Magnetic field of paired coils in Helmholtz arrangement

Aim of the experiment:

1. To measure the magnetic flux density along the x-axis (axial direction) of the circular coils when the distance between them $a = R$ (R = radius of the coils) and when $a > R$ and $a < R$.
2. To measure the radial component of the magnetic flux density density when the distance between coils $a = R$ as a function of axial coordinate.

Apparatus:

1. Pair of Helmholtz coils
2. Power supply, universal
3. Digital multimeter
4. Tesla-meter, digital
5. Hall probe, axial
6. Meter scale, demo, $l = 1000$ mm
7. Barrel base -PASS-
8. Support rod -PASS-, square, l250 mm
9. Connecting cord, 750 mm, blue
10. Connecting cord, 750 mm, red

Theory and Formulae used:

According to Biot-Savart law the magnetic field strength produced by the current element $Id\vec{l}$ at position $\vec{\rho}$ is given as

$$d\vec{H} = \frac{I}{4\pi}\frac{d\vec{l} \times \vec{\rho}}{\rho^3}, \tag{1}$$

where $\vec{\rho}$ is the vector from the conductor element $d\vec{l}$ to the observation point, and $d\vec{H}$ is perpendicular to both these vectors. The field strength along the axis of a circular coil

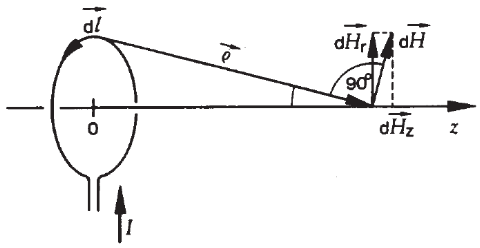

Figure 1: Sketch to aid calculation of the field strength along the axis of a wire loop.

can be calculated using Eq. (1). The vector $\vec{dl}$ is perpendicular to the plane of the sketch as shown in Fig. 1, whereas $\vec{\rho}$ and $d\vec{H}$ lie in its plane. Therefore

$$|d\vec{H}| = \frac{I}{4\pi}\frac{dl}{\rho^2}. \tag{2}$$

$d\vec{H}$ can be resolved into a radial component dH_r and an axial component dH_z. The dH_z components have the same direction for all the conductor elements $\vec{dl}$ and add to each other, whereas the dH_r components of the diametrically opposite conductor elements of length dl cancel each other. Therefore, net radial component of the field along the axis of the coil is

$$H_r = 0. \tag{3}$$

And, the net magnetic field strength along the axis of the coil is the sum total of axial components of the fields

$$\begin{aligned} H &= H_z = \oint_P dH_z = \oint_P \frac{I}{4\pi}\frac{dl\sin\theta}{\rho^2}, && (4)\\ &= \oint_P \frac{I}{4\pi}\frac{dlR}{\rho^3}, && (5)\\ &= \frac{I}{2}\frac{R^2}{(z^2+R^2)^{3/2}}, && (6)\\ & && (7) \end{aligned}$$

where $\oint_P$ represents the integral over the perimeter of the coil and θ is the angle between $d\vec{H}$ and $d\vec{H}_r$ and is same for all the conductor elements of the wire loop. Hence, magnetic flux density along the axis of the wire loop is

$$\begin{aligned} B(z) &= \frac{\mu_0 I}{2}\frac{R^2}{(z^2+R^2)^{3/2}}, && (8)\\ &= \frac{\mu_0 I}{2R}\frac{1}{(1+(z/R)^2)^{3/2}}. && (9) \end{aligned}$$

The magnetic field of a flat coil is obtained by multiplying Eq. (6) by the number of turns N. Therefore, the magnetic flux density along the axis of two identical coils at a distance a apart is

$$B(z, r = 0) = \frac{\mu_0 I N}{2R}\left[\frac{1}{(1 + A_1^2)^{3/2}} + \frac{1}{(1 + A_2^2)^{3/2}}\right], \tag{10}$$

where

$$A_1 = \frac{z + a/2}{R}, \ A_2 = \frac{z - a/2}{R}. \tag{11}$$

When $z = 0$, flux density has a maximum value when $a < R$ and a minimum value when $a > R$. The curves plotted from our measurements also show this when $a = R$, the field is virtually uniform in the range

$$\frac{-R}{2} < z < \frac{+R}{2}. \tag{12}$$

Magnetic flux density at mid-point when $a = R$ is

$$\begin{aligned} B(z = 0, r = 0) &= \frac{\mu_0 I N}{2R}\frac{2}{(5/4)^{3/2}}, &(13)\\ &= 0.716\frac{\mu_0 I N}{2R} &(14)\end{aligned}$$

Procedure:

1. Connect the coils in series and in the same direction, (see Fig. 2). The current must not exceed 3.5 A (operate the power supply as a constant current source). Measure the flux density with the axial Hall probe (measures the component in the direction of the probe stem) as shown in Fig. 3.

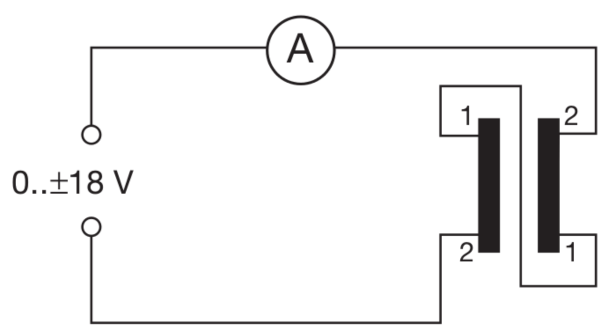

Figure 2: Wiring diagram for Helmholtz coils..

