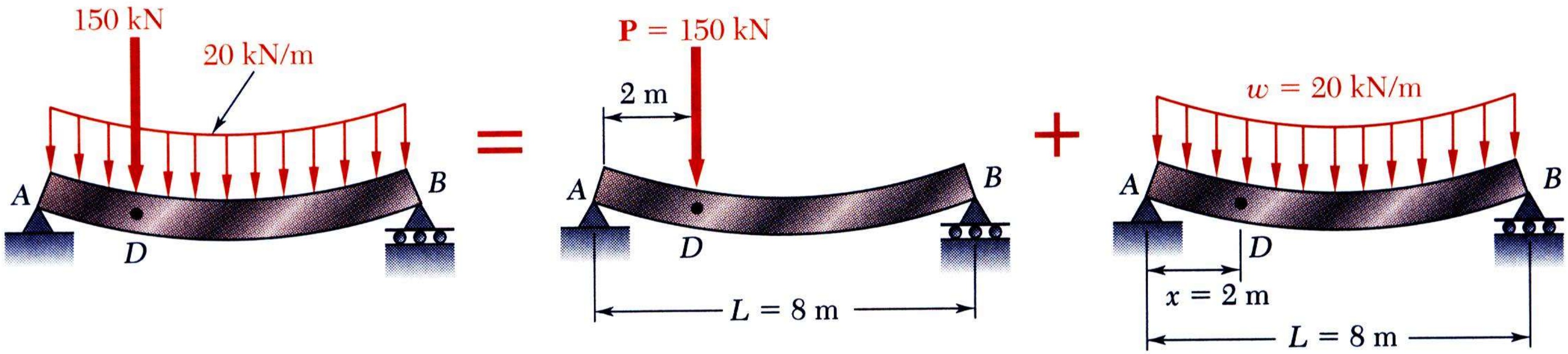
Method of Superposition

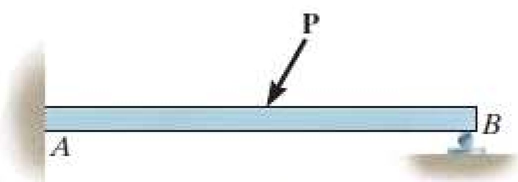
- Deformations of beams subjected to combinations of loadings may be obtained as the linear combination of the deformations from the individual loadings



- Individual problems can be solved by either double integration or moment area method

Statically Indeterminate Beams and Shafts

- if the number of unknown reactions exceeds the available number of equilibrium equations
- We have already discussed methods to obtain support reactions for statically indeterminate shaft and beams
- The additional support reactions on the beam or shaft that are not needed to keep it in stable equilibrium are called redundants
- The number of these redundants is referred to as the degree of indeterminacy



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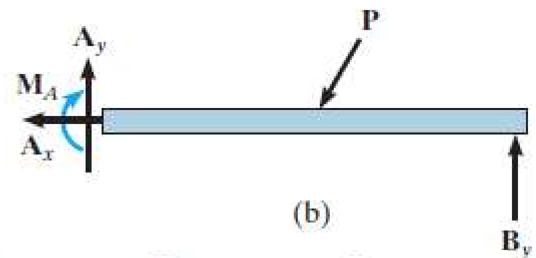

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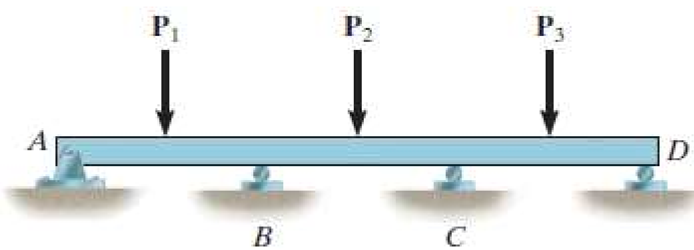
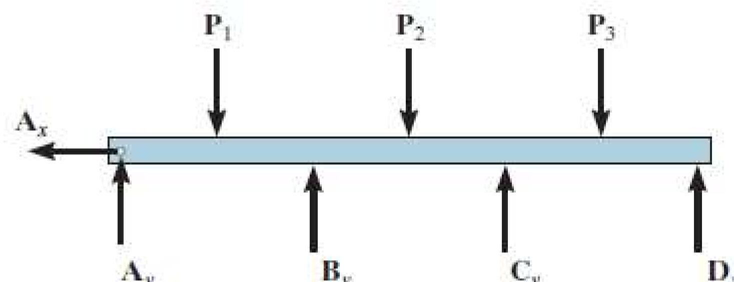




(b)

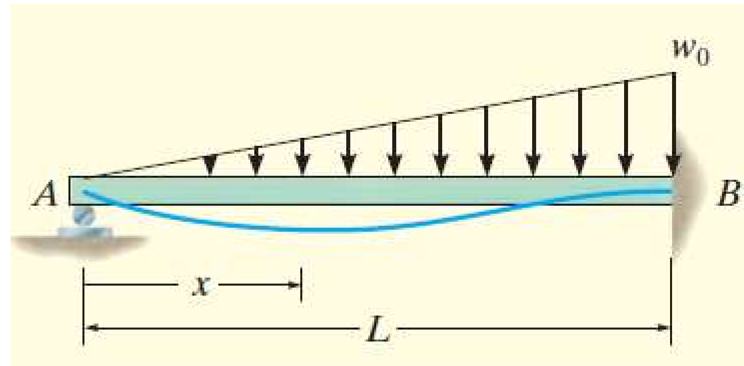
Statically Indeterminate Beams and Shafts

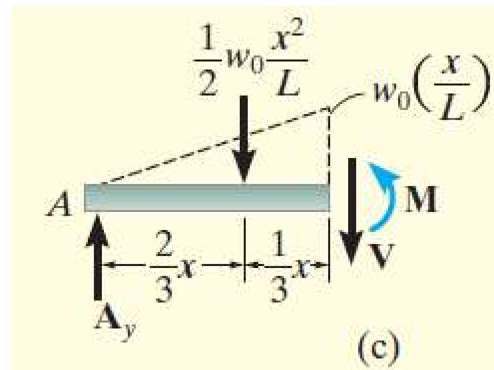
- Double integration method can be used to solve once the internal moment M in the beam is expressed as a function of position x
- There will be two constants of integration along with the redundants to be determined
- These unknowns can always be found from the boundary and/or continuity conditions for the problem

An example of a statically indeterminate beam used to support a bridge deck.

Example

- Determine the reaction at A





Solution:

Moment function:-

$$M = A_y x - \frac{1}{6} w_0 \frac{x^3}{L}$$

$$EI\frac{d^2v}{dx^2} = A_y x - \frac{1}{6} w_0 \frac{x^3}{L}$$

$$EI\frac{dv}{dx} = \frac{1}{2}A_y x^2 - \frac{1}{24} w_0 \frac{x^4}{L} + C_1$$

$$EIv = \frac{1}{6}A_y x^3 - \frac{1}{120} w_0 \frac{x^5}{L} + C_1 x + C_2$$

Example

- Boundary conditions to determine A_Y, C_1 and C_2

$$x = 0, \quad v = 0; \quad x = L, \quad dv/dx = 0; \quad \text{and} \quad x = L, \quad v = 0.$$

$$A_y = \frac{1}{10} w_0 L$$

$$C_1 = -\frac{1}{120} w_0 L^3 \quad C_2 = 0$$

Using A_Y, reactions at B can be obtained using equilibrium equations.

$$B_x = 0, \qquad M_B = w_0 L^2/15.$$

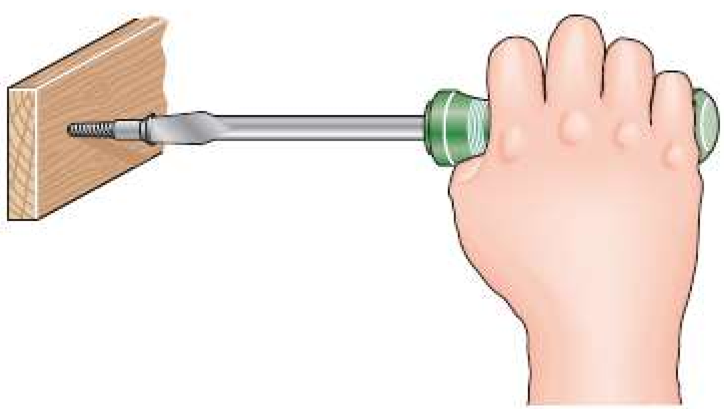
$$B_y = 2w_0 L/5,$$

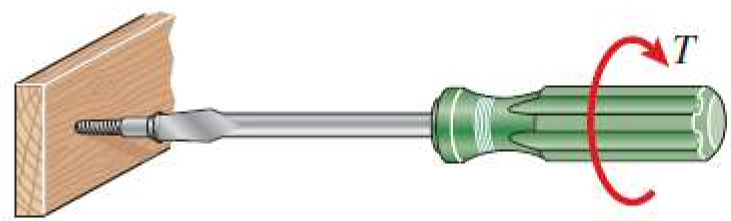
Assignment

- Chapter 12
- All solved example
- F12.6, 12.3,12.12,12.27,12.36,12.41,12.43,12.48
- F12.10,12.57,12.59,12.64,12.74,12.91,12.99,12.107,12.114,12.132

Torsion

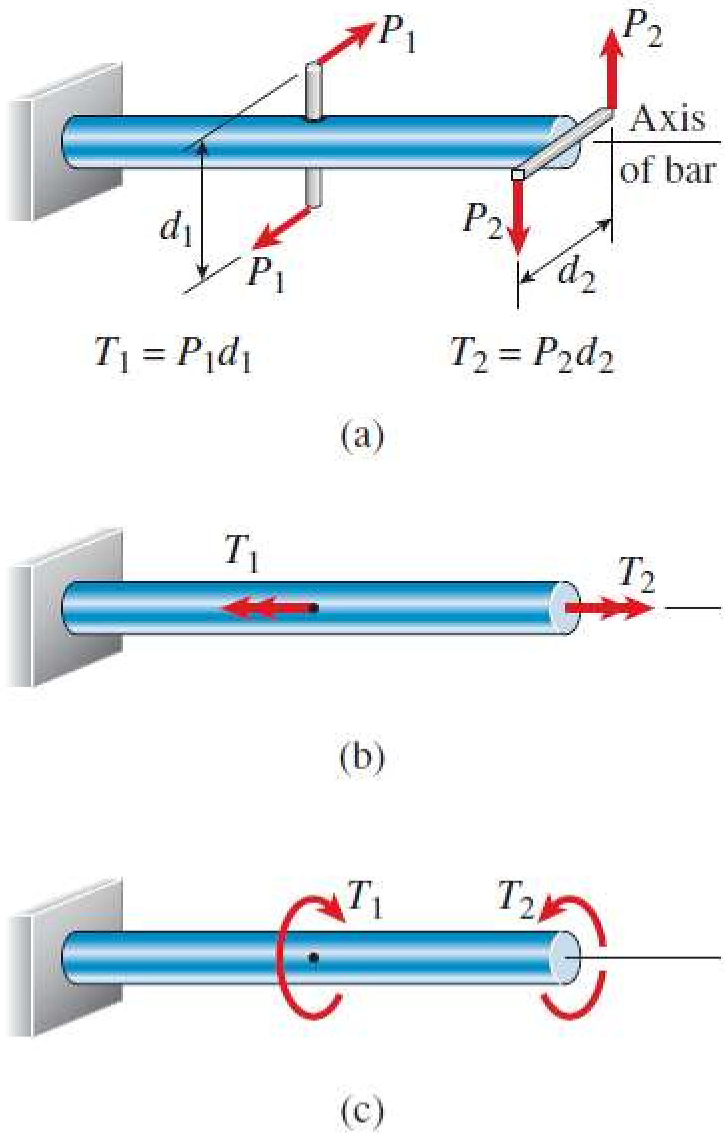
- Torsion refers to the twisting of a straight bar when it is loaded by moments (or torques) that tend to produce rotation about the longitudinal axis of the bar
- Torque is a moment that tends to twist a member about its longitudinal axis





