Arizona State University

Energy-efficient powertrain control of a hybrid vehicle through traffic jams

Project Report

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MAE 598: Design Optimization

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Abstract:

There are about 1.2 billion active vehicle on the road whose number is set to increase to about 2 billion by 2035. To meet stringent emission norms, environmental and energy concerns, vehicles have to reduce their CO_2 emissions. Hybrid Electric Vehicles contributes to fuel savings and emission reduction aims, which ultimately reduces energy consumption. The aim of this project is to develop efficient powertrain control for hybrid vehicles to reduce energy consumption though heavy traffic. The project involves optimization of two subsystems, namely the IC engine and electric motor. The approach is to reduce fuel consumption by switching between electric drive and internal combustion engine during traffic scenario. The change in acceleration helps to find the velocity which in turn determines the position of the following vehicle which is also dependent on the vehicle ahead of the following vehicle. This calculation considers the efficiency of electric drive and internal combustion engine as well at different velocities. We have considered parallel configuration of the drivetrain in the hybrid electric vehicle. In this configuration, both electric motor and IC engine are mechanically coupled to drive the wheels.

Github: https://github.com/harshalshah7b/PowertrainHybridVehicleTraffic

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Nomenclature:

IC	-	-	Internal Combustion
E	watts	-	Energy
P_{ICE}	watts	-	Power of internal combustion engine
η_{ICE}	-	-	Efficiency of internal combustion engine
$\dot{m}_f$	kg/s	-	Mass flow rate of fuel
cv	kJ/kg	-	Calorific value of fuel
τ_{ICE}	N-m	-	Torque of IC engine
ω	rad/s	-	Angular Velocity
x_1	m	-	Position of vehicle ahead of following vehicle
x_2	m	-	Position of following vehicle
x_c	m	-	Distance between following vehicle and vehicle ahead of following vehicle
a_1	m/s^2	-	Acceleration of vehicle ahead of following vehicle
a_2	m/s^2	-	Acceleration of following vehicle
v_1	m/s	-	Velocity of vehicle ahead of following vehicle
v_2	m/s	-	Velocity of the following vehicle
ρ	-	-	Density of vehicles
t	s	-	Time
r	m	-	crankshaft radius
V	m/s	-	Tangential Velocity of the crankshaft

Design Problem Statement:

To minimize energy consumption, the problem is divided into 3 different subsystems:

1. Considering the internal combustion engine only
2. Follow the leader ahead following vehicle approach in IC engine
3. Considering internal combustion engine along with an electric motor drive

First, we are considering only internal combustion engine for optimizing the system by reducing energy consumption. Also, we are considering ”follow the leader” traffic model to get the variables. In this system, we are defining a range for the distance between following vehicle and leading vehicle. If the distance is outside of the range, control algorithm would send control to vehicle accelerator to speed up or down reasonably.

After first part is complete, we moved to the second part of the project which is to include an electric motor drive along with IC engine for vehicle actuation and considering ”follow the leader” traffic model. In this part of the project, efficiencies for both the drives are calculated at different velocities. After getting velocity and efficiency of both the drives for that velocity, best actuation method will be chosen for minimizing the energy consumption.

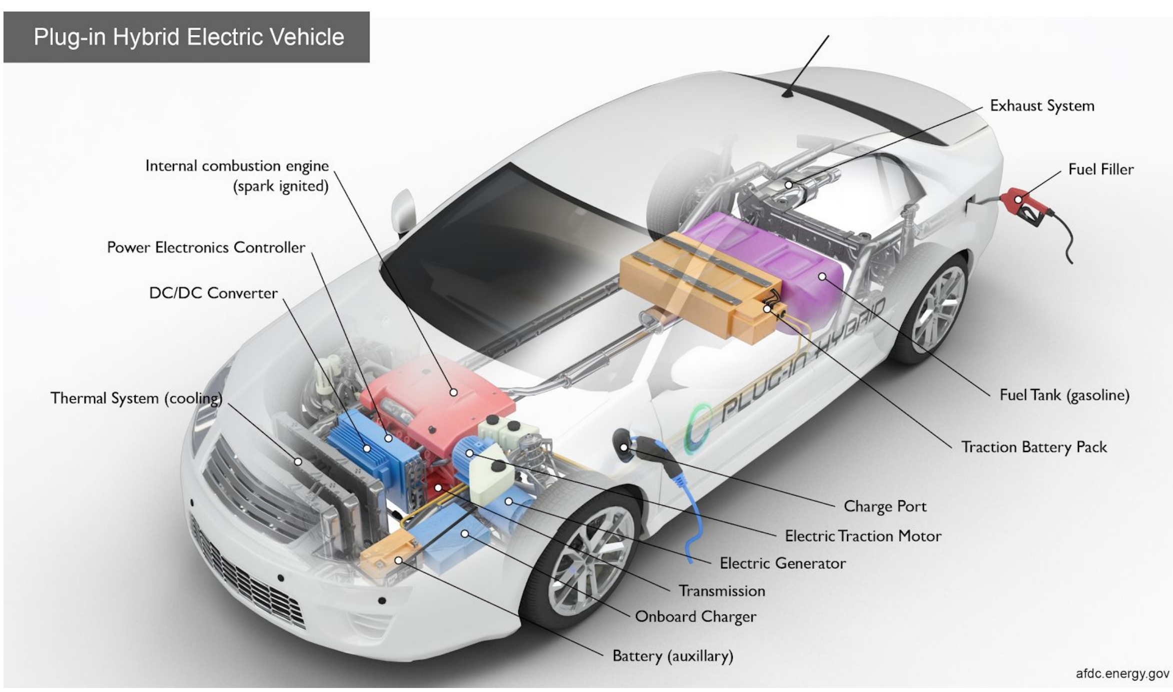

Fig.(1) Hybrid electric vehicle

Types of Power-trains:

