Coordinating Development Under Political Risk

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Abstract

Political risk and coordination failure are two leading causes of low investment and growth in low- and middle-income countries. The international community devotes substantial resources to addressing these challenges. We show that these problems are intertwined: political risk induces coordination failure. We then propose a subsidy program to eliminate politically induced miscoordination. Our program—Guaranteed Return with Profit- and Loss-Sharing (GPLS)—offers a minimum return to investors while clawing back returns above a specified maximum. The design screens out investors who would have invested even without subsidies and induces participation from more hesitant investors at minimal cost. Optimally designed GPLS programs can substantially lower costs relative to natural alternatives, such as guaranteed rate-of-return schemes or Guaranteed Return with Profit-Sharing (GPS) programs. Such optimal subsidies represent a major step toward feasible and sustainable international interventions to coordinate development under political risk.

Keywords: Political Risk, Coordination Failure, Optimal Subsidies, Screening

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1 Introduction

There is a vast and growing global investment gap in achieving sustainable development in low- and middle-income countries.[1] The international community expends enormous resources to address this challenge.[2] How can the international community mitigate these problems and facilitate investment and growth?

Political risk and coordination failure are two leading explanations for low investment[3] and growth in developing countries (World Bank, 2017, 2024). Conflict, policy uncertainty, expropriation and confiscatory taxation, corruption, and weak rule of law are associated with low economic progress (Acemoglu, Johnson, and Robinson, 2001; Alfaro, Kalemli-Ozcan, and Volosovych, 2008; Boix, 2003; Caldara and Iacoviello, 2022; Jensen, 2008; North, 1990; North and Weingast, 1989; Reinhart and Rogoff, 2004).[4] Similarly, countries may be trapped into low levels of investment and growth if investors fail to coordinate and internalize positive externalities in demand, infrastructure, and technology adoption (Bond and Pande, 2007; Buera et al., 2021; Ciccone, 2002; Cooper and John, 1988; Garg, 2025; Hoff and Stiglitz, 2001; Murphy, Shleifer, and Vishny, 1989; Okuno-Fujiwara, 1988; Rodríguez-Clare, 2005; Rodrik, 1996). As the 2017 World Development Report emphasizes "coordination is required to induce investment and innovation. Both depend on firms and individuals believing that others will also invest" (World Bank, 2017, p. 56).

Our key insight is that political risk induces coordination failure among investors, but well-designed subsidy programs can nearly eliminate miscoordination at low cost.[5] Political risk

[1]According to the *World Investment Report 2023*, "the investment gap across all [Sustainable Development Goal] sectors has increased from $2.5 trillion – estimated in [*World Investment Report 2014*]... to more than $4 trillion per year today... The increase is the result of both underinvestment and additional needs" (United Nations Conference on Trade and Development, 2023, p. xv).

[2]For example, the World Bank Group alone committed $117.5 billion towards development projects in the fiscal year 2024 (Multilateral Investment Guarantee Agency, 2024, p. 4).

[3]We use a broad notion of investment, including physical and human capital, technology, and innovation.

[4]The literature is vast. See Abadie and Gardeazabal (2003), Alesina and Perotti (1996), and Cerra and Saxena (2008) for unrest and conflict; Baker, Bloom, and Davis (2016) and Fernández-Villaverde et al. (2015) for policy uncertainty; Boehm and Oberfield (2020) and Ponticelli and Alencar (2016) for weak rule of law; and Fisman and Svensson (2007) and Mauro (1995) for corruption.

[5]Political risk can induce other forms of coordination failure, e.g., between lenders and borrower-investors (Chang, 2010). Our approach applies to such settings as long as subsidies can be conditional on investments.

and coordination failure are intertwined: A larger number of investors reduces political risk, which in turn encourages further investment. For example, Shadmehr (2019) argues that when technology exhibits complementarity between capital and labor, greater investment raises wages, thereby reducing the likelihood of a successful revolution. Similarly, greater investment may yield a more "business-friendly" median voter and encourage more far-sighted policies, reflecting both improved economic conditions and more promising future opportunities (Bernhardt, Krasa, and Shadmehr, 2022; Chang, 2010). Strategic uncertainty, combined with politically induced complementarity, leads to inefficiently low investment. If a potential investor was sure that many others would invest, it would anticipate a lower political risk, and become more optimistic about returns. However, investors remain uncertain about each other's assessments of investment prospects and therefore decisions, leading to miscoordination. If, instead of many, there was one large potential investor, there would be no miscoordination inefficiencies. However, given the vast scales, resolving the investment gap is not feasible for one or a few entities. Even if it were feasible, such a centralized setting would introduce additional frictions, including those associated with monopolies—Lucas (1990) identified monopolies, supported by imperial powers, as a key political risk. Addressing the investment gap therefore requires the participation of many private investors.

Building on the insights of Shen and Zou (2024), we propose a subsidy program[6] that nearly eliminates miscoordination at a lower cost than natural alternatives. The program features voluntary investor participation, guarantees a specified payoff, and claws back any excess profits. It also compensates pre-guarantee losses up to the maximum feasible rate. We call this the optimal *guaranteed return with profit- and loss-sharing* (*GPLS*) program. We show that loss-sharing is essential: without it, any *guaranteed return with profit-sharing* (*GPS*) program can be significantly more expensive—while still better than optimal guarantees without profit-sharing. A restricted version of GPLS (*R-GPLS*), in which the profit- and loss-sharing rates are always equal, is also less cost-effective. We will also refer to R-GPLS

[6]One may view the program as an insurance program. We call it a subsidy program to emphasize that underwriters will transfer funds to participants in expectation.

as the Shen–Zou (SZ) program.[7]

A development agency can entice any investment level by offering sufficiently high payoffs to potential investors to swamp all the risk. However, such a high guarantee can be prohibitively expensive. We maintain the realistic assumption that it will be economically or politically infeasible for development agencies to offer a program that induces investment when the investment environment is sufficiently bad. Even when an agency can induce an aggregate investment level, should it do so by subsidizing investment through a guaranteed payoff? Any program that offers a guaranteed payoff for investment to eliminate miscoordination will attract not only potential investors who would not have invested without the program, but also the sufficiently optimistic potential investors who would have invested even without the program. This significantly raises program costs. To screen out these investors the agency can adjust the program to claw back any profits in excess of the promised return in the event that investments pay off. Now, those who were optimistic enough to invest without the program will not join, reducing the program costs. This is the optimal GPS program.

The optimal GPS program improves upon offering subsidies in the form of a simple guaranteed payoff, but its screening is only partial. If the agency knew the degree of optimism of potential investors, it would provide each investor with just enough funds to induce the desired number of investors: it would not offer any incentives to those who would have invested anyway, continuing toward less pessimistic ones until the desired level is reached. More pessimistic potential investors of course would have needed more funds. However, the agency does not know potential investors' degrees of optimism, and more optimistic ones have incentives to pretend to be pessimistic to receive more favorable conditions from the agency (Börgers, 2015). With the optimal GPS program nearly all investors added to the fold receive more funds from the agency compared to a setting in which the agency knew these investors' degree of optimism, leaving significant room for improvement.

[7] We focus on applying recent theoretical advances to address the global investment gap. Methodologically, the core analysis of Shen and Zou (2024) is in a regime change setting, whereas in our setting net payoffs are continuous. Their Appendix B outlines an extension that encompasses R-GPLS. Our theoretical contribution lies in analyzing both more and less restrictive (GPS and GPLS) programs than their proposed intervention, showing that the optimal GPLS program is more cost-effective—see Sections 5.1 and 6.

The optimal GPLS program addresses the problem by offering a more flexible structure. Instead of offering a large guaranteed return to all participants, the optimal GPLS program offers a smaller guarantee, but picks up a share of an investor's pre-guarantee losses. This makes the program less attractive to more optimistic investors who anticipate to receive less benefits from the program. Consequently, they invest without joining the program. Like GPS, the optimal GPLS has full claw back to deter optimistic investors. We assume that development agencies cannot fully pick up the losses, which would imply the ability to induce investment even in the worst investment conditions. As long as an agency does not fully pick up the losses, GPLS will be more cost effective than R-GPLS (SZ), in which the profit-sharing rate is reduced to match the loss-sharing rate.

Our approach complements the solutions offered in the political risk literature to foster investment and growth, including trade agreements (Büthe and Milner, 2008, 2014), institutional change to introduce checks and balances (Acemoglu, Naidu, et al., 2019; Ferraz and Finan, 2011; La Porta et al., 2004; North, 1990) and legal protection for property rights (Acemoglu and Johnson, 2005; Besley and Ghatak, 2010; Li and Resnick, 2003), as well as transparency, monitoring, and deliberation (Banerjee, Duflo, et al., 2020; Djankov et al., 2010; Ferraz and Finan, 2008; Fujiwara and Wantchekon, 2013; López-Moctezuma et al., 2022; Olken, 2007). It parallels recent microfinance literature on the importance of contract structure and borrower heterogeneity, highlighting the need for better screening to foster development (Balboni et al., 2022; Banerjee, Breza, et al., 2019; Bari et al., 2024; Bryan, Karlan, and Osman, 2024). The growth literature on coordination failure among investors suggests subsidies (Buera et al., 2021; Rodrik, 1996), highlighting that income-based (as opposed to our investment-based) incentive programs are ineffective (Bond and Pande, 2007), but does not focus on identifying efficient programs. Given the vast scale involved in international development, designing optimal programs that achieve the same outcome at lower costs is crucial. We build on recent developments in literature on screening in coordination games (Morris and Shadmehr, 2023; Shen and Zou, 2024) to identify such programs, comparing their costs with natural sub-optimal alternatives.

A broader literature studies contracting and subsidies in the presence of complementarity (see Halac (2025) for a review), including in global games (Luo and Yang, 2023; Sákovics and Steiner, 2012). However, the focus of that literature is not on screening, e.g., Sákovics and Steiner (2012) analyze how to allocate subsidies to ex ante heterogeneous agents based on observable characteristics in a global games setting. Similarly, aside from Morris and Shadmehr (2023), the literature on coordination in conflict does not involve screening (Boix and Svolik, 2013; Bueno de Mesquita, 2010; Bueno de Mesquita and Shadmehr, 2023; Casper and Tyson, 2014; Chen, Lu, and Suen, 2016; Egorov and Sonin, 2021; Gieczewski and Kocak, forthcoming; Little, 2012; Tyson and Smith, 2018).[8]

The international community recognizes political risk as a key obstacle to investment in developing countries (United Nations Conference on Trade and Development, 2025; World Economic Forum, 2024). Development agencies—such as the World Bank Group and the U.S. International Development Finance Corporation—offer various instruments to mitigate political risk and encourage private investment. These include guarantees that cover political losses (e.g., political risk insurance), subsidies (e.g., below-market loans), and blended finance, which combines public and private capital through multiple instruments (Arel-Bundock, Peinhardt, and Pond, 2020; Multilateral Investment Guarantee Agency, 2023).[9] These tools are designed to reduce political risk and thereby stimulate private investment. However, while the international community is aware of coordination problems among investors, these programs are not primarily structured to address coordination failure. In Section 6.1 we examine a sample of these instruments and show how their design compares to the optimal Guaranteed Return with Profit- and Loss-Sharing (GPLS) program. This comparison highlights how widely used programs align with—or diverge from—an approach explicitly aimed at eliminating coordination failure.

