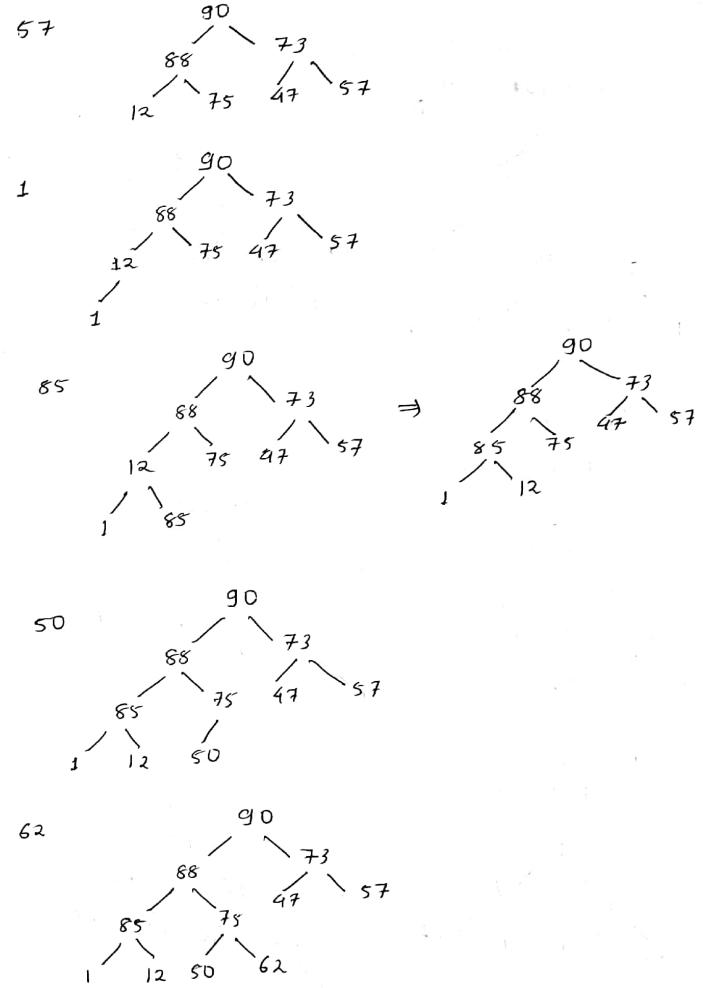
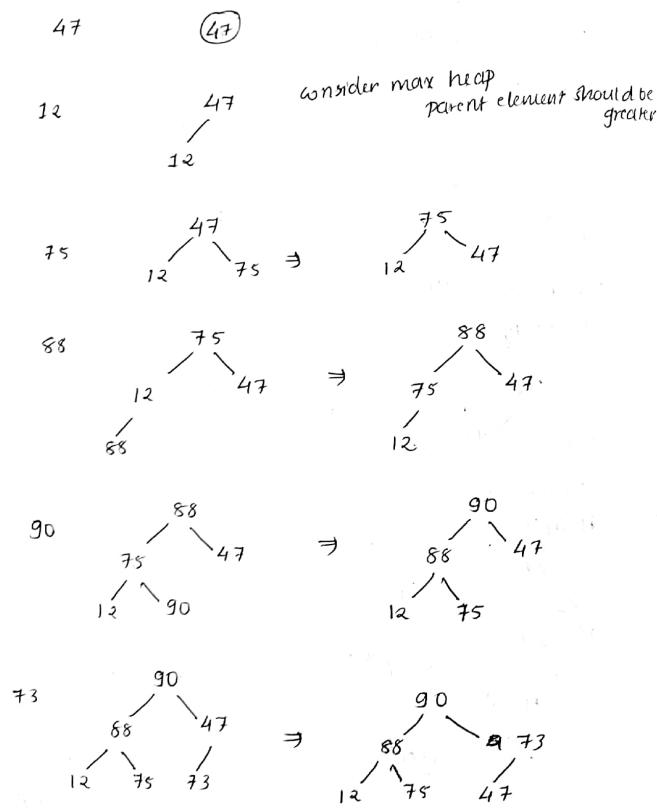
Heapify

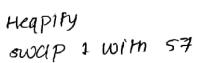
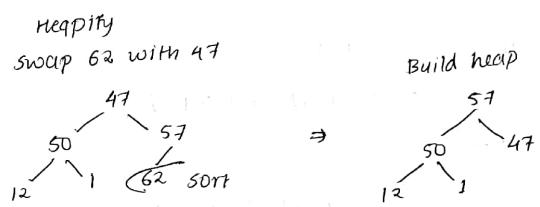
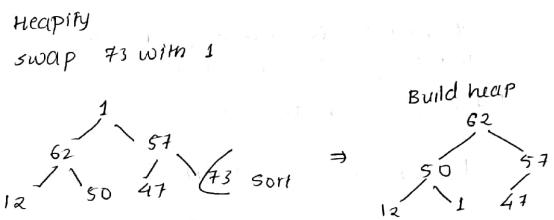
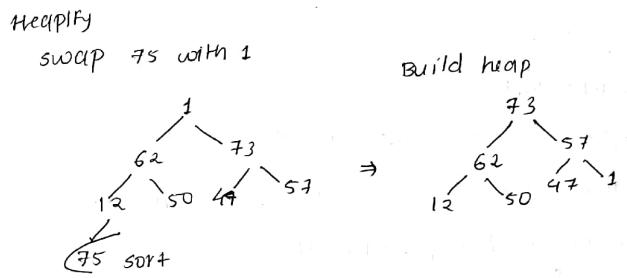
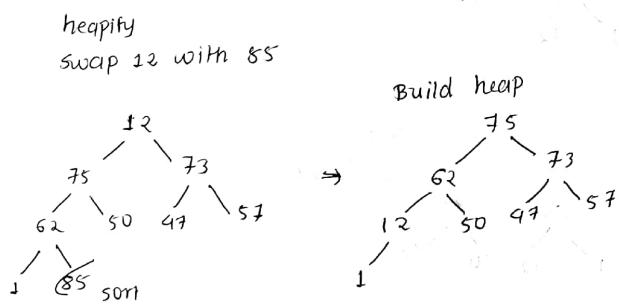
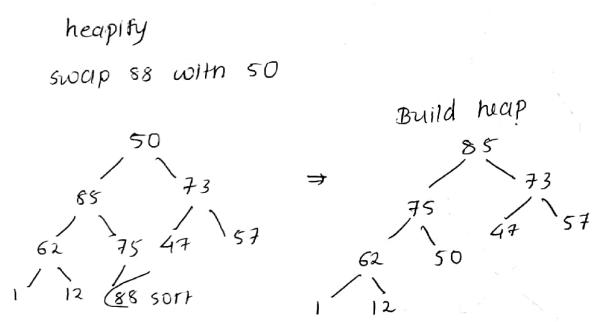
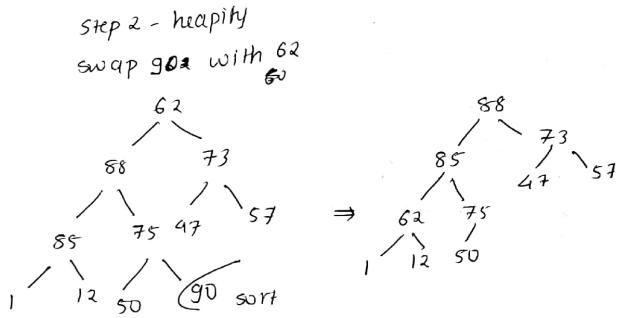
27 sorted