Series Hybrid Drive-trains

Series drivetrains are very simple in terms of configuration. The electric motor is the only means of providing power to the wheels in a series hybrid. The motor receives electric power from either the battery pack or from a generator run by a gasoline engine.

A computer regulates the power magnitude coming from the battery or the engine/generator. Both the engine/generator and the use of regenerative braking recharge the battery pack.

Gasoline and diesel engines are inefficient during stop-and-go traffic whereas series hybrids perform at their best during stop-and-go traffic. The vehicle's computer can opt to power the motor with the battery pack only, saving the engine for situations where it's more efficient.

There is a smaller engine in a series drivetrain because it only has to meet certain power demands; the battery pack is generally more powerful than the one in parallel hybrids in order to provide the remaining power needs. This larger battery and motor, along with the generator, add to the vehicle's cost, making series hybrids more costly as compared to parallel hybrids.

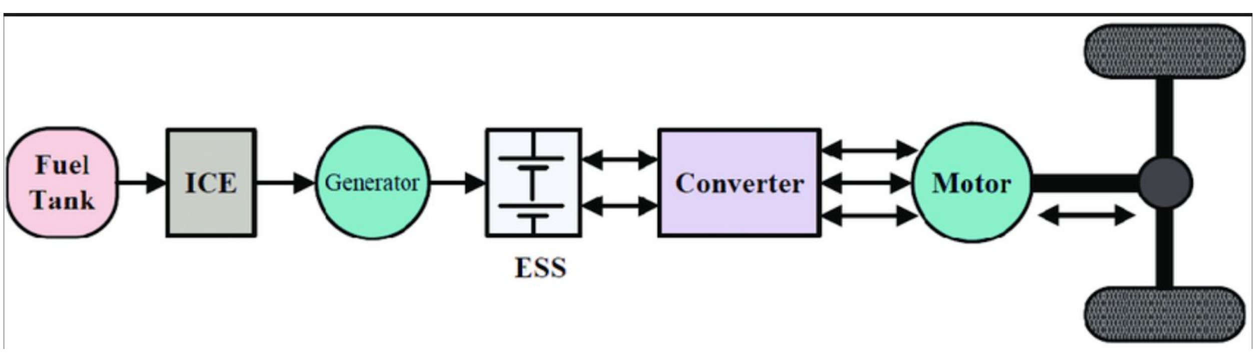

Fig.(2) Series hybrid powertrain

Parallel Hybrid Drivetrains

In parallel hybrid drivetrains, the engine and electric motor work together to generate the power that drives the wheels. Parallel hybrids generally use a smaller battery pack as compared to series drivetrains, charging through regenerative braking. Parallel hybrids use the motor as a generator for supplemental re-charging when power demands are low,, much like an alternator in conventional cars.

Since the engine is connected directly to the wheels in parallel drivetrains, the efficiency of converting mechanical power to electricity and back is eliminated, increasing the efficiency of these hybrids on the highway. This reduces, but does not eliminate the efficiency benefits of having an electric motor and battery in stop-and-go traffic.

