APTITUDE PREPARATION

TABLE OF CONTENTS

Lecture Notes for HCF and LCM

Shortcut for finding factors of a number

For finding the factors of a number, first, we must know about the prime numbers.
Prime Number: ***A number which is greater than unity and only has two divisors; itself and 1.***
NOTE: 1 is not a prime number.

Some properties of Prime numbers

- 2 is the lowest prime and even number.
- The lowest odd prime number is 3.
- When a prime number $p \geq 5$, is divided by 6, it gives remainder is 1 or 5. But, vice versa is not true, i.e. if a number is divided by 6 and gives remainder 1 or 5 then, it need not be prime. For example, 65 is a number which when divided by 6 gives remainder 5, but, 65 is not a prime number. Thus, this can be referred to as a necessary condition but not a sufficient condition.

What do you mean by the factor of a number?

The factor of a number is nothing but the divisor of a number.

Let us take a number as our example,
Number = 20
The factors of 20 are 1, 2, 4 and 5. Like this, if you find out the factors of the numbers 210, 159, 253 etc.it will take a lot of time to do it this way.

Thus, performing the same task in a good way can be:
To find factors of 20:

1×20
2×10
and 4×5

i.e By finding the factors in pairs, *The discovery of one factor will automatically find out another factor*, for example, here, finding out 4 as a factor of 20, it will automatically give you 5 as another factor.

Now, let us take a look again at the "factor pairs" in the example above. If you compare the values in each pair with the square root of 20 (i.e. 4….) you will find that for each pair the number in the left side is lower than the square root of 20, while the number in the right side is higher than the square root of 20. This is always true for all the numbers.
Thus one need not make any effort to find the factors of a number above the square root of the number; these will come automatically. All you need to do is to find the factors below the square root of the number.

Why do we need a factor of a number?

Suppose you are facing a question like this;
The product of two numbers is 192 and their difference is 26. Then find these two numbers?

Sol:
To solve this question we have two approaches. One is a mathematical way of doing and other is a quantitative or logical way of doing things.

Mathematically,

$a \times b = 192$ …………..(1)

And $a-b = 26$ …………..(2)

Here we have two variables and two equations. These two equations lead to a quadratic equation & now, we solve the quadratic equation to get the numbers. But it will take a lot of time.

Logically, make factor pairs of 192,

1×192
2×96
3×64
4×48
6×32
8×42
12×16

According to the second statement, the difference between the two numbers was 26.
You can easily spot your answer from the factors. A factor of pair 6×32 gives the difference between two numbers (32-6=26). Thus two numbers are 6 & 32.
If you find out the factor of 192 like this 1,2,3,4,6…….so on. You can't easily spot your answer and then you have to go through the quadratic equation approach, which is time-consuming.

Shortcut for finding the factors of a number:

Let's take a number: 192

Left side $\times$ **Right side**
1×192
2×96

3×64
4×48
6×32
8×42
12×16

How to approach?

1×192

When 192 is divided by 2 you get 96.
2×96

Now, from this point when you see 3, you have two option for getting $(3 \times .....)$ the right side number. One is, 192 divided by 3 giving you 64, and if you see the previous pair you had 2×96, so from this pair when you see 96 (96 can be broken down into 3×32), if we think of 192 like $2 \times 3 \times 32$ then you can rewrite it as 3×64. It is easier than 192 divided by 3.
3×64

After that when you go to 4, you see 4 which is 2 times "2", So you should have half of 96 on the right side. It is better than dividing 192 by 4.
4×48

As you keep going you will keep getting prime and composite numbers on the left side. When you see the **prime number** on the left side then you try to look at the **larger number of the previous pair** to get the right side of the current pair and **when you see the composite number on the left side, you try to bring the right side from one of the previous numbers on the left side.**

Thus, after 4 you have 5, which is a prime number & 5 does not divide 48 (larger in the previous pair), so 5 is not a factor of 192.
After that 6 is a composite number, from left side 6 is double of 3 & half of 64 is 32 in the left side, thus you get another factor is
6×32

Using this approach you can get all the factors.
8×42
12×16

NOTE: 1. The number on the left side is always less than the square root of the given number.

Let's take a number of 148, then factors are
1×148
2×74
4×37

So, here, 37 is the prime number, thus no further factor of 148.

NOTE: 2. Once you get a prime factor, then no further factor is possible.

NOTE: 3.When we have to check whether a number N is prime or not, we need to only check for its divisibility by prime factors; below the square root of N.

Quant shortcut strategies & Finding Prime number

To build up the speed in aptitude, you have to do these three things:

1. Superior reaction
2. Superior equation solving process
3. Superior calculation

The superior reaction through which you can build up your speed is to improve your reaction to the question situation.

Superior equation solving process, in this you need to know that aptitude mathematics is dominated by linear equations, at max, they become quadratic.
Most of the time you can avoid quadratic equations by just thinking of the number you are dealing with. Eg, $a \times b$ followed by $a + b$

$a \times b$ followed by $a - b$

$a \times b$ followed by $a : b$

Here, you have a list of complete factor pairs then you can easily choose your answer from the list. Thus, somebody solving this problem mathematically can't match your speed.

The superior calculation is to improve your calculation speed as well as accuracy. In this, to speed up the calculations you need to memorize multiplication tables, at least upto 20 and you should use some tricks like, “multiply in parts” (3*74 = 3*70 + 3*4), “it’s ok to approximate”(if options are far apart) etc. The accuracy can only be improved by a lot of practice.

To check whether a number is Prime or not?

Shortcut method :

To check whether a number N is Prime or not, follow these steps:

1. Take the square root of the number.

2. Consider the lower integer after taking the square root. Say this number is x. For example, if you have to check for 241, its square root will be 15.52 Hence, the value of x, in this case, will be 15.
3. Check for divisibility of the number N by all prime numbers below x. If there is no prime number below the value of x which divides N, then the number N will be prime.

For example :

Let the number is 241,

1. $\sqrt{241}$ is 15.52.
2. The value of $\sqrt{241}$ lies between 15 to 16. Hence, take the value of x as 15 (lower integer).
3. Prime numbers less than 15 are 2,3,5,7,11 and 13. 241 is not divisible by any of these. Hence, you can conclude that 241 is a prime number.

NOTE: 1. While checking "Is the number N prime?" We have two things to keep in mind; first, the number N should be odd and second, the number should not end with 5(e.g. 25 is an odd number but it ends by 5, so it is automatically divisible by 5).

Some important shortcut points:

1. For number below 49,
   The only number you would need to check for divisibility with is number "3".
2. A number between 49 and 121,
   You need to check divisibility by "3" and "7" only.
3. A number between 121and 169,
   You need to check divisibility by "3", "7" and "11" only.

Basic concept of HCF

When you talk about HCF, you are always thinking about two or more than two numbers. Similarly in LCM, you also talk about two or more numbers.
HCF (Highest common factor) is also called GCD (Greatest common divisor).

What is HCF?

Let us consider two numbers 20 & 30.
First, write the factors of 20 & 30

Factors of 20	Factors of 30
1×20	1×30
2×10	2×15
4×5	3×10
	5×6

Between 20 & 30 you can observe some factors which are common between the two. So, from the list of the factors, you can see 1, 2, 5, 10 are available in both the factor list of 20 as well as 30. Thus, the common factors between 20 & 30 are 2,5 & 10. Among these factors, the largest/highest one is 10, which is referred to as the HCF.

NOTE: In factors of a number you have prime & composite numbers (Every composite number written as the product of their prime number)

For example, prime factors of 80 and 224 are

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$
$$224 = 2 \times 2 \times 2 \times 2 \times 2 \times 7$$

Rules for Finding the HCF of Two Numbers a & b

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1. Write down the ***prime factors*** of the given numbers.
2. Write down the ***prime factors*** which are common to both.
3. And products of the common factors will give you HCF of the numbers.

Illustration: Find the HCF of 150 & 375.

Step 1: Write down the prime factors of the given numbers.

$$150 = 2 \times 3 \times 5 \times 5$$
$$375 = 3 \times 5 \times 5 \times 5$$

Step 2: Write down the prime factors which are common to 150 & 375.
3,5 & 5.

Step 3: Products of the common factors are $3 \times 5 \times 5$
Hence, HCF = 75.

How do you find out the HCF of more than two numbers?

Let us take three numbers a,b & c.
To find their HCF, what you need to do is, first find out the prime factors of each of the numbers. Say,

$$a = 2^3 \times 3^4 \times 5^1 \times 11^2$$
$$b = 2^5 \times 3^5 \times 5^2 \times 7^3$$
$$c = 2^6 \times 3^4 \times 5^3 \times 7^2$$

HCF (a,b,c) $\rightarrow$ All common prime factors with their ***lowest*** available power.
Thus, HCF of a,b,c will be

$$\text{HCF} = 2^3 \times 3^4 \times 5^1$$

NOTE: Finding LCM is a very similar process to finding the HCF.

LCM(Lowest Common Multiple) of a,b,c $\rightarrow$ All Prime factors with their ***maximum*** available power.
Thus, LCM of a,b,c will be

$$\text{LCM} = 2^6 \times 3^5 \times 5^3 \times 7^3 \times 11^2$$

Shortcut of HCF

Finding the HCF of a set of numbers is however extremely difficult and time taking, through the prime number approach (that was discussed previously). So, we adopt some shortcut for finding the HCF of a set of numbers.

What is the shortcut of HCF?

Let's take two numbers 46 & 76.
Now, the question is "*which are the numbers that would leave the same remainder when they divide both 46 & 76"?*

NOTE: This question statement is useful for shortcuts of HCF.

Properties of remainder:

Let us say two numbers 22 & 39.
Find out the remainder of these two numbers when divided by 7?

$22 \div 7$ gives a remainder 1.
$39 \div 7$ gives a remainder 4.

This remainder "4" can be found by another method. As, 39= 22+17; so, $(22+17) \div 7$ when we look at individual remainders $22 \div 7$ has 1 & $17 \div 7$ has 3, So the total remainder is 4.

Now, let us try to answer the above question "*which are the numbers that would leave the same remainder when they divide both 46 & 76"?* using the remainder property. So, we have these two numbers 46 & 46+30.

Say, you divide these numbers by a divisor D.
Now,

$46 \div D$. Suppose, this gives remainder 'r'

According to question statement if $(46+30) \div D$, you want a remainder 'r' only.
The key element for being overall remainder 'r' will depend on whether $(30 \div D)$ will give remainder "zero" or not, because only in that case, you will get the remainder 'r'.
If $(30 \div D)$ gives you remainder "zero", it is quite obvious that the value of 'D' has to be a factor of 30.
Factors of 30 are 1,2,3,5,6,10,15,30. The value of D will be any of these numbers.
If we take D=5 then,
$(46 \div 5)$ will give you a remainder 1. & $(46+30) \div 5$ will give you a remainder (1+0). No number is possible outside factors of 30, that can have this property.

For example, if you divide 46 & 76 by 7, which is not the factor of 30, then you will get different remainders from 46 and 76.

How does this approach connect to the HCF?

When you talk about the common factor of two numbers X & Y. then the common factor has to leave the same remainder "zero". Which means
Let two numbers X & X+12, the only numbers that will have the possibility of leaving the same remainder zero would be factors of 12.

1×12
2×6
3×4

All the common factors of these two numbers would come in the factors of 12, they can't come from any outer range. And hence, if all the common factors of X & X+12 are inside the factors of 12, So the HCF of X & X+12 would also come from the factors of 12. Which means HCF of X & X+12, can only be one of (1,2,3,4,6 & 12) these numbers.

For example,
Find the HCF of 38 & 50?
Now, you would do HCF without doing prime factors. You would do the HCF by difference method i.e by finding out the factors of difference between the two numbers.
50-38=12, factor of 12 are 12,6,4,3,2&1.
12 → Does Not divide 38, so this is not HCF of these two numbers.
6 → Does Not divide 38, so this is not HCF of these two numbers.
4 → Does Not divide 38, so this is not HCF of these two numbers.
3 → Does Not divide 38, so this is not HCF of these two numbers.
2 → Divide 38, so this is HCF of these two numbers.
Then it is obvious it will divide 38+12 and hence HCF is 2.

NOTE: HCF may vary according to the numbers, but it will always come from the difference.

What about more than two numbers?

Let us consider the numbers are x, x+12, y, z.
For finding HCF of these numbers, take the differences between the numbers. Here, many differences are possible, but you have to choose the smallest difference between any pair of these numbers.
Write the factors of that number and HCF of all these numbers would be from the factor list.

Sometimes you might want to go for prime number difference instead of the smallest difference, For example, suppose the numbers are 44,56 & 93.

So, 56 - 44=12
93 - 56=37
93 - 44=49

Here, a better difference to take here is 37 because 37 is a Prime number, then the factors of 37 are either 1 or 37. So, HCF, in this case, is either 1 or 37. 37 does not divide any number, so, the HCF=1.

Question:
A nursery has 363,429 and 693 plants respectively of 3 distinct varieties. It is desired to place these plants in straight rows of plants of 1 variety only, so that the number of rows required is the minimum. What is the size of each row and how many rows would be required?

Sol:
The size of each row would be the HCF of 363, 429 and 693.
Difference between 363 and 429 =66.
Factors of 66 are 66, 33, 22, 11, 6, 3, 2, 1.
66 need not be checked as it is even and 363 is odd. 33 divides 363, hence would automatically divide 429 and also divides 693.
Hence, 33 plants is the correct answer for the size of each row.
For the number of rows that would be required = Minimum number of rows required = 363/33 + 429/33 + 693/33 = 11 + 13 + 21 = 45 rows.

Problems on HCF

Problem 1: The sides of a pentagonal field are 918, 2160, 2244, 2358 & 1431 meters. Find the greatest length of tape that would be able to exactly measure each of these sides?

Sol: Here in the question, it is required to find the largest tape that would be able to measure all the sides exactly, which means it is talking about HCF.
Sides of pentagons are 918, 2160, 2214, 2358 & 1431.

Step1. Find the smallest difference from all the pairs of numbers. So, the smallest difference comes out between 2214 & 2160, which is 54, as all the other differences are greater than 54.

Step2. Factors of 54 are

1×54
2×27
3×18
6×9

$54 \rightarrow$ Does Not divide 1431
$27 \rightarrow$ Does Not divide 2358
No need to check for 18.(it will not divide odd numbers)
9 will divide all these numbers. Thus, the HCF of these numbers is 9.
And hence, the largest length of the tape that can measure all the sides exactly is 9.

Problem 2: A milkman has the milk of three varieties. He has 403L, 465L & 651L of the three varieties of milk with him.

- **a.** What is the largest size of bottle in which he can bottle each of the three types of milk completely without mixing the milk?
- **b.** What is the minimum number of bottles required?
- **c.** How many different sizes of bottles (with the integral number of litres) can be used in order to bottle all the three varieties of milk?

Sol:

- **a.** Three varieties of milk are 402L, 465L & 651L.
  Largest bottle size means HCF of these numbers.

Step1: Find the smallest difference between all the pairs of numbers. So, the smallest difference comes out between 465 & 403, which is 62, as all other differences are greater than 62.

Step2: Factors of 62 are

1×62

2×31

31 is a Prime number so no further factor is possible.

62 → Does Not divide 403. 31 will divide all the three numbers.
Thus, the largest bottle size is 31L.

b. Let a bottle size of 'b' L.
Minimum No. of bottle required = 403/b + 465/b +651/b ………(1)
To minimise this equation 'b' should be maximised. So, maximum value of b = 31L.
Thus,
Minimum no. of bottle required = 403/31 + 465/31 + 651/31
= 13+15+21
= 49L.

c. This part is basically asking how many factors 31 has. Since, 31 is a prime number, so, it has only two factors 31 & 1. Hence, 2 sizes of bottle can be used.

Problem 3: What is the largest number that would leave the same remainder when it divides 283, 411 & 475.

Sol: To solve this type of question you have to follow the following steps:

1. Arrange all the numbers in increasing order.
2. Then take the difference, pairwise (linked pair i.e. for numbers a,b,c, & d, the differences are (b-a),(c-b) & (d-c)). Let us say that the differences of these pairs are p,q & r respectively.
3. Find the common factors of p,q & r. (common factor of p,q & r gives you the same remainder in all the three cases.)
4. The largest number that leaves the same remainder will then become the HCF of p, q & r.

Some questions for practice:

1. Two equilateral triangles have the sides of lengths 34 and 85 respectively.
(a) The greatest length of tape that can measure both of them exactly is:
(b) How many such equal parts can be measured?
Ans: (a) 17.
(b) 21.

2. A forester wants to plant 44 apple trees, 66 banana trees and 110 mango trees in equal rows (in terms of the number of trees). Also, he wants to make distinct rows of trees (i.e. only one type of tree in one row). The minimum number of rows that the forester will require to plant trees are?
Ans: 10.

3. Find the HCF of
(a) 420 and 1782 (b) 36 and 48
(c) 54, 72, 198 (d) 62, 186 and 279
Ans: (a) 6 (b) 12 (c) 18 (d) 31.

Concept and shortcuts of LCM

LCM: Lets two natural numbers be a & b. *The smallest natural number which will be completely divisible by a & b is called LCM (lowest common multiplication) of a & b*.

For example:

Let's have two natural numbers 4 & 6. Here, 12 is the smallest natural number which will be completely divisible by 4 & 6.
Thus, LCM(4,6) = 12.

Procedure for finding LCM of two numbers:

Step1: Find the prime factor of two numbers a & b.
Step2: Write down all the prime factors that appear at least once in the numbers a & b.
Step3: Write all the prime factors with their highest power.
Step4: Products of all the prime factors with their highest power will give you LCM of a & b.

For example: Let's have two numbers 12 & 80.

Step1: List the prime factors
12= $2 \times 2 \times 3$
80= $2 \times 2 \times 2 \times 2 \times 5$
Step2: Write down all the prime factors that appear, at least once in the numbers: 2,3,5.
Step3: Write all the prime factors with their highest power: $2^4 \times 3^1 \times 5^1$
Step4: The LCM = $2^4 \times 3^1 \times 5^1$
= 240.

Let's take two numbers 4 & 6. When you see the multiples of 4 & 6.
Multiples of 4: 8,12,16,20,........
Multiples of 6: 6,12,18,24,........
Now, the common multiples between 4 & 6 are: 12,24,36…….
Thus, the smallest multiple in the common multiple list is called LCM.

NOTE: Multiplication table of LCM would essentially give the common multiples of all the numbers.

For example, you have three numbers a, b & c and their LCM is 12, then 12, 24, 36, 48, ... all these multiples of 12, would be multiples of a,b & c.

Shortcut for finding the LCM:

As you saw LCM is the product of the highest power of all the prime factors, but that process would be very tedious, especially when the numbers are small.
When the numbers are small the logic of LCM builds around the ***Co-prime numbers***.

Co-prime Number: Two numbers are Co-prime to each other when they have no common factor among each other.
For example: (6, 13), (7, 11), (9, 19) etc.

Three numbers are Co-prime to each other when pairwise, each pair is Co-prime.
For example: Three numbers be a,b and c are Co-prime when,
a,b are Co-prime,
a,c are Co-prime,
& b,c are Co-prime.
All three pairs should be Coprime to each other, only then, a, b and c will be Co-prime.

NOTE: When a & b is Co-prime then the HCF should be 1.

Some important points about the Co-prime numbers:

(i) Two consecutive natural numbers are always co-prime (Example 5, 6; 82, 83; 749, 750 etc.)
(ii) Two consecutive odd numbers are always co-prime (Examples: 7, 9; 51, 53; 513, 515 etc.)
(iii) Two prime numbers are always co-prime (Examples: 13, 17; 53, 71 and so on)
(iv) One prime number and another composite number (such that the composite number is not a multiple of the prime number) are always co-prime (Examples: 17, 38; 23, 49 and so on, but note that 17 and 51 are not co-prime, as 51 is a multiple of 17)

Shortcut for LCM:
Step1: When the numbers are co-prime, then LCM is simply their product.
So, 7, 9 and 11 are co-prime, The LCM is $7 \times 9 \times 11$.

Step2: What to do when you have a mix of prime and Co-prime.

For example, four numbers 42, 44, 18, 25.
(i) If you see any co-prime put them down in your LCM. Here you can see 18 & 25 are Co-prime (and 25,42 ; 25,44 are also Co-prime).
(ii) LCM of these numbers starts with $18 \times 25 \times \ldots$ And
(iii) Now the logic of LCM should contain all the other numbers from the given numbers.
(iv) Out of the LCM, you should be able to construct 42 and 44 also.
(v) The factor of 42= $2 \times 3 \times 7$. Inside 18 you have 2 & 3, But you don't have 7 in 25 and 18. To construct 42, you should have a 7 in your LCM. (LCM= $18 \times 25 \times 7 \ldots$)
(vi) The factor of 44= $2 \times 2 \times 11$. Inside 18, you have one 2, but there is no 11 and other 2 in this LCM; so, to construct 44 you need to introduce 2 & 11 into the LCM.
So, LCM will be = $18 \times 25 \times 7 \times 2 \times 11$.

NOTE: (i). LCM has to be the multiple of HCF.
(ii). For any two numbers a & b, Product of two numbers ($a \times b$) $= LCM \times HCF$
(this formula is valid for two numbers)

HCF & LCM of a Fraction:

HCF of a Fraction:

$$\boxed{\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}}$$

LCM of a Fraction:

$$\boxed{\frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}}$$

For example: LCM & HCF of 1/2 ,5/7 and 8/11 are:
LCM = LCM(1,5,8) / HCF(2,7,11)
HCF = HCF(1,5,8) / LCM(2,7,11)

So,
LCM= 40/1
HCF= 1/(2 × 7 × 11)

Standard questions on LCM

Type 1: Based on the formula: Product of two numbers = HCF × LCM.

Question 1: HCF of two numbers is 75 & their LCM is 1800. If one of the numbers is 600, then what will be the other number?

Sol: Let the required number is b, and the given question says a=600.
So, According to formula:

$$HCF \times LCM = a \times b$$
$$75 \times 1800 = 600 \times b$$

b= 225. This is the required number.

But be careful of traps in this question.
For example:
HCF of two numbers is 75 & their LCM is 900. If one of the numbers is 600 then, what will be the other number?

Sol:
Let the required number is b, and the given question says a=600.
So, According to the formula:

$$HCF \times LCM = a \times b$$
$$75 \times 900 = 600 \times b$$

b= 112.5.
So, 600 & 112.5 don't have an HCF of 75 and LCM of 900.

Another trap in this type of question is:
For example:
HCF of two numbers is 75 & their LCM is 400. If one of the numbers is 100 then, what will be the other number?
Sol:
Let the required number is b, and the given question says a=100.
So, According to the formula:

$$HCF \times LCM = a \times b$$
$$75 \times 400 = 100 \times b$$

b= 300.

So, 100 & 300 don't have an HCF of 75 and LCM of 400. Also in this question, LCM is not a multiple of HCF.
So, be careful of these types of traps.

Type 2: Bell tolling question

Question 1: 4 Bells toll together at 9:00 A.M. They toll at an interval of 7, 8, 11 and 12 seconds respectively. After 9:00 A.M., at what time will these bells toll together for the first time & how many times will they toll together again in the next 3 hours?

Sol: 4 Bells toll at an interval of 7, 8, 11 & 12 sec.
7-sec bell tolls at multiples of 7; 8-sec bell tolls at multiples of 8; 11-sec bell tolls at multiples of 11 and 12-sec bell tolls at multiples of 12.
To find, at what time will these bells toll together the first time after 9 AM, you need to find the LCM of these intervals.

So, LCM(7,8,11 & 12) = 1848 sec.
Thus, after 9 AM bells toll together for the first time at 9:30:08.

In 3 hr i.e. 10800 sec:
Number of times the bells toll together in the next 3 hours= 10800/1848
= 5.84 $\approx$ 5 times.

Type 3:
Question: What is the smallest number greater than 1, that leaves a remainder 1, when divided by 1, 2, 3, 4, 5, 6, 7, 8, 9 & 10.
Sol:
To understand the logic of this question first you have to understand this logic:
For example: What is the smallest number greater than 1, that leaves a remainder 1, when divided by 3 & 4.
LCM of 3 & 4 is 12. And 12+1=13 is the smallest number.
So, in this type of question, the logic is LCM+Remainder

So, now come to the question
LCM of 1,2,3,4,5,6,7,8,9 & 10 is 2520. And the smallest number that leaves a remainder 1, is LCM+1 i.e. 2520+1=2521.

Some question for practice:

Question1: Find the HCF of
(a) 420 and 1782 (b) 36 and 48
(c) 54, 72, 198 (d) 62, 186 and 279
Ans: (a) 6 (b) 12 (c) 18 (d) 31.

Question2: Find the LCM of
(a) 13, 23 and 48 (b) 24, 36, 44 and 62
(c) 22, 33, 45, and 72 (d) 13, 17, 21 and 33
Ans: (a) 14352 (b) 24552 (c) 3960 (d) 51051.

Question3: The LCM of two numbers is 936. If their HCF is 4 and one of the numbers is 72, the other is?
Ans: 52.

Question 4: Three runners running around a circular track can complete one revolution in 2, 4 and 5.5 hours respectively. When will they meet at the starting point?
Ans: 44.

Question 5: The HCF and LCM of two numbers are 33 and 264 respectively. When the first number is divided by 2, the quotient is 33. The other number is?
Ans: 132.

Lecture Notes for Average

Intro To Averages

You can find out the application of averages across different chapters of aptitude, like Time & Work, Time Speed and Distance, Ratio and Proportion, Percentages etc. So, from the aptitude point of view, the average is an important chapter.

What is an Average?

Average is a number that measures the central tendency of a set of numbers. In other words, it is an estimate where the centre point of the set of numbers lies. Average is also known as the mean. In mathematics, the average is equal to the sum of the set of numbers divided by numbers of

$$\text{Average} = \frac{\text{Sum of the numbers}}{\text{Number of numbers}}$$

values in the set.

The formula for the average for the set of numbers:-

For Example:

Let x1, x2, x3, … , xn be a set of n numbers and "An" be their average.

So,

An = (x1 + x2 + x3 + … + xn)/n .

Another meaning of average is, average is that single number, that can replace each of the given numbers present in the set with the average number and still get the same total.

For Example:

The average of 5 numbers 11, 14, 17, 18 and 20 is:

Average = (11 + 14 + 17 + 18 + 20)/5 = 80/5=16

This means that if you replace all the 5 numbers with 16 (average number), even then the sum will be 80, there would be no change in the total.

The average also refers to one of the 3 central tendencies as we have studied in statistics for any group of numbers. The three central tendencies are: 1. Mean (Average)

2. Median
3. Mode

Mean (Average) we have already discussed above.

Median: It is defined as the middle term of the group of numbers arranged in order.
For Example : The numbers 3, 1, 5, 7, 6, 4, 2 are not arranged in order. So for the median, first, you have to arrange these numbers in order.
1, 2, 3, 4, 5, 6, 7 are arranged in order. Thus, Median = 4.(i.e. 4 is the middle number)

Application : 1. It is used to measure the distribution of earning.
2. To find the middle age, from the class of students.

Mode : In a set of numbers the number that occurs with the greatest frequency(most often) is the mode of that set.
For Example : Let a set of numbers 10, 15, 19, 19, 7, 11, 15, 19, 12, 11, 19, 23
So, Mode = 19(Because it occurs 4 times).

Application : 1. It is used to measure the influx of public transport.
2. It is used to measure the number of games succeeded by any team of players.

NOTE : 1. Average of first n natural numbers = (n+1)/2
2. Average of first n even numbers = n+1
3. Average of first n odd numbers = n

Assumed Average Approach

Assumed average approach is a way to find out the average of a set of numbers by assuming the average.

What is the assuming average approach?

We already know that, Average is that one number that can replace each of the numbers in a group of numbers and still keep the same total.
By using this concept the assumed average approach is a bypass for getting the average of the numbers.
Let us say 6, 10, 7 & 5 are the 4 numbers. So, Average is;

$$\text{Average} = (6 + 10 + 7 + 5)/4 = 7$$

7 can replace all the 4 numbers.
If you see the deviation between the numbers and their average (between left column and right column), the direction should be: left column - right column

Left column	Right column	Deviation
6	7	- 1
10	7	+3
7	7	0
5	7	- 2

The net sum of all these deviations is 0 (-1+3+0-2 = 0). This means the average value is correct.

The following are some steps to calculate correct average from the assumed average:
Step1. You have to assume an average.
Step2. Calculate how much the given numbers deviate from assumed average.
Step3. Calculate the sum of all the deviations (i.e. Total deviation).
Step4. Calculate the average deviation with the help of the following formula :

$$\text{Average Deviation} = \frac{\text{Total Deviation}}{\text{Number of numbers}}$$

Step5. Now, the correct average will be equal to the sum of assumed average and average deviation. i.e.

Correct average = Assumed average + Average Deviation.

To understand the assumed average approach, consider an example:

Let 37, 75, 83, 94 & 46 are 5 numbers. You don't know the average and you want to find out the average for these numbers without doing the sum of these numbers.

Step1. For this example, assume an average of let us say, 60.
Step2. Deviation calculation
60 to 37 there is a deviation of -23.
60 to 75 there is a deviation of +15.
60 to 83 there is a deviation of +23.
60 to 94 there is a deviation of +34.
60 to 46 there is a deviation of -14.
Step3. Total deviation = -23+15+23+34-14 = 35.
Step4. Average deviation = 35/5 = 7.
Step5. Correct average = 60+7 = 67.

You can assume any value of average, but assumed value should be nearly equal to the one of the given value for simple calculation.

In the above example, let us say you assume the average to be 70 instead of 60.
Step1. Assumed average = 70.
Step2. Deviation calculation
70 to 37 there is a deviation of -33.
70 to 75 there is a deviation of +5.
70 to 83 there is a deviation of +13.
70 to 94 there is a deviation of +24.
70 to 46 there is a deviation of -24.
Step3. Total deviation = -33+5+13+24-24 = -15.
Step4. Average deviation = -15/5 = -3.
Step5. Correct average = 70+(-3) = 67.
So you can see that the answer will be the same irrespective of what average you take.

The benefit of the assumed average method is that it is much faster in the case when numbers are bigger and they are clustered (for example,a group of numbers between the range of 300 to 400), then your calculation is much faster than what you normally do.

Some questions for practice:

1. Find the average of the following numbers using assumed average approach:
   250, 225, 275, 281, 294
   Ans : 265.

2. Find the average of the following numbers using assumed average approach:
   35, 72, 81, 93, 49
   Ans : 66.

3. Find the average of the following numbers using assumed average approach:
   792, 775, 724, 765
   Ans : 764.

Standard Language In Average

Every chapter has standard language inside it. You can also observe that there is some standard language inside the Average chapter.

You can understand the standard language on average by the help of some examples. So, here we understand the standard language by the help of following examples:

Example 1:

Statement : The average of 5 numbers is 12.
When you see this statement two reactions come to mind. The 1st one is that, $5 \times 12 = 60$. and 2nd is that, add 12 five times i.e. 12 + 12 + 12 + 12 + 12 = 60.
So, there are two approaches to tackle this statement.

Example 2:

Statement 1: The average age of 24 students and principle is 15.
Solution: When you look at the statement you realise that there are 25 people with an average of 15. Your reaction is $25 \times 15 = 375$, that means the total age of 25 people is 375.

Statement 2: The average age of the students is 14.
Solution: The total age of the students = $24 \times 14 = 336$.
From these two statements the difference between 375 & 336 (375-336=39) will give you the average age of principle.
This is the total difference approach.
Any question has still not been asked to you, but still you have calculated a lot of things in this question in your mind.

Questions that may be asked from you, is to find the age of principle. So, in this type of question solving while reading, what will happen, you will know the answer much before you actually see what is asked. This is the best kind of solving in the exams. This will definitely increase your solving skills.

Example 3:

Statement 1: The average score of a batsman after 9 innings is x.
Solution: Your reaction is 9x i.e. Total run in 9 innings = 9x.
Statement 2: In the 10th inning he scored 100 and increased his average by 8 runs.
Solution: In the 2nd statement two reactions come in your mind. 1st one is that,
Total run after 10th innings = 9x+100. ………….(1)
And 2nd is, total run after 10th inning = 10(x+8). ………..(2)
Statement 3: 1. Find the original average?
2. Find a new average?
3. Find the total run scored by him in 9 innings?

4. Find the total run scored by him in 10 innings?

Solution: 1. Equate eq(1) & (2) you will get;

$$9x+100 = 10(x+8)$$

$$x = 20$$, This is the total average.

2. New average = x+8 = 20+8 = 28.
3. Total run scored in 9 innings = $9x = 9 \times 20 = 180$.
4. Total run scored in 10 innings = $9x+100 = (9 \times 20) + 100 = 280$. Or
   $= 10(x+8) = 10(20+8) = 280$.

As you have deduced the statements before even knowing the questions given to you this makes your solving faster.

Example 4:

Statement 1: A boy who has earned an average salary of Rs 4200 per month during his 1st 11 months in India.

Solution: Total income in 11 months = $11 \times 4200 = 46200$.

Statement 2: He wants to ensure an annual average income of Rs 5000 per month. How much did he earn in 12th month?

Solution: Total income in 12 months = $12 \times 5000 = 60000$.

In the 12th month he earns = 60000-46200 = 13800.

So, you can see 3 different questions, each of these questions using the same language. This type of statement is one of the standard statements.

Standard Situation In Averages

Standard situation 1:

This chapter is about identifying those standard situations that are generally asked in exams with the help of some examples. In the above 3 examples (example No. 2, 3 & 4) one thing is common that one new number is entering into the group.
In example 2. Group of 24 students and principle added to it.
In example 3. Group of 9 innings and added 10th innings to it.
In example 4. It has an 11 months income and added 12th month income into it.
Here the situation is entering a new number.

Let us say you got 5 numbers with an average of 12 and 6th number entered and the average of all 6 numbers becomes 15. What is the 6th number?
Solution : There are two ways of solving this type of question.

The 1st way;

6th number = Total of 6 numbers - Total of 5 numbers

$= 6 \times 15 - 5 \times 12 = 30$

The 2nd way;

Addition of a 6th number, increases the average by 3.

$12 + 3 = 15$
$12 + 3 = 15$
$12 + 3 = 15$
$12 + 3 = 15$
$12 + 3 = 15$

The +3 appearing 5 times is due to the 6th number, which is able to maintain the average of 15 first, and then 'give 3' to each of the first 5.
Hence, the 6th number in this case = **maintain + contribute**

$= 15 + 3 \times 5 = 30$

This is another way of solving these types of questions. You can solve the example 2,3 & 4 by this approach.

Let's take example 3;
The average score of a batsman after 9 innings is x. In the 10th inning he scored 100 and increased his average by 8 runs. Find the original average?
Solution : After 10th inning, increases average by 8 runs.

$x \rightarrow x+8$
$x \rightarrow x+8$
$x \rightarrow x+8$

$$x \rightarrow x+8$$
$$x \rightarrow x+8$$
$$x \rightarrow x+8$$
$$x \rightarrow x+8$$
$$x \rightarrow x+8$$
$$x \rightarrow x+8$$

In this question, 100 contribute to two things. It maintains an average of (x+8) and +8, nine times. So,

$$100 = (x+8)+(8 \times 9)$$
$$x = 20.$$

Standard situation 2:

The 2nd standard situation is about what happens if more than one number enters. This situation can also be explained with the help of examples:

Let us say 8 numbers with an average of 10. Two new numbers enter due to that the average becomes 13. What is the average value of these two numbers?

Solution: There are two approaches to solve this question;

1. Total difference approach:
   Total of two numbers = $10 \times 13 - 8 \times 10 = 50$
   Thus, the average of two numbers = 50/2 = 25.

2. 2nd approach;
   Addition of a 2 numbers, increases the average by 3.

Average of 8 numbers	Average after 2 number entry
10	13
10	13
10	13
…	…
…	…
8 times…	10times…

Average of two number = **maintain + average contribution**

$$= 13 + (3 \times 8)/2$$
$$= 13 + 24/2 = 25$$

The contribution 24 has to be brought by these two together.

You can understand this situation like when you and your friend go to a hotel and you are going to be paid equally. Bill comes out of 24, then you will divide the bill into 2. So, each individual will pay 12.

Example:

After 120 innings batsman has an average of 55. And he realizes that he is going to play 180 innings more and he wants an average of 100 runs per inning. So what should be the average of the remaining 180 innings?

Solution: Average increases by 45runs.

Average in first 120 innings	Average after 300 innings
55	100
55	100
55	100
…	100
…	…
120 times…	300 times…

180 new innings maintain the average 100 and make the average contribution in 120 innings.

Average of remaining 180 innings = maintain + average contribution

$= 100 + (45 \times 120)/180$

$= 130$

Example :

After 83 innings batsman has an average of 48 runs. In the 84th inning he scored 100 runs. What's its new average?

Solution:

100 maintained 48 runs average and remaining 52 runs are going to be equally divided in 84 innings. So,

New average = 48 +52/84 = 48.61 runs.

2nd method:

Total runs in 83 innings = $83 \times 48 = 3984$.

Total runs in 84 innings = 3984 + 100 = 4084

New average = 4084/84 = 48.61

Standard situation 3:

The 3rd standard situation that you will see in the average chapter is replacement of a number.

Example 1:

A set of 5 numbers with an average of 13 and one number is replaced. Average is increased by 4. The outgoing number is 32, then find the replaced number?

Solution:

In this situation there is an outgoing number and an incoming number and average changes by 4 for 5 numbers. The difference in the total = $(5 \times 17) - (5 \times 13) = 20$.

Incoming number how much larger = (change in average) $\times$ (number of numbers)

$$= \mathbf{4 \times 5 = 20}$$

If the average increases then it is obvious that the incoming number is larger.

Incoming number - outgoing number = difference in total

Incoming number = 20 + 32 = 52.

NOTE: If average increases then; Ni - No = Difference in total. (Ni = incoming number, No = outgoing number), If average decreases then, No - Ni = Difference in total.

Example 2:

The average age of an office is 32 with 10 people in office. One person is replaced and due to this, the average age decreases by 3. Replaced person age was 52. What is the age of the incoming person?

Solution: Average age decreases means the incoming person's age is smaller than outgoing person's age. So,

$$No - Ni = \text{Difference in total}$$

$$52 - Ni = (10 \times 32) - (10 \times 29)$$

$$Ni = 52 - 30 = 22$$

Example 3:

Average temperature on Monday, Tuesday and Wednesday was 37 degrees. And the average temperature of Tuesday, Wednesday and Thursday was 39 degrees. What is the temperature on thursday?

Solution :

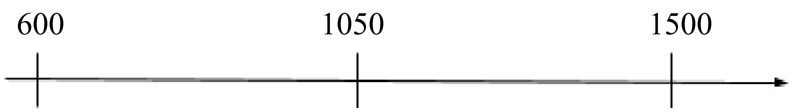
Monday is replaced by Thursday and the average increases by 2. So,

Temperature on thursday = (change in average) $\times$ (number of days)

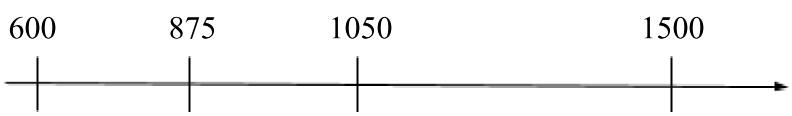
$= 2 \times 3 = 6$ degrees more than monday.

Concept Of Weighted Average

Concept of a weighted average can be understood with the help of an example.
Suppose I had to buy a T-shirt and jeans and let us say that the average cost of a T-shirt was 600, while that of jeans was 1500.
In such a case, the average cost of a T-shirt and jeans would be given by (600 + 1500)/2 = 1050.
This can be observed on the number line as:
(midpoint) = answer.



From the figure it is observed that the average occurs at the midpoint of the two numbers.
Now, let us try to modify the situation:
Suppose I had to buy 3 T-shirts and 1jeans. In such a case I would end up spending (600 + 600 + 600 +1500) = 3300 in buying a total of 4 items. So,
Average = 3300/4 = 825. Clearly, the average has shifted.
On the number line :



It is clearly visible that the average has shifted towards 600 (which was the cost price of the T-shirts, the larger purchased item.)
In a way, this shift is similar to the way a two pan weighing balance shifts when the weights are put on it. The balance shifts towards the pan containing the larger weight.
Similarly, in this case, the correct average (875) is closer to 600 than it is to 1500. Since, this is very similar to the system of weights, we call this a weighted average situation.

Formula for weighted average:

Let say, we have k groups with averages A1, A2 ... Ak and having n1, n2 ... nk elements then the weighted average is;

$$A_w = \frac{n1A1+n2A2+n3A3+..................+nkAk}{n1+n2+n3+..........+nk}$$

Situation involving weighted average

Situation 1: Purchasing two kinds or k varieties of something and mixing them together, to form composite.

Suppose I purchase 30Rs/kg rice and 70Rs/kg rice in the ratio 2:3. What is the average price of rice?

Solution : Average price = (n1A1 +n2A2)/(n1+n2)

Here, A1= 30 , A2 = 70 , n1 = 2 , n2 = 3

Average price = $(2 \times 30 + 3 \times 70)/(2 + 3)$

= 270/5 = 54Rs/kg.

Situation 2:

Let's say you drive a car 30km/hr and 70km/hr and drive it for 2hr and 3 hr respectively. Find the average speed?

Solution : Average speed = (total distance)/(total time)

= $(2 \times 30 + 3 \times 70)/(2 + 3)$

= 270/5 = 54 km/hr.

Situation 1 and 2 are the same but the story is different.

Situation 3:

Let say you invest 2 lac and give 30% return. Investment of 3 lac rupees, give70% return. What is the average % return?

Solution : Average % return = (n1A1 +n2A2)/(n1+n2)

Here, A1= 30% , A2 = 70% , n1 = 2 Lac , n2 = 3 Lac

Average % return = $(2 \times 30 + 3 \times 70)/(2 + 3)$

= 270/5 = 54 %.

Important thing in weighted average :

Let us say 30Rs/kg rice: 2kg and 70Rs/kg rice: 3kg.

And for average we did = $(2 \times 30 + 3 \times 70)/(2 + 3)$ = 54 Rs/kg ………….**(1)**

Suppose I changed the numbers in the question, let say 30Rs/kg rice: 12kg and 70Rs/kg rice: 18kg. Find the average price.

So, using formula $(n1A1 + n2A2)/(n1+n2)$
Average price = $(12 \times 30 + 18 \times 70)/(12 + 18)$
$= (360+1260)/30$
$= 1620/30 = 54$Rs/kg.
Same average price as i obtained with the above numbers.
From this, we observed that, in the given formula we don't need to insert the exact value of n1 & n2. Instead of 12 and 18 kg, If we simply use 2 & 3 and calculate it by equation **(1)**, we would definitely get the same answer. So, in the formula you should always use the ratio of the quantity.

Situation 3:

There are two sections, in section 1 there are 20 students who scored 30 marks on an average in exam, while in section 2 there are 30 students who scored 70 marks on an average in exam. What is the average marks of both the sections?
Solution : Ratio of the quantities 20:30 = 2:3
So, Average marks = $(2 \times 30 + 3 \times 70)/(2 + 3)$
$= 270/5 = 54.$

This situation can be modified into Boys and Girls in a class with ratio 2:3 and Boys average marks is 30 and Girls average marks is 70. So what are the average marks of the class?
Average marks of the class will be 54.

Situation 4: Alloys and Mixture

Let say two water and milk solutions of 2L and 3L, In one solution milk is 30% and other solution milk is 70% respectively. Mix both the solutions then what is the % of milk in the mixture?
Solution : % of milk in the mixture = $(2 \times 30 + 3 \times 70)/(2 + 3)$
$= 270/5 = 54\%.$
Instead of water milk solution, we can take gold and copper alloy, 2kg gold and copper alloy with 30% of gold & 3kg gold and copper alloy with 70% of gold. If both the alloys are mixed and a new alloy is formed, then what is the % of gold in the new alloy?
Solution : % of gold in the new alloy = $(2 \times 30 + 3 \times 70)/(2 + 3)$
$= 270/5 = 54\%.$

These are some important situations that are used in weighted averages.