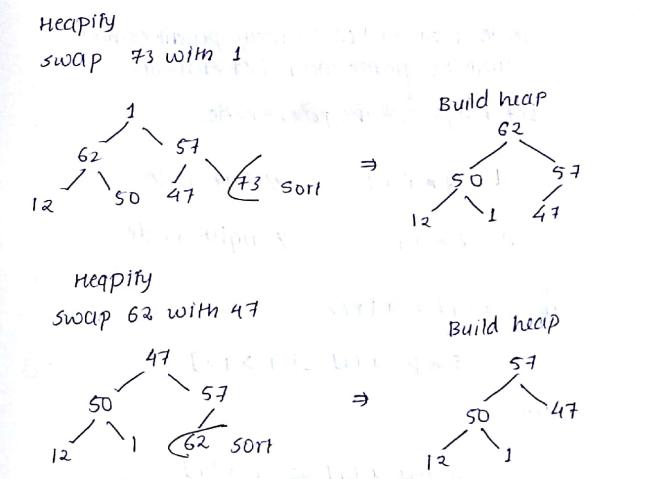
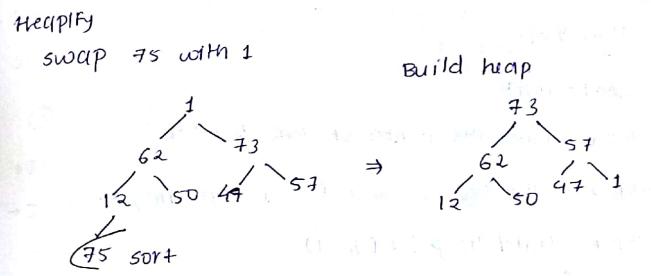
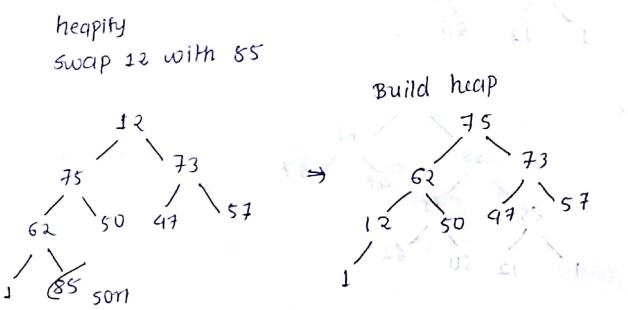
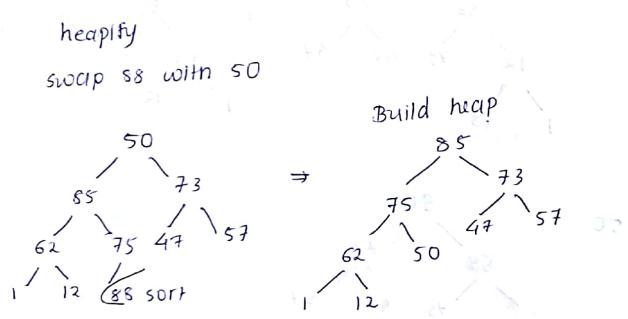
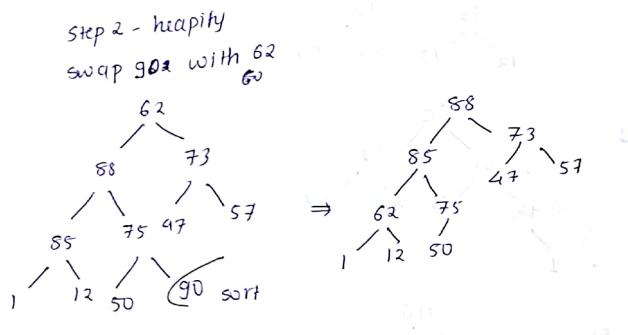


→ 47, 12, 75, 88, 90, 73, 57, 1, 85, 50, 62

Step 1 - build heap







Heapify
swap 1 with 57

Algorithm:-

Step 1 - Start
Step 2 - Enter the number of elements i.e. n

Step 3 - Enter the elements and store into array x.
Step 4 - Build heap (x[], n)

{

 Create a binary tree so that parent element
 must be greater than child elements

 Let i represent the parent node

 l = 2 * i + 1 // left node

 r = 2 * i + 2 // right node

 if (x[l] > x[r])

 swap x[i] with x[l]

 else

 swap x[i] with x[r]

}

Step 5 - Heapify (x[], n)

{ for (i = n-1; i >= 1; i--)
 swap x[0] with x[i]

 Build heap (x[], i)

}

Step 6 - Display sorted elements from array x.

Step 7 - Stop.

Quick Sort-

→ It is also called as partition exchange sort.
→ It follows divide and conquer strategy that consists
of three steps - i) divide
 ii) conquer
 iii) combine

i) Divide -

→ An array of elements are divided into partitions.

ii) Conquer -

→ Elements from each partition are separately sorted.

iii) Combine -

→ sorted partitions are combined to get final sorted elements

Ex - Pass 1 down quick(x, 0, 7)
 partition(x, 0, 7)
 (32) 25 62 18 57 15 27 37
 pivot

 (32) 25 down 18 57 15 up 27 37

∴ down < up (2 < 6)

x[down] ←→ x[up]

 25 ←→ 27
 (32) down 18 57 15 up 62 37

(32) 25 27 18 ^{down} 57 ^{up} 15 62 37
PIVOT

$\therefore \text{down} < \text{up}$ ($4 < 5$)

$57 \longleftrightarrow 15$

(32) 25 27 18 ^{down} 15 ^{*} 57 ^{up} 62 37

32 25 27 18 ^{up} 15 ^{down} 57 62 37

$\therefore \text{down} > \text{up}$ ($5 > 4$)

$x[\text{up}] \longleftrightarrow \text{pivot}$

$15 \longleftrightarrow 32$

15 25 27 18 32 57 62 37
Sort
 $j=4$

Pass 2

partition ($x, 0, j-1$) $x, 0, 3$

partition ($x, j+1, 7$) $x, 5, 7$

15 25 27 18
pivot

15 ^{up} 25 ^{down} 27 ^{up} 18

$\therefore \text{down} > \text{up}$ ($1 > 0$)