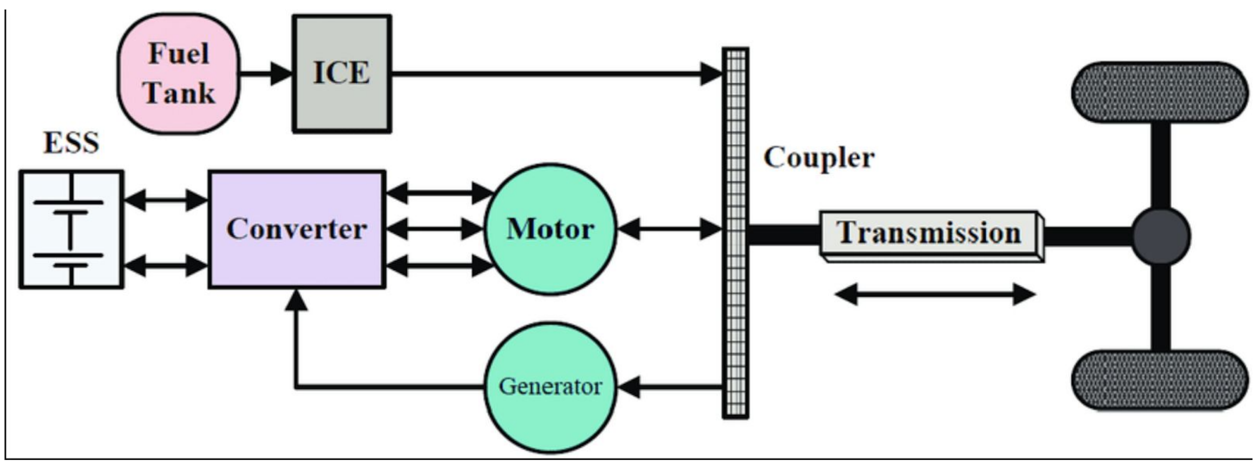

Fig.(3) Parallel hybrid powertrain

Series/Parallel Drive trains

Series/parallel drivetrains merge the advantages and complications of the parallel and series drivetrains. By combining the two designs, the engine can both drive the wheels directly(as in parallel drivetrain), and be effectively disconnected, with only electric motor providing power (as in the series drivetrain). The Toyota Prius helped make series/ parallel drivetrains a popular design.

With gas only and electric only options, the engine operates at near optimum efficiency more often. At lower speeds it operates more as a series vehicle, while at high speeds, where the series drivetrains is less efficient, the engine takes over and energy loss is minimized.

This system incurs higher costs than pure parallel hybrid since it requires a generator, a larger battery pack, and more computing power to control the dual system. Yet its efficiencies mean that the series/ parallel drivetrain can perform better-and use less fuel-than either the series or parallel systems alone.

State of Charge (SoC) State:

It is the equivalent of a fuel gauge for the batter pack in a battery electric vehicle (BEV), hybrid vehicle (HV), or plug-in hybrid electric vehicle (PHEV). The units of SoC are percentage points (0% = empty; 100% = full). An alternate form of the same measure is the depth of discharge (DoD), the inverse of SoC (100% = empty; 0% = full). SoC is normally used when discussing the current state of a battery in use, while DoD is most often seen when discussing the lifetime of the battery after repeated use.

Determining SoC

Usually, SoC cannot be measured directly but it can be estimated from direct measurement variables in two ways: offline and online. In offline techniques, the battery desires to be charged and discharged in constant rate such as Coulomb-counting. This method gives precise estimation of battery SoC, but they are protracted, costly, and interrupt main battery performance.

Voltage method for SoC

This method converts a reading of the battery voltage to SoC, using the known discharge curve (voltage vs. SoC) of the battery. However, the voltage is more significantly affected by the battery current (due to the battery's electrochemical kinetics) and temperature. This method can be made more accurate by compensating the voltage reading by a correction term proportional to the battery current, and by using a look-up table of batteries open circuit voltage vs. temperature.

In fact, it is a stated goal of battery design to provide a voltage as constant as possible no matter the SoC, which makes this method difficult to apply.

Mathematical model:

The energy consumed by the vehicle is the power generated by the internal combustion engine ($P_{ICE}(t)$). The power of the engine will vary between maximum and zero.

$$0 \leq P_{ICE}(t) \leq P_{ICE_{max}}, \ \forall \ t$$

We also have to take into account the efficiency (η_{ICE}) of the IC engine. The efficiency value ranges from 0 to 1.

$$0 \leq \eta_{ICE} \leq 1$$

The energy (E) consumed by an IC engine with respect to time can be found out by calculating the integral,

The new Mathematical model,

$$E = \int_0^t \frac{P_{ICE}(t)}{\eta_{ICE}} dt \tag{1}$$

$$\begin{aligned} P_{ICE} &= \dot{m}_f * CV \qquad \textit{where}, CV = \text{calorific value} \\ &= \tau_{ICE} * \omega * CV \end{aligned}$$

The overall objective function becomes

$$E = \int_0^t \frac{\tau_{ICE} * \omega * CV}{\eta_{ICE}} dt$$

η_{ICE} & CV are constant so the new function is defined as

$$E = f(\tau_{ICE}, \omega)$$

The objective function is derived with the help of given engine data of Toyota Prius and these data points are fit into the curve fitting app in Matlab to get the objective function.

The state of charge of the battery is represented as SOC. The SOC varies will vary between the maximum and minimum value.

$$SOC_{min} \leq SOC(t) \leq SOC_{max}, \ \forall \ t$$

When the power is supplied from the electric motor, it will be the energy consumed by the vehicle.

Taking into account the efficiency of the electric motor,

$$E = \int_0^t \frac{P_{EM}(t)}{\eta_{EM}} \cdot dt \qquad 0 \leq \eta_{EM} \leq 1 \qquad (2)$$

Assumptions:

- The vehicle will be traveling in the same lane throughout this drive cycle
- The maximum speed of the traffic to be less than 65 mph (29.057 m/s)
- There is a traffic density model which is being defined for the vehicles in the system
- The gear ratio is constant
- The data for the traffic density model is taken as random variables to generate the outputs and the code uses this data to get various output plots
- The test vehicle and the following vehicle always have a safe distance between them and this is maintained throughout the traffic model

Objective function:

Our objective is to minimize the fuel consumption and that can be achieved by minimizing the energy (power) consumption of the IC engine. Energy consumption of IC engine is Power delivered by IC engine divided by IC engine efficiency over the time.

$$min\,(fuel\ consumption) \Rightarrow min\,(energy\ consumption)$$
$$\Rightarrow min\ E = f(\tau_{ICE}, \omega)$$

Power of IC engine(P_{ICE}) is the product of mass flow rate of IC engine($\dot{m}_f$) and calorific value of the fuel (cv). Again, mass flow rate($\dot{m}_f$) is the product of torque delivered by IC engine(τ_{ICE}) and angular velocity of IC engine(ω) divided by IC engine efficiency(η_{ICE}). Here we divide torque by efficiency in order to consider the losses due to the efficiency of the engine.

