Power calculations in AC circuits

Instantaneous power : Product of time-domain voltage and current at any point of time.

Instantaneous power may be useful in determining safe operating limits for some circuit elements.

$$p(t) = v(t)\,i(t) \quad \cdots\cdots (3)$$

Let us consider the circuit in Fig. 12, where a sinusoidal voltage source drives a series combination of resistance and inductance.

$v_s(t) = V_m \cos \omega t$

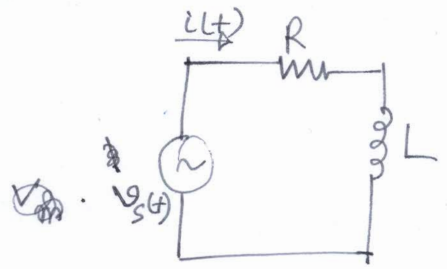

Fig. 12 : Power calculation

The current is given by,

$i(t) = I_m \cos(\omega t + \phi)$,

where $I_m = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$; $\cdots\cdots (4)$

$\phi = -\tan^{-1} \dfrac{\omega L}{R}$ $\cdots\cdots (5)$

[Note: $\bar{I} = \dfrac{\bar{V}}{R + j\omega L} = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \angle -\tan^{-1}\dfrac{\omega L}{R}$]

Instantaneous power delivered to the circuit is given by,

$$\begin{aligned} p(t) = v(t)\,i(t) &= V_m I_m \cos(\omega t + \phi) \cos \omega t \\ &= \frac{V_m I_m}{2}\left[\cos(2\omega t + \phi) + \cos\phi\right] \\ &= \frac{V_m I_m}{2}\cos(2\omega t + \phi) + \frac{V_m I_m}{2}\cos\phi \\ &= V_{rms} I_{rms} \cos\phi + V_{rms} I_{rms} \cos(2\omega t + \phi) \quad \cdots (6) \end{aligned}$$

First part of 6 is a constant term, and the second part varies at twice the source frequency. If we take the average of the instantaneous power over a cycle, the sinusoidal part vanishes. Hence, the average power is given by,

$$P = V_{rms} I_{rms} \cos\phi \quad \ldots\ldots \quad (7)$$

This is called the average real power.

~~For $v(t) = V_m \cos(\omega t + \phi_v)$~~
~~and $i(t) = I_m \cos(\omega t + \phi_i)$,~~
~~The average power is given by,~~

$$\cancel{P = \frac{V_m I_m}{2} \cos(\omega t + \phi_v - \phi_i)} \quad \ldots\ldots \quad (8)$$

Example 1. Find the ~~averge~~ average real power being delivered to an impedance $\bar{Z}_L = 8 - j11\ \Omega$ by a current $I = \cancel{5\angle 20^\circ}\ \frac{5}{\sqrt{2}}\angle 20^\circ$

Solution. Real power is absorbed by the resistance only.

Hence $P_{av} = I_{rms}^2 \cdot 8 = \frac{5^2}{2} \times 8 = 100\ W$.

Example 2: Calculate the average real power delivered to an impedance $6\angle 25^\circ\ \Omega$ by the current $\bar{I} = 2 + j5\ A$.

Solution.

Rms value of the current is given by,

$$I_{rms} = \frac{\sqrt{2^2 + 5^2}}{\sqrt{2}} = \frac{\sqrt{29}}{\sqrt{2}}\ A$$

$$P_{av} = I_{rms}^2 R = \frac{29}{2} \times 6\cos 25^\circ$$
$$= 78.85\ W$$

Maximum power transfer

Consider the circuit in Fig. 13, where a ~~sour~~ load impedance $\bar{Z}_L$ is connected with a ~~se~~ Thevenin equivalent of the rest of the system.

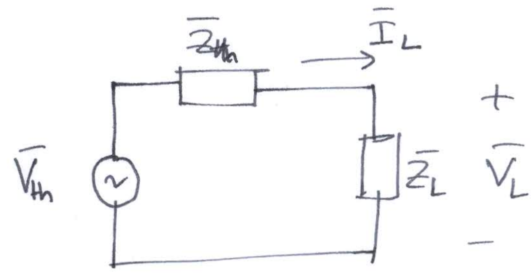

Fig. 13: Circuit to illustrate maximum power transfer.

