GE 104: Introduction to electrical engineering

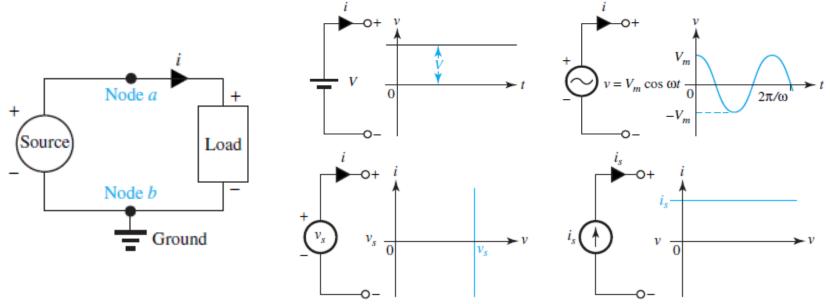
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1. Energy and Power



Power
$$p = \frac{dw}{dt} = \left(\frac{dw}{dq}\right) \left(\frac{dq}{dt}\right) = vi$$

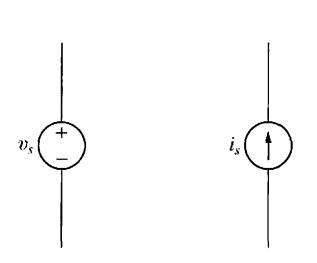
2. Sources and loads



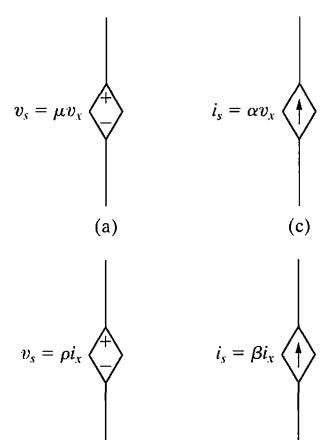
DC voltage; AC voltage; Ideal voltage and current source

3. Sources





Idea Voltage and current source



Dependant sources- (a) Voltage controlled voltage (b) Voltage controlled current (c) current controlled Voltage (d) current controlled current

4. Lumped Circuit Elements



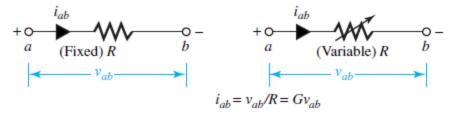
- Resistance
- Inductance
- Capacitance

$$i = v/R = Gv$$
, or $v = iR$

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A}$$

Temperature dependency on resistivity

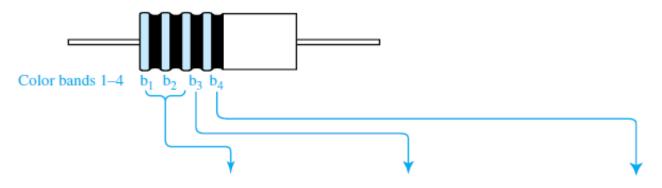
$$\rho_{T2} = \rho_{T1} \left(\frac{T_2 + T}{T_1 + T} \right)$$



Туре	Material	$\rho(\Omega \cdot \mathbf{m})$
Conductors	Silver	16×10^{-9}
(at 20°C)	Copper	17×10^{-9}
	Gold	24×10^{-9}
	Aluminum	28×10^{-9}
	Tungsten	55×10^{-9}
	Brass	67×10^{-9}
	Sodium	0.04×10^{-6}
	Stainless steel	0.91×10^{-6}
	Iron	0.1×10^{-6}
	Nichrome	1×10^{-6}
	Carbon	35×10^{-6}
	Seawater	0.25
Semiconductors	Germanium	0.46
(at 27°C or 300 K)	Silicon	2.3×10^{3}
Insulators	Rubber	1×10^{12}
	Polystyrene	1×10^{15}

5. Lumped Circuit Elements (Resistance)



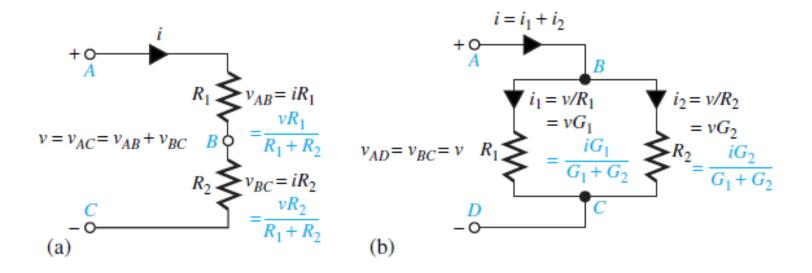


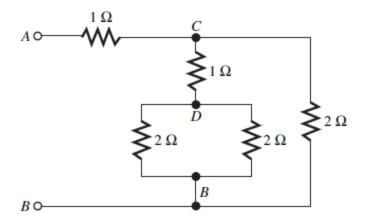
Color of Band	Digit of Band	Multiplier	% Tolerance in Actual Value
Black	0	100	_
Brown	1	10 ¹	_
Red	2	10^{2}	_
Orange	3	10^{3}	_
Yellow	4	10 ⁴	_
Green	5	10 ⁵	_
Blue	6	10^{6}	_
Violet	7	10 ⁷	_
Grey	8	10^{8}	_
White	9	_	_
Gold	_	10^{-1}	± 5%
Silver	_	10^{-2}	$\pm 10\%$
Black or no color	_	_	± 20%

Resistance value = $(10b_1 + b_2) \times 10^{b_3} \Omega$.