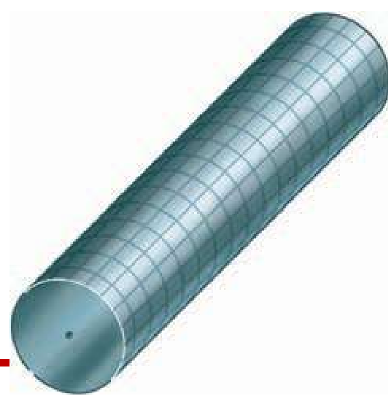
Torsion

- Different modes

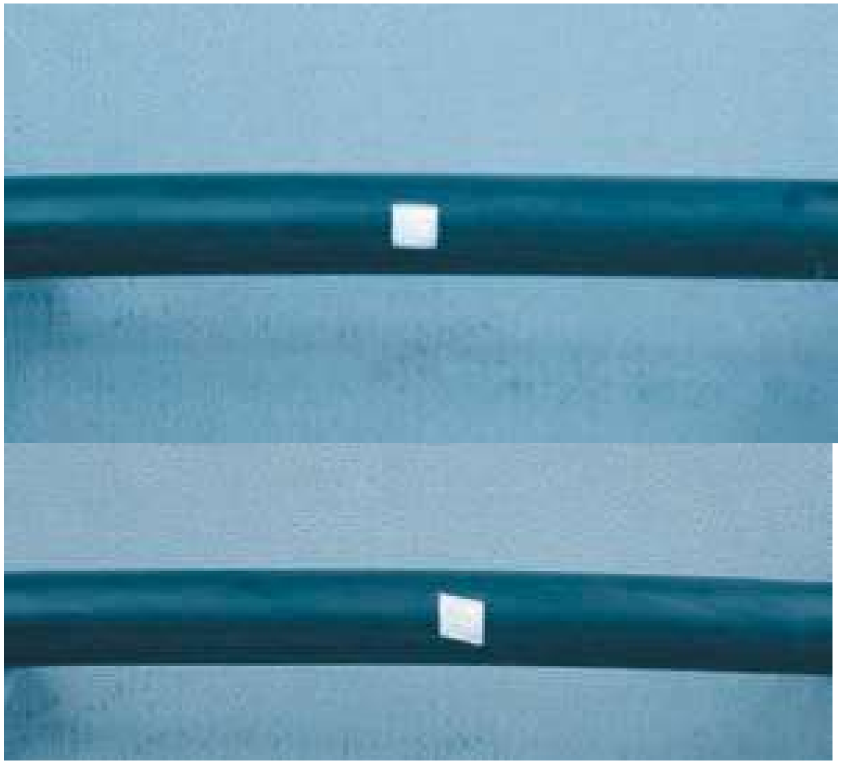


Torsion

- What exactly happens??
  - Circles remain circles
  - longitudinal grid line deforms into a helix that intersects the circles at equal angles
  - Cross sections from the ends along the shaft will remain flat, they do not warp or bulge in or out
  - Radial lines remain straight during the deformation
  - we can assume that if the angle of twist is small, the length of the shaft and its radius will remain unchanged



Torsion



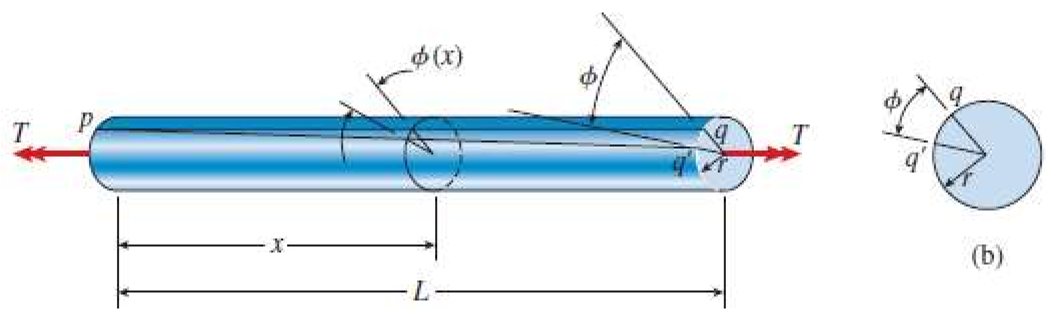
- Length of lines remain same.
- The angle between the lines changes – only shear and no normal strains.

Torsional deformation of circular bar

- Consider a prismatic bar of circular cross section twisted by torques T acting at the ends
- Every cross section of the bar is identical and subjected to internal torque T, we say that the bar is in pure torsion
- Angle of rotation is small so no change in length or radius
- Cross-sections remain plane and circular and radii straight
- Left end of the bar is fixed so right end will rotate with respect to left on the application of torque

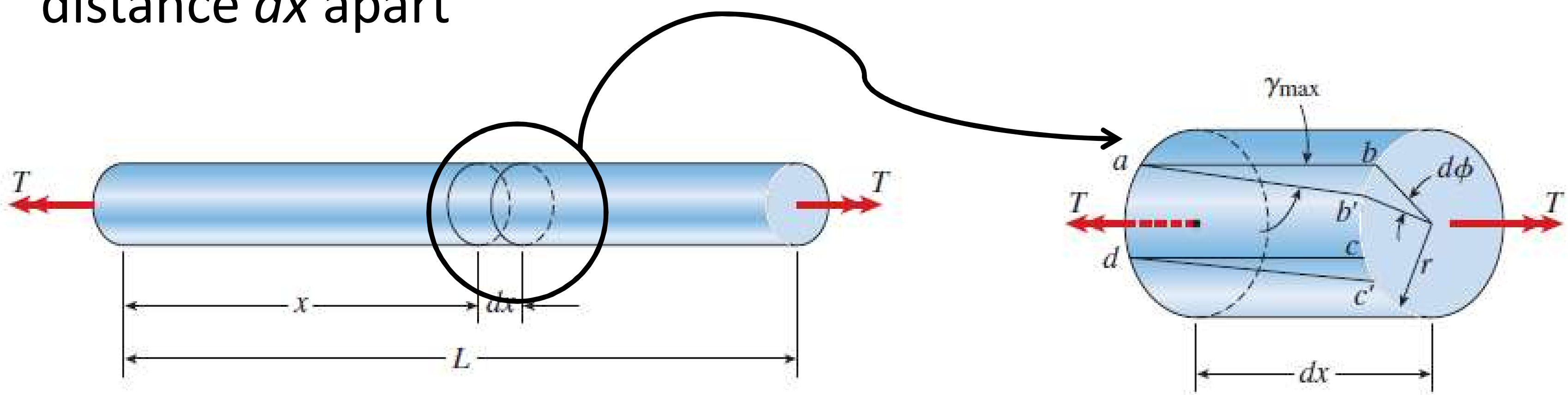
Angle of rotation or twist – ϕ

Angle of twist will change with the length of the bar.



Torsional deformation of circular bar

- Consider an element of the bar between two cross sections distance *dx* apart



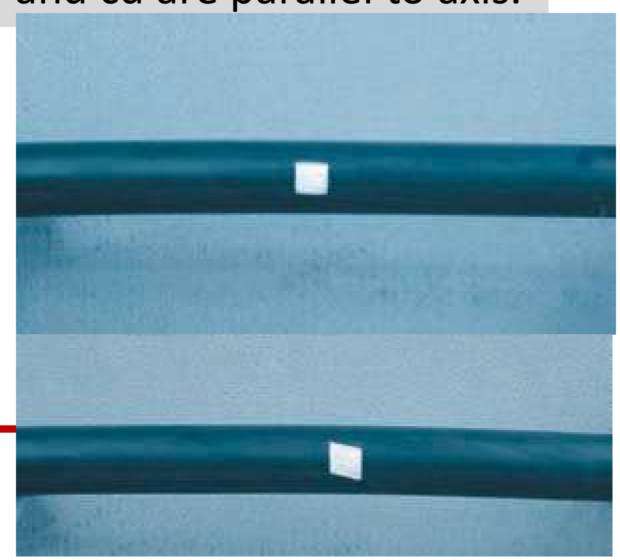
Sides ab and cd are parallel to axis.

abcd changes to ab’c’d after twisting by small angle $d\phi$.

Angle between corners do not remain 90º.

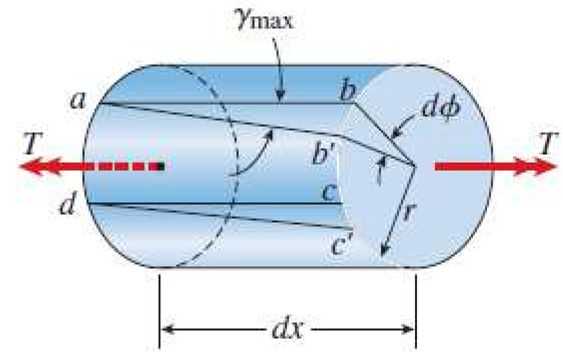
What does it indicate???

Only shear strains and no normal strains.



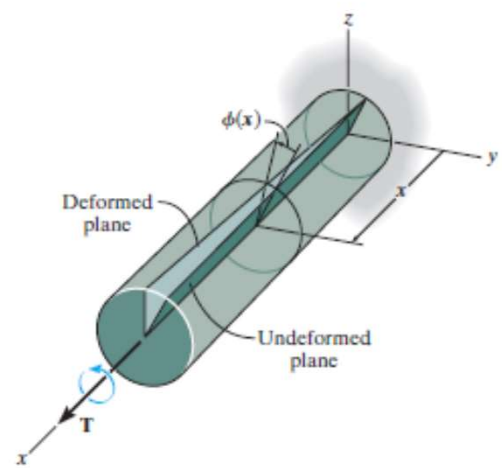
Torsional deformation of circular bar

- Shear strain at outer surface
  - Sides *ab* and *cd* are parallel to axis
  - *abcd* changes to *ab′c′d* after twisting
  - Right side cross section rotates by angle $d\phi$
  - Length of the element do not change
  - Let us denote shear strain at outer surface γ_{max}
  - How to calculate it??



$$\gamma_{max} = \frac{bb'}{ab}$$

$$\gamma_{max} = \frac{r\,d\phi}{dx}$$



The angle of twist $\phi(x)$ increases as x increases.