Some questions for practice:

1. The average of a batsman after 25 innings was 56 runs per innings. If after the 26th inning his average increased by 2 runs, then what was his score in the 26th inning?
   Ans : 108.

2. The average age of a class of 30 students and a teacher reduces by 0.5 years if we exclude the teacher. If the initial average is 14 years, find the age of the class teacher.
   Ans : 29 years.
3. The average marks of a group of 20 students on a test is reduced by 4 when the topper who scored 90 marks is replaced by a new student. How many marks did the new student have?
   Ans : 10.

4. The mean temperature of Monday to Wednesday was 27 °C and of Tuesday to Thursday was 24 °C. If the temperature on Thursday was 2/3rd of the temperature on Monday, what was the temperature on Thursday?
   Ans : 18.

5. A school has only 3 classes that contain 10, 20 and 30 students respectively. The pass percentage of these classes are 20% , 30% and 40% respectively. Find the pass % of the entire school?
   Ans : 33.33%.

Lecture Notes For Alligations

Concept of alligation is closely related to the weighted average.

Alligations is a visual approach to solve weighted averages, involving the mixing of two groups.

For example:

Two varieties of rice at 50 per kg and 80 per kg are mixed together in the ratio 3 : 7. Find the average price of the resulting mixture.

Solution : By using weighted average formula; $Aw = (n1A1 + n2A2) / (n1+n2)$

$$\text{Average price} = (3 \times 50 + 7 \times 80) / (3 + 7)$$
$$= 710 / 10$$
$$= 70.$$

Weighted average approach is slightly slower than, if we see the same situation through alligations. Alligations is a faster approach.

Mathematical formula for alligation:

In the case of a situation where just two groups are being mixed, we can write weighted average formula:

$$Aw = (n1A1 + n2\,A2) / (n1 + n2)$$

Here, we have 2 groups with averages A1, A2 and having n1 and n2 elements respectively.

Rewriting this equation we get:

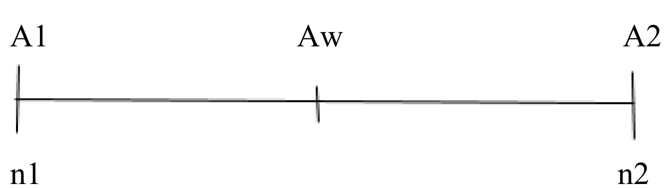
$$(n1 + n2)\,Aw = n1A1 + n2A2$$
$$n1(Aw - A1) = n2\,(A2 - Aw) \text{ or}$$
$$n1/n2 = (A2 - Aw)/(Aw - A1) \text{ ……….. The alligation equation.}$$

As a convenient convention, we take $A1 < A2$. Then, by the principal of averages,

we get $A1 < Aw < A2$.

Straight line approach:

Positions of A1, A2, Aw, n1 and n2 on number line are;

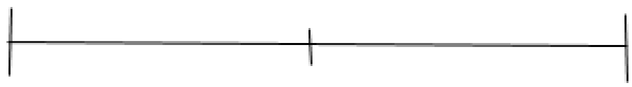


On the number line the points A1, Aw and A2 are in order from left to right because we have the condition: A1 < Aw < A2

Now, according to the given alligation equation; n1/n2 = (A2 – Aw)/(Aw – A1):
(a) n2 is responsible for the distance between A1 and Aw or n2 corresponds to Aw – A1
(b) n1 is responsible for the distance between Aw and A2. or n1 corresponds to A2 – Aw
(c) (n1 + n2) is responsible for the distance between A1 and A2. or (n1 + n2) corresponds to A2 – A1.

Solving the above example of rice mixing by this approach;

50 Aw 80

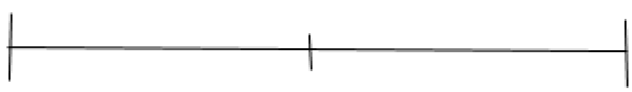


3 7

Since the total distance = (80 - 50) = 30. If we split 30 into 3:7, the value of 3 parts and 7 parts are 9 and 21 respectively.
As, the distance between Aw and 50 is corresponding to n2 (i.e. 7) and 7 parts are 21. So;
I.e. Aw - 50 = 21 ≫ Aw = 71.
OR the distance between 50 and Aw is corresponding to n1(i.e. 3) and 3 parts are 9. So;
80 - Aw = 9 i.e . Aw = 71.

For practice, try yourself:

30 Aw 70 **Answers**

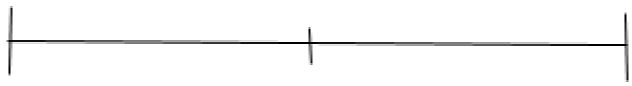


n1	n2	
1	3	60
3	1	40
3	5	55
5	3	45
7	1	35

By this approach you can handle fraction situations too:

For example;

30 Aw 70



5 8

Since the total distance = (70 - 30) = 40. If we split 40 into 5:8,

Aw - 30 = $(8/13) \times 40$

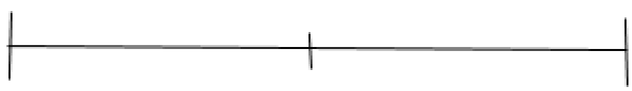
Aw - 30 = 24.62;

So, Aw = 54.62.

Alligation has essential three situations:

Situation 1: When A1, A2, n1 and n2 are known and Aw is unknown.

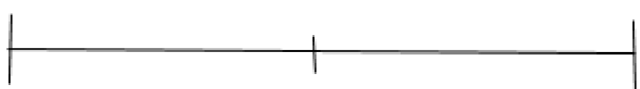
A1 Aw A2



n1 n2

For example:

50 Aw 80



3 7

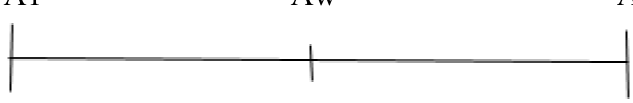
Since the total distance = (80 - 50) = 30. If we split 30 into 3:7, the value of 3 parts and 7parts are 9 and 21 respectively.

Thus the distance between Aw and 50 is corresponding to n2 (i.e. 7) and 7 parts are equal to 21.

I.e. Aw - 50 = 21 ≫ Aw = 71.

Situation 2: When A1, A2 and Aw are known and n1: n2 is unknown.

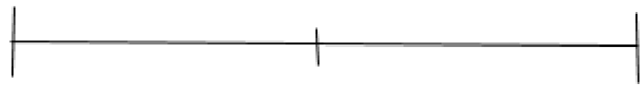
A1 Aw A2



n1 n2

For example:

70 80 84



n1 n2

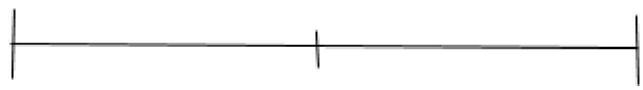
By using alligation equation,

n1/n2 = (A2 – Aw)/(Aw – A1)

n1:n2 = 4:10 or 2:5.

Situation 3: When A1, Aw and n1:n2 are known and A2 is unknown.

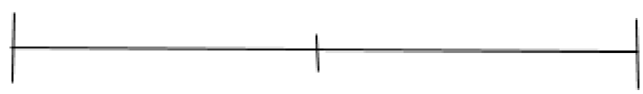
A1 Aw A2



n1 n2

For example:

30 70 A2



3 5

By using alligation equation,

n1/n2 = (A2 – Aw)/(Aw – A1)

3/5 = (A2 - 70)/(70-30)

A2 = 94.

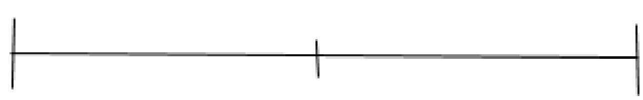
Problems Where You Can Use Alligations-1

Example 1:

Two varieties of rice at 40 per kg and 50 per kg are mixed together in the ratio 2 : 3. Find the average price of the resulting mixture.

Solution :

40 Aw 50

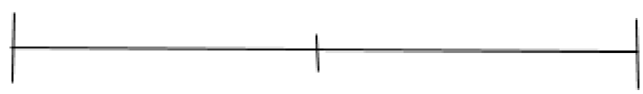


2 3

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2 (i.e. 3) and 3 parts are equal to 6.
I.e. Aw-40 = 6
≫ Aw = 46.
Hence, the average price of the resulting mixture is at 46 per kg.

Example 2:
A man has driven a car at 40kmph and 50kmph. He has driven for 2 hours and 3 hours respectively. Find the average speed of a car?
Solution :

40 Aw 50



2 3

Here, Aw is the average speed of the car.
Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.
i.e. Aw-40 = 6
≫ Aw = 46.
Hence, the average speed of the car is 46kmph.

These two questions are on the surface different from each other, the first one was talking about average price and the other is talking about the average speed, But structurally both are the same.
Equation in 1st question ;

$$\text{Average price} = (n1A1 + n2A2) / (n1 + n2).$$

Here, n1= 2kg, n2= 3kg , A1= 40per kg, A2= 50 per kg.
So,

$$\text{Average price} = (2*40 + 3*50)/(2+3)$$

Equation in 2nd question ;

$$\text{Average speed} = (t1S1 + t2S2) / (t1 + t2).$$

Here t1= 2hr, t2= 3hr , S1= 40kmph, S2= 50kmph.
So,

$$\text{Average speed} = (2*40 + 3*50)/(2+3)$$

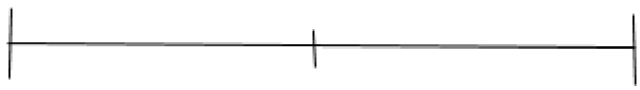
By looking at these two equations you will observe that these both are the same, only difference is in variables.

Example 3:

Class1 has 20 students having average marks of 60 and class2 has 30 students having average marks of 70. Find the average marks of two classes combined ?

Solution :

60 Aw 70



20 30

In weighted average and in alligation we take ratio of the quantities. So, n1 : n2 is 2 : 3.

Since the total distance = (70 - 60) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.

Thus the distance between Aw and 60 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.

i.e. Aw - 60 = 6

≫ Aw = 66.

Hence average marks of two classes is 66.

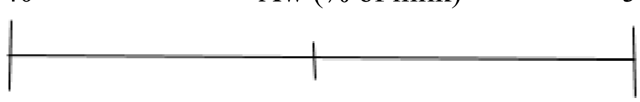
Problems Where You Can Use Alligations-2

Example 1:

We have two mixtures of milk and water, the 1st mixture contains 40% milk & 60% water and the 2nd mixture contains 50% milk & 50% water. These two mixtures are mixed in ratio 2:3, then find the % of milk in the mixture?

Solution : Using milk %

40 Aw (% of milk) 50



2 3

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.

Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.

i.e. Aw-40 = 6

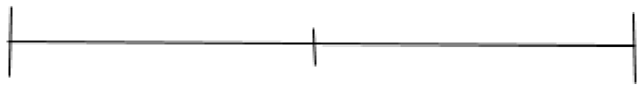
≫ Aw (% of milk) = 46%.

Another way to solve this question is by using water %

The 1st mixture has 60% water and the 2nd mixture has 50% water.

According to convention, we need A1< Aw < A2 and the ratio of 1st mixture to 2nd mixture is 2:3, this will be inverted here because we have to flip the % here to make it according to the given convention.

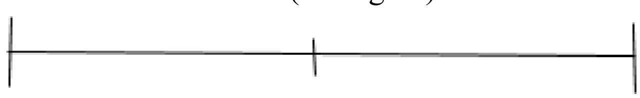
50 Aw (% of water) 60



3 2

Since the total distance = (60 - 50) = 10. If we split 10 into 3:2, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 50 is corresponding to n2(i.e. 2) and 2 parts are equal to 4.
i.e. Aw-50 = 4
≫ Aw (% of water) = 54%.
Thus; % of milk = 100 - 54 = 46%.

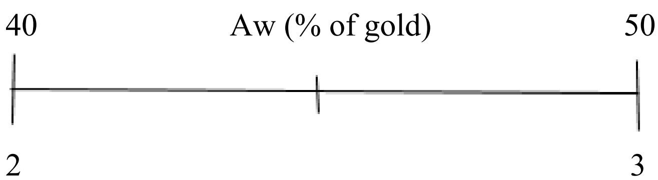
Example 2:
Anjali mixes 2 alloys of gold and copper in ratio 2:3. The 1st alloy contains 40% gold and the 2nd alloy contains 50% gold. Find the gold % in the mixture?
Solution :
40 Aw (% of gold) 50



2 3

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.
i.e. Aw-40 = 6
≫ Aw (% of gold) = 46%.
Another way to solve this question is by using copper %.

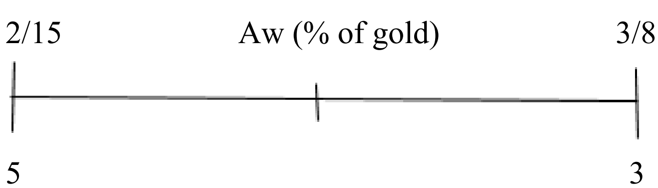
Example 3:
Two alloys of gold and copper mix in ratio 2:3. The 1st alloy contains gold and copper in ratio 2:3 and 2nd alloy contains gold and copper in ratio 1:1. What is the ratio of gold and copper in the final mixture?
Solution :
Ratio 2:3 means 40% gold and 60% copper. & ratio 1:1 means 50% gold and 50% copper.



Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6..
I.e. Aw-40 = 6
≫ Aw (% of gold) = 46%. So, % of copper = 54%
Hence, the ratio of gold and copper in the mixture is 46:54 i.e. 23:27.

The ratio in a particular question can be converted into percentage composition only if the percentages are easy to get.

Example 4:
Two varieties of gold and copper alloy mixed in ratio 3:5. The 1st alloy contains gold and copper in ratio 3:8 & 2nd alloy contains gold and copper in ratio 2:13. What is the final ratio of gold and copper in the mixture?
Solution :
In this situation if you try to put this into alligation, the numbers will not support you because the ratio 2:13 gets converted into % is difficult and putting on a number line is also difficult.
If you still want to do this question through alligation, you will do it. The thought is that the fraction of gold in 1st alloy is 3/11 and in 2nd alloy is 2/15.
Since, 2/15< 3/8 , so the ratio will flip.



Finding the 3 parts and 5parts of total distance is not going to be a very easy calculation and hence alligation in this situation structurally does apply but it's not a good approach for such a type of question.

So, what you have to do in this type of question;

Alloy 1		**Alloy 2**
3	:	5

G:C		**G:C**
3:8		2:13

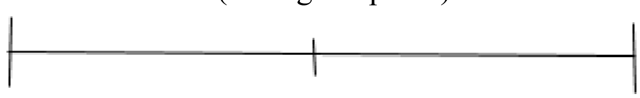
1st you have to take LCM of (3+8) = 11 and (2+13)= 15. Thus , LCM (11&15) = 165.
This means 165 kg of 1st alloy and 165 kg of 2nd alloy in 1 part.

Alloy 1		**Alloy 2**
3	:	5
$165 \times 3 = 495$	:	$165 \times 5 = 825$
G:C		G:C
3:8		2:13

You have 3:8 divisions of 495kg & 2:13 divisions of 825kg. So,
Total gold in the mixture = $(3/11) \times 495 + (2/15) \times 825$
$= 135 + 110 = 235.$
Total mixture = 495 +825 = 1320.
Total copper in the mixture = 1320 - 235 = 1075.
Final ratio of gold and copper in the mixture will be 235:1075 i.e. 49:215.

Problems Where You Can Use Alligations-3

Example 1:
A shopkeeper sold chairs and tables. The ratio of cost price of chair and table is 1:2. He sold chairs at 30% profit and tables at 60% profit. What is the average % profit?
Solution :
30 Aw(average % profit) 60



1 2
Since the total distance = (60 - 30) = 30. If we split 30 into 1 : 2, the value of 1 part and 2 parts are 10 and 20 respectively.
Thus the distance between Aw and 30 is corresponding to n2(i.e. 2) and 2 parts are equal to 20.
I.e. Aw-30 = 20
≫ Aw (average % profit) = 50%.

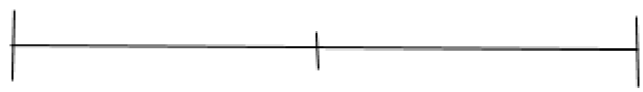
Example 2:

A shopkeeper sold chairs and tables. He sold tables at 20% profit and chairs at 30% loss. Thereby he made no profit or no loss in the transaction. What is the cost price ratio of table to chair?

Solution :

Chair **Table**

-30 0 20



n1 n2

n1/n2 = (A2 – Aw)/(Aw – A1)

Here A1= -30, A2= 20, Aw= 0.

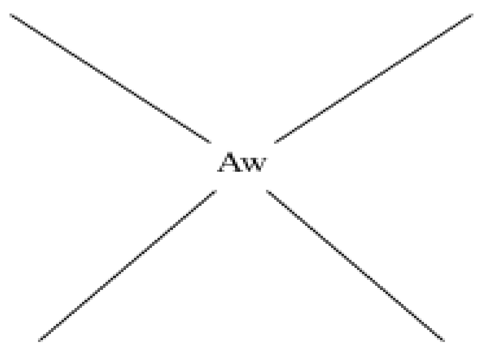
n1/n2 = (20 – 0)/(0 – (-30))

n1/n2 = 20/30 i.e. n1:n2 = 2:3.

Thus, table to chair cost price ratio = 3:2.

Cross diagram approach:

A1 A2



A2-Aw : Aw-A1

n1 : n2

Note : That the cross method yields nothing but the alligation equation. Hence, the cross method is nothing but a graphical representation of the alligation equation.

As we have seen, there are five variables in the alligation equation.

The three averages A1,A2 and Aw. and the two weights n1 and n2.

Example 1:

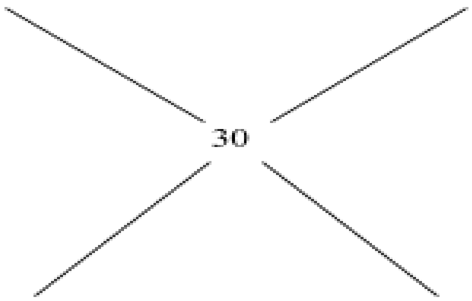
On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

(a) The ratio of students in the classes

(b) The number of students in the first class if the second class had 30 students.

Solution :

25 40



A2-Aw : Aw-A1

10 : 5

(a) The ratio of students in class is 10:5 i.e 2:1.

(b) If the ratio is 2 : 1 and the second class has 30 students, then the first class will have 60 students.

NOTE: 1. A1, A2 and Aw are always rate units, while n1 and n2 are quantity units.

2. All percentage values represent the average values.

Some questions for practice:

---

1. If 5 kg of salt costing 5/kg and 3 kg of salt costing 4/kg are mixed, find the average cost of the mixture per kilogram.
   Ans:4.625kg.
2. Two types of oils having the rates of 4/kg and 5/ kg respectively are mixed in order to produce a mixture having the rate of 4.60/kg. What should be the amount of the second type of oil if the amount of the first type of oil in the mixture is 40 kg?
   Ans:60kg.
3. How many kilograms of sugar worth 3.60 per kg should be mixed with 8 kg of sugar worth 4.20 per kg, such that by selling the mixture at 4.40 per kg, there may be a gain of 10%?
   Ans:4kg.
4. Ravi lends 3600 on simple interest to Harsh for a period of 5 years. He lends a part of the amount at 4% interest and the rest at 6% and receives 960 as the amount of interest. How much money did he lend on a 4% interest rate?
   Ans:1200.
5. 400 students took a mock exam in Delhi. 60% of the boys and 80% of the girls cleared the cut off in the examination. If the total percentage of students qualifying is 65%, how many girls appeared in the examination?

Ans:100 girls.

Lecture Notes for Percentages

Percentages is an important chapter. As a chapter, not only from the point of view of its own questions but apart from that, percentages is a base chapter for a lot of other chapters. E.g. profit and loss, interest these two chapters are completely built on percentages and chapters like time and work, time speed and distance are exclusively built on percentages plus ratio.
Percentage and ratio & proportion are two chapters which will be the base of all these 4 chapters. If you can master these 6 chapters together you build up an ability to handle aptitude exams easily.

What is the percentage?

Basic definition of percentage is essentially out of 100. Percentage is derived from French word 'cent'. Meaning of 'cent' in French is 100.
Percentage is used to compare data and numbers.
For example:
(a) If there are 5 (A,B,C,D,E) students who have taken the 12th board exam from five different boards. The percentages they get is a defined thing i.e. comparison between 5 diverse students in 5 diverse boards.

A	B	C	D	E
86%	92%	94%	78%	52%

By seeing the percentage of these students we can compare which student is better.

(b) GDP defines how the world is doing in terms of Global world economies. GDP compares different countries' economies in terms of their percentage.

Mathematically;
Any ratio if you multiply by 100, it gives you its percentage value. Percentage is denoted by a sign "%".
Why when the ratio is multiplied by 100, gives you a percentage value? You can see that from the unitary method.
Unitary method: It is a method which talks about a situation where two variables are moving linearly w.r.t. each other.

For example:

1. You bought 10 bananas for 30 rupees then, how many rupees will you need to buy 15 bananas?

Solution: let x rupees you will need to buy 15 bananas.

10 bananas = 30Rs

15 bananas = x Rs

Cross multiply and equate;

$10*x = 15 \times 30$

$x = 45.$

So, 45 rupees is the amount that you will need to buy 15 bananas.

2. You scored 10 out of 20 in a quiz and you want to put it in % then, how much out of 100 did you score?

Solution:

10 out of 20.

x out of 100.

So, by unitary method;

$20 \times x = 10 \times 100$

$x = (10/20) \times 100$

$x = 50\%$

NOTE: Any fraction multiplied by 100 gives its percentage value.

Concept of percentage change:

Percentage always happens when you go from one number to the next number.

Basic structure of percentage change will always be in the situation, where you are talking about the difference between two numbers.

Let say we have number x becoming y. The percentage change between x to y.

$$x \longrightarrow y$$

Formula for percentage change:

Percentage change = (change/original value)*100

1st you have to identify which number is the original number that depends on which direction you are looking at percentage change. So, percentage change is always a **directional input**.
If x changes to y the percentage change going from x to y, will be having x as the original value.

$$x \longrightarrow y$$

If y changes to x. So, in this situation the percentage change will have to be seen from y to x and will be having y as the original value.

$$y \longrightarrow x$$

For example;
If you have two numbers 20 & 40. So, going from 20 to 40.

$$20 \longrightarrow 40$$

Here change = 40 - 20 = 20, and original value = 20.
% change = (20/20) × 100 = 100%
The change is +ve. So, % change is increasing by 100%.

20 to 40 have different % change than coming from 40 to 20.

$$20 \longleftarrow 40$$

Here change = 20 - 40 = -20, and original value = 40.
% change = (-20/40) × 100 = -50%
So, % change is decreasing by 50%.

NOTE: 1. In percentage change there should be two numbers.
2. You need to understand which number is the original number.

People make a very common mistake in % change calculation.
In the question given that 50 to 75, instead of this they calculated 75 to 50. Because the language of % change can get complex sometimes, where language structures are used especially in DI.

Percentage Change Graphics(PCG):

It is an important concept in percentage change and important for chapters like interest, profit and loss etc. As the name suggests, percentage change graphics means the graphical method of doing the percentage change.

Basics of percentage change:

1. 100% of a number is a number itself.
2. 10% of a number is a shift of 1 decimal point on the number towards left.
3. 1% of a number is a shift of 2 decimal points on the number towards left.
4. 0.1% of a number is a shift of 3 decimal points on the number towards left and so on….

For example:
Let say a number N=52123.
100% of the number N is 52123
10% of the number N is 5212.3
1% of the number N is 521.23
0.1% of the number Nis 52.123

PCG has two structures:

Structure 1:
Given the starting value and the ending value. You have to calculate:
1. Absolute change (below the arrow).
2. % change(above the arrow).

$$S \xrightarrow[\text{Absolute change}]{\text{\% change}} E$$

Example:
Let us say, 40 changing into 52.

$$40 \xrightarrow{\qquad\qquad} 52$$

Absolute change = 52 - 40 = +12. Absolute change is +ve that means an increase in % change.
10% of the number 40 is 4, and the number 12, is 3 times the number 4 which means that the percentage increases by 30% (3 * 10% = 30%).

$$40 \xrightarrow[+12]{30\% \uparrow} 50$$

Examples:

(1). 50 changing into 70.

Absolute change = 70 - 50 = +20. Absolute change is +ve that means an increase in % change.

10% of the number 50 is 5, and the number 20, is 4 times the number 5 which means that the percentage increases by 40%(4 * 10% = 40%).

$$50 \xrightarrow[+20]{40\% \uparrow} 70$$

(2). 60 changing into 42.

Absolute change = 42 - 60 = -18. Absolute change is -ve that means a drop in % change.

10% of the number 60 is 6, and the number 18, is 3 times the number 6, which means that the percentage decreases by 30%(3 * 10% = 30%).

$$60 \xrightarrow[-18]{30\% \downarrow} 42$$

(3). You can have situations which are a little bit more calculative than this.

E.g. 33 changes into 47.

$$33 \longrightarrow 47$$

Absolute change = 47 - 33 = +14.

10% of 33 is 3.3. If you keep adding 3.3, 4 times you get 13.2.(3.3+3.3+3.3+3.3=13.2).

In 14, you definitely have 40% of 33 included.

14 = 13.2 + 0.8.

0.8 as the percentage of 33. As we know, 1% of 33 is 0.33.

If you add 0.33, 2 times the value you get lies under 0.8 that means 2% increment further on the 40%.

Hence, the % change is in between 42 to 43%.

$$33 \xrightarrow[+14]{42\text{-}43\% \uparrow} 47$$

Structure 2:

1. Starting value is given to you,
2. Percentage change is given to you.
3. Absolute change you need to calculate.
4. And calculating the ending value.

Example1:
There is a number 40 that has to be increased by 30%.

$$40 \xrightarrow{30\% \uparrow}$$

Solution:
We were doing this problem by unitary method.
40 is 100%
x is 130%
Cross multiply and equate;
x = (40 × 130)/100.
Rather than this, much easy calculation is done through percentage change graphics.

$$40 \xrightarrow{30\% \uparrow}$$

10% of 40 is 4. 30% increase means adding 4, 3 times. 4+4+4 = 12 i.e adding 12 in 40 so the ans is 52.

$$40 \xrightarrow[4+4+4=12]{30\% \uparrow} 52$$

Example2:
A number 37 has to be increased by 13%.

$$37 \xrightarrow{13\% \uparrow}$$

Solution: In this question you have to build up 13% by;
10% of 37 is 3.7
1% of 37 is 0.37
1% of 37 is 0.37
1% of 37 is 0.37
So, 13% of 37 is 4.81(3.7+0.37+0.37+0.37 = 4.81), adding 4.18 in 37. So, the answer is 41.81

$$37 \xrightarrow[+4.81]{13\% \uparrow} 41.81$$

PCG applied to percentage change:

The 1st structure under which you can use the percentage change in quantitative aptitude is product change situation.

Example1:

Let say a product $x \times y$. x is increased by 20% and y is increased by 30%. You want to find out what is the % change in the product?

Solution:

x would become x(1+ (20/100)) = $x \times 1.2$

y would become y(1+ (30/100)) = $y \times 1.3$

So, in product; $x \times 1.2 \times y \times 1.3$= 1.56xy. This means, 56% change.

Same question can be done by PCG. If you assume your original product to be 100. And this product will go through two changes, 20% increase in x and 30% increase in y. You have to put two arrows,

One for ‘x’ and other for ‘y’.

$$100 \xrightarrow[\text{‘x’}]{20\%} \qquad \xrightarrow[\text{‘y’}]{30\%}$$

If x increases by 20% the product also increases by 20% and then if y increases by 30% the product also increases by 30%.

$$100 \xrightarrow[+20]{20\%} 120 \xrightarrow[+36]{30\%} 156$$

i.e. 56% increase in the product.

Example2:

A product $x \times y \times z$. Where x increases by 10%, y increases by 20% and z decreases by 30%. What is the % change in product?

Solution:

Let us assume the original product is 100.

$$100 \xrightarrow[+10]{10\%\uparrow} 110 \xrightarrow[+22]{20\%\uparrow} 132 \xrightarrow[-39.6]{30\%\downarrow} 92.4$$

10% increase in 100 = 110 then, 20% increase in 110 = 132 and then, 30% decrease in 132 = 92.4.

92.4 - 100 = -7.6 i.e 7.6% decrease in product.

NOTE: You can order the arrow according to what you want.

For this situation;

$$100 \xrightarrow[-30]{30\% \downarrow} 70 \xrightarrow[+14]{20\% \uparrow} 84 \xrightarrow[+8.4]{10\% \uparrow} 92.4$$

92.4 - 100 = -7.6 i.e 7.6% decrease in product.

Some question for practice:

---

1. A product $x \times y \times z$. Where x increases by 5%, y increases by 20% and z decreases by 10%. What is the % change in product?
   Ans: 13.4% increase.
2. A product $x \times y \times z$. Where x decreases by 30%, y increases by 20% and z increases by 20%. What is the % change in product?
   Ans: 0.8% increase.
3. A product $x \times y \times z$. Where x decreases by 20%, y increases by 30% and z increases by 30%. What is the % change in product?
   Ans: 35.2% increase.

Problems on percentage change:

Area and volume based problem:

Problem 1:

The length of a rectangle goes up by 30% and the breadth of the rectangle comes down by 10%. What is the percentage change in area?

Solution:

Area = $l \times b$ and now it becomes a product change situation.

Assume the original area = 100. Makes two arrows one for length and other is for breadth.

$$100 \xrightarrow[\text{'l'}]{30\% \uparrow} \quad \xrightarrow[\text{'b'}]{10\% \downarrow}$$

$$100 \xrightarrow[+30]{30\% \uparrow} 130 \xrightarrow[-13]{10\% \downarrow} 117$$

Hence 17% is the increase in the area of the rectangle.

Problem 2:

The length of a rectangle is decreased by 20% and the breadth of the rectangle is increased by 23%. What is the percentage change in area?

Solution:

Area = $l \times b$ and now it becomes a product change situation.

Assume the original area = 100. Makes two arrows one for length and other is for breadth.

$$100 \xrightarrow[\text{‘b’}]{23\% \uparrow} \quad \xrightarrow[\text{‘l’}]{20\% \downarrow}$$

For easy calculation we put breadth on the 1st arrow and length on the 2nd arrow.

$$100 \xrightarrow[+23]{23\% \uparrow} 123 \xrightarrow[-24.6]{20\% \downarrow} 98.4$$

Hence, 1.6% is the decrease in the area of the rectangle.

We can do the same problem with the help of following formula;

Percentage change = (a + b + ab/100)

Let us say, x increases by 20% and y increases by 10%. Then the percentage change;

Percentage change = 20 + 10 + (20 × 10)/100

= 32%.

But rather than this PCG is a more easy way to solve this problem.

One other problem to this formula, if $x \times y \times z$ situation occurs then the formula can not take a change 3 components of the product. PCG is always better for these problems.

Problem 3:

Length, Breadth and Height of a cuboid are decreased by 30%, increased by 20% and increased by 20% respectively. What is the percentage change in the volume of cuboid?

Solution :

Volume of cuboid = $l \times b \times h$

Assume the original volume = 100.

$$100 \xrightarrow[-30]{30\%\downarrow} 70 \xrightarrow[+14]{20\%\uparrow} 84 \xrightarrow[+16.8]{20\%\uparrow} 100.8$$

0.8% increment in the volume of cuboid.

Expenditure and revenue problem:

Problem 1:

Price of a commodity has gone up by 20% and a person reduces its consumption by 10%. What is the % change in the expenditure?

Solution :

Price $\times$ consumption = expenditure.

Assume the original expenditure = 100. Makes two arrows one for price and other is for consumption.

$$100 \xrightarrow[\text{‘P’}]{20\%\uparrow} \quad \xrightarrow[\text{‘C’}]{10\%\downarrow}$$

$$100 \xrightarrow[+20]{20\%\uparrow} 120 \xrightarrow[-12]{10\%\downarrow} 108$$

Hence, 8% is the increase in the expenditure of the commodity.

Problem 2:

A shopkeeper selling chairs, reduces the price of chairs by 20% due to which he gets an increment of 60% in the sale. What is the percentage change in the revenue?

Solution :

Price $\times$ sale = revenue.

Assume the original revenue = 100. Makes two arrows one for price and other is for sale.

$$100 \xrightarrow[\text{‘P’}]{20\%\downarrow} \quad \xrightarrow[\text{‘S’}]{60\%\uparrow}$$

$$100 \xrightarrow[-20]{20\%\downarrow} 80 \xrightarrow[+48]{60\%\uparrow} 128$$

Hence, 28% is the increment in the revenue.

Problem 3:

Speed of a car increases by 30%, and the time for which it travels is increased by 40%. How much percent is the increment in the distance?

Solution :

Speed $\times$ Time = Distance.

Assume the original distance = 100. Makes two arrows one for speed and other is for time.

$$100 \xrightarrow[\text{‘S’}]{30\%\uparrow} \qquad \xrightarrow[\text{‘T’}]{40\%\uparrow}$$

$$100 \xrightarrow[+30]{30\%\uparrow} 130 \xrightarrow[+52]{40\%\uparrow} 182$$

Hence, 82% is the increment in the distance.

NOTE: Anywhere you have a product relationship between two variables, you will always be able to use PCG on it.

PCG applied to product constancy:

Product constancy is after the series of changes, you need to come back to the original value. Product constancy is applied in a lot of questions directly.

$$100 \longrightarrow \qquad \longrightarrow \qquad \longrightarrow 100$$

Problem 1:

Price of a commodity has gone up by 25% and the consumption is reduced such that the expenditure remains constant.

Solution :

Price $\times$ consumption = expenditure.

Let 100 be the original expenditure after two change one on price and other on consumption, the expenditure should be back at 100.

After a 25% increment in price, expenditure becomes 125. So, 125 should be reduced by 25 to keep expenditure constant i.e consumption reduced by 20%.

$$100 \xrightarrow[+25]{25\%\uparrow} 125 \xrightarrow[-25]{20\%\downarrow} 100$$

25% increase in price is offset by 20% decrease in consumption to keep expenditure constant.

Problem 2:

The length of a cuboid has increased by 20%, the breadth has increased by 50%. How much should you reduce the height to keep the volume constant?

Solution :

Volume = $l \times b \times h$

After 20% and 50% increment in length and breadth respectively, volume becomes 180. So, 180 should be reduced by 80 to keep volume constant i.e height dropped by 44.44%.

Drop in height = (80/180) $\times$ 100

= (4/9) $\times$ 100 = 44.44%

$$100 \xrightarrow[+20]{20\%\uparrow} 120 \xrightarrow[+60]{50\%\uparrow} 180 \xrightarrow[-80]{44.4\%\downarrow} 100$$

Ratio-percentage equivalence:

1/2 = 50%	1/11 = 9.09%
1/3 = 33.3%	1/12 = 8.33%
1/4 = 25%	1/13 = 7.69%
1/5 = 20%	1/14 = 7.14%
1/6 = 16.67%	1/15 = 6.66%
1/7 = 14.28%	1/16 = 6.25%
1/8 = 12.5%	1/17 = 5.88%
1/9 = 11.11%	1/18 = 5.55%
1/10 = 10%	1/19 = 5.26%
	1/20 = 5%

These percentage values you should have to remember to make the calculations easier and faster.

Product constancy table:

Product = $x \times y$

To make the product constant one component increases then, the other component should be decreased.

	'x' increases(%)	**'y' decreases(%)**
Standard Value 1	9.09	8.33
Standard Value 2	10	9.09
Standard Value 3	11.11	10
Standard Value 4	12.5	11.11
Standard Value 5	14.28	12.5
Standard Value 6	16.66	14.28
Standard Value 7	20	16.66
Standard Value 8	25	20
Standard Value 9	33.33	25
Standard Value 10	50	33.33
Standard Value 11	60	37.5
Standard Value 12	66.66	40
Standard Value 13	75	42.85
Standard Value 14	100	50

For example:

Standard value 7 ;

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[-20]{16.67\% \downarrow} 100$$

The fractional view to the product constancy table :

There is a fractional view of the product constancy table also.

Product = $x \times y$

To make the product constant one component increases then, the other component should be decreased.

'x' increases	**'y' decreases**
1/2 ↑	1/3 ↓
1/3 ↑	1/4 ↓

1/4 ↑	1/5 ↓
1/5 ↑	1/6 ↓
1/7 ↑	1/8 ↓
1/9 ↑	1/10 ↓
1/11 ↑	1/12 ↓
1/12 ↑	1/13 ↓ …...and so on.

Decrease in ‘y’ means the denominator of the fractional part of ‘y’ is more than the denominator of the fractional part of ‘x’.

Generic form of this table, If ‘x’ increases by **1/a**, then ‘y’ will drop by **1/(a+1)**.

The advantage of a fractional table is, it is easier to remember than the % view of the product constancy table.

For example:

If I gave you any non standard value. Such as 1/26 is growth, you know that 1/27 has to be dropped.

It is not necessary that the fraction would be in 1/a form.

For example:

x increased by 3/14 then y will decrease by 3/17.

It is not 1/a or 1/(1+a).

If ‘x’ increases by **x/a**, ‘y’ will drop by **x/(a+x)**.

Problem 1:

A man travelling over a certain distance. He is going from Delhi to Chandigarh at a certain speed and takes a certain time to cover the distance. If speed increases by 3/23. What will be the reduction of time so that distance remains constant?

Solution :

$Distance = speed \times time$

Speed is increased by 3/23. So, for distance being constant time will reduce by 3/26.

Problem 2:

A man is going at a certain speed and he takes 260 min to reach his destination. If he increases the speed by 3/23, how much time will he take to cover the journey?

Solution :

$Distance = speed \times time$

Speed is increased by 3/23. So, for distance being constant time will reduce by 3/26.

Original time = 260min and it is reduced by 3/26. So, time = $(3/26) \times 260$ = 30min. Time will drop by 30min. Hence, he will take 230min to cover the same journey.

Problems on product constancy :

Already we have discussed what is product constancy and we did some standard problems based on product constancy. Some more problems on product constancy are:

Problem 1: The price of a commodity has gone up by 25%. To keep the total expenditure on the commodity constant, by what percentage you have to reduce consumption?

Solution :

Price $\times$ consumption = expenditure.

Let 100 be the original expenditure after two change one on price and other on consumption, the expenditure should be back at 100.

After a 25% increment in price, expenditure becomes 125. So, 125 should be reduced by 25 to keep expenditure constant i.e consumption reduced by 20%.

$$100 \xrightarrow[+25]{25\% \uparrow} 125 \xrightarrow[-25]{20\% \downarrow} 100$$

25% increase in price is offset by 20% decrease in consumption to keep expenditure constant.

Problem 2: Speed of a car has gone up by 50%. How much would the time come down to covering the same distance?

Solution :

$Distance = speed \times time$

Let 100 be the original distance.

$$100 \xrightarrow[+50]{50\% \uparrow} 150 \xrightarrow[-50]{33.33\% \downarrow} 100$$

Reduce in time = 50/150 = 1/3, fraction 1/3 is equivalent to 33.33%.

After 50% increment in speed, distance becomes 150. So, 150 should be reduced by 50 to keep distance constant i.e time reduced by 33.33%.

Problem 3: In a triangle the length has increased by 40% and you want to restrict breadth percentage change, such that increase in area of the triangle is limited to 60%. What is the maximum percentage change in the breadth?

Solution :

Area of triangle = 1/2 × *l* × *b*

Let 100 be the original area of the triangle and area of the triangle limited to 60% that means the final area would be 160.

$$100 \xrightarrow[\substack{+40 \\ \text{‘l’}}]{40\% \uparrow} 140 \xrightarrow[\substack{+20 \\ \text{‘b’}}]{14.28\% \uparrow} 160$$

Here the last part is 160. This can be described as targeted % change in the product.

% change in breadth = 20/140 = 1/7, fraction 1/7 is equivalent to 14.28%.

After a 40% increment in length, there is an 14.28% increment in breadth to restrict the area 160.

Problem 4: Price of a commodity has gone up by 40% and Shubham wants to limit his expenditure increase to 5%. What is the reduction in consumption, so that expenditure increases 5%?

Solution :

Price × consumption = expenditure.

Let 100 be the original expenditure and expenditure limited to 5% that means the final expenditure would be 105.

$$100 \xrightarrow[+40]{40\% \uparrow} 140 \xrightarrow[-35]{25\% \downarrow} 105$$

% change in consumption = 35/140 = 1/4, fraction 1/4 equivalent to 25%.

Hence consumption dropped by 25%, so that expenditure was limited to a 5% increase.

PCG applied on successive percentage change :

Successive percentage change use of PCG is structurally very similar to product change use of PCG. One small difference is that in product change we have seen that the arrows are interchangeable w.r.t. each other but in successive percentage change use of PCG we can not interchange the arrows because sometimes we need intermediate value, if we interchange the arrows then we do not get the exact intermediate value. You can understand that difference through some examples/problems.

Problem 1:

Population of the town goes up by 20% in 1st year, comes down by 10% in 2nd year and goes up by 5% in 3rd year. What is the % change in population after 3 years ?

Solution :

Let the population of the town is 100. Population after one year becomes 120 with an increase of 20%. Population after 2 year will become 108 and after the 3rd year the population will become 113.4.

$$100 \xrightarrow[+20]{20\%\uparrow} 120 \xrightarrow[-12]{10\%\downarrow} 108 \xrightarrow[+5.4]{5\%\uparrow} 113.4$$

% change in the population after 3 years is 13.4%. But the intermediate value is important, if anyone asks what is the % change in population after 2 years.

If you interchange the arrows e.g 10% is placed on the last arrow.

$$100 \xrightarrow[+20]{20\%\uparrow} 120 \xrightarrow[+6]{5\%\uparrow} 126 \xrightarrow[-12.6]{10\%\downarrow} 113.4$$

Final value does not make a difference. But after two year the population value is wrong. If the question is built on intermediate value then you will go wrong if you do not keep the arrow constant as they are, that is the only difference in this.

Problem 2:

A shopkeeper successively marks his goods by 20% increase, 30% increase and 50% increase and then gives a discount of 10% to his customers. What is the percentage profit to the shopkeeper?

Solution :

Let the original cost is 100. He is marking up 3 times successively, 1st mark up on 1st arrow,2nd markup on 2nd arrow, 3rd markup on 3rd arrow and discount on the 4th arrow.

$$100 \xrightarrow[+20]{20\%\uparrow} 120 \xrightarrow[+36]{30\%\uparrow} 156 \xrightarrow[+78]{50\%\uparrow} 234 \xrightarrow[-23.4]{-10\%\downarrow} 210.6$$

% profit of the shopkeeper is 110.6%.

Problem 3:

A product $a \times b \times c \times d$, a increased by 20%, b is increased by 30%, c is increased by 50% and d decreased by 10%. What is the percentage change in the product?

Solution :

Let the original product is 100. So;

$$100 \xrightarrow[\substack{+20 \\ \text{'a'}}]{20\% \uparrow} 120 \xrightarrow[\substack{+36 \\ \text{'b'}}]{30\% \uparrow} 156 \xrightarrow[\substack{+78 \\ \text{'c'}}]{50\% \uparrow} 234 \xrightarrow[\substack{-23.4 \\ \text{'d'}}]{-10\% \downarrow} 210.6$$

% change in product is 110.6%.