5. Resistance (Parallel & series)



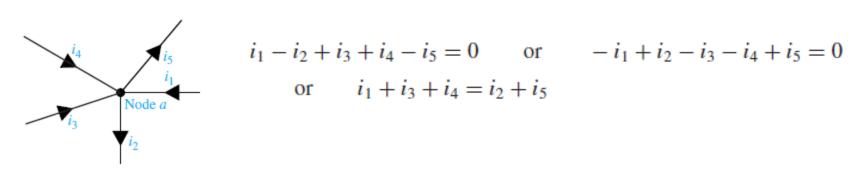




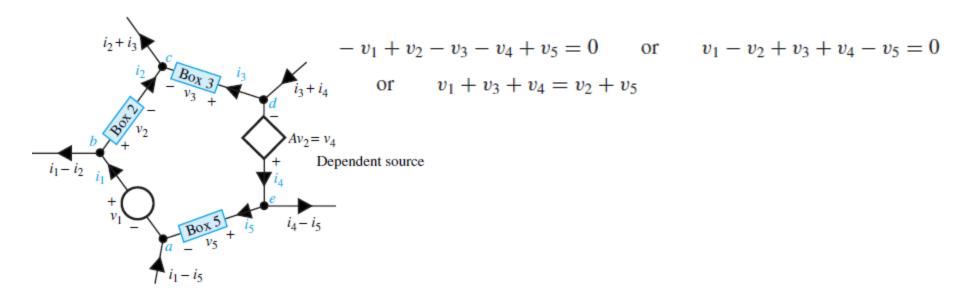
6. Kirchhoff's Laws



kirchhoff's Current Law (KCL)

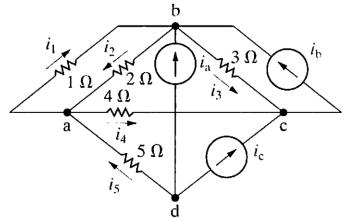


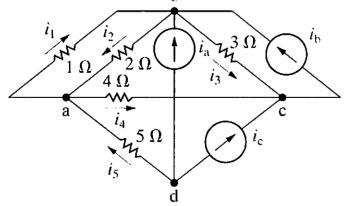
kirchhoff's Voltage Law (KVL)

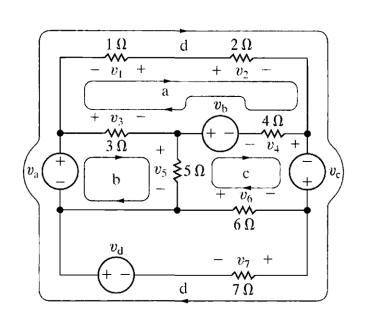


6. Example









node a
$$i_1 + i_4 - i_2 - i_5 = 0$$
,

node b
$$i_2 + i_3 - i_1 - i_b - i_a = 0$$
,

node c
$$i_b - i_3 - i_4 - i_c = 0$$
,

node d
$$i_5 + i_a + i_c = 0$$
.

path a
$$-v_1 + v_2 + v_4 - v_b - v_3 = 0$$
,

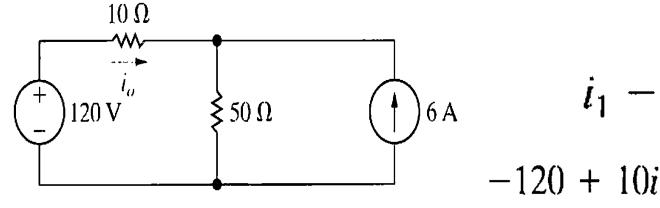
path b
$$-v_a + v_3 + v_5 = 0$$
,

path c
$$v_b - v_4 - v_c - v_6 - v_5 = 0$$
,

path d
$$-v_a - v_1 + v_2 - v_c + v_7 - v_d = 0$$
.

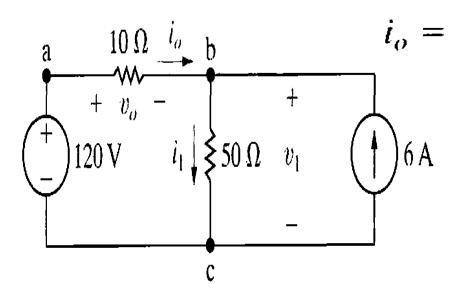
6. Example





$$i_1 - i_o - 6 = 0.$$

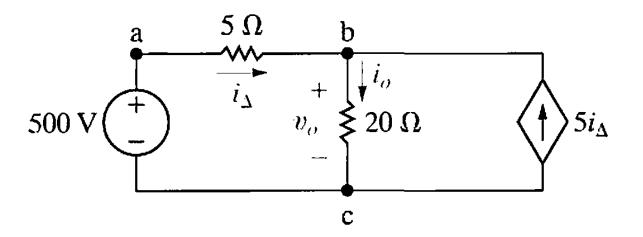
$$-120 + 10i_o + 50i_1 = 0.$$



 $i_0 = -3 \text{ A}$ and $i_1 = 3 \text{ A}$.

6. Example -Dependant source





$$500 = 5i_{\Delta} + 20i_{o}$$
.

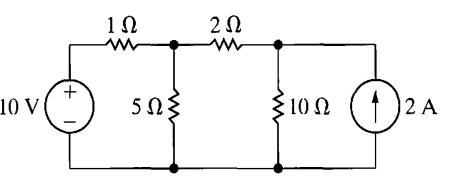
$$i_o = i_\Delta + 5i_\Delta = 6i_\Delta.$$