Rate of twist or angle of twist per unit length

$$\theta = \frac{d\phi}{dx}$$

Rate of change of angle of twist with respect to length.

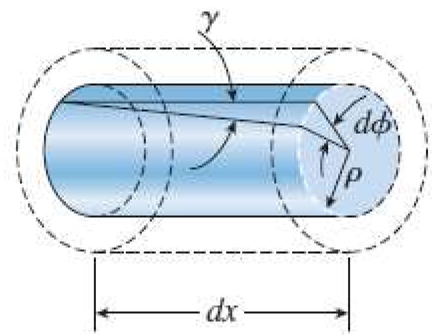
Shear strain

- We can re-write relation for shear strain

$$\gamma_{\max} = \frac{r\,d\phi}{dx} = r\theta$$

For special case of pure torsion → $\theta = \phi/L$

$$\gamma_{\max} = r\theta = \frac{r\phi}{L}$$



- How about shear strains with in the bar
  - Using similar geometry and noting that radii remain straight and undistorted

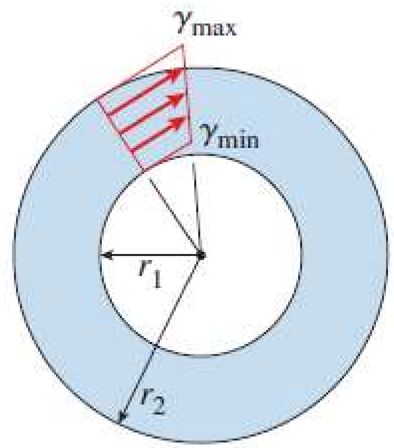
$$\gamma = \rho\theta = \frac{\rho}{r}\gamma_{\max}$$

Shear strain

- For circular tubes

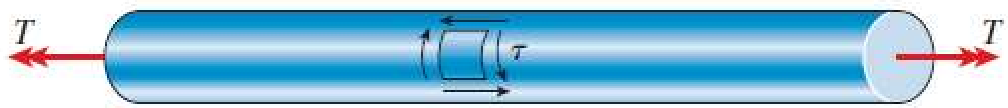
$$\gamma_{max} = \frac{r_2 \phi}{L}$$

$$\gamma_{min} = \frac{r_1}{r_2}\gamma_{max} = \frac{r_1 \phi}{L}$$



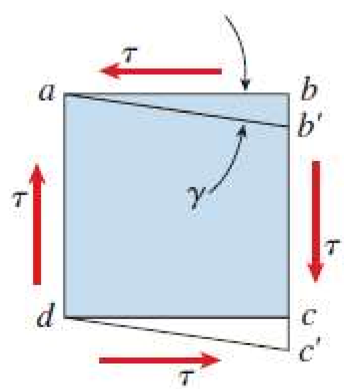
Bars of linear elastic material

- Need to determine the corresponding stresses
- What kind of stresses will be induced??



(a)

How about distribution of stress??



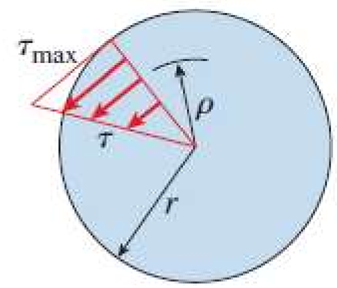
(b)

Bars of linear elastic material

- Using Hooke's law in shear

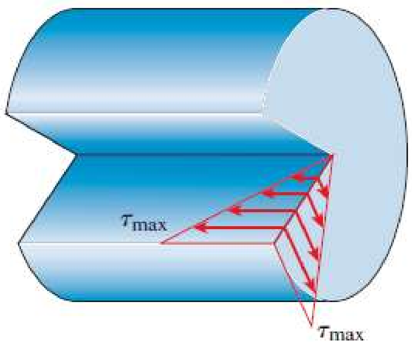
$$\tau = G\gamma$$

$$\boxed{\tau_{max} = Gr\theta} \qquad \boxed{\tau = G\rho\theta = \frac{\rho}{r}\tau_{max}}$$



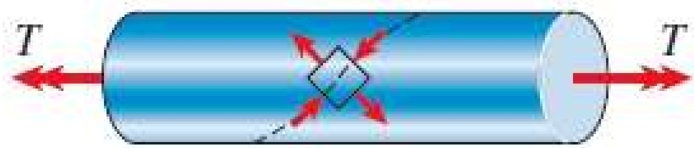
(c)

- Existence of cross shears – Shear stresses acting on cross-sectional plane are accompanied by equal shear stresses along longitudinal planes
  - Failure of wooden members along the length under torque



Bars of linear elastic material

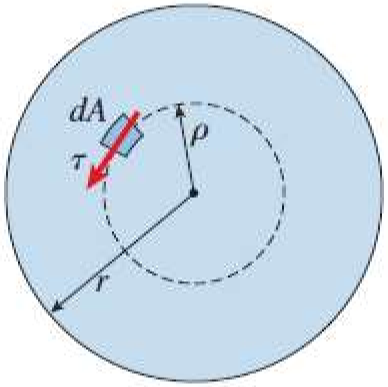
- Stresses on a plane oriented with respect to the axis of shaft



- How stresses are related to torque *T*?
  - The stresses act continuously around the cross section
  - They have a resultant in the form of a moment
  - A moment equal to torque T acting on the shaft

Bars of linear elastic material

- The torsion formula
  - Consider an element with area dA located at radial distance ρ



Shear force acting on this element = $\tau\, dA$

Moment of this force about center = $\tau \rho dA$

$$dM = \tau \rho dA = \frac{\tau_{\max}}{r} \rho^2 dA$$

Total torque

$$T = \int_A dM = \frac{\tau_{\max}}{r} \int_A \rho^2 dA = \frac{\tau_{\max}}{r} I_P$$

$$I_P = \int_A \rho^2 dA$$

Polar moment of inertia.

$$I_P = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

Bars of linear elastic material

- On re-arranging terms, we can write

$$\tau_{\max} = \frac{Tr}{I_P}$$

Torsion formula.

For a circular cross-section:

$$\tau_{\max} = \frac{16T}{\pi d^3}$$

Stress at any cross-section:

$$\tau = \frac{\rho}{r}\,\tau_{\max} = \frac{T\rho}{I_P}$$

Generalized torsion formula.

Bars of linear elastic material

- Angle of twist

$$\tau = G\rho\theta$$

$$\tau = \frac{\rho}{r}\tau_{\max} = \frac{T\rho}{I_P}$$

$$\theta = \frac{T}{GI_P}$$

Rate of twist is directly proportional to torque *T* and torsional rigidity GI_P.

- For a bar in pure torsion, angle of twist $\phi = \theta L$

$$\phi = \frac{TL}{GI_P}$$

GI_P/L → Torsional stiffness, torque required to produce unit angle of twist.

Bars of linear elastic material

- We can define torsional stiffness and torsional flexibility as

$$k_T = \frac{GI_P}{L} \qquad f_T = \frac{L}{GI_P}$$

- Which are similar to axial stiffness and axial flexibility

$$k = EA/L \qquad f = L/EA$$

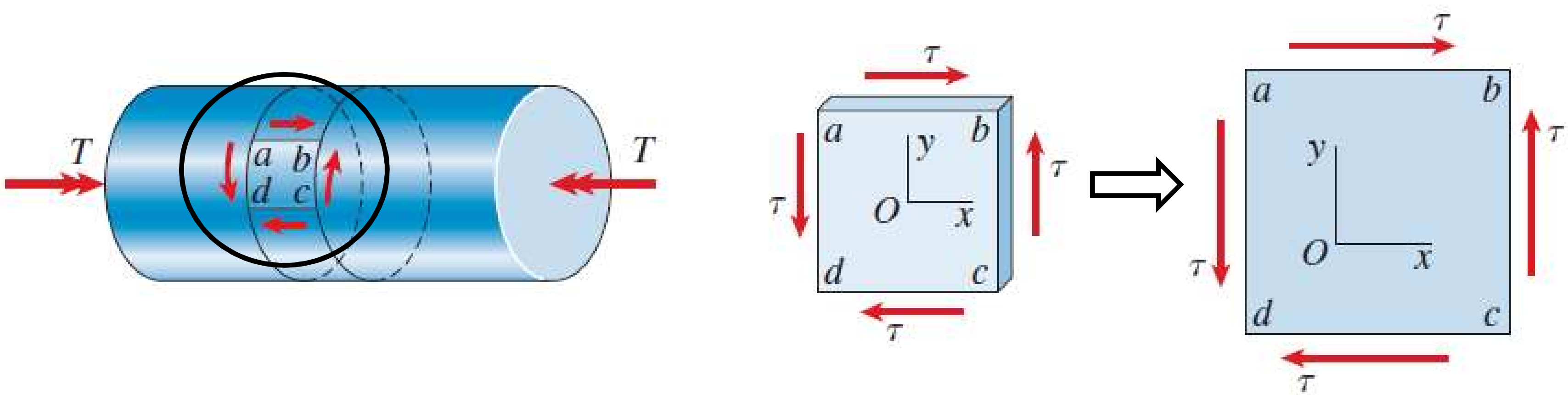
- For circular tubes

$$I_P = \frac{\pi}{2}(r_2^4 - r_1^4) = \frac{\pi}{32}(d_2^4 - d_1^4) \qquad (r_1 + r_2)/2 = r \qquad (d_1 + d_2)/2 = d$$

$$I_P = \frac{\pi r t}{2}(4r^2 + t^2) = \frac{\pi d t}{4}(d^2 + t^2) \qquad r_2 - r_1 = t$$

Stresses and strain in pure shear

- Detailed analysis of stresses and strains produced during twisting of a bar
- Direction of shear stresses will depend on the direction of *T*
- Let *T* rotate the right hand end of bar clockwise when viewed from right

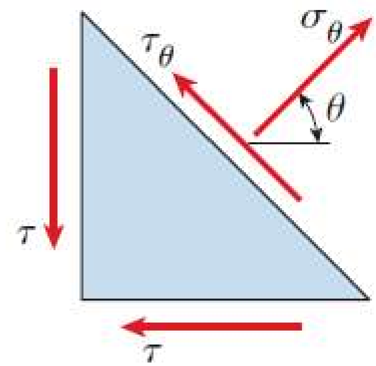


2D view.

We want to calculate stresses on inclined plane.

Stresses and strain in pure shear

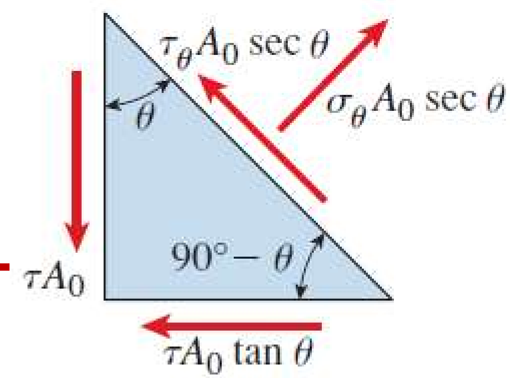
- Lets cut an element having one face oriented at an angle θ to the x-axis.
- The normal and shear stresses will act on the element
- How to determine τ_θ and σ_θ??



Using equilibrium of triangular element!!