$x[\text{up}] \longleftrightarrow \text{pivot}$

$15 \longleftrightarrow 15$

15 25 27 18
sort
 $j=0$

partition ($x, 0, j-1$) $x, 0, -1$

partition ($x, j+1, 3$) $x, 1, 3$

$P(x, 0, -1)$ sort

$P(x, 1, 3)$

↓

$P(x, 5, 7)$

57 62 37
pivot

57 down 62 up 37

$\therefore \text{down} < \text{up}$ ($6 < 7$)

sort

62 \longleftrightarrow 37

57 down 62 up

57 up 37 down 62

down > up ($7 > 6$)

$x[\text{up}] \longleftrightarrow \text{pivot}$

37 \longleftrightarrow 57

57 37 62
sort

$\begin{matrix} 5 \\ 37 \end{matrix}$ $\begin{matrix} 6 \\ 57 \end{matrix}$ $\begin{matrix} 7 \\ 62 \end{matrix}$
sort
 $j=6$

partition ($x, 05, j-1)$ x_{10}, x_{15}, x_5

partition ($x, j+1, 7)$ $x, 7, 7$

$\left. \begin{matrix} P(x, 5, 5) \\ P(x, 7, 7) \end{matrix} \right\}$ sorted

PASS 3

$P(x, 1, 3)$
down $\begin{matrix} 1 \\ 25 \end{matrix}$ $\begin{matrix} 2 \\ 27 \end{matrix}$ $\begin{matrix} 3 \\ 18 \end{matrix}$ up
pivot

$\begin{matrix} 25 \\ 27 \end{matrix}$ down $\begin{matrix} 1 \\ 18 \end{matrix}$ up

\therefore down < up ($2 < 3$)

$27 \longleftrightarrow 18$
down $\begin{matrix} 1 \\ 25 \end{matrix}$ $\begin{matrix} 2 \\ 18 \end{matrix}$ $\begin{matrix} 3 \\ 27 \end{matrix}$ up
pivot

$\begin{matrix} 25 \\ 18 \end{matrix}$ up $\begin{matrix} 1 \\ 27 \end{matrix}$ down

\therefore down > up ($3 > 2$)

$18 \longleftrightarrow 25$

$\begin{matrix} 18 \\ 25 \\ 27 \end{matrix}$
sort
 $j=2$

$\left. \begin{matrix} P(x, 1, 1) \\ P(x, 3, 3) \end{matrix} \right\}$ sorted

\therefore sorted elements are ~~32~~

$\rightarrow 15 \ 18 \ 25 \ 27 \ 32 \ 37 \ 57 \ 62$

X $\begin{matrix} 54 \\ 26 \\ 93 \\ 17 \\ 77 \\ 31 \\ 44 \\ 55 \\ 20 \end{matrix}$
Pass 1 down
pivot $\begin{matrix} 54 \\ 26 \\ 93 \\ 17 \\ 77 \\ 31 \\ 44 \\ 55 \\ 20 \end{matrix}$ up

$\begin{matrix} 54 \\ 26 \\ 93 \\ 17 \\ 77 \\ 31 \\ 44 \\ 55 \\ 20 \end{matrix}$ down up
pivot $\begin{matrix} 54 \\ 26 \\ 93 \\ 17 \\ 77 \\ 31 \\ 44 \\ 55 \\ 20 \end{matrix}$ up

\therefore down < up ($2 < 8$)

$93 \longleftrightarrow 44 \ 20$

$\begin{matrix} 54 \\ 26 \\ 44 \\ 20 \end{matrix}$ down up
pivot $\begin{matrix} 17 \\ 77 \\ 31 \\ 44 \\ 55 \\ 20 \\ 93 \end{matrix}$ up

$\begin{matrix} 54 \\ 26 \\ 44 \\ 17 \\ 77 \\ 31 \\ 44 \\ 55 \\ 20 \\ 93 \end{matrix}$ down up
pivot $\begin{matrix} 54 \\ 26 \\ 44 \\ 17 \\ 77 \\ 31 \\ 44 \\ 55 \\ 20 \\ 93 \end{matrix}$ up

\therefore down < up ($4 < 5$)

$77 \longleftrightarrow 31 \ 44$

$\textcircled{54} \quad 26 \quad 44 \quad 17 \quad 31 \quad 20 \quad 44 \quad 33 \quad \text{down} \quad \text{up} \quad 77 \quad 55 \quad 93 \quad 88$
 $\textcircled{54} \quad 26 \quad 44 \quad 17 \quad 31 \quad 20 \quad 44 \quad 33 \quad \text{down} \quad 77 \quad 55 \quad 93 \quad 88$

down > up ($88 > 33$)

$x[\text{up}] \leftrightarrow \text{pivot}$

$17 \leftrightarrow 54$

$\underbrace{17}_{\text{down}} \quad \underbrace{26}_{\text{up}} \quad \underbrace{44}_{\text{up}} \quad \underbrace{\textcircled{54}}_{\text{sort}} \quad \underbrace{31}_{\text{up}} \quad \underbrace{77}_{\text{up}} \quad \underbrace{93}_{\text{up}} \quad \underbrace{55}_{\text{up}} \quad \underbrace{20}_{\text{up}}$
 $j=3$

$P(x, 0, j-1) \quad P(x, 0, 2)$

$X \quad P(x, j+1, 8) \quad P(x, 4, 8)$

$\rightarrow \textcircled{54} \quad \begin{matrix} \text{down} \\ \text{pivot} \end{matrix} \quad 26 \quad 93 \quad 17 \quad \text{Quick } C(x, 0, 8) \quad \text{Partition } (x, 0, 5) \quad 77 \quad 31 \quad 44 \quad 55 \quad 20$
 $\textcircled{54} \quad 26 \quad 93 \quad 17 \quad 77 \quad 31 \quad 44 \quad 55 \quad 20 \quad \text{up}$

$\therefore \text{down} < \text{up}$

$93 \leftrightarrow 20$

$\textcircled{54} \quad 26 \quad 20 \quad 17 \quad 77 \quad 31 \quad 44 \quad 55 \quad 93 \quad \text{up}$
 $54 \quad 26 \quad 20 \quad 17 \quad 77 \quad 31 \quad 44 \quad 55 \quad 93 \quad \text{down}$
 $\therefore \text{down} < \text{up}$
 $77 \mapsto \dots$