Problems on successive percentage change :

We have already discussed successive markup and discount problems and also discussed population problems.

Problem 1:

A man's salary is 100Rs out of his salary he spends 20% on food, 30% of the remaining on household expenses, 10% of total on entertainment and saves the rest.

(1). What percentage of income does he save?

(2). What percentage of income does he spend?

(3). What is the ratio of household expense to the entertainment expense?

Solution :

Total salary = 100.

Food expense is 20% of 100 = 20Rs.

Remaining salary = 100 - 20 = 80Rs

Household expense is 30% of remaining salary i.e. 30% of 80 = 24Rs

Entertainment expense is 10% of 100 = 10Rs.

Total expense = 20+24+10 = 54Rs

Saving = 100 - 54 = 46.

(1). Saving percentage = 46%

(2). % of income he spend = 54%

(3). Ratio of household expense to the entertainment expense = 24/10 = 12:5.

If a question is asked on absolute value, you can not be answering that because you do not know any value in this situation.

Let's say he save rupees 9200

The connection between the value in the left box and value in the right box, 46 becoming 9200. Give a multiplier of **200** (9200/46 = 200),then you allow to multiply any of the other numbers in the left box by 200 to answer any question asked about absolute value.

	Left box	**Right box** (Actual value)
Income	100	………

Food expense	20	………
Household expense	24	…….....
Entertainment expense	10	…........
Saving	46	9200

If you have one connector value then you can find any value.
For example here:
Total income = $100 \times 200 = 20000$.
Food expense = $20 \times 200 = 4000$.
Household expense = $24 \times 200 = 4800$.
Entertainment expense = $10 \times 200 = 2000$.

Problem 2:
A machine depreciates in value by 10% every year for 3 years consecutively before repair and maintenance in the 4th year increases the value by 10%. What is the final value of the machine after the 4th year?
Solution :

$$100 \xrightarrow[-10]{10\%\downarrow} 90 \xrightarrow[-9]{10\%\downarrow} 81 \xrightarrow[-8.1]{10\%\downarrow} 72.9 \xrightarrow[+7.29]{10\%\uparrow} 80.19$$

Final value of machine after 4th year = 80.19
If, the actual value of the machine after two year was seen as 162000 then;
(1). What is the original value?
(2). What is the value after 1st year?
(3). What is the value after 3 years?
(4). What is the value at the end of 4th year?

Ans: Here, 81 becomes 162000. 162000/81 gives a multiplier of 2000.
(1). Original value = $100 \times 2000 = 200000$.
(2). Value after 1st year = $90 \times 2000 = 180000$.
(3). Value after 3 years = $72.9 \times 2000 = 145800$.
(4). Value at the end of 4th year = $80.19 \times 2000 = 160380$.

Problem 3:
A shopkeeper gives 3 successive discounts of 20%, 30% and 50%. What is the equivalent total single discount?

Solution :
Let the original value of markup price of products is 100.

$$100 \xrightarrow[-20]{20\%\downarrow} 80 \xrightarrow[-24]{30\%\downarrow} 56 \xrightarrow[-28]{50\%\downarrow} 28$$

Final price of the product is 28 when its original markup price is 100. 100 coming down to 28. Hence, equivalent single discount = 100 - 28 = 72%.

A to B to A problems (compare two numbers) :

Very often we face a situation, where we compare two numbers, say A and B. In such cases, if we are given % change from A to B, then the reverse relationship can be determined by using PCG in the same way as the product constancy.

Problem 1:
A’s salary is 25% more than B’s salary. By what percent is B’s salary less than A’s salary?
Solution :
Let B’s salary = 100.

$$100(B) \xrightarrow[+25]{25\%\uparrow} 125(A) \xrightarrow[-25]{20\%\downarrow} 100(B)$$

A drop of 25 on 125 gives a 20% drop.
Hence B’s salary is 20% less than A’s.
NOTE: Product constancy table is also useful for this situation.

Problem 2:
B gets 20% more marks than A and C gets 50% more marks than B, then how much % less than C does A get?
Solution :
Lets A’s marks = 100.

$$\underset{\text{‘A’}}{100} \xrightarrow[+20]{20\%\uparrow} \underset{\text{‘B’}}{120} \xrightarrow[+60]{50\%\uparrow} \underset{\text{‘C’}}{180}$$

Coming back from C to A, a drop of 80 on 180 i.e 80/180 = 4/9. The fraction 4/9 is equivalent to 44.44%. Hence, A gets 44.44% marks less than C.

Problem 3:
A shopkeeper marks up his goods by 25% and the selling price for his goods is 192. What was its cost price?
Solution :
So, come back from 192 to the cost price and we know standard pair 25% increase means 20% decrease.

$$\text{Cost price} \quad \underset{20\% \downarrow}{\overset{25\% \uparrow}{\rightleftarrows}} \quad 192$$

20% drop on 192 = - (192 × 20)/100 = -38.4.
Hence, cost price = 192 - 38.4 = 153.6.

Problem 4:
A shopkeeper increases the price of his goods by 3/13 and the selling price for his goods is 320. What was its cost price?
Solution :
We know the fractional implication of the product constancy table. If we go from A to B 3/13 increase, we will have to come back with a 3/16 decrease.

$$\text{Cost price} \quad \underset{3/16 \downarrow}{\overset{3/13 \uparrow}{\rightleftarrows}} \quad 320$$

1/16 of 320 = 20 and 3/16 of 320 = 60.
Hence, cost price = 320-60 = 260.

Some questions for practice :

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1. Mr. Navdeep is worried about the balance of his monthly budget. The price of petrol has increased by 40%. By what percent should he reduce the consumption of petrol so that he is able to balance his budget?
   Ans : 28.56%.

2. In Question 1, if Mr. Navdeep wanted to limit the increase in his expenditure to 5% on his basic expenditure on petrol then what should be the corresponding decrease in consumption so that expenditure exceeds only by 5%?
   Ans : 25%.

3. Ram sells his goods 25% cheaper than Shyam and 25% dearer than Ghanshyam. How much percentage of Ghanshyam's goods are cheaper than Shyam's?
   Ans : 40%.
4. In an election between 2 candidates, Shubham gets 65% of the total valid votes. If the total votes were 6000, what is the number of valid votes that the other candidate Anjali gets if 25% of the total votes were declared invalid?
   Ans : 1575.
5. In a medical certificate, by mistake a candidate gave his height as 25% more than normal. In the interview panel, he clarified that his height was 5 feet 5 inches. Find the percentage correction made by the candidate from his stated height to his actual height.
   Ans : 20%.
6. Arjit generally wears his father's coat. Unfortunately, his cousin Shaurya poked him one day that he was wearing a coat of length more than his height by 15%. If the length of Arjit's father's coat is 120 cm then find the actual length of his coat.
   Ans : 104.34.
7. A number is mistakenly divided by 5 instead of being multiplied by 5. Find the percentage change in the result due to this mistake.
   Ans : 96%.
8. If 65% of x = 13% of y, then find the value of x if y = 2000.
   Ans : 400.
9. In a mixture of 80 litres of milk and water, 25% of the mixture is milk. How much water should be added to the mixture so that milk becomes 20% of the mixture?
   Ans : 20L.
10. 50% of a% of b is 75% of b% of c. Which of the following is c?
    Ans : 0.667a.

Lecture Notes for Ratio, Proportion & Variation

Ratio, Proportion and Variation in an important chapter like Percentages. It is the base chapter for some other chapters such as Time and work; Time, Speed and Distance.

Concept of Ratio :

The ratio is a method to compare quantities. When you compare the quantities the first thing that comes to mind is that the quantities should be in the same unit.

For example :
20kmph and 30kmph are the two quantities which are in the same unit.
So,
Ratio = 20/30 = 2/3
= 2:3.
If quantities are in different units, then they can't be compared.
For example:
20 kmph and 18Rs/kg are the two quantities in different units. So, these two quantities can't be compared.

NOTE: 1. Ratio is always a unitless quantity.
2.The numerator is called the antecedent and the denominator is called the consequent of the ratio.

Ratios can be expressed as percentages. To express the value of a ratio as a percentage, we have to multiply the ratio by 100.
Thus, 4/5 = 0.8 = 80%

Some important properties of ratio :

1. If we multiply the numerator and the denominator of the ratio by the same number, the ratio does not change.
   Thus, , multiplying 'm' by both numerator and denominator of the same ratio gives,
   a/b = ma/mb
   For example :
   For Ratio = 3/4
   Multiply the numerator and the denominator by 6 i.e 3/4 = $(3 \times 6)/(4 \times 6)$ = 18/24
   Here 3/4 is the **lowest/basic form** of a ratio. This lowest/basic form gives the infinite number of ratio values.
   For example :
   3/4=6/8=15/20=18/24=......................so on.

NOTE: In the lowest form of ratio the numerator and the denominator are always coprime numbers.

2. If we divide the numerator and the denominator of a ratio by the same number, then the ratio does not change. Thus;
   Dividing 'd', by both numerator and denominator or ratio a/b gives,
   a/b = ($a \div d$)/ $b \div d$)

3. Dividing one ratio by another ratio can be expressed as a new ratio.
   Let the 2 ratios be 'a/b' and 'c/d'. Therefore,
   (a/b) ÷ (c/d) OR
   a/b:c/d = ad/bc
   For example:
   2/3:4/5 = (2 × 5)/(4 × 3)
   = 10/12.

4. The multiplication of two ratios a/b and c/d gives:
   a/b × c/d = ac/bd.

5. If a/b = c/d = e/f = k then;
   (a+c+e)/(b+d+f) = k.
   For example : 2/3 = 4/6 = 10/15 = 200/300 = k then,
   (2+4+10+200) / (3+6+15+300) = 216/324 = 2/3.

6. When numbers are added in both numerator and denominator to maintain equality, then the numbers should have the same ratio as that of the original ratio in which we are adding.
   Let say ratio = 400/800
   400/800 = (400+2)/(800+4) i.e a/b = (a + c)/(b + d) if and only if c/d = a/b.

7. In a ratio, if we add two numbers such that their ratio is larger than the original ratio, then the final ratio becomes larger.
   Let say a ratio = 400/800.
   (400+5)/(800+7).Here, ratio 5/7 is larger than the original ratio(400/800 =1/2).
   i.e c/d > a/b then (a + c)/(b + d) > a/b
   i.e. (400+5)/(800+7) > 400/800
   In case you add a smaller ratio than your final ratio will be less than original ratio.
   Let say a ratio = 400/800.
   (400+3)/(800+7). Here, the ratio 3/7 is smaller than original ratio.
   i.e. c/d < a/b then (a + c)/(b + d) < a/b
   i.e. (400+3)/(800+7) < 400/800

8. If, some ratio is in fractional form, then to convert it into integral ratio, multiply all fractions by LCM of their denominators.
   For example:
   1/2 : 3/5 : 7/6 to convert this ratio into integral ratio, multiply all the fractions by LCM of their denominators (2,5&6). LCM(2,5,6) = 30.
   i.e 30/2 : $(3\times30)/5$: $(7\times30)/6$ = 15:18:35.

Chain Ratio :

Chain ratio is a ratio in which one to next, next to the next and next to next ratios are given.
Let say A:B, B:C and C:D are chain ratios given and convert these ratios into A:B:C:D.
For example :
A:B = 3:5, B:C = 7:8 then, convert chain ratios into a single ratio A:B:C.
Here B being a common element in both the ratios. To equate 5 & 7, take LCM of 5 & 7.
LCM(5,7) = 35. To make common element 35. Multiply the ratios A:B and B:C by 7 and 5 respectively. Thus, A:B will become 21:35 and B:C will become 35:40. B is the same in both cases.
Hence A:B:C is 21:35:40.

If there are 4 and 5 ratios in this case the LCM process will become tedious.
Let us say, A:B = 3:5, B:C = 7:8 and C:D = 9:13. Find A:B:C:D?
Solution :
We have already calculated A:B:C is 21:35:40 and we have C:D is 9:13. C is a common element in both the ratio. To equate 40 and 9, take LCM of 40 & 9.
LCM(40,9) = 360. To make common element 360. Multiply the ratio A:B:C and C:D by 9 and 40 respectively. Thus; A:B:C will become 189:315:360 and C:D will become 360:520. C is the same in both cases.
Hence A:B:C:D is 189:315:360:520.
If D:E is also there this will become even longer to do, because you will have to take LCM 3 times.

Bypass Method :
There is a bypass to this without doing LCM to convert it into a single ratio.
Let us say A:B is N1:D1, B:C is N2:D2, C:D is N3:D3 and D:E is N4:D4. Find A:B:C:D:E.
The value of A would correspond to the multiplication of all numerators. So, A would be N1N2N3N4.
Value of B would be D1N2N3N4.
Value of C would be D1D2N3N4.
Value of D would be D1D2D3N4.
And the value of E would be D1D2D3D4.

A B C D E

N1N2N3N4 : D1N2N3N4 : D1D2N3N4 : D1D2D3N4 : D1D2D3ND

For example :

A:B is 3:5, B:C is 7:8, and C:D is 9:13. Find A:B:C:D.

Solution :

A B C D

N1N2N3: D1N2N3: D1D2N3: D1D2D3

A B C D

$3 \times 7 \times 9 \ : \ 5 \times 7 \times 9 \ : \ 5 \times 8 \times 9 \ : \ 5 \times 8 \times 13$

A B C D

189 : 315 : 360 : 520

Problem :

There are three sections A, B and C in a school. Section A & B have a student ratio 5:7. Section B & C have a student ratio 8:11. The number of students in section C is 154. What is the total no of students in all sections?

Solution :

Given A:B is 5:7 and B:C is 8:11. A:B:C will be;

A B C

$5 \times 8 \ : \ 7 \times 8 \ : \ 7 \times 11$

A:B:C is 40: 56: 77.

Number of students in section C is 154.

Assume A= 40x, B = 56x and C=77x.

We have C = 154. Thus; 77x = 154, x = 2.

Students in section A = 40×2 = 80. Students in section B = 56×2 = 112.

Total number of students in all sections = 80 + 112 + 154 = 346.

Multiplier logic :

It is an important construct of thinking in a ratio situation.

In the last topic, we had a question of 3 sections in a class. In that we had a ratio 40:56:77. And the number of students in section C was 154.

We assumed 3 numbers were 40x,56x and 77x.

We had C = 154. Thus; 77x = 154,

x = 2. **Here x = 2 is a multiplier.**

Students in section A = 40×2 = 80. Students in section B = 56×2 = 112.

Total number of students in all sections = 80 + 112 + 154 = 346.

1st way in which a multiplier could be communicated to you:

Sometimes this multiplier will be communicated to you by giving you an individual value of one of the given numbers.

Let us say 3 children have toys in the ratio 3:4:9. The child with the largest number of toys is 36 toys.
i.e 9 is 36, Which means a multiplier of 4.
Hence, the number of toys with each child will be $3 \times 4 = 12$, $4 \times 4 = 16$ and $9 \times 4 = 36$.

2nd way in which a multiplier could be communicated to you:
Let us say the salary of three people is 5:7:13 and the total is 225.
Total of ratio 5:7:13 is 25. And the total in the actual number running parallel to the given ratio is 225. i.e 25 is 225, which means multiplier of 9.
Hence the numbers are $5 \times 9 = 45$, $7 \times 9 = 63$ and $13 \times 9 = 117$.

3rd way in which a multiplier could be communicated to you:
If a ratio 5:7:13 is given. If the difference between the smaller two numbers is 18.
Difference between smaller two numbers = 7-5 = 2. So, 2 is 18, which means a multiplier of 9.
Hence the numbers are $5 \times 9 = 45$, $7 \times 9 = 63$ and $13 \times 9 = 117$.

Concept Of Proportion :

Proportion basically equates to two or more ratios. When two ratios are equal, the four quantities composing them are said to be proportionals. Thus if a/b = c/d, then a, b, c, d are proportionals. The proportion can be written as;
a:b::c:d, that means a is to b as c is to d. Also, it can be written as a:b = c:d.

NOTE: The terms a and d are called the extremes while the terms b and c are called the means.

- ➢ If four quantities are in proportion then the product of extremes and product of means are equal.
  Let a,b,c and d are in proportion. Then ; $a \times d = b \times c$ i.e, ad = bc.
- ➢ Sometimes the mean proportion is the same.
  Let say a:b::b:c is referred to as a continued proportion. Thus, the product of extremes is equal to the product of means.
  $a \times c = b \times b$ i.e $b^2 = ac$ or we can say that $b = \sqrt{ac}$. So, b is called a geometric mean between a & c.

NOTE: Mean proportion is always the geometric mean of extremes.

Example 1:
Let us say 2:3::a:33. What is the value of a?
Solution :

the product of extremes = the product of means

$2 \times 33 = 3 \times a$

a = 22.

Proportion is not used too often in questions but it is a more supportive structure to the ratio chapter.

Some proportion operations :

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1. **Invertendo:** If a/b = c/d then b/a = d/c
2. **Alternando:** If a/b = c/d, then a/c = b/d
3. **Componendo:** If a/b = c/d, then (a+b)/b = (c+d)/d.
4. **Dividendo:** If a/b = c/d, then (a-b)/b = (c-d)/d.
5. **Componendo and Dividendo:** If a/b = c/d, then (a + b)/(a – b) = (c + d)/(c – d)

Lecture Notes For Profit And Loss

Profit and loss is an important topic of the arithmetic section of quantitative aptitude. You will find this chapter's application in certain DI questions as well. It is used to determine the price of a commodity in the market and understand how to profit an organization. Every product has a cost price and selling price. Based on these values we can calculate profit and loss of a product.

The basic concept of profit and loss:

- **Cost price:** The price at which an item is purchased is called its cost price (C.P).
- **Selling price:** The price at which an item is sold is called its selling price (S.P).
- **Profit:** If the selling price of an item is more than its cost price, then there is a profit/gain on that item. i.e SP - CP = Profit/Gain.
- **Loss:** If the cost price of an item is more than its selling price, then there is a loss on that item. i.e CP -SP = Loss.

Positive profit is a negative loss and negative profit is a positive loss.
For example :
If CP = 20 and And SP = 18. Then, Profit = 18 - 20 = -2. i.e negative profit is a positive loss.

NOTE: If the cost price and selling price of an item is equal then there is no loss and no profit on that item.

Basic formulas for Profit and loss :

1. Profit = SP – CP
2. SP = Profit + CP
3. CP = SP – Profit
4. Percentage Profit = (Profit/CP) × 100
5. SP = CP + Gain
   = CP + (Gain%/100) × CP
   = (1 + Gain%/100) × CP
6. Loss = CP – SP
7. SP = CP – Loss
8. CP = SP + Loss
9. Loss% = (Loss/CP) × 100

NOTE : Profit percent and Loss percent are always calculated on the base of cost price (CP).

Marked Price: The price that is marked on the article in shops is called as the Marked Price of that article, abbreviated as M.P.

Between cost price and selling price, there is a % markup or markup % is defined.
If CP = 100 and markup by 30% then MP should be 130. But when you sell you might also give a discount while selling.

Discount: Discount is the amount given on the marked price by lowering the price.
S.P = M.P – discount
Or, Discount = M.P – S.P
You have cost price and has markup% on that, and you get the marked price from there. Based on mark price, you offered a discount as a shopkeeper, which is in % or absolute value. And you get a selling price.

$$\text{CP} \xrightarrow{\text{markup\%}} \text{MP} \xrightarrow{\text{Discount\%}} \text{SP}$$

In some situations, MP, as well as discounts, can be successive in nature.
Let us say, a shopkeeper bought an item of 40Rs and successively marked up by 20% and 30%. He offers successive discounts of 10% and 20%. What is the selling price of that item?
Solution :
Using the PCG structure;

$$40 \xrightarrow[+8]{20\%\uparrow} 48 \xrightarrow[+14.4]{30\%\uparrow} 62.4 \xrightarrow[-6.24]{10\%\downarrow} 56.16 \xrightarrow[-11.232]{20\%\downarrow} 44.928$$

Problem :
A shopkeeper bought 10 mangoes for 80Rs and sold 8 mangoes for 96 Rs. What is the percentage profit?
Solution :
In such a situation when the number of units bought and sell are different, then the first thing you will have to think is profit % can only be calculated when;
Number of units bought = number of units sold
For calculating profit % either calculate the selling price of 10 mangoes or you would have to look at the cost price of 8 mangoes.
10 mangoes bought for 80 Rs and 8 mangoes sell for 96 Rs.
CP of 1 mango = 8 Rs
SP of 1 mango = 12 Rs
Profit = 12 - 8 = 4 Rs/mango
% Profit = (4/8) × 100 = 50% or
CP of 10 mangoes = 80 Rs

SP of 10 mangoes = 120 Rs
Profit = 120 - 80 = 40 Rs
% Profit = (40/80) × 100 = 50%.

Problems in Profit & Loss :

Type 1: Simple question based on profit and loss
Problem 1:
You bought an item of 800 Rs and you sold the item at a profit of 10%. What are the selling price and absolute profit?
Solution :
CP = 800Rs
% profit = 15. 15% of 800 = 800 × 15/100 = 120.

$$800 \xrightarrow[+120]{15\% \uparrow} 920$$

Hence SP = 920Rs. And absolute profit = 920 - 800 = 120Rs.

Problem 2:
A shopkeeper sold goods for 2000 at a profit of 25%. Find the cost price for the shopkeeper.
Solution :
SP = 2000Rs
%profit = 25.
% Profit = (SP - CP)/CP *100
CP = SP × 100/125. CP = 2000 × 100/125 = 1600Rs.

Type 2: Problem on markup price and Discount
Problem 1:
The cost price of an article was 800 and it is sold at a discount of 10% and at a profit of 12.5%. What is the selling price and mark price?
Solution :
Using the PCG structure;

$$\begin{array}{ccccc} 800 & \xrightarrow[+100]{12.5\% \uparrow} & 900 & \xleftarrow[-100]{10\% \downarrow} & 1000 \\ CP & & SP & & MP \end{array}$$

CP = 800 , %Profit = 12.5, SP = CP + CP × % Profit , SP = 800 + 800 × 12.5/100 = 900.
Let Mark price = x , Discount = 10% , SP = MP - MP × Discount%
SP = x - x × 10/100 = 0.9x and we have SP = 900.

Hence 900 = 0.9x , x = 1000.

Problem 2:

An item was sold at 639 after giving a discount of 10%. What is the original mark price of the item?

Solution :

$$\text{MP} \xrightarrow{10\% \downarrow} \underset{\text{SP}}{639}$$

This type of situation we have seen in % chapter. In PCG structure going from one side to the other side between 2 numbers. Here drop of 10% going from left to right side then there is an increment of 11.11% going from right to left.

$$\text{MP} \underset{11.11\% \uparrow}{\overset{10\% \downarrow}{\rightleftarrows}} \underset{\text{SP}}{639}$$

11.11% equivalent to 1/9. So, 1/9 of 639 = 71.

Hence mark price = 639 +71 = 710.

Type 3:

Problem 1:

An item is sold at a profit of 16%. If it was sold at 20Rs more. The net profit would have been 20%. Find the cost price of the item?

Solution :

Let original cost price = x.

% profit = 16%.

$$\underset{\text{CP}}{x} \xrightarrow[+0.16x]{16\% \uparrow} \underset{\text{SP}}{1.16x}$$

New SP = 1.16x + 20

New profit = 20%

$$\underset{\text{CP}}{x} \xrightarrow[+0.2x]{20\% \uparrow} \underset{\text{SP}}{1.2x}$$

Hence 1.16x + 20 = 1.2x , x = 500.

2nd method;

Let CP = 100.

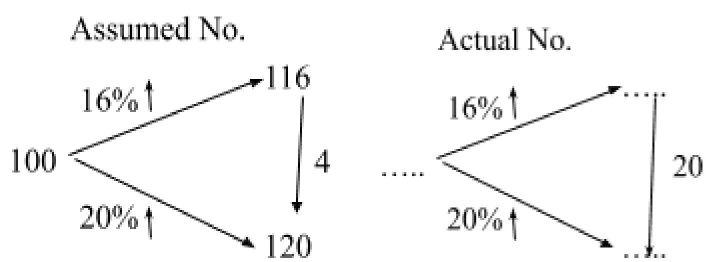
SP in 1st case when profit = 16%

$$\underset{CP}{100.} \xrightarrow[+16]{16\% \uparrow} \underset{SP}{116}$$

P in 2nd case when profit = 20%.

$$\underset{CP}{100.} \xrightarrow[+20]{20\% \uparrow} \underset{SP}{120}$$

Difference between two SP = 120 - 116 = 4.
These problems always have a parallel actual set of numbers.
Parallel number to 100 which is not known.
Parallel number to 116 which is not known.
Parallel number to 120 which is also not known.
There is a parallel number to 4 which is 20.



Between 4 & 20 there is a multiplier of 5. You can apply a multiplier of 5 to any of these numbers to find which is asked.
Hence CP = 100 × 5 = 500.

Type 4: Multiple transaction question
Problem 1:
A manufacturer who sells his items to a wholesaler at a profit of 20% and wholesaler sells it to a shopkeeper at a profit of 20% and shopkeeper sells it to a customer at a loss of 15%. What % above the manufacturer cost were the items sold at?
Solution :
Let manufacturer CP = 100.

$$\underset{CP}{100} \xrightarrow[+10]{10\% \uparrow} 110 \xrightarrow[+22]{20\% \uparrow} 132 \xrightarrow[-19.8]{15\% \downarrow} 112.2$$

Hence, items sold 12.2% more than the manufacturer cost.
If in this question it is given that the customer bought the items for 56100.
112.2 would correspond to 56100 then, the multiplier will be 56100/112.2 = 500.

The multiplier would be constant between assumed value and actual value.
CP of manufacturer = 100 × 500 = 50000.

Type 5: Dishonest shopkeeper question
Problem 1:
A shopkeeper professes to sell at CP and he cheats the customer by 10% (on weight) while selling. What is % profit to the shopkeeper?
Solution :
Assume that he sells 1kg = 1000gm and the price of each gm is 1Rs. CP of 1000gm = 1000Rs. His SP for 1000gm is also 1000 Rs.
But the only problem is while selling 1000gm, he only gives 900gm because he cheats the customer by 10%.
SP of 900gm is 1000Rs.
In profit and loss problem if money is equated, Money got = Money given, **then you can use the formula for % profit;**
% Profit = (Goods left / Goods sold) × 100.
Hence % profit to shopkeeper = (100/900) × 100 = 11.11%. OR
CP for 1000 gm = 1000 Rs
SP for 900 gm = 1000 Rs
So, CP for 900 gm = 900 Rs.
Hence % profit to shopkeeper = (100/900) × 100 = 11.11%.

Problem 2:
A man sells 2 items 1 at a profit of 20% and other at a loss of 20% and SP of both the items are equal. What is his % profit or loss?
Solution :
If a man sells two items at the same price in which he sells one at a profit of x% and the other one at a loss of x%, then **the result will always be a loss percent of [x/10]^2 %**
Here x is 20. Hence, the answer = (20/10)^2 = 4% Loss.

Some Questions for Practice :

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1. A shopkeeper incurs a loss of 10%, by selling a watch for 495. Find the C.P. of the watch for the shopkeeper.
   Ans: 550.
2. By selling a cap for 34.40, a man gains a profit of 7.5%. What will be the cost price of the cap?
   Ans: 32.

3. A cellular phone when sold for 4600 gains a profit of 15%. Find the CP of the cellular phone.
   Ans: 4000.
4. A machine costs 375. If it is sold at a loss of 20%, what will be its cost price as a percentage of its selling price?
   Ans: 125%.
5. A shopkeeper sold goods for ` 2400 and made a profit of 25% in the process. Find his profit percent if he had sold his goods for 2040.
   Ans: 6.25%.

INTEREST

Chapter of interest is an application of percentages. Interest is calculated as a percentage of a loan (or deposit) balance, paid to the lender periodically for the advantage of using their money. Interest can be calculated for periods that are longer or shorter than one year.

Interest is of two types:

1. Simple interest
2. Compound interest

The basic difference between simple interest and compound interest is the compounding factor that is often talked about in all economic and finance.

Simple interest :
Simple interest is the interest that is paid only on the amount borrowed (or invested), and not on past interest.
Compound interest :
Compound interest is the interest on capital invested as well as interest on the interest.

For example :
If you invested 100Rs @ 10% per annum on simple interest for 3 years.
Interest after 1st year = 10, after 2nd year = 10 and after 3rd year also be 10.
Amount after 1st year = 100 + 10 = 110
Amount after 2nd year = 110 + 10 = 120
Amount after 3rd year = 120 + 10 = 130

In the case of compound interest
Let say you invested 100 Rs @ 10% per annum on compound interest for 3 years.
Interest after 1st year = 10, Amount after 1st year = 100 + 10 = 110

Interest after 2nd year on 110 @ 10% = 11, Amount after 2nd year = 110 + 11 = 121
Interest after 2nd year on 121 @ 10% = 12.1, Amount after 3rd year = 121 + 12.1 = 133.1
Difference between compound interest and simple interest starts from 2nd year not from 1st year (after 1st year CI & SI both are same) it is illustrated as;

A sum of 100 at 10% per annum will have

Simple interest		Compound interest
10	After First year	10
10	After Second year	11
10	After Third year	12.1

NOTE: 1. Simple interest is generally used only on short-term i.e duration of less than one year.
2. Compound interest is used for a longer period.

Some important terms :

1. The man who borrows the money is **Debtor** and the man who lends money is the **Creditor**
2. The initially borrowed amount of money is known as the **Capital or Principal money**.
3. The extra money that will be paid or received for the use of the principal after a certain period is called the **Total interest on the capital**.
4. The sum of the principal and the interest at the end of any time is called the **Amount**.
5. The period for which money is deposited or borrowed is called **Time**.

Hence, **Amount = Principal + Total Interest.**
Rate of Interest is the rate at which the interest is calculated and it is always specified in terms of percentage.

Concept of Simple Interest :

Simple interest is the interest that is paid only on the amount borrowed (or invested), and not on past interest.
The formula for simple interest:

$$I = P \times r \times t / 100.$$

Here I = total interest, P = Principal amount, r = rate%, t = time period
Since the Amount = Principal + Total interest

NOTE: The half-yearly rate of interest is half the annual rate of interest.

Problem 1:

You are investing 100 Rs @ 10% per annum. What is the simple interest for 3 years?

Solution :

Here P = 100, r = 10%, t = 3 years

Interest = $P \times r \times t/100$.

= $100 \times 10 \times 3/100 = 30$ Rs.

Another simpler way to solve this problem without using formula.

You can simply solve inside your mind, you can simply think that on 100 Rs, 10% means 10Rs for 1 year. In simple interest for every time period interest will be constant.

So,you can simply do interest calculation = $10 \times 3 = 30$ Rs.

Problem 2:

You are investing 90 Rs @ 6% per half-year, then after 3.5 years what will be the total amount?

Solution :

Here P = 90, r =6% , t = 1/2 year , For one year rate will be $6 \times 2 = 12\%$.

I = $P \times r \times t/100$, I = $90 \times 12 \times 1/100 = 10.8$

Interest for 3.5 year = $10.8 \times 3.5 = 37.8$ Rs.

Total amount = P +total interest = 90 + 37.8 = 127.8 Rs

OR

12% of 90 = $90 \times 12/100 = 10.8$.

Total interest for 3.5 year = $10.8 \times 3.5 = 37.8$.

Total amount = P +total interest = 90 + 37.8 = 127.8 Rs

Problems on Simple interest :

Some standard problems on simple interest are;

Problem 1:

An amount of 4000 Rs is invested at a rate of 8% per annum simple interest and after a certain time period, it becomes 5920 Rs. What is the time period?

Solution :

Total amount = Principal + total interest.

5920 = 4000 + total interest

Total interest = 5920 - 4000 = 1920 Rs.

Annual interest = 8% of 4000 = $4000 \times 8/100 = 320$.

No of time period = total interest / annual interest

Total time period = 1920 / 320 = 6 years.

Hence time period = 6 years.

Problem 2:

A sum of money lends a simple interest. Sum of money after 2 years is 2394 Rs and after another 3 years is 2835 Rs. What is the sum, annual interest and the rate of interest?

Solution :

Let 'i' be the interest for 1 year.

Sum of money after 2 years;

Sum = P + total interest after 2 years

2394 = P + i + i, 2394 = P +2i …………….(1).

And sum after another 3 years;

Here P = 2394 Rs

Sum = P + total interest after 3 years

2835 = 2394 + 3i

3i = 2835 - 2394 = 441, i = 441/3 = 147.

Hence annual interest = 147.

Put i = 147 in equation (1).

2394 = P +2 × 147, P = 2394 - 294 = 2100.

Annual rate = (interest / Principal) × 100

= (147 / 2100) × 100 = 7%.

Total sum = P + interest after 5 years

= 2100 + 5i

= 2100 + 5 × 147 = 4185 Rs.

Problem 3:

An amount becomes 1240 Rs after 4 years and the same amount after 10 years will be 1600 Rs. Money is invested at a simple interest at a certain rate per annum. What are the annual interest and a rate of interest?

Solution :

Let 'i' be the interest for 1 year.

Sum of money after 4 years;

Sum = P + total interest after 4 years

1240 = P + i + i+i+i, 1240 = P +4i …………….(1).

Sum of money after 10 years;

Sum = P + total interest after 10 years

1600 = P + 10i …………….(2)

Subtract (1) from (2)

6i = 360 , i = 60 Rs.

Hence annual interest = 60 Rs.

Put the value of i in (1).

1240 = P + 4 × 60, P = 1000.

Annual rate = (interest / Principal) $\times$ 100

= (60 /1000) $\times$ 100 = 6%

Hence annual rate = 6%.

Problem 4:

A lent B Rs 6000 for 2 years and to C he lent Rs 1500 for 4 years. Together he earned a total interest of Rs 900. What is the rate of interest?

Solution :

Mathematically;

A lent B Rs 6000 for 2 years. So;

I = $P \times r \times t/100$, I = 6000 $\times$ r $\times$ 2/100 = 120r …………(1)

A lent C Rs 1500 for 4 years. So;

I' = 1500 $\times$ r $\times$ 4/100 = 60r ………...(2)

And total interest = 900 i.e I + I' = 900

120r + 60r = 900, 180r = 900

r = 5%

Hence rate of interest = 5%.

Another way to do this question;

i.e; 6000 for 2 years $\equiv$ 12000 for 1 year ………..(1)

1500for 4 years $\equiv$ 6000 for 1 year ………..(2)

Form (1) and (2);

18000 for 1 year and total interest earned is 900.

Hence annual rate of interest = (900/18000) $\times$ 100 = 5%.

Problem 5:

A certain sum of money doubles in 10 years at simple interest.what is the rate of interest per annum?

Solution :

Let if money was 100 it has become 200 after 10 years.

So; interest earned in 10 years is 100%.

Hence interest earned in 1 year = 100/10 = 10%.

Problem 6:

Shubham invested 800 Rs at a rate of 6% per annum for 9 years at simple interest. What is the interest he earned in 9 years?

Solution :

Here 6% interest for one year. So; interest for 9 years = 6 $\times$ 9 = 54%.

Thus; using PCG structure;

54% of 800 = 54 × 800/100 = 432.

$$800 \xrightarrow[+432]{54\% \uparrow} 1232$$

Hence interest he earned = 432.
And total amount = 1232

Problem 7:

Sum of money at simple interest tripled in 6 years. In how many years would it become 12 times itself?

Solution :

Let if money was 100 it has become 300 after 6 years. That means an addition of 200 in 6 years and money became 12 times itself i.e 1200.

$$100 \xrightarrow[\text{In 6 years}]{+200} 300 \xrightarrow[?]{+900} 1200$$

6 years interest is 200 and for another 6 years interest would be again 200 because annual interest is the same. Hence in every 6 years, you will add 200.
So; after 12 years the amount will become = 300+200 = 500.
After 18 years the amount will become = 500+200 = 700.
After 24 years the amount will become = 700+200 = 900.
After 30 years the amount will become = 900+200 = 1100.
Now you need 100 Rs interest more.
200 Rs interest in 6 years. So; 100 Rs Interest in 3years.
So; after 33 years the amount will become = 1100+100 = 1200
Hence 1200 will become in 33 years.

Concept of Compound interest :

Compound interest is the interest on capital invested as well as interest on the interest.
Let say you invested 100 Rs @ 10% per annum on compound interest for 3 years.
In compound interest every year you will get the interest on the amount.
Interest after 1st year = 10, Amount after 1st year = 100 + 10 = 110
Interest after 2nd year on 110 @ 10% = 11, Amount after 2nd year = 110 + 11 = 121
Interest after 2nd year on 121 @ 10% = 12.1, Amount after 3rd year = 121 + 12.1 = 133.1

Formula :

Case 1: Let principal = P, time = ‘n’ years and rate = r% per annum and let A be the total amount at the end of n years, then

A = P $\times$ (1 + $r/100)^t$

Let say a man invested 1000 Rs @ 20% per annum. What will be the amount in 3 years?

P = 1000 Rs, r = 20% , t = 3 years.

A = P $\times$ (1 + $r/100)^t$

A = 1000 $\times$ (1 + $20/100)^3$ = 1000 $\times$ ($1.2)^3$ = 1000 $\times$ 1.728

A = 1728 Rs.

Case 2: When compound interest is half-yearly then,

If the annual rate is r% per annum and is to be calculated for n years.

Here, rate = r/2 % half-yearly and time = (2n) half-years.

From the above we get

A = P $\times$ (1 + $(r/2)/100)^t$

In case of quarterly, rate = r/4 % and time = (4n) quarter years.

Let say a man invested 1000 Rs @ 10% per 6 months. What will be the amount after 2 years?

A = P $\times$ (1 + $(r/2)/100)^t$

Rate = 6% half-yearly, t = 2 years means 4 half years. Hence t = 4.

A = 1000 $\times$ (1 + $10/100)^4$ = 1000 $\times$ ($1.1)^4$ = 1000 $\times$ 1.4641

Hence amount = 1464.1 Rs.

In the given formula what you notice is that the power in the formula, if it goes to 4 or 5 it becomes slightly complex to calculate the amount because you might not know the value.

To solve this question think about PCG structure.

invested 1000 Rs @ 10% per 6 months for 2years.

$$1000 \xrightarrow[+100]{10\%\uparrow} 1100 \xrightarrow[+110]{10\%\uparrow} 1210 \xrightarrow[+121]{10\%\uparrow} 1331 \xrightarrow[+133.1]{10\%\uparrow} 1464.1$$

You should solve all compound interest questions through PCG structure.

Problems On CI :

Problem 1:

What principal amounts to 270.40 Rs in 2 years at the 4% compound interest per annum?

Solution :

As we know;

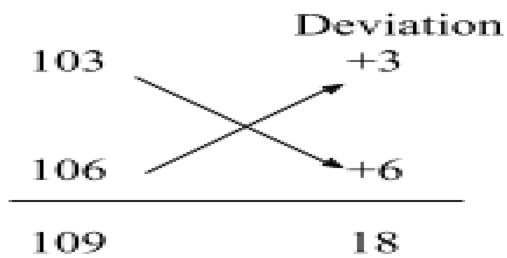
A = P $\times$ (1 + $r/100)^t$

270.40 = P $\times$ (1 + $4/100)^2$

270.40 = P $\times$ ($104/100)^2$

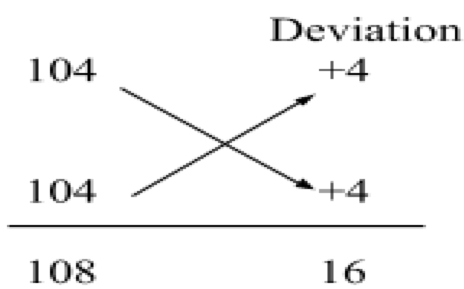
Method of multiplying 2 numbers when they are close to 100, that is very useful in CI.
For example :
You multiply 103 and 106.
In this method, you have to take the base value as 100.
103 is a deviation of +3 from 100.
106 is a deviation of +6 from 100.
Answer to multiplication will consist of two parts; the last two digits and the starting digit.

1. The value of two digits of this multiplication is obtained by multiplying the deviation +3 and +6.
2. And across the diagonal, you will have to get the initial digits. Whether you do 103+6 or 106+3, you will get the same number in both additions.



Hence $103 \times 106 = 10918$

In this question, we have 104×104



Hence $104 \times 104 = 10816$.
$270.40 = P \times (1.04)^2 = P \times 1.0816$
$P = 270.40/1.0816 = 250$ Rs.
Here the calculation of P is not easy. So; solve these type of problems from the options given to you.
Let us say the options for this problem are

a) 220 b) 200 c) 250 d) 225

Let's say you try from 220.

$$220 \xrightarrow[+8.8]{4\%} 228.8 \xrightarrow[+9.15]{4\%} 237.95$$

Hence 220 gets rejected. 200 & 225 are also rejected because 200 is less than 220 and 225 is not far away from 220.
Now try for 250.

$$250 \xrightarrow[+10]{4\%} 260 \xrightarrow[+10.4]{4\%} 270.4$$

This is exactly what was required.
Problem 2:
On a certain principal, the compound interest is Rs 132 for the 2nd year and rate of interest 10% per annum. What was the principal?
Solution :
Solve by PCG structure; let say P is the principal

$$P \xrightarrow{10\%} x \xrightarrow[+132]{10\%}$$

After one year the amount not given to us. Let say it is x. The interest for 2nd year is 132.
It is obvious interest on x at a rate of interest is 132 that means x must be 1320.
So the starting principal is ;
$P \times 1.1 = 1320$, $P = 1200$.
Hence principal amount = 1200 and the amount after two years is;

$$1200 \xrightarrow[+120]{10\%} 1320 \xrightarrow[+132]{10\%} 1452$$

Problems On SI & CI :

Problem 1:
Compound interest on a sum of money for 2 years is 615. While the SI for the same period is Rs 600. Find the principal and rate of interest.
Solution :
Let's say P is the principal. Here IS for 2 years is 600 that means annual interest is 300.
In the case of SI :

$$P \xrightarrow[+300]{\text{1st year}} \xrightarrow[+300]{\text{2nd year}}$$

In the case of CI :

CI is 615 for 2 years. As we know CI in the 1st year is the same as the SI in the 1st year.

CI for 1st year = 300. And for 2nd year = 600 - 300 = 315.

$$P \xrightarrow[+300]{\text{1st year}} \xrightarrow[+315]{\text{2nd year}}$$

Let the annual rate of interest is x%.

x% of P = 300 ……….(1)

In case of CI :

x% of (P + 300) = 315

x% of P + x% of 300 = 315

300 + x% of 300 = 315 …………(2)

So; from this equation x% of 300 should be equal to 15 to satisfy the equation.

x% of 300 = 315. Hence x = 5%.

For calculation of Principal from eq (1)

5% of P = 300.

Hence P = 6000 Rs.

Problem 2:

Difference between CI and SI of a certain sum of money for 2 years at 20% per annum is Rs 48. What is the sum of money?

Solution :

Let say x is the original amount.

SI @ 20% for 2 year on x = 0.2x + 0.2x = 0.4x

CI @ 20% for 2 year on x;

$A = x \times (1 + 20/100)^2 = 1.44x$

CI = 1.44x - x = 0.44x.

Difference between CI & SI = 48

0.44x - 0.4x = 48

x = 1200.

2nd method :

Assume principal= 100 Rs.

In the case of SI :

SI on 100 @ 20% for 2 year is;

I = 20 + 20 = 40.
In the case of CI :

$$100 \xrightarrow[+20]{20\%} 120 \xrightarrow[+24]{20\%} 144$$

A = 144.   CI = 144 - 100 = 44.
Difference between CI & SI = 48
44 - 40 = 48
4 ≡ 48.
Using the multiplier logic, 4 to 48 the multiplier is 12. Multiply all the assumed values by 12 you will get the actual value.
Hence Principal amount = 100 × 12 = 1200.
3rd method :
The difference can also be calculated by a formula which is $p \times (r/100)^2$. This gives you the difference between CI & SI for 2 years for a principal amount P @ a rate "r".
Difference between CI & SI = 48 = $p \times (20/100)^2$
48 = p × 1/25
P = 1200.
NOTE : $p \times (r/100)^2$ This work on the difference between CI & SI for 2 year.