IC engine efficiency(η_{ICE}) is known to us. Also, the angular velocity of IC engine(ω) can be found by the following equation,

$$\omega = v/r \qquad (3)$$

where,

$v = v_2 =$ following vehicle velocity

$r =$ Wheel radius

The calorific value(cv) is the value for the fuel used in the IC engine. It is a constant value. The calorific value for diesel is 45.8 MJ/kg and for gasoline is 45.5 MJ/kg and $\dot{m}_f$ is defined as the mass flow rate of fuel in the IC engine.

Constraints:

The torque of the vehicle will be within the range of the maximum and the minimum,

$$T_{min} \leq T(t) \leq T_{min} , \forall t \tag{4}$$

Similarly, the RPM(N) will also be within the range of the maximum and the minimum,

$$N_{min} \leq N(t) \leq N_{min} , \forall t \tag{5}$$

The distance between the test vehicle and the vehicle ahead of it is defined in a range,

$$x_2 - x_1 = [x_c] \tag{6}$$

x_1 is the position of the following vehicle

x_2 is the position of the vehicle ahead of the leading vehicle

x_c is the distance between the following vehicle and the leading vehicle

Where $[x_c] = [3 : 5]$ meters. We have defined a range for x_c so that distance between two vehicles can be used for control algorithm to define acceleration or deceleration. For example, if x_c is reaching close to the lower limit, the vehicle would have to decrease the speed and is if x_c is reaching close to the upper limit, the vehicle would have to increase the speed.

Variable:

Acceleration of the following vehicle is variable. Acceleration of the following vehicle depends on the accelerator pedal of the vehicle. The position of the pedal will decide the rate of acceleration of the vehicle. The position range of pedal lies between [0,1] where 0 means the pedal is at its original position and 1 being the angle of absolute displacement.

By integrating acceleration, we can find the velocity of the vehicle for a given time.

$$v_2 = \int a_2 \, dt \qquad (7)$$

where,
v_2 - velocity of the following vehicle
a_2 - acceleration of following vehicle
We can use v_2 in equation (iv) to get angular velocity which will eventually give us IC engine power.

By integrating velocity, we can find the position variable x_2 for given

$$x_2 = \int v_2 dt$$

where,
x_2 - position of the following vehicle

We use this x_2 in equation (vi) in constraints to get the distance between vehicles.

We are varying torque of the engine in simulation to get the optimized mass flow rate in traffic.

Partial Differential Equation (PDE) for traffic flow:

We obtain the PDE model for traffic flow from the Lighthill-Whitham conservation law describing traffic flow. Mentioned below are a few characteristics of the model

- Alone on the road ⇒ drive with speed limit
- Bumper to bumper ⇒ complete clogging
- In between, a linear function is followed

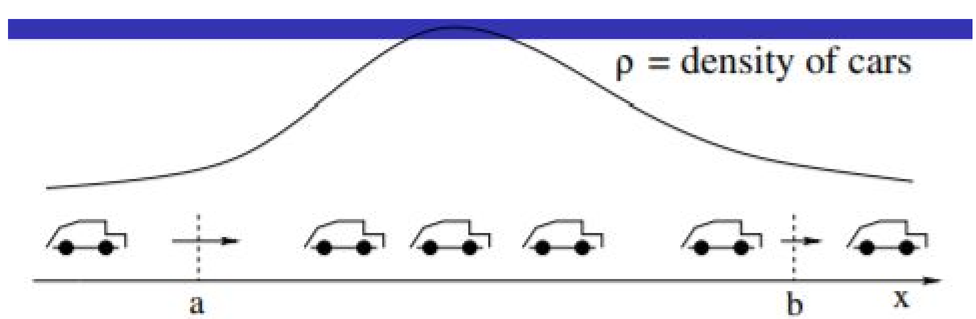

Fig. 4: Traffic Density Model

The traffic density model is depicted only by the number of cars entering and leaving the system and independent of the total cars in the system. The entry point is defined as ‘a’ and the exit point is defined as ‘b’ and the cars within those ranges are considered for the model.