$$i_{\Delta} = 4 A$$

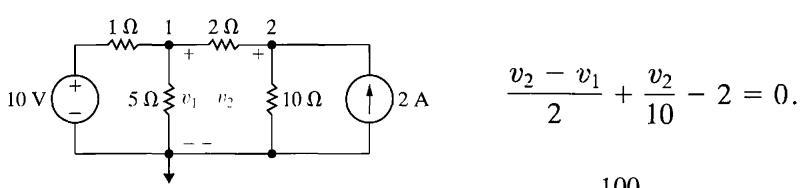
$$i_o = 24 \text{ A}.$$

7. Nodal Analysis





$$\frac{v_1-10}{1}+\frac{v_1}{5}+\frac{v_1-v_2}{2}=0.$$

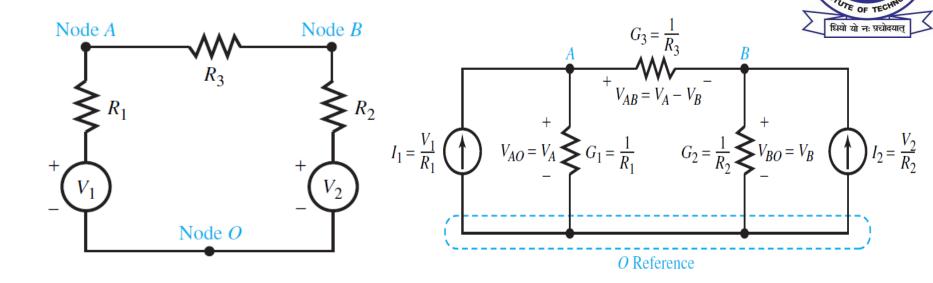


$$\frac{v_2-v_1}{2}+\frac{v_2}{10}-2=0$$

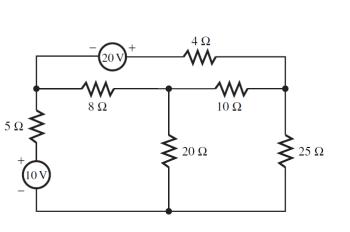
$$v_1 = \frac{100}{11} = 9.09 \text{ V}$$

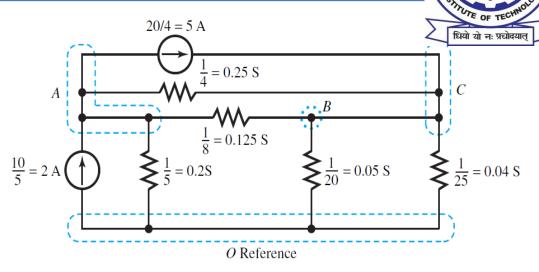
$$v_2 = \frac{120}{11} = 10.91 \text{ V}.$$

7. Nodal Analysis



7. Nodal Analysis – Example





Node A:
$$(0.2 + 0.125 + 0.25)V_A - 0.125V_B - 0.25V_C = 2 - 5 = -3$$

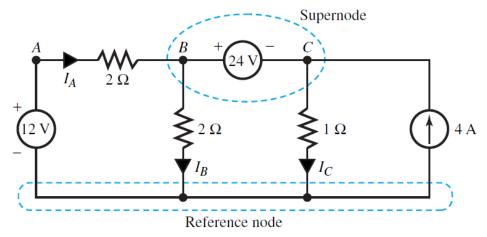
Node B:
$$-0.125V_A + (0.125 + 0.05 + 0.1)V_B - 0.1V_C = 0$$

Node C:
$$-0.25V_A - 0.1V_B + (0.25 + 0.1 + 0.04)V_C = 5$$

$$V_A = 4.34 \text{ V}; \qquad V_B = 8.43 \text{ V}; \qquad V_C = 17.77 \text{ V}$$

7. Nodal Analysis – Example





$$\int_{AA} V_B - V_C = 24 \text{ V}$$

$$I_A - I_B - I_C + 4 = 0$$
 or $\frac{12 - V_B}{2} - \frac{V_B}{2} - \frac{V_C}{1} + 4 = 0$

$$V_B + V_C = 10$$

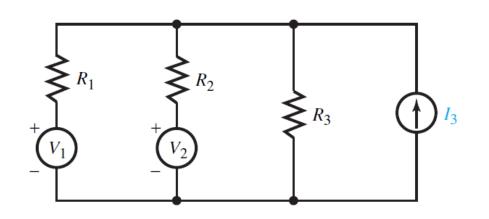
$$V_B = 17 \text{ V} \quad \text{and} \quad V_C = -7 \text{ V}$$

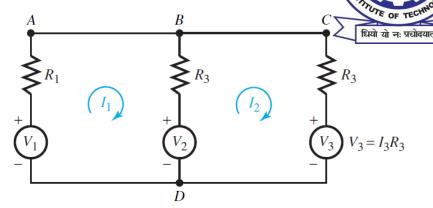
$$I_A = \frac{12 - V_B}{2} = \frac{12 - 17}{2} = -2.5 \text{ A}$$

$$I_B = \frac{V_B}{2} = \frac{17}{2} = 8.5 \text{ A}$$

$$I_C = \frac{V_C}{1} = \frac{-7}{1} = -7 \text{ A}$$

7. Mesh current Analysis

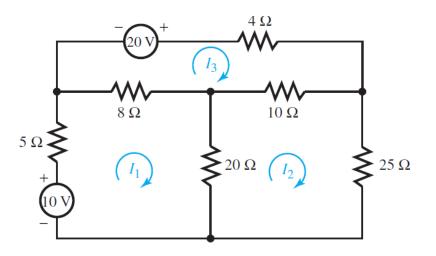




$$R_{11}I_1 - R_{12}I_2 - \cdots - R_{1N}I_N = V_1 \\ -R_{21}I_1 + R_{22}V_2 - \cdots - R_{2N}I_N = V_2 \\ \vdots & \vdots & \vdots \\ -R_{N1}I_1 - R_{N2}V_2 - \cdots + R_{NN}I_N = V_N$$

7. Mesh current Analysis - Example





```
Loop 1 with mesh current I_1: (5+8+20)I_1 - 20I_2 - 8I_3 = 10

Loop 2 with mesh current I_2: -20I_1 + (20+10+25)I_2 - 10I_3 = 0

Loop 3 with mesh current I_3: -8I_1 - 10I_2 + (4+10+8)I_3 = 20
```

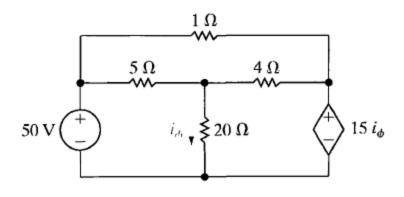
$$33I_1 - 20I_2 - 8I_3 = 10$$

 $-20I_1 + 55I_2 - 10I_3 = 0$
 $-8I_1 - 10I_2 + 22I_3 = 20$

$$I_1 = 1.132 \text{ A}; \qquad I_2 = 0.711 \text{ A}; \qquad I_3 = 1.645 \text{ A}$$