The ~~maximum~~ average real power delivered to $\bar{Z}_L$ becomes maximum when $\bar{Z}_L$ is the complex conjugate of $\bar{Z}_{th}$, i.e.,

$$\bar{Z}_L = \bar{Z}_{th}^{*} \quad \text{- - - - - (8)}$$

Apparent power and power factor

Voltage and current at the terminals of a load $\bar{Z}_L$ shown in Fig. 14 are given by,

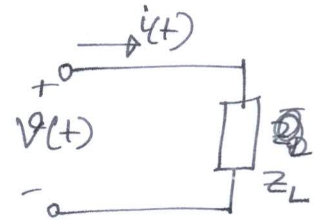

Fig. 14. Simple 2-terminal circuit

$$\left.\begin{aligned} v(t) &= V_m \cos(\omega t + \theta_v) \\ i(t) &= I_m \cos(\omega t + \theta_i) \end{aligned}\right\} \text{— (9)}$$

The corresponding phasors are given by,

$$\left.\begin{aligned} \bar{V} &= \frac{V_m}{\sqrt{2}} \angle \theta_v \\ \bar{I} &= \frac{I_m}{\sqrt{2}} \angle \theta_I \end{aligned}\right\} \text{- - - - (10)}$$

The average real power is given by,

$$\begin{aligned} P &= \frac{V_m I_m}{2} \cdot \cos(\theta_v - \theta_i) \\ &= V_{rms} I_{rms} \cos(\theta_v - \theta_i) \\ &= S \cdot \cos(\theta_v - \theta_i) \quad \text{- - - - - (11)} \end{aligned}$$

where $S = V_{rms} I_{rms}$ . . . . . (12)

The term S is known as the apparent power, and is given a unit VA (volt-amperes), to differentiate from the unit of real power, watt (W).

The ratio of the real power to the apparent power is called the power factor. (pf). Hence,

$$pf = \frac{P}{S} = \frac{P}{V_{rms} I_{rms}} = \cos(\theta_v - \theta_i) \quad \ldots (13)$$

The angle $(\theta_v - \theta_i)$ by which the voltage leads the current, is known as the pf angle.

Complex power

The ~~concept~~ quantity called 'complex power' is used to simplify the power calculations. Using the voltage and current phasors described in (10), the complex power $\bar{S}$ is defined as,

$$\bar{S} = \bar{V} \bar{I}^*$$

$$= V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$= V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i) \quad \ldots (14)$$

~~The~~ [Note: The magnitude of the complex power is the apparent power, and the phasor angle is the power factor angle]

The first part of (14) is equal to the real power, as shown in (11). The second part is given a term 'Reactive power', and is represented by the symbol 'Q'. Hence, ~~form~~ (14) can be rewritten as,

$$\bar{S} = P + jQ \quad \ldots (15)$$

The reactive power, Q, is given a unit 'volt-ampere-reactive' or VAR. Physically, the reactive power represents the rate of change of energy exchange between the source and the reactive components of the load. Note that, for resistive loads, $Q = 0$; for inductive load, $Q > 0$; and for capacitive loads, $Q < 0$.

Power triangle

A graphical way of representing the complex power is the use of 'power triangle', similar to the 'impedance triangle', as shown in Fig. 15.