[8] In Gieczewski and Shadmehr (2024)'s analysis of election fraud, rewards are provided only if the project succeeds, and agents are motivated to take the risky action solely to obtain these designed rewards.

[9] See https://ida.worldbank.org/en/financing/ida-private-sector-window for information on the International Development Association's Private Sector Window and https://www.ifc.org/en/what-we-do/sector-expertise/blended-finance/how-blended-finance-works for the International Finance Corporation's Blended Finance.

2 Model

There is a continuum 1 of investors, indexed by $i \in [0, 1]$, each endowed with $\overline{K}$ units of capital. They simultaneously decide whether to invest their capital in the global market or in a developing country. The expected return on capital in the global market is r. The return from investing in the country depends on factor productivity and the resolution of political risk. We model political risk as an effective tax rate $T \in [0, 1]$. This effective tax reflects not only formal taxation but also the likelihood of expropriation, destruction from conflict, corruption costs (Shleifer and Vishny, 1993), and weak rule of law and contract enforcement.

Let $k_j \in \{0, 1\}$ denote an investor j's decision, where 0 means investing in the global market and 1 means investing in the developing country, so that the aggregate invested capital is $K = \overline{K} \int k_j dj$. The production technology is $Y = A(\underline{K} + K)$ where $A > 0$ is the total factor productivity, and $\underline{K} > 0$ is an exogenous, immobile capital in the country, e.g., land. A share $\alpha \in (0, 1)$ of the output is divided equally among the investors. An investor's payoff from investing in the country is the after-tax return on her capital: $(1 - T)\alpha A\overline{K}$.

Empirical studies emphasize that greater investment and better economic conditions tend to reduce political risk (Bazzi and Blattman, 2014; Blattman and Miguel, 2010; Brückner and Ciccone, 2010; Caldara and Iacoviello, 2022; Johns and Wellhausen, 2016; Malesky, 2009). In our model, T captures political risk. To be consistent with this empirical regularity, we could directly assume that T decreases in output Y, and hence in aggregate capital K. Instead, we provide a micro-foundation for this relationship, which we use throughout the paper. Specifically, after investment and production occur, the representative citizen in the country receives a share $(1 - \alpha)$ of output and chooses the tax rate T. The citizen's payoff is

$$u_c \; = \; (1 - T)(1 - \alpha)Y \; + \; 2\sqrt{\delta \, TY},$$

where TY is the tax revenue and δ captures both the productivity of tax revenues and the citizen's ideological or cultural attitude toward taxation.[10]

[10] An interpretation of the representative citizen is the median voter in a setting with a continuum 1 of

The productivity A in the developing country is uncertain. Investors share a prior that $A \sim U[\underline{A}, \overline{A}]$. Each investor i receives a noisy private signal x_i about A, where $x_i = A + \sigma\epsilon_i$, $\sigma > 0$, with $\epsilon_i \sim iid\ U[-1, 1]$. The fundamental A and noise ϵ_is are independent.

The game proceeds as follows. First, nature determines A and ϵ_is, and investors observe their signals, x_is. Next, investors simultaneously make investment decisions $(k_i)_{i\in[0,1]}$. Then, production takes place and players receive their shares. Then, the effective tax rate T is chosen. Payoffs are received and the game ends.

An investor i's strategy $\sigma_i : [\underline{A} - \sigma, \overline{A} + \sigma] \to \{0, 1\}$ is a mapping from her signal x_i to her investment decision k_i. The representative citizen's strategy $\sigma_c : \mathbb{R}_+ \to [0, 1]$ is a mapping from the output to an effective tax rate $T \in [0, 1]$. The equilibrium concept is Perfect Bayesian Nash equilibrium. We focus on symmetric monotone strategies in which an investor i invests if and only if i's signal x_i is above a threshold $x^* \in \mathbb{R}$.

3 Inefficient Investment and Strategic Uncertainty

We begin by analyzing the citizen's choice of effective tax rate. The citizen's problem is:

$$\max_{T\in[0,1]} (1 - T)(1 - \alpha)Y + 2\sqrt{\delta(TY)} \tag{1}$$

Lemma 1. *The equilibrium level of effective tax rate is*

$$T^* = \min\left\{\frac{\delta}{(1-\alpha)^2}\frac{1}{Y}, 1\right\} = \min\left\{\frac{\delta}{(1-\alpha)^2}\frac{1}{A(\underline{K}+K)}, 1\right\}. \tag{2}$$

All omitted proofs are in Online Appendix B. The tax rate is decreasing in output, and hence in the total factor productivity A and in the aggregate investment K. This generates

workers, each providing 1 unit of labor inelastically, receiving the labor share $(1-\alpha)$ equally, and then voting on a tax rate T that finances the public good $g = TY$. In other settings, the representative citizen may instead be interpreted as the government, or a fictitious agent encompassing the government, citizens, and rebels. Our micro-foundation in the text should be viewed as a modeling tool. What matters for our results is that, in equilibrium, T will be decreasing in output Y, and hence in aggregate capital K—see Lemma 1.

a force for strategic complementarity among the investors.

For a given aggregate capital, the tax rate is a random variable because productivity A is random. When more investors invest in the country, this reduces the tax rate in the FoSD sense: the distribution of tax rate shifts to left, raising an investor's incentives to also invest in the country.

We now consider the investors' decisions. Given productivity A, aggregate capital K, and tax rate T^*, an investor's net rate of return from investing in the country versus in the global market is

$$\alpha\ \pi(K, A) := (1 - T^*)\alpha A - r = \alpha A - \frac{\alpha\delta}{(1-\alpha)^2}\frac{1}{\underline{K} + K} - r, \tag{3}$$

where we substituted T^* from Lemma 1, assuming that T^* is interior, and recognizing that investors anticipate the tax rate for any given productivity A and aggregate capital K.

To ease exposition and have unique equilibrium, we maintain the following assumption.

Assumption 1. *Let* $A_l = \underline{A} - \pi(\overline{K}, \underline{A}) = \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2(\underline{K}+\overline{K})}$ *and* $A_h = \overline{A} - \pi(0, \overline{A}) = \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2\underline{K}}$.

1. $\underline{A} > \frac{\delta}{(1-\alpha)^2\underline{K}}$
2. $\underline{A} < A_l - \sigma$
3. $\overline{A} > A_h + \sigma$

Part 1 ensures that the tax rate is always less than 1. Parts 2 and 3 ensure the existence of lower and upper dominance regions needed for uniqueness. To ease exposition, when indifferent, we assume that a potential investor will invest in the country.

Proposition 1 (Complete Information Benchmark)**.** *Suppose productivity A is known.*

- *If $A < A_l$, then there is a unique equilibrium in which no one invests.*
- *If $A \geq A_h$, then there is a unique equilibrium in which everyone invests.*
- *If $A \in [A_l, A_h)$, then there are multiple equilibria.*

When the productivity is sufficiently large, $A > A_h$, the rate of return is higher in the country than in the global market even if the aggregate investment is negligible. Then, there is a unique equilibrium in which all potential investors invest in the country, because it is a dominant strategy. Similarly, when the productivity is sufficiently small, $A < A_l$, the rate of return is lower in the country even if all potential investors invest in the country. Then, there is a unique equilibrium in which no potential investor invests in the country. In between, there are multiple equilibria, including one in which all invest, and one in which no one invests.

We now turn to the incomplete information setting with uncertain productivity and information asymmetry. Investors use their information to assess the expected net payoff from investing. They have uncertainty about the productivity and they have strategic uncertainty about other investors' behavior. For a given realization of productivity A and other investors' cutoff x^* for investing, the aggregate invested capital in the country is

$$K(A; x^*) = \overline{K}\ Pr(x_i \geq x^* | A), \tag{4}$$

which is increasing in productivity. Thus, higher productivity directly increases incentives to invest by raising the expected pre-tax returns, and it indirectly increases incentives by reducing the effective tax and hence the expected after-tax returns. It does so because higher productivity reduces the tax rate by raising the output, both directly through the productivity channel A and indirectly through increasing the aggregate capital $\underline{K} + K(A; x^*)$.

An investor i with signal x_i thus estimates i's net expected payoff $\mathbb{E}[\alpha\pi(K(A; x^*), A) \mid x_i]$, investing if and only if $x_i \geq x^*$. The investor must be indifferent between investing in the country and in the global market at the threshold signal $x_i = x^*$. It follows that any x^* that satisfies $\mathbb{E}[\pi(K(A; x^*), A) \mid x_i = x^*] = 0$ constitutes an equilibrium. Obviously, if a solution exists, we must have $x^* \in [A_l, A_h]$: an investor with signal $x_i > A_h$ has a dominant strategy to invest in the country; and one with signal $x_i < A_l$ has a dominant strategy not to.[11] To assess her net expected payoff from investing, $\mathbb{E}[\alpha\pi(K(A;x^*),A) \mid x_i]$, the marginal investor with the threshold signal $x_i = x^*$ must estimate the tax rate and, hence, aggregate investment $K(A;x^*)$. A key observation is that when $\sigma < \min\{\overline{A} - A_h, A_l - \underline{A}\}$, so that $x^* \pm \sigma$ is away from the boundaries, we have:

$$\Pr(x_j \geq x^*|A) \mid x_i = x^* \sim U[0,1], \tag{5}$$

so that the marginal investor always believes that the aggregate (new) capital in the country, $K(A;x^*)$, is uniformly distributed on $[0,\overline{K}]$. Then, the indifference condition becomes

$$\alpha\mathbb{E}[A|x_i = x^*] - \alpha\mathbb{E}[A \cdot T^*|x_i = x^*] = \alpha x^* - \int_0^{\overline{K}} \frac{\alpha\delta}{(1-\alpha)^2}\frac{1}{\overline{K}}\frac{dK}{\underline{K}+K} - r = 0, \tag{6}$$

yielding a unique solution. In fact, it is straightforward to confirm that the setting satisfies the standard assumptions for the common value setting in Morris and Shin (2003). The above arguments adopt their proof of Proposition 2.2 to the setting with uniform distributions.[12]

Proposition 2 (No Intervention Benchmark)**.** *Let $\overline{\sigma} = \min\{\overline{A} - A_h, A_l - \underline{A}\}$. If $\sigma < \overline{\sigma}$, there is a unique symmetric equilibrium in cutoff strategies in which an investor i with signal x_i invests in the country if and only if $x_i \geq x^*$. Moreover, x^* is the unique solution to $\int_0^{\overline{K}} \pi(K,x^*)dK = 0$:*

$$x^* = \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2}\frac{1}{\overline{K}}\log\left(\frac{\overline{K}+\underline{K}}{\underline{K}}\right).$$

Comparative statics are intuitive.