Some Questions for Practice :

---

1. The Simple Interest on a sum of money is 25% of the principal amount, and the rate per annum is equal to the number of years. Find the rate %.
   Ans: 5%
2. The rate of interest for the first 3 years is 6% per annum and for the next 4 years it is 7 per cent per annum and for the period beyond 7 years, 7.5 per cent per annum. If a man lent out 1200 for 11 years, then, find the total interest earned by him?
   Ans: 912.
3. A sum of money doubles itself in 12 years. Find the rate of % per annum.
   Ans: 8.33%
4. A certain sum of money amounts to 704 in 2 years and 800 in 5 years. Find the principal.
   Ans: 640.
5. A sum of money was invested at simple interest at a certain rate for 3 years. Had it been invested at a 4% higher rate, it would have gained 480 more. Find the principal.
   Ans: 4000.

Lecture Notes For Time And Work

Time and work is an important chapter of quantitative aptitude. Time and work has constant performance in all exams under aptitude. Best way to understand time and work is through examples.

Intro to Time and Work :

Work is defined as something which has an effect or outcome. The basic concept of Time and Work is similar to that across all Arithmetic topics, i.e. the concept of Proportionality.

Problem based on two or more people work together :
Problem 1:
Two people A and B, can do a piece of work in 12 and 15 days respectively. In how many days will they complete the work together?

Solution :
There are 3 methods to solve this question.

1. Fraction method.
2. Percentage method.
3. LCM method.

1. Fraction method :

Let the total work = 1unit.
A can finish the work in 12 days and B can finish the work in 15 days.
A's per day work = 1/12 unit.
B's per day work = 1/15 unit.

In time & work the basic equation is;
Rate of work × Time = work done

Rate of work = 1/12 + 1/15 = 9/60 unit
9/60 × t = 1. Therefore t = 60/9 = 6.66 days.
Time is reciprocal of rate of work.

It is a very combusive method. One advantage of this method is in the last step you just take the reciprocal of the value you got.

2. Percentage method :

Let the total work = 100%
A can finish the work in 12 days and B can finish the work in 15 days.
A's per day work = 1/12 i.e. 8.33%
B's per day work = 1/15 i.e. 6.66%

Rate of work × Time = work done
Rate of work = 8.33 + 6.66 = 15%
15 × t = 100. Therefore t = 100/15 = 6.66%.
It is a better method than fraction, but this method has only the problem of decimal work.

For example; A can finish the work in 5 days and B can finish the work in 9 days.
A's per day work = 1/5 i.e. 20%
B's per day work = 1/9 i.e. 11.11%
Rate of work = 20+11.11 = 31.11%. So in this case numbers are not supporting you.

3. LCM method :

A can finish the work in 12 days and B can finish the work in 15 days.
Assume total work be the LCM of 12 &15.
LCM(12,15) = 60.
A's per day work = 60/12 = 5 unit.
B's per day work = 60/15 = 4 unit.
One day total work = 5+4 = 9unit.
Total time required = total work / per day work
= 60/9 = 6.33 days.
This is the better method to work upon by avoiding the use of decimal work

Time and work :

People come and go type problem :

Problem 1:

A can do a piece of work in 10 days. B can also do the same work in 12 days and C can do the same work in 15 days. A & B start the work and work for 2 days and then B leave and after 1 more day C join A to complete the work. In how many days will the work be completed?

Solution :

Total work = LCM(10,12,15) = 60 units.
A's per day work = 60/10 = 6 units.
B's per day work = 60/12 = 5 units.
C's per day work = 60/15 = 4 units.
A+B per day work = 6+5 = 11 units.
Work in 2 days = 11 × 2 = 22 units.

On the 3rd day A is working alone and B left.
3rd work = 6 units.
Total work in 3 days = 22+6 = 28 units.
So; work left = 60-28 = 32 units. This work has to be done by A & C.
A+C per day work = 6+4 = 10 units. Therefore remaining work 32 units will take 32/10 = 3.2 days more.
Hence total days required = 3 + 3.2 = 6.2 days.

Problem 2 :

A can do a piece of work in 10 days. B can also do the same work in 12 days and C can do the same work in 15 days. A & B start the work and work for 2 days and then B leave. C joined A on the 4th day and A left one day before the work was completed. In how many days will the work be completed?

Solution :

Total work = LCM(10,12,15) = 60 units.
A's per day work = 60/10 = 6 units.
B's per day work = 60/12 = 5 units.
C's per day work = 60/15 = 4 units.
A+B per day work = 6+5 = 11 units.
Work in 2 days = 11 × 2 = 22 units.

On the 3rd day A is working alone and B left.
3rd work = 6 units.
Total work in 3 days = 22+6 = 28 units.
On the last day C is alone and C did 4 units of work on the last day.
Total work is done = 28+4 = 32 units. Remaining work = 60 - 32 = 28 units and remaining work is done by A & C together.
A+C per day work = 6+4 = 10 units. Therefore remaining work 28 units will take 28/10 = 2.8 days more.

Hence total days required = 4 + 2.8 = 6.8 days.

Pipe & Cistern Problem :

Problem 1:

2 pipes A & B are filling a tank. A can fill it in 12 hours and B can fill it in 15 hours. How much time will they take to fill an empty tank?

Solution :

A can fill the tank in 12 hours and B can fill the tank in15 hours.
Assume total capacity of the tank be the LCM of 12 &15.
LCM(12,15) = 60 L.
A's per hour filling = 60/12 = 5 L.
B's per hour filling = 60/15 = 4 L.
In one hour total filling = 5+4 = 9 L.
Total time required = total capacity / per hour filling
= 60/9 = 6.33 hours.

Problem 2:

2 pipes A & B filling a tank. A can fill it in 12 hours and B can fill it in 15 hours. How much time will they take to fill a half filled tank?

Solution :

A can fill the tank in 12 hours and B can fill the tank in15 hours.
Assume total capacity of the tank be the LCM of 12 &15.
LCM(12,15) = 60 L.
A's per hour filling = 60/12 = 5 L.
B's per hour filling = 60/15 = 4 L.
In one hour total filling = 5+4 = 9 L.
Given that tank is half filled i.e. 30 L. So; remaining capacity = 60 -30 = 30 L
Total time required to fill remaining half tank = remaining capacity / per hour filling
= 30/9 = 3.33 hours.

Problem 3:

2 pipes A & B filling a tank. A can fill it in 12 hours and B can fill it in 15 hours. C can empty the tank in 10 hours. How much time will they take to fill the tank?

Solution :

A can fill the tank in 12 hours and B can fill the tank in15 hours.

Assume total capacity of the tank be the LCM of 12 &15.

LCM(12,15) = 60 L.

A's per hour filling = 60/12 = 5 L.

B's per hour filling = 60/15 = 4 L.

C's per hour filling = 60/10 = -6 L.

In one hour total filling = 5+4-6 = 3 L.

Total time required to fill the tank = total capacity / per hour filling

= 60/3 = 20 hours.

Problem 4:

A & B can do a piece of work in 6 days. A & C can do the work in 9 days. B &C can do the same work in 15 days. In how much time the work will complete if A,B & C work together?

Solution :

Assume total work = LCM(6,9,15) = 90 units.

Work rate of A+B = 90/6 = 15 units.

Work rate of A+C = 90/9 = 10 units.

Work rate of B+C = 90/15 = 6 units.

And A+B+A+C+B+C = 15+10+6 = 31units/day

2(A+B+C) = 31units/day

Work rate of A+B+C = 31/2 units/day.

Total days required = $90 \times 2/31$ = 180/31 days.

Time and work (man days) :

Here we will discuss that the work is measured in terms of man day or man hours.

Let 20 men work on a project for 8 days. Work done can be measured in such a case, as multiplication of 20×8 and units used here man-days. i.e $20 \times 8 = 160$ man-days.

We use the concept of work equivalence in such situation means;

20 men working for 8 days is same as 10 men working for 16 days is same as 1 men working for 160 days i.e $20 \times 8 \equiv 10 \times 16 \equiv 1 \times 160$.

Problem 1:

A certain number of people can complete a piece of work in 55 days. If there were 6 more men added, the work could get done in 11 days less.what is the number of men initially?

Solution :

Assume in the starting there are x number of men.

Total work done by x men = $x \times 55$ man-day.

6 men more join & work is done in 55 - 11 = 44 days.

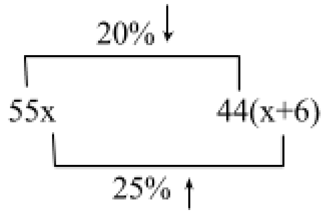
So; according to work equivalence ;

$x \times 55 = (x+6) \times 44$

$55x = 44x + 264 \Rightarrow x = 24$ men.

We can do this question by product constancy also.

Numerical component of the product is going down by 20% and the other component going up by 25%.



$$x \xrightarrow[+6]{25\% \uparrow} x+6$$

+6 present 25% increase on x. 25% is 6 and 100% is $6/25 \times 100 = 24$.

Hence the number of men = 24 men.

Problem 2:

10 men working 6 hour a day can complete a work in 18 days. In how many hours a day should 15 men work for 12 days. So that they can complete double the work?

Solution :

Original work = $10 \times 18 \times 6$ man-days.

New work = $10 \times 18 \times 6 \times 2$

Let x hours per day 15 men take.

According to work equivalence;

$10 \times 18 \times 6 \times 2 = 15 \times 12 \times x$

Therefore x = 12 hr/day

Problem 3:

The work done by (x - 1) men in (x+1) days to the work done by (x+2) men in (x - 1) days is 9:10. What is the value of x?

Solution :
1st work = (x - 1)(x+1) man-days
2nd work = (x+2)(x - 1) man-days
1st work / 2nd work = 9/10
(x - 1)(x+1) / (x+2)(x - 1) = 9/10; (x+1) / (x+2) = 9/10
Therefore x = 8.

Time and work (man days)-2 :

Problem 1:
A contractor undertakes to complete a job in 100 days and employs 200 men to complete the work. After 50 days he finds that only 40% of the work is completed. To complete the work in time how many men should he hire?

Solution :
Work to be done in 50 days = 200 × 50 =10000 man-days
10000 man-days is only 40% of the work .
Remaining work = 100 - 40 = 60%
40% work = 10000 man-days
60% work = (10000/40) × 60 = 15000 man-days.
You have only 50 more days left. Let n be the number of men required to complete the work.
Therefore; 50 × n = 15000 and n = 300 men.
Hence; 300 - 200 = 100 men need to hire.

The Specific Case of Building a Wall :

Building of a wall of a certain length, breadth and height.
In such cases, the following formula applies:

$$\frac{M1 \times D1 \times T1}{M2 \times D2 \times T2} = \frac{L1 \times B1 \times H1}{L2 \times B2 \times H2}$$

where L, B and H are respectively the length, breadth and height of the wall to be built, while m, t and d are respectively the number of men, the amount of time per day and the number of days. Further, the suffix 1 is for the first work situation, while the suffix 2 is for the second work situation.

Problem 2:

12 men working 8 hours a day can completely build a wall of length 12ft, breadth 40 ft and height 4ft in 10 days. How many days will 10 men working 6 hours a day require to build a wall of length 24ft, breadth 60ft and height of 2ft.

Solution :

Using formula;

$$\frac{M1 \times D1 \times T1}{M2 \times D2 \times T2} = \frac{L1 \times B1 \times H1}{L2 \times B2 \times H2}$$

Here, L1 is 12ft	L2 is 24ft
B1 is 40ft	B2 is 60ft
H1 is 4ft	H2 is 2ft
while M1 is 12 men	M2 is 10 men
D1 is 10 days	D2 is unknown
and T1 is 8 hours a day	T2 is 6 hours a day

$$\frac{12 \times 10 \times 8}{10 \times D2 \times 6} = \frac{12 \times 40 \times 4}{24 \times 60 \times 2}$$

16/D2 = 2/3 ,   D2 = 24 days

Men, Women & Children :

Problem 1:

20 women can do work in 16 days while 16 men can do it in 15 days. What is the ratio of capacity of a man and a woman?

Solution :

Total work to be done = 20 × 16 = 320 woman-days.

or total work to be done = 16 × 15 = 240 man-days.

Since, the work is the same, we can equate 240 man-days = 320 woman-days.

Hence, 3 man-days = 4 woman-days or 1 man-day = 1.33 woman-days.

Assume total work = 12 unit

1 man-day work rate = 4 units.

1 woman-day work rate = 3 units.

Therefore the work rate of man to woman = 4:3.
Answer is not 3:4, answer is 4:3 because 3 man-days doing the same work as 4 woman-days. So; the work rate of a man must be higher than the work rate of a woman.

Problem 2:
18 men or 36 boys can complete a work in 24 days if they work 6 hours per day. How many days would be required if 24 men and 24 boys work for 9 hours per day to the same job?

Solution :
Total work to be done = $36 \times 24 \times 6$ boy-hours.
18 men or 36 boys can do the same work. So;
1 man work $\equiv 2$ boys work.
24 men work $\equiv 48$ boys work.
Therefore;
24 men & 24 boys $\equiv 72$ boys. 72 boys working 9 hours/day for 'n' days to complete the same job.
Total work = $72 \times 9 \times n$ boy-hours.
Since the work done is the same. So;
$36 \times 24 \times 6 = 72 \times 9 \times n$
2n = 16 $\Rightarrow$ n = 8 days.
Hence 8 days will be required.

Problem 3:
2 men and 3 boys can do a piece of work in 10 days and 3 men and 2 boys can do it in 8 days. How many days are required for 2 men and 1 boy to finish that work?

Solution :
Total work to be done = 2 × 10 man-days + 3 × 10 boy-days
or total work to be done = 3 × 8 man-days + 2 × 8 boy-days
Since work is the same. So;
20 man-days + 30 boy-days = 24 man-days + 16 boy-days
4 man-days = 14 boy-days or 1 man-day = 3.5 boy-days
Now, if 2 men and 1 boy are working on the work
1 man = 3.5 boy , 2 man = 3.5×2 boys.

Effectively 7+1 = 8 boys are working when 2 men and 1 boy are working.
Work done = 20 man-days + 30 boy-days
= 20×3.5 boy-days + 30 boy-days =100 boy-days.

Let 8 boys work for n days.
Therefore; $8 \times n = 100 \Rightarrow n = 100/8 = 12.5$ days.

Some question for practice :

---

1. If 12 men and 16 boys can do a piece of work in 5 days and 13 men and 24 boys can do it in 4 days, compare the daily work done by a man with that done by a boy?
Ans : 2:1.

2. A can do work in 10 days and B can do the same work in 20 days. They work together for 5 days and then A goes away. In how many more days will B finish the work?
Ans : 5 days.

3. 30 men working 5 hr a day can do work in 16 days. In how many days will 20 men working 6 h a day do the same work?
Ans : 20days.

4. A can do a piece of work in 10 days and B can do the same work in 20 days. With the help of C, they finish the work in 5 days. How long will it take for C alone to finish the work?
Ans : 20 days.

5. 10. A can do a piece of work in 20 days. He works at it for 5 days and then B finishes it in 10 more days. In how many days will A and B together finish the work?
Ans : 8 days.

(Ref: Quantitative Aptitude by Arun Sharma)

Lecture Notes For Reasoning

Logical reasoning can be verbal or non-verbal. It consists of aptitude problems that have some logical level analysis to solve them. And most of the problems are concept based.

Sequence and Series :

It is an important chapter because there are a lot of questions you see in various exams. Sequence and Series having a standard structure and pattern.
All sequence & Series questions are essentially about pattern recognition. Patterns based on alphabets and number series.

Alphabet pattern :
As we know English alphabets have 26 letters.

Forward order position	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Alphabets	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Backward order position	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

You should understand the numerical position, maybe you do not remember the whole table but you can keep a few markers in your mind w.r.t. 4,5. eg. E as 5th, J as 10th, M&N as 13, 14 and W as 23rd.

For example :
You know N is 14 and you want to extract what Q is?
As you know the position of N so you can simply go N, O, P, Q rather than starting from A.you will start with N-14, O-15, P-16 and hence Q-17.

NOTE: The sum of going forward and backward alphabets position, series will always be 27 for any alphabets (Z is 26th in the forward direction and 1 in the backward direction. So; sum = 26+1 =27).

For example :
Do you want to extract G from backward direction?
If you go reverse it would be 20th. If you know the position of the forward alphabet you can easily extract backward position alphabets.

Problem 1:
DELHI coded as CCIDD and code JAIPUR will be?

Solution :
As

D	E	L	H	I
↓− 1	↓− 2	↓− 3	↓− 4	↓− 5
C	C	I	D	D

Similarly,

J	A	I	P	U	R
↓− 1	↓− 2	↓− 3	↓− 4	↓− 5	↓ -6
I	Y	F	L	P	L

NOTE : While solving this type of question first you have to write alphabets as reference.

Recognising Numerical Pattern :

The numerical pattern is also based on pattern recognition. Number series based questions are tougher than alphabet patterns. Number series is a very diverse set of patterns.

1. Single Logic patterns :

(a) Simple addition/subtraction :
For example : **3,7,11,15,19,------.**
In the given series, the difference between two consecutive numbers is the same i.e. 4.
In the given series, the number added to each term is in increasing order.

3 7 11 15 19 21
→ → → → →
+4 +4 +4 +4 +4

(b) Simple multiplication/division :

In the given series, the ratio between two consecutive numbers is the same.

For example : **3,6,12,24,48,-------.**

In the given series the previous element is multiplied by 2 to obtain the next element therefore the ratio between two consecutive numbers is the same.

3 6 12 24 48 96

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

$\times 2$ $\times 2$ $\times 2$ $\times 2$ $\times 2$

(c) Progressive addition series :

For example : 1) **5,8,12,17,23,-------.**

In the given series, the difference between two consecutive numbers is in increasing order.

5 8 12 17 23 30

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

+3 +4 +5 +6 +7

2) **5,8,13,20,29**

In the given series, the difference between two consecutive numbers is in increasing order.

5 8 13 20 29 40

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

+3 +5 +7 +9 +11

(d) Progressive subtraction series :

For example : **29,20,13,8,5,-------.**

Here the difference between two consecutive numbers is in decreasing order.

29 20 13 8 5 4

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

-9 -7 -5 -3 -1

(e) Addition/subtraction of progressive squares :

For example : 1) **10,11,15,24,40,-------.**

In the given series, the difference between two consecutive numbers is in increasing order squares.

10 11 15 24 40 65

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

+1 +4 +9 +16 +25

1^2 2^2 3^2 4^2 5^2

2) **4,5,21,102,358,-------.**

In the given series, the difference between two consecutive numbers is in increasing order squares of squares.

4 5 21 102 358 983

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

+1 +16 +81 +256 +625

1^2 4^2 9^2 16^2 25^2

1^{2^2} 2^{2^2} 3^{2^2} 4^{2^2} 5^{2^2}

(f) Addition/subtraction of progressive cubes :

1) **5,6,14,41,105,-------.**

In the given series, the difference between two consecutive numbers is in increasing order cubes.

5 6 14 41 105 230

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

+1 +8 +27 +64 +125

1^3 2^3 3^3 4^3 5^3

(g) Progressive multiplication and division :

1) **160,80,120,300,1050,-------.**

In the given series, the ratio between two consecutive numbers is in increasing order and numbers are multiplied by the numbers in increasing order.

160 80 120 300 1050 4725

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

$\times 0.5$ $\times 1.5$ $\times 2.5$ $\times 3.5$ $\times 4.5$

1) **600,300,100,25,5,-------.**

In this given series, the elements are divided by 2,3,4,5, and 6 respectively to obtain the next element.

600 300 100 25 5 5/6

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

$\div 2$ $\div 3$ $\div 4$ $\div 5$ $\div 6$

2. Mixed patterns :

---

a) Simple multiplication/division combined with addition/subtraction :

For example :

1) **3,7,15,31,63,--------.**

In the given series, the two consecutive numbers are increasing by a combination of multiplication by 2 and addition by 1.

3 7 15 31 63 127

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

$\times 2+1$ $\times 2+1$ $\times 2+1$ $\times 2+1$ $\times 2+1$

2) **1,7,43,259,---------.**

In the given series, the two consecutive numbers are increasing by a combination of multiplication by 6 and addition by 1.

1 7 43 259 1555

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

$\times 6+1$ $\times 6+1$ $\times 6+1$ $\times 6+1$

b) Progressive multiplication/division with addition/subtraction :

For example :

1) 1,3,11,47,--------.

In the given series, the two consecutive numbers are increasing by a combination of multiplication and addition is in increasing order.

1 3 11 47 239

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

$\times 2+1$ $\times 3+2$ $\times 4+3$ $\times 5+4$

2) 4,13,40,135,--------.

1 13 40 135 552

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

$4\times 1+1\times 9$ $13\times 2+2\times 7$ $40\times 3+3\times 5$ $135\times 4+4\times 3$

This type of pattern is very difficult to recognise, but you have to be aware that this pattern could happen inside the series.

c) alternating multiplication/division and addition/subtraction and square/cube :

For example :

5,14,3,18,-3,-------.

5 14 3 18 -3 4

$\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$

$\times 2+2^2$ $\div 2-2^2$ $\times 3+3^2$ $\div 3-3^2$ $\times 4+4^2$

d) Addition/subtraction some set of the previous term of the series to form next term :
For example :
1) 2,3,5,8,13,21,---------.
5 = 2+3, 8 = 3+5, 13 = 5+8, 21 = 13+8 and next term = 21+13 = 34.
This pattern is called a Fibonacci series.

2) 1,1,1,3,5,9,17,31,---------.
3 = 1+1+1, 5 = 1+1+3, 9 = 1+3+5, 17 = 3+5+9, 31 = 5+9+17 and next term = 9+17+31 = 57.

Intro To Syllogisms :

It is an important chapter of logical reasoning and hence, working knowledge of its rules is expected from a candidate. In this chapter, questions are based on some statements and their conclusions. We are not supposed to apply extra information except for the information given in statements while drawing the conclusion.

To understand the syllogism first you have to understand some standard statements.
1. All A's are B's :
This statement has two possible pictures. The primary statement is A's circle inside B's circle.

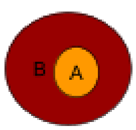


Fig. 1

In this case, some B's are A's while some B's are not A's.
The secondary statement is one circle for A's and B's.

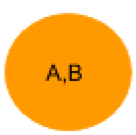


Fig. 2

In this case, it also follows that all B are A.

2. No A is B :

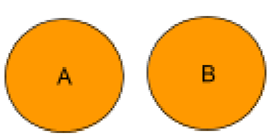
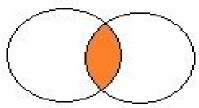


Fig. 1

The conclusion No B is A is a valid conclusion.

3. Some A's are B's :
This can be understood by the following Venn diagram.



Although the above figure also supports conclusion-some B's are not A's, this cannot be taken as a definite conclusion. This is because, when we say that Some A's are B's, it does not mean that there have to be some B's that are not A's.

4. Some A's are not B's :
This can be understood by the following Venn diagrams.

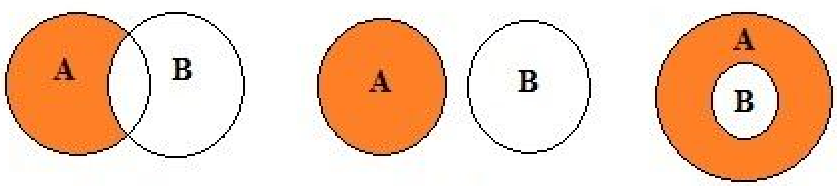


Fig. 1 Fig. 2 Fig. 3

In syllogism when you solve through a Venn diagram you have tested the conclusion. At that time you see many variant pictures for each of the structures might be one way in which you reject the conclusion.

Problem Solving In Syllogism :

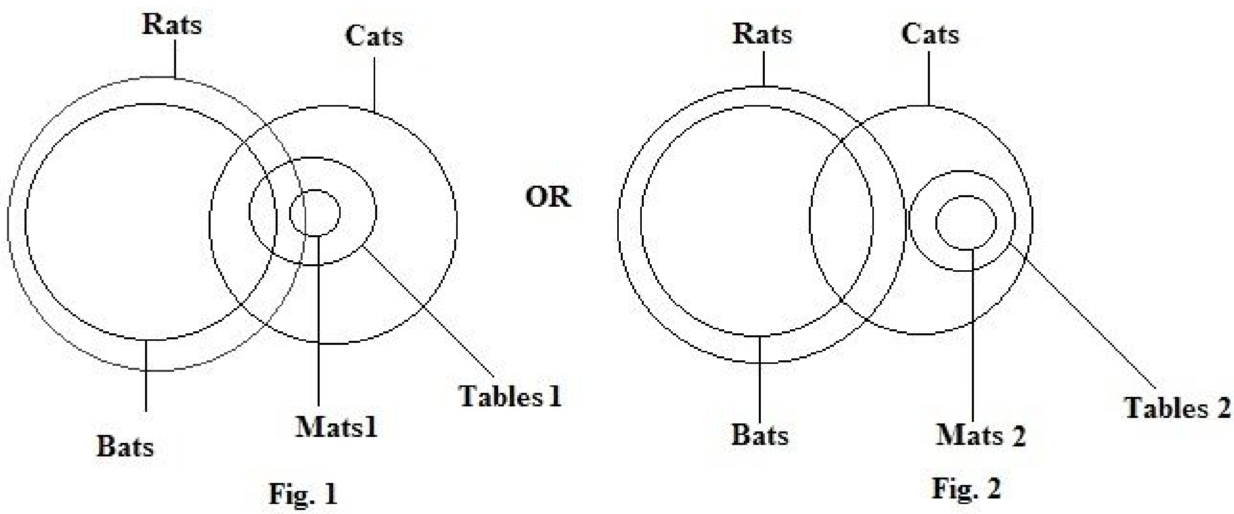
---

1. Statements:
1. Some cats are bats.
2. All bats are rats.
3. All tables are cats.
4. All mats are tables.

Conclusion:
a. Some mats are rats.
b. Some tables are bats.
c. Some cats are rats.

d. None of these.

Explanation:



Conclusion (a): Some mats are rats in fig. 1 but it is not necessary because no mats are rats in Fig. 2. So; conclusion (a) not follows.

Conclusion (b): Some tables are bats in fig. 1 but it is not necessary because no tables are bats in Fig. 2. So; conclusion (b) not follows.

Conclusion (c): Some cats are rats true in both the Figures.
Therefore, the conclusion (c) follows.

2. Statement:
Some ships are boats. All boats are submarines. Some submarines are yachts.
Conclusion:
a. Some yachts are boats.
b. Some submarines are boats.
c. Some submarines are ships.
d. Some yachts are ships.
1. All follow.
2. Only ‘b’ and ‘c’ follow.
3. Only ‘c’ follows.
4. Only ‘d’ follows.

Explanation:

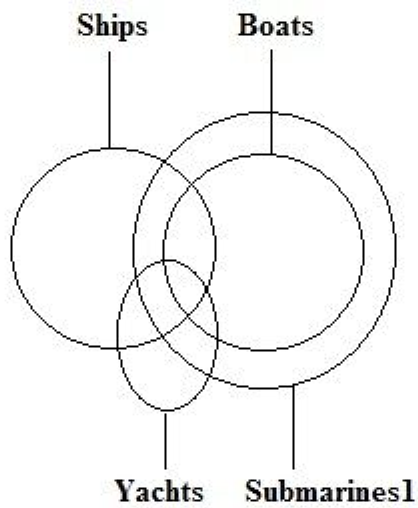


Fig. 1

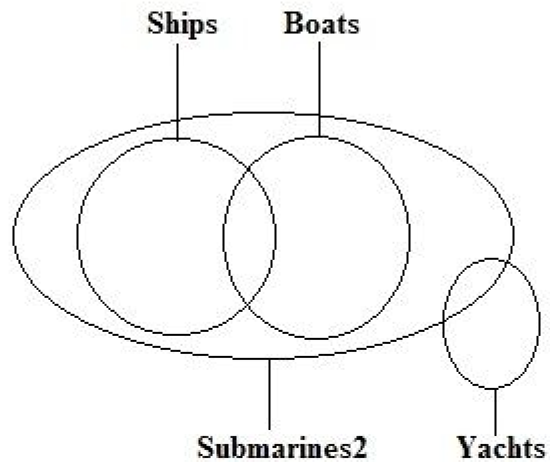


Fig. 2

Conclusion (a): No yachts are boats.
Conclusion (b): Definite conclusion.
Conclusion (c): Definite conclusion.
Conclusion (d): No yachts are ships.
Therefore only ‘b’ & ‘c’ follows.

3. Statement:
1. All cats are dogs.
2. All dogs are brown.

Conclusion:
a. All cats are brown.
b. All brown are dogs.
c. Some brown are not dogs.

Explanation:

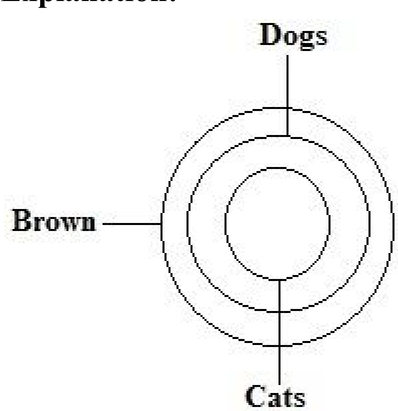


Fig. 1

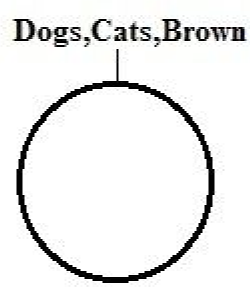


Fig. 2

Conclusion (a): Definite conclusion.
Conclusion (b): To reject this conclusion in fig. 1 some brown are not dogs.
Conclusion (c): To reject this conclusion in fig. 2 all brown are dogs.

4. Statement:
All computers are Pentiums and some Pentiums are machines.
Conclusion:
a. Some computers are machines.
b. Some machines are computers.

Explanation:

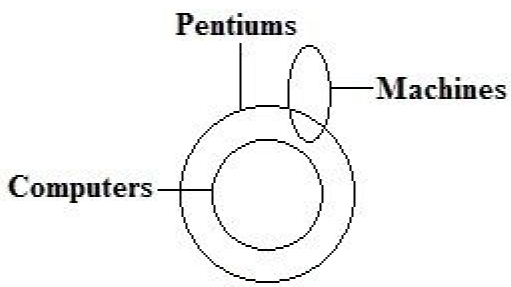
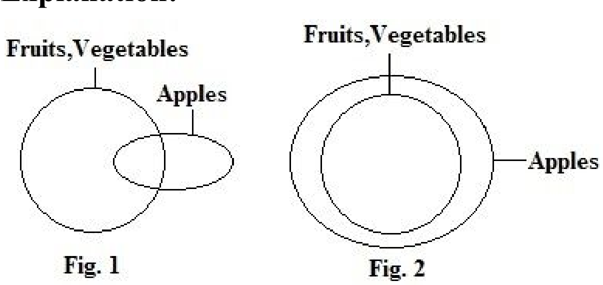


Fig. 1

Conclusion (a): To reject this conclusion in fig. 1 no computers are machines.
Conclusion (b): To reject this conclusion in fig. 1 no machines are computers.

5. Statement:
1. Some apples are fruits.
2. All vegetables are fruits.
3. All fruits are vegetables.
Conclusion:
a. Some apples are vegetables.
b. All vegetables are fruits,
c. All fruits are apples.
d. All vegetables are apples.

Explanation:



Conclusion (a): Definite conclusion.
Conclusion (b): Definite conclusion.
Conclusion (c): To reject this conclusion in fig. 1 some fruits are not apples.
Conclusion (d): To reject this conclusion in fig. 1 some vegetables are not apples.

6. Statements:
1. Some cats are animals.
2. Some animals are mammals.
3. Some mammals are earthlings.

Conclusion:
a. Some earthlings are cats.
b. Some mammals are cats.
c. Some earthlings are animals.
d. Some cats are earthlings.

Explanation:

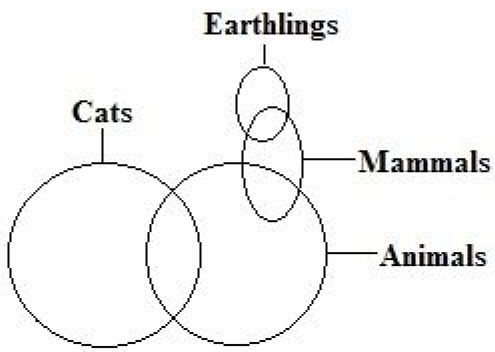


Fig. 1

Conclusion (a): To reject this conclusion in fig.1 no earthlings are cats.
Conclusion (b): To reject this conclusion in fig.1 no mammals are cats.
Conclusion (c): To reject this conclusion in fig.1 no earthlings are animals.
Conclusion (d): To reject this conclusion in fig.1 no cats are earthlings.

Remember that in syllogism you have to disprove rather than prove. The approach has to visualize the first 2 or 3 circles, draw them and then visualize the next one without drawing it. Just try to fix that one so that you can reject the conclusion.

Coding-Decoding pattern 1:

You always get coding-decoding questions in various exams.

Type 1: Interchange of letters inside the words

Problem 1:

GLORIOUS coded as GOLRIOSU. What will be the codes for JUDICIAL?

Solution:

In GLORIOUS 2nd and 3rd letters are interchanged and after that, the next 3 letters are the same and the last two letters are also interchanged.

GLORIOUS → GOLRIOSU

Do the same thing in JUDICIAL. Therefore, JUDICIAL → JDUICILA.

Problem 2:

SUDHIR is coded as HIRSUD. What will be the code for VISHES.

Solution:

In SUDHIR 1st 3 letters are interchanged with the last 3 letters.

SUDHIR → HIRSUD

Do the same thing in VISHES. Therefore, VISHES → HESVIS.

Problem 3:

PURPOSE is coded as UPPRSOE. What will be the code for SUDHI?

Solution:

In PURPOSE interchanging of 1st and 2nd letters, interchanging of 3rd and 4th letters, interchanging of 5th and 6th letters and last letters kept as it is.

PURPOSE → UPPRSOE

So, in the case of SUDHI

1st two letters interchange to ‘US’, ‘DH’ interchange to ‘HD’ and last letter ‘I’ will be as it is.

Therefore, SUDHI → USHDI

Type 2: Word coding

Problem 1:

SUDHIR is coded as QTBGGQ. What will be code for RAVI?

Solution:

S	U	D	H	I	R
$\downarrow -2$	$\downarrow -1$	$\downarrow -2$	$\downarrow -1$	$\downarrow -2$	$\downarrow -1$
Q	T	B	G	G	Q

Therefore, RAVI $\rightarrow$ PZTH

R	A	V	I
$\downarrow -2$	$\downarrow -1$	$\downarrow -2$	$\downarrow -1$
P	Z	T	H

Type 3: Replace letters with numbers

Problem 1:

PAINT is coded as 74128 & EXCEL is coded as 93596. What will be the code for ACCEPT?

Solution:

P $\rightarrow$ 7	E $\rightarrow$ 9
A $\rightarrow$ 4	X $\rightarrow$ 3
I $\rightarrow$ 1	C $\rightarrow$ 5
N $\rightarrow$ 2	E $\rightarrow$ 9
T $\rightarrow$ 8	L $\rightarrow$ 6

Therefore, ACCEPT is coded as:

A $\rightarrow$ 4
C $\rightarrow$ 5
C $\rightarrow$ 5
E $\rightarrow$ 9
P $\rightarrow$ 7
T $\rightarrow$ 8
ACCEPT $\rightarrow$ 455978.

Type 4: Numeric pattern based

Problem 1:

1000 is coded as 1728 and 125 coded as 343. What will be code for 512?

Solution:

1000 $\rightarrow$ 1728 & 125 $\rightarrow$ 343

$10^3 \rightarrow 12^3$ & $5^3 \rightarrow 7^3$

That means there is an increment of +2 on the cube.

512 is a cube of 8. And +2 increment on the cube will be 10^3.
Therefore, 512 is coded as 1000
512 $\rightarrow$ 1000

Problem 2:
L is coded as 12 and G is coded as 7. What will be the code for 9?

Solution:
Code is just an alphabet order placed.

ABCDEFGHIJKL…..Z

$\downarrow$ $\downarrow$ $\downarrow$

7 9 12

Therefore, 9 is coded as I.

Type 5: Language coding

Problem 10:
"Tee See Pee" means "drink fruit juice", "See Kee Lee" means " juice is sweet", "Lee Ree Mee" means "he is intelligent". What is the code for the word "sweet"?

Solution:
In the first and second statement, the common word is **'juice'** and the common code word is **'See'**. So **'See'** means **'juice'**.
In the second and third statements, the common word is **'is'** and the common code is **'Lee'**. So,**'Lee'** means **'is'.**
Thus, in the second statement, the remaining word is **'sweet'** which is coded as **'Kee'**.
Hence, code for the word **'sweet'** is **'Kee'**

Problem 2:
In a certain code, '786' means 'study very hard', '958' means 'hard work pays' and '645' means 'study and work'. Which of the following is the code for 'very' ?

Solution:
In the first and second statements, the common word is **'hard'** and the common code digit is **8**. So, **'8'** means **'hard'**.
In the first and third statements, the common word is **'study'** and the common code digit is **'6'**. So, **'6'** means **'study'**.
Thus, in the first statement, **'7'** means **'very'**.

Problem 3:
In a certain code language
“lu ja ka hu” means ‘we provide study material’,
“fa ka la ju” means ‘we score maximum selection’,
“la fu ja ju “ means “study score the selection”
“ju lu na fu” means “selection of the material”.
What is the code of “ provide of maximum”?

Solution :
In the 1st and 4th statement, the common word is **'material'** and the common code word is **'lu'**. So **'lu'** means **'material'**.

In the 3rd and 4th statements, the common words are **'selection & the'** and the common codes are **'ju & fu'**. Thus, in the 4th statement **‘of’** means **‘na’.**

In the 2nd and 3rd statement, the common word is **'selection'** and the common code word is **'ju'**. So **'selection'** means **'ju'**. Thus, in the 4th statement **‘the’** means **‘fu’**

In the 2d and 3r statements, the common word is **'score'** and the common code is **'la'**. So,**'score'** means **'la'.** Thus, in the 3d statement, the remaining word is **'study'** which is coded as **'ja'**.

In the 1st and 2nd statement, the common word is **'we'** and the common code word is **'ka'**. So **'ka'** means **'we'**.Thus, in the 1st and 2nd statement, the remaining words are **'provide & maximum'** which are coded as **'hu & fa'** respectively.
Therefore, the code of “ provide of maximum” is “ hu na fa”.

Some Questions For Practice:

Coding-Decoding questions:

1. If in a certain language MYSTIFY is coded as NZTUJGZ, how is NEMESIS coded in that language?
   Ans: OFNFTJT.

2. In a certain code, SIKKIM is written as THLJJL. How is TRAINING written in that code?
   Ans : UQBHOHOF.

3. If in a certain language, MADRAS is coded as NBESBT, how is BOMBAY coded in that code?
Ans: CPNCBZ.

4. In a certain code, TRIPPLE is written as SQHOOKD. How is DISPOSE written in that code?
Ans: CHRONRD.

5. If in a code language. COULD is written as BNTKC and MARGIN is written as LZQFHM, how will MOULDING be written in that code?
Ans: LNTKCHMF.

Syllogism questions:

Question 1 to 3:
Give answers (a) if only the conclusion I follow.
(b) if only conclusion II follows.
(c) if either I or II follows.
(d) if neither I nor II follows
(e) if both follow

1. Statements:
(A) All cats are dogs.
(B) All dogs are brown.
Conclusions:
I. All cats are brown.
II. All brown are dogs.
Ans : a.

2. Statements:
(A) All computers are Pentiums.
(B) Some Pentiums are machines.
Conclusions:
I. Some computers are machines.
II. Some machines are computers.
Ans : d.

3. Statements:
(A) Some apples are fruit.
(B) Some fruits are sour.

Conclusions:

I. Some apples are sour.

II. Some sours are fruit.

Ans: c.

4. Statements: (A) Some apples are fruits.

(B) All vegetables are fruits.

(C) All fruits are vegetables.

Conclusions: I. Some apples are vegetables.

II. All vegetables are fruits.

III. All fruits are apples.

IV. All vegetables are apples.

(a) Only I and II follow.

(b) Only II follows.

(c) Only I and IV follow.

(d) Only II and IV follow.

(e) None of these.

Ans: a.

5. Statements: (A) Some cars are four-wheelers.

(B) All four-wheelers are vehicles.

(C) Some vehicles are SUVs.

Conclusions: I.Some SUVs are four-wheelers.

II. Some vehicles are four-wheelers.

III. Some vehicles are cars.

IV. Some SUVs are cars.

(a) All follow

(b) Only II & III follow

(c) Only III follows

(d) Either III or IV follows

(e) None of these

Ans : b.

Notes For Permutation and Combination

Intro To Premutation And Combination:

Permutation and combination are all about counting and arrangements made from a certain group of data. You have a counting situation which requires formulas. If count is small you do not require formulas but if count is large you require formulas for counting.
For example:
If you have to count 1 to 10, you can easily do this, but if you have to count upto 10255 it will require formulas.

Permutation: In mathematics, permutation relates to the act of arranging all the things of a set into some sequence or order.

Combination: Combinations can be defined as the number of ways in which ‘r’ things at a time can be selected from amongst ‘n’ things available for selection.

This chapter gives you counting situations which are mapped to the use of certain formulas and you have to know which formula is used in which situation.

Every P & C question will always end with asking you to “Find the numbers of ways?’ doing something. Whenever you identify that the question is a P & C question, you 1st ask yourself if it is a selection question, distribution question, or it is an arrangement question then you go with an appropriate formula.

This chapter splitted into 3 parts:

1. Selection **2.** Distribution **3.** Arrangement

Selection:

Selection can be defined as the number of ways in which r things at a time can be selected from amongst n things available for selection.
Let say select two people for 4 people A,B,C,D and count the number of different ways i which one can make the selection.

Count physically;
1st selection is AB, 2nd selection is AC, 3rd selection is AD, 4th selection is BC, 5th selection is BD, 6th selection is CD.
Hence the number of possible selections = 6.
But if you have to select 8 people from the 16 people. You can not physically count the number of selections because there are so many possible cases which are not possible to visualize. Hence in order to handle this situation you need the **nCr** formula.

This formula tells us if you have 'n' "distinct" objects from them select 'r' objects and you want to count the number of selections.

Thus, **nCr = n! /[r! (n–r)!]** ; where $n \geq r$.

Example 1:
Selection of 2 people from 4 people.
Here n = 4 and r = 2
According to formula, $4C2 = 4!/2! \times 2! = 6$ ways.

Example 2:
Selection of 8 objects from 16 objects.
Here n = 8 and r = 16
According to formula, $16C8 = 16!/8! \times 8!$ways.

Factorial: The product of an integer and all the integers below it.ie $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
$n! = 1 \times 2 \times 3 \times \ldots\ldots\ldots\ldots\ldots \times n$ **OR** $n \times (n-1) \times (n-2) \times \ldots\ldots\ldots\ldots\ldots \times 3 \times 2 \times 1$

NOTE : Factorial only defined for the whole number.

value of 0! Is always 1.
7! Can be written as $7 \times 6!$ Or $7 \times 6 \times 5!$
$8C3 = \frac{8\times7\times6}{3\times2\times1} = 56$. And $7C2 = \frac{7\times6}{2\times1} = 21$.