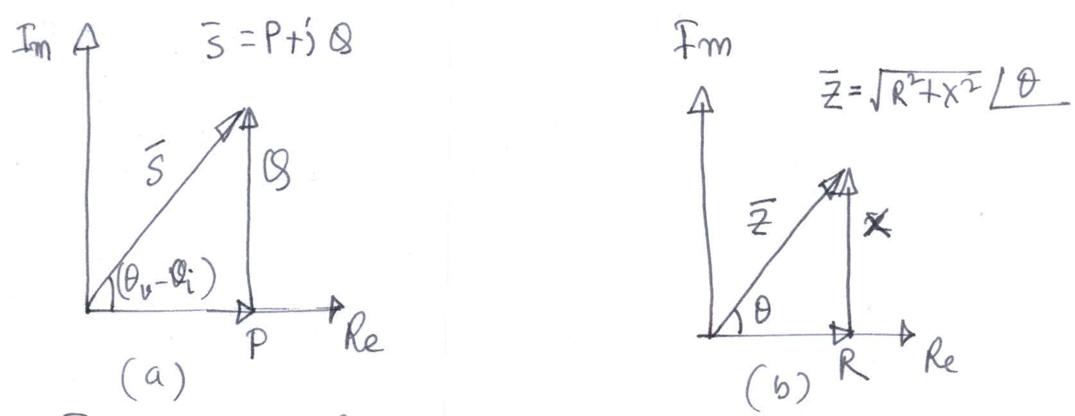

Fig. 15. (a) Power triangle. (b) Impedance triangle

~~Power measurement~~

~~Real power is measured by Wattmeter.~~
~~Reactive power is measured by VAR Varmeter.~~

Power factor correction

Real power is the power that is utilized by the load. Reactive power flows back and forth between the source and the load, and cannot be utilized by the consumer. Due to additional flow of current corresponding to the reactive power, line heating ~~increas~~ and voltage drop increas.

To avoid these problems, loads are, in general, desired to ~~draw~~ maintain ~~for~~ their power factors close to unity. The process of improving the power factor of the load without altering its original voltage and current is known as power factor correction.

Let us consider an inductive load shown in Fig. ~~16(a)~~, 16 (a), and the phasor diagram in Fig. 16 (b). It draws a lagging ~~power facto~~ current from the source.

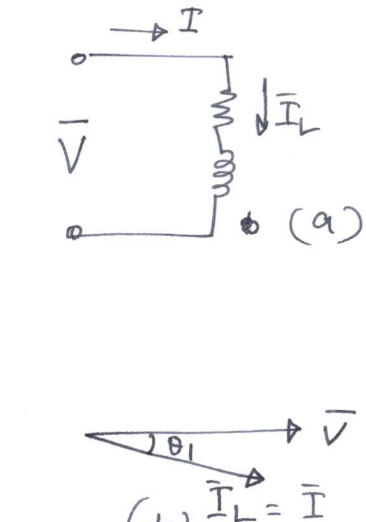

Fig. 16: Inductive load and phasor diagram

To improve the power-factor, let us put a capacitor in parallel with the load, as shown in Fig. 17(a). The current through the load and the capacitor are $\bar{I}_L$ and $\bar{I}_c$ respectively, such that

$$\bar{I} = \bar{I}_L + \bar{I}_c \quad \ldots (16)$$

The corresponding phasor diagram is shown in Fig. 17. The new pf is $\cos\theta_2$, compared to the earlier pf of $\cos\theta_1$. Since, $\theta_2 < \theta_1$, there is a clear improvement in the power factor.

Improving power factors by adding shunt capacitors is very common, since most of the loads are ~~ф~~ inductive in nature.

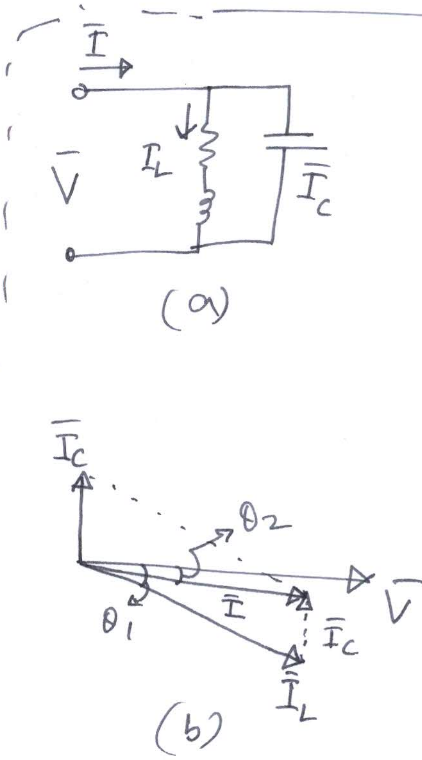

Fig. 17: Power factor correction with a capacitor