$\textcircled{54} \quad 26 \quad 20 \quad 17 \quad 44 \quad \text{down} \quad 31 \quad \text{up} \quad 77 \quad 55 \quad 93$
 $\textcircled{54} \quad 26 \quad 20 \quad 17 \quad 44 \quad \text{up} \quad \text{down} \quad 31 \quad 77 \quad 55 \quad 93$

pivot

$\therefore \text{down} > \text{up}$

$x[\text{up}] \leftrightarrow \text{pivot}$

$31 \leftrightarrow 54$

$\textcircled{31} \quad 26 \quad 20 \quad 17 \quad 44 \quad \underline{\textcircled{54}} \quad 77 \quad 55 \quad 93$
 $\text{sort} \quad j=5$

$P(x, 0, j-1) \quad P(x, 0, 4)$
 $P(x, 0, j+1, 8) \quad P(x, 6, 8)$

PASS 2

$\textcircled{31} \quad \begin{matrix} \text{down} \\ \text{pivot} \end{matrix} \quad 26 \quad 20 \quad 17 \quad \text{up} \quad 44$

$\textcircled{31} \quad 26 \quad 20 \quad 17 \quad \text{up} \quad \text{down} \quad 44$

$\therefore \text{down} > \text{up}$

$17 \leftrightarrow 31$

$\textcircled{17} \quad 26 \quad 20 \quad \underline{\textcircled{31}} \quad 44$
 $\text{sort} \quad j=3$
 $P(x, 0, j-1) \quad P(x, 0, 2)$
 $P(x, j+1, 4) \quad P(x, 4, 4) - \text{sort}$

$P(x, 6, 8)$
 down up
 77 55 93
 pivot
 (77) 55 93
 down up down
 down > up
 55 \longleftrightarrow 77

55 77 93
 sort
 $j = 7$

$P(x, 6, 6)$ } sorted
 $P(x, 8, 8)$

pass 3

$P(x, 0, 2)$
 down 2¹ 2⁰ 2^{up}
 (17) 26 20
 pivot
 up down 2⁰
 17 26 20
 down > up
 17 \leftrightarrow 17

2⁰ 1¹ 2⁰ 1¹ 2¹ 2⁰
 sort
 $j = 1$
 $P(x, 0, 2)$ } sorted
 $P(x, 0, 2)$ } sorted

pass 4 $P(x, 1, 2)$

down 2¹ 2⁰ 2^{up}
 26 20
 P(x, 0, -1) - sort
 $j = 0$
pass 4 $P(x, 1, 2)$
 down 2¹ 2⁰ 2^{up}
 20 26 20 26

summary -

- First element pivot
- First element index down, last index up
- Increment down and
Find greater element than pivot
- Decrement up and find lower element than pivot
- If down < up then swap $x[down] \longleftrightarrow x[up]$ and procedure is continued
- If down > up then swap $x[up] \longleftrightarrow$ pivot elements
pivot element gets sorted at position $\underline{\underline{j}}$
- call partition ($x, lb, j-1$)
partition ($x, j+1, ub$)

Algorithm -

Step 1 - start

Step 2 - Enter the number of elements i.e n .

Step 3 - Enter the elements and store into array x .

Step 4 - quick (x, lb, up)
 $\{$

if ($lb < ub$)
 $j = \text{partition}(x, lb, ub)$

quick ($x, lb, j-1$)

quick ($x, j+1, ub$)

$\}$

$\frac{5}{37}$ $\frac{6}{57}$ $\frac{7}{62}$
sort
 $j=6$

partition ($x, 85, j-1$) $x, 10, 5, 5$

partition ($x, j+1, 7$) $x, 7, 7$

$P(x, 5, 5)$ }
 $P(x, 7, 7)$ } sorted

PASS 3

$P(x, 1, 3)$
down up
 $\frac{1}{(25)}$ $\frac{2}{27}$ $\frac{3}{18}$
pivot

$\frac{1}{25}$ down up
27 18

\therefore down < up ($2 < 3$)

$27 \leftrightarrow 18$
down up
 $\frac{1}{(25)}$ $\frac{2}{18}$ $\frac{3}{27}$

$\frac{1}{25}$ up down
18 27

\therefore down > up ($3 > 2$)

$18 \leftrightarrow 25$

$\frac{1}{18}$ $\frac{2}{25}$ $\frac{3}{27}$
sort
 $j=2$

$P(x, 1, 1)$ }
 $P(x, 3, 3)$ } sorted

\therefore sorted elements are 32

$\rightarrow 15 \ 18 \ 25 \ 27 \ 32 \ 37 \ 57 \ 62$

$\rightarrow 54 \ 26 \ 93 \ 17 \ 77 \ 31 \ 44 \ 55 \ 20$

X Pass 1 down up
 $(\frac{5}{4})$ 26 $\frac{1}{93}$ $\frac{2}{17}$ $\frac{3}{77}$ $\frac{4}{31}$ $\frac{5}{44}$ $\frac{6}{55}$ $\frac{7}{20}$
pivot

$(\frac{5}{4})$ down up
26 $\frac{1}{93}$ $\frac{2}{17}$ $\frac{3}{77}$ $\frac{4}{31}$ $\frac{5}{44}$ $\frac{6}{55}$ $\frac{7}{20}$

\therefore down < up ($2 < 8$)

$93 \leftrightarrow 44 \ 20$

$(\frac{5}{4})$ down up
pivot 26 $\frac{1}{44}$ $\frac{2}{20}$ $\frac{3}{17}$ $\frac{4}{77}$ $\frac{5}{31}$ $\frac{6}{44}$ $\frac{7}{55}$ $\frac{8}{20}$ $\frac{9}{93}$

$\frac{5}{4}$ 26 $\frac{1}{44}$ $\frac{2}{17}$ down up
up $\frac{3}{77}$ $\frac{4}{31}$ $\frac{5}{44}$ $\frac{6}{55}$ $\frac{7}{20}$ $\frac{8}{93}$ $\frac{9}{40}$

\therefore down < up ($4 < 5$)

$77 \leftrightarrow 31 \ 44$

(54) 26 20 44 17 31 44 down up 31 77 55 93

(54) 26 44 17 31 down 77 93 55 20

down > up (8 > 3)

$x[\text{up}] \leftrightarrow \text{pivot}$

17 \leftrightarrow 54

17 26 44 54 31 77 93 55 20
sort j=3

$P(x, 0, j-1)$

$P(x, 0, 2)$

X $P(x, j+1, 8)$

$P(x, 4, 8)$

↳ (54) down Quick ($x, 0, 8$) Partition ($x, 0, 8$)
pivot 26 93 17 77 31 44 55 20