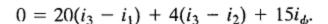
7. Mesh current Analysis -Example





$$50 = 5(i_1 - i_2) + 20(i_1 - i_3),$$

$$0 = 5(i_2 - i_1) + 1i_2 + 4(i_2 - i_3),$$



$$i_{\phi}=i_1-i_3,$$

$$50 = 25i_1 - 5i_2 - 20i_3,$$

$$0 = -5i_1 + 10i_2 - 4i_3,$$

$$0 = -5i_1 - 4i_2 + 9i_3.$$

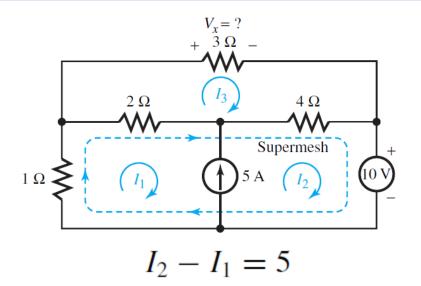


$$i_2 = 26 \text{ A},$$

$$i_3 = 28 \text{ A}.$$

7. Mesh current Analysis –Home work





$$1I_1 + 2(I_1 - I_3) + 4(I_2 - I_3) + 10 = 0$$

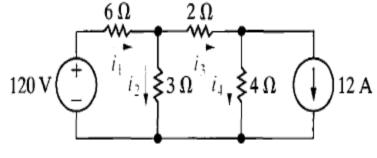
KVL equation for mesh 3

$$3I_3 + 4(I_3 - I_2) + 2(I_3 - I_1) = 0$$

8. Super position theorem



$$f(Kx) = Kf(x)$$
 (homogeneity)
 $f(x_1 + x_2) = f(x_1) + f(x_2)$ (additivity)

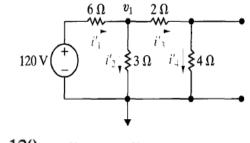


$$i_1 = i'_1 + i''_1 = 15 + 2 = 17 \text{ A},$$

$$i_2 = i'_2 + i''_2 = 10 - 4 = 6 \text{ A},$$

$$i_3 = i_3' + i_3'' = 5 + 6 = 11 \text{ A},$$

$$i_4 = i'_4 + i''_4 = 5 - 6 = -1 \text{ A}.$$



$$\frac{v_1 - 120}{6} + \frac{v_1}{3} + \frac{v_1}{2 + 4} = 0,$$

$$v_1 = 30 \text{ V.}$$

 $i'_1 = \frac{120 - 30}{6} = 15 \text{ A,}$

$$i_2' = \frac{30}{3} = 10 \text{ A},$$

$$i_3' = i_4' = \frac{30}{6} = 5 \text{ A}.$$

$$\begin{array}{c|c}
6\Omega & 2\Omega \\
\downarrow i_1'' & \downarrow i_2'' \\
\downarrow i_2'' & \downarrow 3\Omega & \downarrow i_4'' \\
\downarrow \downarrow 4\Omega & \downarrow 12A
\end{array}$$

$$\frac{v_3}{3} + \frac{v_3}{6} + \frac{v_3 - v_4}{2} = 0,$$

$$\frac{v_4 - v_3}{2} + \frac{v_4}{4} + 12 = 0.$$

$$i_1'' = \frac{-v_3}{6} = \frac{12}{6} = 2 \text{ A}$$

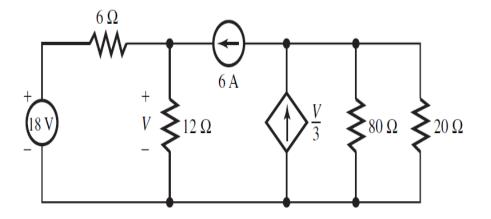
$$i_2'' = \frac{v_3}{3} = \frac{-12}{3} = -4 \text{ A}.$$

$$i_3'' = \frac{v_3 - v_4}{2} = \frac{-12 + 24}{2} = 6 \text{ A}$$

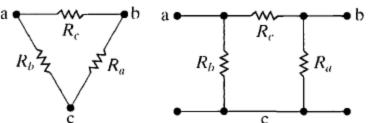
$$i_4'' = \frac{v_4}{4} = \frac{-24}{4} = -6 \text{ A}$$

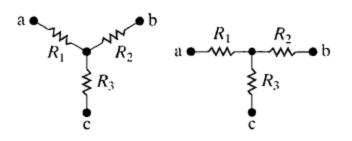
8. Super position theorem- example





9. Star-delta and delta-star





$$R_b$$
 R_c
 R_a

$$R_{ab} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2,$$

$$R_{\rm bc} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3,$$

$$R_{\rm ca} = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} = R_1 + R_3.$$

$$R_{1} = \frac{R_{b}R_{c}}{R_{a} + R_{b} + R_{c}},$$

$$\Delta \text{-to-Y} \qquad R_{2} = \frac{R_{c}R_{a}}{R_{a} + R_{b} + R_{c}},$$

$$R_{3} = \frac{R_{a}R_{b}}{R_{a} + R_{b} + R_{c}}.$$

$$R_{a} = \frac{R_{1}R_{2} + R_{2}R_{3} + R_{3}R_{1}}{R_{1}},$$

$$Y^{\text{-to-}} \Delta$$

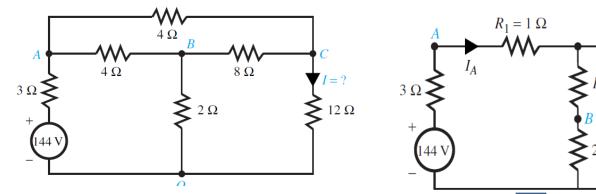
$$R_{b} = \frac{R_{1}R_{2} + R_{2}R_{3} + R_{3}R_{1}}{R_{2}},$$