Here, t = time, x = space variable along road, ρ = ρ(t, x) = density of vehicles

flux = [number of vehicles crossing the point x per unit time]
= [density] × [velocity]

Assumption: velocity of vehicles depends only on their density: v = v(ρ)

The total number of vehicles conserved:
The model is defined as the integral of traffic density function ranging from the entry to the exit point.

$$\frac{d}{dt}\int_a^b \rho\,(t,x)\,dx = \text{[flux of vehicles entering at a]} - \text{[flux of vehicles exiting at b]}$$

$$= \rho(t, a) \cdot v(\rho(t, a)) - \rho(t, b) \cdot v(\rho(t, b))$$

$$\int_a^b \frac{\partial}{\partial t}\rho\, dx = - \int_a^b \frac{\partial}{\partial t}\,[\rho\; v(\rho)]\,dx$$

$$\therefore\; \frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}\,[\rho \cdot\; v(\rho)] = 0 \tag{8}$$

By solving the above equation, we will get the velocity of flow ($v(\rho)$) as traffic density.

- The velocity of the vehicle is determined by the formula below

$$v = v_{max} * (1 - \frac{\rho}{\rho_{max}}) \tag{9}$$

Modal Approach:

The traffic model plays a pivotal role in defining the whole system apart from the vehicle specifications. The data for both have been mentioned below:

Vehicle data:

Vehicle Name = Toyota Prius
Vehicle length = 4.48 meter
Vehicle weight = 1805 kg
Gear ratio =2.683
Final drive ratio = 3.267
Wheel radius = 0.3175 m
Maximum Power = 73 kW

PDE traffic model assumptions:

Number of iterations (time steps) = 100
Road length for the system = 500 meter
The safe distance between 2 vehicles in the traffic flow = 3 meter
Maximum vehicle velocity = 29.0576 m/s = 65 mph
Engine Efficiency = 0.80

Mathematical Equations:

- **Net Gear Ratio(NGR) = Gear Ratio (GR) x Differential Gear Ratio (FDR)**
  Both the gear ratios data has been taken from the Prius specifications found online.

  $$NGR = GR * FDR \qquad (10)$$

- **Wheel rpm = (Velocity x 60)/(2 x π x Wheel radius)**
  The velocity is extracted from the PDE model and the wheel radius which can be referenced from the vehicle data will combine to give the rpm of the individual wheel.

  $$N = \frac{v*60}{2\pi r} \qquad (11)$$

- **Engine rpm = Wheel rpm x Net Gear Ratio**
  The engine rpm is defined by the wheel rpm and net gear ratio, both of which have been derived above.
  $$N_{engine} = N * NGR \qquad (12)$$

- **Engine torque = Power (P) x Engine rpm**
  The power of the engine is taken from the data and maximum power is assumed for the given optimization along with the engine rpm.
  $$\tau_{engine} = p * N_{engine} \qquad (13)$$

- **Wheel torque = Engine Torque x Net Gear ratio x 0.5**
  All the variables are derived above and 0.5 is a constant value from which we get the wheel torque.
  $$\tau_{wheel} = \tau_{engine} * NGR * 0.5 \qquad (14)$$

- **Velocity of follower vehicle:**
  The velocity of vehicle can be calculated from the following formula:
  $$v_f = \frac{\tau}{NGR}\frac{2\Pi}{60} * r_\omega \qquad (15)$$

- **Distance between vehicles:**
  The distance 'd' is the distance between two consecutive vehicles is defined as
  $$d = (v * t_r) \qquad (16)$$

We obtain the velocity of the leading vehicle from the PDE traffic model and the velocity of our car from optimization.

The time step is taken of one second. The distance is calculated by multiplying velocity with time step.

$$d = \int v * (dt) \ = \sum v * (1) = v \ meters$$

The inequality constraints for the problem are listed below:

Let the distance between two vehicles be greater or equal to safe distance(3 meters in our case)

$$d_l - d_f \geq 3$$

To make the constraint into null negative form, we multiply both sides by negative sign,

$$-d_l + d_f \leq -3 \tag{17}$$

Also let's assume that the maximum distance between two cars is constrained to some distance (5 meters in our case);

$$d_l - d_f < 5 \tag{18}$$

We write the constraints in equation (17) and (18) in the form of system of linear equations,

$$Ax = B$$

where,

$$A = \left[-d_l \quad d_f \ ; \ d_l \ -d_f\right]$$

$$B = [-3; \ 5]$$

These are our linear inequality constraints for optimization.

We got our objective fun $f(\tau_{ICE}, \omega)$ from curve fitting app in Matlab. We used "enginedata.mat" file given by instructor and fed it to the curve fitting app.

We are using **fmincon** function in matlab to optimize our model.

Results and inferences:

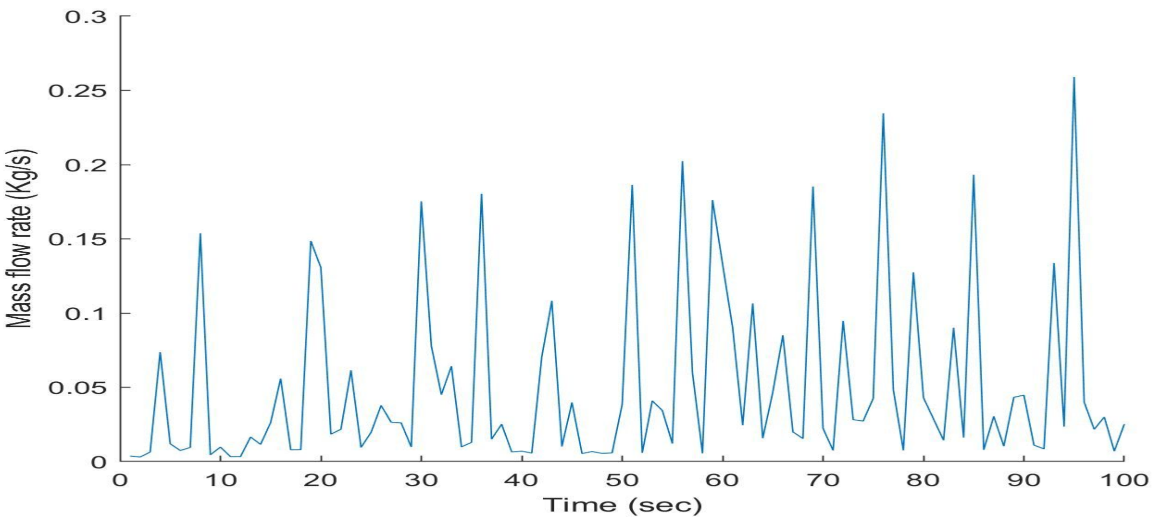

Fig.(5) Mass flow rate v/s time

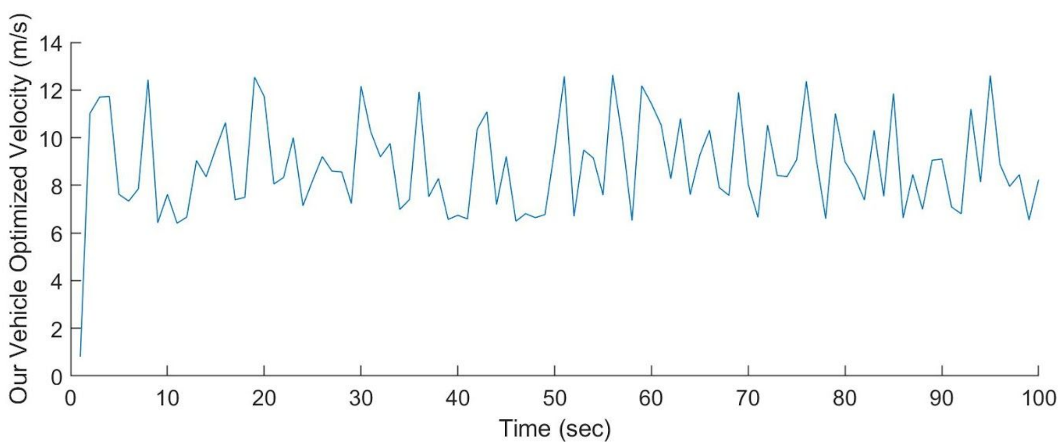

Fig.(6) Drive cycle (Vehicle velocity v/s time)

Our graphs shows that as road traffic increases, velocity of the vehicle decreases and so does the mass flow rate. Mass flow rate increases as velocity of the vehicle increases.

Our maximum allowable velocity on road traffic is 29.057 m/s and optimized velocity in moderate traffic (we constrained generation of random variables to get some specific range of density) is equal to or below 12.6316 m/s.

ECMS Results:

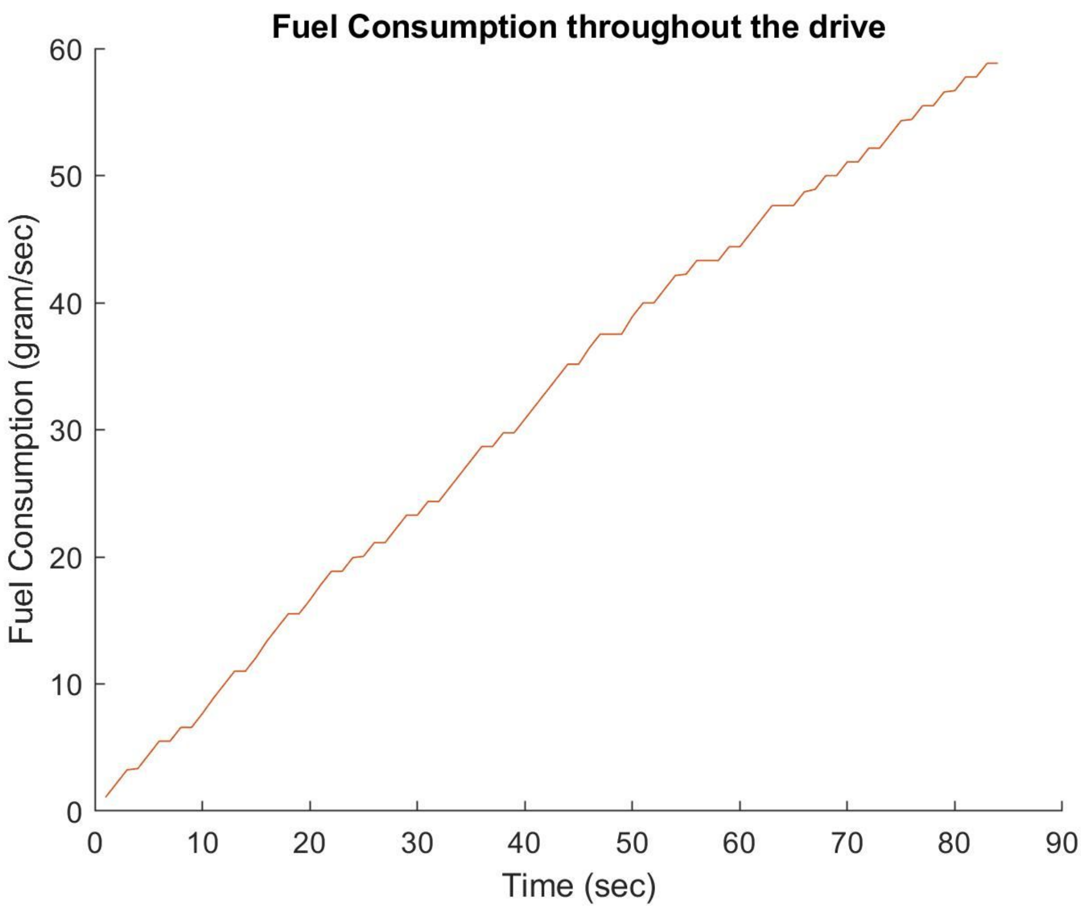

Fig.(7) ECMS fuel consumption v/s time

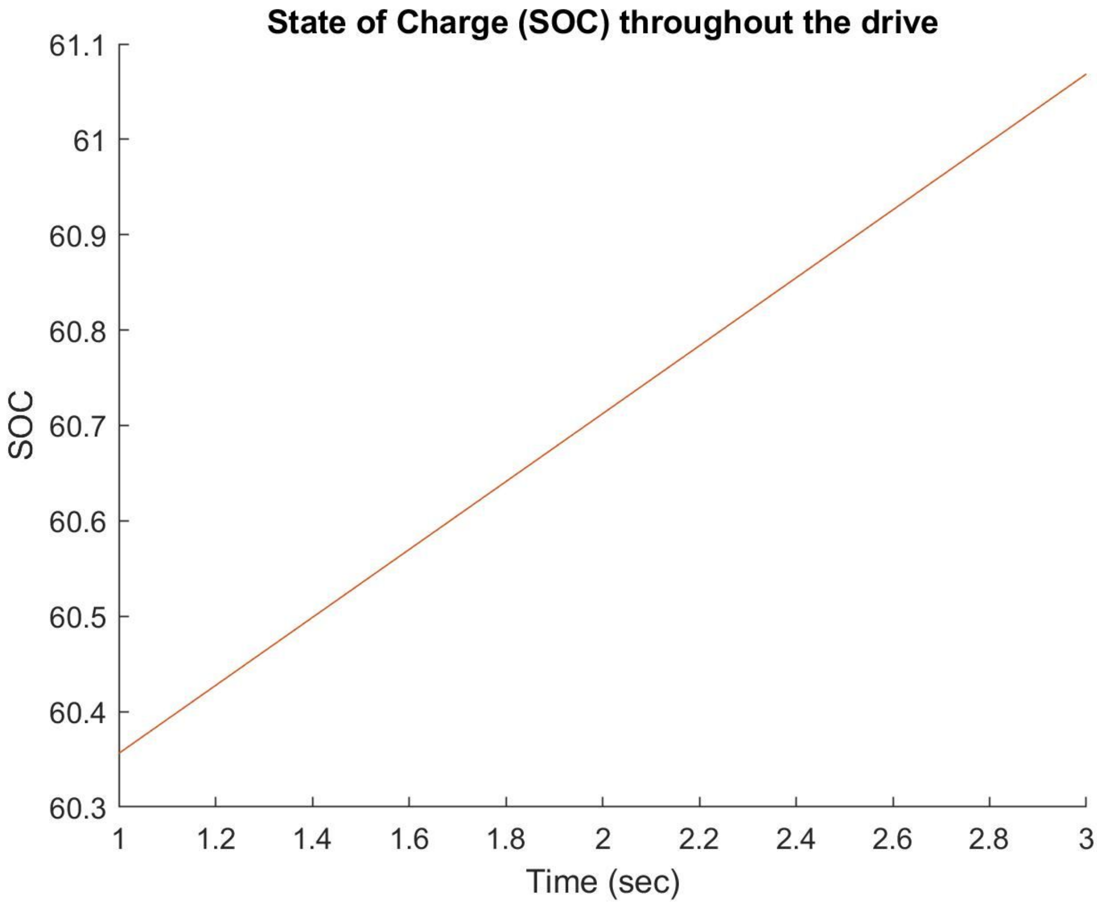

Fig.(8) ECMS State of Charge v/s time

Our fuel consumption is increasing in small steps because it is cumulation of overall fuel consumption and when regenerative braking is active, we can see that there are small horizontal steps in the plot and thus this fair explanation of drive cycle in stop and go traffic driving.