(54) 26 down 93 17 77 31 44 55 up 20

\therefore down < up

93 \leftrightarrow 20

(54) 26 down 20 17 77 31 44 up 55 93

54 26 20 17 down 77 31 44 up 55 93
 \therefore down < up
77 \leftrightarrow 44

(54) 26 20 17 down 44 up 31 77 55 93

(54) 26 20 17 44 up 31 down 77 55 93

\therefore down > up

$x[\text{up}] \leftrightarrow \text{pivot}$

31 \leftrightarrow 54

31 26 20 17 44 54 77 55 93
sort j=5

$P(x, 0, j-1)$

$P(x, 0, 4)$

$P(x, 4, j+1, 8)$

$P(x, 6, 8)$

PASS 2

(31) down 26 20 17 44 up
pivot

(31) 26 20 17 up down 44

\therefore down > up

17 \leftrightarrow 31

17 26 20 31 44
sort j=3
 $P(x, 0, j-1)$ $P(x, 0, 2)$
 $P(x, j+1, 4)$ $P(x, 4, 4) - \text{sort}$

$P(x, 6, 8)$
 down up
 6 7 8
 77 55 93
 pivot
 (77) 55 93
 down > up
 55 \longleftrightarrow 77

55 77 8
sort
 j=7

$P(x, 6, 6)$ } sorted
 $P(x, 8, 8)$

PASS 3

$P(x, 0, 2)$
 down up
 17 26 20
 pivot
 17 26 20
 down > up
 17 \longleftrightarrow 17

26 17 20
sort
 j=1
 P(x, 0, 2)
 P(x, 0, 2) - sort

PASS 4 $P(x, 1, 2)$

down up

Summary -

- First element pivot
- First element index down, last index up
- Increment down and find greater element than pivot
- Decrement up and find lower element than pivot
- If down < up then swap $x[\text{down}] \longleftrightarrow x[\text{up}]$ and procedure is continued
- If down \geq up then swap $x[\text{up}] \longleftrightarrow$ pivot elements
pivot element gets sorted at position $\underline{\underline{j}}$

call partition ($x, lb, j-1$)
 partition ($x, j+1, ub$)

Algorithm -

Step 1 - start

Step 2 - Enter the number of elements i.e. n.

Step 3 - Enter the elements and store into array x.

Step 4 - quick (x, lb, up) initially

if ($lb < ub$)
 j = partition (x, lb, ub)

quick ($x, lb, j-1$)

quick ($x, j+1, ub$)

}

```

Step 5 - partition (x, lb, ub)
{
    down = lb
    up = ub
    pivot = x[lb] or x[down]

    while (down < up)
    {
        while (x[down] < pivot)
            down++

        while (x[up] > pivot)
            up--

        if (down < up)
        {
            int temp = x[down]
            x[down] = x[up]
            x[up] = x[down] temp
        }
    }

    temp = x[up]
    x[up] = pivot
    pivot = temp
}
}

Step 6 - sorted display elements from array x
Step 7 - Stop

```

Bubble sort -

It is also called ~~extended~~^{exchange} sort,
in which one element is sorted in each pass.
Hence to sort n elements, it requires $n-1$ passes.

Example -

25 67 12 5 4 62 52

Pass-1

25 12 5 4 62 52 67
sort

Pass-2

25 12 5 4 25 52 62 67
sort

Pass-3

5 4 12 25 52 62 67
sort

Pass-4

4 5 12 25 52 62 67
sort

Pass-5

4 5 12 25 52 62 67
sorted

Algorithm-

Step 1 - start

Step 2 - Enter the number of elements i.e.

Step 3 - Enter the elements and store into array x.

Step 4 - Bubble ($\alpha[], n$)

5

```
flag = 1;
```

```
for(i=0; i<n && flag == 1; i++)
```

3

`flag = 0;`

```
for (j=0; j < n-i; j++)
```

5

if ($x[j] > x[j+1]$)

۱

flag = 1

7

Step 5 - Display sorted elements from array x.

Step 6 - STOP

Merge sort -

→ This sorting follows divide and conquer strategy that consists of three steps -
1) divide
2) conquer
3) combine

i) divide -

→ An array of elements are divided into partitions.

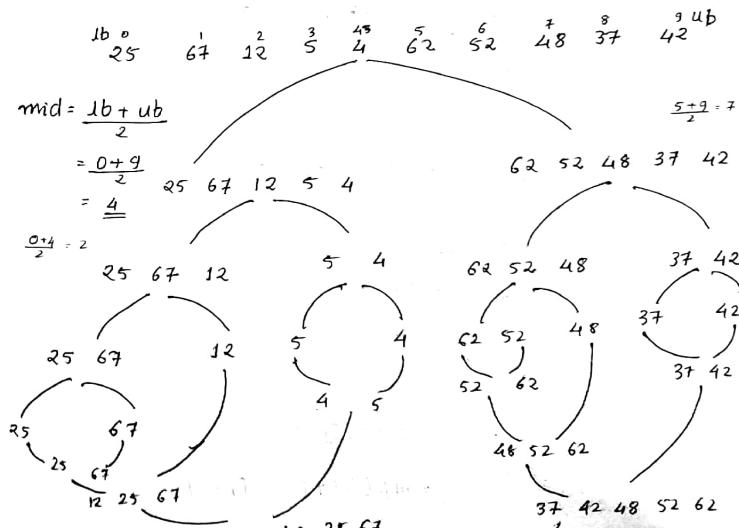
i) Conquer-

→ Elements from each partition are separately sorted.

iii) combine-

→ sorted partitions are combined to get final sorted elements.

Example -



Algorithm-

Step 1 - start

Step 2 - Enter the number of elements i.e. n.

Step 3 - Enter the elements and store into array x.

Step 4 - mergesort (x[], lb, ub)

{

if (lb < ub)

{

$$[mid = \frac{lb + ub}{2}] \quad mid = (lb + ub)/2;$$

mergesort (x, lb, mid);

mergesort (x, mid+1, ub);

merge (x, lb, mid, ub);

}

}

merge (x[], lb1, ub1, ub2)

{

i = lb1;

j = ub1 + 1;

k = 0;

while (i <= ub1 && j <= ub2)

{

if (x[i] < x[j])

temp[k++] = x[i++];

else

temp[k++] = x[j++]

}

while (i <= ub1) // remaining elements

temp[k++] = x[i++]

while (j <= ub2) // remaining elements

temp[k++] = x[j++]

copy elements from temp [] to x[]

i.e. $x[] \leftarrow temp[]$

Step 5 - Display sorted elements from array x

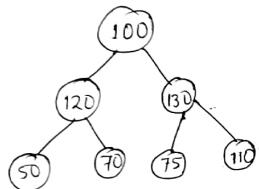
Step 6 - Stop.

Tree and Graph

Binary Tree-

→ It is a type of tree in which every node have atmost two sub trees i.e left sub tree and right sub tree.