2. The magnetic field of the coil arrangement is rotationally symmetrical about the axis of the coils, which is chosen as the z-axis of a system in cylindrical coordinates (z, r, ϕ). The origin is at the center of the system. The magnetic flux density does not depend on the angle ϕ so only the components $B_z(z, r)$ and $B_r(z, r)$ are measured.

3. Clamp the Hall probe on to a support rod with barrel base, level with the axis of the coils. Secure two rules to the bench (parallel or perpendicular to one another, see Figs. 3 and 4). The spatial distribution of the magnetic field can be measured by pushing the barrel base along one of the rules or the coils along the other one. Always push the barrel base bearing the Hall probe along the rule in the same direction.

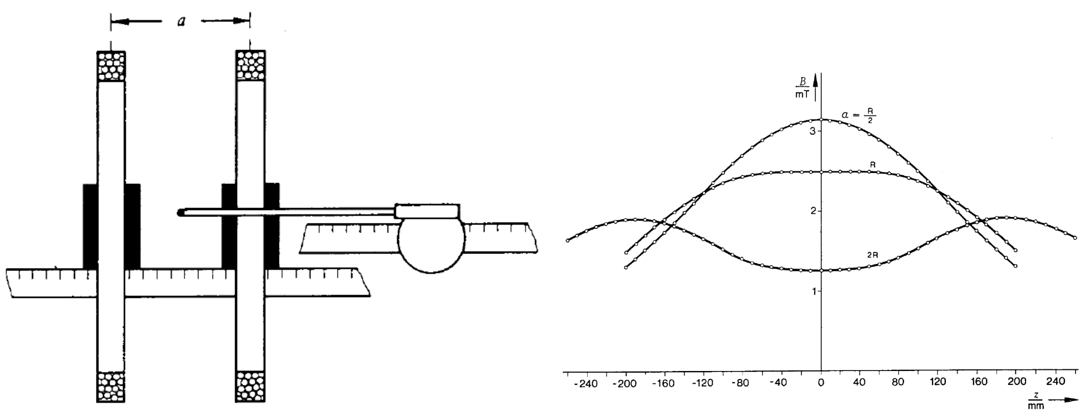

Figure 3: Figure on the left shows how to set up the coils, probe and rules to measure the axial component of the field. The edge of the bench can be used instead of the lower rule if required. Measure $B(z, r = 0)$ when the distance between the coils $a = R/2, R, 2R$ to obtain the plot as shown in figure on the right.

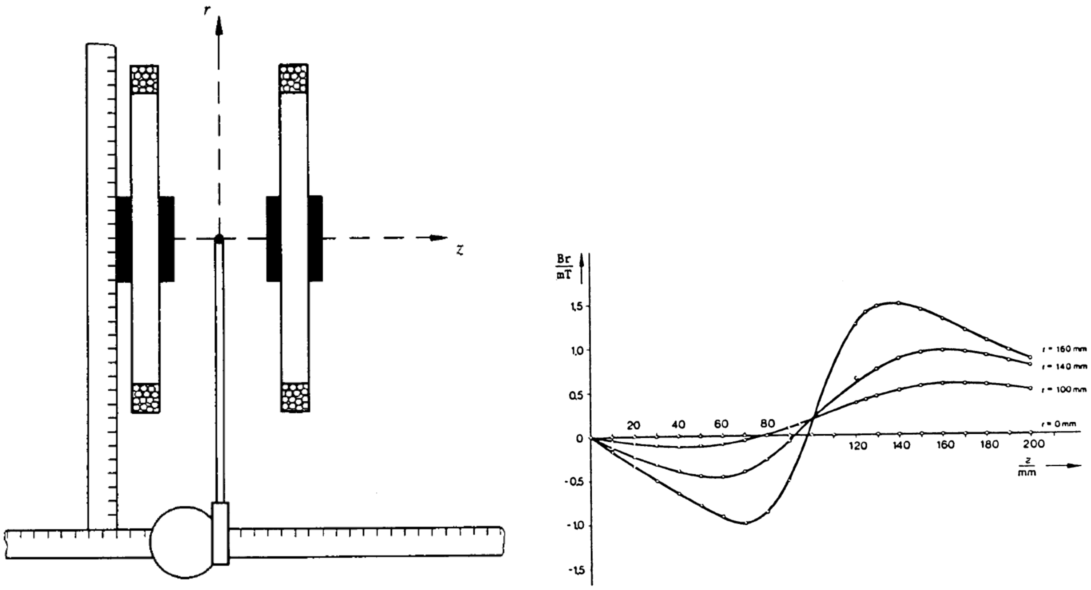

Figure 4: Figure on the left shows how to set up the coils, probe and rules to measure the radial component of the field. Measure $B_r(z, r = r_0)$ as a function of z at different radial distances r_0 from the axis when the distance between the coils $a = 2R$ to obtain the plot as shown in figure on the right.

Table I						
Sr. No.	$a = R$	$a = R$	$a = R/2$	$a = R/2$	$a = 2R$	$a = 2R$
	z mm	B_z (mT)	z mm	B_z (mT)	z mm	B_z (mT)
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
⋮	⋮	⋮	⋮	⋮	⋮	⋮
25						

Table II					
Sr. No.	z mm	$r = 0$ mm, B_z (mT)	$r = 100$ mm, B_z (mT)	$r = 140$ mm, B_z (mT)	$r = 160$ mm, B_z (mT)
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
25					

Precautions:

1. Keep the Tesla-meter at zero before starting.
2. Current in the coil should not exceed 3.5 Ampere.
3. The scale should be kept straight.