Here in the denominator we just talk about the factorial of r and in the numerator start with 8 and take terms equal to 'r'.

nCr = nC(n-r). This formula comes from the following logic,

Number of selection of ‘r’ things from ‘n’ distinct things $\equiv$ number of ways of rejecting (n-r) things for ‘n’ things.
Let say form 7 people you ask to select 5.
Select 5 from 7 $\equiv$ reject 2 from 7

Eg. 16C13 = 16C3 if the value of ‘r’ is large then use the formula **nCr = nC(n-r).**

Value of **nC0 = 1** and **nC1 = n.**

Questions on Selection

Case1: A,B,C,D are 4 ‘distinct’ people and you have to select 2 people.
Case2: 4 ‘identical’ objects and you have to select 2 objects.

In both cases we are talking about the number of selections. In 1st case the selection process is differ at each step i.e. AB,AC,AD,BC,BD and CD. In the 2nd case there is only one selection because objects are identical.

NOTE: Selecting ‘r’ things from ‘n’ identical things, number of selections is always one.

Problem 1:
There is a room with 12 people and everyone shakes hands with each other. What is the number of handshakes?

Solution:
To understand this lets take a scenario where 3 people A,B,C and count the number of handshakes.

1st hand shake between A- B.
2nd hand shake between A - C
3rd hand shake between B - C
This is similar to selecting 2 people from 3 i.e. 3C2.
Hence, in the given question the number of handshake will be 12C2 = 12 × 11/2 = 66.

This question may asked in different way,

Problem 2:

In a room there are 8 men, and 6 women and a handshake is held between 1 man and 1 woman.What is the number of handshakes?

Solution:

To visualize this take a small case, 3 men A,B,C and 2 women D, E in a room and they start handshake with each other.

Then, 1. A handshake with D.
2. A handshake with E.
3. B handshake with D.
4. B handshake with E.
5. C handshake with D.
6. C handshake with E.

Hence the total number of handshakes is 6.

This is similar to selecting a man and a woman. Number of handshake = 3C1 × 2C1 = 3 × 2 = 6. Hence, in the given questions selecting a man out of 8 men and a woman out of 6 women, then the number of handshake will be 8C1 × 6C1 = 8 × 6 = 48.

Problem 3:

In a room there are a certain number of people and everybody handshake with each other. It was found that the number of handshakes was 153. Find the number of people in the room?

Solution:

Let's say in the room there are n people and everybody handshake with each other.

Total number of handshake = nC2 = 153

$$n \times (n-1)/2 = 153$$

$$n^2 - n - 306 = 0$$

Therefore n = 18,-17 but the number of people can not be -ve. So, n = 18 people.

Problem 4:

In a room with men and women everybody handshake with each other. The number of handshakes between 2 men is 153 and the number of handshakes between 1 man and 1 woman is 180. Find the total number of handshakes in the room?

Solution:

Let 'n' be the number of men in the room and 'w' be the number of women in the room.

Number of handshakes between 2 men = 153 i.e. nC2 = 153

$$n \times (n-1)/2 = 153$$

$$n^2 - n - 306 = 0, \text{ hence } n = 18 \text{ men in the room.}$$

Handshakes between a man and a woman = 180. i.e. nC1 × wC1 = 180

$$n \times w = 180 \text{ and hence, } w = 180/18 = 10 \text{ women.}$$

Therefore the total number of people in the room = 18+10 = 28.

Total number of handshakes = 28C2 = 28 × 27/2 = 378.

Questions on selection-2

Problem 1:

In how many ways can a team of the 3 players be selected from 11 players?

Solution:

Here the value of n = 11. And value of r = 3.

Thus, 11C3 = $\frac{11\times10\times9}{3\times2\times1}$ = 165.

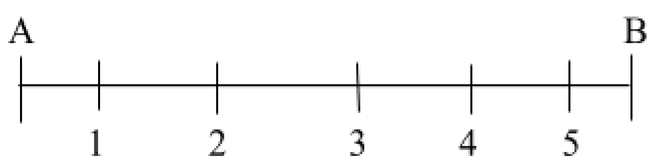
Similarly form a cricket team from 16 players = 16C11

Problem 2:

A train going from station A to B with 5 stations in between A to B and 6 people get into the train during the journey (not at A) with different tickets. How many different sets of tickets?

Solution:

We have 5 stations in between A and B.



If somebody gets in at station 1. He will have tickets available to station 2,3,4,5 and B.

So, people who get in at station 1 will have a choice of 5 tickets.

Likewise a person who gets in at station 2 will have a choice of 4 tickets.

A person who gets in at station 3 will have a choice of 3 tickets.

A person who gets in at station 4 will have a choice of 2 tickets.
A person who gets in at station 5 will have a choice of 1 ticket.
Total choice = 5+4+3+2+1 = 15 tickets.

From 15 tickets you are selecting 6 because 6 people have got on the journey with 6 different tickets.
Hence selecting 6 from 15 = 15C6.

Problem 3:
8 collinear points on a plane, with these points how many 1. Triangle 2. Quadrilateral 3. Straight lines can be formed?

Solution:
1. To form a triangle you need to select any 3 points out of 8.
So, number of triangle = 8C3 = $\frac{8\times7\times6}{3\times2\times1}$ = 56.

2. To form a quadrilateral you need to select any 4 points out of 8.
So, number of quadrilateral = 8C4 = $\frac{8\times7\times6\times5}{4\times3\times2\times1}$ = 70.

3. To form a straight line you need to select any 2 points out of 8.
So, number of straight line = 8C2 = $\frac{8\times7}{2\times1}$ = 28.

Formulae For Selection

Already we have discussed two formulae for selection,

1. **nCr =** $\frac{n!}{r!(n-r)!}$
2. **nCr = nC(n-r)**
3. **Total number of selections of zero or more things out of n different things**
   nCo + nC1 + nC2 + … + nCn
   nCo + nC1 + nC2 + … + nCn = 2^n

For example:
Let's say A,B,C are 3 different objects and you want to select any number of objects(including 0), then the total number of selections are?

Solution:

You have the following choice to select the objects are,

Select 0 object or select 1 object or select 2 object or select 3 object

(here 'or' refers to +)

i.e. 3C0 + 3C1 + 3C2 + 3C3

According to formula; nCo + nC1 + nC2 + … + nCn = 2^n

Here n = 2.

Therefore number of selection are = 2^3 = 8.

4. **The number of selections of 1 or more things out of n different things**
   nC1 + nC2 + … + nCn = 2^n - 1

For example:

Number of different values of exact change that you can pay if you have one coin each of 1 Rs, 5Rs, 10Rs and 50Rs.

Solution:

You can pay money by selecting;

Selecting 1 coin or Selecting 2 coins or Selecting 3 coins or Selecting 4 coins or selecting 5 coins.

eg. If you have to pay 3 Rs, you can pay by 2 coins (1Rs and 2Rs coins). Likewise 6Rs, 7Rs, 12Rs, only pay by using 2 coins.

Values like 65 Rs you can pay only by using 2 coins (5Rs, 10 Rs and 50 Rs coins)

i.e. 5C1 + 5C2 + 5C3 +5C4 + 5C5

According to formula; nC1 + nC2 + … + nCn = 2^n - 1

Here n = 5.

Therefore number of different values = 2^5 - 1 = 31.

5. Total number of selections of zero or more things out of n identical things = n + 1 (include zero thing)

6. Total number of selections of 1 or more things out of n identical things = **n**

Question On Selection-3

Type 1: Question involving pre selection

Problem 1:

In a cricket team there are 16 players and select 11 players such that the captain is always selected. Find the total number of selections?

Solution:

Here given that the captain always be selected (i.e. preselected) now you have to select only 10 players from 15 players.
Therefore, selection of 10 from 15 = 15C10.

Problem 2:

A hostel warden who has a hostel with 12 students living inside it. He selects 3 students for a committee every week and he always wants to select his favourite student in the committee. How many weeks can he continue with selecting the same group again?

Solution:

Let's say his favourite student is A and has to be in the committee. Now he has to select only 2 students from 11 students.
Therefore, selection of 2 from 11 = 11C2.

Type 2: Constraint based selection

Problem 1:

Out of 6 men and 4 women and you have to select a committee of 3 with at least one woman. In how many different ways can it be done?

Solution:

You have committee with at least 1 woman are,
1 women and 2 men or 2 women and 1 man or 3 women and no man
4C1 × 6C2 + 4C2 × 6C1 + 4C3 × 6C0

2nd method:

Committee of all men subtracted from total number of committee i.e. 10C3 - 6C3

From 10 people if you want to draw a committee of 3, will be 10C3.

If divide 10 people into 6 men and 4 women and you have to make committee of 3 and do not given any constraint in case you decide to do this problem using how many men and how many women then you have to write all possible committee i.e.

3 men & no woman or 2 men & 1 woman or 1 man & 2 women or no man & 3 women i.e.
6C3 × 4C0 + 6C2 × 4C1 + 6C1 × 4C2 + 6C0 × 4C3

Problem 2:
A plane with 12 points all are non-collinear except 5 points that lie on the same line. How many triangles, quadrilaterals and straight lines can be formed?

Solution:
Let's say ABCDE are the collinear points and FGHIJKL are non collinear points.
1. To form a triangle take
2 points from collinear & 1 point from non collinear or 1 point from collinear & 2 points from non collinear or no points from collinear & 3 points from non collinear

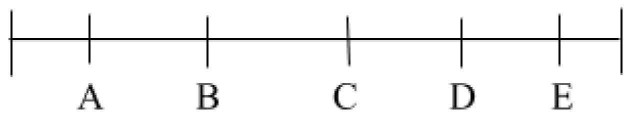
Number of triangle = 5C2 × 7C1 + 5C1 × 7C2 + 5C0 × 7C3

2. To form a quadrilateral take
2 points from collinear & 2 points from non collinear or 1 point from collinear & 3 points from non collinear or no points from collinear & 4 points from non collinear

Number of quadrilateral = 5C2 × 7C2 + 5C1 × 7C3 + 5C0 × 7C4

3. To form straight lines
1 point from collinear & 1 point from non collinear or no point from collinear & 2 points from non collinear

Number of straight line = 5C1 × 7C1 + 5C0 × 7C2 + 1
Add 1 because in collinear points when you select any two points from these, you form the same line whether you select AB or DE. Hence ABCDE lines do not get counted when you select one point from collinear and one point from non collinear.



Distribution of identical objects

Distribution can happen of identical objects or distinct objects.

Number of ways of distributing n identical things among r persons when each person may get any number of things = (n + r – 1) C(r–1)

Problem 1:

If you have 4 identical objects to give between two friends X & Y. What are the number of distributions?

Solution:

	X	**Y**
1st distribution	4	0
2nd distribution	3	1
3rd distribution	2	2
4th distribution	1	3
5th distribution	0	4

Therefore total number of distributions = 5

According to formula;

Here n = 4 and r = 2

So, the total number of distributions = (4+2-1)C(2-1) = 5C1 = 5.

Problem 2:

If x+y+z = 20 and x,y,z are whole numbers. How many solutions does x+y+z = 20 have?

Solution:

x+y+z = 20 is the same as distributing 20 objects between x,y and z.

Here n = 20 and r = 3.

So, the total number of solutions = (20+3-1)C(3-1) = 22C2 = 231 .

If x,y,z are natural numbers, in this case this formula does not work directly because in this case zero is not allowed.

Problem 2:
20 identical chocolates are distributed amongst A,B,C such that each person gets at least 1 chocolate. What are the number of distributions?

Solution:
In this case we do not use the formula **(n + r – 1) C(r–1)** because it includes the 20, 0, 0 and 19, 1 , 0 amongst A,B,C respectively.
From 20 chocolates first you have to give 1 to each of A,B,C, then you left with 17 chocolates, now you are allowed to give those 17 chocolates freely to these 3 people as you want including zero distribution.

	A	B	C
1st distribution	1	1	1

Now n = 17 and r = 3.
So total number of distributions = (17+3-1)C(3-1) + 1= 19C2 + 1

Problem 3:
A+B+C = 20, A,B,C ≥ 2 and all are integers. How many solutions does it have?

Solution:
A+B+C = 20, this is the same as 20 identical chocolate distributed amongst 3 people A,B,C with minimum 2 chocolate each.

	A	B	C
1st distribution	2	2	2

Now you left with 14 and these 14 distribute among 3.
Here n = 14 and r = 3
So total number of distributions = (14+3-1)C(3-1) + 1 = 16C2 + 1
This approach is called a modified 'n' approach.

Problem 4:
20 identical chocolates are distributed amongst A,B,C such that A gets minimum 3, B gets minimum 5 chocolates. What are the number of distributions?

Solution:
20 identical chocolate distributed amongst 3 people A,B,C A with minimum 3 and B with minimum 5 chocolates.

	A	B	C
1st distribution	3	5	0

Now you left with 12 and these 12 are distributed among 3.
Here n = 12 and r = 3
So total number of distributions = (12+3-1)C(3-1) + 1 = 14C2 + 1

Formulae For Arrangement

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1. MNP Rule

It tells us if you have 3 tasks to do and there are M ways of doing the first thing, N ways of doing the second thing and P ways of doing the third thing then there will be M × N × P ways of doing all the three things together.

This formula is used to do problems on arrangements and also used for distribution of distinct objects.

Problem 1:
Shubham wants to go from Mumbai to Pune and Pune to Delhi and Delhi to Kolkata. There are 6 trains from Mumbai to Pune, 5 trains from Pune to Delhi and 8 trains from Delhi to kolkata. Find the total number of ways of travelling?

Solution:

$$\text{Mumbai} \xrightarrow[6\,T]{} \text{Pune} \xrightarrow[5\,T]{} \text{Delhi} \xrightarrow[8\,T]{} \text{Kolkata}$$

So, total number of ways of travelling = 6 × 5 × 8 = 240.

2. r! Formula

If you have ‘r’ distinct things and you want to place them in ‘r’ places, then the total number of ways **= r!**

Problem 1:
6 people ABCDEF and you want to sit them on 6 chairs. Find the total number of ways of sitting?

Solution:
The 1st chair can be filled by 6 people.
The 2nd chair can be filled by 5 people.

The 3rd chair can be filled by 4 people.
The 4th chair can be filled by 3 people.
The 5th chair can be filled by 2 people.
The 6th chair can be filled by 1 person.
So the total number of ways = 6! = $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.
r! Nothing but the MNP rule used for 'r' distinct objects in 'r' places.

3. r! modified for arrangement of identical objects

Number of arrangements of 'n' things out of which P1 are alike and are of one type, P2 are alike and are of a second type and P3 are alike and are of a third type and the rest are all different
= **n!/ P1! P2! P3!**

For example:
AAA BB CCC and you want to be placed in 8 places.
AAA are 3 alike things, BB are two alike things and CCC are three alike things.

So, total number of ways = $8!/3! \times 2! \times 3!$

4. nPr formula

nPr = number of arrangements of 'n' distinct things taken r at a time.
nPr = $n!/ (n - r)!$; $n \geq r$

For example:
Six people ABCDEF arrange in 3 places = 6P3 = 6!/3! = 120.
Similar situation is getting handled using the MNP rule. So, according to MNP rule, 6 people arranging in 3 places = $6 \times 5 \times 4 = 120$

The Relationship Between Permutation & Combination:

When we look at the formulae for Permutations and Combinations and compare the two we see that,
nPr = $r! \times$ nCr
i.e. The arrangement of r things out of n is nothing but the selection of r things out of n followed by the arrangement of the r selected things amongst themselves.

Generic Questions On Arrangements

Problem 1:
In how many ways can you send 5 letters, if you have 4 servants. Any servant be used any number of times.

Solution:
You have to send L1&L2&L3&L4&L5. Each of these 5 distributions you have 4 ways of it because you have 4 servants.
So, total number of ways = 4^5

Problem 2:
In how many ways in which to wear 6 distinct rings in 4 fingers, if any finger has any number of rings.

Solution:
Each of these 6 rings can have 4 fingers.
So, total number of ways = $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

these type of question always confusing about weather it 4^6 or 6^4.

Let's say 8 servants and 5 letters. Answer will be either 8^5 or 5^8 Which one will you choose? Between servants and letters one of them is repeatable and the other is non repeatable. If you think about servants and letters you can send the servant again and again but you can not send letters again and again. So the repeatable aspect is servants and the non repeatable aspect is letters.

NOTE : In this type of question answer will be R^{NR}. Here R = repetible and NR = Non repetible.

Problem 3:
A team of 16 players, making a batting order of 11 players, such that the captain always selected. Find the total number of ways?

Solution:
This question can be done by selection and arrangement.
1 player is preselected and out of 15 players we have to select 10 players.
No of selection = 15C10 and no of arrangement = 11!.
So total number of ways = 15C11 $\times$ 11!.

Problem 4:
In how many ways 7 people A,B,C,D,E,F,G are placed in 7 places such that A & B are together?

Solution:
A&B are together. So, A&B counted as one person and 5 people separately, effectively there are 6 people.
Arrangement of 6 people is 6! And arrangement of AB = 2!.
Therefore total number of ways = $6! \times 2!$.

Problem 5:
In how many ways 7 people A,B,C,D,E,F,G are arranged in a straight line in 7 places such that A is always in the middle?

Solution:
Middle place is fixed by A and the remaining 6 places are filled by 6 people.
So, total number of ways = 6!.

Problem 6:
In how many ways 7 people A,B,C,D,E,F,G are arranged in 7 places such that no two of A,B,C are together?

Solution:
A,B,C in 3 places is 3! And D,E,F,G in 4 places is 4!
Total number of ways = $3! \times 4!$.

Questions On Word Formation

Type 1: Word formation question
Problem 1:
How many words can be formed with the word PATNA, LUCKNOW and JAIPUR which have
1. No restrictions.
2. Total number of new words
3. Start with the first letter.
4. Start and end with vowels.

Solution:
PATNA

1. Total number of letters - P,T,N occurs once while A occurs twice.
So, the total number of words that can be formed = 5!/2! = 60

2. Total number of new words = 60 - 1 = 59.

3. We can arrange only 4 letters (as place of P is restricted) in which A occurs twice.
So, the total number of words that can be formed = 4!/2!

4. In the word PATNA in which we have 2 vowels(A,A).
So, the total number of words that start with A and end with A = 3!

LUCKNOW

1. Total number of distinct letters = 7.
So, the total number of words that can be formed = 7!

2. Total number of new words = 7! - 1.

3. We can arrange only 6 letters (as place of L is restricted)
So, the total number of words that can be formed = 6!

4. In the word LUCKNOW in which we have 2 vowels(U,O). Arrangement of two vowel = 2!
So, the total number of words that can be formed = $2! \times 5!$

JAIPUR

1. Total number of distinct letters = 6.
So, the total number of words that can be formed = 6!

2. Total number of new words = 6! - 1.

3. We can arrange only 5 letters (as place of J is restricted)
So, the total number of words that can be formed = 5!

4. In the word JAIPUR in which we have 3 vowels(A,I,U). We have to select 2 vowels and arrange them amongst 1st and last place = $3C2 \times 2!$and also arrange 3 consonants and 1 vowel = 4!
So, the total number of words that can be formed = $3C2 \times 2! \times 4!$.

Type 2: Dictionary position question

Problem 1:
What is the dictionary position of the word RUPAJI that can be formed by letters of the word JAIPUR?

Solution:
1st arrange all the letters of the word JAIPUR in alphabetically order for reference.
A-I-J-P-R-U
Number of words starting with A = 5!
Number of words starting with I = 5!
Number of words starting with J = 5!
Number of words starting with P = 5!
Number of words starting with R = 5!
Number of words starting with U = 5!

You are looking for the word RUPAJI. In this word letter 'U' will come only after the letter 'R'. so, the words starting with letter 'U' are not considered. RUPAJI one of the word inside words start with letter 'R'

Before the words start with the letter 'R' we have words = 5! + 5! +5! + 5! = 480 words.

Words start with the letter 'R'

Number of words starting with RA = 4!
Number of words starting with RI = 4!
Number of words starting with RJ = 4 !
Number of words starting with RP = 4!
Number of words starting with RU = 4!

RUPAJI one of the word inside the words start with letters 'RU'
Before the words start with the letters 'RU' we have words = 480 + 4! + 4! + 4! + 4! = 480 + 96 = 576 words.

Words start with the letter 'RU'
Number of words starting with RUA = 3!
Number of words starting with RUI = 3!
Number of words starting with RUJ = 3!
Number of words starting with RUP = 3!

RUPAJI one of the word inside the words start with letters 'RUP'

Before the words start with the letters ‘RUP’ we have words = $480 + 96 + 18 = 594$ words.
Remaining lettres A,I,J six words can be form from A,I,J

AIJ,AJI,IAJ,IJA,JAI,JIA. So out of six the 2nd word AJI will complete the word RUPAJI
Therefore the position of the word RUPAJI = 594 + 2 = 596.

Questions On Number Formation

Forming numbers with and without replacement:

Problem 1:
How many 4 digit numbers can be formed by using digit 1,2,3,4,5,6 and 7 with replacement of digit allowed?

Solution:
To forming a 4 digit number with replacement;
1st place can be filled with any of the 7 digits.
2nd place can be filled with any of the 7 digits.
3rd place can be filled with any of the 7 digits.
4th place can be filled with any of the 7 digits.
Therefore total number of ways = $7 \times 7 \times 7 \times 7 = 7^4$

Problem 2:
How many 4 digit numbers can be formed by using digit 0,1,2,3,4,5 and 6 with replacement of digit allowed?

Solution:
1st place cannot be filled with zero because it makes 4 digit numbers in 3 digit numbers.
So, To forming a 4 digit number with replacement;
1st place can be filled with any of the 6 digits.
2nd place can be filled with any of the 7 digits.
3rd place can be filled with any of the 7 digits.
4th place can be filled with any of the 7 digits.
Therefore total number of ways = $6 \times 7 \times 7 \times 7 = 6 \times 7^3$

Problem 3:
How many 4 digit numbers can be formed by using digit 1,2,3,4,5,6 and 7 without replacement of digits?

solution:
To forming a 4 digit number without replacement;
1st place can be filled with any of the 7 digits.
2nd place can be filled with any of the 6 digits.
3rd place can be filled with any of the 5 digits.
4th place can be filled with any of the 4 digits.
Therefore total number of ways = $7 \times 6 \times 5 \times 4$

Problem 4:
How many 4 digit numbers can be formed by using digit 0,1,2,3,4,5 and 6 without replacement of digits?

Solution:
1st place cannot be filled with zero because it makes 4 digit numbers in 3 digit numbers.
So, To forming a 4 digit number without replacement;
1st place can be filled with any of the 6 digits.
2nd place can be filled with any of the 6 digits.
3rd place can be filled with any of the 5 digits.
4th place can be filled with any of the 4 digits.
Therefore total number of ways = $6 \times 6 \times 5 \times 4$

Limit based question:

Problem 1:
How many 4 digit numbers can be formed by using digit 0,1,2,3,4 such that the numbers are not greater than 4000?

Solution:
In this question we can think that numbers are not greater than 4000. So, numbers are starting with digit 1,2 and 3. First place cannot be filled with zero because it makes 4 digit numbers in 3 digit numbers.

Numbers starting with 1
1st place can be filled with 1 digit i.e 1.
2nd place can be filled with any of the 5 digits.
3rd place can be filled with any of the 5 digits.

4th place can be filled with any of the 5 digits
So, the number of ways = $1 \times 5 \times 5 \times 5 = 125$.

Numbers starting with 2
1st place can be filled with 1 digit i.e. 2.
2nd place can be filled with any of the 5 digits.
3rd place can be filled with any of the 5 digits.
4th place can be filled with any of the 5 digits
So, the number of ways = $1 \times 5 \times 5 \times 5 = 125$.

Numbers starting with 3
1st place can be filled with 1 digit i.e. 3.
2nd place can be filled with any of the 5 digits.
3rd place can be filled with any of the 5 digits.
4th place can be filled with any of the 5 digits
So, the number of ways = $1 \times 5 \times 5 \times 5 = 125$.
And number 4000 itself will get counted.
Therefore total 4 digit numbers = 125+125+125+1 = 376.

NOTE : When in number formation nothing is mentioned about weather repetition allowed or not, in that case default is repetition allowed.

Problem 2:
How many 4 digit numbers can be formed by using the digits 0,1,2,3,4,5 and 6 which are divisible by 5.
1. With repetition allowed.
2. With repetition not allowed.

Solution:
(a) 1. With repetition:
Divisibility rule of 5 is that the last digit can be 0 or 5. So, the last digit can be filled by 0 or 5.
Numbers end with zero = $6 \times 7 \times 7 = 294$.
Numbers end with 5 = $6 \times 7 \times 7 = 294$.
Therefore total numbers = 294+294 = 588.

2. Without repetition
Numbers end with zero = $6 \times 5 \times 4 = 120$.
Numbers end with 5 = $5 \times 5 \times 4 = 100$.

Therefore total numbers = 120+100 = 220.

Problem 3:
How many 4 digit numbers can be formed by using the digits 0,1,2,3,4 and 5 which are divisible by 4.

Solution:
Divisibility rule of 4 is that the last 2 digits are divisible by 4.
Numbers end with last 2 digits 0,0 = 5×6
Numbers end with last 2 digits 0,4 = 5×6
Numbers end with last 2 digits 1,2 = 5×6
Numbers end with last 2 digits 2,0 = 5×6
Numbers end with last 2 digits 2,4 = 5×6
Numbers end with last 2 digits 3,2 = 5×6
Numbers end with last 2 digits 3,6 = 5×6
Numbers end with last 2 digits 4,0 = 5×6
Numbers end with last 2 digits 5,2 = 5×6
Therefore total numbers = $9 \times (5 \times 6) = 270$.

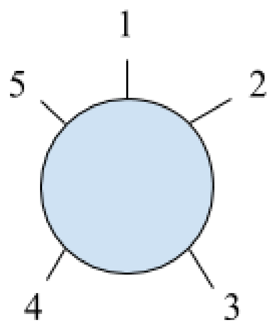
Circular Arrangements

In this chapter you just need to understand a couple of things. On a circle every position is the same, unlike straight lines every position is different.

1. **Number of ways of placing 'r' distinct objects on 'r' places is equal to (r-1)!**
2. **If there is a reference point on a circle no need to do minus 1.**

For example:
How many ways of arranging 5 people on seats in a circular table (seat 1 is a reference point)?

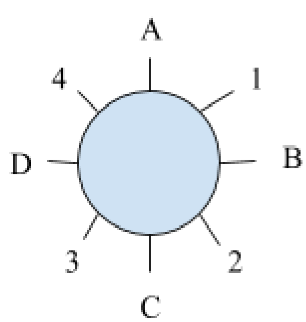
Solution:



Seat 1 is a reference point. So, the number of arrangements = 5!

Problem 1:
In how many ways 4 Indian and 4 European sit in alternate places around a circle?

Solution:



Let say 4 Indian sit in A,B,C,D places around a circle. Now you have a circle with a reference point.
Number of ways of arranging 4 Indian = (4-1)! = 6 and Number of ways of arranging 4 European = 4! = 24
Therefore total number of ways = 6 × 24 = 144.

3. **'N' objects arrange around a circle where clockwise is equal to anticlockwise, then the number of arrangements = (n-1)!/2**

Some Practice Questions

1. How many numbers of 3-digits can be formed with the digits 1, 2, 3, 4, 5 (repetition of digits not allowed)?
Ans : 60.

2. In how many ways can a person send invitation cards to 6 of his friends if he has four servants to distribute the cards?
Ans : 4^6 **.**

3. In how many ways can the letters of the word 'EQUATION' be arranged so that all the vowels come together?
Ans : 4! $\times$ **5!.**

4. How many straight lines can be formed from 8 non-collinear points on the X-Y plane?
Ans : 28.

5. For the arrangements of the letters of the word PATNA, how many words would start with the letter P?
Ans :12.

Lecture Notes For Reading Comprehension

We can see reading comprehension for all aptitude exams. A reading comprehension puts to test your reading as well as writing skills. One has to be very very careful about how one is reading and how one is solving. First of all, remember the different methodology of reading and keeping in mind what is essential.

Read effectively for reading comprehension:

To read a comprehension you have to follow the following points:

1. Skimming
2. Scanning
3. Main idea
4. Tone and style
5. Vocabulary
6. References
7. Conclusion

1. Skimming: When you read something and go fast over the details. If there are a lot of facts related things etc. which would not be read so much, you can go quickly over the details.

2. Scanning: We will go with the essential points and go a little slow in that area, meaning you are reading carefully. In this case there are the points from where the main idea comes and from where the author shows the change, showing an important example or trying to put an argument. So, the points from which question has been asked. These are the points which you have to scan while reading.

3. Main idea: The main idea is essence. The main idea is something around which the whole passage is revolving.

4. Tone and style: When you talk about tone and style, you have to look at the purpose of writing. Basically, why has something being written, automatically you try to find the tone and style.
Eg. Informational style of writing, entertainment style of writing, argumentative style of writing or analytical style of writing.

5. Vocabulary: Vocabulary in context means according to passage how the particular word is used. As you start reading the passage, you are likely to come across new words. Do not be frightened. Try to guess the meaning with the help of context and move ahead.

6. References: Point of references from where the author asked the questions. Point of references could be a fact, any information, any data, any vocabulary word, an example and a person.

7. Conclusion: The conclusion is what you learnt from the passage.

How to find main idea

Before trying to find out the main idea, also look at the kinds of topics that one needs to be comfortable with while reading.

TOPICS:

1. Philosophy
2. Psychology
3. Genetic- biology, animal life, ecology and environment.
4. Science and technology
5. Politics and sociology
6. Civilization and History
7. Economics
8. Biography

These topics you can read from- bbc.com, history.com, Hindu editorial, epw.in, national geographic magazine, encyclopedia, economist magazine etc. these are the places from where you can read and become comfortable with all these topics.

What you have to keep in mind while reading:

1. What is the topic?
2. What is the most essential point about the topic and i.e. the main idea.

The positive argument, negative argument, by supporting, by opposing, by explaining etc. if everything is pointing towards the one thing, i.e. your main idea.

Whatever you think of the main idea, if you remove it from the story and you find that the whole story changes you got the right main idea.

Note that the author doesn't need to write the entire main idea of a paragraph in one sentence. Sometime the main idea might be distributed across two or three sentences in the paragraph.

Passage 1:

"In the first weekend of every August, the town of Twinsburg, Ohio, holds a parade, Decorated floats, cars and lorries roll slowly past neat, white houses and clipped lawns, while thousands of onlookers clap and wave flags in the sunshine. The scene is a perfect little slice of America. There is, though, something rather strange about the participants: they all seem to come in pairs. Identical twins of all colours, shapes, ages, and size are assembling for the world's largest annual gathering of their kind.

The Twinsburg is of interest to more people than just the twins themselves. Every year, the festival attracts dozens of scientists who come to prod, swab, sample and question the participants. For identical twins are natural clones: the odd mutation aside, they share 100% of their genes. That means studying them can cast light on the relative importance of genetics and environments in sharing particular human characteristics."

Answer:
The topic is 'identical twins' and the main idea as expressed in the second last and last sentences of the second paragraph is that studying identical twins can help us understand better,the relative importance of genetics and environment is shaping particular human characteristics.'

Passage 2:

In Pakistan - they are kachi bodies, in Cuba - focus isalubers, in India - bustees and Brazil - favelas. Whatever the local name of slums, there are a lot of them and they are growing fast. A new report, "The Habitat," by the United Nations agency responsible for "human settlement", says that in 2001, just under a billion people were living in slums -- about a third of the world's city dwellers. In the last decade, the urban population in less developed regions increased by a third. On present trends, says the UN report, 2 billion people could be living in slums by 2023.

In Africa, many parts of the Middle East, Latin America and Asia, migrants are leaving farmland which is unable to support them, and arriving in cities which are unprepared to deal with them. This has been a long term trend and is unlikely to abate no matter how awful the slums become. In 1800, only 2 % of the world's population was urbanized; by 2008, more than half of the world will be. Because such migration is so predictable and long-established, it might seem surprising that many governments are ill-equipped for it. But there is little new in that either: the much-rich countries fared just as badly when their cities first began to grow rapidly.

1. Topic.
2. Main idea.

Answer:

1. Spiraling slums. (Spiraling means increasing very very fast)
2. Main idea: whatever the local name of slums, there are alot of them and they are growing fast.

Passage 3:

They have a dismal track record when it comes to predicting economic growth, exchange rates or the direction of the stock market. So, you might have expected economists to despair at the thought of forecasting sports results. Not at all. Efforts to work out the numbers of medals which countries are likely to get in the Athens Olympics, which start on August 13th, are well underway.

Answer:
The topic is forecasting sports results and the main idea: Efforts to work out the numbers of medals which countries are likely to get in the Athens Olympics.

Passage 4:

"You want rubes! We can do business!" the anonymous caller promised a lucrative deal over the phone: stumps up dollars in advance, and he would produce a glittering hoard in Johannesburg. But your correspondence refused, so the caller instead produced some colourful abuse, sneered at British muscle prowess, and hung up.

It is rare for any attempted African scam to be executed so inelegantly. Advance-fee frauds often lure victims to part with tens of thousands of dollars on the promise of huge, but somehow plausible, later gains. Nigerian, especially are renowned for elaborate and persuasive tables: "my uncle the president, died leaving me a million to smuggle to your country; let me use your bank account to hide the cash, and you will get a slice; oh, and pay me a few thousand dollars in advance for handling fees".

Answer:
Topic: Advance fee frauds in Africa.
Main idea: What is done in these frauds.

Theory of reading comprehension:

The best way to approach the reading comprehension section is to begin reading for the reading comprehension, to begin to interact with the text. When you read every day, start to ask yourself questions:

- What is the main idea of this passage?
- What is the author's position and tone (neutral? positive? critical? etc.)?
- What is the structure of the argument(two opposing or one-sided, informational or author's opinion)?

You can start by reading some publications available online:

The Economist, The Washington Post, Scientific American, Businessweek, the Hindu etc. these are some places from which you can read, and they give you an excellent insight about how to understand different topics.

You will learn how to read the different sections of passages more freely and learn how the other parts of texts are interwoven.

Therefore:
- Try to practice active reading once every day.
- Read CRT exam level publications.
- Practice determining the main idea, tone, type and structure of text.
- Pay attention to how the text flows.notice change in opinion or content.

Different question type in reading comprehension

- General
- Specific
- Direct
- Indirect
- Main idea
- Style and tone
- Title

General: General question is basically from where you can read the whole passage and answer. Question like:

- Which of the following authors agree with?
- Which of the following authors disagree with?
- Which of the following is mentioned in the passage?
- Which of the following is not mentioned in the passage?

These kinds of questions are your general questions.

Specific: Specific questions are from a particular point of reference; you just have to go to that point and give the answer.
Specific points - could be a fact, could be an argument, could be information, could be a vocabulary word etc.

Direct: Question based on fact, information and data are the direct question.

Indirect: These are the basic assumption, inference, conclusion question. Indirect questions are also called as which of the following can be implied, concluded, inferred or assumed.

Main idea: The main idea is usually either at the starting idea, as a concluding idea, somewhere in the middle and sometimes inferential, based on the passage you have to find out the essence.

The main idea can be direct and general, can be indirect and general, can be indirect and specific.

Style and tone - We already talked about style and tone.
Title: Some time this question comes. So, for this question, you have to be related to the topic in their idea. The title has to be related to the passage.

One thing is better to read the question first or read the passage first. If it is a long passage, it is better to read the question first because then you know which part of the passage you can skim and which part you have to scan.

Solving reading comprehension

Passage 1:

Ordinarily, nothing upset the equilibrium of the Pundit. But the loss of the necklace, which his wife had borrowed from a neighbour, and the fact that he had to replace it worried him. He spent all his time in devising ways and means to replace the debt. Six months later, he gave his wife a gold necklace. It was exactly what he had yearned for. "There is no need to repay it ", she said.

The Pundit was surprised. She explained, 'the necklace was not stolen. That was the only way I could think of making you get me one'.

1. The Pundit was almost never upset as he

a. Had the courage to face a problem.
b. Had a balanced attitude.
c. Was indifferent to ordinary incidents.
d. Always succeeded in finding a solution to his problem.
e. There always existed a solution.

Ans : had a balanced attitude.

2. What worried the pandit most was that?

a. His wife had lost the necklace.
b. He had to replace the necklace.
c. His wife was so foolish and careless.
d. The necklace was very expensive.
e. His wife had hidden the necklace.

Ans: he had to replace the necklace.

3. The Pundit struggle for six months so that

a. He could present a necklace to his wife.
b. He could lend the necklace to the others.
c. His wife could replace the lost necklace.
d. His wife could satisfy her desire to own a necklace.
e. He could buy a new necklace for his wife.

Ans : his wife could replace the lost necklace.

4. The reason why the Pundit's wife refused to return the necklace was that

a. She had always wanted one like it.
b. The real owner did not expect it back.
c. She had not lost any necklace.
d. She was tempted to keep this one for herself.
e. The real owner gifted it to her.

Ans : she had not lost the necklace.

5. The Pundit's wife had told him a lie in order to

a. Trick him to satisfying her yearning.
b. Punish him.
c. Make him repay a debt.

d. Force him to work hard to earn.
e. None of these.

Ans : Trick him to satisfying her yearning.

Passage 2:

Pablo Picasso showed his truly exceptional talent from a very young age. His first word was lapis(Spanish for pencil), and he had to learn to draw before he could talk. He was the only son in the family, and he was good- looking, so he was thoroughly spoilt. He hated school and often refused to go unless his doting parents allowed him to take one of his father's pet pigeons with him.

Apart from pigeons, his great love was art, and when in 1891 his father, who was an amateur artist, got a job as a drawing teacher at a college, Pablo went with him to the college. He often watched his father paint and sometimes was allowed to help. One evening his father was painting a picture of their pigeon when he had to leave the room. He returned to find that Pablo had completed the picture, and it was so amazingly beautiful and lifelike that he had given to his son his own palette and brushes and never painted again. Pablo was just 13.

1. As a boy Pablo Picasso was

a. Ordinary looking but talented.
b. Loved by one and all.
c. Handsome and studious.
d. Handsome and hardworking.
e. Glamorous and happening.

Ans : loved by one and all.

2. He was spoilt mostly because he was

a. A smart boy.
b. Loved by one and all.
c. The only son in the family.
d. Always surrounded by notorious boys.
e. Of the money power.

Ans : the only son in the family.

3. Picasso went to school only when

a. His friends accompanied him.’
b. His father went to him.’
c. He was allowed to paint at school.
d. He was allowed to carry a pet with him.

e. He was allowed to carry a new school bag.

Ans : he was allowed to carry a pet with him.

4. When his father painted his college, Pablo

a. Never helped him.
b. Rarely helped him.
c. Always helped him.’
d. Invariably helped him.
e. Occasionally helped him.

Ans : occasionally helped him.

5. Pablo’s father gave up painting because he

a. Did not like this job.
b. Retired from the college.
c. Was impressed by his son’s talent.
d. Lost interest in painting.
e. He thought about changing his profession.

Ans : was impressed by his son’s talent.

Passage 3:

Nature challenges humans in many ways, through disease, weather, and *famine*. For those living along the coast, one unusual phenomenon capable of *catastrophic* destruction is the tsunami (pronounced "tsoo-NAH-mee"). A tsunami is a series of waves generated in a body of water by an impulsive disturbance. Earthquakes, landslides, volcanic eruptions, explosions and even the *impact* of meteorites can create tsunamis. Starting at sea, a tsunami slowly approaches land, growing in height and losing energy through bottom friction and turbulence. Still, just like any other water wave, tsunamis *unleash* tremendous energy as they *plunge* onto the shore. They have great erosion potential, stripping beaches of sand, undermining trees, and *flooding* hundreds of meters inland. They can easily crash cars, homes, vegetation, and anything they *collide* with. To minimize the devastation of a tsunami, scientists are constantly trying to *anticipate* them more accurately and more quickly. Because many factors come together to produce a life-threatening tsunami, foreseeing them is not easy. **Despite this,** researchers in meteorology *preserve* in studying and predicting tsunami behaviour.

1. Which sentence best expresses the essential information of this passage?

a. Tsunamis become a new source of usable energy in the next hundred years.
b. Tsunamis do more damage to the land than flooding.
c. Tsunamis can have an especially catastrophic impact on coastal communities.

d. Scientists can predict and track tsunamis with a fair degree of accuracy, reducing their potential impact.

Ans : tsunamis can have an especially catastrophic impact on coastal communities.

2. In the first sentence, why does the author mention the weather?

a. Because tsunamis are caused by bad weather.
b. Because tsunamis are more destructive than weather phenomena.
c. As an example of a destructive nature force.
d. As an introduction to the topic of coastal storm.

Ans : as an example of a destructive nature force.

Lecture Note For Sentence Completion/Fillups

Fill in the blanks also in the same case is called sentence completion. It is basically a combination of both reading skill and grammar knowledge.

Sentence completion is of three types:

1. Single blank
2. Double blank
3. Cloze test

1. Single blank: It is basically one sentence with one blank that you have to fill.
2. Double blank: It is a longer sentence with two blanks that you have to fill.
3. Cloze test: It is like a paragraph having some blanks. Actually it a combination of both fill in the blanks and reading comprehension.

What all factor kept in mind:

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1. First of all, you should always have a mental answer when you are trying to solve a problem.
2. With the mental answer, match it with the option skill.
3. Vocabulary should be very very strong.

4. **The idea of the sentences**:
   Every sentence has an idea and each sentence also communicates ideas.

   For example:
   Sentences are either positive or negative. If the positive sentence the blank word will be positive and if the sentence is negative the blank word will be negative.
   Whether sentences are formal or informal. Let us say friend is a formal word and pal/buddy is an informal word.

5. **Proactive solving:**
   Usually, sentences go through the option first and try to somewhat how to fit into the blanks, this way of approach is called **reactive solving** and this is likely to cause errors.

A better way would be proactive solving means acting in anticipation. In other words, try to guess the answer without solving.

6. Identify the clues present in the sentence. A positive sentence, negative sentence, formal sentence, informal sentence these all are clues in the sentence.

7. Pay special attention to introductory and transitional words. **Introductory** means this thing or that thing is talking about one thing or many things. **Transitional** words are like, but, although, however, yet, even, in spite off, despite off etc.

   For example:
   Ravi is a good boy but his brother is a bad boy.
   If the 1st part is positive and the 2nd part will be negative and vice versa.

8. Be sure your choice is both logically and grammatically correct. Make sure your grammar matches with the sentence, otherwise, grammar is not matching even if the meaning of the word is correct, grammatically the sentence will be wrong.

9. If you do not know words use elimination and educated guessing. Which means you are able to make one or more choices that are definitely wrong or guessing from context when you know a related word.

There are of several types of sentence completion:

---

1. Restatement
2. Comparison
3. Contrast
4. Cause and effect

1. Restatement: Restatement means repeating the same things again and again. So, if it's a positive one, it will be positive and if it’s a negative one, it will be negative.

For example:
The city council formed a committee to simplify several dozen ______ city ordinances that were unnecessarily complicated and out-of-date.

a. feckless b. empirical c. byzantine d. Slovenly e. Pedantic

Answer :
Here we are talking about something which was very complex and has been simplified. So, here the answer is ‘c’ i.e. byzantine that means very complicated.

2. Comparison: Two things are being compared. eg. Ram is a good boy similarly his brother is also a good boy.
In this case if it is positive it will remain positive and vice versa.
Similarly, likewise, and just as etc. are used for comparison.

3. Contrast: If contrast is there then but, although, despite, however, though, or etc. words you will be seen.
eg. Ram is a good boy but Shyam is a bad boy.

4. Cause and effect: Cause and effect mean one thing is the reason for others. Words like cause, leads to, because etc. when you have these words then you know there is a **cause & effect.** Even without these words, we can have cause & effect.