To get FBD of triangular element, we need to determine forces.

To get forces we need to find the area of each side. Let us say the area of vertical face is A_0.



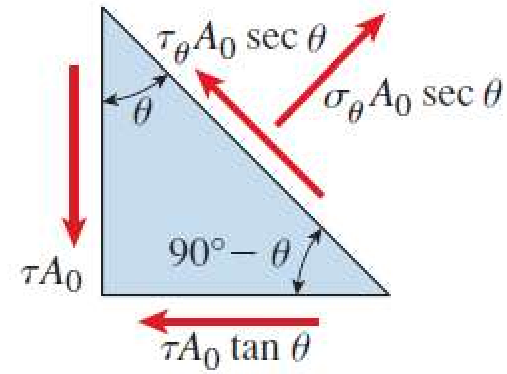
How to get normal and shear stresses???

Stresses and strain in pure shear

- Summing up the forces along normal and shear stress direction, we get

$$\sigma_\theta A_0 \sec\theta = \tau A_0 \sin\theta + \tau A_0 \tan\theta \cos\theta$$

$$\tau_\theta A_0 \sec\theta = \tau A_0 \cos\theta - \tau A_0 \tan\theta \sin\theta$$



$$\boxed{\begin{aligned} \sigma_\theta &= 2\tau \sin\theta \cos\theta \\ \tau_\theta &= \tau(\cos^2\theta - \sin^2\theta) \end{aligned}}$$

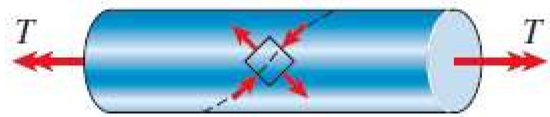
$$\sigma_\theta = \tau \sin 2\theta$$

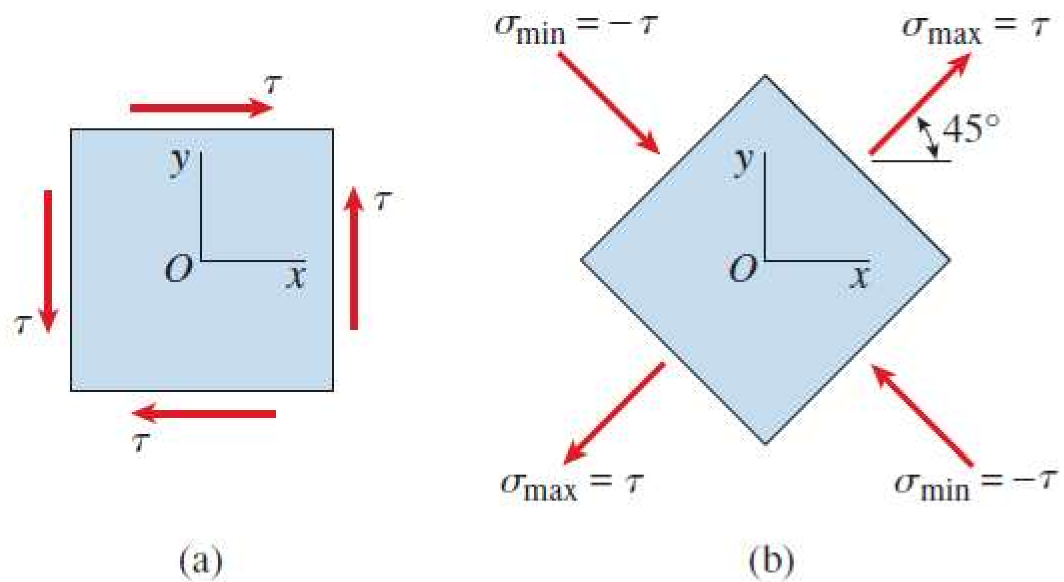
$$\tau_\theta = \tau \cos 2\theta$$

At θ = 0 and ±45°

Stresses and strain in pure shear

- What happens at θ = ±45°

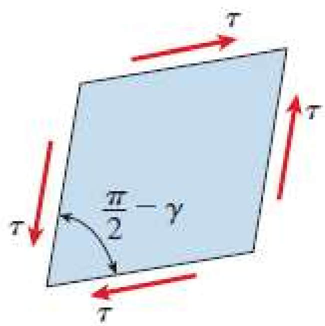




You want to try with chalk?

Stresses and strain in pure shear

- How about strains??
- Distortion of element oriented at 0 and 45^{o} to x-axis?



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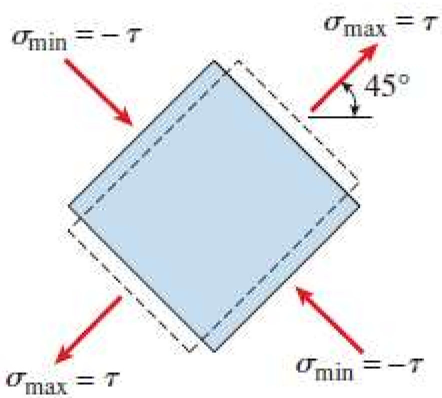


(b)

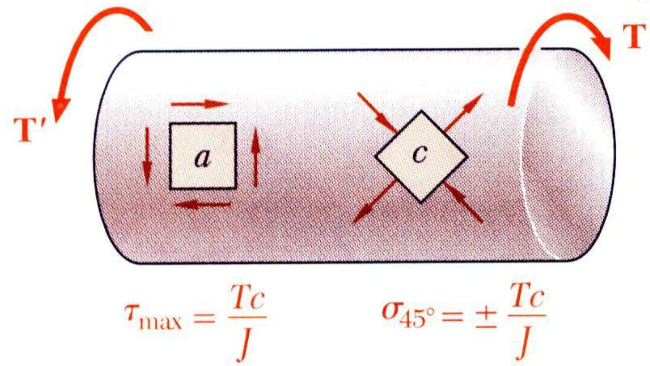
$$\gamma = \frac{\tau}{G}$$

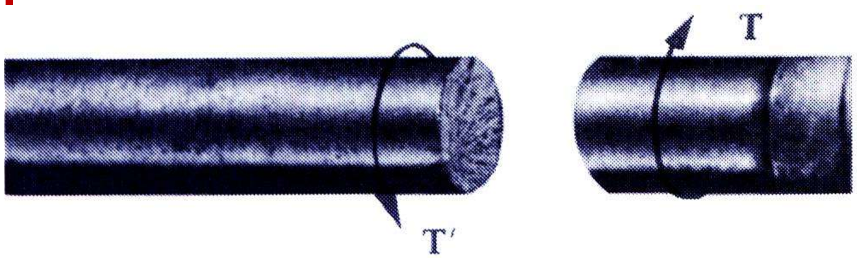
Using generalized Hooke's law, we get

$$\epsilon_{max} = \frac{\tau}{E} + \frac{\nu\tau}{E} = \frac{\tau}{E}(1 + \nu)$$

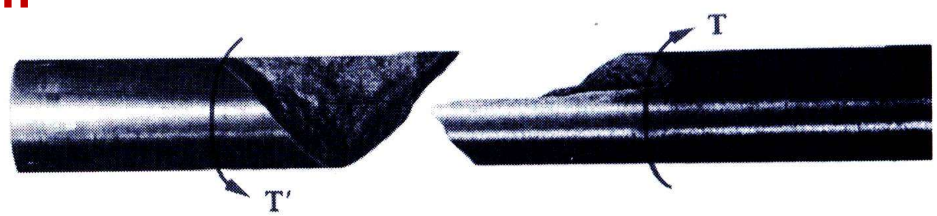
Torsional Failure Modes

- Ductile materials generally fail in shear. Brittle materials are weaker in tension than shear.





- When subjected to torsion, a ductile specimen breaks along a plane of maximum shear, i.e., a plane perpendicular to the shaft axis.



- When subjected to torsion, a brittle specimen breaks along planes perpendicular to the direction in which tension is a maximum, i.e., along surfaces at 45° to the shaft axis.

Bars of linear elastic material

- Validity of relation derived
  - Circular cross section
  - Material must remain in elastic region

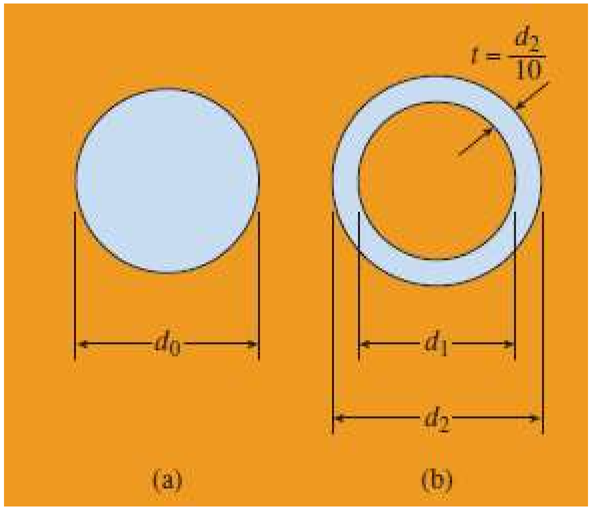
Example

- A steel shaft is to be manufactured either as a solid circular bar or as a circular tube. The shaft is required to transmit a torque of 1200 Nm without exceeding an allowable shear stress of 40 MPa or an allowable rate of twist of 0.75°/m. (The shear modulus of elasticity of the steel is 78 GPa.)

(a) Determine the required diameter d_0 of the solid shaft.

(b) Determine the required outer diameter d_2 of the hollow shaft if the thickness t of the shaft is specified as one-tenth of the outer diameter.

(c) Determine the ratio of diameters (that is, the ratio d_2/d_0) and the ratio of weights of the hollow and solid shafts.



Example

- For solid shaft
  - Allowable stress

$$d_0^3 = \frac{16T}{\pi \tau_{\text{allow}}} = 152.8 \times 10^{-6}\ \text{m}^3$$

$$d_0 = 0.0535\ \text{m} = 53.5\ \text{mm}$$

  - Allowable rate of twist

$$I_P = \frac{T}{G\theta_{\text{allow}}} = 1175 \times 10^{-9}\ \text{m}^4$$

$$d_0^4 = \frac{32 I_P}{\pi} = 11.97 \times 10^{-6}\ \text{m}^4$$

$$d_0 = 0.0588\ \text{m} = 58.8\ \text{mm}$$

- For hollow shaft $d_2 = 67.1\ \text{mm}$

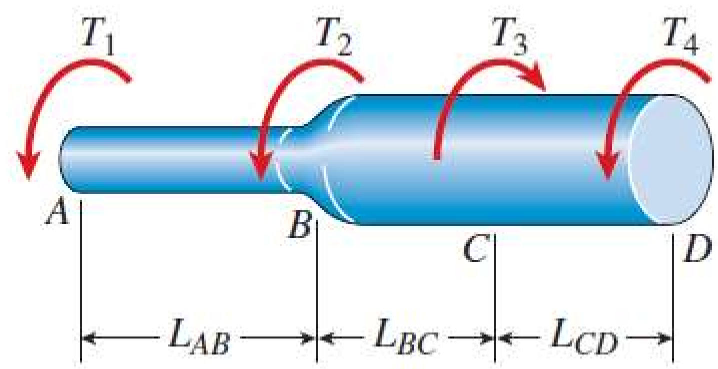
$$\frac{d_2}{d_0} = \frac{67.1\ \text{mm}}{58.8\ \text{mm}} = 1.14 \qquad \frac{W_{\text{hollow}}}{W_{\text{solid}}} = 0.47$$

Nonuniform torsion

- Pure torsion – refers to torsion of prismatic bar subjected to torques acting only at ends
- Non-uniform torsion
  - Non prismatic cross section
  - Applied torque may act anywhere along the axis of bar/shaft
- Case 1 – bar consisting of prismatic segments with constant torque throughout each segment

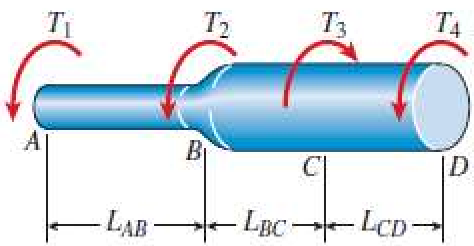
How to solve for such cases??