$$R_{c} = \frac{R_{1}R_{2} + R_{2}R_{3} + R_{3}R_{1}}{R_{2}}.$$



9. Star- Delta transform Example





$$R_1 = \frac{4 \times 4}{4 + 8 + 4} = 1 \ \Omega$$

$$R_2 = \frac{4 \times 8}{4 + 8 + 4} = 2 \ \Omega$$

$$R_3 = \frac{4 \times 8}{4 + 8 + 4} = 2 \ \Omega$$

$$R_{1} = 1 \Omega$$

$$R_{2} = 2 \Omega$$

$$I_{A}$$

$$R_{3} = 2 \Omega$$

$$R_{3} = 2 \Omega$$

$$R_{3} = 2 \Omega$$

$$R_{3} = 2 \Omega$$

$$R_{4} = 0$$

$$R_{3} = 0$$

$$R_{4} = 0$$

$$R_{3} = 0$$

$$R_{4} = 0$$

$$R_{5} = 0$$

$$R_{7} = 0$$

$$R_{8} = 0$$

$$R_{1} = 0$$

$$R_{2} = 0$$

$$R_{3} = 0$$

$$R_{3} = 0$$

$$R_{4} = 0$$

$$R_{3} = 0$$

$$R_{4} = 0$$

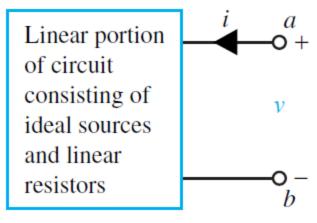
$$R_{5} =$$

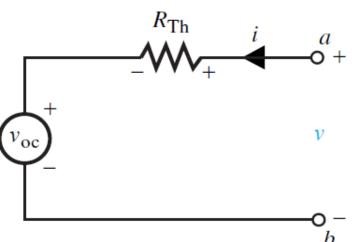
$$I_A = \frac{144}{(3+1)+(4\|14)} = \frac{81}{4} \text{ A}$$

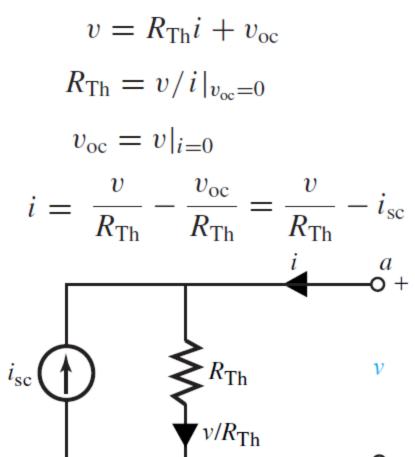
$$R_3 = \frac{4 \times 8}{4 + 8 + 4} = 2 \Omega$$
 $I = \frac{81}{4} \times \frac{4}{18} = \frac{9}{2} = 4.5 \text{ A}$