For example:
After a brief and violent _______ that ousted the president, General Mosanto declared himself the dictator of the country.

a. nurance b. Coup c. solicitation d. upbraiding e. lament

Answer:
In this sentence outage is a clue. Outage means to remove. Here the answer is ‘b’ coup that means to take over any government.

Questions On Sentence Completion:

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a) Single blanks question:

1. She had not eaten all the day, and by the time she got home she was _______ .

a. blighted b. confutative c. ravenous
d. Ostentatious e. Blissful

Answer: c.

Explanation:
Here the mental answer is hungry and the word ravenous means hungry.

Blissful - very happy, ostentatious - showy and confutative - the act of refuting someone's point forcefully and blighted - spoil.

2. The movie offended many of the parents of its younger viewer by including unnecessary _______ in the dialogue.

a. vulgarity b. verbosity c. vocalization
d. garishness e. Tonality

Answer: a.

Explanation:
Offended means irritating. So, vulgarity offended many parents.
Verbosity - many words, garishness - very bold, tonality - music and vocalization - way of speaking.

3. His neighbours find his _______ manner bossy and irritating and they stop inviting him to backyard barbeques.

a. insentinent b. magisterial c. reparatary
d. restorative e. modest

Answer: b.

Explanation:
Find something which talks about his manner is bossy and irritating. So, magisterial is the answer that means dominating.
Insentient - can not sense anything, Reparatory - repayment, Restorative - having the ability to restore health and modest - very humble.

4. Shubham is always _______ about showing off work because he feels that tardiness is a sign of irresponsibility.

a. legible b. Tolerable c. punctual
d. literal e. Belligerent

Answer: c.

Explanation:
Tardiness means unpunctual or lazy. So, the answer is punctual.
Legible - handwriting, Tolerable - something you can tolerate, Literal - taking words in their usual sense and belligerent - a war like happening.

5. Anjali would _______ her little sister into an argument by teasing her and calling her names.

a. advocate　　b. provoke　　c. perforate

d. lament　　e. Expunge

Answer: b.

Explanation:

Her sister made her angry. So, the answer is, provoke that means anger.

Advocate - incorrect, perforate - make holes, lament - very sad and expunge - remove.

6. The dress Ariel wore _______ with small, glassy beads, creating a shimmering effect.\

a. titillated　　b. reiterated　　c. scintillated

d. enthralled　　e. Striated

Answer: c.

Explanation:

The dress Ariel wore decorated. The answer is scintillated that means shinny or decorated.\

Titillated - excite, Reiterated - repeat again and again, Enthralled - very happy about something and Striated - having striped.

Cloze test:

In the cloze test, the whole paragraph has to be taken into concentration. Sometimes clues are given later on also, so it is a good idea to read the whole paragraph and then keep filling it as and when you can.

Text 1

Insects do not have (1)_____ as sharp as that of mammals or birds. The insect compound eye is more familiar to movement and so it cannot(2) ______ position distant objects. So, insects tend to take a rather unsteady flight path to navigate to a particular object. For example, in order to locate the caterpillar, the wasp needs to balance the odour signals(3)______ by its two antennae.

1. A) wings B) vision C) absorption
2. A) hilarious B) precisely C) tangibly
3. A) sacrificed B) accompanied C) received

Answer :

1. Vision.

The clue is sharp. Sharp could be eye side or could be the vision.

2. Precisely.
3. Received.

The clue is antennae, antenna received signal.

Text 2.

A last attempt is being made to move the beetles to a specially designed pile of ribble that (1)_____ their existing habitat. But experts stress that is only a slim chance that the (2)_______ will succeed.

1. A) succumb B) replicates C) resonate.
2. A) formation B) migration C) translocation.

Answer:

1. Replicate.

The clue is an existing habitat.

2. Translocation.

Beetles location change . migration means permanent shifting.

Text 3.

Giant pandas are black-and-white Chinese beers that are on the verge of (1)_____. These large, cuddly-looking mammals have a big head, a heavy body, rounded ears, and a short tail. Most bears' eyes have round pupils. The (2)______ is the giant panda, whose pupils are vertical slits, like cats' eyes, these unusual eyes (3) _______ the Chinese call the panda "giant cat bear."

1. A) indication B) accommodation C) extinction.
2. A) dimension B) exception C) speculation
3. A) inspired B) predicated C) reversed

Answer:

1. Extinction.

The clue is on the verge.

2. Exception.

The clue is pupils are vertical slits.

3. Inspired.

Chinese inspired by the looking of giant pandas.]

Text 4.

Although the population of England in the nineteenth century was rising at a (1) ______ rate, that of the city was increasing by leaps. This was due to the effect of the industrial revolution; people were (2)______ into the towns and cities in search of employment; for the same, it was also the call of the unknown, (3)______ and a better way of life. This period is known to be the beginning of many new things.

1. A) crepuscular B) unprecedented C) reprehensible

2. A) flocking B) abrogating C) ensconcing
3. A) escapade B) pliable C) abstruse

Answer:
1. Unprecedented
Means like never before.
2. Flocking.
Means moving. The clue is town.
3. Escapade
Means adventurous

Some Questions For practice:

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1. Suresh was ______ criticised for his rude behaviour.
   A) apparently   B) vehemently
   C) severely   D) glaringly

2. We should not ______ our emotions openly.
   A) yield   B) render
   C) provoke   D) display

3. The manager begins to doubt the ______ of his assistant.\
   A) credits   B) credit
   C) credential   D) chances

4. You must ______ ourselves to changing circumstances.
   A) adopt   B) adept
   C) adapt   D) accept

5. The hawkers are selling their ______ in the street.
\   A) wire   B) wares
   C) warn   D) wear

Answers: 1. C 2. D 3. C 4. C 5. B

Cloze test:

Text 1.

The thermometer is an instrument for measuring temperature. The (1) _____ form consists of a (2) ______ tube with a fine (3) ______ . one end of the (4) ______ is blown to form (5) ______ bulb and the other is closed.

1. A) seen B) unseen
   C) heard D) common

2. A) brass B) glass
   C) plastic D) metal

3. A) boar B) cavity
   C) mole D) bore

4. A) tube B) edge
   C) center D) place

5. A) that B) a
   C) all D) an

Text 2.

There (6) ______ in the city of Ujjain a poor tailor (7) ______ Rampal. As he was very poor, he (8) ______ lived and worked in one little room with his wife and (9) ______ their small children. The children fought (10) ______ each other and made sp noise.

6. A) lived B) stayed
   C) inhabited D) existed

7. A) famous B) named
   C) known D) titled

8. A) wished B) liked
   C) had D) wanted

9. A) few B) a few
   C) their D) some

10. A) to B) upon
C) for D) with

Answer: 1. D 2. B 3. D 4. A 5. B 6. A 7. B 8. C 9. C 10. D

Lecture Notes For Probability

Probability is one of the most important mathematical concepts that we use in our daily life. *Probability means possibility of something.* It is a mathematical tool which deals with the occurrence of random events. Value of the probability lies in between 0 and 1.

Probability of an event is defined by the number of ways in which the event occurs divided by the number of outcomes in the sample space.

P(event) = n(E)/n(S)

Sample space: *Sample space of an event is the set of all possible outcomes of that event.*

For example:

1. You tossed a coin. Your sample space is head or tail.
   P(H) = 1/2.
2. You throw a dice. Your sample space {1,2,3,4,5,6}
   P(6) = 1/6.
3. England and India play a one day match.
   In this case 3 events will happen. 1. England wins 2. India wins 3. Match tie.
   P(tie) $\neq$ 1/3 because the possible outcomes of the India Vs England match is not the sample space in this situation.

Two things happen to form a sample space;
1. Exhaustive or complete list of all possible outcomes.
2. A list to become a sample space is that the outcome should be equally likely.

So, in India Vs England match, tie is not an equally like outcome. Hence it is not in the sample space.

1st kind of questions based on coins:

Problem 1:
A coin tossed three times. What is the probability of a) All heads. b) Exactly two heads. c) Minimum two heads. d) At Least one head.

Solution:

List of the possible outcomes {HHH,HHT,HTH,THH,TTH,THT,HTT,TTT}

Total number of outcomes = 8. i.e. n(S) = 8.

a) All heads n(E) = 1.

P(All heads) = n(E)/n(S) = 1/8.

b) exactly two heads n(E) = 3

P(Exactly two heads) = 3/8.

c) minimum two heads n(E) = 4.

P(Minimum two heads) = 4/8.

d) P(At least 1 head) = 1 - P(not heads)

= 1 - P(all tails) = 1 - 1/8 = 7/8.

NOTE: 1. None event in probability is denoted by $\overline{E}$ or E' and P(E) + P($\overline{E}$) = 1.

2. The probability of all events in a sample space is 1.

2nd method: Without forming sample space

a) All heads.

If you do not want to form a sample space, you can define this in 3 events.

Event definition: All heads.

H&H&H i.e. $1/2 \times 1/2 \times 1/2 = 1/8$.

b) Exactly two heads.

Event definition: Exactly two heads.

H&H&T or H&T&H or T&H&H i.e. $1/2 \times 1/2 \times 1/2 + 1/2 \times 1/2 \times 1/2 + 1/2 \times 1/2 \times 1/2 = 1/8 + 1/8 + 1/8 = 3/8$.

Biased coin question

Problem 1:

A coin toss three times, what is the probability of getting 2 Heads and 1 Tail if the probability of head is 0.6 and tail is 0.4?

Solution:
2 Heads and 1Tail.
Event definition: 2 Heads and 1 Tail
H&H&T or H&T&H or T&H&H i.e. $0.6 \times 0.6 \times 0.4 + 0.6 \times 0.6 \times 0.4 + 0.6 \times 0.6 \times 0.4 = 3(0.6 \times 0.6 \times 0.4) = 3 \times 0.144 = 0.432$.

Probability based on dice

Single dice situation is very simple. Normally we get in dice question type is the 2 dice situation. In such cases normally questions are asked on the sum of the dice.

In a 2 dice situation you need to understand that there is a certain pattern for different numbers.
For example:

Sum 2 can happen in only 1 way.(i.e. 1,1)	$\leftarrow$ Sum 12 can happen in 1 way.
Sum 3 can happen in 2 ways.	$\leftarrow$ Sum 11 can happen in 2 ways.
Sum 4 can happen in 3 ways.	$\leftarrow$ Sum 10 can happen in 3 ways.
Sum 5 can happen in 4 ways.	$\leftarrow$ Sum 9 can happen in 4 ways.
Sum 6 can happen in 5 ways.	$\leftarrow$ Sum 8 can happen in 5 ways.
Sum 7 can happen in 6 ways.	

The pair which have same number of ways;

$2 \Leftrightarrow 12$ **sum of each pair = 14.(i.e. 2+12=14).**
$3 \Leftrightarrow 11$
$4 \Leftrightarrow 10$
$5 \Leftrightarrow 9$
$6 \Leftrightarrow 8$

Problem 1:
Two dice are thrown together. Find the probability of :

1. Getting a number greater than 10.
2. Getting a sum of 5.
3. Getting a sum is prime.
4. Getting a multiple of 3 or 4.

Solution:

1. Total number of possible outcome = 36
   Getting a number greater than 10 means we want 11 or 12.

Sum 11 can happen in 2 ways or Sum 12 can happen in 1 way.
Number of events of getting number greater than 10 = 2+1=3
There for probability of Getting a number greater than 10
P(E) = 3/36 = 1/12.

2. Total number of possible outcome = 36
   Getting a sum of 5:

Sum 5 can happen in 4 ways.
Number of events of getting sum of 5 = 4.
There for probability of getting a sum of 5
P(E) = 4/36 = 1/9.

3. Total number of possible outcome = 36

Getting a sum is prime. In this case we will go and search situations for sum2,sum3,sum5,sum7 and sum 11.
Sum 2 can happen in only 1 way.
Sum 3 can happen in 2 ways.
Sum 5 can happen in 4 ways.
Sum 7 can happen in 6 ways.
Sum 11 can happen in 2 ways.
Number of events of getting sum is prime = 1+2+4+6+2 =15
There for probability of getting a sum is prime
P(E) = 15/36 = 5/12.

4. Total number of possible outcome = 36

Getting a sum is multiple of 3or4. Multiple of 3 or 4 is 3,4,6,8,9,12
Sum 3 can happen in 2 ways.
Sum 4 can happen in 3 ways.
Sum 6 can happen in 5 ways.
Sum 8 can happen in 5 ways.
Sum 9 can happen in 4 ways.
Sum 12 can happen in 1 way.
Number of events of getting sum is multiple of 3or4 = 2+3+5+5+4+1 = 20
There for probability of getting a sum is multiple of 3or4
P(E) = 20/36 = 5/9.

Probability based on cards

Some basic information about cards:

1. Pack of cards = 52
2. There are 4 suits in a pack of 52 cards.(**clubs,spades,diamonds,hearts)**
3. 13 cards in each of the 4 suits.
4. Each of 4 suits has an ace,2,3,4………,10,jack.queen,king.
5. Clubs and spades are in black color.
6. Diamonds and hearts are in red color.
7. Jack is at the same time in problems also referred to as Knave.
8. Jack,Queen and King are face cards.

Problem 1:

A card is drawn from a pack of 52 cards. Find the probability:

1. A spade.
2. A king.
3. A Black card.
4. A king or a queen.
5. A face card.
6. A king or a spade.

Solution:

1. Total number of possible outcomes = 52.

Number of events of drawing a spade = 13.

Therefore the probability of a spade

P(E) = 13/52.

2. Total number of possible outcomes = 52.

Number of events of drawing a king = 4.

Therefore the probability of a king

P(E) = 4/52.

3. Total number of possible outcomes = 52.

Number of events of drawing a black card = 26.

Therefore the probability of a black card

P(E) = 26/52.

4. Total number of possible outcomes = 52.

Number of events of drawing a king or queen = 4+4=8.
Therefore the probability of a spade
P(E) = 8/52.

5. Total number of possible outcomes = 52.

Number of events of drawing a face card = 12.
Therefore the probability of a face card
P(E) = 12/52.

6. Total number of possible outcomes = 52.

A king or a spade: there are 4 kings(among 4 king one king of spades) out of 52 cards and 13 cards of spades.

Number of events of drawing a king or a spade = 4+12 = 16
Therefore the probability of a king or a spade
P(E) = 16/52.

Problem 2:
Two cards are drawn at random **without replacement** from a pack of 52 cards. Find the probability of:

1. 1 queen and 1 king.
2. 1 red and 1 black.

Solution:

1. **1 queen and 1 king :**

Total number of possible outcomes = 52.
From a pack of 52 cards probability of queen = 4/52.
From a pack of 52 cards probability of king = 4/52.

1 queen and 1 king :
In this case 1st is queen & 2nd is king or 1st is king and 2nd is queen
Q&K or K&Q i.e. $4/52 \times 4/51 + 4/52 \times 4/51 = 8/(52 \times 51)$.
Therefore P(1Q & 1K) = $8/(52 \times 51)$.

2. **1 red and 1 black :**

Total number of possible outcomes = 52.
From a pack of 52 cards probability of red = 26/52.
From a pack of 52 cards probability of black = 26/52.

1 red and 1black :
In this case 1st is red & 2nd is black or 1st is black and 2nd is red
R&B or B&R i.e. $26/52 \times 26/51 + 26/52 \times 26/51 = 52/(52 \times 51)$.
Therefore P(1Q & 1K) = 1/51.

Probability based on balls from boxes

Problem 1:
A box contains 10 red, 5 blue and 1 black. All the balls are identical and 1 ball drawn at random. What is the probability that :

1. Ball is red.
2. Ball is blue.
3. Ball is black.

Solution:
Total number of balls = 10+5+1=16. i.e. n(S) = 16.

1. **Ball is red:**

n(E) = number of ways of drawing red balls = 10.
Therefore probability of drawing red balls
p(E) = n(E)/n(S) = 10/16.

2. **Ball is blue:**

n(E) = number of ways of drawing blue balls = 4.
Therefore probability of drawing blue balls
p(E) = n(E)/n(S) = 5/16.

3. **Ball is black:**

n(E) = number of ways of drawing black balls = 1.
Therefore probability of drawing black balls
p(E) = n(E)/n(S) = 1/16.

One ball question is very simple, but the main question here draws two balls. In such cases there are two kinds of questions.

1. Ball drawn with replacement.
2. Ball drawn without replacement.

Problem 2:

A box contains 10 red, 5 blue and 1 black. All the balls are identical and 3 balls drawn at random one after the other with replacement. What is the probability that all 3 balls are red?

Solution:

Total number of balls = 10+5+1=16. i.e. n(S) = 16.
n(E) = number of ways of drawing red balls = 10.
Probability of a red ball = 10/16
Therefore probability of drawing 3 red balls with replacement
1st red & 2nd red & 3rd red
$10/16 \times 10/16 \times 10/16$.

Problem 3:

A box contains 10 red, 5 blue and 1 black. All the balls are identical and 3 balls drawn at random one after the other with replacement or without replacement. What is the probability that all 3 balls are of the same color?

Solution:

Total number of balls = 10+5+1=16. i.e. n(S) = 16.
Probability of a red ball = 10/16.
Probability of a blue ball = 5/16.
Probability of a black ball = 1/16.

With replacement:

R - Red ball, b - Blue ball , B - Black ball
P(all 3 balls of same color) = R&R&R or b&b&b or B&B&B
$= 10/16 \times 10/16 \times 10/16 + 5/16 \times 5/16 \times 5/16 + 1/16 \times 1/16 \times 1/16$

Without replacement:

R - Red ball, b - Blue ball , B - Black ball
In this case we have only 1 black ball.
P(all 3 balls of same color) = R&R&R or b&b&b
$= 10/16 \times 9/15 \times 8/14 + 5/16 \times 4/15 \times 3/14$

Problem 4:

A box contains 10 red, 5 blue and 1 black. All the balls are identical and 3 balls drawn at random one after the other without replacement. What is the probability that all 3 balls are of the different color?

Solution:
Total number of balls = 10+5+1=16. i.e. n(S) = 16.
Probability of a red ball = 10/16.
Probability of a blue ball = 5/16.
Probability of a black ball = 1/16.

R - Red ball, b - Blue ball , B - Black ball
Arrangement of three different balls = 3!

P(all 3 balls of different color) = R&b&B × 3!
= (10/16 × 5/15 × 1/14) × 3!

Draw 1 ball from 2 boxes or 3 boxes

Problem 1:
A box contains 10 red, 5 blue and 2 black and another box contains 5 red, 7 blue and 8 black. 1 ball drawn at random from any of the 2 boxes. Find the probability that the ball is black?

Solution:
Probability of black ball form 1st box = 2/16
Probability of black ball from 2nd box = 8/20.
Selection of 1st box = 1/2
Selection of 2nd box = 1/2
P(Ball is black) = 1st box & Black ball or 2nd box & Black ball
= 1/2 × 2/16 + 1/2 × 8/20
= 1/16 + 8/40.

Word based question on probability

Problem 1:
What is the probability that there are 53 Sundays in a normal non leap year?

Solution:
In a non leap year = 365 days.
365 days has 52 complete weeks and 1 day.
The 365 days calendar will start on 1st january and 1st week will end on 7th january ……. And so on …. the 52nd week will end on 30th december.

For 53 sundays in a non leap year, the last day of the year 31th december has to be a sunday and probability of 31th dec being a sunday = 1/7.
Hence the answer = 1/7.

Problem 2:
What is the probability that there are 53 Sundays in a leap year?

Solution:
In a leap year = 366 days.
366 days has 52 complete weeks and 2 days.

The 52nd week would end on the 364th day of the year and that day would be 29th december.
For 53 sundays in a non leap year, the last 2 days of the year would be

1. Sunday or Monday
2. Saturday or sunday
3. Monday or Tuesday
4. Tuesday or Wednesday
5. Wednesday or Thursday
6. Thursday or Friday and
7. Friday or Saturday

Last 2 days of the year out of 7 cases. Out of 7 cases only 2 cases have Sundays in them.
Hence probability of 53 sundays in a leap year = 1/7.

Problem 3:
N1,N2,N3,N4 and N5 are the natural numbers. What is the probability that the product of these numbers ends in an odd number that is not a multiple of 5.

Solution:
Product of 5 numbers to be odd, 1st of all the last digit should not be even and last digit should not be 5.
Probability of any unit place = 1/10
All the numbers should end with 1,3,7 and 9 i.e. 4 numbers.

Probability of number P(N1) = 4/10.
Probability of number P(N2) = 4/10.
Probability of number P(N3) = 4/10.

Probability of number $P(N4) = 4/10$.

$$P(N1,N2,N3,N4 \text{ and } N5) = P(N1) \times P(N2) \times P(N3) \times P(N4) \times P(N5)$$
$$= 4/10 \times 4/10 \times 4/10 \times 4/10 \times 4/10 = 0.4^5$$

Problem 4:

Amit orders a gift from 4 different websites for his friend's birthday. The probability of the sites delivering on time are 0.9,0.8,0.7 and 0.6 respectively. What is the probability that the friend would get the gift on time?

Solution:

Lets say $\overline{E}$ be the event that a friend doesn't get the gift on time.

$P(E) = 1 - P(\overline{E})$.

1st site should fail deliver on time & 2nd site should fail deliver on time & 3rd site should fail deliver on time & 4th site should fail deliver on time.

P(1st site fail) = 1 - 0.9 = 0.1
P(2nd site fail) = 1 - 0.8 = 0.2
P(3rd site fail) = 1 - 0.7 = 0.3
P(4th site fail) = 1 - 0.6 = 0.4

$P(\overline{E}) = 0.1 \times 0.2 \times 0.3 \times 0.4 = 0.0024$

$P(E) = 1 - P(\overline{E}) = 1 - 0.0024 = 0.9976$.

Problem 5:

Probability of a man living for 50 years from today is 0.6 and the probability for his wife to live for 50 year from today is 0.5. Find the probability that both are alive after 50 years and one of them is dead ?

Solution:

P(man alive) = 0.6 and P(wife alive) = 0.5

P(both are alive) = P(man alive) & P(wife alive)
$= 0.6 \times 0.5 = 0.3$.

P (man not alive) = 1 - 0.6 = 0.4 and P (wife not alive) = 1 - 0.5 = 0.5.

P(one of them is dead) = (man alive & wife dead) or (man dead & wife alive)
$= 0.6 \times 0.5 + 0.4 \times 0.5 = 0.5$.

Problem 6:

Probability that India wins the match is 0.6 and probability that England wins the match is 0.4. India and England play 3 one-day matches. What is the probability that India wins the series?

Solution:

Events:

India can win the series by 3-0 or 2-1

3-0 means India wins 1st match & 2nd match & 3rd match

2-1 means India wins 2 matches and England wins 1 match and 3 arrangements.

P(India wins series) = $0.6 \times 0.6 \times 0.6 + (0.6 \times 0.6 \times 0.4) \times 3 = 0.648$.

Some questions for practice

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1. What is the probability of getting a number greater than 9, in a throw of two normal unbiased dice having 6 faces?
   Ans : 1/6.

2. In a throw of two dice, find the probability of getting one prime and one composite number.
   Ans : 1/3.

3. There are two bags containing white and black balls. In the first bag, there are 8 white and 6 black balls and in the second bag, there are 4 white and 7 black balls. One ball is drawn at random from any of these two bags. Find the probability of this ball being black.
   Ans : 41/77.

4. The letters of the word LUCKNOW are arranged among themselves. Find the probability of always having NOW in the word.
   Ans : 1/42.

5. Out of 13 applicants for a job, there are 5 women and 8 men. Two persons are to be selected for the job. The probability that at least one of the selected persons will be a woman is:
   Ans : 25/39.

6. The probability that A can solve the problem is 2/3 and B can solve it is 3/4. If both of them attempt the problem, then what is the probability that the problem gets solved.
   Ans : 11/12.

Lecture Notes For Time, Speed & Distance

Introduction

Time, Speed and Distance is an important chapter for the purpose of the Maths section in aptitude exams. The basic concepts of Time, Speed and Distance are used in solving questions based on straight-line motion, relative motion, circular motion, problems based on trains, problems based on boats, clocks, races, etc.

Time, Speed and Distance is a situation related to the motion of a body. If a person is moving from point ‘x’ to point ‘y’, this journey is described by three variables and every Time, Speed and Distance question has only 3 variables in it (time, speed and distance).

Time, Speed & Distance formula :

(a) Distance = Speed × Time
(b) Time = Distance/Speed
(c) Speed = Distance/Time

Units:

Speed: m/sec, km/hr and in some case, you will see km/min, m/min, feet/sec and feet/hr.
Time: min, hour and sec
Distance: km, meter and miles
Whenever you will use Speed × Time = Distance formula, units of all three Time, Speed and Distance should be consistent with each other, which means if speed is in kmph(km/hr), you can’t take time in sec or min, time will have to be in “hour” and distance will have to be in “km”.

Conversion:

1 km = 1000 meters = 0.6214 mile
1 mile = 1.609 km
1 hr = 60 min = 60*60 seconds = 3600 seconds
1 km/hr = 5/18 m/s
1 m/s = 18/5 km/hr
1 km/hr = 5/8 miles/hour

A car is travelling at 40 kmph from point ‘x’ to observer ‘o’ for a distance of 80 km.

40 kmph can be described as the rate at which a car is approaching the observer. So, every hour the car will keep coming 40 km closer to the observer.
If a journey is of 80km, so the car will take 2 hr to reach the observer.
Another way of looking it is;

The rate at which the car is moving away from the observer. And in this case, the car will reach the point x in 2hrs if the speed and distance are kept the same

The proportionality in the TSD equation:

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1. **s ∝ d** if time is constant.
2. **t ∝ d** if speed is constant.
3. **s ∝ 1/t** if the distance is constant.

1. s ∝ d if time is constant.

In the first proportionality, time should be constant in both motions, whether the two bodies are moving or two different journeys by the same car. After observing both the motions, if the time required is the same for both of them then, you can say that this is a constant time situation.
In time constant proportionality, if the speed increases then distance also increases in the same manner.

For example:
If train 1 starts from X and train 2 starts from Y and they start moving towards each other at the same time. They meet at a point somewhere in between.

Solution:
Let's say they start at 1 pm and meet at 3 pm.
So, here we can see that there are two motions and for these motions, the value of time is 2 hours.
Let say Sx and Dx be the speed and time respectively for train 1.
& Sy and Dy be the speed and time respectively for train 2.
In this case, the following ratio will be valid:

$$\frac{Sx}{Dx} = \frac{Sy}{Dy}$$

2. t ∝ d if speed is constant.

Example:
A car moves for 4 hours at a speed of 25 kmph and another car moves for 5 hours at the same speed. Find the ratio of distances covered by the two cars.
Solution:
Since the speed is constant, we can directly conclude that time ∝ distance.
Hence $\frac{Ta}{Tb} = \frac{Da}{Db}$
Since the times of travel are 2 and 3 hours respectively, the ratio of distances covered is also 4/5.

3. s ∝ 1/t if distance is constant.

Example:
A man goes from Delhi to Karnal and Comes back. In this case distance for Delhi to Karnal and Karnal to Delhi is the same i.e distance is constant. Hence, the speed will be inversely proportional to the time.
If the distance is constant it is also a product constancy situation ($s \times t$ = constant). Hence you can use any of the product constancy structures.
In this case, the following ration will be valid;
$\frac{Sa}{Sb} = \frac{Tb}{Ta}$

Problem Based On Proportionality:

Problem 1:
Abhishek walks at 3/4th of his normal speed and he is 16 minutes late in reaching office. Find his normal time of reaching office.

Solution:
Let S1 = s and T1= t be its normal speed and time respectively.
And S2 = $3/4 \times s$ and T2 = t+16.
Here distance is the same i.e distance constancy situation.
Speed from 's' to $3/4 \times s$ i.e. speed is reduced by 1/4th and time from 't' to t+16 i.e. time would be increased by 1/3rd as speed is reduced by ¼.
($s \times t$ = constant, is 's' reduced by 1/4 then 't' increased by 1/3)

time from ‘t’ to t+16 i.e. time is increased by 1/3rd means 1/3rd of normal time ‘t’ = 16 min
Therefore, Normal time = 16×3 = 48 min.

2nd method:
We know ratio;
$\frac{S1}{S2} = \frac{T2}{T1}$
T1 = $\frac{S1}{S2} \times$ T2
t = $\frac{3}{4} \times$ (t+16)
Therefore, the normal time ‘t’ = 48 min.

Problem 2:
Two people X and Y travelled the same distance at speeds of 6 kmph and 10 kmph respectively. If X takes 1 hour longer than Y then, what is the distance being travelled?

Solution:
Lets ‘t’ be the time taken by Y. So, time taken by X is t+1.
Speed of X = 6 kmph and speed of Y = 10 kmph.
We can solve this problem by following methods:
Method 1:
Here given that;
Difference of time = 1
d/6 - d/10 = 1
10d - 6d = 60, d = 15 km.
Therefore distance travelled = 15 km

Method 2:
Distance is constant so;
S1 × t1 = S2 × t2
6(t+1) = 10t
t = 3/2 hr
Therefore distance = speed × time
= 10 × 3/2 = 15 km

Problem 3:
Rohit walks at speed of 12 kmph and he reaches the railway station 10 min after the train has gone and by walking at 15 kmph, he reaches at railway station 10 min before the train has gone. Find the distance from his home to the railway station.

Solution:
Let original time of reaching = t min
We have;
S1 = 12 kmph and S2 = 15 kmph
t1 = t+10 min and t2 = t - 10 min

Method 1:
Here distance is constant so;
S1 × t1 = S2 × t2
12(t+10) = 15(t-10)
15t - 12t = 120 + 150
t = 90 min
Therefore distance = speed × time
= 12(90+10)/60 = 20 km
Method 2:
Difference between time = 20 min
d/12 - d/15 = 20/60
5d - 4d = 20 , d = 20 km
Therefore distance = 20 km.

Concept Of Relative Speed:

We already discussed the movement of a body with respect to a stationary point. And now, we need to determine the movement and its relationships with respect to a moving point/body. In such situations, we have to take into account the movement of the body w.r.t. which we are trying to determine relative motion.
"Relation motion of a body is the motion of one body/point with respect to other body/point"

Case 1: Two cars C1 & C2 are moving in opposite directions. C1 moving at S1 kmph and C2 moving at S2 kmph.
So, Relative speed S = S1+S2.

Some problems based on Case 1:
Problem 1:
Two cars C1 & C2 are moving towards each other. C1 at 50 kmph and C2 at 30 kmph. The initial distance between them is 280 km. After how much time they will meet?

Solution:

S1 = 50 kmph

S2 = 30 kmph

The speed with which they are approaching S = S1+S2

S = 50+30 = 80 kmph

They have to approach each other and reach the meeting point.

So, approaching distance/Relative distance= 280 km

Hence, Relative Speed × Time = Relative Distance

$80 \times t = 280$

$t = 3.5$ hours.

Therefore; they will meet after 3.5 hours.

Problem 2:

Two cars C1 & C2 are moving towards each other. C1 at 50 kmph and C2 at 30 kmph. The initial distance between them is 280 km. When will the distance between them next become 280 km?

Solution:

1st they have to approach each other and reach the meeting point.

So, approaching distance/Relative distance= 280 km

From the meeting point, they again have to separate by 280 km.

So, Separating Distance = 280 km.

Hence, net distance covered = 280+280 = 560 km

S1 = 50 kmph

S2 = 30 kmph

The speed with which they are approaching S = S1+S2

S = 50+30 = 80 kmph

So, Relative Speed × Time = Relative Distance

$80 \times t = 560$

$t = 7$ hours.

Therefore, the distance between them next becomes 280 after 7 hours.

Problem 3:

Two cars C1 & C2 are moving towards each other. C1 at 50 kmph and C2 at 30 kmph. The initial distance between them is 280 km. They start at 1 pm, after some time it was found that the distance between them was 200 km, then at what time could it be?

Solution:

$S1 = 50$ kmph

$S2 = 30$ kmph

The speed with which they are approaching $S = S1+S2$

$S = 50+30 = 80$ kmph

Initial distance between them = 280 km

Final distance between them = 200 km

So, the total approaching Distance = 280 - 200 = 80 km

Relative Speed × Time = Relative Distance

$80 \times t = 80$

$t = 1$ hour.

Therefore, this situation will happen at 2 pm.

But in this question, it is not saying that they have not met.

The initial distance between them = 280 km

After the meeting point, they have to cover a distance of 200 km.

So, total distance = 280+200 = 480 km

Relative Speed × Time = Relative Distance

$80 \times t = 480$

$t = 6$ hours.

Therefore, this situation will happen at 7 pm.

Case 2: Two bodies are moving in the same direction.

So, the Relative Speed $S = S1 - S2$

Problem 1:

Two cars C1 & C2 are moving in the same direction at a speed 50 kmph and 30 kmph respectively from the same point and they start moving at 2 pm. After how many hours will C1 be 140 km ahead of C2?

Solution:

$S1 = 50$ kmph

$S2 = 30$ kmph

The Relative Speed $S = S1-S2$

$S = 50 - 30 = 20$ kmph

Relative Distance = 140 km

So, Relative Speed × Time = Relative Distance

$20 \times t = 140$

t = 7 hours.
Therefore, after 7 hours C1 ahead 140 km of C2.

Problem 2:
Two cars C1 & C2 are moving in the same direction. Car C2 going at 30 kmph and C1 catching up at 50 kmph, starting distance between them is 120 km. In how many hours does C1 catch C2?

Solution:
S1 = 50 kmph
S2 = 30 kmph
The Relative Speed S = S1-S2
S = 50 - 30 = 20 kmph
Relative Distance = 120 km
So, Relative Speed × Time = Relative Distance
$20 \times t = 120$
t = 6 hours.
Therefore, in 7 hours C1 catches C2.

Problem 3:
Two cars C1 & C2 are moving in the same direction. Car C2 is going at 30 kmph and C1 is catching up at 50 kmph, starting distance between them is 120 km. When will the next time they will be at 120 km distance?

Solution:
S1 = 50 kmph
S2 = 30 kmph
The Relative Speed S = S1-S2
S = 50 - 30 = 20 kmph
Approaching distance = 120 km
Separating distance = 120km
Hence, net distance = 120+120 = 240 km.
So, Relative Speed × Time = Relative Distance
$20 \times t = 240$
t = 12 hours.
Therefore, after 12 hours they will be at 120 km again.

Question-Based On Relative Motion:

Type 1: Policeman and theft question

Problem 1:

The theft is committed at 2 A.M and the thief after committing the theft starts escaping at a speed of 80 kmph. The theft is discovered at 6 A.M and the policeman gives pursuit of the thief at 100 kmph. Find at what time the policeman will catch the thief?

Solution:

Speed of Thief = 80 kmph
Speed of policeman = 100 kmph
According to question,
Distance between thief and policeman after 4 hours (2 A.M to 6 A.M) = $80 \times 4 = 320$ km.
Speed at which policeman approaches thief = $100 - 80 = 20$ kmph
So, Relative Speed × Time = Relative Distance
$20 \times t = 320$
$t = 16$ hours
Therefore, police caught thief at 10 P.M (6 A.M + 16 hours =10 P.M)

Problem 2:

At what distance from the original point did the thief get caught?

Solution:

To answer this question we have to find out the policeman's journey.
Speed of policeman = 100 kmph
Time taken by the policeman to catch the thief = 16 hours.
So, distance = $100 \times 16 = 1600$ km.

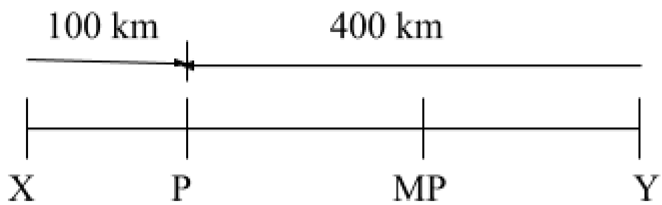
Type 2: Train question

Problem 1:

Two trains T1 and T2, T1 starting from point X to Y and T2 starting from point Y to X 2 hours later. T1 moving at 50 kmph and T2 at 30 kmph. Distance between point X and Y is 500 km. Find the distance from X, after which they will meet.

Solution:

Speed of T1 = 50 kmph
Speed of T2 = 30 kmph
Distance between X and Y = 500 km
Train T1 starts 2 hours before train T2.
Distance covered by T1 in 2 hours = 50 × 2 = 100 km
Let us say T1 reaches at point P in 2 hours.



Distance left from point P to Y = 500 - 100 = 400 km
Speed at which they approaching = 50+30 = 80 kmph
Approach required to get the meeting point (MP), the total distance they have to approach together = 400 km
So, Relative Speed × Time = Relative Distance
80 × t = 400
t = 5 hours
So, distance from P to MP = 50 × 5 = 250 km
Therefore, they will meet 350 km (100 + 250 = 350) from point X.

Concept Of Circular Motion:

The movement of an object along a circle is called circular motion. When we talk about circular motion, there are 3 variables inside the questions. 1. Speed 2. Circumference 3. Time.

Units:

Speed: m/sec, kmph or it can also be measured in %/sec,%/min and %/hr.
Circumference: meter, km or % (if circle as 100%)

Problem 1:
Three people A, B and C running around a circle, whose circumference is 100 km. Speed of A is 20 kmph and the speed of B is 15 kmph and speed of C is 12 kmph.

a. After how much time they will meet at the starting point.
b. How many rounds were done by A?
c. The time required for the first meeting at any point.

Solution:

Speed of A = 20 kmph

Speed of B = 15 kmph

Speed of C = 12 kmph

Circumference = 100 km

(a) Let Ta, Tb and Tc be the time taken by A, B and C respectively to cover the circle.

So, Ta = 100/20 = 5 hours

Tb = 100/15 = 20/3 hours

Tc = 100/12 = 25/3 hours

Time required to meet at starting point = LCM(Ta,Tb,Tc)

We know, LCM of fraction = LCM of numerator / HCF of denominator

= LCM (5,20/3,25/3) = 100/1 = 100 hours

Hence, they meet at the starting point after 100 hours.

(b) A done one round in 5 hours.

So, In 100 hours A done = 100/5 = 20 rounds.

(c) A is fastest, A would be overlapping each of B & C after some time.

Let Tab and Tac be the time in which A overlap B and C respectively.

The time required for the first meeting at any point = LCM(Tab, Tac)

Relative speed between A and B 'Sab' = 20-15 = 5 kmph

Relative speed between A and C 'Sac' = 20-12 = 8 kmph

So, Tab = 100/5 = 20 hours and Tac = 100/8 = 12.5 hours.

LCM (20,25/2) = 100 hours

Hence, they will meet at any point after 100 hours.

Some questions for practice:

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1. Ram starts walking from a place at a uniform speed of 2 km/h in a particular direction. After half an hour, Shyam starts from the same place and walks in the same direction as Ram at a uniform speed and overtakes Ram after 1 hour 48 minutes. Calculate the speed of Shyam.
   Ans: 23/9 kmph.

2. Shubham and Navneet travel the same distance at the rate of 6 km per hour and 10 km per hour respectively. If Shubham takes 30 minutes longer than Navneet, the distance travelled by each is
   Ans: 7.5 km.

3. Two trains for Kota leave Mumbai at 6: 00 a.m. and 6: 45 am and travel at 100 kmph and 136 kmph respectively. How many kilometres from Mumbai will the two trains be together?
Ans: 283.33 km.

4. Walking at 3/4 of his normal speed, a man takes 2(1/2) hours more than the normal time. Find the normal time.
Ans: 7.5 hours.

5. A motor car does a journey in 17.5 hours, covering the first half at 30 km/h and the second half at 40 km/h. Find the distance of the journey.
Ans: 600 km.

Blood relations

Introduction to Blood relations

Blood relation is one of the most important topics of logical reasoning and found its importance in almost every entrance exam. This topic tests the analytical skills of the students and their solution approach. The questions asked in this chapter depend upon 'Relations'. You should have a sound knowledge of the blood relation in order to solve the questions.

To remember easily, the relation may be divided into two forms:

Relation of the paternal side

Father's father	Grandfather
Father's mother	Grandmother
Father's brother	Uncle
Father's sister	Aunt
Children of uncle	Cousin
Wife of uncle	Aunt
Children of aunt	Cousin
Husband of aunt	Uncle

Relation of the maternal side

Mother’s father	Maternal Grandfather
Mother’s mother	Maternal Grandmother
Mother’s brother	Maternal Uncle
Mother’s sister	Aunt
Children of maternal uncle	Cousin
Wife of maternal uncle	Maternal Aunt
Children of the maternal aunt	Cousin
Husband of the maternal aunt	Maternal Uncle

Others

Son’s wife → Daughter-in-law
Daughter’s husband → Son-in-law
Husband’s (or) wife’s father → Father-in-law
Husband’s (or) wife’s mother → Mother-in-law
Husband’s (or) wife’s brother → Brother-in-law
Husband’s (or) wife’s sister → Sister-in-law
Sister’s husband → Brother-in-law
Brother’s (or) sister’s son → Nephew
Brother’s (or) sister’s daughter → Niece

Son's wife	Daughter-in-law
Daughter's husband	Son-in-law
Husband's (or) wife's father	Father-in-law
Husband's (or) wife's mother	Mother-in-law
Husband's (or) wife's brother	Brother-in-law
Husband's (or) wife's sister	Sister-in-law
Sister's husband	Brother-in-law
Brother's (or) sister's son	Nephew
Brother's (or) sister's daughter	Niece

Relations from one generation to other

Generation1: Grandfather, Grandmother, Maternal grandfather, Maternal grandmother
Generation 2: Mother, Father, Uncle, Aunt, Maternal uncle, Maternal aunt
Generation 3: Self, Sister, Sister-in-law, Brother, Brother-in-law
Generation 4: Son, Daughter, Nephew, Niece

Symbols

1. '+' for male 2. '-' for female 3. ' ⇔' for couples

Types of problem statements

Type 1: Statement based relationship questions

Problem 1:
Pointing to a lady on the stage, Sonali said, “She is the sister of the son of the wife of my husband.” How is the lady related to Sonali?
Solution:
My husband = Sonali’s husband
Wife of my husband = is me = Sonali
Son of the wife of my husband = My Son
Sister of the Son of the wife of my Husband = My Son’s Sister = My daughter
So, the lady on the stage is Sonali’s daughter.

Problem 2:
Eeshas father was 34 years of age when she was born. Her younger brother, Shashank, now that he is 13, is very proud of the fact that he is as tall as her, even though he is three years younger than her. Eeshas mother, who is shorter than Eesha, was only 29 when Shashank was born. What is the sum of the ages of Eeshas parents now? **(asked in TCS)**

a) 92
b) 76
c) 66
d) 89

Answer: a) 92
Solution:Let Eesha's present age be x.
Eesha's father's present age = x + 34
Shashank's age = 13
Eesha's present age = 13 + 3 = 16
Eesha's mother's present age = 29 + 13 = 42
Sum of the ages of Eeshas parents now = 42 + 16 + 34 = 92

Problem 3:
Pointing to a lady a man said, “Her husband is the only son of my mother”. How is the lady related to the man?
Solution:

My mother's only son = is me (man)
Her husband = is me
So, the lady is Man's wife.

Problem 4:
Pointing to Alex, Lita says, "I am the daughter of the only son of his grandfather." How Lita is related to Alex? **(Asked in Sapient)**

a) Niece
b) Daughter
c) Sister
d) Cannot be determined

Answer: C) Sister
Solution:
Lita is the daughter of the only son of Alex's grandfather. Hence, it's clear that Lita is the sister of Alex.