We have solved similar problems under axial loading.



Nonuniform torsion

- We can divide the bar into prismatic sections and subjected to constant torque
- Relations derived previously can be separately applied to each section



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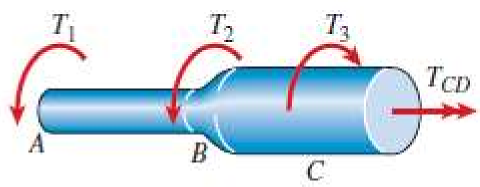

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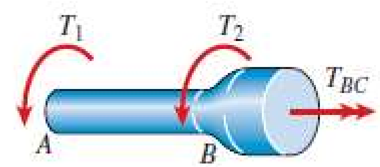


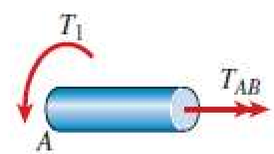
(c)

Using equilibrium equations:

$$T_{CD} = -T_1 - T_2 + T_3$$

$$T_{BC} = -T_1 - T_2$$

$$T_{AB} = -T_1$$



(d)

Each of these torques are constant throughout the length of section.

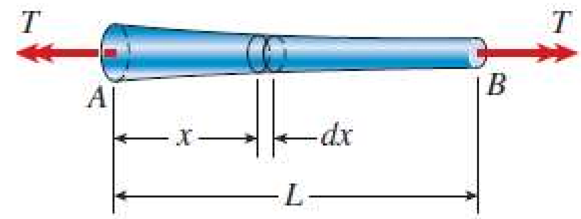
Nonuniform torsion

- Shear Stress
  - When calculating the stress, we need only the magnitude of these torques as the direction of stress is not of interest
- Twist
  - To get the angle of twist for entire bar, we must know the direction of twist in each segment
- So we need to define a sign convention
  - An internal torque is positive when its vector points away from the cut section and negative when its vector points toward the section
  - If the calculated torque turns out to have a positive sign, it means that the torque acts in the assumed direction; if the torque has a negative sign, it acts in the opposite direction
  - total angle of twist – using algebraic sum

$$\phi = \phi_1 + \phi_2 + \ldots + \phi_n \quad \Longrightarrow \quad \phi = \sum_{i=1}^{n} \phi_i = \sum_{i=1}^{n} \frac{T_i L_i}{G_i (I_P)_i}$$

Nonuniform torsion

- Case 2: bar with continuously varying cross section and constant torque
- Maximum stress at minimum *d*



$$\tau_{\max} = \frac{16T}{\pi d^3}$$

We just need to find the location of minimum *d*.

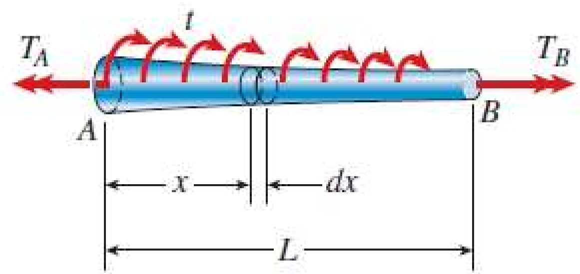
- Angle of twist

$$d\phi = \frac{T\,dx}{GI_P(x)}$$

$$\phi = \int_0^L d\phi = \int_0^L \frac{T\,dx}{GI_P(x)}$$

Nonuniform torsion

- Case 3: Bar with continuously varying cross sections and continuously varying torque
  - Internal torque will vary with distance $T(x)$
  - Polar moment of inertial will too $I_P(x)$

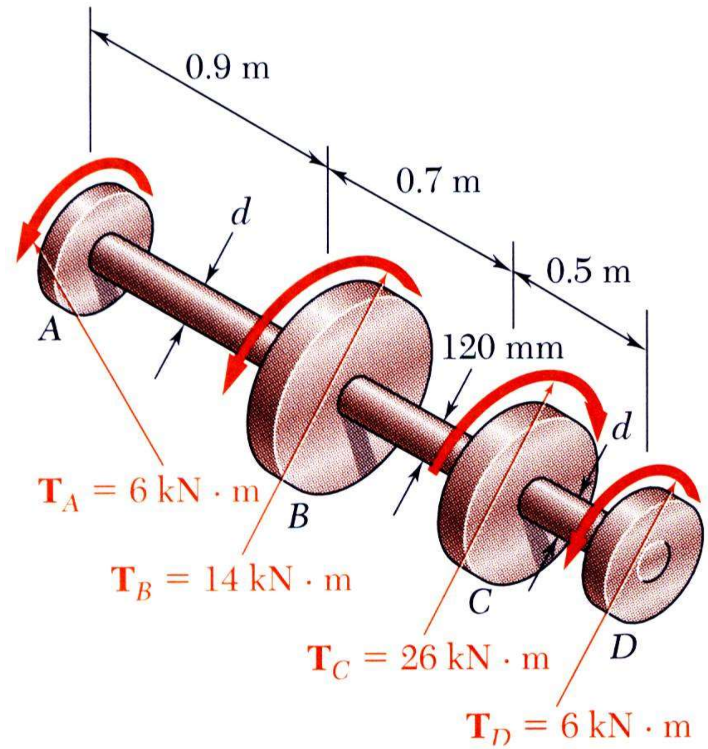


- Knowing both the torque and polar moment of inertia as functions of *x*, we can use the torsion formula to determine how the shear stress varies along the axis of the bar
- The cross section of maximum shear stress can then be identified, and the maximum shear stress can be determined
- Angle of twist

$$\phi = \int_0^L d\phi = \int_0^L \frac{T(x)\,dx}{GI_P(x)}$$

Example

Shaft *BC* is hollow with inner and outer diameters of 90 mm and 120 mm, respectively. Shafts *AB* and *CD* are solid of diameter *d*. For the loading shown, determine (*a*) the minimum and maximum shearing stress in shaft *BC*, (*b*) the required diameter *d* of shafts *AB* and *CD* if the allowable shearing stress in these shafts is 65 MPa.



Steps:

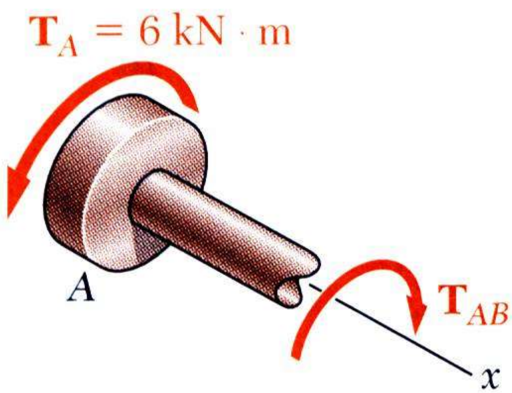
1- cut sections.

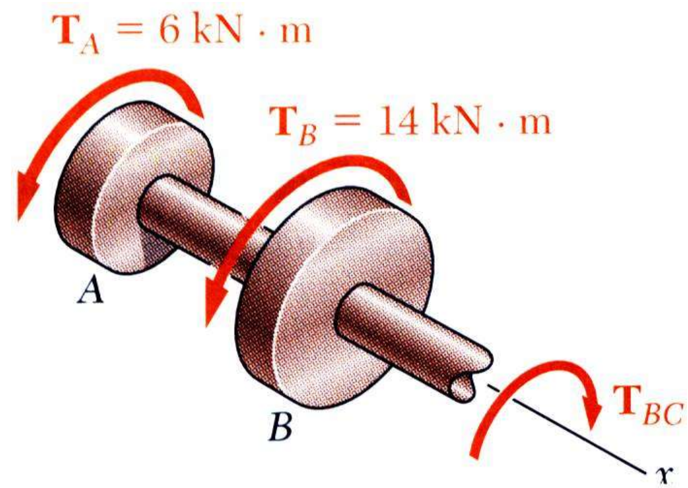
2- using torsion equation, find shear stresses.

3- inverting the relation find the value of *d*.

Example

- Cut sections through shafts AB and BC and perform static equilibrium analysis to find torque loadings





$$\sum M_x = 0 = (6\,\text{kN}\cdot\text{m}) - T_{AB}$$

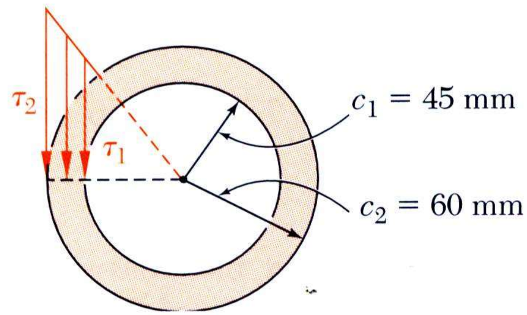
$$T_{AB} = 6\,\text{kN}\cdot\text{m} = T_{CD}$$

$$\sum M_x = 0 = (6\,\text{kN}\cdot\text{m}) + (14\,\text{kN}\cdot\text{m}) - T_{BC}$$

$$T_{BC} = 20\,\text{kN}\cdot\text{m}$$

Example

- Apply elastic torsion formulas to find minimum and maximum stress on shaft BC



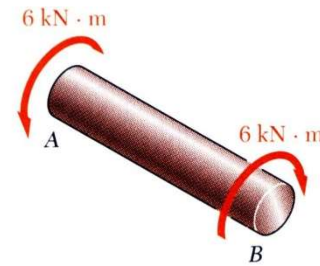
$$I_P = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}\left[(0.060)^4 - (0.045)^4\right]$$
$$= 13.92 \times 10^{-6}\,\mathrm{m}^4$$

$$\tau_{max} = \tau_2 = \frac{T_{BC}c_2}{J} = \frac{(20\,\mathrm{kN \cdot m})(0.060\,\mathrm{m})}{13.92 \times 10^{-6}\,\mathrm{m}^4}$$
$$= 86.2\,\mathrm{MPa}$$

$$\frac{\tau_{min}}{\tau_{max}} = \frac{c_1}{c_2} \qquad \frac{\tau_{min}}{86.2\,\mathrm{MPa}} = \frac{45\,\mathrm{mm}}{60\,\mathrm{mm}}$$
$$\tau_{min} = 64.7\,\mathrm{MPa}$$

$$\boxed{\begin{aligned}\tau_{max} &= 86.2\,\mathrm{MPa}\\ \tau_{min} &= 64.7\,\mathrm{MPa}\end{aligned}}$$