10. The Thevenin and norton theorem





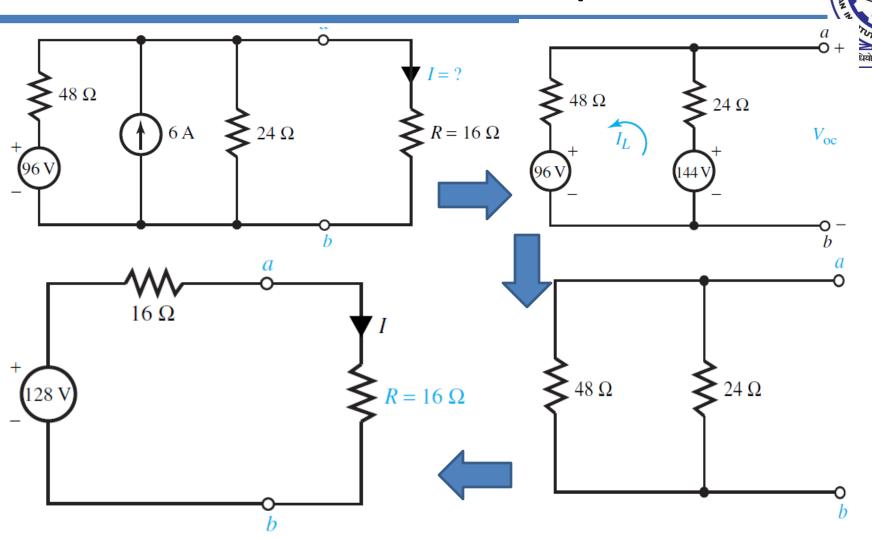




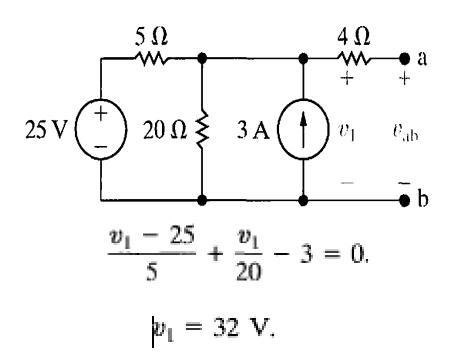
Thevenin Equivalent

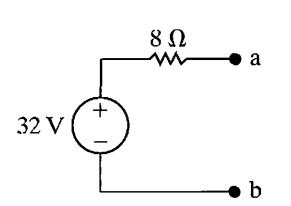
Norton Equivalent

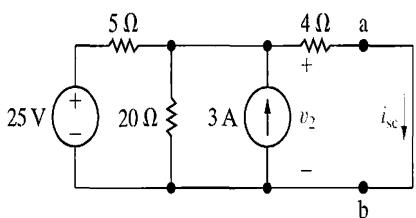
10. The Thevenin theorem - example



10. The Thevenin theorem - example







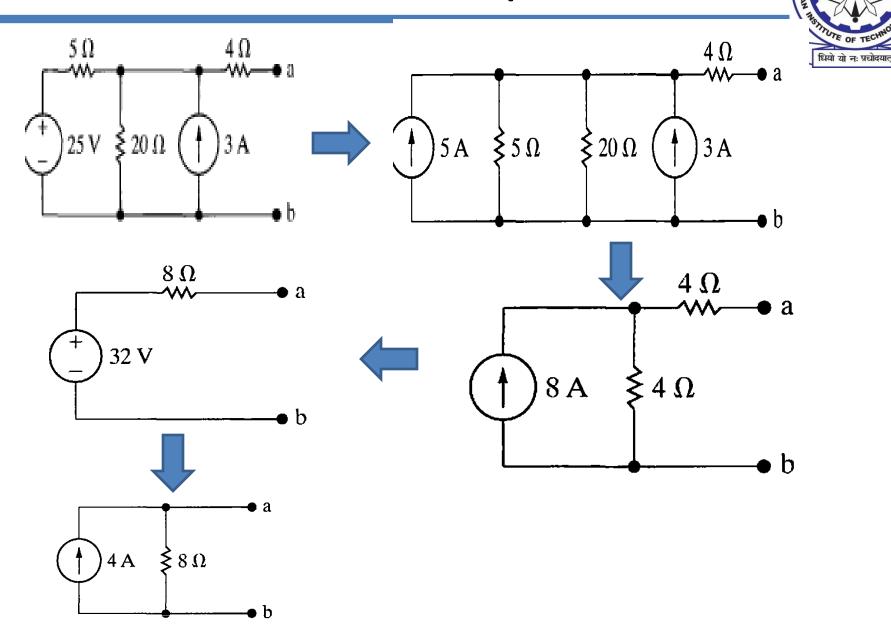
$$\frac{v_2-25}{5}+\frac{v_2}{20}-3+\frac{v_2}{4}=0.$$

$$v_2 = 16 \text{ V}.$$

$$i_{\rm sc} = \frac{16}{4} = 4 \text{ A}.$$

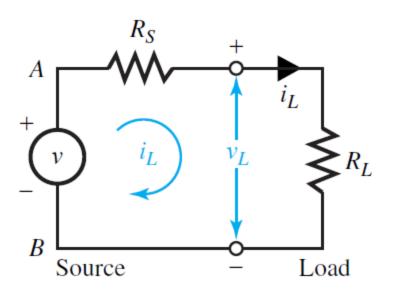
$$R_{\rm Th} = \frac{V_{\rm Th}}{i_{\rm re}} = \frac{32}{4} = 8 \ \Omega.$$

10. The Norton theorem - example



11. Maximum power transfer theorem





$$P_L = i_L^2 R_L$$

$$i_L = \frac{v^2}{R_S + R_L}$$

$$P_L = \frac{v^2}{(R_S + R_L)^2} R_L$$

$$\frac{dP_L}{dR_L} = \frac{v^2(R_L + R_S)^2 - 2v^2R_L(R_L + R_S)}{(R_L + R_S)^4} = 0$$

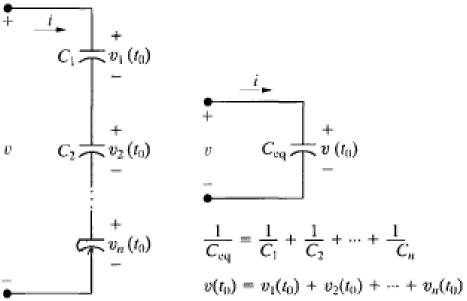
$$(R_L + R_S)^2 - 2R_L(R_L + R_S) = 0$$

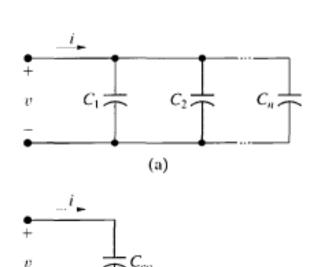
$$R_L = R_S$$

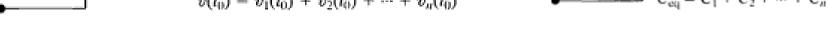
12. Capacitor Principle



$$\bullet \longrightarrow \stackrel{C}{\longleftarrow} \qquad i = C \frac{dv}{dt},$$



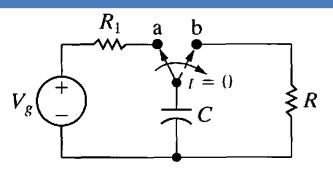




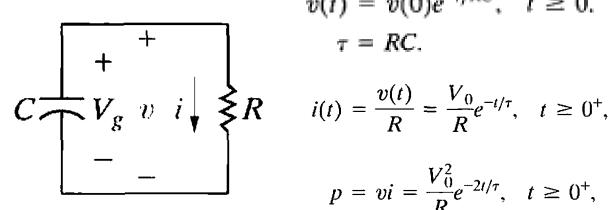
https://www.youtube.com/watch?v=f MZNsEqyQw

12. Natural response of RC circuit





$$\begin{cases}
R \\
C \frac{dv}{dt} + \frac{v}{R} = 0.
\end{cases}$$



$$v(t) = v(0)e^{-t/RC}, \quad t \ge 0.$$

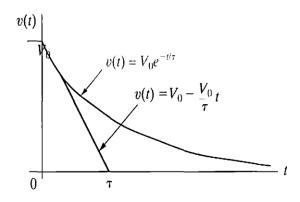
 $\tau = RC.$

$$i(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}, \quad t \ge 0^+,$$

$$p = vi = \frac{V_0^2}{R}e^{-2t/\tau}, \quad t \ge 0^+,$$

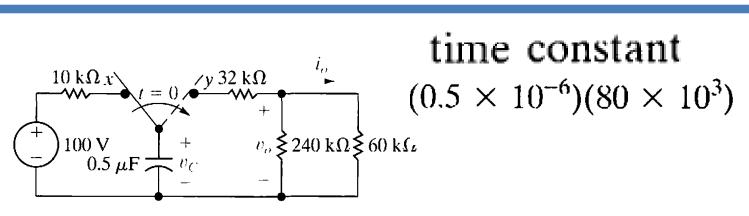
$$w = \int_0^t p \, dx = \int_0^t \frac{V_0^2}{R} e^{-2x/\tau} \, dx$$

$$=\frac{1}{2}CV_0^2(1-e^{-2t/\tau}), \quad t\geq 0.$$



12. Natural response of RC circuit - example





$$(0.5 \times 10^{-6})(80 \times 10^{3})$$

$$v_C(t) = 100e^{-25t} \,\mathrm{V}, \quad t \ge 0.$$

$$v_o(t) = \frac{48}{80}v_C(t) = 60e^{-25t} \text{ V}, \quad t \ge 0^+.$$

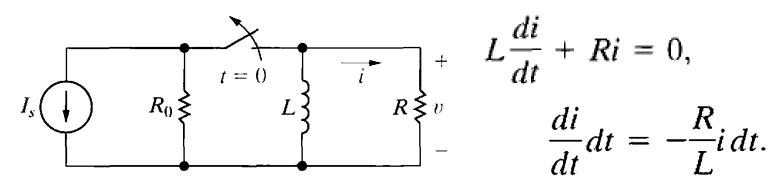
$$i_o(t) = \frac{v_o(t)}{60 \times 10^3} = e^{-25t} \,\mathrm{mA}, \quad t \ge 0^+.$$

$$p_{60k\Omega}(t) = i_o^2(t)(60 \times 10^3) = 60e^{-50t} \text{mW}, \quad t \ge 0^+.$$

$$w_{60\text{k}\Omega} = \int_0^\infty i_o^2(t)(60 \times 10^3) dt = 1.2 \text{ mJ}.$$

