Propositional Logic

and incorporating it into programming

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Outline

Acknowledgement and disclaimer

All mistakes (if any) are mine.
I have used several other sources which I have referred to in the appropriate places.

Section 1

Galileo and some background about him

Galileo (and his curiosity)

Galileo Galilei (15 February 1564 – 8 January 1642), was an astronomer, physicist and engineer, sometimes described as a polymath, from Pisa, in modern-day Italy. '

Galileo has been called the father of observational astronomy, modern physics, scientific method, and modern science.

Galileo was deliberately kept away from mathematics *since a physician earned a higher income than a mathematician.* However, after accidentally attending a lecture on geometry, he talked his reluctant father into letting him study mathematics instead of medicine. [1]

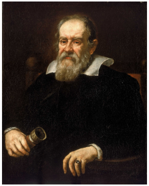

Figure: 1636 portrait by Justus Sustermans

[1] Source: Wikipedia

Some Trivia

- Although Galileo seriously considered the priesthood as a young man, at his father's urging he instead enrolled in 1580 at the University of Pisa for a medical degree. [2]
- In 1581, when he was studying medicine, he noticed a swinging chandelier, which air currents shifted about to swing in larger and smaller arcs.
- To him, it seemed, by comparison with his heartbeat, that the chandelier took the same amount of time to swing back and forth, no matter how far it was swinging. Animation for this
- When he returned home, he set up two pendulums of equal length and swung one with a large sweep and the other with a small sweep and found that they kept time together.

[2] Reston, J. (2000). Galileo: A Life. Beard Books. Gilbert, N. W. (1963). "Galileo and the School of Padua". Journal of the History of Philosophy.

Section 2

Proving statements via logic

Symbolic Logic

- Logic is the study of consequence.
- Given a few mathematical statements or facts or propositions, we would like to be able to draw some conclusions.
- A proposition is simply a declarative statement/sentence that is either true or false, but not both. Examples:
  - Mangoes are tasty.
  - The evening is beautiful.
  - What a beautiful evening! No.
  - Do your homework. No.
- Propositional logic does not care about the content of the statements. Therefore the claims, "if you give me four mangoes then pigs will fly" and "if an elder person is telling me something then it is necessarily true" are the same.

Guess?

My poor attempt at capturing the movement of celestial bodies across the sky at night (Chail, September, 2015).

The geocentric theory and Aristotle

- The ancient Greek philosophers, whose ideas shaped the worldview of Western Civilization leading up to the Scientific Revolution in the sixteenth century, had conflicting theories about why objects moved across the sky.
- One camp thought that the planets orbited around the Sun, but Aristotle, whose ideas prevailed, believed that the planets and the Sun orbited Earth.
- Aristotle saw no sign that the Earth was in motion:
  - No perpetual wind blew over the surface of the Earth; and
  - A ball thrown straight up into the air doesn't land behind the thrower

  as Aristotle *assumed* it would, if the Earth were moving.
- For Aristotle, this meant that the Earth had to be stationary, and the planets, the Sun, and the fixed dome of stars rotated around Earth.[3]

[3] https://earthobservatory.nasa.gov/features/OrbitsHistory

Section 3

Arguments

Making cogent arguments

- An argument is a set of propositions, one of which is called the conclusion and the rest of which are called premises.
- An argument is said to be valid if the conclusion must be true whenever the premises are all true.

Let's examine Aristotle's argument

Aristotle's reasoning for the Earth to be stationary, and the planets, the Sun, and the fixed dome of stars rotated around the Earth:

- Argument: If no perpetual wind blew over the surface of the Earth, then the Earth is stationery.
- Premise: No perpetual wind blew over the surface of the Earth.
- Conclusion: The Earth is stationery.

or

- Argument: If a ball thrown straight up into the air doesn't land behind the thrower, then the Earth is stationery.
- Premise: A ball thrown straight up into the air doesn't land behind the thrower.
- Conclusion: The Earth is stationery.

Section 4

Propositional Logic

Formalizing things ...

Did you see the pattern? It's okay if you didn't.