- Given allowable shearing stress and applied torque, invert the elastic torsion formula to find the required diameter



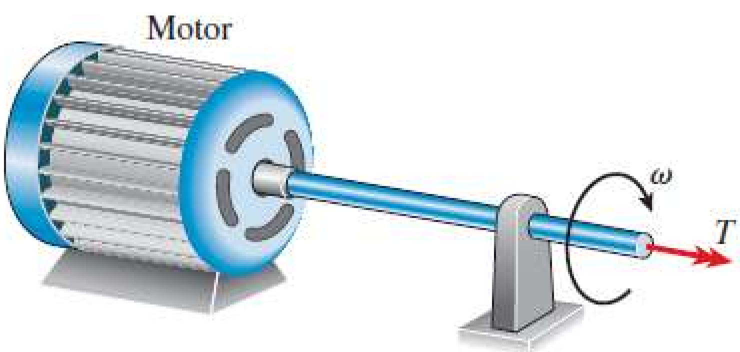
$$\tau_{max} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} \qquad 65MPa = \frac{6\,\mathrm{kN \cdot m}}{\frac{\pi}{2}c^3}$$
$$c = 38.9 \times 10^{-3}\,\mathrm{m}$$
$$\boxed{d = 2c = 77.8\,\mathrm{mm}}$$

Transmission of power

- Power transmission is the most important use of circular shafts e.g. automobile, propeller shafts of ship.
- The power is transmitted through rotary motion of shaft
- Amount of power depends on magnitude of torque *T* and rotational speed ω
- Design problem is to determine the size of shaft so that it can transmit a specified amount of power at a given speed

If *T* is the torque and ψ is the angle of rotation, then work done *W*

$$W = T\psi$$



Transmission of power

- Power is the rate of work done

$$P = \frac{dW}{dt} = T\frac{d\psi}{dt}$$

Angular velocity ω.

$$P = T\omega$$

$$\omega = 2\pi f$$

$$\boxed{P = 2\pi f T}$$

1 HP = 760 watt

$$\boxed{P = \frac{2\pi n T}{60}} \quad (n = \text{rpm})$$

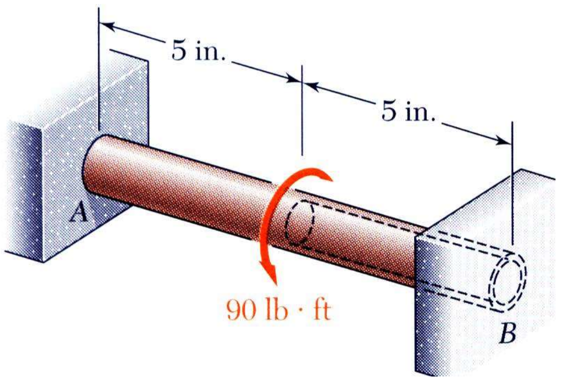
$$n = 60\,f$$

Dia of shaft:

$$\tau_{\max} = \frac{16T}{\pi d^3}$$

Statically Indeterminate Shafts

- Similar to axial bars, we may also have situation which statically indeterminate during torsion of shaft
- Given the shaft dimensions and the applied torque, find the torque reactions at A and B.



- From a free-body analysis of the shaft,

$$T_A + T_B = 90\,\mathrm{lb \cdot ft}$$

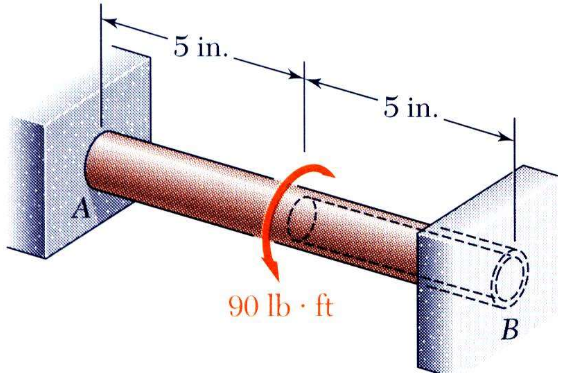
which is not sufficient to find the end torques. The problem is statically indeterminate.

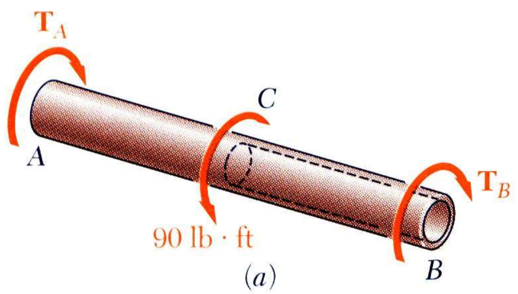
The steps to solve such problems is same as we discussed in axial loading cases.

We use compatibility equations and principal of superposition.

Statically Indeterminate Shafts

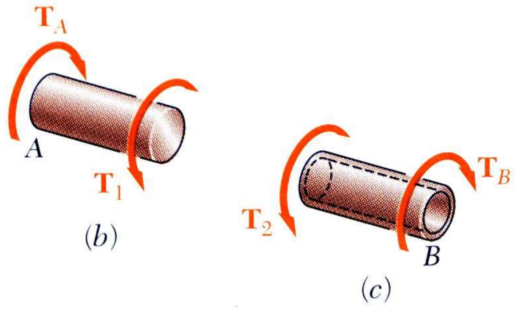
- Case 1
  - Using FBD





$$T_A + T_B = 90\,\mathrm{lb}\cdot\mathrm{ft}$$

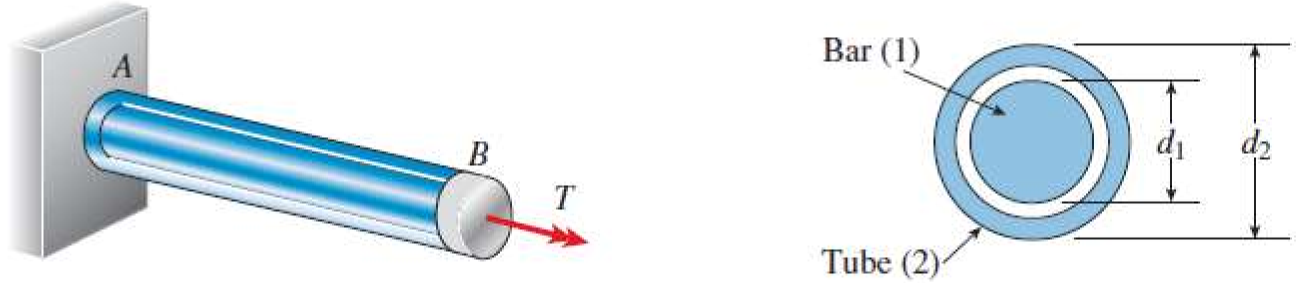
Total twist:

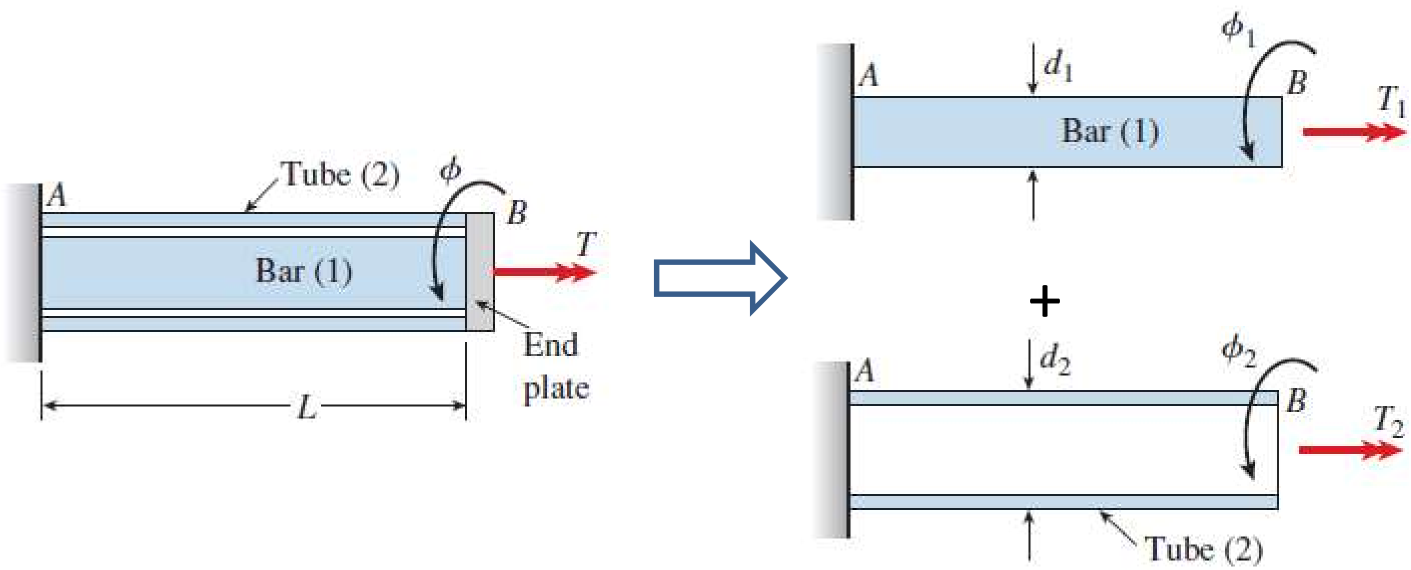


$$\phi = \phi_1 + \phi_2 = \frac{T_A L_1}{J_1 G} - \frac{T_B L_2}{J_2 G} = 0 \qquad T_B = \frac{L_1 J_2}{L_2 J_1} T_A$$

Statically Indeterminate Shafts

- Case 2





$$T_1 + T_2 = T$$

$$\phi_1 = \phi_2$$

$$\frac{T_1 L}{G_1 I_{P1}} = \frac{T_2 L}{G_2 I_{P2}}$$

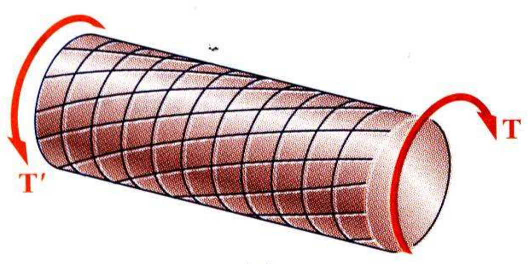
Statically Indeterminate Shafts

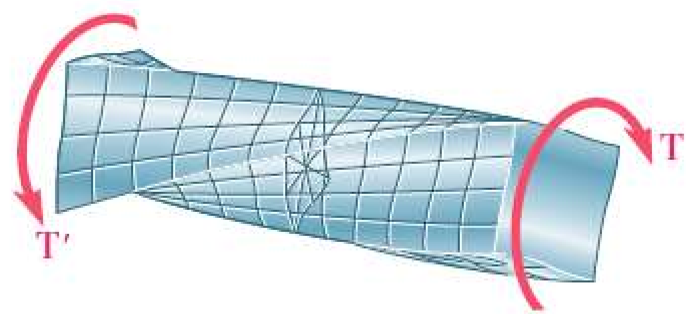
- Case 2
  - Solving two equation, we can derive

$$T_1 = T\left(\frac{G_1 I_{P1}}{G_1 I_{P1} + G_2 I_{P2}}\right) \qquad T_2 = T\left(\frac{G_2 I_{P2}}{G_1 I_{P1} + G_2 I_{P2}}\right)$$