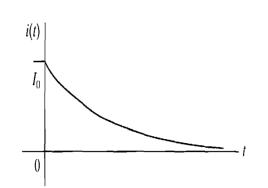
13. Natural response of RL circuit





$$i(0) = I_s \uparrow \begin{cases} \frac{1}{i} \\ L \end{cases} + \begin{cases} \frac{di}{i} = -\frac{R}{L}dt. & \ln \frac{i(t)}{i(0)} = -\frac{R}{L}t. \end{cases}$$

$$\frac{di}{i} = -\frac{R}{L}dt. \quad \ln\frac{i(t)}{i(0)} = -\frac{R}{L}t$$



$$i(t) = i(0)e^{-(R/L)t}$$
, $i(0^{-}) = i(0^{+}) = I_0$,

$$i(t) = I_0 e^{-(R/L)t}, \quad t \ge 0,$$

13. Natural response of RL circuit



$$i(0) = I_s \uparrow \begin{cases} 1 & 1 \\ 1 & 1 \end{cases}$$

$$p = I_0^2 R e^{-2(R/L)t}, \quad t \ge 0^+.$$

$$p = vi$$
, $p = i^2 R$, or $p = \frac{v^2}{R}$.

$$w = \int_0^t p dx = \int_0^t I_0^2 R e^{-2(R/L)x} dx$$
$$= \frac{1}{2(R/L)} I_0^2 R (1 - e^{-2(R/L)t})$$

 $=\frac{1}{2}LI_0^2(1-e^{-2(R/L)t}), \quad t\geq 0.$

13. Natural response of Parallel RLC circuit



$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} + C\frac{d^2v}{dt^2} = 0.$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0. \quad v = Ae^{st},$$

$$As^2e^{st} + \frac{As}{RC}e^{st} + \frac{Ae^{st}}{LC} = 0,$$

$$Ae^{st}\bigg(s^2+\frac{s}{RC}+\frac{1}{LC}\bigg)=0,$$

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0.$$

$$s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}},$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}.$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2},$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2},$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\zeta = \frac{\alpha}{\omega_0}$$

$$\begin{array}{c} 600 \\ 400 \\ -20$$

13. Natural response of series RLC circuit



$$\begin{array}{c|c}
R & L \\
\downarrow & I_0 \\
\downarrow & V_0 \\
-
\end{array}$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0.$$

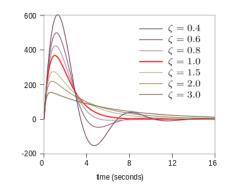
$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0.$$

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}},$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}.$$

$$\alpha = \frac{R}{2L} \text{ rad/s}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}.$$

$$\omega_0^2 < \alpha^2, \omega_0^2 > \alpha^2$$
, or $\omega_0^2 = \alpha^2$, overdamped, underdamped, or critically damped



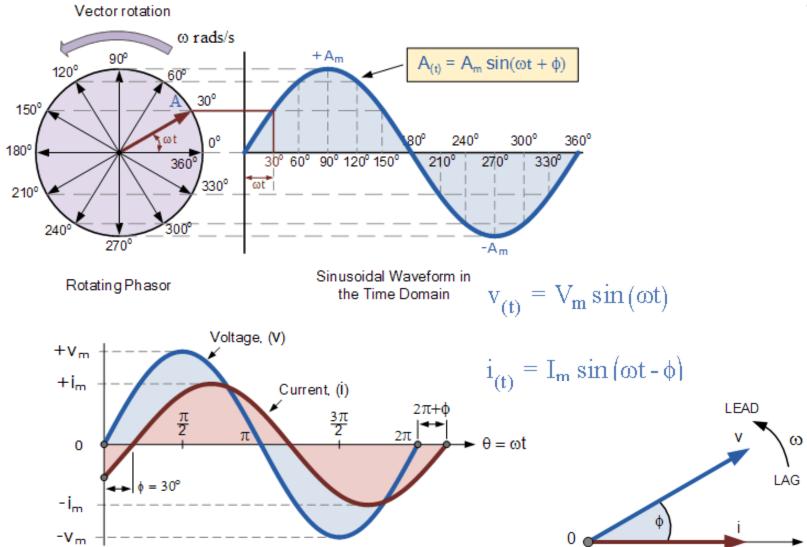
$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \text{ (overdamped)},$$

$$i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$$
 (underdamped).

$$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$
 (critically damped).