Corollary 1. *Higher global market returns r and public good productivity δ reduce investment incentives, while higher immobile capital $\underline{K}$ and available investment $\overline{K}$ raise investment incentives—because they reduce taxes: $\frac{dx^*}{d\overline{K}}$, $\frac{dx^*}{d\underline{K}} < 0 < \frac{dx^*}{dr}$, $\frac{dx^*}{d\delta}$. Moreover, there is a $\widehat{\alpha} \in$*

[11] When no one else invests ($K = 0$), i invests whenever her private signal is greater than A_h, because $\mathbb{E}[\pi(0,A)|x_i = A_h] = 0$. Similarly, when everyone else invests ($K = \overline{K}$), i does not invest if her private signal is below A_l, because $\mathbb{E}[\pi(\overline{K},A)|x_i = A_l] = 0$.

[12] It follows that focusing on monotone strategies is without loss of generality and that the unique equilibrium is the only one surviving the iterated deletion of dominated strategies.

$(0, 1)$ *such that* $\frac{dx^*}{d\alpha} < 0$ *if and only if* $\alpha < \widehat{\alpha}$.

The effect of public good productivity δ, immobile capital $\underline{K}$, and available investment $\overline{K}$ are all through taxes. Higher capital share α has conflicting effects: the direct effect raises investment incentives, but the indirect, strategic effect through higher taxes reduces incentives. When capital share is small, the direct effect tends to dominate, but when it is large, the strategic effect tends to dominate.

Inefficiency If there was only one large investor with capital stock $\overline{K}$ and signal x_i, the investor would invest in the country if and only if

$$\mathbb{E}[\pi(\overline{K}, A) \mid x_i] \geq 0, \quad \text{that is} \quad x_i \geq A_l. \tag{7}$$

However, when there are many investors, strategic uncertainty prevents investors from coordinating their investment decisions efficiently. The following result is immediate from the inspection of x^* and A_l.

Proposition 3 (Strategic Uncertainty and Inefficiency)**.** *The investment threshold is lower in the centralized setting with one large investor than in the decentralized setting:* $A_l < x^*$. *Moreover,* $x^* - A_l$ *is increasing in* α, δ, *and decreasing in* $\underline{K}$.

Intuitively, higher capital share α, public good productivity δ, and lower immobile capital $\underline{K}$ all increase the marginal effect of higher capital investment in reducing tax, e.g., from Lemma 1, we have $\frac{\partial}{\partial \underline{K}} \left| \frac{\partial T^*}{\partial K} \right| < 0$. Similarly, higher available capital $\overline{K}$ implies decisions to invest in the country have higher impact on reducing the effective tax rate.

Alternatively, we note that it is socially optimal for all potential investors to invest when $A \geq A_l$. In contrast, in the decentralized setting, all investors invest if and only if $A \geq x^* + \sigma$. Thus, there is an *inefficiency interval*, $A_I := [A_l, x^* + \sigma)$, such that when productivity is realized within it, $A \in A_I$, there is inefficiently low investment in the country due to strategic uncertainty that hinders coordination among investors.[13]

[13]If one large potential investor with signal x_i decides whether to invest, its expected payoff will be

4 Guaranteed Return with Profit-Sharing (GPS)

Proposition 3 suggests that there is room for intervention by the international community to mitigate the inefficiencies due to politically-induced coordination failure. Subsidy programs can increase investment. For example, from (3), a return subsidy $s > r - \alpha \underline{A} + \frac{\alpha\delta}{(1-\alpha)^2} \frac{1}{\underline{K}}$ will make investment the dominant strategy. However, this induces investment even when investment is inefficient and it is prohibitively expensive because *all* potential investors will always invest to take advantage of government subsidies.

We aim to design subsidy programs that eliminate miscoordination inefficiency in the least costly manner. Such cost-effective programs should, at a minimum, discourage participation by investors who would have invested even without the subsidies. This suggests that optimal subsidy structure must impose some expected participation costs in addition to providing guaranteed returns.

We begin by considering Guaranteed Return with Profit-Sharing (GPS) programs. GPS programs are described by a guaranteed return $s \in [0, \overline{s}]$, and a profit-sharing rate $t \in [0, 1]$ that a participating investor must pay to the guarantor if the investment turns a profit. This is equivalent to programs in which a minimum return is guaranteed and investor pays a premium that is increasing in the profit and is paid only if there is a positive profit. The participants pay the guarantors when profits are sufficiently high in exchange for payments when profits are lower.

When GPS programs are offered, the investors' decisions include whether to participate in the program. Accordingly, investor i's choice set is $k_i \in \{0, 1, 2\}$, where $k_i = 0$ denotes not investing, $k_i = 1$ denotes investing with participation in the program, and $k_i = 2$ denotes investing without participation.

Consider an $(\hat{s}, t)$ program, where $\hat{s}$ is the guarantee and t is the profit-sharing rate. Given productivity A and aggregate investment level K, a potential investor i's returns from not

$\max\left\{r, \alpha\mathbb{E}[A|x_i] - \frac{\alpha\delta}{(1-\alpha)^2} \frac{1}{\underline{K}+\overline{K}}\right\} \cdot \overline{K}$, so there is no risk-aversion and no demand for insurance. The only underlying reason for intervention in our setting would be miscoordination inefficiency.

investing, and from investing with and without participation in the program are r, $(1-T^*)\alpha A + \hat{s} - t\max\{(1-T^*)\alpha A - r, 0\}$, and $(1-T^*)\alpha A$, respectively. It is convenient to work with an affine transformation by subtracting the payoffs by r and dividing by α, so that the program becomes $(s,t) = (\hat{s}/\alpha, t)$. Thus, (an affine transformation of) the investor i's returns from participating in a (s,t) program is

$$\pi(1, K, A) = \pi(K, A) - t \cdot \max\{\pi(K, A), 0\} + s, \tag{8}$$

where $\pi(K, A)$ is defined in (3). The corresponding payoffs from not investing is $\pi(0, K, A) = 0$, and from investing without participation in the program is $\pi(2, K, A) = \pi(K, A)$.

Consistent with the previous section, we assume that an investor invests in the country when she is indifferent between investing and not; and does not participate in the program when indifferent between investing with participation and investing without participation.

The program makes potential investors more optimistic about their returns from investing in the developing country. Therefore, the lower dominance region (where the dominant strategy is investing in global markets) shrinks. The following assumption ensures the existence of a lower dominance region and the uniqueness of equilibrium with the GPS program. Substantively, it captures the realistic feature that the guarantor does not have so much resources to induce investment even in the worst state of the world $A = \underline{A}$.

Assumption 2. *If productivity is sufficiently low, not investing in the country is the dominant strategy under GPS programs:* $\pi(\overline{K}, \underline{A}) + \overline{s} = \underline{A} - A_l + \overline{s} < -\sigma$.

The condition $\pi(\overline{K}, \underline{A}) + \overline{s} < 0$ suffices for the existence of a lower dominance region. The stricter condition in Assumption 2 ensures a unique equilibrium for a given $\sigma > 0$.

Equilibrium An investor i's strategy $\sigma_i : [\underline{A} - \sigma, \overline{A} + \sigma] \to \{0, 1, 2\}$ is a mapping from her signal x_i to her investment and program participation decisions. As in the benchmark, we focus on equilibria in symmetric monotone strategies. A monotone strategy is (weakly) increasing, and hence it is characterized by two cutoffs: there are two thresholds $x' \leq x''$

such that

$$\sigma_i(x_i) = \mathbf{1}_{\{x_i' \leq x_i < x_i''\}} + 2\ \mathbf{1}_{\{x_i'' \leq x_i\}},$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. That is, investor i with signal x_i does not invest ($k_i = 0$) in the developing country if $x_i < x'$, invests and participates in the program ($k_i = 1$) if $x_i \in [x', x'')$, and invests without program participation ($k_i = 2$), if $x_i \geq x''$. Without loss of generality $x', x'' \in [\underline{A} - \sigma, \overline{A} + \sigma]$. If it is never (i.e., except on a measure-0 set) optimal for investors to take action 0, we set $x' = \underline{A} - \sigma$. If it is never optimal for investors to take action 2, we set $x'' = \overline{A} + \sigma$. Naturally, if investors never take action 1, then $x' = x''$.

Given aggregate investment K and productivity A, investor i's relative payoffs from a higher versus lower action are

$$\delta^1(K, A) = \pi(1, K, A) - \pi(0, K, A) = \pi(K, A) - t \cdot \max\{\pi(K, A), 0\} + s \tag{9}$$

$$\delta^2(K, A) = \pi(2, K, A) - \pi(1, K, A) = t \cdot \max\{\pi(K, A), 0\} - s. \tag{10}$$

Action and state monotonicity hold. In particular, actions are strategic complements. Given A, both $\delta^1(K, A)$ and $\delta^2(K, A)$ are increasing in K. Similarly, given K, they are increasing in A. Moreover, signals x_is and the fundamental A are affiliated. Lemma 2 then follows from standard arguments (Frankel, Morris, and Pauzner, 2003, Van Zandt and Vives, 2007).

Lemma 2. *Player i's best response to a monotone strategy profile is a monotone strategy.*

A cutoff pair (x', x'') constitutes a symmetric monotone equilibrium if and only if

$$x' = \min\{x \ s.t. \ \Delta^1(x; x') \geq 0\} \ \text{ and } \ x'' = \max\{\min\{x \ s.t. \ \Delta^2(x; x') \geq 0\}, x'\}, \tag{11}$$

where $\Delta^1(x; x') = \mathbb{E}[\delta^1(K(A; x'), A)|x_i = x]$ and $\Delta^2(x; x') = \mathbb{E}[\delta^2(K(A; x'), A)|x_i = x]$ and we take the min to be $\overline{A} + \sigma$ if the set is empty, i.e., if no $x \in [\underline{A} - \sigma, \overline{A} + \sigma]$ satisfies the corresponding inequalities. For sufficiently low signals, Assumption 1.2 and Assumption 2 ensure that $\Delta^2(x_i; x')$, $\Delta^1(x_i; x') < 0$, respectively. Therefore, to find equilibrium cutoffs, first, we look for solutions to $x' = \Delta^1(x'; x')$; pick the minimum when there are multiple solutions, and pick $x' = \overline{A} + \sigma$ when there is no solution. Then, given the solution x' we have found, we do the same for $x'' = \Delta^2(x''; x')$. Finally, if $x'' < x'$, then we set $x'' = x'$.