Problem 5:
Pointing to a man Manisha said, "He is the youngest son of my father-in-law's only son". How is Manisha related to this youngest son's father?

a) Sister
b) Sister-in-law
c) Wife
d) Mother

Solution:
Manisha's father in law's only son = Manisha's husband
The youngest son of my father-in-law's only son is my husband's son = My son = Manisha's son
So, Manisha is the **wife** of the youngest son's father

Type 2: Puzzle type questions with a family relationship component

Problem 1:

A family consists of a husband and wife, their three sons and two daughters, three wives of three sons. How many females are in this family? **(Wipro hiring 2018)**

Solution:

Husband wife (female)

Three sons = S1 S2 S3 and two daughter = D1 D2

Son's wives = W1 W2 W3

So, the total number of females = wife + D1 + D2 + W1 + W2 + W3 = 6 females.

Directions for problem 2 to 6:

If a + b means, a is the daughter of b,

a - b means, a is the husband of b,

a×b means, a is the brother of b.

Problem 2:

What does the relation p×q - r show?

(a) p is the son-in-law of r
(b) p is the brother of r
(c) r is the wife of p
(d) None of these

Solution:

p×q means p is the brother of q

q - r means, q is the husband of r i.e.

p is the brother-in-law of r or r is the sister-in-law of p.

So the answer to this question is an option (d).

Problem 3:

If h+i×j+k×l+m×n, then what is the present generation of h. Assume that the oldest generation of this group is 1st generation.

(a) 2nd (b) 3rd
(c) 1st (d) 4th

Solution:

Here symbol '+' is for a generation change.

m×n = m is the brother of n (1st generation)

l+m = l is the daughter of m (2nd generation)

k×l = k is the brother of l

j+k = j is the daughter of k (3rd generation)

i×j = i is the brother of j
h+i = h is the daughter of i (4th generation)
Hence, present generation of ‘h’ = 4th generation i.e. option (d)

Problem 4:
Which of the following options does not hold?
(a) a+b×c
(b) a-b×c
(c) a+b+c
(d) a+b-c

Solution:
(a) a+b×c, here ‘b’ is the brother of ‘c’ i.e ‘b’ is a male and ‘a’ is the daughter of ‘b’. This option is correct.
(b) a-b×c, here ‘b’ is the brother of ‘c’ i.e ‘b’ is a male and ‘a’ is the husband of ‘b’ This option can not hold. ‘a’ can’t be the husband of ‘b’, because ‘b’ comes out a male.

Problem 5:
From the statement a×b×c×d, which of the following statements is not necessarily true?
(a) ‘b’ is the brother of ‘a’
(b) ‘c’ is the brother of ‘a’
(c) ‘d’ is the brother of ‘c’
(d) a,b,c are male

Solution:
a×b×c×d, here ‘c’ is the brother of ‘d’, ‘b’ is the brother of ‘c’ and ‘a’ is the brother of ‘b’
So, here a,b,c are males.
Option (c) ‘d’ is the brother of ‘c’ is not necessarily true because we don’t know whether ‘d’ is male or not.

Problem 6:
From the statement p-q+r×s, how is ‘q’ related to ‘s’?
(a) Niece
(b) Sister
(c) Daughter
(d) Brother

Solution:

r×s = ‘r’ is the brother of ‘s’ (‘r’ is male)

q+r = ‘q’ is the daughter of ‘r’(‘q’ is a female)

p-q = ‘p’ is the husband of ‘q’

So from the above conclusion, ‘q’ is the niece of ‘s’ i.e. option (a) is the correct answer.

Directions for questions 7 to 8.

a*b means ‘a’ is the brother of ‘b’

a@b means ‘a’ is the daughter of ‘b’

a$b means ‘a’ is the sister of ‘b’

Problem 7:

Which of the following show the relationship ‘p’ is the paternal uncle of ‘c’?

(a) n $ o @ p
(b) n @ o $ p
(c) All of the above
(d) None of these

Solution:

(a) n $ o @ p

o @ p = ‘o’ is the daughter of ‘p’ and n $ o = ‘n’ is the sister of ‘o’

So, here ‘p’ is either the father or the mother of ‘n’.

(b) n @ o $ p

o $ p = ‘o’ is the sister of ‘p’ and n @ o = ‘n’ is the daughter of ‘o’

So, ‘p’ is either uncle or aunt of ‘n’ because the gender of p can not be determined.

Hence, the answer will be an option (d).

Problem 8:

abc@d@e*f*g, then how many males and females are there respectively?

(a) 4,3
(b) 3,4
(c) 5,2
(d) Can’t be determined

Solution:

f*g = ‘f’ is the brother of ‘g’ (i.e. ‘f’ is a male)

e*f = ‘e’ is the brother of ‘f’ (i.e. ‘e’ is a male)

d@e = ‘d’ is the daughter of ‘e’ (i.e. ‘d’ is a female)

c@d = ‘c’ is the daughter of ‘d’ (i.e. ‘c’ is a female)

b$c = ‘b’ is the sister of ‘c’ (i.e. ‘b’ is a female)

a$b = ‘a’ is the sister of ‘b’ (i.e. ‘a’ is a female)

Here we can not find the gender of ‘g’.

Here 4 women and 2 men but we can't find the gender of one person.

So, the answer is can’t be determined, option(d)

Lecture Notes For Application Of TSD:

Application 1: Trains

Problems based on trains are a special case in questions related to time, speed and distance because they have their own theory and distinct situations.
Problems based on trains are of two types. One is a train crossing an object having no length and another is a train crossing an object having a length.

For example:

a. A train crossing a man, tree or pole is considered as “no length” objects. In this case, the distance travelled by train would be equal to the length of the train.
b. A train crossing a bridge, a platform of another train is considered as an object having a length. In this case, the distance travelled by train would be the sum of the length of the object and the length of the train.

For each of the following situations we have to use some specific notations:
S_t = Speed of train S_o = Speed of object t = time
L_t = Length of train L_o = Length of the object

a. Train crossing a stationary object with zero length:
$S_t \times t = L_t$

b. Train crossing a moving object with zero length:
- In opposite direction: $(S_t + S_o) \times t = L_t$
- In the same direction: $(S_t - S_o) \times t = L_t$

c. Train crossing a stationary object with length
$S_t \times t = L_t + L_o$

d. Train crossing a moving object with length:
- In opposite direction: $(S_t + S_o) \times t = L_t + L_o$
- In the same direction: $(S_t - S_o) \times t = L_t + L_o$

Problem 1:
A train crosses a pole in 10 seconds. If the speed of the train is 18 kmph, find the length of the train.

Solution:
This is the case when a train crosses a stationary object without length.
Here, S_t = 18 kmph, t = 10 sec
So, $S_t \times t = L_t$
$18 \times 5/18 \times 10 = L_t$
L_t = 50 m.
Therefore, the length of the train is 50 meters.

Problem 2:
Two trains are moving towards each other having a ratio of length 4:3. One train is travelling at 54 kmph, other is travelling at 72 kmph. They cross each other in 30 sec. Find the length of two trains individually.

Solution:
Here, S_t = 54 kmph = $54 \times 5/18$ = 15 m/sec.
S_o = 72 kmph = $72 \times 5/18$ = 20 m/sec.
t = 30 sec.
So, $(S_t + S_o) \times t = L_t + L_o$
$(15+20) \times 30 = L_t + L_o$
$L_t + L_o = 1050$
And given that, $L_t : L_o = 4:3$
So, $L_t = 1050 \times 4/7$ = 600 meters and $L_o = 1050 \times 3/7$ = 450 meters.

Problem 3:
Two trains T1 and T2, T1 is travelling at 54 kmph and T2 is coming from the opposite direction at 72 kmph. A man is sitting in T2. T1 crosses the man in 10 sec. Find the length of train T1.

Solution:
This is the case in which a train crosses a moving object with zero length.
Here, S_t = 54 kmph = $54 \times 5/18$ = 15 m/sec.
S_o = 72 kmph = $72 \times 5/18$ = 20 m/sec.
t = 30 sec.
So, $(S_t + S_o) \times t = L_t$
$(15+20) \times 10 = L_t$

L_t = 350 meters.
Therefore the length of the train T1 is 350 meters.

Application 2: Boats and Streams

The problems of boats and streams are also dependent on the Speed, Time and distance formula:
Speed × Time = Distance
S_B = Speed of the boat in still water.
S_S = Speed of stream.

a. In still water, the speed of movement is S_B.
b. While moving upstream (against the flow of the water), the speed of movement is:
   $S_U = S_B - S_S$
c. While moving downstream (with the flow of the water), the speed of movement is:
   $S_D = S_B + S_S$

NOTE: Speed of Boat is an average of S_U & S_D.

Problem 1:
A boat whose downstream speed is 10 kmph and upstream speed is 6 kmph. Find the speed of Boat and Stream.

Solution:
We know,
$S_B = (S_D + S_U) / 2$
S_B = (10 + 6)/2 = 8 kmph.
$S_S = S_D - S_B$
S_S = 10 - 8 = 2 kmph.

Application 3: Clocks

Problems on clocks are based on the relative movement between the minute hand and the hour hand. You can think hands in the clock as two runners (minute hand and hour hand), the minute hand is running at a speed of 360^o per hour (here we assume distance in degree) and the hour hand is running at a speed of 30^o per hour.
Since the minute hand and the hour hand both are moving in the same direction, the relative speed of the minute hand w.r.t the hour hand is 330^o per hour. In one hour the minute hand either is approaching the hour hand or it is leaving it behind (separation).

Some useful information:

1. In every hour there are two instances of right angles when the hands of the clock are at right angles.
2. There are two instances on the clock when the hands of the clock make a straight line. It happens whether the hands are coinciding or pointing opposite to each other.

Problem 1:
At what time between 1 to 2 the hands of the clock will form a straight line?

Solution:
At 1'oclock situation hour hand 30^o ahead of the minute hand. The minute hand has to approach the hour hand by 30^o i.e. distance is 30^o
We know the relative speed between the hour hand and the minute hand is 330^o per hour.
So,
Relative speed $\times$ time = distance
$330^o \times t = 30^o$
t = 30/330 = 1/11 hours.
So, the number of hours required to form a straight line will be 1/11 hours.

Convert into minutes:
1 hour = 60 minutes
1/11 hours = 60 $\times$ 1/11
60/11 minutes = 5(5/11) minutes.
Into seconds:
1 minute = 60 seconds
5/11 minutes = 60 $\times$ 5/11 seconds
300/11 seconds = 27.27 seconds
Hence, the required answer is 1:05:27.27 seconds.

Problem 2:
At what time between 4 to 5 the hands of the clock will form a straight line?

Solution:
At 1'oclock situation hour hand 120^o ahead of the minute hand. The minute hand has to approach the hour hand by 120^o i.e. distance is 120^o
We know the relative speed between the hour hand and the minute hand is 330^o per hour.
So,
Relative speed $\times$ time = distance

$330^o \times t = 120^o$
t = 120/330 = 4/11 hours.
So, the number of hours required to form a straight line will be 4/11 hours.
Convert into minutes:
1 hour = 60 minutes
4/11 hours = 60 × 4/11
240/11 minutes = 21(9/11) minutes.
Into seconds:
1 minute = 60 seconds
9/11 minutes = 60 × 9/11 seconds
540/11 seconds = 49.09 seconds
Hence, the required answer is 4:21:49.09 seconds.

NOTE: Straight line happens at 0^o, 180^o behind and 180^o ahead. In an hour you will get only 2 cases.

If a right angle form, the distance either 90^o ahead, 90^o behind, 270^o ahead or 270^o behind, only these 4 cases will happen.

Problem 3:
At what time between 4–5 is the 1st and 2nd right angle formed by the hands of the clock?

Solution:
At 4'oclock the minute hand behind the hour hand by 120^o. The minute hand is going to approach the hour hand.
Also, the 1st right angle between 4-5 is formed when the minute hand is 90^o behind the hour hand.
So, the minute hand has to cover 30^o.
We know the relative speed between the hour hand and the minute hand is 330^o per hour.
So,
Relative speed × time = distance
$330^o \times t = 30^o$
t = 30/330 = 1/11 hours.

So, the number of hours required to form 1st right angle will be 1/11 hours.
Convert into minutes:
1 hour = 60 minutes
1/11 hours = 60 × 1/11

60/11 minutes = 5(5/11) minutes.
Into seconds:
1 minute = 60 seconds
5/11 minutes = 60 × 5/11 seconds
300/11 seconds = 27.27 seconds
Hence, the 1st right angle is formed at 4:05:27.27 seconds.

Also, the 2nd right angle between 4-5 is formed when the minute hand is 90^o ahead the hour hand.
So, the minute hand has to cover 210^o.
$330^o \times t = 210^o$
t = 210/330 = 7/11 hours.

So, the number of hours required to form 2nd right angle will be 7/11 hours.
Convert into minutes:
1 hour = 60 minutes
7/11 hours = 60 × 7/11
420/11 minutes = 38(2/11) minutes.
Into seconds:
1 minute = 60 seconds
2/11 minutes = 60 × 2/11 seconds
120/11 seconds = 10.90 seconds
Hence, the 2nd right angle is formed at 4:38:10.90 seconds.

Application 3: Races & Games of skill

These questions are completely based on the unitary method.

Problem 1:
In a 200 meter race, A can give B a start of 20 meters and B can give C a start of 30 meters. In a 1 km race, how much of a start can A give C?

Solution:
A gives B a start of 20 meter means when A does 200 m, B does 180 m.
B gives C a start of 30 meter means when B does 200 m, C does 170 m.
When B at 200, C will be at 170.
When B at 1, C will be at 170/200.

When B at 180, C will be at $180 \times 170/200 = 153$ m
Hence, when A is doing 200 m, C is doing 153 m.
In 200 m race A beats C by 47m
So, in 1000 m race A beats C by $1000 \times 47/200 = 235$ m.
Therefore, A can give C 235 m.

Problem 2:
In a 200 meter race, A beats B by 20 meters and in 300-meter race B beats C by 30 meters. In a 1 km race, how much A beats C?

Solution:
In a 200 meter race, when A does 200 m, B does 180 m.
In a 300 meter race, when B does 300 m, C does 270 m.
When B does 1 m, C does 270/300.
When B does 180 m, C does $180 \times 270/300 = 162$ m.
Hence, when A is doing 200 m, C is doing 162 m.
In 200 m race A beats C by 38 m
So, in 1000 m race A beats C by $1000 \times 38/200 = 190$ m.
Therefore, A beats C by 190 m.

Problem 3:
In a game of billiards, A can give B 20 points in 200. While B can give C 30 points in 300. How much A can give C in a 600 point game?

Solution:
In a 200 point game, when A does 200 points, B does 180 points
In a 300 points game, when B does 300 points, C does 270 points
When B does 1 point, C does 270/300 points.
When B does 180 points, C does $180 \times 270/300 = 162$ ponits

Hence, when A does 200 points, C is doing 162 points
In the 200 points game, A gives C 38 points.
So, in 600 points game, A gives C $600 \times 38/200 = 114$ points
Therefore, A gives C 114 points.

Some question for practice:

1. A train crosses a pole in 8 seconds. If the length of the train is 200 metres, find the speed of the train.
Ans: 90 kmph.
2. A train crosses a man travelling in another train in the opposite direction in 8 seconds. However, the train requires 25 seconds to cross the same man if the trains are travelling in the same direction. If the length of the first train is 200 metres and that of the train in which the man is sitting is 160 metres, find the speed of the first train.
Ans: 59.4 kmph.

3. A boat sails down the river for 10 km and then up the river for 6 km. The speed of the river flow is 1 km/h. What should be the minimum speed of the boat for the trip to take a maximum of 4 hours?
Ans: 4 kmph.

4. At what time between 2–3 p.m. is the first right angle in that time formed by the hands of the clock?
Ans : 2:27:16.36 seconds.

5. Vinay runs 100 metres in 20 seconds and Ajay runs the same distance in 25 seconds. By what distance will Vinay beat Ajay in a hundred-metre race?
Ans: 25 m.

(Reference: https://schoolbag.info/mathematics/cat/16.html)

Lecture Notes For Data Interpretation

Data interpretation, as the name suggests, is all about the analysis of data. Data interpretation is the process of making sense out of the collection of data. Data may be collected in the form of bar graphs, line charts and tabular forms and hence some kind of interpretation that we need.

What is data?

Data is the number that comes from the occurrence of any event - physical, social, economic, graphical and other kinds of events.
A number value by itself represents nothing. Thus if we imagine a number, say 40, it means nothing by itself. The number starts to gain some significance when any unit attaches to it, say 40 crores. However, just by saying that the number represents crores does not complete the description of the number. It has to be further qualified by specific descriptions, that is the sales revenue of Coding ninjas for the year 2019-20.

Thus, three facts attached to the number:

a. The number which represents the sales revenue.
b. It refers to a company Coding ninjas.
c. In the year 2019-20.

What is data interpretation?

The interpretation of data is the process through which some information is drawn about the data available for analysis.
Let say Coding ninjas in 2019-20 has sales revenue of Rs 40 crores and in 2020-21 has sales revenue of Rs 50 crore.

From these two sales revenue, you get certain information:

a. Company sales have grown by 10 crores.
b. Company % growth has been 25%.
c. The ratio of sales revenue for 2019-20 to 2020-21 is 5:4.

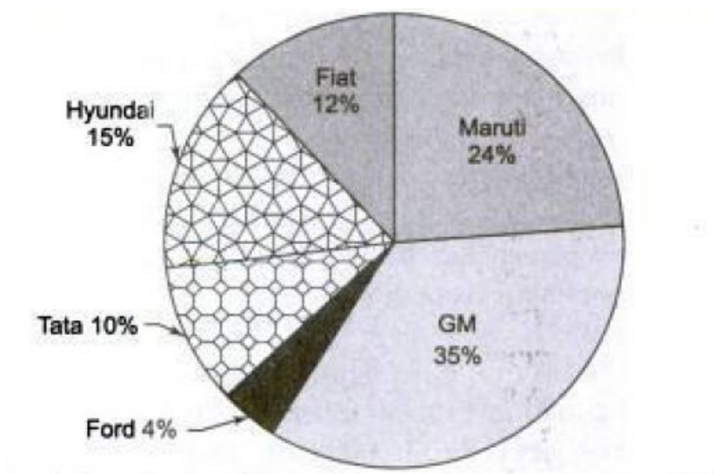
You make out these types of deduction by interpreting the data.

Data does not make any sense when it is in random form or it is difficult to draw out information from random data. So, you have to represent the data in some standard forms like a line graph, pie chart, bar chart, tables and caselet.

How To Read Pie Charts

Pie chart is a specific type of data presentation where data is presented in the form of a circle and pie charts essentially divide 100% of value within a circle. The circle is divided into various subparts. Each subpart represents a certain percentage of total. In the pie chart, the value of the individual pie chart will be an additive construct.

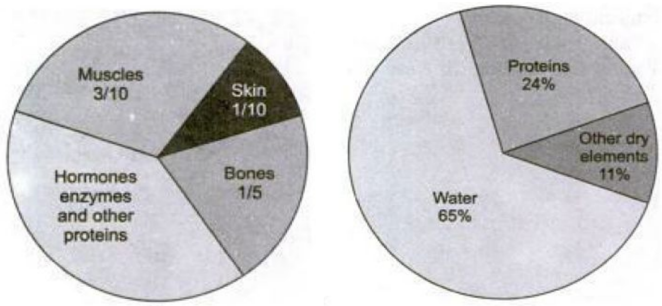
For example: A pie chart showing the distribution of car sales between six companies.



In this pie chart, Maruti has 24% of market share, while GM has 35% of market share, ford has 4% of market share, tata has 10% of market share, Hyundai has 15% of market share and fiat has 12% of market share.

The basic component here is car sales and divided into six companies. The pie chart is a circle, so it is also equal to 360^o or 100%. Thus, 1% is 3.6^o on a pie chart.

For example, The following pie chart figures (a) and (b) gives the information about the distribution of weight in the human body w.r.t. different kinds of components.



(a) (b)

In this case, the kind of information that we can extract by interpreting what is given:
Here muscles are 3/10 means 30%, the skin is 1/10 means 10%, bones 1/5 means 20% and hormones and enzymes and other proteins is 40%.

Let's say a person whose weight is 40kg. So, we can extract information about the components.
Thus, weight of the muscles = 30% of 40 = 12 kg,
Weight of skin = 10% of 40 = 4 kg
Weight of bones = 20% of 40 = 8 kg
Weight of hormones and enzymes and other proteins = 40% of 40 = 16 kg
Weight of protein = 24% of 40 = 9.6 kg
Weight of other dry elements = 11% of 40 = 4.4 kg
Weight of water = 65% of 40 = 26 kg.

The question may be asked, what is the difference between water weight of a 40 and 60 kg person?
Water weight of 40 kg = 65% of 40 = 26 kg.
Water weight of 60 kg = 65% of 60 = 39 kg.
Difference between water weight = 39 - 26 = 13 kg.

In DI once you start understanding the variable, you start understanding the extraction or deduction you make. So, understanding the variable is the most important construct in DI.

How To Read Bar Charts

Data is always about variables, variables are either continuous or discrete.

***For example*,** Sales revenue of company coding ninjas is 40 crores in the year 2019-20. Inside this statement there are few variables, which are running. The running variables are as follow:

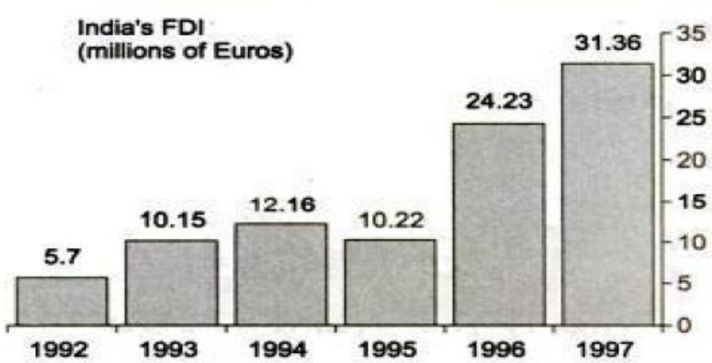
1. Number 40 crore is a sales revenue, which is a variable.
2. The year 2019-20 is also a variable because it could be 2020-21.
3. Company Coding Ninjas is also a variable.

Sales revenue is a continuous variable because it could be 40.01,40.12 etc. whereas the year 2019-20 and company coding ninjas are discrete variables.

Simple Bar Chart:

The simple bar chart is the simplest bar chart which has one continuous variable charted along with one discrete variable.

***For example*:**

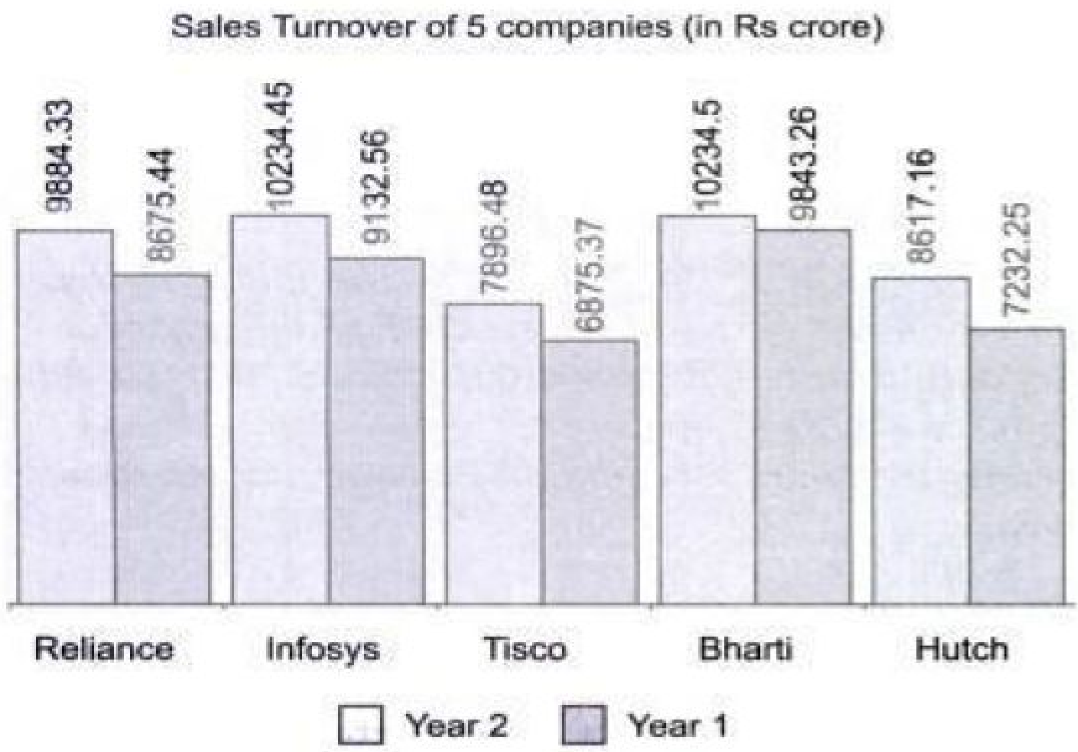


To read this Bar chart, we have to focus on the variables involved.
The year is a discrete variable.
Country India is also a discrete variable.
India's FDI is a continuous variable.
In 1992, number 5.7 meant 5.7 million euros. In this bar chart, we can see the trends of what is happening to India's FDI.

Composite Bar Charts:

In the composite Bar chart, we have two or more continuous variables that are represented.

***For example*:** The following figure shows a Composite Bar Chart.



To read this Composite Bar chart, we have to focus on variables involved.

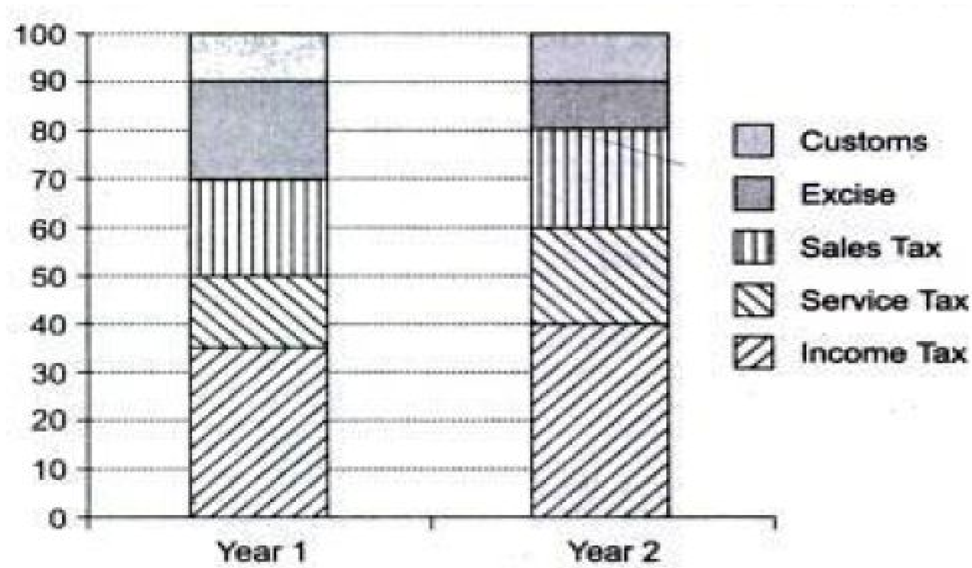
1. Year is a discrete variable.
2. Company names are also a discrete variable.
3. Sales turnover is a continuous variable.

This bar chart gives two or more information about the same discrete variable, for Reliance in Year 1 the sales turnover was 8675.44 crores and in the Year 2 was 9884.33 crores.

Stacked Bar Charts:

Stacked Bar charts represent multiple continuous variables. Sometimes stacked Bar chart can also be used to represent the break-up of some continuous variables.

***For example*:**



Representing Percentage on Stacked Bar Chart

To read this Composite Bar chart, we have to focus on variables involved.

1. Year is a discrete variable.
4. Percentage is also a discrete variable.
5. Taxes i.e. customs, excise, sales tax, service tax and income tax are five continuous variables.

In the Stacked Bar Chart defining different types of taxes into their percentage component breakdown for Year 1 and Year 2.

How To Read Tables and X-Y charts

Tables:

Tables refer to the representation of data in form horizontal and vertical columns. Tables are one of the more versatile methods of representation of data. In tables, we can have any number of continuous variables over any number of discrete variables. The data that can be represented on any type of chart can also be represented on a table.

***For example*:** Representation of state-wise Literacy and Population growth on a table.

State	Percentage increase in		
	Total Literacy (From 1981 to 1991)	Female Literacy (From 1981 to 1991)	Change in % Population Growth Rate (From 1981 to 1991)
Andhra Pradesh	25.17	23.32	+ 0.09
Bihar	22.34	19.48	– 0.04
Gujarat	27.21	26.20	– 0.53
Haryana	29.19	28.67	– 0.11
Himachal Pradesh	31.06	31.00	– 0.24
Karnataka	27.52	26.63	– 0.47
Kerala	30.17	31.20	– 0.43
Madhya Pradesh	25.58	22.86	+ 0.13
Maharashtra	25.87	25.92	+ 0.10
Manipur	29.61	29.68	– 0.25

To read this Composite Bar chart, we have to focus on the variables involved.

1. Three continuous variables: (a) total literacy (b) female literacy (c) change in % population growth rate.
2. States are discrete variables.
3. Year is also a discrete variable.

Total literacy of Andhra Pradesh 25.17% (from 1981 to 1991) means literacy rate 10 years later increased by 25.17%.
Change percentage growth rate 0.09 means percentage growth rate 10 years later increased by 0.09.

Some following type of questions may arise after reading this table:

1. Which state has the highest % growth in literacy?
   Ans: % growth literacy highest for Himachal Pradesh (31.06%)
2. Which state shows the lowest % growth in female literacy?
   Ans: Bihar (19.48%)
3. How many states may have negative growth in population growth rate while having more than 20% growth in both total literacy and female literacy?
   Ans: Gujrat, Himachal Pradesh, Haryana, Karnataka, Kerala and Manipur i.e. 6 states.

***Example 2*:** Shows courier charges (in Rs) for sending a parcel of 1 kg from one city to another city.

Courier Charges For Sending Parcel:

Cities	Allahabad	Mumbai	Kolkata	Delhi	Lucknow
Allahabad	–	10	5	15	10
Mumbai	10	–	7	25	20
Kolkata	5	7	–	20	15
Delhi	15	25	20	–	10
Lucknow	10	20	15	10	–

In this table, sending parcels from Allahabad to Mumbai costs 10 Rs. and sending parcels from Lucknow to Mumbai costs 25 Rs. Similarly, we can see the costs for other cities.
In this table what kind of question can be asked, Minimum cost, maximum cost, % difference in cost or cost of the parcel from Mumbai to Kolkata and then from Kolkata to Delhi.
In this table from Mumbai to Delhi and Delhi to Mumbai has the same courier cost and this is true for every pair.

***Example 3*:** Employees working in various departments of Hoola Moola Boola, Inc.

Years	Departments (Number of Employees)				
	Production	Marketing	Corporate	Accounts	Research
1999	150	25	50	45	75
2000	225	40	45	62	70
2001	450	65	30	90	73
2002	470	73	32	105	70
2003	500	80	35	132	74
2004	505	75	36	130	75

Variables are; Year, Departments and Number of employees. Let's say if we want to extract in 2004 the number of employees in Research, then it is 75 employees.

X-Y Charts:

As the name itself suggests the X-Y Charts will be, in which discrete variables against the continuous variables.

X-Y charts are also useful in determining the trends, rate of change and for illustrating comparison w.r.t some time series.

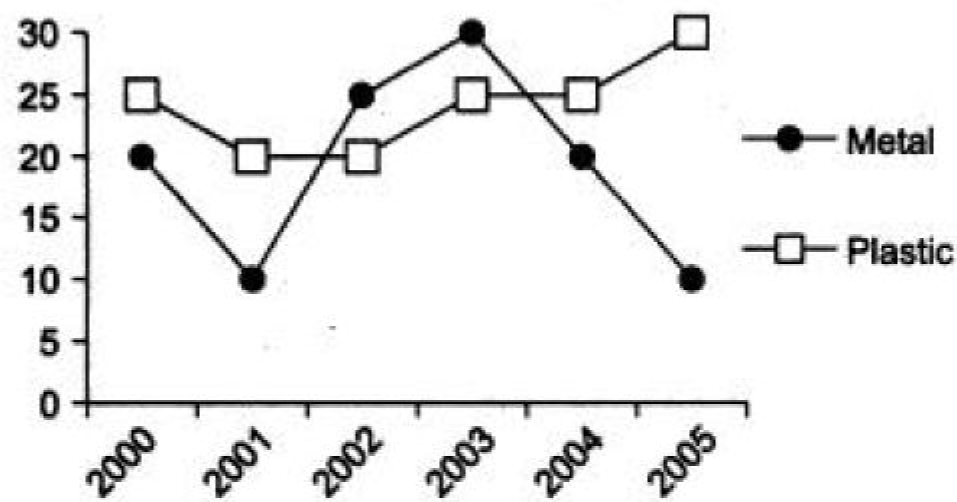
***Feo example*:** The X-Y Chart of Consumer Price Index In 1993-94.

The continuous variable in this set is the consumer price index for the year 1993-94 and the discrete variable is the name of months.

Consumer price index in 1993-94, Jan was 335.

In X-Y charts we also have multiple continuous variables:

For example, The following graph shows the trends of consumption of metals and plastic in the production of cars between 2000-2005.

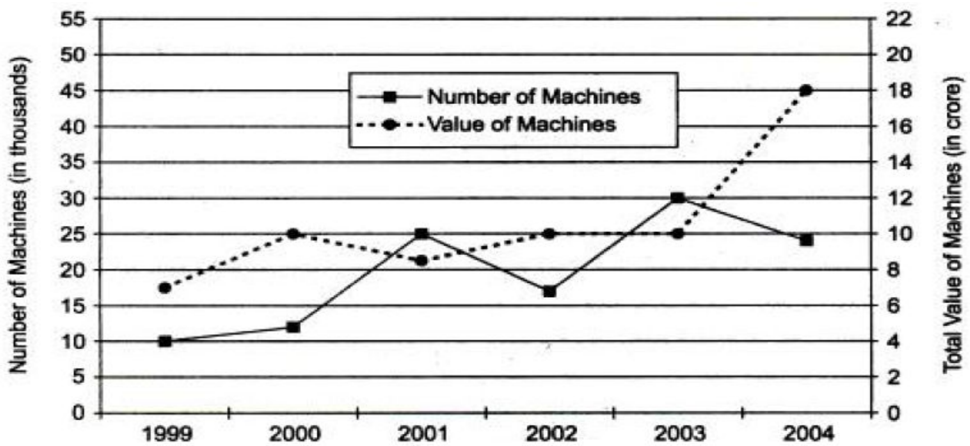


Consumption of metals versus plastic in given years for car manufacturing (in thousand tons)

In 2000 the metal used in cars was 10 k tons and plastic used in cars was 20k tons.
There is a small difference between Line cheats and X-Y charts, in Line charts we draw the lines and in X-Y charts the line will not be there, only points will be marked.

Some Question For Practice

Directions for Questions 1-3: the following graph gives us information about the number of washing machines produced by HLL.



1. What was the value of each machine in 2000?

(a) Rs 20,000 (b) Rs 83,33.33

(c) Rs 2000 (d) Rs 833.33

2. What was the percentage drop in production of the number of machines from 2001-2002?

(a) 20% (b) 25%

(c) 27% (d) 32%

3. What was the difference in the value per machine between the years 2000 and 2003?

(a) Rs 2000 (b) Rs 5000

(c) Rs 4000 (d) None of these

Direction for questions 4-6: Study the following table and answer the questions.

Age-wise Brand Ownership for Television Sets

Brand	1 year old	1–2 years old	2–5 years old	More than 5 years old
LG	15%	45%	40%	
Samsung	5%	15%	25%	55%
BPL	10%	10%	10%	70%
Videocon	25%	55%	20%	
Sony	15%	50%	20%	15%

4. If 1,00,000 TVs were sold last year, how many LG sets were sold?

(a) 10,000 (b) 12,500

(c) 15,000 (d) Can not be determined

5. If the total BPL sets sold to date are 500,000, how many are more than one year old?

(a) 500,000 (b) 450,000

(c) 50,000 (d) Can not be determined

6. When did Videocon capture the maximum market?

(a) Last year (b) 2 years ago

(c) 5 years ago (d) Can not be determined

Answers: 1. (b) 2. (d) 3. (b) 4. (d) 5. (b) 6. (d)

(Reference: Quantitative Aptitude by Arun Sharma)

Lecture Notes For Set Theory, Mensuration & Logarithms

Intro To Logs:

Questions based on this chapter are not so frequent in aptitude exams. You will find some questions based on logs, to solve those questions you have to learn some basic formulae.

Definition of "log":

Let 'a' be a positive real number and a^b = c. then 'b' is called the logarithm of 'c' to the base 'a' and written as $log_a c$ and vice versa, if $log_a c$ = b, then a^b = c.

NOTE: Log of a negative base is not defined.

$log_a c$ = b is possible if and only if a>0 and c>0.

Formulae for log:

1. $log_b a + log_b c = log_b (a \times c)$
2. $log_b a - log_b c = log_b \frac{a}{c}$
3. $log_a 1 = 0$ for all a > 0
4. $log_a a = 1$ for all a > 0
5. $log_c a^b$ = b $log_c a$

Base change rule:

Till now all the formulae are in logarithm with the same base. However, there are a lot of situations in Logarithm problems where you have to operate on logs having different bases. Those situations are:

1. $log_y x = log_z x / log_z y$
2. $log_y x = log_x x / log_x y = 1/ log_x y$
3. $log_{(y^z)} x$ = (1/z) $log_y x$

Logs Problem Solving

Problem 1:

$log_3 x = log_{12} y$ = a, where x,y are real positive numbers. If G is the geometric mean of x and y. What is the value of $log_6 G$?

Solution:

From the statement, $log_3 x = log_{12} y$ = a, we have

$log_3 x$ = a and $log_{12} y$ = a

By definition of the log;

$log_3 x$ = a, x = 3^a and $log_{12} y$ = a, y = 12^a

G is the geometric mean of x and y. So, G = $\sqrt{xy}$

G = $\sqrt{3^a.12^a}$ = $\sqrt{36^a}$ = 6^a

Now; $log_6 G = log_6 6^a$ = a $log_6 6$ = a

Hence, $log_6 G$ = a.

Problem 2:

X is a real number such that $log_3 5 = log_5(2+x)$, which of the following is true.

a. 0<x<3
b. 23<x<30
c. x>30
d. 3<x<23

Solution:

Given, $log_3 5 = log_5(2+x)$ ……….(1)

We know;

$log_3 3 = 1$, $log_3 9 = 2$

So, we can conclude that the value of $log_3 5$ lies between 1 and 2.

Hence, $log_3 5 = 1.46$

So, from eq(1)

$log_5(2+x) = 1.46$ …………(2)

Now, $log_5(2+x)$

If x = 2. Then, $log_5(2+5) = log_5 5 = 1$

If x = 23. Then, $log_5(2+23) = log_5 25 = 2$.

But from eq(2) it is clear that $log_5(2+5)$, can not be 2. Hence, x should be greater than and less than 23.

Hence, option (d) is the answer.

Problem 3:

$\frac{log\, x}{b-c} = \frac{log\, y}{c-a} = \frac{log\, z}{a-b}$, which of the following is correct;

a. xyz = 1
b. $x^a y^b z^c = 1$
c. $x^{b+c} y^{c+a} z^{a+b} = 1$
d. All are correct

Solution:

Let $\frac{log\, x}{b-c} = \frac{log\, y}{c-a} = \frac{log\, z}{a-b} = \text{k}$

$\frac{log\, x}{b-c} = \text{k}$, $\frac{log\, y}{c-a} = \text{k}$ and $\frac{log\, z}{a-b} = \text{k}$

$log\, x$ = k(b-c) and $log\, y$ = k(c-a) and $log\, z$ = k(a-b) and

x = $10^{k(b-c)}$ y = $10^{k(c-a)}$ z = $10^{k(a-b)}$

Now,

xyz = $10^{k(b-c)} \times 10^{k(c-a)} \times 10^{k(a-b)}$

xyz = $10^{k(b-c+c-a+a-b)} = 10^0 = 1$

Hence, xyz = 1.

Problem 4:

$\frac{1}{log_1 n} + \frac{1}{log_2 n} + \frac{1}{log_3 n} + \text{..........} + \frac{1}{log_{43} n} = ?$

Solution:

Use base change rule:

$log_n 1 + log_n 2 + log_n 3 + \text{...........} + log_n 43 = log_n (1.2.3.....43)$

$= log_n 43!$

Hence, $\frac{1}{log_1 n} + \frac{1}{log_2 n} + \frac{1}{log_3 n} + \text{..........} + \frac{1}{log_{43} n} = log_n 43!$

Problem 5:

Find the minimum value of 2 $log_{10}x$ - $log_x(0.01)$, if x>1.

Solution:

Given statement; 2 $log_{10}x$ - $log_x(0.01)$

$log_x(0.01) = log_x 10^{-2}$ = -2 $log_x 10$ = -2/ $log_{10}x$ (using base change rule)

Now given statement becomes 2 $log_{10}x$ + 2/ $log_{10}x$ = 2($log_{10}x$ + 1/ $log_{10}x$)

Since, x>1, we can conclude that the minimum value of this expression would come when x = 10.

$2(\log_{10}10 + 1/\log_{10}10) = 2(1+1) = 4$.
If we try any value of x other than 10, we will always get a value greater than 4.

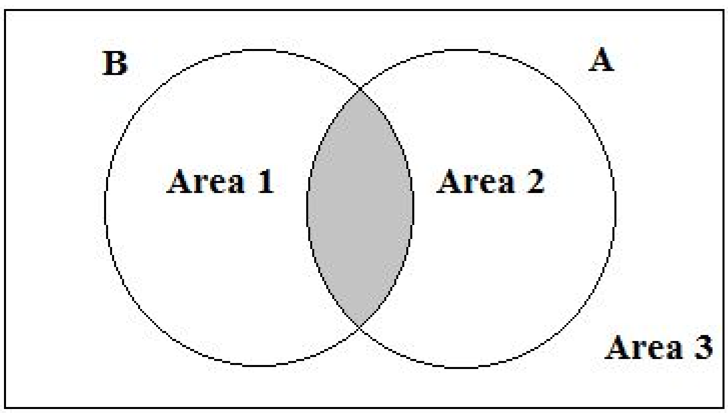
Set theory

Set theory is important both from a mathematical point of view as well as a reasoning point of view. You will see a lot of questions based on set theory in a lot of aptitude exams. Set theory questions have two ways of solving.

1. Formula approach.
2. Venn Diagram approach.

Two attributes situation:

Let's have a situation where two attributes A and B. A refers to those people who passed Physics and B refers to those people who passed Chemistry.



The rectangular box represents a universal set.
Area 1: People who passed only Physics.
Area 2: People who passed only Chemistry.
Area 3: People who passed neither Physics nor Chemistry.

Formula: $A \cup B = A + B - A \cap B$.

Problem 1:
In a school of 350 students, 100 are in the Band, 200 are in the Sports team and 50 are in both Band and Sports team.

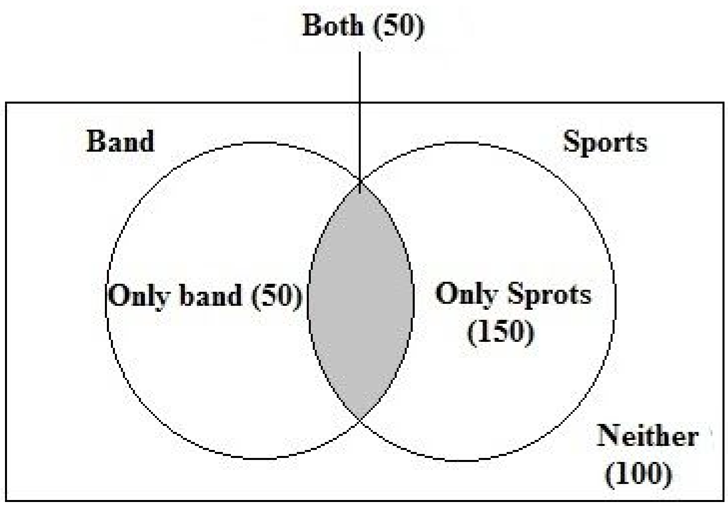
1. How many students are involved neither in Band nor in Sports?

