

Ideal Rotor with wake rotation

Assume

Tip speed ratio = 7

Number of Blades = 3

Airfoil used: NACA 63215

It can be seen that the max (Cl/Cd) occurs at an angle of attack of 5. Hence for the current airfoil design "Cl" is taken to be 0.722, corresponding to $\alpha=5$.

Hence $Cl_\alpha = 0.722$

Two separate programs were run to complete the given assignment. The first one was used to calculate the section pitch and the chord variation with radius for the given "TSR", and then to find the turbine performance for the geometry found at different "TSRs".

The value of "a" and "a'" at different radius and "TSRs" were calculated by iteration and the formulae below were used for the same.

$$\begin{aligned}\phi &= \tan^{-1}((1-a)/((1+a')*\lambda r)) \\ a &= 1/(1+4*\sin^2(\phi)/(\sigma(Cl*\cos(\phi)+cd*\sin(\phi)))) \\ a' &= 1/(1+4*\sin(\phi)*\cos(\phi)/(\sigma(Cl*\cos(\phi)+cd*\sin(\phi))))\end{aligned}$$

The iterations were run until a converged value for both "a" and "a'" was achieved.

Code:

Program 1:

```
clear all; close all; clc;
tic
lambda=10; %design tip speed ratio
R=10; %Total Radius
r=1:0.5:10; %local Radius
rr=[r(1:length(r)-1)+0.25 R]; %Control points
r_bar=rr/R;
lamda_r=r_bar*lambda; %local speed ratio
N=3; %number of blades
alpha=5; %design alpha
phi=2*radtodeg(atan(1./lamda_r))/3;
Cl_alpha=0.722; %design Cl
theta=phi-alpha;
chord=16*pi().*r_bar.*sin(degtorad(phi/2)).^2./(N*Cl_alpha);
sig=N.*chord./(2*pi().*r_bar); %solidity
sectionpitch=theta-theta(length(theta));
Output= [r_bar' lamda_r' phi' theta' chord' sectionpitch'];

figure(1)
subplot(2,1,1)
plot(r_bar,sectionpitch)
title('section pitch vs rbar')
xlabel('rbar')
ylabel('section pitch degrees')
subplot(2,1,2)
```

Submitted by

Harshal Kaushik – AM13M025 and
Antony Samuel B –AM13S022

```

plot(r_bar, chord)
title('chord vs rbar')
xlabel('rbar')
ylabel('chord')

TSRrange=3:1:15;

for TSR = TSRrange
    LTSR(TSR,:)=r_bar*TSR;
    for i=1:length(r_bar)
        k=1;
        a(TSR,i,k)=0.33;
        ap(TSR,i,k)=0.01;
        err=1;
        while err>0.01
            phi(TSR,i,k)=radtodeg(atan((1-
a(TSR,i,k))./(LTSR(TSR,i)*(1+ap(TSR,i,k)))));
            alp(TSR,i,k)=phi(TSR,i)-theta(i);
            [cl(TSR,i) cd(TSR,i)]=clcdcalc('naca63215',alp(TSR,i));
            cd(TSR,i)=0; %<==activate if drag is assumed to be zero (cd=0)
            %else put a '%' sign before it (to obtain fig 4&5)
a(TSR,i,k+1)=1/(1+4.*sin(degtorad(phi(TSR,i))).^2/(sig(i)*(cl(TSR,i).*cos(d
egtorad(phi(TSR,i)))+cd(TSR,i).*sin(degtorad(phi(TSR,i))))));

ap(TSR,i,k+1)=1/(4.*cos(degtorad(phi(TSR,i))).*sin(degtorad(phi(TSR,i)))/(s
ig(i)*(cl(TSR,i).*sin(degtorad(phi(TSR,i)))-
cd(TSR,i).*cos(degtorad(phi(TSR,i))))) -1);
            err1=abs(a(TSR,i,k+1)-a(TSR,i,k))./a(TSR,i,k);
            err2=abs(ap(TSR,i,k+1)-ap(TSR,i,k))./ap(TSR,i,k);
            err=max(err1,err2);
            k=k+1;
            if k>1000
                break;
            end
        end
        a_stor(TSR,i)=a(TSR,i,k-1);
        ap_stor(TSR,i)=ap(TSR,i,k-1);
        phi_stor(TSR,i)=phi(TSR,i,k-1);
        alp_stor(TSR,i)=alp(TSR,i,k-1);

    end

Cp(TSR)= 8*TSR^2*trapz([0 ap_stor(TSR,:).*(1- a_stor(TSR,:)).*r_bar.^3 ],[0
r_bar ] );

end

figure(2)
subplot(2,1,1)
plot(r_bar,a_stor(lambda,:))
title('a vs radius')
xlabel('radius')
ylabel('a')
subplot(2,1,2)
plot(r_bar,ap_stor(lambda,:))
title('a'' vs radius')
xlabel('radius')
ylabel('a''')

figure(3)