Torsion of Noncircular Members

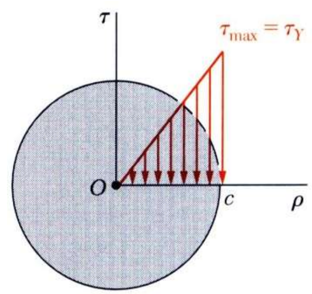
- In the previous discussions, we assumed that the cross section of the member remained plane and undistorted
  - Validity of this assumption depends upon the *axisymmetry* of the member
  - its appearance remains the same when it is viewed from a fixed position and rotated about its axis through an arbitrary angle
  - Circular cross section – axisymmetric, linear variation in strain, uniform strain at all points on the same radius – cross section do not deform
  - Square cross section – same if we change angle by 90 or 180^o

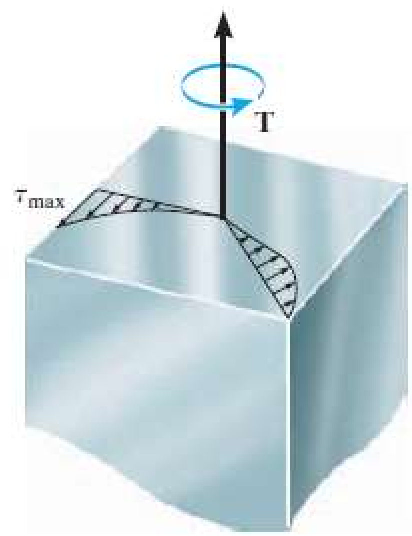




Torsion of Noncircular Members

- Warping or bulging of cross section is observed in non circular cross sections





- It is caused by diff in shear stress distribution in circular and non circular cross sections

Torsion of Noncircular Members

- Distribution of shear stress

Distribution on the face perpendicular to y-axis:

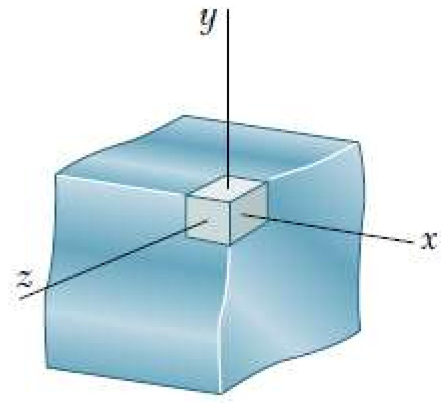
$$\tau_{yx} = 0 \qquad \tau_{yz} = 0$$

Distribution on the face perpendicular to z-axis:

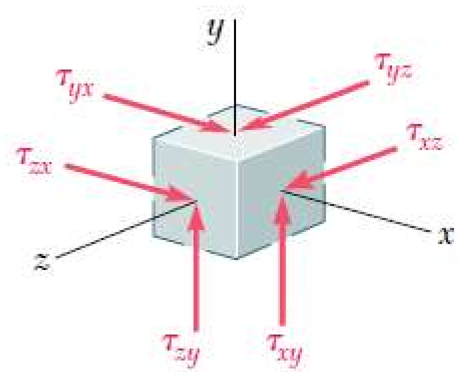
$$\tau_{zx} = 0 \qquad \tau_{zy} = 0$$

All this implies that:

$$\boxed{\tau_{xy} = 0 \qquad \tau_{xz} = 0}$$



(a)

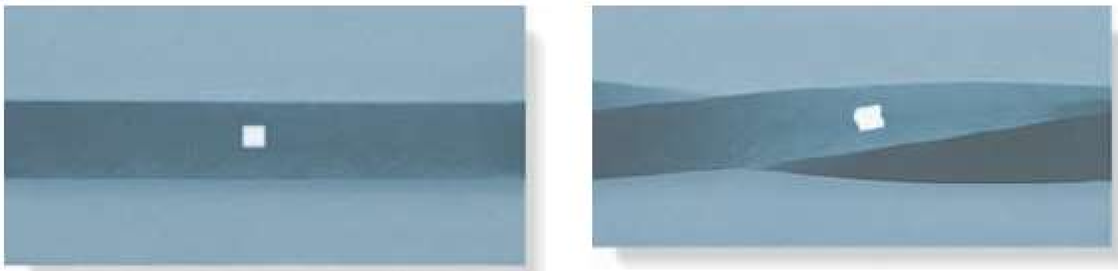


(b)

No shearing stress at the corners of the cross section of the bar.

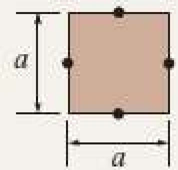
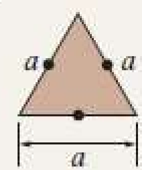
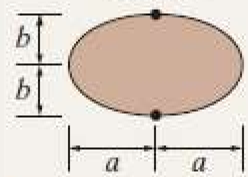
Torsion of Noncircular Members

- Patch test with square cross section



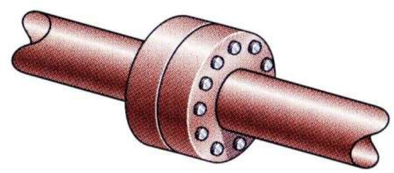
- For non circular shafts the maximum shear stress occur at a point on the edge of cross section that is closest to the center axis of shaft.

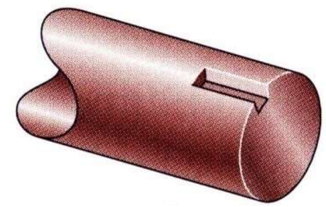
TABLE 5–1

Shape of cross section	τ_{max}	ϕ
Square 	$\frac{4.81\,T}{a^3}$	$\frac{7.10\,TL}{a^4G}$
Equilateral triangle 	$\frac{20\,T}{a^3}$	$\frac{46\,TL}{a^4G}$
Ellipse 	$\frac{2\,T}{\pi ab^2}$	$\frac{(a^2+b^2)TL}{\pi a^3b^3G}$

Stress Concentrations

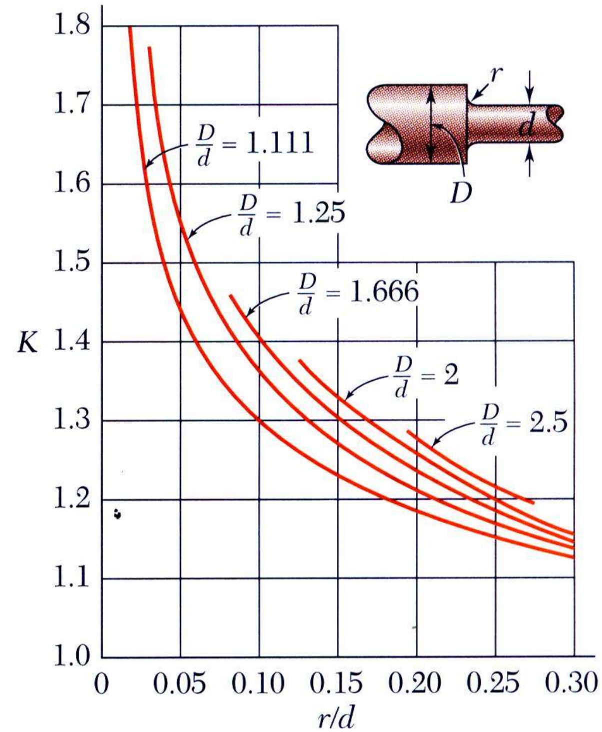
- While deriving the torsion formula, we assumed a circular shaft with uniform cross-section





$$\tau_{\max} = \frac{Tr}{I_P}$$

- The use of flange couplings, gears and pulleys attached to shafts by keys in keyways, and cross-section discontinuities can cause stress concentrations

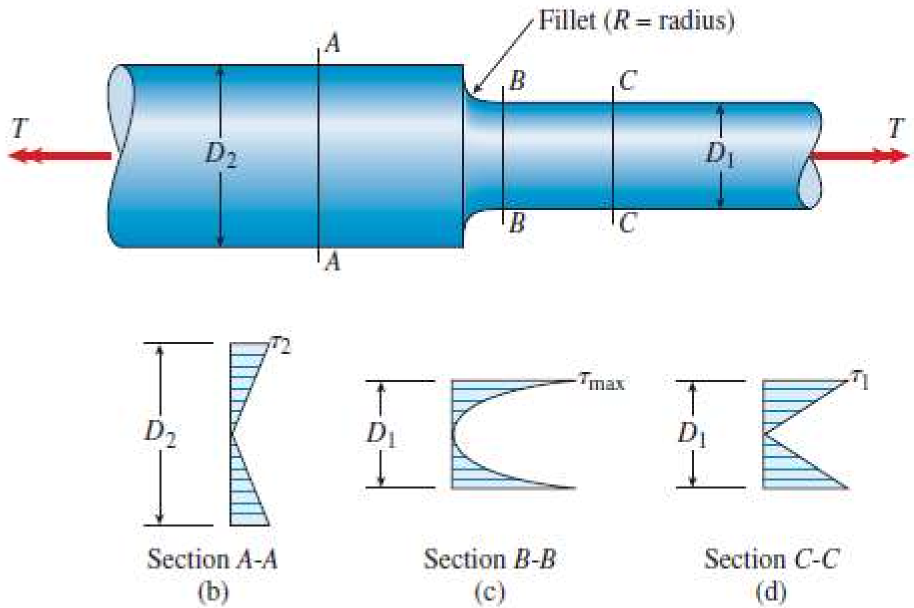


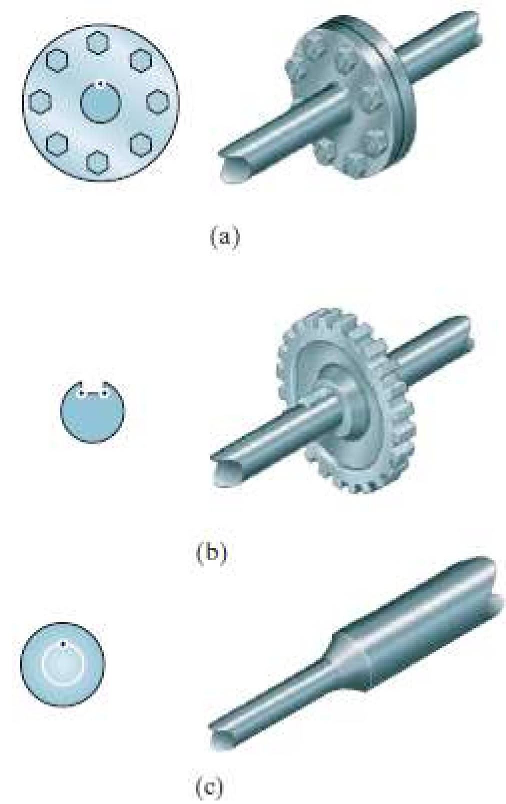
- Experimental or numerically determined concentration factors are applied as

$$\tau_{\max} = K\frac{Tr}{I_P}$$

Stress concentration

- Various design situations which can cause stress concentration





Assignment – 4 & 5

- All solved example of chapter 4 and 5
- F4.1,4.2
- 4.10,4.12,4.15,4.19,4.20,4.27,4.32,4.37,4.40,4.45, 4.69,4.70,4.85, 4.88,4.92,4.94,4.102,4.105
- F5.4,5.5,5.6
- 5.7,5.9,5.11,5.14,5.23,5.30,5.32,5.42
- F5.8,5.10
- 5.48,5.52,5.55,5.58,5.62,5.64,5.75,5.78,5.79,5.82,5.91,5.94,5.98,5.99,5.101,5.102,5.108,5.119,5.120,5.122,5.124, 5.129,5.130,5.135,5.141