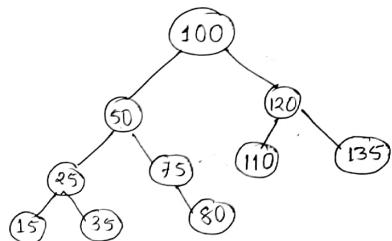
Expt -



Binary search Tree (BST)-

→ It is a binary tree i.e every node have atmost two sub trees i.e left and right sub tree but left sub tree data must be less than parent node and right sub tree data must be greater than or equal to parent node.

Expt * 100, 50, 120, 25, 75, 35, 110, 135, 15, 80



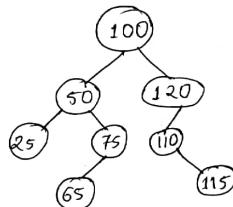
Representation of Binary Tree -

There are two ways - i) Array representation
ii) Linked list representation

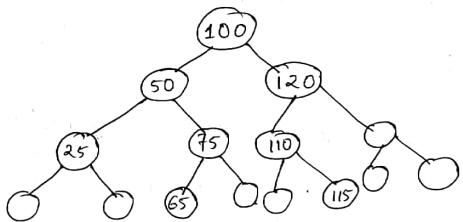
i) Array representation -

First, convert the given binary tree into complete binary tree i.e every node must have either 0 or 2 subtrees.

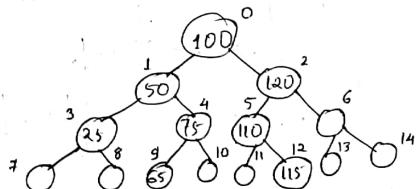
for expt -



→ complete ^{binary} tree can be drawn as follows -



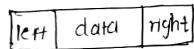
→ Assign index from root node



$\rightarrow x[] = [100 \ 50 \ 120 \ 25 \ 75 \ 110 \ 10 \ 10 \ 10 \ 65 \ 10 \ 10 \ 115 \ 10 \ 10]$

2) Linked list representation-

Every node is divided into three parts - i) data
ii) left pointer and
iii) right pointer

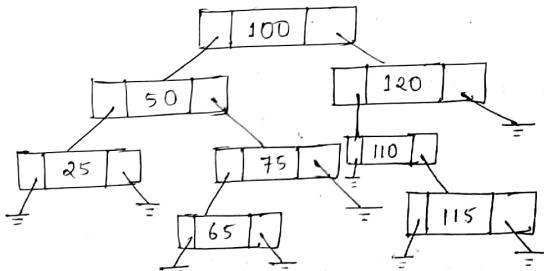


→ Structure representation of a node -

```
struct node
{
    int data;
    struct node *left, *right;
};
```

} self rep referential structure

→ Linked list representation for above binary tree can be drawn as follows-



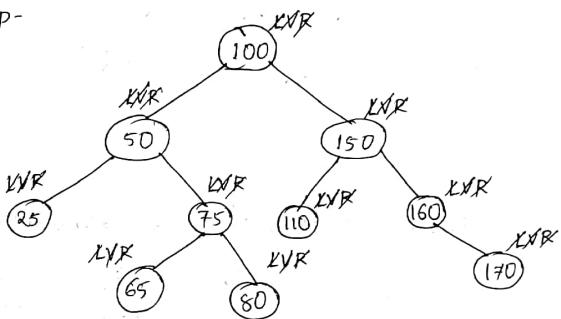
→ If there are n elements then number of null pointers are n+1, hence memory waste to represent binary tree by using linked list.

Binary Tree Traversal -

i) Inorder Traversal -

It follows LVR policy i.e Left Visit Right

Ex:-



∴ Inorder traversal is : 25, 50, 65, 75, 80, 100, 110, 150, 160, 170

Implementation -

→ Inorder (struct node * P)

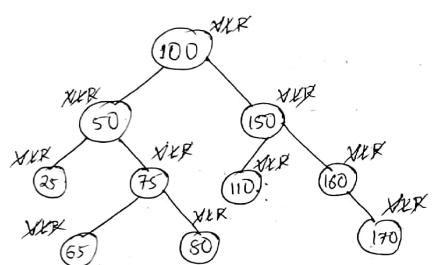
{

```
if (P != null)
{
    inorder (P->left)
    print (P->data)
    inorder (P->right)
}
```

ii) Preorder Traversal-

It follows VLR policy i.e visit left right

Ex:-



Preorder Traversal is : 100, 50, 25, 75, 65, 80, 150, 110, 160, 170

Implementation-

```

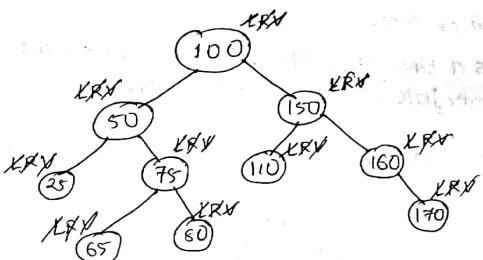
→ Preorder (struct node * P)
{
    if (P == null)
    {
        print (P → data)
        preorder (P → left)
        preorder (P → right)
    }
}
  
```

root node always comes first

Postorder Traversal-

It follows LRV policy i.e Left Right Visit

Ex:-



Postorder Traversal is : 25, 65, 80, 75, 50, 110, 170, 160, 150, 100

root node always comes last

Implementation -

```

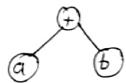
→ postorder (struct node * P)
{
    if (P == null)
    {
        postorder (P → left)
        postorder (P → right)
        print (P → data)
    }
}
  
```

Expression tree -

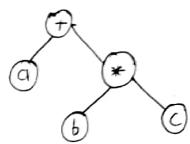
- This tree is used to represent mathematical expression in the form of tree.
- It is a binary tree whose leaf nodes represent operands and intermediate node represent operators.

EXP -

i) $a + b$

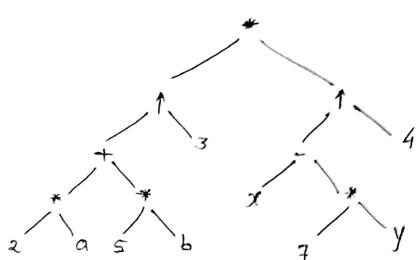


ii) $a + b * c$



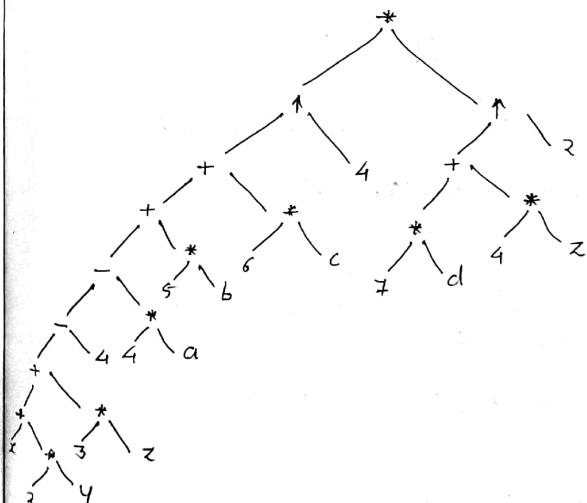
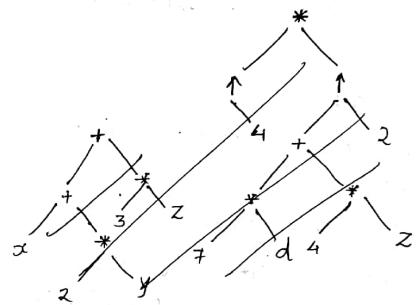
iii) $(2a + 5b)^3 (x - 7y)^4$