```

```
plot(TSRrange,Cp((length(Cp)-length(TSRrange)+1):length(Cp)))  
title('Cp vs TSR')  
xlabel('TSR')  
ylabel('Cp')  
toc
```

Output 1:

Cd=0

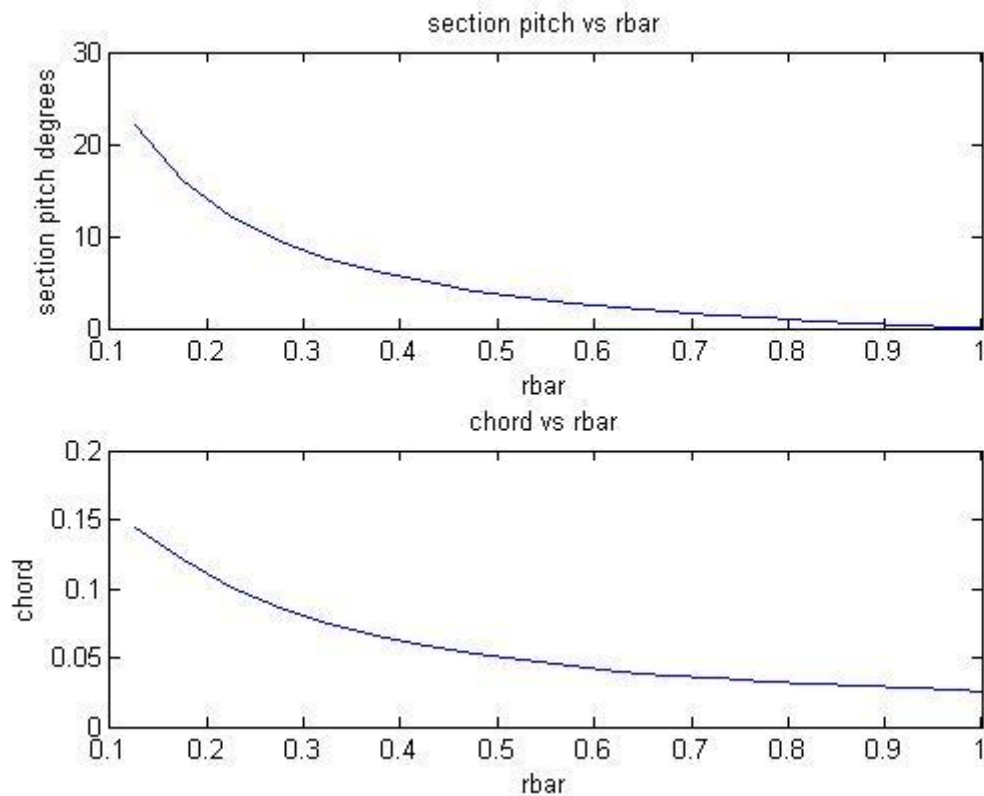


Figure 1

The variation of section pitch and chord with radius is shown here.

r	chord	section pitch
0.125	0.14	21.97
0.175	0.12	16.02
0.225	0.10	12.17
0.275	0.09	9.52
0.325	0.07	7.59
0.375	0.07	6.15
0.425	0.06	5.02
0.475	0.05	4.12
0.525	0.05	3.38
0.575	0.04	2.77
0.625	0.04	2.25

0.675	0.04	1.81
0.725	0.04	1.43
0.775	0.03	1.09
0.825	0.03	0.80
0.875	0.03	0.54
0.925	0.03	0.31
0.975	0.03	0.10
1	0.03	0.00

The variation of “a” and “a’” with radius is shown below at a TSR of 10 (Cd=0)