2. How many people involved at least one of the two?
3. What is the ratio of people who participate only in the band to only in sports?

Solution:

50 students are in both Band and Sports. So, 100 - 50 = 50 students are in Band only and 200 - 50 = 150 students are in Sports only.

Total students 350 and 350 - 250 = 100 students are neither in Band nor in Sports.



1. Students are involved neither in Band nor in Sports = 100.
2. Students involved at least one of the two = 50+50+150 = 250.
3. Students only in Band = 50 and students only in Sports = 150

   Hence, the Ratio of students only in the band to only in sports = 50:150 = 1:3.

Problem 2:

There are 60 students in a class, 60% fail in English and 30% pass in Maths and 20% pass in both English and Maths. How many students fail in either of 2 subjects or at least in one subject?

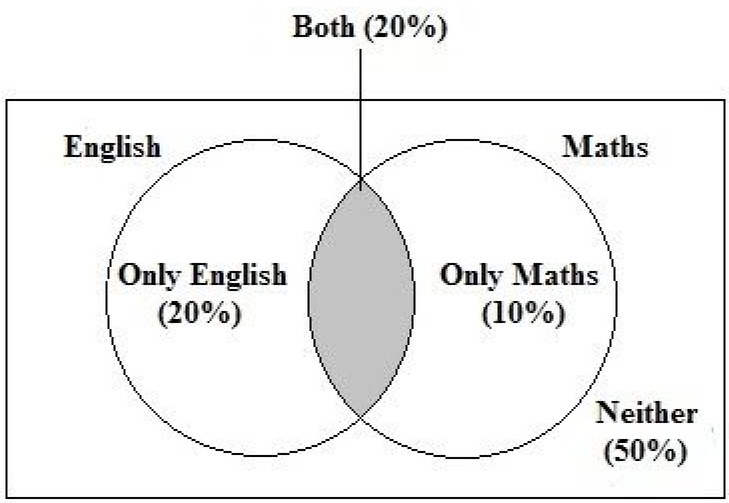
Solution:

20% of students pass in both English and Maths. So, 30% - 20% = 10% of students pass in maths only and 60% fail in english means 40% pass in english and 40% - 20% = 20% of students pass in English only.

Total students 100% and 100 - 50 = 50% of students neither pass in english nor pass in maths.

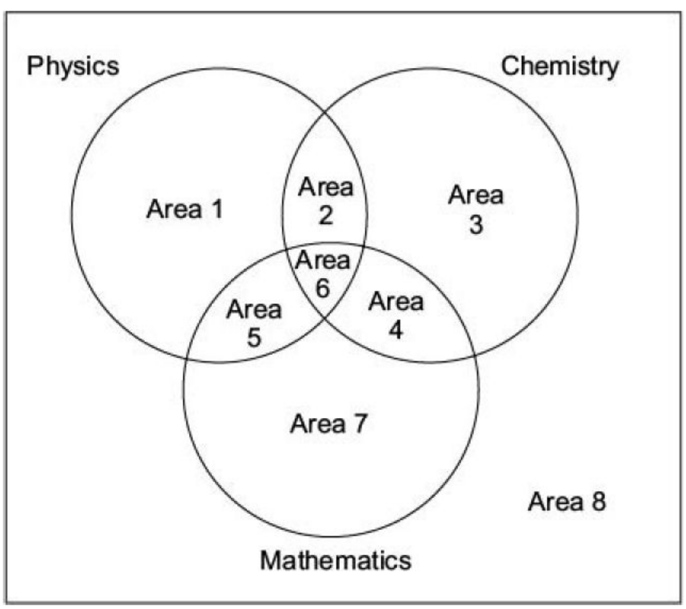
Number of students fail in either of two subjects = 20% + 10% = 30% i.e 30% of 60 = 18 students.

Number of students fail in at least one subject = 20 + 10 + 50 = 80% i.e 80% of 60 = 48 students.



Three attributes situation:

Let's have a situation where there are three attributes being measured. Suppose we are talking about people who passed Physics, Chemistry and Mathematics.



Area 1: People who passed in Physics only
Area 2: People who passed Physics and Chemistry but not Maths.
Area 3: People who passed Chemistry only

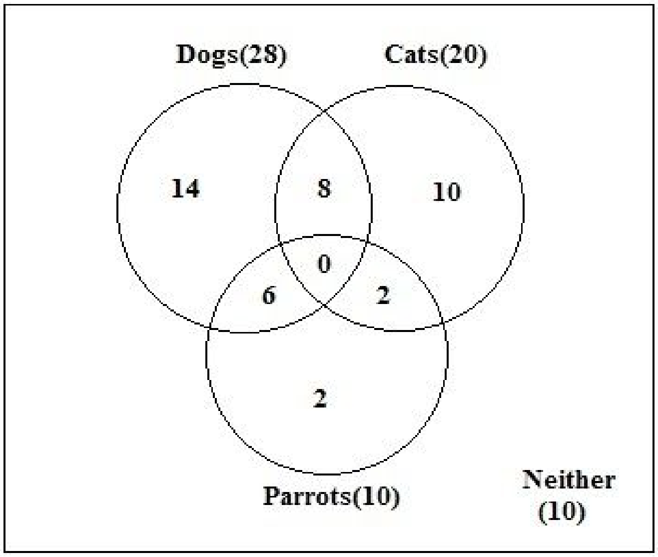
Area 4: People who passed Chemistry and Maths but not physics.
Area 5: People who passed Physics and Maths but not in Chemistry.
Area 6: People who passed Physics, Chemistry and Maths
Area 7: People who passed Maths only
Area 8: People who passed in no subjects.

People passing Physics and Chemistry: Represented by the sum of areas 2 and 6
People passing Physics and Maths: Represented by the sum of areas 5 and 6
People passing Chemistry and Maths: Represented by the sum of areas 4 and 6
People passing Physics: Represented by the sum of the areas 1, 2, 5 and 6
People passing at least 2 subjects = area 6 + area 2/4/5
People passing exactly 2 subjects: represented by area 2,4 and 5.

Problem 1:
A veterinary doctor surveyed 52 people. He discovered that 28 have dogs, 20 have cats and 10 have parrots, 8 have dogs and cats, 6 have dogs and parrots and 2 have cats and parrots. No one has all three pets.

1. How many people have only a dog?
2. How many people have at least 2 pets among dogs, cats and parrots?
3. How many people have none of the 3 pets?

Solution:
8 people have dogs and cats, 6 people have dogs and parrots. 28 - (8+6) = 14 people have only dogs.
8 people have dogs and cats, 2 people have cats and parrots. 20 - (8+2) = 10 people have only cats.
6 people have dogs and parrots, 2 people have cats and parrots. 10 - (6+2) = 2 people have only parrots.



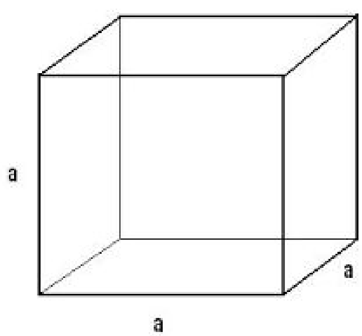
1. People have only a dog = 14.
2. People have at least 2 pets = 6+8+2= 16.
3. People have none of the 3 pets = 10.

Cubes And Cuboids

Cubes and cuboids are a chapter of mensuration. Mensuration is a measurement of 2-D and 3-D figures. Cubes and cuboids are 3-D shapes which consist of 6 faces, 8 vertices and twelve edges.

Cube:

Cube is a 3-D shape. It consists of 6 faces, 8 vertices and twelve edges. All faces of the cube are square-shaped and have equal dimensions.

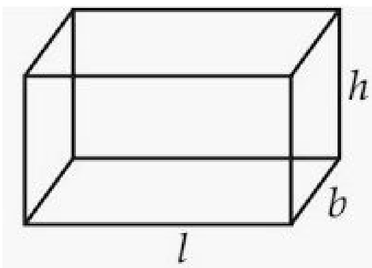


Some formulae of cube:

1. The total surface area of cube = $6a^2$
2. The volume of cube = a^3
3. Length of diagonal = $\sqrt{3}a$
4. Perimeter of cube = 12a

Cuboid:

The cuboid is also a 3-D shape. It consists of 6 faces, 8 vertices and twelve edges. All faces are not equal in dimensions.



Some formulae of cuboid:

1. The total surface area of cuboid = $2[\ l \times b + b \times h + l \times h\ ]$
2. The volume of cube = $l \times b \times h$
3. Length of diagonal = $\sqrt{l^2 \times b^2 \times h^2}$
4. The perimeter of cube = $4[\ l + b + h\ ]$

Problem 1:

The surface area of a cube is 216 cm^2. Find its volume?

Solution:

The surface area of cube = 216

$$6a^2 = 216$$

$$a = 6.$$

So, the volume of the cube = $a^3 = 6^3 = 216\ cm^3$.

Problem 2:

3 cubes of volume 1 cm^3, 8 cm^3 and 27 cm^3. These are melted to form a new cube. Find the side of a new cube.

Solution:

Volume of a new cube = sum of volume of 3 cubes

$$= 1+8+27 = 36\ cm^3$$

Volume of cube = $a^3 = 36$
Hence, side of new cube = $\sqrt[3]{36}\ cm$

Problem 3:
A room with sides $6\,cm$, $4\,cm$ and $3\,cm$. What is the longest length of stick that can placedt in this room?

Solution:
Length (l)= 6 cm
Breadth (b)= 4cm
Height (h)= 3 cm
The longest stick that can be placed will be along the diagonal of the room.

Length of the Diagonal $= \sqrt{l^2 \times b^2 \times h^2}$

$= \sqrt{6^2 \times 4^2 \times 3^2} = \sqrt{36 \times 16 \times 9} = \sqrt{61}$ cm
Hence, the length of the longest stick that can be placed = $\sqrt{61}$ cm

Problem 4:
4 equal squares are placed side by side in a row. Find the ratio of the total surface area of the resulting cuboid and to the sum of the surface area of all cubes.

Solution:
Let the side of a cube = a
Total surface area of one cube = $6a^2$
Total surface area of 4 cubes = $4\times6a^2 = 24a^2$

If 3 cubes are placed side by side, dimension of resulting cuboid is
length = 4a
breadth = a
height = a
Total surface area of cuboid =2[$l \times b + b \times h + l \times h$]
= $2(4a\times a + a\times a + a\times4a)$
= $2(4a^2 + a^2 + 4a^2)$
= $2 \times 9a^2$
= $18a^2$

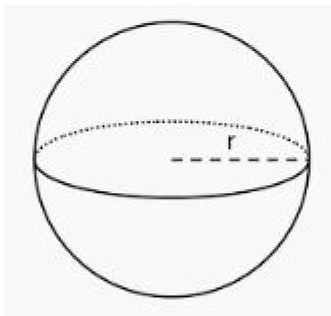
Ratio = $\frac{total\ surface\ area\ of\ cuboid}{total\ surface\ area\ of\ cubes}$ = 18/24 = 3:4

Hence, the ratio is 3:4.

Sphere And Cylinder

Sphere:

A sphere is solid as a ball with radius ‘r’.



Basic measurement in sphere:

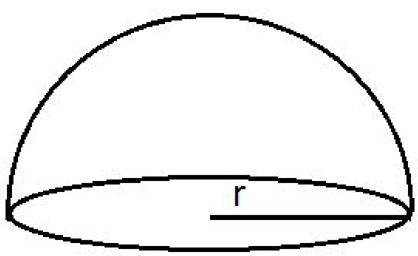
1. Radius of sphere.
2. The surface area of the sphere.
3. The volume of the sphere.

Formulae:

1. Surface area of Sphere = $4\pi r^2$
2. Volume of Sphere = $4/3(\pi r^3)$

Hemisphere:

When a plane cuts across the sphere at the centre then it forms a hemisphere.

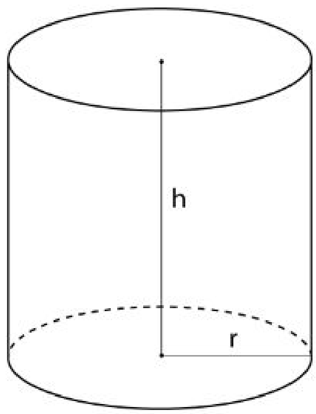


Formulae:

1. Surface area of hemisphere = $2\pi r^2$
2. Surface area of top = πr^2
3. Total surface area of hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Cylinder:

A cylinder is a solid which has both its ends in the form of a circle. Its dimensions are defined in the form of the radius of the base ‘r’ and the height ‘h’.



Formulae:

1. Volume of cylinder = $\pi r^2 h$
2. Total surface area of cylinder = $2\pi r h + 2\pi r^2 = 2\pi r$ (r+h)
3. The curved surface area of cylinder = $2\pi r h$

Problem 1:

From a sphere of radius 2 cm, how many spheres of radius 0.2 can be made?

Solution:

Volume of original sphere = $4/3(\pi r^3)$

$= 4/3(\pi 2^3) = 32/3(\pi)\ cm^3$

Volume of new sphere = $4/3(\pi r^3)$

$= 4/3(\pi (0.2)^3) = 0.032/3(\pi)\ cm^3$

Number of sphere can be formed = $\frac{volume\ of\ original\ sphere}{volume\ of\ new\ sphere}$

$= \frac{32/3(\pi)}{0.032/3(\pi)} = 1000$ spheres.

Problem 2:

Sphere has the same volume as the cylinder of height 10cm and radius 4 cm. Find the radius of the sphere.

Solution:

Volume of the sphere = $4/3(\pi r^3)$
Volume of the cylinder = $\pi r^2 h$
$= \pi 4^2 \times 10 = 160\pi$
According to question;
Volume of sphere = volume of cylinder
$4/3(\pi r^3) = 160\pi$
$r^3 = 120$
Hence, radius of sphere = $\sqrt[3]{120}$ cm.

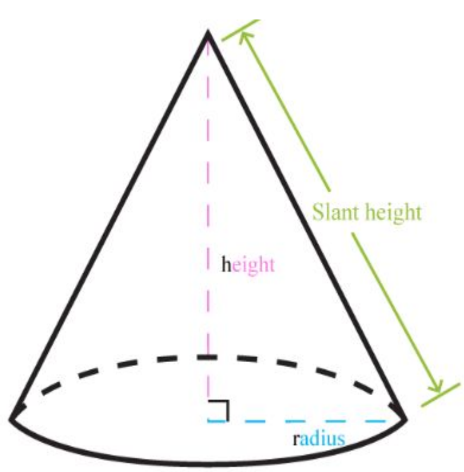
Problem 3:
Two right circular cylinders have equal volume and their height are in ratio 1:2. What is the ratio of their radii?

Solution:
Let r_1 and r_2 be the radii of two cylinders. And h_1 and $2h_1$ be their heights.(ratio of height is 1:2 given)
Volume of both the cylinders are equal
$\pi r_1^2 h_1 = \pi r_2^2 \times 2h_1$
$r_1^2 / r_2^2 = 2$
$r_1 : r_2 = \sqrt{2} : 1$

Cones, Prisms and Pyramids

Cone:

Cone is an object of circular base and its lateral sides converse to a single point at the top.



Measurement in cone:

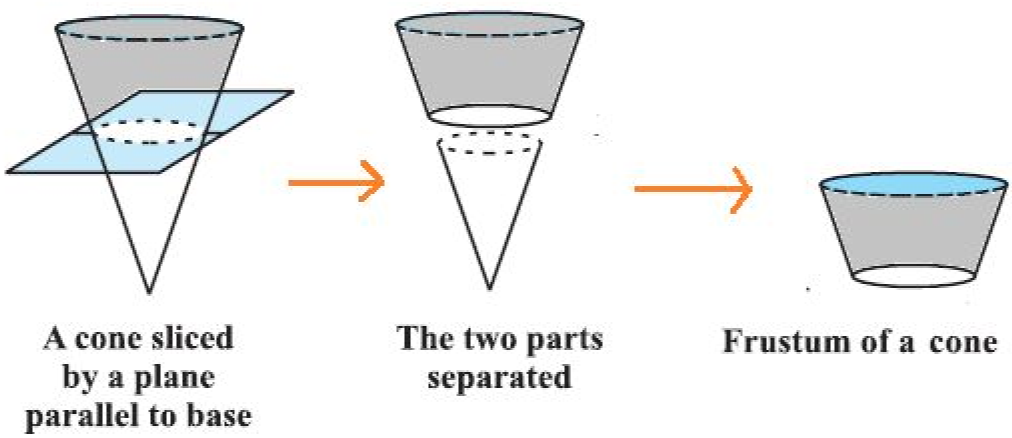
1. The diameter of the base.
2. Height of the cone.
3. Slant height of the cone.
4. Volume of the cone.
5. Curved surface area of cone.
6. Total surface area of cones.

Formulae:

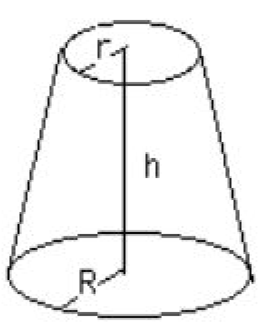
1. Curved surface area of cone = πrl
2. Total surface area of cone = $\pi rl + \pi r^2 = \pi r(r + l)$
3. Volume of cone = $1/3(\pi r^2 h)$

Frustum of a cone:

When a cone is cut by a parallel plane is called the frustum of a cone.



Formulae:



1. Slant surface of the frustum of a cone = π (r + R)l; where l is the slant height.
2. Volume of the frustum of a cone = $\frac{1}{3}\pi h(r^2 + rR + R^2)$

Similarity concept:

Cone on top which we will cut out and the cone which was originally, because of their angle, they are similar to each other.

Let say we cut the cone from half of the height i.e. h/2
In the similarity concept, all the ratios have to be similarly maintained. This tells us the two cones are similar and hence,

1. Area is **"Square of the length"**.
2. Volume is **"cube of the length"**.

***For example*:**
When we cut a cone from between.
Let the surface area of the original cone is 'x'.
Hence, the surface area of cut out = Area × square of the length

$= x \times (1/2)^2 = (1/4)x.$

Now, slant area of frustum = **surface area of the original cone - surface area of cut out**

$= x - (1/4)x = (3/4)x\ cm^2$

Similarly, volume of frustum = **volume of original cone - volume of cut out**

Prism:

A prism is solid and has the same geometrical shape as polygon at both its ends. Its dimensions are defined by the dimensions of the polygon at its ends and its height.

1. Lateral surface area of a prism = Perimeter of base × height
2. Volume of a prism = area of base × height

Pyramids:

A pyramid is a solid which can have any polygon as its base and its outer surfaces are triangular and converge to a single point at the top.

1. Slant surface of a pyramid = 1/2 × Perimeter of the base × slant height
2. Whole surface of a pyramid = Slant surface + area of the base
3. Volume of a pyramid = $(\frac{Area\ of\ base}{3}) \times height$

Problem On Cones And Pyramids

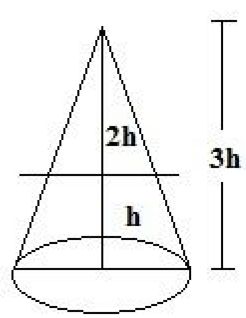
Problem 1:

A cone is cut at 1/3rd of height from the base to make a frustum. What is the ratio of original cone to the frustum of the cone?

Solution:

Let take the original height = 3h and the cone cut 1/3rd from the bottom.

The ratio of the height of the original cone and the cut out cone = 3:2



All length ration should be 3:2. Hence, by similarity concept;

The ratio of the volume of the original cone to cut out cone = $3^3 : 2^3 = 27:8$

Let the volume of the original cone = 27x and volume of the cutout cone = 8x

Hence, volume of the frustum of cone = 27x - 8x = 19x.

Therefore, the ratio of the original cone volume to the frustum of cone = 27:19.

Problem 2:

The radius of a cone is 7 cm, slant height is 25 cm. Find the volume of the cone.

Solution:

The volume of the cone = $1/3(\pi r^2 h)$

We have, r = 7cm and l = 25 cm.

Height of the cone 'h' = $\sqrt{l^2 + r^2}$

$$h = \sqrt{25^2 + 7^2} = 24 \text{ cm}$$

Hence, volume of the cone = $1/3(\pi 7^2 24) = 392\,\pi\ cm^3$

Problem 3:

Height and radius of the conical vessel is ‘h’ and ‘r’ respectively. Vessel has a capacity of 10 litres of water. What is the capacity of a cylinder having the same height ‘h’ and radius ‘r’?

Solution:

Volume of the cone = $1/3(\pi r^2 h)$

Volume of the cylinder = $\pi r^2 h$

Given, the volume of the cone = 10

$$1/3(\pi r^2 h) = 10$$

$$\pi r^2 h = 30$$

Hence, volume of a cylinder = 30 cm^3

Problem 4:

A map of a country of height 6 feet requires 10 litres of paints. How much paint would be required for a map of the same country of height 12 feet?

Solution:

Heights are in the ratio of 6:12 or 1:2

Therefore area will be in the ratio of 1:4

Therefore, to paint a map of 12 feet height $=4 \times 10 = 40$ litre

Problem 5:

A circular iron ball of radius 1 m and weight 20 kg. Find the weight of another iron ball of radius 3 m.

Solution:

Weight of a ball is a function of volume.

Radius is in the ratio of 1:3

Area will be in the ratio of 1:9

Therefore, volume in the ratio = 1:27

Hence, weight of iron ball of radius 3m $= 20 \times 27 = 540$ kg.

Some questions for practice

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1. Calculate: log2 (2/3) + log4 (9/4) = log2
   Ans : 0.

2. If log102 = 0.301 find log10125.
   Ans: 2.097.

3. If log10a = b, find the value of 103b in terms of a.
   Ans: a^3 **.**

4. In the Fun club, all the members participate either in the Tambola or the Fete. 320 participate in the Fete, 350 participate in the Tambola and 220 participate in both. How many members does the club have?
   Ans: 450.

5. A solid wooden toy in the shape of a right circular cone is mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.
   Ans: 266.11 cm^2

(Ref: Quantitative Aptitude by
Arun Sharma)

Lecture Notes For Important Topics

In this chapter, we will discuss the word-based problems on the number system, Arithmetic progression, Geometry progression, Remainder theorem and Unit digits. All these chapters are very important for all aptitude exams.

Word Based Problem On Number System

We can solve these problems with the help of:

1. Linear equations.
2. Identify the variable in the equation.
3. Think numerically and logically.

Problem 1:
5 year ago Anjali was 5 times as old as her son. 5 years hence her age will be 8 less than three times the corresponding age of her son. Find the present age of Anjali.

a. 30 years
b. 35 years
c. 40 years
d. 45 years

Solution:

	5 Years Ago	**Present**	**5 Years Later**
Anjali	5x	5x+5	5x+10
Son	x	x+5	x+10

Now it is given that after 5 years her age will be 8 less than three times her Son's age.
Hence, $5x+10+8 = 3(x+10)$
$3x+30 = 5x+18$
$2x = 12$
$x = 6$.
Therefore, Anjali’s present age = $5 \times 6+5 = 35$ years.

2nd method:

We can check from the options.
Let's take the option (a) 30 years.
So, Anjali's present age = 30 years
5 year ago her age = 30 - 5 = 25 years and her son's age 5 years ago =(1/5) × 25 = 5years

Present age of son = 5+5 = 10.
5 years later age of her son = 10+5 = 15 and Anjali's age = 35
5 years later her age will 3 times of her son's age = 15 × 5 = 45 years.

Anjali,s age 8 years less than her son's age after 5 years, that means Son's age 8 year more than 35 years. So, 35+8 = 43 years
Hence this option is wrong.

The same way we can check for option(b).

	5 Years Ago	Present	5 Years Later
Anjali	30	35	40
Son	6	11	16

5 year age Son's age = (1/5) × 30 = 6.
After 5 years Anjali's age is 3 times of her son's age i.e. 16 × 3 = 48.
And after 5 years Anjali's age is 8 years less than 3times of her son's age i.e. 16 × 3 - 8 = 40 years.

Problem 2:

Girish's youth lasted one-sixth of his life. He grew a beard after one twelfth more. After one seventh more of his life, he married. 5 year later, he and his wife had a son. The son lived exactly one half as long as his father and Girish died four years after his son. How many years did Girish live?

a. 76 years
b. 80 years
c. 84 years
d. 88 years

Solution:

Lets the Girish final age = x
Final age = youth lasted age + grew beard age + marriage + 5 + son's age + 4
$x = x/6 + x/12 + x/7 + 5 + x/2 + 4$

X = 84 years.

2nd method:
Girish youth lasted = 14/6 of his life.
So, His life should be divisible by 6 because age always be an integer. Only option (c) 84 will be divisible by 6.
Hence, the age of Girish = 84 years.

Problem 3:
5 children who are born at an interval of 4 years and sum of their ages is 60. What is the age of the oldest child?

Solution:
Children are born at a 4-year interval. It means it is an A.P. of +4.
Sum of their ages = 60.
Average of the ages = 60/5 = 12
There are only 5 children, so an average of 5 will be the middle term.

1st children	2nd children	3rd children	4th children	5th children
4	8	**12**	16	20

Hence, the age of the oldest children = 20 years.

2nd method:
Let the age of 1st children = a
Hence, the age of 2nd children = a+4
age of 3rd children = a+8
age of 4th children = a+12
age of 5th children = a+16

Sum of their ages = 60
a+a+4+a+8+a+12+a+16 = 60
5a + 40 = 60
a = 4.
Hence, the age of oldest children = 4+16 = 20 years.

Arithmetic Progressions (AP)

Any series that has the property of the same number getting added to get the next number every time is called Arithmetic Progression (AP).

***For example*:**

1. 5,8,11,14,17,20
   Here 3 is added to get the next number.
2. 3,10,17,24,31
   Here 7 is added to get the next number.

This number getting added is referred to as the ***Common Difference*** of an A.P. and it is denoted by **'d'.** OR ***common difference*** is the number which is the difference between two consecutive terms of an A.P.

Important notations:

a. ***'a'*** is defined as the first term of an A.P.
b. ***'n'*** is the total number of terms in an A.P.
c. **'** a_n **'** denotes the nth term of an A.P.

Formulae:

1. n^{th} term of an A.P, a_n = a + (n-1)d
2. Sum of an A.P is $S_n = \frac{n}{2}$ (2a+(n-1)d)
3. $S_n = \frac{n}{2}$ (a+l) here, 'l' is the last term of an A.P.
4. Number of terms 'n' = $\frac{D}{d}$ + 1, here, D is the difference of 1st and last term of an A.P and 'd' is the common difference.

Problem 1:
A series 3,10,17,...........up to 20 terms. Find the 20th term of this sequence.

Solution:
Here, a = 3 , n = 20 and d = 10-3 = 7
Last term, a_{20} = a + (n-1)d
a_{20} = 3 + (20-1)7
a_{20} = 3+133 = 136.

Problem 2:
A series 3,10,17,.........,381. Find the number of terms.

Solution:
a = 3, d = 7 and a_n = 381.
a_n = a + (n-1)d
381 = 3+(n-1)*7
Hence, n = 55.

Or we can calculate by n = $\frac{D}{d}$ + 1, D = 381-3 = 378 and d = 7.

n = $\frac{378}{7}$ + 1 = 54+1 = 55
Hence, the total number of terms is 55.

Problem 3:
A sequence 3,7,11,15,19,23. Find the sum of this sequence.

Solution:
Addition of 1st and last term = 3+23 = 26.
Addition of 2nd and 2nd last term = 7+19 = 26
Addition of 3rd and 3rd last term = 11+15 = 26
Avg of each pair = 26/2 = 13
And 13 is an average of this A.P. and there are 6 terms in this A.P
Hence, sum of the A.P = 6 × 13 = 78.

In an A.P. when n is even: Then the average of A.P. comes from two middle terms.
eg 3,5,7,9,11,13
7 and 9 are two middle terms. So, Average = (7+9)/2 = 8.

In an A.P. when n is odd: Then the middle term, itself is an average.
eg. 2,5,8,11,14
8 is the middle term. So, average = 8.

NOTE: Sum of an A.P. = n × average.

Problem 4:

How many numbers between 100 and 200 leave a remainder 3, when divided by 7 and what are some of the numbers?

Solution:

This forman A.P of common difference = 7

101,108,115,...........,192,199

D = 199 - 101 = 98 and d = 7

Number of terms = $\frac{D}{d} + 1$

$= \frac{98}{7} + 1 = 14+1 = 15.$

Average = (101+199)/2 = 150

And, sum of these number = 15 × 150 = 2250

Problem 5:

Two A.P's are 3,10,17,24,.........up to 200 terms and 2,10,18,26,............,up to 200 terms. How many common terms exist between these A.P's?

Solution:

In 1st AP, a_{200} = 3 + (200-1)7 = 1396

In 2nd AP, a_{200} = 2 + (200-1)8 = 1594

1st common term is 10

Common terms between the AP will themselve form an AP and the common difference of this AP is LCM (d1, d2).

d1 = 7 and d2 = 8

d = LCM(7,8) = 56.

So, AP formed by common terms is;

10,66,122,178,.................

This series is 56n+10 and we have to limit this series to 1396(because last term would be less than 1396)

Let take n = 20, 56 × 20+10 = 1130 and the next terms are 1130,1186,1242,1298,1354.

Now, number of terms = $\frac{D}{d} + 1$, D = 1354-10 = 1344 and d = 56

$= \frac{1344}{56} + 1 = 25.$

Hence, the total number of common terms = 25.

Geometric Progressions (GP)

Geometric progression is a sequence in which any number after the first number is obtained by multiplying the proceeding number by a constant value, then the sequence is called geometric progression(GP).
And that constant value is called the common ratio, which is denoted by ‘r’.

***For example*:** The sequence 4,12,36,108,324……. Is a GP with common ratio 3.

Important notations:

a. ***‘a’*** is defined as the first term of a G.P.
b. ***‘n’*** is the total number of terms in a G.P.
c. ‘a_n’ denotes the nth term of a G.P.

Formulae:

1. n^{th} term of a G.P, $a_n = a \times r^{(n-1)}$
2. Sum of ‘n’ terms
   a. $S_n = \frac{a(r^n-1)}{(r-1)}$, when r > 1.
   b. $S_n = \frac{a(1-r^n)}{(1-r)}$, when 0 < r < 1.
3. Sum of infinite terms: $S_\infty = \frac{a}{(1-r)}$, when 0 < r < 1.
4. The geometric mean between two quantities: GM = $\sqrt{a \times b}$

NOTE: Common ratio can be negative. eg 10,-20,40,-80,160.., here r = -2

Problem 1:
Find the sum of the series: 5,10,20,40,...........up to 18 terms.

Solution:
Here, a = 5, r = 2 and n = 18.
Now, sum of this G.P is
$S_{18} = \frac{5\times(2^{18}-1)}{(2-1)}$

Problem 2:

A ball is dropped from a height of 400 ft and bounce back half of its height and drops again and keeps bouncing and coming back to half of its height until it comes to rest. What is the total distance covered by the ball before it comes to rest?

Solution:

In this case, we find two GP,

1. G.P when the ball dropped: 400+200+100+................

   Here, a = 400 , r = 1/2. Then the sum is, $S_\infty = \frac{a}{(r-1)}$

   $S_\infty = \frac{400}{(1-1/2)} = 800.$
2. G.P when the ball bounced back: 200,100,50,................

   Here, a = 200 , r = 1/2. Then the sum is, $S_\infty = \frac{a}{(r-1)}$

   $S_\infty = \frac{200}{(1-1/2)} = 400.$

Hence, total distance covered by the ball = 800+400 = 1200 ft.

Eliminate the formula ($S_\infty = \frac{a}{(r-1)}$):

We have,

1st term	**Common Ratio**	S_∞
a	1/2	2a
a	1/3	3a/2
a	2/5	5a/3

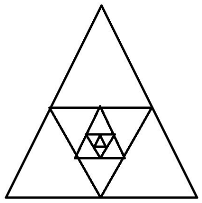
So, we can conclude that denominator common ratio becomes a multiplier for ‘a’ and difference between denominator and numerator of the common ratio becomes the denominator of the sum. So, without using the formula for infinite terms sum, we can directly calculate the sum of infinite terms.

Problem 3;

Midpoints of a triangle join to form another triangle, whose midpoints again join to form another triangle and process are repeating infinite times to form infinite triangles by continuously joining midpoints. P is the perimeter of the 1st triangle and A is the area of the 1st triangle. What is the sum of all perimeters and all areas?

Solution:

By joining midpoints of a triangle we will get the following figure.



By the midpoints theorem, the perimeter of the next triangle becomes ½ of the first triangle. So, we have G.P

P, P/2, P/4, P/8, P/16,.................
Here, a = P and r = 1/2
So, S_∞ = 2P

In case of area, all the triangles are similar to each other, So, using the similarity concept, if the perimeter becomes half then lengths also becomes half and area will become 1/4th. So, we have G.P
A, A/4, A/16, …………...
Here, a = A and r = 1/4
So, S_∞ = 4P/3
Hence, the sum of all perimeters = 2P and the sum of all areas = 4A/3.

Remainder Theorem

The logic of remainder theorem:

Consider a question, 17×23
We want to find the remainder of this expression when divided by 7.
$\frac{17\times23}{7}$ = 391/7 = 6/7.
Hence, the remainder of this expression is 6.
This is the school time approach and this is time-consuming. To solve this question we have to use the Remainder theorem.

We can write this as:
$17 \times 23 = (14+3) \times (21+2)$
Which can be expanded:
$(14+3) \times (21+2) = 14 \times 21 + 14 \times 2 + 3 \times 21 + 3 \times 2$
When you divide this expression by 7, you will realise that remainder depends on last terms.

Thus, $\frac{14\times21 + 14\times2 + 3\times21 + 3\times2}{7} = \frac{6}{7}$

Hence, the remainder of this expression is 6.

Remainder theorem transformation:

The remainder theorem transformation denote it by the sign $\xrightarrow{R}$

If we use the remainder theorem transformation, then we can take the remainder of individual number 17 and 23 (the above question data) when divided by 7. When 17 divided by 7 gives remainder 3 and when 23 divided by 7, gives remainder 2 and then by multiplying these remainder you will get the remainder of the original expression.

$$\frac{17\times23}{7} \xrightarrow{R} \frac{3\times2}{7} \xrightarrow{R} \frac{6}{7}$$

Hence, the remainder is 6.

Problem 1:

Find the remainder of 1421 × 1423 × 1425, when divided by 12.

Solution:

The individual remainder are,

		Remainder
1.	$\frac{1421}{12}$	5
2.	$\frac{1423}{12}$	7
3.	$\frac{1425}{12}$	9

$$\frac{1421\times1423\times1425}{7} \xrightarrow{R} \frac{5\times7\times9}{7} = \frac{35\times9}{7} \xrightarrow{R} \qquad \xrightarrow{R} \frac{11\times9}{7} \quad \frac{3}{7}$$

Hence, the remainder of this expression is 3.

Using Negative Remainder:

Consider the following question:

Find the remainder when: 53 × 54 divided by 55.

$$\frac{53\times54}{55} \xrightarrow{R} \frac{-2\times-1}{55} = \frac{2}{55}$$

Hence, the remainder is 2.

Some you might find a question which does not allow simple calculation and that will involve long calculations. Hence, the principle is that you should use negative remainders wherever you can.

When Answer Comes Negative:

Find the remainder when: $52 \times 53 \times 54$ divided by 55.

$$\frac{52\times53\times54}{55} \xrightarrow{R} \frac{-3\times-2\times-1}{55} = \frac{-6}{55}$$

But we know that remainder can't be negative i.e. -6. So, the remainder of this expression will be 55-(-6) = 49.

Use Of Cutting In Remainder Theorem problem:

Find the remainder when: $42 \times 31 \times 17$ divided by 12.

When we go with remainder theorem,

$$\frac{42\times31\times17}{12} \xrightarrow{R} \frac{6\times7\times5}{12} = \frac{210}{12}$$, hence remainder is 6.

Instead of doing this, if we write this expression as:

$$\frac{42\times31\times17}{12} = \frac{7\times31\times17}{2} \xrightarrow{R} \frac{1\times1\times1}{2} = \frac{1}{2}$$

Hence, the remainder, in this case, is 1. But we can see the answer is not the same.

We have transformed $\frac{42}{12}$ into $\frac{7}{2}$ by dividing the numerator and the denominator by 6. The result is that the original remainder 6 is also divided by 6 giving us 1 as the remainder. Thus to get the original remainder 1 is multiplied by 6.

Dealing With Large Power:

a. +1 remainder rule:

For example $\frac{17}{8} \xrightarrow{R} +1$

$$\frac{17\times17}{8} \xrightarrow{R} \frac{1\times1}{8} \xrightarrow{R} +1$$

$$\frac{17\times17\times17}{8} \xrightarrow{R} \frac{1\times1\times1}{8} \xrightarrow{R} +1$$

Thus, $\frac{17^{5555}}{8} \xrightarrow{R} \frac{1^{5555}}{8} \xrightarrow{R} +1$

So, we can say that there is no use of power.

You will not always get the situation of remainder 1.

For example $\frac{10^{800}}{7}$ in this situation +1 rule has to be used in an oblique way to get the point where you can use +1 remainder rule.

Step 1: $\frac{10^{800}}{7} \xrightarrow{R} \frac{10\times10\times10\times10.......}{7} \xrightarrow{R} \frac{3\times3\times3\times3.......}{7}$

$\frac{10^{800}}{7} \xrightarrow{R} \frac{3^{800}}{7}$

Step 2: Try to find out the power of 3 when divided by 7 gives the remainder 1.

		Remainder
1.	$\frac{3^1}{7}$	3
2.	$\frac{3^2}{7}$	2
3.	$\frac{3^3}{7}$	6
4.	$\frac{3^4}{7}$	4
5.	$\frac{3^5}{7}$	5
6.	$\frac{3^6}{7}$	1

Thus, $\frac{3^{6^{133}}\times3^2}{7}$ [because 800/6, gives quotient 133 and remainder 2]

$\frac{3^{6^{133}}\times3^2}{7} \xrightarrow{R} \frac{1^{133}\times3^2}{7} \xrightarrow{R} \frac{3^2}{7} \xrightarrow{R} +2$

Hence, the remainder for this expression is +2.

b. +1 remainder rule:

1. $\frac{16^1}{17} \xrightarrow{R} -1$
2. $\frac{16^2}{17} \xrightarrow{R} \frac{-1\times-1}{8} \xrightarrow{R} +1$
3. $\frac{16^3}{17} \xrightarrow{R} \frac{-1\times-1\times-1}{8} \xrightarrow{R} -1$
4. $\frac{16^4}{17} \xrightarrow{R} \frac{-1\times-1\times-1\times-1}{8} \xrightarrow{R} +1$

This situation where you have $\frac{a^{power}}{a}$ The remainder in such a situation depends upon the value of power.

a. If power is odd: the remainder would be -1, then the original remainder would be a+1-1 = a

b. If power is even: the remainder would be +1.

Find Power Which Leaves Remainder 1:

$\frac{A^{P-1}}{P}$ always gives remainder 1, if P is a prime number and A should not be multiple of P.

Problem 1:

$\frac{3^P}{7}$ what power of 3 gives remainder 1?

Solution:

$\frac{3^P}{7}$ compare with standard form $\frac{A^{P-1}}{P}$

Thus, P = 7 - 1 = 6.

Hence, 3^6 gives a remainder 1 when divided by 7.

Unit Digits

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3 kinds of cyclicity of unit digits we need to understand.

1. Cyclicity of 1 value in unit digit:

a. Number ending in 1 and raised to any power, the unit digit is always 1.
e.g: $(91)^{12}$ the unit digit will be 1.

b. Number ending in 5,6 and 0 and raised to any power, the unit digit remains the same.
e.g: $(25)^{12}$, the unit digit will be 5.
$(26)^{12}$, the unit digit will be 6.
$(20)^{12}$, the unit digit will be 0.

2. Cyclicity of 2 value in unit digit:

a. Number ending in 4

$54 \rightarrow$ unit digit is 4

$54 \times 54 \rightarrow$ unit digit is 6

$54 \times 54 \times 54 \rightarrow$ unit digit is 4

$(4)^{odd} \rightarrow$ unit digit remains the same.
$(4)^{even} \rightarrow$ unit digit is 6.

b. Number ending in 9

$(9)^{odd} \rightarrow$ unit digit is 1.
$(9)^{even} \rightarrow$ unit digit remains the same.

3. Cyclicity of 4 values in unit digit:

a. Number ending in 2

Number	Unit digit
2^1	2
2^2	4
2^3	8
2^4	6
2^5	2
2^6	4 ……….. And so on

We conclude that 2^{4n+1} gives unit digit 2, 2^{4n+2} gives unit digit 4, 2^{4n+3} gives unit digit 6, 2^{4n+4} gives unit digit 8.
Hence, 2^{power} and power $\div$ 4,then check for 4n+1,4n+2,4n+3 and 4n+1.

b. Number ending in 3

Number	Unit digit
3^1	3
3^2	9
3^3	7
3^4	1
3^5	3
3^6	9 ……….. And so on

We conclude that 3^{4n+1} gives unit digit 3, 3^{4n+2} gives unit digit 9, 3^{4n+3} gives unit digit 7, 3^{4n+4} gives unit digit 1.
Hence, 3^{power} and power $\div$ 4,then check for 4n+1,4n+2,4n+3 and 4n+1.

c. Number ending in 7

Number	Unit digit
7^1	7
7^2	9
7^3	3
7^4	1
7^5	7
7^6	9 ……….. And so on

We conclude that 7^{4n+1} gives unit digit 7, 7^{4n+2} gives unit digit 9, 7^{4n+3} gives unit digit 3, 7^{4n+4} gives unit digit 1.
Hence, 7^{power} and power $\div$ 4,then check for 4n+1,4n+2,4n+3 and 4n+1.

d. Number ending in 8

Number	Unit digit
8^1	8
8^2	4
8^3	2
8^4	6
8^5	8
8^6	4 ……….. And so on

We conclude that 8^{4n+1} gives unit digit 8, 8^{4n+2} gives unit digit 4, 8^{4n+3} gives unit digit 2, 8^{4n+4} gives unit digit 6.
Hence, 8^{power} and power $\div$ 4,then check for 4n+1,4n+2,4n+3 and 4n+1.

Summary: Unit digit

	If the value of the power is			
Number ending in	**4n + 1**	**4n + 2**	**4n + 3**	**4n**
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1

8	8	4	2	6
9	9	1	9	1

Some Question For Practice

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1. Find the value of the expression:
1 – 6 + 2 – 7 + 3 – 8 +……… to 100 terms
Ans: -250.

2. Find a_{10} and S_{10} for the following series:
1, 8, 15,22,29…
Ans : 64,325.

3. Find the sum to 200 terms of the series
1 + 4 + 6 + 5 + 11 + 6 +…
Ans : 30200.

4. How many terms are there in GP 5, 20, 80, 320,… 20480?
Ans: 7.

5. If the fifth term of a GP is 81 and the first term is 16, what will be the 4th term of the GP?
Ans: 54.

6. Find the remainder when 73 × 75 × 78 × 57 × 197 is divided by 34.
Ans: 22.

7. Find the remainder when 4177 is divided by 7
Ans: 6.

8. Find the remainder when 73 + 75 + 78 + 57 + 197 is divided by 34.
Ans: 4.

9. Find the Units digit in 67 × 37 × 43 × 91 × 42 × 33 × 42.
Ans: 4.

10. Find the Units digit in 67 × 35 × 45 × 91 × 42 × 33 × 81.
Ans : 0.