$$=((2 * a + 5 * b) \uparrow 3) * ((x - 7 * y) \uparrow 4)$$



IV) $(x + 2y + 3z - 4 - 4a + 5b + 6c)^4 (7d + 4z)^2$

$$=((x + 2 * y + 3 * z - 4 - 4 * a + 5 * b + 6 * c) \uparrow 4) * ((7 * d + 4 * z) \uparrow 2)$$



→ Construct a binary tree for the following traversal.

Inorder: 25, 50, 60, 75, 80, 100, 110, 115, 120, 125

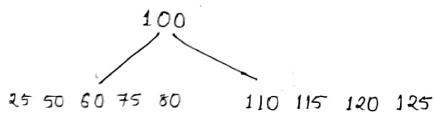
Preorder: 100, 50, 25, 75, 60, 80, 120, 110, 115, 125

100

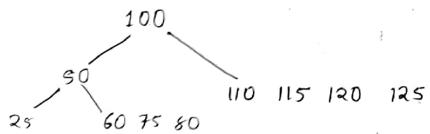
→ In preorder traversal, root node always comes first
∴ 100 is the root node

Find these root node in given inorder traversal.

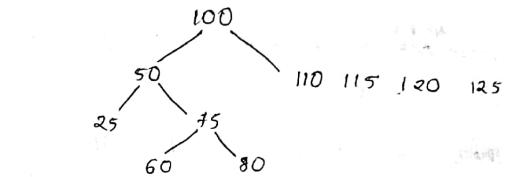
Nodes which are present on the left side of 100 are attached as a left sub tree and the nodes which are present on the right side of 100 are attached as the right side sub tree.



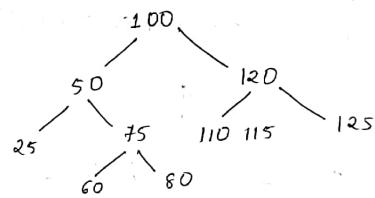
Next node in preorder traversal is 50



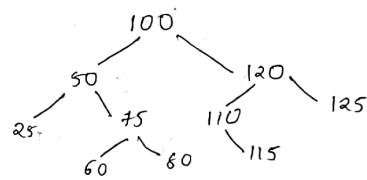
Next node is 25 (already arranged) ∴ its 75



Next preorder 60, 80 are arranged ∴ its 120



Next preorder is 110



Next preorder 115, 125 are arranged

→ Construct a binary tree for following traversal

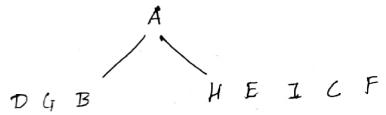
Inorder- D, G, B, A, H, E, I, C, F

post order- G, D, B, H, I, E, F, C, A

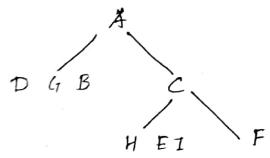
→ In post order traversal; last element root node always comes last
 $\therefore A$ be the root node

Find these root node in inorder traversal

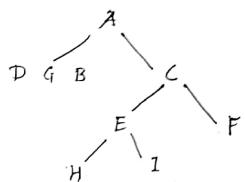
Nodes which are present on left side of A are attached as left sub tree and the nodes which are present on right side of A are attached as the right sub tree



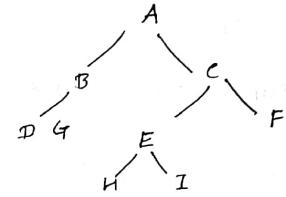
Next inorder node in postorder traversal is C



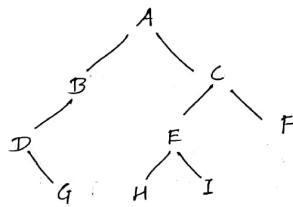
Next node in postorder traversal is F (its arranged)
 \therefore it's E



Next node in postorder traversal is B
 \therefore its E



Next post order traversal is E D



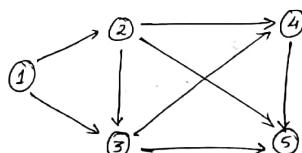
Graph-

→ It is the a collection of vertices and edges denoted by
 $G = (V, E)$, where
 V = set of vertices, E = set of edges

i) Directed Graph:-

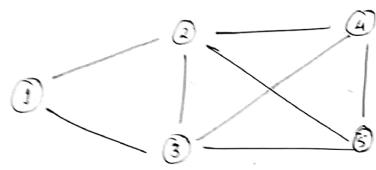
Flow of direction can be consider in one way.

Expt-



iii) Undirected or bidirected graph:-

Flow of direction can be consider in two way.

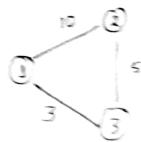


Minimum spanning Tree (MST)-

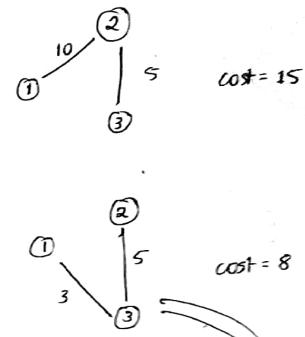
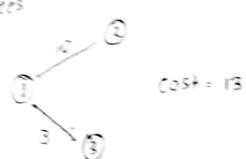
It is a subset of graph in which every vertex is reachable from other vertices but cycle should not be formed.

For any graph there will be multiple spanning tree and a Spanning tree whose cost is minimum is called as Minimum Spanning Tree (MST).