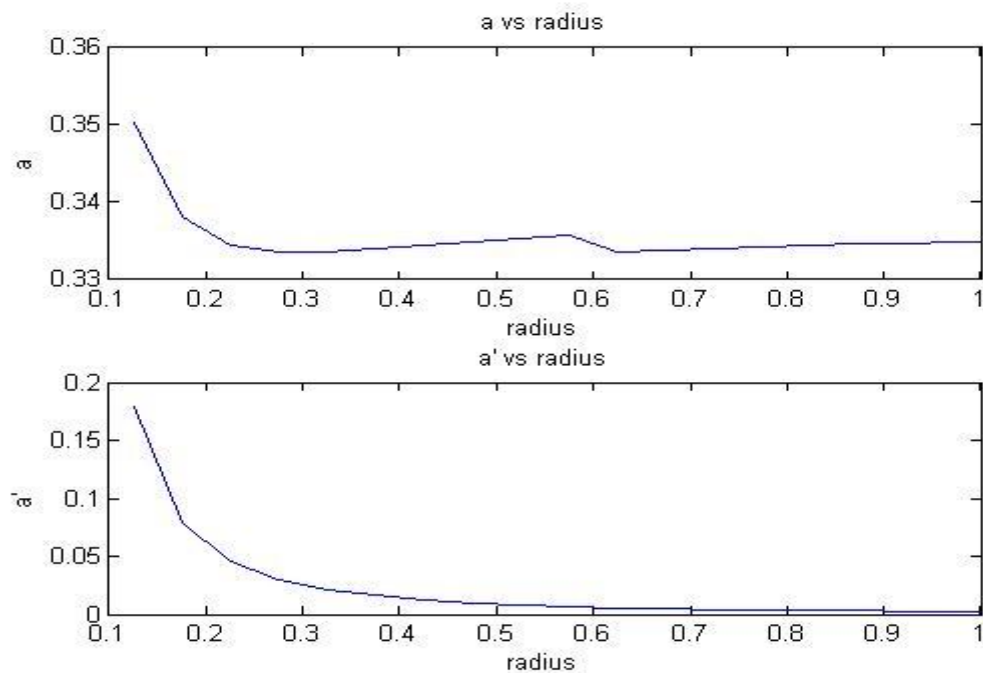


Figure 2: Variation of a and a’ with radius, finite drag

The variation of C_p with TSR is shown in figure 3 (Cd =0).

When the program 1 is run without setting Cd as 0 we find that 2nd and 3rd figures we obtained earlier, changes. This has been shown below as figure 4 and 5. It can be seen that C_p Obtained this way is lower than that observed when the Cd is set to be 0. Cd value is taken from the aerofoil data, along with Cl.