Proposition 4. *Suppose assumptions 1 and 2 hold and a GPS program (s, t) is in place. As the noise becomes vanishingly small, there is a unique equilibrium.*[14] *In equilibrium, $x' = A'(s, t) \leq x^*$, where*

$$A'(s,t) = \begin{cases} x^* - s & ; A_l \geq x^* - s \\ \min\{\tilde{A}(s,t), x^*\} & ; A_l \leq x^* - s, \end{cases}$$

and $\tilde{A}$ is the unique solution to

$$\tilde{A} = x^* - s + t\ \frac{\delta}{(1-\alpha)^2\overline{K}} \left\{ \frac{\tilde{A} - r/\alpha}{A_l - r/\alpha} - \log\left(\frac{\tilde{A} - r/\alpha}{A_l - r/\alpha}\right) - 1 \right\}.$$

Moreover,

- *if $s \geq t\ (\overline{A} - A_l)$, then $x'' = \overline{A}$, so that all who invest will participate in the program.*
- *if $s < t\ (\overline{A} - A_l)$, then $x'' = A''(s, t) < \overline{A}$, where*

$$A''(s,t) = \begin{cases} A_l + s/t & ; s > t\ (A'(s,t) - A_l) \\ A'(s,t) & ; s \leq t\ (A'(s,t) - A_l), \end{cases}$$

  so that some who invest will not participate in the program.

When $s = t = 0$, Proposition 4 implies $A' = x^*$ and $A'' = \overline{A}$. The choice of A'' is inconsequential because there is no difference between participating in such a program or not. Thus, there is indeterminacy in A'', and we have picked a convenient A'' for this special case. For example, if $s, t \to 0$, with $s/t < \overline{A} - A_l$, then A'' approaches a threshold strictly less than $\overline{A}$.

Proposition 4 has intuitive features. The threshold for investing in the country, A', is weakly lower than the threshold absent any program; strictly so when the guarantee s is sufficiently large. Figure 1 demonstrates—in a setting where $\overline{s} > x^* - A_l$. When the guarantee s

[14]We characterize the equilibrium under a GPS program with finite noise, $0 < \sigma < \min\{A_l - \underline{A}, \overline{A} - A_h\}$, (Proposition 9) in Online Appendix A.

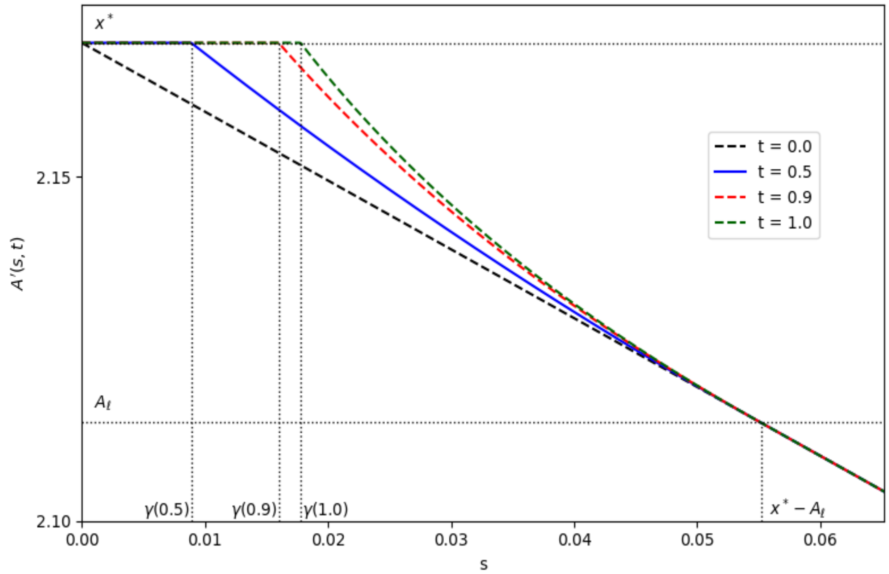

Figure 1: The equilibrium investment threshold $A'(s,t)$ as a function of the GPS program's guarantee s and profit-sharing rate t. Note that $A'(s = x^* - A_l, t) = A_l$, where A_l and x^* are defined in Assumption 1 and Proposition 2, respectively. Parameters: $\delta = 0.1$, $\overline{s} = 0.15$, $\alpha = 0.5$, $r = 1$, $\underline{K} = \underline{A} = 1.5$, $\overline{K} = 2$, $\overline{A} = 2.27$.

is sufficiently small ($s \approx 0 < x^* - A_l$), we have $A' = x^*$ regardless of the profit-sharing magnitude t, and the program has no effect, because no one participates. As the guarantee s increases, A' falls below x^* along $\tilde{A}(s,t)$, reaching $A' = A_l$ (at $s = x^* - A_l$), the productivity threshold below which investors have a dominant strategy not to invest absent programs. At this threshold, the coordination failure is fully resolved. Once the guarantee s exceeds the threshold $x^* - A_l$, the investment threshold becomes $A' = x^* - s < A_l$, falling with s.

The logic is that the marginal investor with threshold signal believes that the fraction of investors who will invest is uniformly distributed on $[0,1]$. From (6), the anticipated (normalized) tax payment (per unit of capital) associated with that belief is $x^* - r/\alpha$. When $t = 0$, as the guarantee s increases, it compensates for both this tax and for a lower productivity, thereby pushing down the investment threshold:

$$\begin{aligned}\mathbb{E}_A[\pi(1, K(A; A'), A) \mid x_i = A'] &\approx A' - \frac{\mathbb{E}_K[\text{tax payment}]}{\alpha}\bigg|_{K\sim U[0,\overline{K}]} - \frac{r}{\alpha} + s = A' - x^* + s \\ &= A' - x^* + s, \end{aligned} \tag{12}$$

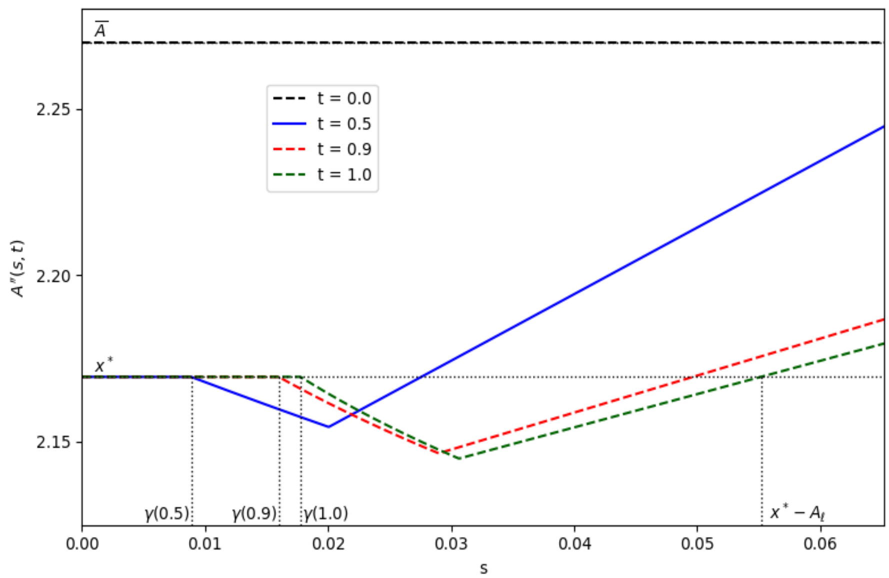

Figure 2: The equilibrium participation threshold $A''(s,t)$. Parameters: $\delta = 0.1$, $\overline{s} = 0.15$, $\alpha = 0.5$, $r = 1$, $\underline{K} = \underline{A} = 1.5$, $\overline{K} = 2$, $\overline{A} = 2.27$.

where tax payment is per unit of capital. At $s = 0$, the investment threshold at which the investor is indifferent between investing in the country and not is x^*. As s increases, this threshold falls linearly with s.

Figure 1 also shows how the profit-sharing rate t affects the investment threshold A'. When there is profit-sharing $t > 0$, the threshold tends to be higher for a given guarantee, because some of an investor's profit will be clawed back. However, the profit-sharing magnitude t is irrelevant when the guarantee is so small that no one participates, or so large that the marginal investor anticipates to make no profit to share, investing only to collect the guarantee. In the middle, where $A' \in (A_l, x^*)$, higher profit-sharing naturally increases the investment threshold A'. Figure 2 shows how the threshold for investment without participation varies with the program features (s,t). We will later describe the intuition in the context of optimal GPS.

To reach $A' = A_l$, the program needs a sufficiently high guarantee $s = x^* - A_l$. However, Assumption 2 places an upper bound on the guarantee, preventing the guarantor from inducing investment in the worst state of the world: when the noise is vanishingly small, it requires that $s \leq \bar{s} < A_l - \underline{A}$. Assumption 3 ensures that, for a fixed noise level σ, it is feasible to resolve the coordination failure, without violating Assumption 2: the guarantor cannot induce investment in the worst state of the world, but it can fully remove coordination failure.[15]

Assumption 3. *It is feasible to remove the inefficiency interval:* $\sigma < A_l - (\underline{A} + x^*)/2$

Because $x^* > A_l$ by Proposition 3, the bound $A_l - (\underline{A} + x^*)/2 > 0$ for sufficiently small $\underline{A}$, so that Assumption 3 would hold when noise is small.[16]

To eliminate coordination failure, the guarantor must induce potential investors to invest whenever their signals are above the threshold A_l—when the noise is vanishingly small. Proposition 4 shows that this can be achieved by providing a guarantee $s = x^* - A_l$, regardless of the profit-sharing magnitude—see Figure 1. It follows that the cheapest program that can eliminate miscoordination must set the guarantee $s = x^* - A_l$ and the profit-sharing magnitude $t = 1$, both to maximize the clawback and to disincentivize program participation by those who invest in the country even without additional incentives. Proposition 5 formalizes this result and computes the associated cost of the optimal GPS program that eliminates miscoordination.

Proposition 5. *Suppose assumptions 1, 2, and 3 hold. As the noise becomes vanishingly small, the optimal GPS that eliminates miscoordination features* $(s, t) = (x^* - A_l, 1)$*, and its expected cost is* $\frac{1}{2}\frac{(x^* - A_l)^2}{\overline{A} - \underline{A}}$.

The intuition builds on that of Proposition 4 around Equation (12). Absent programs, the opportunity cost of investing for the marginal investor with the threshold signal x^* includes the normalized tax payment (per unit of capital) associated with the belief that the fraction

[15] Given σ, there is no coordination failure whenever $x' \leq A_l - \sigma$.