- Argument: $A \rightarrow B$, if A, then B
- Premise: A
- Conclusion: B

Compound Proposition

A proposition that can be broken down into primitive (also known as atomic) propositions.

Examples:

A: ‘The mangoes are cold’

B: ‘I eat mangoes’

- Example 1: Connectives “if ... then”
  If the mangoes are cold, then I eat mangoes.
- Example 2: Connectives “and”
  The mangoes are cold and I eat mangoes.
- Example 3: Connectives “if & only if”
  I eat mangoes if and only if the mangoes are cold.

Take a couple of minutes to read this.

In English	Math term	Symbol
"and"	conjuction	$\wedge$
"or"	disjunction	$\vee$
"not"	negation	$\neg$
"or ... but not both"	XOR	$\oplus$
"if ... then"	implication	$\rightarrow$
"if & only if", "iff"	equivalence	$\Leftrightarrow$

Truth Tables

Table: truth table

A	B	$A \wedge B$	$A \vee B$	$A \oplus B$	$A \rightarrow B$	$A \Leftrightarrow B$
1	1	1	1	0	1	1
1	0	0	1	1	0	0
0	1	0	1	1	1	0
0	0	0	0	0	1	1

The implication $(A \rightarrow B)$ and the equivalence $(A \Leftrightarrow B)$ need to be looked into.

Implication: if ... then statement

$A \rightarrow B$ or A implies B

If A is True, then B has to be True.

But not vice versa, i.e. if B is True, A can be False. So ...

The statement 'If the mangoes are cold, then I eat mangoes' means that:

- If you get me cold mangoes, I will definitely eat them.
- However, if you see me eating mangoes (i.e. B is True), that does not mean that the mangoes are cold (i.e. A is True). The mangoes could be warm as well.

Equivalence: if & only if statement

$A \Leftrightarrow B$ or A is logically equivalent to B

A is True if and only if B is True.

Also vice versa, i.e. B is True if and only if A is True. So ...

The statement 'I eat mangoes if and only if the mangoes are cold.' means that:

- If you see me eating mangoes (i.e. B is True), they are definitely cold (i.e. A is True).
- And, if you get me cold mangoes (i.e. A is True), I will definitely eat them (i.e. B is True).

Let's complete Aristotle's logic ...

Aristotle's Geocentric Theory (simplified): The Earth is stationery and the celestial bodies rotate around the Earth.

A: The Earth is stationery.
B: The celestial bodies rotate around the Earth.
Connective: 'and'

A was proved earlier. Let's look at B.

- Argument: If the Earth is stationery and objects moved across the sky then the celestial bodies rotate around Earth.
- Premise (from earlier): The Earth is stationery and objects moved across the sky.
- Final Conclusion: The celestial bodies rotate around Earth.

Q1

Given the Argument: If the Earth is stationery and objects moved across the sky then the celestial bodies rotate around Earth.

Can you break the Premise into into the primitive propositions?

- A: The Earth is stationery.
- B: Objects moved across the sky.
- C: The celestial bodies rotate around Earth.
- What is the relation between the primitive propositions?
- $(A \wedge B) \to C$

Copernicus and the Heliocentric theory

For nearly 1,000 years, Aristotle's view of a stationary Earth at the center of a revolving universe dominated the studies of the physical world. A geocentric worldview became ingrained in Christian theology, making it a doctrine of religion as much as natural philosophy.

In 1515, a Polish priest named Nicolaus Copernicus proposed that the Earth was a planet like Venus or Saturn, and that all planets circled the Sun. The theory gathered few followers, and for a time, some of those who did give credence to the idea faced charges of heresy. Italian scientist Giordano Bruno was burned at the stake for teaching, among other heretical ideas, Copernicus' heliocentric view of the Universe.

When Galileo pointed his telescope into the night sky in 1610, he saw for the first time in human history that moons orbited Jupiter. So what did this mean?

- Conclusion (from Aristotle): The Earth is stationery and the celestial bodies rotated around Earth.
- Argument: If Jupiter's moon is a celestial object, then Jupiter's moon should rotate around the Earth.
- Premise: Jupiter's moon is a celestial object.
- Conclusion (according to Aristotle's theory): Jupiter's moon should rotate around the Earth.
- But physical proof suggested otherwise. So one of the premises was incorrect? Which one? Bam!