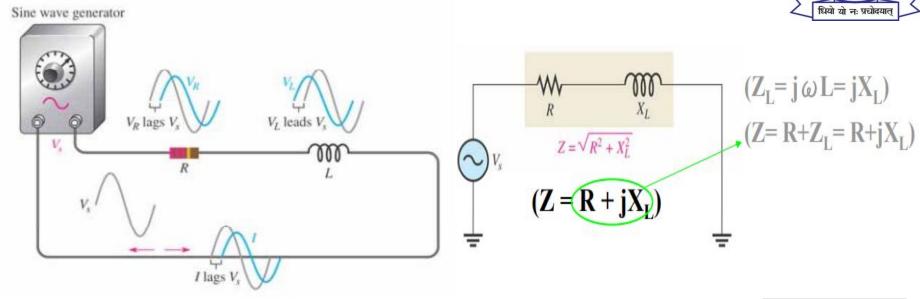
14. Single phase AC circuits

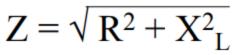


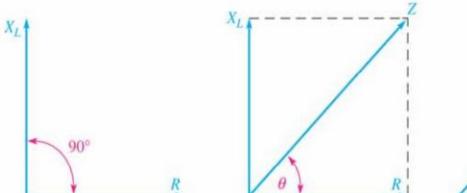


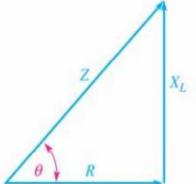
15. Series RL circuits











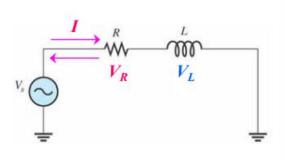
$$\theta = \tan^{-1}(X_L/R)$$

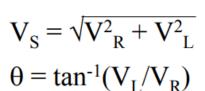
15. Relationship I and V in series RL circuit

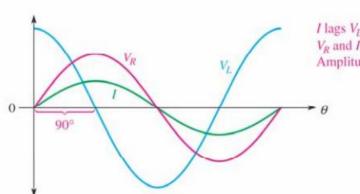


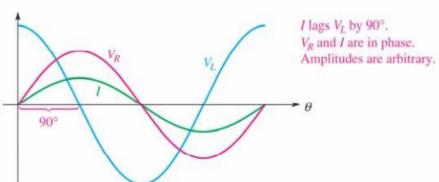
 $V_S = IZ = I(R + jX_L)$

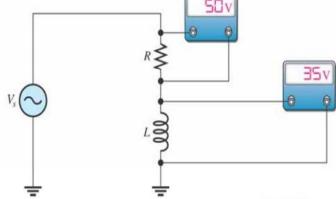
 $V_R = IR$





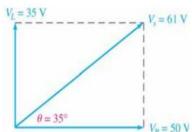




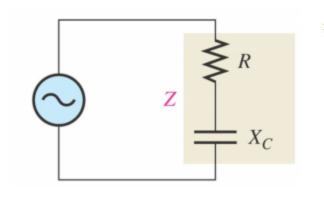


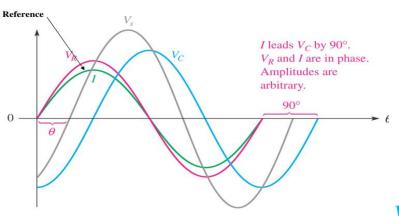
$$V_S = \sqrt{(50)^2 + (35)^2} = 61 \text{ V}$$

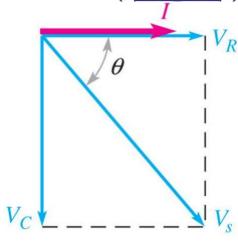
 $\theta = \tan^{-1}(35/50) = \tan^{-1}(0.7) = 35^\circ$



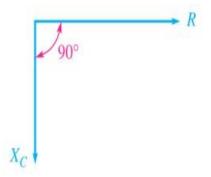
15. Series RC circuits

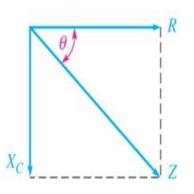


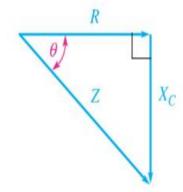




I_t is the used as the Reference Wave





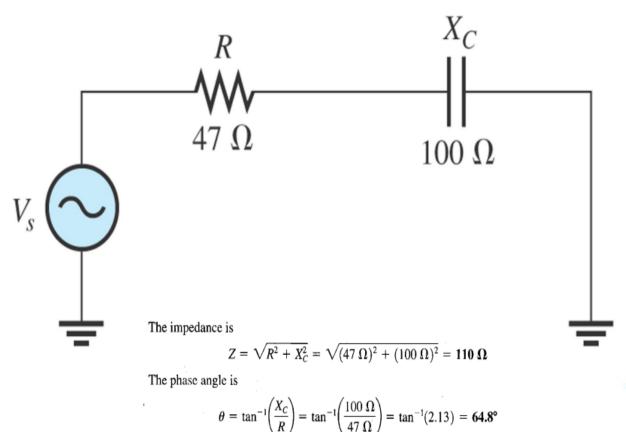


$$Z = \sqrt{R^2 + X^2_C}$$

$$\theta = \tan^{-1}(X_C/R)$$

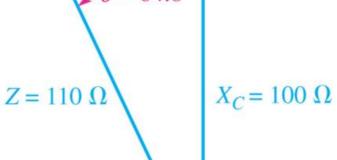
15. Series RC circuits- example





(11)

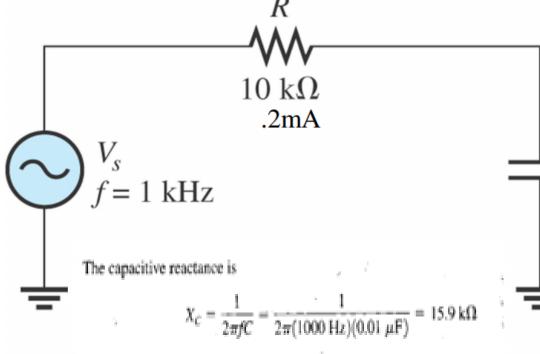
The source voltage lags the current by 64.8 Degrees



 $R = 47 \Omega$

15. Series RC circuits- example





The impedance is

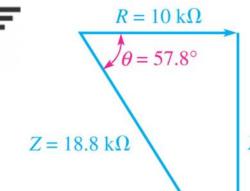
$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(10 \text{ k}\Omega)^2 + (15.9 \text{ k}\Omega)^2} = 18.8 \text{ k}\Omega$$

Applying Ohm's law yields

$$V_s = IZ = (0.2 \text{ mA})(18.8 \text{ k}\Omega) = 3.76 \text{ V}$$

The phase angle is

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{15.9 \text{ k}\Omega}{10 \text{ k}\Omega}\right) = 57.8^{\circ}$$



 $0.01 \, \mu F$

 $X_C = 15.9 \text{ k}\Omega$