Creep and Fatigue

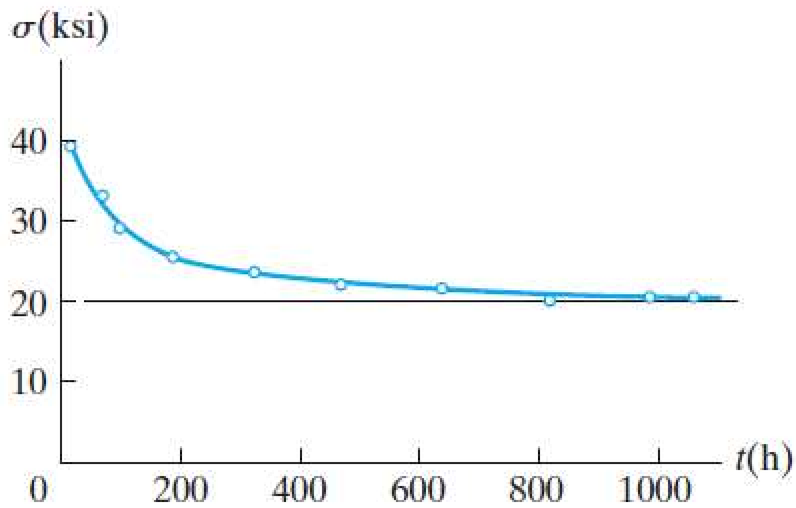
- Creep
  - Observation

Creep and Fatigue

- Creep
  - Time-dependent permanent deformation is known as *creep*
  - A rubber band will not return to its original shape after being released from a stretched position in which it was held for a very long period of time – elastic bands becomes useless after certain time
  - Or for a shorter time but at higher temperature
  - Both stress and/or temperature play a significant role in the rate of creep
  - An important mechanical property that is used in this regard is called the creep strength
    - the highest stress the material can withstand during a specified time without exceeding an allowable creep strain
    - The creep strength will vary with temperature, and for design, a given temperature, duration of loading, and allowable creep strain must all be specified. For example, a creep strain of 0.1% per year has been suggested for steel in bolts and piping.

Creep and Fatigue

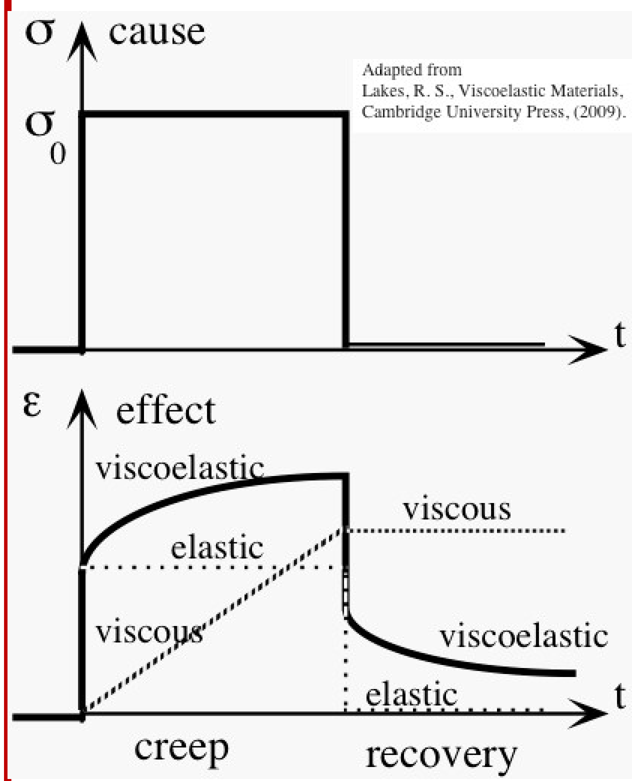
- Testing several specimens simultaneously at a constant temperature and different axial stress
- By measuring the time needed to produce either an allowable strain or the fracture strain for each specimen,
  - Normally these tests are run to a maximum of 1000 hours
    - stainless steel at a temperature of 1200°F and prescribed creep strain of 1%

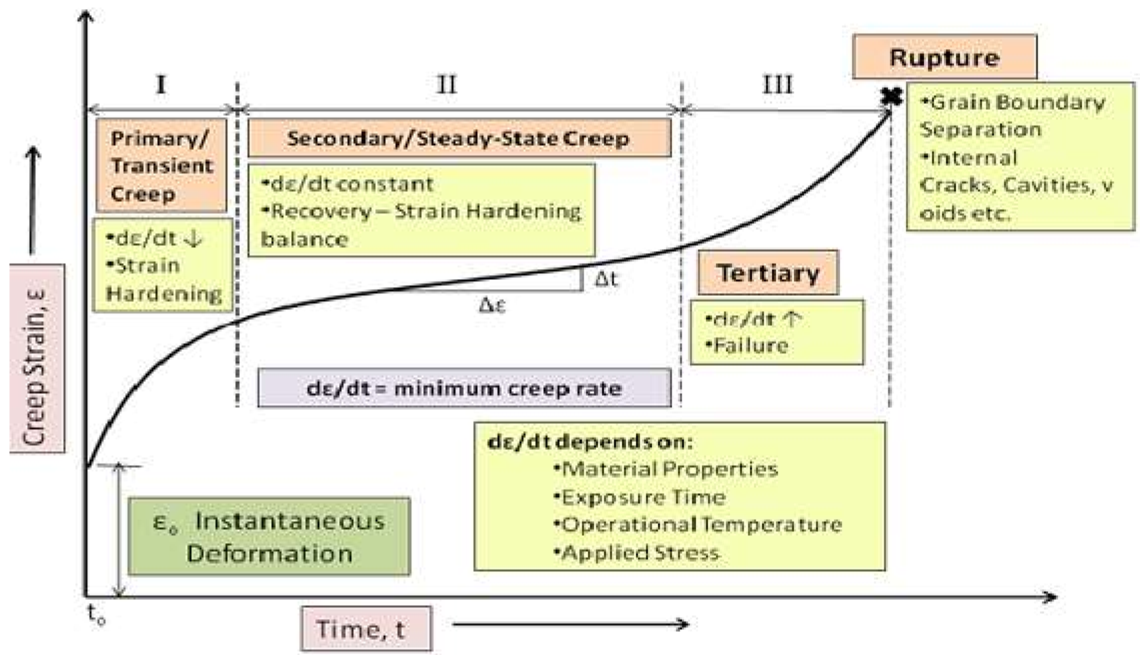


Creep strength??

Creep and Fatigue

- Creep – a little more





Creep and Fatigue

- Fatigue
  - observations

Creep and Fatigue

- Fatigue
  - When a metal is subjected to repeated cycles of stress or strain, it causes its structure to break down, ultimately leading to fracture
  - responsible for a large percentage of failures in connecting rods and crankshafts of engines; steam or gas turbine blades; connections or supports for bridges, railroad wheels, and axles; and other parts subjected to cyclic loading
  - In all these cases, fracture will occur at a stress that is less than the material's yield stress
  - Happens due to the presence of microscopic imperfections at the surface of the member
  - localized stress becomes much greater than the average stress acting over the cross section

Creep and Fatigue

- To specify a safe strength for a metallic material under repeated loading, it is necessary to determine a limit below which no evidence of failure can be detected after applying a load for a specified number of cycles
- This limiting stress is called the *endurance or fatigue limit*
- Using a testing machine, a series of specimens are each subjected to a specified stress and cycled to failure
- The results are plotted as a graph representing the stress *S (or* σ*)* and the number of cycles-to-failure *N. This graph* is called an *S–N diagram* or *stress–cycle diagram*
- Most often the values of *N* are plotted on a logarithmic scale since they are generally quite large

Creep and Fatigue

- *S*-*N* diagram
  - When the stress is reduced below the *endurance limit*, fatigue failures do not occur for any number of cycles
  - How to define endurance limit for materials like Al?

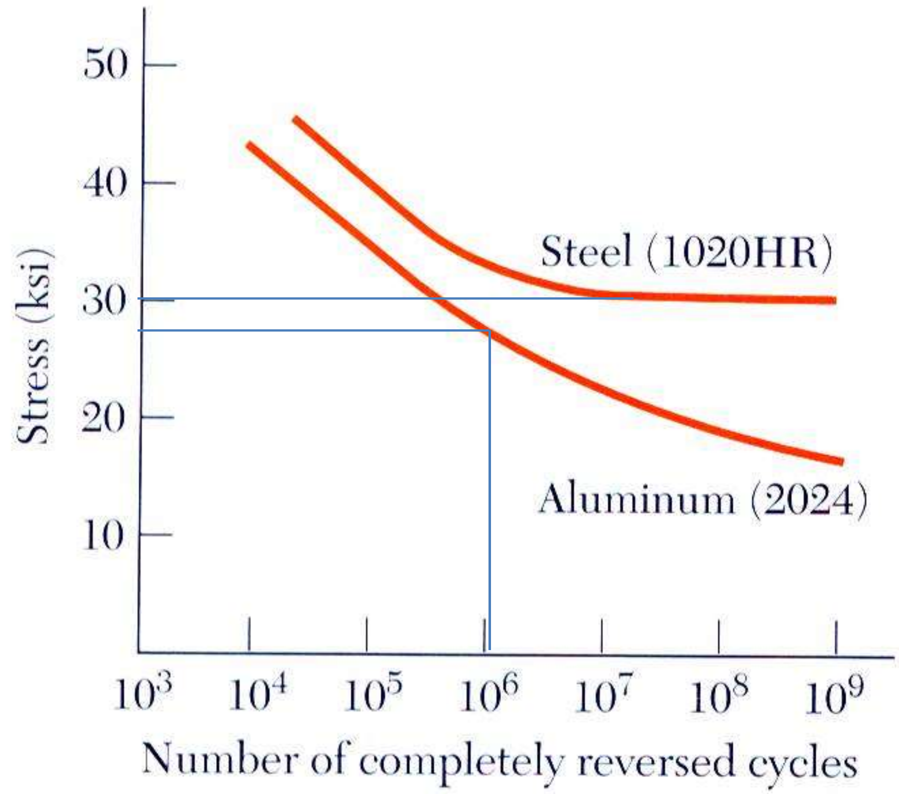


Fig. 2.21