[16] Assumption 3 tends to require that $\underline{A}$ be small, while Assumption 1.1 tends to require that it be large—to ensure interior tax rate. These assumptions can be satisfied simultaneously when, e.g., r and $\overline{A}$ are sufficiently large that $\frac{r}{\alpha} > \frac{\delta}{(1-\alpha)^2}\left\{\frac{1}{\overline{K}+\underline{K}}\frac{\overline{K}-\underline{K}}{\underline{K}} + \frac{1}{\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right)\right\} + 2\sigma$ and Assumption 1.3 hold, respectively.

of investors who will invest is uniformly distributed on $[0, 1]$:

$$\left.\frac{\mathbb{E}_K[\text{tax payment}]}{\alpha}\right|_{K\sim U[0,\overline{K}]} = x^* - \frac{r}{\alpha}.$$

The guarantor wants the marginal investor to behave *as if* she believes that all other potential investors will invest, so that the associated tax payment is only

$$\left.\frac{\mathbb{E}_K[\text{tax payment}]}{\alpha}\right|_{K=\overline{K}} = A_l - \frac{r}{\alpha}.$$

The guarantor pays the difference, $x^* - A_l$, via the guarantee. Moreover, because there is no (pre-program) profit at $A = A_l$, profit-sharing rate is irrelevant for the marginal investor:[17]

$$\mathbb{E}_A[\pi(K(A; A_l), A)|x_i = A_l] < \mathbb{E}_A[\pi(\overline{K}, A)|x_i = A_l] = 0.$$

While the magnitude of profit-sharing rate t does not change the marginal investor's decision whether to invest in the country, it does change program participation decisions of those who invest. Therefore, it plays a critical role in program costs and hence the design of optimal GPS programs. This is apparent in Figure 2: among GPS programs with guarantee $s = x^* - A_l$, which resolve miscoordination, (non-)participation threshold A'' varies with the profit-sharing rate t. A higher threshold A'' means that a larger fraction of investors participate in the program. In particular, when $t < 1$, so that $A'' > x^*$, investors with signals $x_i \in (x^*, A'')$ participate, even though they would have invested in the country even absent any program. The participation of these investors does not help with miscoordination, but

[17]To see the generality of these arguments, consider a general setting in which an investor's utility is $u(A, K)$ (Frankel, Morris, and Pauzner, 2003; Morris and Shin, 2003). In the limit as $\sigma \to 0$, $E[u(A, K)|x_i = A_l] < 0 = E[u(A, \overline{K})|x_i = A_l] = E[u(A, K)|x_i = x^*] < E[u(A, \overline{K})|x_i = x^*]$. The guarantor must pay $s = -E[u(A, K)|x_i = A_l]$ to make type A_l indifferent between investing and not. Now, suppose payoffs are additively separable: $u(\theta, K) = u_1(\theta) + u_2(K)$, let $\hat{u} = \int u_2(K)dK$, and recall that the marginal investor (the one with the threshold x^* above which players invest) believes that the fraction of investors is $U[0, 1]$. We have $E[u(A, K)|x_i = x^*] = u_1(x^*) + \hat{u} = 0$. Thus, $s = -E[u(A, K)|x_i = A_l] = -(u_1(A_l) + \hat{u}) = -u_1(A_l) + u_1(x^*)$, which yields $x^* - A_l$ for linear $u_1(\cdot)$. Moreover, $E[u(A, \overline{K})|x_i = x^*] = u_1(x^*) + u_2(\overline{K}) = u_1(x^*) - u_1(A_l)$, where we used $E[u(A, \overline{K})|x_i = A_l] = u_1(A_l) + u_2(\overline{K}) = 0$. That is, the most pessimistic investor without the program (with signal x^*) is indifferent between participating and investing without participation.

raises the costs of the program. Profit-sharing is crucial to screen them out.

Remarkably, by setting $t = 1$ and thereby claiming all subsequent (pre-program) profits, a GPS program can nearly perfectly screen out investors who would have invested even without the program—inducing the participation threshold $A'' = x^*$. Intuitively, absent programs, the marginal investor with signal x^* is indifferent between investing in the country or in global markets. Participation in the program $(s, t) = (x^* - A_l, 1)$, provides this marginal investor (with signal x^*) the payoff $s = x^* - A_l$, regardless of any profits which will be fully taken away at the profit-sharing rate of 1. By contrast, not participating in the program, when it exists, simply makes this marginal investor more optimistic about aggregate investment and hence the anticipated tax payment: the marginal investor with signal x^* believes that almost all investors will invest, because $x^* > A_l$, the new investment threshold induced by the program. The difference in the anticipated tax payment is exactly $x^* - A_l$ as we discussed above. That is, with program $(s, t) = (x^* - A_l, 1)$, the marginal investor with the threshold signal x^* is indifferent between investing with and without participation in the program. It follows that reducing the profit-sharing rate from $t = 1$ tips the balance in favor of participation, raising the participation threshold A'' above x^*, and hence program costs.

These intuitions also allow us to provide a heuristic derivation of the program costs. Because investors with signals $x_i \in [A', A'')$ participate in the program, when the noise is vanishingly small, almost all investors participate in the program with probability $\frac{A''-A'}{\overline{A}-\underline{A}} = \frac{x^*-A_l}{\overline{A}-\underline{A}}$. Moreover, an investor i with signal $x_i \in (A', A'')$ obtains a net transfer

$$s - t\ \pi(\overline{K}, A) \approx (x^* - A_l) - 1\ (x_i - A_l) = x^* - x_i = A'' - x_i > 0, \tag{13}$$

so that $\mathbb{E}[A'' - x_i \mid x_i \in (A', A'')] \approx \mathbb{E}[x^* - A \mid A \in (A_l, x^*)] = x^* - \frac{A_l+x^*}{2} = \frac{x^*-A_l}{2}$. Therefore, the expected costs are

$$\Pr(x_i \in (A', A''))\ \mathbb{E}[A'' - x_i \mid x_i \in (A', A'')] \approx \frac{1}{2}\frac{(x^* - A_l)^2}{\overline{A} - \underline{A}},$$

as specified in Proposition 5.

Corollary 2 is immediate from Proposition 3 and Proposition 5.

Corollary 2. *The expected cost of the optimal GPS that eliminates miscoordination is increasing in public good productivity δ and capital share α, and it is decreasing in the immobile capital level $\underline{K}$.*

This result suggests that eliminating miscoordination via GPS programs is cheaper when capital is invested in industries with less advanced automation technology (Acemoglu and Restrepo, 2018) (lower α), or where there is abundant immobile capital (higher $\underline{K}$).

5 Guaranteed Return with Profit- and Loss-Sharing (GPLS)

GPS programs have appealing features. An optimal GPS program takes a natural form: the guarantor effectively rents capital from investors who are willing to participate in the program, and then invests it in the developing country. Those who are more optimistic about the country's investment prospects invest without participating in the program. In fact, the program screens out all investors who would have invested absent the program.

However, as equation (13) showed, even the optimal GPS program offers net positive subsidy to all participants except the most *optimistic* one with signal $x_i = A''$, who is made indifferent between investing with and without participation. In fact, all participants will end up with the same, strictly positive net payoff $s = x^* - A_l$ from investing in the country versus global markets. This feature is problematic for two reasons. First, ideally a GPS program should make participants indifferent between investing and not investing in the country, instead of leaving them with net positive payoffs that the program must then pay. Second, taking a mechanism design approach, more optimistic investors have more incentives to misreport their types (signals) and therefore should receive (weakly) more rent (Börgers, 2015, Ch. 1). Given that the optimal GPS program already required profit-sharing at rate 1, any adjustment must reduce the guarantee. However, the guarantee is already at the minimum amount required to eliminate miscoordination inefficiencies. This suggests that the GPS program's instruments are too coarse. To reduce costs, a program should either (1) allow some miscoordination inefficiencies, or (2) introduce new instruments that can provide different compensation to different investors depending on their types (signals) or equivalently, their expected pre-program payoffs.

We thus extend the GPS program to Guaranteed Return with Profit- and Loss-Sharing (GPLS) program by introducing loss-sharing at a rate t^-. GPLS programs are described by a guaranteed return $s \in [0, \bar{s}]$, a profit-sharing rate $t^+ \in [0, 1]$, and a loss-sharing rate $t^- \in [0, \bar{t}]$ with $\bar{t} < 1$, which specifies the fraction of losses covered by the guarantor. The upper bound $\bar{t} < 1$ captures realistic settings in which the guarantor is unable to induce investment in the worst state of the world. It may also reflect political constraints that preclude a 100% compensation of losses.

Payoffs from not investing and from investing without participation in the program as well as our tie-breaking rules are the same as before. Likewise, Assumption 2 is modified to

Assumption 4. *If productivity is sufficiently low, not investing in the country is the dominant strategy under GPLS program:* $\pi(\overline{K}, \underline{A}) + \frac{\bar{s}}{1-\bar{t}} = \underline{A} - A_l + \frac{\bar{s}}{1-\bar{t}} < -\sigma$.

Because GPLS programs offer loss-sharing in addition to the guarantee, Assumption 4 implies Assumption 2, by setting $\bar{t} = 0$.

Given productivity A and aggregate investment K, the incremental payoffs from taking higher versus lower actions under a GPLS program (s, t^+, t^-) are

$$\begin{aligned} \delta^1_{gpls}(K, A) &= \pi(K, A) - t^+ \max\{\pi(K, A), 0\} - t^- \min\{\pi(K, A), 0\} + s \\ \delta^2_{gpls}(K, A) &= t^+ \max\{\pi(K, A), 0\} + t^- \min\{\pi(K, A), 0\} - s. \end{aligned} \tag{14}$$

Action and state monotonicity continue to hold, and hence best responses to monotone strategies are also monotone. Similarly, we can compute the equilibrium cutoffs x' and x'' under a GPLS program analogously to a GPS program, with δ^j_{gpls} replacing δ^j, $j \in \{1, 2\}$.

The following proposition is analogous to Proposition 4.