References:

1. http://www.iitg.ac.in/scifac/qip/public_html/cd_cell/chapters/uk_saha_internal_combustion_engine/qip-ice-05-engine%20efficiencies.pdf
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11. https://www.google.com/search?q=series+hev+drivetrain&source=lnms&tbm=isch&sa=X&ved=0ahUKEwiTp9z404LfAhUHE3wKHUMYBhoQ_AUIDygC&biw=1536&bih=754#imgrc=KRuyLJEjesBx1M:

MATLAB Code:

Contents


```
clear; clc;
```

Data

```
% load('enginedata.mat');
% % Data from (TOYOTA PRIUS):
https://media.toyota.co.uk/wp-content/files_mf/1329489972120216MTOYOTAPRIUSTECHNICALSPECIFICATIONS.
pdf

Vehicle_length = 4.48; % meter
Vehicle_mass = 1805; % kg
Gear_ratio = 2.683; % Forward Gear Ratio
Differential_drive_ratio = 3.267; % Differential Gear Ratio
Wheel_radius = 0.3175; % m => 12.5 inch tire radius (195/65r15 tire);
Max_power = 73000; % watt

% Assumptions:
time = 1:100;
random_values = 0.2*rand(length(time),1) + 0.6; % limiting range of random variables
Road_length = 500; % meter
Vehicle_safe_distance = 3.0 ; % meter
Vehicle_max_velocity = 29.057; % m/s => 65 mph
Engine_efficiency = 0.80;
```

PDE model for Vehicle Density

```
Road_density_max = Road_length / (Vehicle_length + Vehicle_safe_distance);
Road_density_random = random_values * Road_density_max;
```

```
velocity_vehicle = Vehicle_max_velocity * (1 - (Road_density_random / Road_density_max));
```

Vehicle Modeling

```
Net_Gear_ratio = Gear_ratio * Differential_drive_ratio;
Wheel_rpm = (velocity_vehicle*60)./(2*pi*Wheel_radius);
Engine_rpm = Wheel_rpm * Net_Gear_ratio;
Engine_torque = Max_power ./ Wheel_rpm;
Wheel_torque = Engine_torque .* Net_Gear_ratio * 0.5;
```

Optimization

```
% Polynomial derived from curve fitting app using given engine data
% X axis: eng_consum_spd
% Y axis: eng_consum_trq
% Z axis: eng_fuel_map_gpkWh
% Polynomial order: (5,5)

p00 = 976.7;
p10 = -12.14;
p01 = -3.574;
p20 = 0.1646;
p11 = -0.02367;
p02 = 0.01739;
p30 = -0.0009203;
p21 = 0.000117;
p12 = 4.708e-06;
p03 = -2.651e-05;
p40 = 2.344e-06;
p31 = -2.78e-07;
p22 = -4.89e-09;
p13 = 6.132e-10;
p04 = 1.604e-08;
p50 = -2.234e-09;
p41 = 2.812e-10;
p32 = -4.378e-11;
p23 = 2.251e-11;
p14 = -6.75e-12;
p05 = -1.881e-12;

% x0 = [Engine_rpm(1) Engine_torque(1)];
```

```
x0 =[100 100];

bsfc = @(x) (1.5*(p00 + p10*x(1) + p01*x(2) + p20*x(1)^2 + p11*x(1)*x(2) + p02*x(2)^2 + p30*x(1)^3 +
p21*x(1)^2*x(2) ...
            + p12*x(1)*x(2)^2 + p03*x(2)^3 + p40*x(1)^4 + p31*x(1)^3*x(2) + p22*x(1)^2*x(2)^2 ...
            + p13*x(1)*x(2)^3 + p04*x(2)^4 + p50*x(1)^5 + p41*x(1)^4*x(2) + p32*x(1)^3*x(2)^2 ...
            + p23*x(1)^2*x(2)^3 + p14*x(1)*x(2)^4 + p05*x(2)^5));
dl_0 = 0;
df_0 = 0;
dt = 1;

for i = time

  dl(i) = velocity_vehicle(i)*dt + dl_0; % Distance(position) of Leading Vehicle
  dl_0 = dl(i);

end

for j = time

LB = [min(Engine_rpm) min(Engine_torque)];
UB = 8*[Engine_rpm(j) Engine_torque(j)];
A = [-dl(j) df_0; dl(j) -df_0];
B = [-3; 5]; %[Safe_Distance; max_allowed_distance_for_following_purpose]

x = fmincon(bsfc,x0,A,B,[],[],LB,UB);

x1(j) = x(1);
x2(j) = x(2);

mf(j) = abs(bsfc(x))/(3.6e9*Engine_efficiency)*0.8;
vel(j) = ((x2(j)./Net_Gear_ratio)*(2*pi/60))*(Wheel_radius);

df(j) = vel(j)*dt + df_0; % Distance(position) of following vehicle
df_0 = df(j);

x0=x;

end

figure(1);
hold on;
subplot(2,1,1);
```

```
plot(time,mf)
xlabel("Time (sec)");
ylabel("Mass flow rate (Kg/s)");
hold off;

hold on;
subplot(2,1,2);
plot(time,vel)
xlabel("Time (sec)");
ylabel("Our Vehicle Optimized Velocity (m/s)");
hold off;

cyc_mph(:,1) = time;
cyc_mph(:,2) = vel;
save('cyc_mph.mat','cyc_mph')
```