While Galileo did not share Bruno's fate, he was tried for heresy under the Roman Inquisition and placed under house arrest for life.

Section 5

Control flow in Python

Translating propositional logic for control flow in Python

Programs are written to solve real-world problems. A collection of simple statements is not sufficient to do this.
We, therefore, need to introduce programming constructs that can replicate real-world elements. We will discuss some of these today.

- Branching statements;
- Loops; and
- Loop control statements (break and continue).

Branching statements: if-else block

```
if Boolean expression:
    Block of code
else:
    Block of code
```

If-Else Example

```
x = 10
if x%2 == 0:
    print ('Even')
else:
    print ('Odd')
print('After conditional') #
```

Remember that == is used for comparison, since = is reserved for assignment.

Indentation is semantically meaningful in Python.

Branching statements: if-elif-else block

```
if Boolean expression:
    Block of code
elif Boolean expression:
    Block of code
.
.
.

elif Boolean expression:
    Block of code
else:
    Block of code
```

Example: if-elif-else

```
x = 6
if x%2 == 0:
    print ('Divisible by 2')
elif x%3 == 0:
    print ('Divisible by 3')
else:
    print ('Not divisible by 3 or 2')
```

There is a problem with the logic in the above code. Can you guess what it is?

Branching statements: Nested blocks

When either the true block or the false block of a branching statement contains another branching statement, the branching statements are said to be nested.

```
if x%2 == 0:
    if x%3 == 0:
        print ('Divisible by 2 and 3')
    else:
        print ('Divisible by 2 and not by 3')
elif x%3 == 0:
    print ('Divisible by 3 and not by 2')
```

Loops

- Let us say that you want to print numbers from 0 to 100 on the terminal. How do you do it?
- One way is putting down 100 print statements. But this repetitive and boring.
- Fortunately, most programming languages have loops that handle repetitive computations.

While loop

```
while Boolean expression:
    block of code
```

- A generic iteration (also called looping) mechanism begins with a `test`.
- If the test evaluates to True, the program executes the loop body once, and then goes back to reevaluate the `test`.
- This process is repeated until the `test` evaluates to False, after which control passes to the code following the iteration statement.

Example: While loop

```
x = 3
ans = 0
itersLeft = x
while (itersLeft != 0):
    ans = ans + x
    itersLeft = itersLeft - 1
```

Doing dry runs of programs

In the beginning it helps to hand-simulate the code, i.e., execute the program using pencil and paper. It is an excellent way to understand how a program behaves.

```
x = 3
ans = 0
itersLeft = x
while (itersLeft != 0):
    ans = ans + x
    itersLeft = itersLeft - 1
```

Table: truth table

step #	x	ans	itersLeft
1	3	0	3
2	3	3	2
3	3	6	1
4	3	9	0

Iteration: for loop

```
for variable in sequence:
    code block
```

The variable following for is bound to the first value in the sequence, and the code block is executed.
The variable is then assigned the second value in the sequence, and the code block is executed again.

Using range

The sequence of values bound to variable is most commonly generated using the built-in function range, which returns a sequence containing an arithmetic progression. The range function takes three integer arguments: start, stop, and step.

```python
#range(start, stop, step)
for i in range(0, 10, 2):
    print(i)
    #0, 2, 4, 6, 8
for i in range(1, 5):
    print(i)
    #1, 2, 3, 4
for i in range(5):
    print(i)
    #0, 1, 2, 3, 4
```

Control flow: break statement

What do to if we want to end the iterations early, we use the break statement. The control passes on to the next statement after the for loop.

```
for val in "string":
    if val == "i":
        break
    print(val)

print("The end")
```

Control flow: continue statement

The continue statement is used to skip the rest of the code inside a loop for the current iteration only. Loop does not terminate but continues on with the next iteration.

```python
for val in "string":
    if val == "i":
        continue
    print(val)

print("The end")
```

What did we learn today?

1. Galileo and some background about him
2. Proving statements via logic
3. Arguments
4. Propositional Logic
5. Control flow in Python

Thank you!