Proposition 6. *Suppose assumptions 1 and 4 hold and a GPLS program (s, t^+, t^-) is in place. As the noise becomes vanishingly small, there is a unique equilibrium.*[18] *In equilibrium, $x' = A'(s, t^+, t^-) \leq x^*$, where*

$$A'(s,t^+,t^-) = \begin{cases} x^* - \frac{s}{1-t^-} & ; A_l \geq x^* - \frac{s}{1-t^-} \\ \min\{\tilde{A}(s,t^+,t^-), x^*\} & ; A_l \leq x^* - \frac{s}{1-t^-}, \end{cases}$$

and $\tilde{A}$ is the unique solution to

$$\tilde{A} = x^* - \frac{s}{1-t^-} + \frac{t^+ - t^-}{1-t^-} \, \frac{\delta}{(1-\alpha)^2 \overline{K}} \left\{ \frac{\tilde{A} - r/\alpha}{A_l - r/\alpha} - \log\left(\frac{\tilde{A} - r/\alpha}{A_l - r/\alpha}\right) - 1 \right\}.$$

Moreover,

- *if $s \geq t^+ \, (\overline{A} - A_l)$, then $x'' = A''(s, t^+, t^-) = \overline{A}$, so that all who invest will participate in the program.*
- *if $s < t^+ \, (\overline{A} - A_l)$, then $x'' = A''(s, t^+, t^-) < \overline{A}$, where*

$$A''(s,t^+,t^-) = \begin{cases} A_l + s/t^+ & ; s > t^+ \, (A'(s,t^+,t^-) - A_l) \\ A'(s,t^+,t^-) & ; s \leq t^+ \, (A'(s,t^+,t^-) - A_l), \end{cases}$$

  so that some who invest will not participate in the program.

Proposition 6 becomes Proposition 4 by setting $t^- = 0$ and $t^+ = t$. From Proposition 6, $A'' = \overline{A}$ whenever $s = t^+ = 0$ regardless of t^-: When $t^- > 0$, loss-sharing benefits the participants. When $t^- = 0$, participation has no effect. Thus, there is indeterminacy in A'', and we have picked a convenient A'' for this special case as we did for GPS programs.

To reach $A' = A_l$, GPLS programs incentivize investors by a sufficiently attractive combination of guarantee and lost-sharing: $s/(1-t^-) = x^* - A_l$.[19] The logic is similar to our

[18] We characterize the equilibrium under GPLS programs with finite noise, $0 < \sigma < \min\{A_l - \underline{A}, \overline{A} - A_h\}$, in Proposition 10 in Online Appendix A.

[19] Assumption 4 places an upper bound on the guarantee. Assumption 3 ensures that $x^* - A_l < A_l - \underline{A}$,

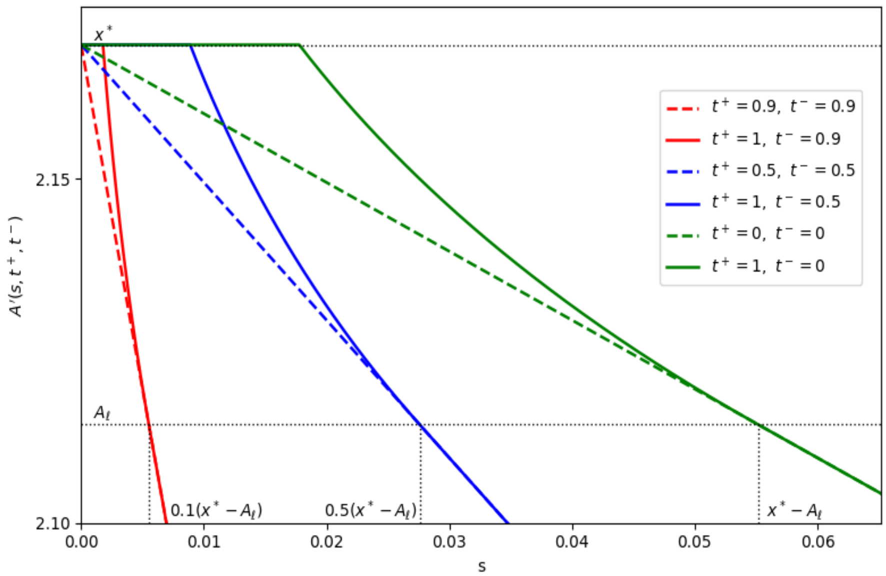

Figure 3: The equilibrium investment threshold $A'(s,t^+,t^-)$ as a function of the GPLS program's guarantee s, profit-sharing parameter t^+, and loss-sharing t^-. Parameters: $\delta = 0.1,\ \overline{s} = 0.15,\ \alpha = 0.5,\ r = 1,\ \underline{K} = \underline{A} = 1.5,\ \overline{K} = 2,\ \overline{A} = 2.27,\ \overline{t} = 0.9$.

discussion following Proposition 5. The guarantor must pay $x^* - A_l$ to the marginal investor, so that she invests when her signal is just above A_l. With GPLS programs, the guarantor can do so through two channels: guarantee and loss-sharing. With a loss-sharing rate of t^-, the guarantor transfers $t^-\ (x^* - A_l)$ to the marginal investor through loss-sharing, requiring a guarantee of $s = (1-t^-)(x^* - A_l)$ to cover the rest.

Any combination of guarantee s and risk-sharing rate t^- such that $s/(1-t^-) = x^* - A_l$ will eliminate miscoordination as it makes no difference to an investor to be compensated through guarantee or risk-sharing. As Figure 3 illustrates, $A' = A_l$ at $s = (1-t^-)(x^* - A_l)$ for $t^- = 0,\ 0.5$, or 0.9. Importantly, the guarantor does not know which potential investor is more optimistic and hence will require less subsidy to invest in the country. Moreover, optimistic investors have incentives to pretend they are pessimistic (have lower signals) to extract more subsidy. To screen them out and save on costs, the guarantor must maximize the loss-sharing rate and choose the guarantee just high enough to induce the marginal

so that miscoordination can be eliminated even when the guarantor cannot induce coordination is the worst state of the world.

investor to invest. More optimistic investors anticipate lower or no losses and hence lower or no subsidy through loss-sharing. In contrast, every participant expects to receive the same guarantee. It follows that the optimal GPLS program features $t^- = \bar{t}$, and $s = (1-\bar{t})(x^* - A_l)$. Mirroring the logic of the GPS programs, the guarantor also chooses $t^+ = 1$.

Proposition 7. *Suppose assumptions 1, 3, and 4 hold. As the noise becomes vanishingly small, the optimal GPLS program that eliminates miscoordiantion is* $(s, t^+, t^-) = ((1-\bar{t})(x^* - A_l), 1, \bar{t})$*. Its expected cost is* $\frac{(1-\bar{t})^2}{2}\frac{(x^*-A_l)^2}{\bar{A}-\underline{A}}$*, which is strictly smaller than the expected costs of the optimal GPS program if and only if* $\bar{t} > 0$*.*

The optimal GPLS program's expected cost is lower than that of the optimal GPS program and coincides with it for $t^- = 0$. Remarkably, when loss-sharing rate is near 100%, the program costs are negligible—in the limit as the noise vanishes. The intuition is that the guarantor does not need to provide a guarantee to induce the marginal investor to invest: when $t^- = \bar{t} \approx 1$, the optimal guarantee $s = (1 - \bar{t})(x^* - A_l) \approx 0$. With negligible optimal guarantee, and a positive profit-sharing rate $t^+ = 1$, nearly no investor participates in the program: $A'' \approx A' \approx A_l$: investors with signals slightly more optimistic than the marginal investor—those with signals $x_i \in (A_l, A_l + \epsilon)$—anticipate a positive payoff, $\pi(\overline{K}, A_l + \epsilon) \approx \epsilon > 0$, and hence no loss-sharing subsidy and nearly no guarantee $s \approx 0$, but they have to share their profit as $t^+ = 1$.[20]

5.1 Restricted GPLS (Shen-Zou Program)

GPLS programs allow for different rates for profit- and loss-sharing. We now analyze the additional costs to the guarantor introduced if we restrict the rates to be equal: $t^+ = t^- \in (0, \bar{t}]$, with $\bar{t} \in [0, 1)$. We refer to such programs as Restricted GPLS (R-GPLS) programs.

[20]We will discuss realistic restrictions on $\bar{t}$ in the next subsection. Here, we clarify the technicalities concerning $\bar{t} \approx 1$ for completeness. Assumption 4 rules out $\bar{t} = 1$ for substantive reasons: the guarantor should not be able to induce investment even in the worst state of the world. If it was allowed, $\bar{t} = 1$, $s = 0$, and any $t^+ > 0$ would induce a continuum of equilibria with $A'' = A_l$ and $A' \in [\underline{A}, A_l]$ without our tie-breaking rule, and $A' = \underline{A}$ with it. Then, an optimal GPLS program along the following path $\bar{t} \to 1$, $s = (1 - \bar{t})(x^* - A_l) \to 0$, and any $t^+ > 0$ can be viewed as selecting the equilibrium with $A' = A'' = A_l$ when first $\sigma \to 0$ and then $\bar{t} \to 1$. However, for any given $\bar{t} < 1$, the optimal guarantee $s > 0$, and hence the optimal profit-sharing $t^+ = 1$.

An R-GPLS program is analogous to the intervention that Shen and Zou (2024) suggest in the context of bank runs, and therefore we also refer to them as SZ programs.

Proposition 8. *Suppose $\bar{t} > 0$, and assumptions 1, 3, and 4 hold. As the noise becomes vanishingly small, the optimal R-GPLS (SZ) program that eliminates miscoordination is $(s, t^+, t^-) = ((1-\bar{t})(x^* - A_l), \bar{t}, \bar{t})$. Its expected cost is $\frac{1}{\bar{t}} \frac{(1-\bar{t})^2}{2} \frac{(x^*-A_l)^2}{\bar{A}-\underline{A}}$, which is strictly larger than the expected costs of the optimal GPLS program if and only if $\bar{t} < 1$.*

Intuitively, for any $\bar{t} < 1$, the optimal program requires $s > 0$, creating incentives for optimistic investors to participate. To screen them out, the guarantor wants to choose the highest profit-sharing rate t^+, and so it chooses its upper bound $\bar{t} < 1$. In the GPLS program this upper bound is 1—without violating any substantive assumption, e.g., Assumption 4. Therefore, the costs of R-GPLS programs are higher.

6 Comparisons: GPS, GPLS, SZ, and Common Programs

Propositions 5, 7, and 8 allow us to compare the costs associated with the programs that aim to eliminate miscoordination inefficiencies.

Corollary 3. *Suppose Assumptions 1 to 4 hold. Let $C(i)$ be the expected costs of the optimal program $i \in \{GPS, GPLS, SZ\}$. As the noise becomes vanishingly small,*

$$C(GPLS) = \bar{t} \cdot C(SZ) = (1-\bar{t})^2 \cdot C(GPS).$$

That is, the costs of optimal SZ and GPS programs are larger than that of the optimal GPLS program by factors of $1/\bar{t}$ and $1/(1-\bar{t})^2$, respectively.