Ex:-



Spanning trees

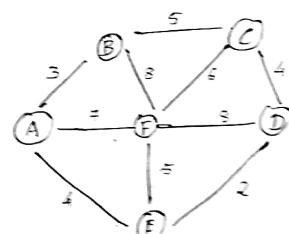


∴ Min. span tree is

MST Algorithms-

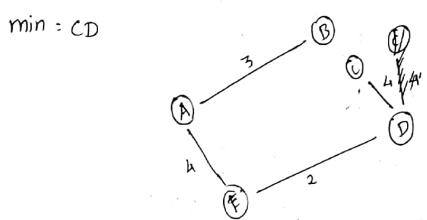
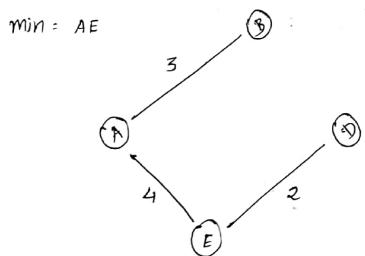
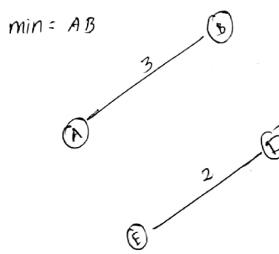
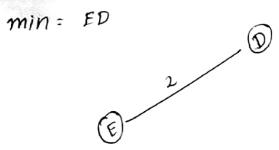
→ Kruskal's Algorithm • and the Prim's Algorithm

→ Kruskal's Algorithm-



Arrange all the edges in ascending order-

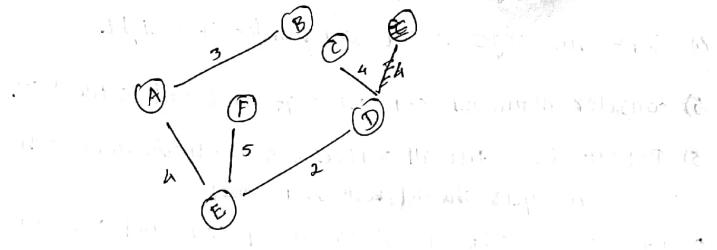
ED, AB, AF, CD, BC, EF, CF, AE, BF, DF



$\min = BC$

since, it form cycle, reject BC

$\min = EF$



$\min = CF$

since, it form cycle, reject CF

$\min = AF$

since it form cycle, reject AF

$\min = BF$

since, it forms cycle, reject BF

$\min = FD$

Since, it forms cycle, reject FD

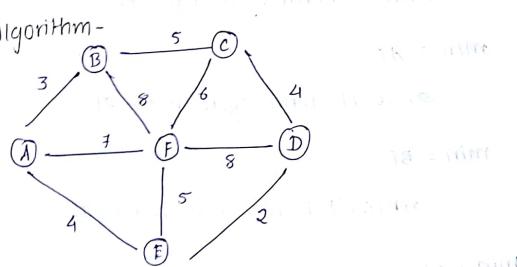
Hence, above graph be the min spanning tree

cost = 18

Algorithm-

- 1) Start
- 2) Enter the no. of vertices and edges.
- 3) Enter the edges among the vertices.
- 4) Sort the edges in ascending order of weight.
- 5) consider minimum cost edge and add to the MST.
- 6) Repeat step 5 till all vertices are reachable from each other but cycle should not be formed.
- 7) Add the weights of all considered edges and display it
- 8) STOP

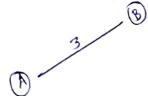
Prim's Algorithm-



Let A be the source code

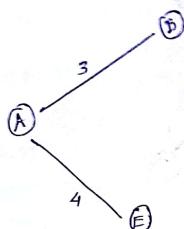
Edges are {AB, AE, AF}

$\therefore \min = AB$



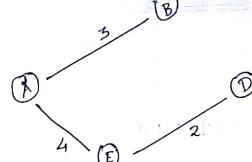
Edges connected to A, B = {AE, AF, BC, BF}

$\min = AE$



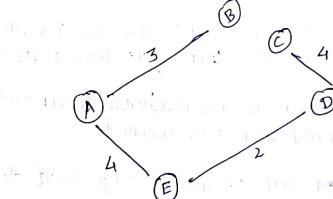
Edges connected to A, B, E = {AF, EF, BF, BC, ED}

$\min = ED$



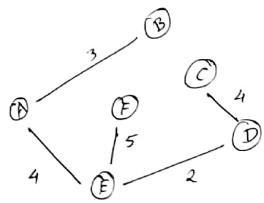
Edges connected to A, B, D, E = {AF, EF, BF, BC, DF, DC}

$\min = DC$



Edges connected to A, B, C, D, E = {AF, BF, BC, CF, EF, DF}

$$\min = EF$$



Hence, above graph be the min spanning tree.

$$\underline{\text{cost}} = 18$$

Algorithm-

- 1) start.
- 2) Enter the no. of vertices and edges.
- 3) Enter the edges among vertices.
- 4) Consider source node; find the connected edges and add minimum cost edge to the MST.
Mark vertices as visited.
- 5) Find the edges which are connected to the visited vertices and add min cost edge to the MST but cycle should not be formed.
- 6) Repeat step 5 till all vertices are reachable from each other but cycle should not be formed.
- 7) Add the weight of all considered edges and display it.
- 8) STOP

Graph Traversals-

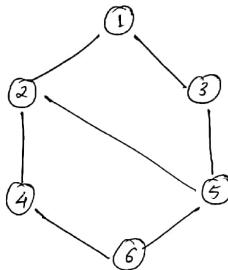
→ Depth First Search (DFS)

→ Breadth First search (BFS)

1) Depth First search (DFS)-

This traversal starts from any vertex and uses stack data structure.

Exp.-



Let's start from vertex 1

push 1

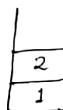


visit 1

top = 1

adj. vertex are 2, 3

push 2, visit 2



top = 2

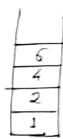
adj. vertex are 4, 5
push 4, visit 4



top = 4

adj. vertex are 6

push 6, visit 6



top = 6

adj. vertex is 5

push 5, visit 5



top = 5

adj. vertex is 3

push 3, visit 3



top = 3

since, vertex 3, doesn't have unprocessed adjacent vertices

∴ pop 3



since vertices 1, 2, 4, 6 and 5 doesn't have unprocessed adjacent vertices

∴ pop these vertices

as since stack is empty stop the traversal.

∴ DFS traversal is 1, 2, 4, 6, 5, 3

Algorithm-

- 1) START
 - 2) Enter the no. of vertices and edge's.
 - 3) Enter the edges among the vertices.
 - 4) Select any vertex from which DFS traversal starts.
 - 5) Push selected vertex on to the stack and mark as visited.
 - 6) Find top of stack and its adjacent vertices.
 - 7) Select any adjacent vertex and push on to the stack. Mark as visited.
 - 8) If top of stack (top) vertex doesn't have unprocessed adjacent vertices then pop these vertex.
 - 9) Repeat steps 7 and 8 until all vertices are visited or stack becomes empty.
 - 10) Display DFS traversal in the order vertices are visited.
 - 11) STOP
- 2) Breadth First search - (BFS)-

The traversal starts from any vertex and uses Queue datastructure