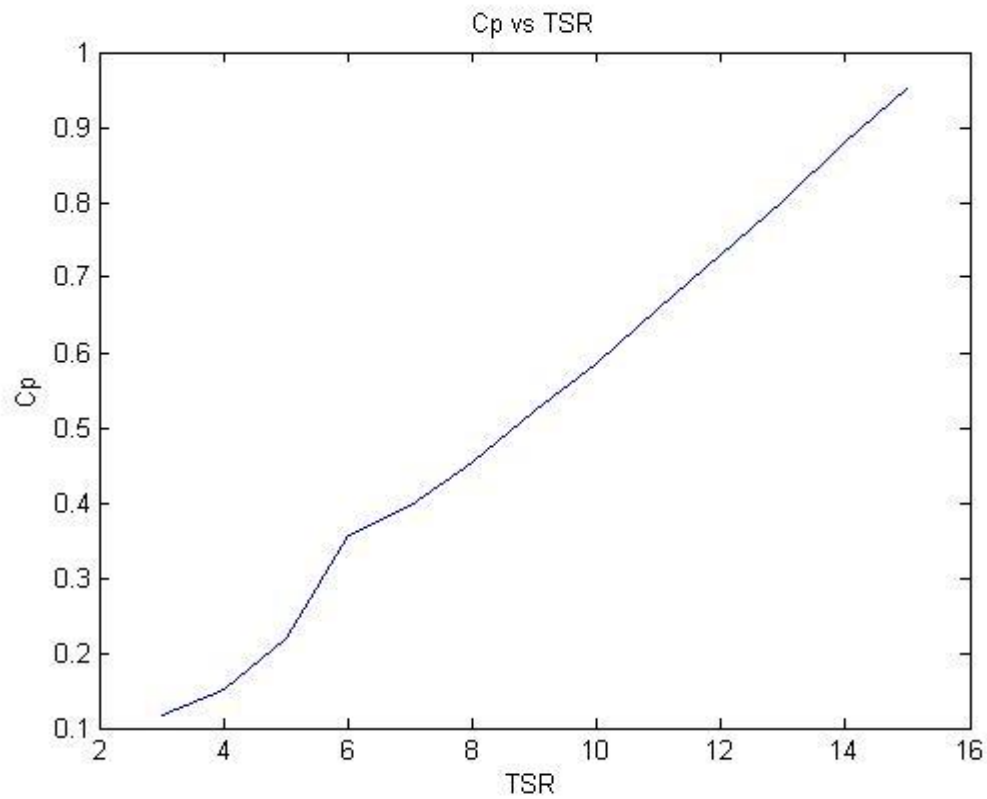


Figure 3

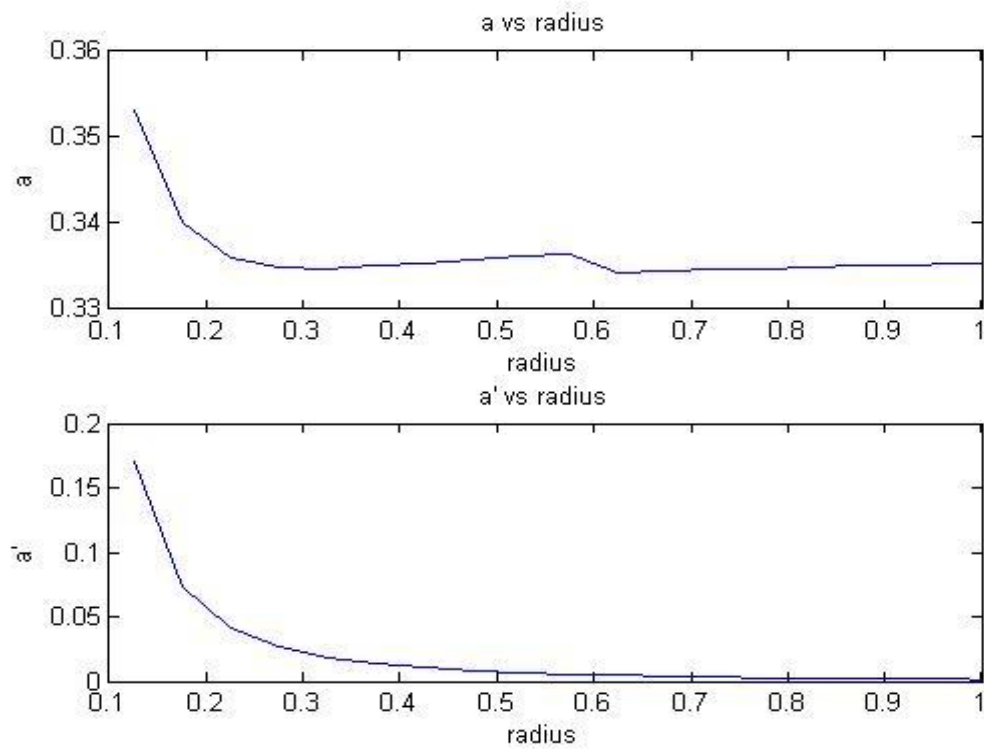


Figure 4

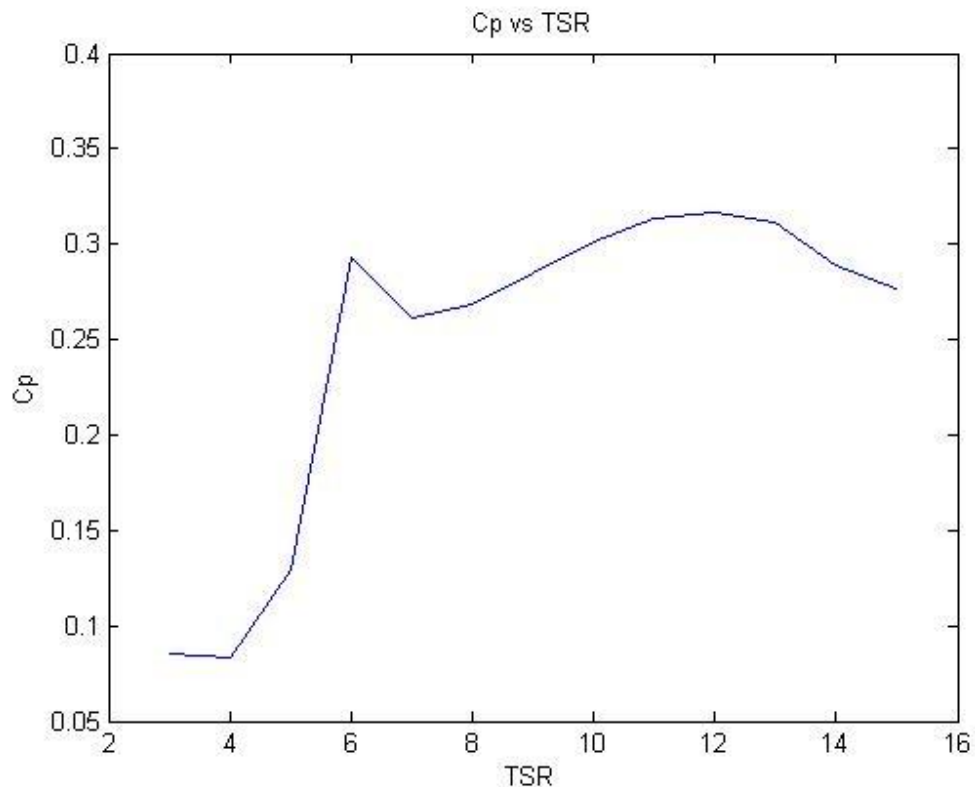


Figure 5: Variation of Cp with TSR, finite drag.

Program 2:

The variation of chord and pitch angle, if the blade is designed for different tip speeds is found using the below program.

```
clear all; close all; clc;

for lambda=3:1:12;                                %design tip speed ratio
    R=10;                                           %Total Radius
    r=1:0.5:10;                                    %local Radius
    rr=[r(1:length(r)-1)+0.25 R];                %Control points
    r_bar=rr/R;
    lamda_r=r_bar*lambda;                          %local speed ratio
    N=3;                                           %number of blades
    alpha=5;                                       %design alpha
    phi(lambda,:)=2*radtodeg(atan(1./lamda_r))/3;
    Cl_alpha=0.722;                               %design Cl
    theta(lambda,:)=phi(lambda,:)-alpha;

    chord(lambda,:)=16*pi().*r_bar.*sin(degtorad(phi(lambda,:)/2)).^2./(N*Cl_alpha);
    sig=N.*chord(lambda,:)/(2*pi().*r_bar);
    sectionpitch(lambda,:)=theta(lambda,:)-theta(lambda,length(theta));
    Output=[r_bar' lamda_r' phi(lambda,:) ' theta(lambda,:) '
    chord(lambda,:) ' sectionpitch(lambda,:) '];

figure(1)
```

```
plot(r_bar,sectionpitch(lambda,:))
title('section pitch vs rbar')
text(r_bar(1),sectionpitch(lambda,1),sprintf(' TSR=%1.0f',lambda),
'HorizontalAlignment','left');
xlabel('rbar')
ylabel('section pitch degrees')
hold on
figure(2)
plot(r_bar,chord(lambda,:))
title('chord vs rbar')
text(r_bar(1),chord(lambda,1),sprintf(' TSR=%1.0f',lambda),
'HorizontalAlignment','left');
xlabel('rbar')
ylabel('chord')
hold on
end
```

Results:

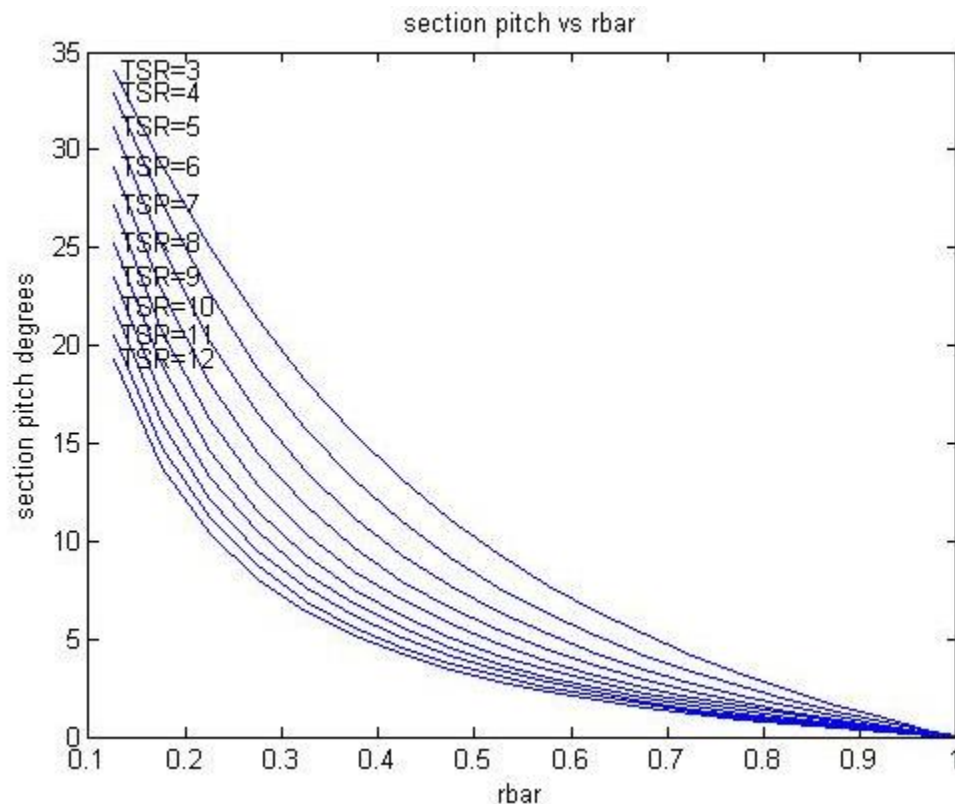


Figure: 6

The above figure shows the variation of sectionpitch for different tip speed ratios. The below one shows the variation of chord for different tip speed atios. It can be seen that for higher TSRs the chord and pitch value obtained is smaller.

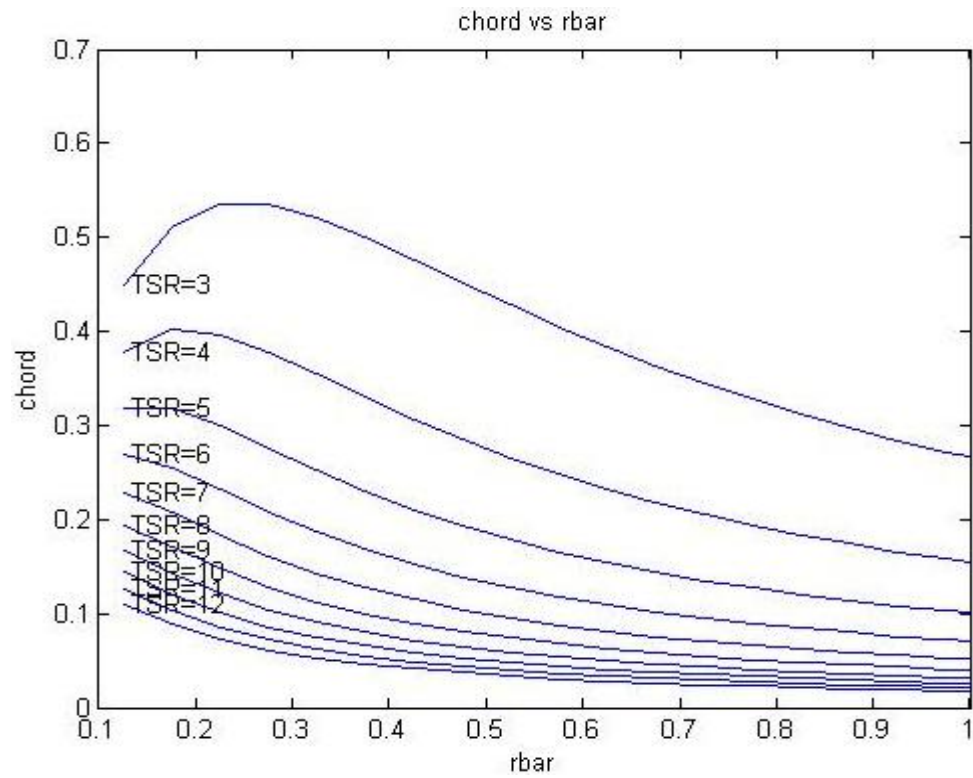


Figure 7