When there is nearly complete loss-sharing, GPLS and SZ (R-GPLS) coincide: both $t^- \approx t^+ = 1$. However, this would imply that the guarantor can induce investment regardless of how bad the state of the world will be—recall Assumption 4. A more realistic assumption is that $\bar{t}$ is bounded away from 1. Then, Corollary 3 shows that restricted GPLS programs

return	$A < A_l$	$A > A_l$
r_{gps}	$r - \alpha(A_h - x^*) - \alpha(A_l - A)$	$r + \alpha(x^* - A_l)$
r_{gpls}	$(1-\bar{t})\, r_{\text{gps}} + \bar{t}\, r$	$r + (1-\bar{t})\, \alpha(x^* - A_l)$
r_{np}	$r - \alpha(A_h - A)$	$r + \alpha(A - A_l)$

Table 1: Investor absolute returns under optimal GPS and GPLS programs.

(SZ programs) by requiring $t^- = t^+$ raises expected program costs by a factor of $1/\bar{t}$.[21]

Conversely, relaxing the R-GPLS (SZ) programs by allowing full profit-sharing while maintaining the realistic assumption of restricted loss-sharing rate to a maximum of $\bar{t} < 1$ can generate significant savings, more so when $\bar{t}$ is lower or when miscoordination is a more significant problem, so that $x^* - A_l$ is larger; that is when public good productivity δ or capital share α are higher, or the immobile capital level $\underline{K}$ is lower—see Proposition 3.

We performed the analysis in normalized and net returns, because they determine strategic behavior. We end by showing the investors' absolute returns in equilibrium (when the noise is vanishingly small) under the optimal GPS and GPLS programs.

Corollary 4. *Let r_{gps} and r_{gpls} denote an investor's (absolute) return from participating in the optimal GPS and GPLS programs, respectively, and let r_{np} denote the corresponding return for a non-participating investor. Then, returns are given in Table 1.*

Figure 4 illustrates. Optimal GPS and GPLS programs both feature a minimum guaranteed return. Importantly, they both also feature a *maximum* return. Participating investors get a higher payoff than non-participating ones if and only if the productivity shock A is sufficiently low. For the optimal GPLS program this threshold is higher than A_l only by a margin of $(1-\bar{t})s_{gps}$, where s_{gps} is the guarantee under the optimal GPS program. Therefore, when $\bar{t}$ is only slightly below 1, potential investors who believe that productivity is at least slightly above A_l will not participate, reducing the program costs. Critically, while the programs

[21] SZ (R-GPLS) programs are more cost effective than GPS programs if and only if $\bar{t}$ is above a threshold. The threshold is the solution to $(1-x)^2 = x$ in the interval $(0, 1)$.

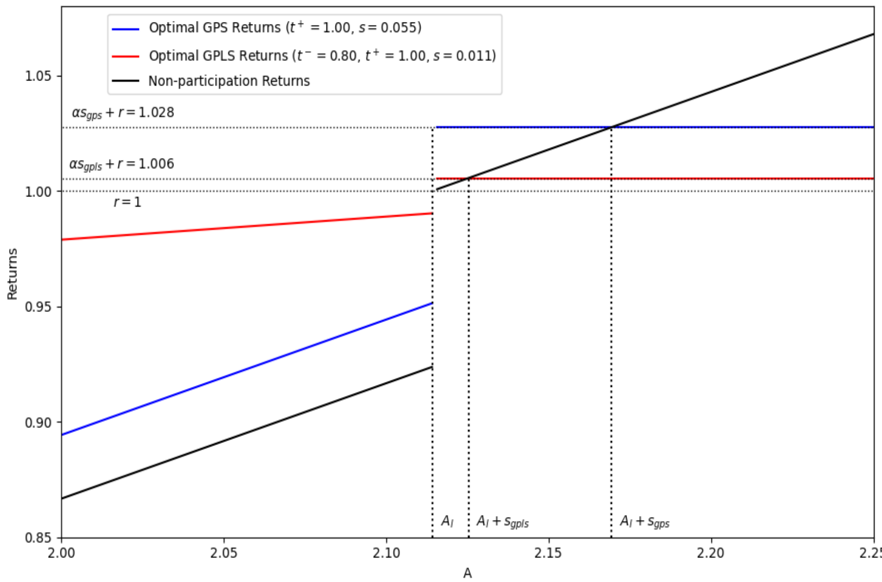

Figure 4: Absolute returns as a function of productivity A under optimal GPS and GPLS programs with guarantees s_{gps} and s_{gpls}, respectively. Parameters: $\delta = 0.1,\ \bar{s} = 0.15,\ \alpha = 0.5,\ r = 1,\ \underline{K} = \underline{A} = 1.5,\ \overline{K} = 2,\ \overline{A} = 2.27,\ \bar{t} = 0.8, A' = A_l$.

look qualitatively similar, as Corollary 3 shows, even at a loss-sharing rate of $t^- = 0.5$, the optimal GPLS program costs only a quarter of the optimal GPS program. The difference is in better screening: the optimal GPLS program will have far less participants than the optimal GPS program, underlying its efficiency.

6.1 Common Programs: Political Risk Insurance, Concessional Loans, and Blended Finance

International organizations, national governments, and private-sector organizations, provide instruments to mitigate political risk. While the international community is aware of the coordination problem among investors (the academic literature goes back to at least Rosenstein-Rodan (1943)) and the reports of development agencies engage with the issue (World Bank, 2017), available instruments focus on addressing political risk for one or few investors and creditors, overlooking the coordination problem. We present an overview of these instruments and show how their structures relate to the optimal GPLS program structure. This comparison provides a benchmark that highlights the features of the optimal GPLS program, and brings to focus how existing programs could be adjusted to address miscoordination inefficiencies.

Two classes of instruments stand out: political risk insurance and concessional loans.[22]

Political Risk Insurance compensates for losses caused by covered adverse political events (e.g., breach of contract or political violence) at a premium. These products are not subsidies in general, and the agencies that offer them are self-sustaining. Examples include the Multilateral Investment Guarantee Agency MIGA's *Guarantees* and the U.S. International Development Finance Corporation DFC's *Political Risk Insurance*.[23] (Multilateral Investment Guarantee Agency, 2024; World Bank, 2016).

Concessional Loans to investors below the market rate. For example, the International Finance Corporation IFC's blended finance instrument offers concessional loans to investors through the International Development Association (IDA) Private Sector Window (PSW).[24]

Instruments may be combined in what is known as *blended finance*, in which, for example, the International Finance Corporation provides loans, guarantees, and equity investments—some or all of which may be offered at a discount.[25]

A concessional loan reduces an investor's cost of capital and resembles the guaranteed return in our setting, which is $\alpha s_{gpls} = \alpha(1 - t^-)(x^* - A_l)$ for the optimal GPLS program. Political risk insurance compensations or guarantees are similar to loss-sharing. In Table 1, we can

[22]Other policies include providing technical assistance, making equity investment, and offering grants, although major agencies tend to offer grants only to governments.

[23]See https://www.dfc.gov/what-we-offer/our-products for DFC and https://www.miga.org/products for MIGA.

[24]See https://ida.worldbank.org/en/financing/ida-private-sector-window for PSW and https://www.ifc.org/en/what-we-do/sector-expertise/blended-finance/how-blended-finance-works for IFC's Blended Finance.

[25]These discounts are sometimes referred to as concessionality, e.g., see: https://www.ifc.org/en/what-we-do/sector-expertise/blended-finance/how-blended-finance-works#concessionality. For an example, see *Ghana Sankofa Gas Project* (2020), in which the World Bank Group used several instruments to help mobilize about $8 billion in private investments for an offshore natural gas development project in Ghana.

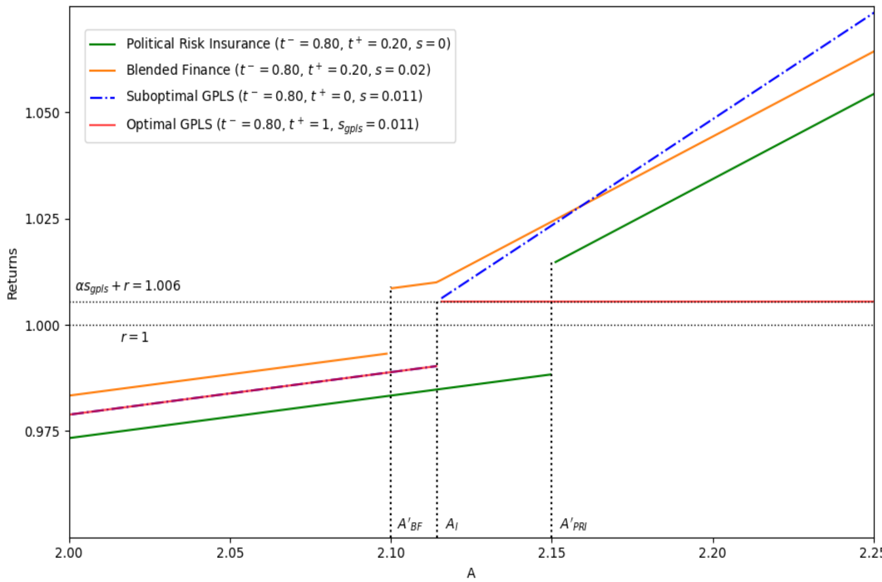

Figure 5: Absolute returns as a function of productivity A for Political Risk Insurance (PRI), Blended Finance (BF), a suboptimal GPLS, and the optimal GPLS. A'_{BF} and A'_{PRI} denote the investment thresholds (A's) under BF and PRI, respectively. Parameters: $\delta = 0.1$, $\overline{s} = 0.15$, $\alpha = 0.5$, $r = 1$, $\underline{K} = \underline{A} = 1.5$, $\overline{K} = 2$, $\overline{A} = 2.27$, $\overline{t} = 0.8$.

write r_{gpls} when $A < A_l$ (so there is a loss) as $r + \alpha s_{gpls} - (1-t^-)\alpha(A_h - A)$. Then $\alpha(A_h - A)$ may be viewed as the loss and $\overline{t}$ as the loss-sharing rate.[26] Similarly, when $A > A_l$, so there is a profit, r_{gpls} can be written as $r_{np} + \alpha s_{gpls} - \alpha(A - A_l)$, so that $\alpha(A - A_l)$ resembles the premium paid to the agency. However, "premium" here is state-contingent, not a fixed, predetermined amount.

In sum, one may think of political risk insurance and guarantee programs, in a stylized sense, as featuring $t^-, t^+ \in (0,1)$ with $s = 0$, and concessional loans as $s > 0$ with $t^- = t^+ = 0$.

Figure 5 shows how a participating investor's absolute return under the optimal GPLS program—designed to eliminate miscoordination—compares with alternatives that are suboptimal for addressing coordination risk. For example, political risk insurance and guarantees without concessional loans ($s = 0$) cannot eliminate miscoordination. Programs with $t^+ < 1$ are significantly more expensive: not only they offer higher payments to each participant,

[26] If the program accounts for "concessionality" in the loss-sharing, the rate is $\frac{t^-\alpha(A_h-A)}{r-(\alpha s_{gpls}-(1-\overline{t})\alpha(A_h-A))}$.

but importantly, screening is impaired, leading to a larger number of participants. They are ineffective on both intensive and extensive margins of saving. Finally, programs that combine concessional loans with loss- and profit-sharing in a suboptimal way could be doubly inefficient: they not only induce coordination when investment should be avoided (see $A'_{BF} < A_l$ in Figure 5), but also impose higher costs due to ineffective screening.

7 Conclusion

Political risk and coordination failure are considered two primary hindrances for growth in developing countries. We showed how political risk can induce coordination failure, and proposed subsidy programs to mitigate it at minimal cost compared to natural alternatives, including guarantees. The key is effective screening of potential investors and designing the program such that only the minimum necessary investors find it beneficial to take advantage of the program, while others invest, anticipating that the program encourages participation.

Several directions for future research stand out. An alternative approach to providing subsidy programs is that development agencies directly invest as large players. As Corsetti et al. (2004) show, even one large player can influence coordination. A natural question is whether and when international agencies can mitigate miscoordination at a lower cost by diverting funds from subsidies to direct investment. Another direction is to study whether and how international agencies can provide information that facilitates coordinating investment (Basak, Deb, and Kuvalekar, 2024; Basak and Zhou, 2020). Third, our analysis was focused on settings with ex ante homogeneous investors (cf. Sákovics and Steiner, 2012), in which there is little information asymmetry among investors. Analyzing the more general setting is left for future research. Fourth, we also restricted attention to settings where development agencies can observe investors' profits. In practice, investors may seek to conceal profits or overstate losses, while agencies can engage in costly monitoring. It remains to analyze how the optimal program is modified in the presence of reporting fraud and monitoring costs. Finally, as an intermediate step toward implementing the proposed program in practice, lab experiments can provide guidance on its effectiveness. There is already a large experimental literature on different aspects of coordination and global games (Avoyan, 2024). Investigating subsidies in that context is a feasible and fruitful direction for future research.

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Online Appendix A: GPS and GPLS with Finite Noise

In Proposition 4 and Proposition 6 in the text, we characterize the equilibrium as the noise becomes vanishingly small. Here, we present propositions characterizing the equilibrium under a GPS and GPLS program for finite noise, $0 < \sigma < \min\{A_l - \underline{A}, \overline{A} - A_h\}$. The proofs of these propositions are in Online Appendix B.

For a fixed noise, $\sigma \in (0, \min\{A_l - \underline{A}, \overline{A} - A_h\})$, and a lower threshold x', let $A_0(x', \sigma)$ denote the value of A at which investor i's payoff is equal to zero, i.e., $\pi(K(A_0(x',\sigma); x'), A_0(x',\sigma)) = A_0(x',\sigma) - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A_0(x',\sigma);x'))} = 0$ where $K(A_0(x',\sigma); x') = \overline{K}Pr(x_i \geq x' | A_0(x',\sigma))$. Then,

$$A_0(x',\sigma) = \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2}\frac{1}{(\underline{K} + \overline{K}Pr(x_i \geq x'|A_0(x',\sigma)))}$$

Substituting the value of $Pr(x_i \geq x'|A_0(x',\sigma))$,

$$A_0(x',\sigma) = \begin{cases} A_h & ; x' > A_0(x',\sigma) + \sigma \\ \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2}\frac{1}{\left(\underline{K}+\frac{\overline{K}}{2}\left[1-\left(\frac{x'-A_0(x',\sigma)}{\sigma}\right)\right]\right)} & ; x' \in [A_0(x',\sigma) - \sigma, A_0(x',\sigma)+\sigma] \\ A_l & ; x' < A_0(x',\sigma) - \sigma \end{cases} \tag{15}$$

For $x' \in [A_0(x',\sigma) - \sigma, A_0(x',\sigma) + \sigma]$, solving for $A_0(x',\sigma)$ yields

$$A_0(x',\sigma) = \left(\frac{x'}{2} + \frac{r}{2\alpha} - \sigma\frac{\underline{K}}{\overline{K}} - \frac{\sigma}{2}\right) \pm \sqrt{\left(\frac{x'}{2} - \frac{r}{2\alpha} - \sigma\frac{\underline{K}}{\overline{K}} - \frac{\sigma}{2}\right)^2 + 2\sigma\frac{\delta}{\overline{K}(1-\alpha)^2}}.$$

$A_0(x',\sigma)$ is,

$$A_0(x',\sigma) = \left(\frac{x'}{2} + \frac{r}{2\alpha} - \sigma\frac{\underline{K}}{\overline{K}} - \frac{\sigma}{2}\right) + \sqrt{\left(\frac{x'}{2} - \frac{r}{2\alpha} - \sigma\frac{\underline{K}}{\overline{K}} - \frac{\sigma}{2}\right)^2 + 2\sigma\frac{\delta}{\overline{K}(1-\alpha)^2}}.$$

The other root yields $|A_0(x',\sigma) - x'| > \sigma$, which is not possible.

We can now present the equilibrium characterization. The following proposition states the result for a GPS program.

Proposition 9. *Suppose assumptions 1 and 2 hold and a GPS program (s,t) is in place. There is a unique equilibrium wherein $x' = x'(s,t) \le x^*$, where*

$$x'(s,t) = \begin{cases} x^* - s & ; A_l \ge x^* - s + \sigma \\ \min\{\tilde{x}(s,t), x^*\} & ; A_l \le x^* - s + \sigma, \end{cases}$$

and $\tilde{x} \in X' := \{x \in \mathbb{R} : A_0(x,\sigma) - \sigma \le x < A_0(x,\sigma) + \sigma\}$ is the unique solution to

$$\tilde{x} = \frac{t}{2\sigma}\left[\frac{(\tilde{x}+\sigma)^2 - A_0(\tilde{x},\sigma)^2}{2} - \frac{r(\tilde{x}+\sigma - A_0(\tilde{x},\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{\tilde{x}-A_0(\tilde{x},\sigma)}{\sigma}\right)\right)}\right)\right]$$
$$+ x^* - s.$$

Moreover,

- *if $s \ge t\ (\overline{A} - A_l)$, then $x'' = \overline{A} + \sigma$, so that all who invest will participate in the program.*
- *if $s < t\ (\overline{A} - A_l)$, then $x'' = x''(s,t) = \max\{x', x^\diamond(s,t)\}$, where $x^\diamond$ is the minimum x that solves*

  $$s = t \cdot E[\max\{\pi(K(A;x'),A),0\}|x_i = x],$$

  so that some who invest will not participate in the program.

The following proposition characterizes the equilibrium under a GPLS program when noise is finite.

Proposition 10. *Suppose assumptions 1 and 4 hold and a GPLS program (s,t^+,t^-) is in place. There is a unique equilibrium wherein $x' = x'(s,t^+,t^-) \le x^*$, where*

$$x'(s,t^+,t^-) = \begin{cases} x^* - \frac{s}{1-t^-} & ; A_l \ge x^* - \frac{s}{1-t^-} + \sigma \\ \min\{\tilde{x}(s,t^+,t^-), x^*\} & ; A_l \le x^* - \frac{s}{1-t^-} + \sigma, \end{cases}$$

where $\tilde{x} \in X' := \{x \in \mathbb{R} : A_0(x,\sigma) - \sigma \le x < A_0(x,\sigma) + \sigma\}$ *uniquely solves*

$$\begin{aligned}\tilde{x} = {} & \frac{t^+}{2\sigma}\left[\frac{(\tilde{x}+\sigma)^2 - A_0(\tilde{x},\sigma)^2}{2} - \frac{r(\tilde{x}+\sigma - A_0(\tilde{x},\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{\tilde{x}-A_0(\tilde{x},\sigma)}{\sigma}\right)\right)}\right)\right] \\ & + \frac{t^-}{2\sigma}\left[\frac{A_0(\tilde{x},\sigma)^2 - (\tilde{x}-\sigma)^2}{2} - \frac{r(A_0(\tilde{x},\sigma) - \tilde{x} + \sigma)}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{\tilde{x}-A_0(\tilde{x},\sigma)}{\sigma}\right)\right)}{\underline{K}}\right)\right] \\ & + x^* - s.\end{aligned}$$

Moreover,

- *if* $s \ge t^+ \, (\overline{A} - A_l)$*, then* $x'' = \overline{A} + \sigma$*, so that all who invest will participate in the program.*
- *if* $s < t^+ \, (\overline{A} - A_l)$*, then* $x'' = x''(s,t^+,t^-) = \max\{x', x^\diamond(s,t^+,t^-)\}$*, where* $x^\diamond$ *is the minimum* x *that solves*

$$s = t^+ \cdot E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x\big] + t^- \cdot E\big[\min\{\pi(K(A;x'),A),0\}|x_i = x\big],$$

  so that some who invest will not participate in the program.

Online Appendix B: Proofs[27]

Proof of Lemma 1. The first-order condition for the worker's problem (1) is $-(1-\alpha)Y + \sqrt{\delta Y/T} = 0$, i.e., $T = \frac{\delta}{(1-\alpha)^2}\frac{1}{Y} > 0$. The objective is strictly concave. Thus, the optimal tax rate $T^* = \min\left\{\frac{\delta}{(1-\alpha)^2}\frac{1}{Y}, 1\right\}$. $\square$

Proof of Proposition 1. Given the aggregate investment in the country is K, investor i invests in the country if and only if $\alpha A\left(1 - \frac{\delta}{(1-\alpha)^2 A(\underline{K}+K)}\right) - r \ge 0$. If $A < A_l$, then even if every other investor invests, i.e., $K = \overline{K}$, investor i does not invest because her returns from investing is $\alpha A\left(1 - \frac{\delta}{(1-\alpha)^2 A(\underline{K}+\overline{K})}\right) - r = \alpha(A - A_l) < 0$. Thus, not investing is the dominant strategy; in equilibrium, no investor invests when $A < A_l$. If $A \ge A_h$,

[27] We are currently refining the proofs and appreciate any comments.

even if no other investor invests ($K = 0$), investor i's returns from investing in the country is $\alpha A\left(1 - \frac{\delta}{(1-\alpha)^2\underline{K}}\right) - r = \alpha(A - A_h) \geq 0$. Thus, investing is the dominant strategy; in equilibrium, all investors invest when $A \geq A_h$. For $A \in [A_l, A_h)$, both the equilibria - one where no one invests and the other where everyone invests, exist. $\square$

Proof of Proposition 4. For the lower cutoff x', consider the following cases:

Case 1: $s \geq x^* - A_l$

From Proposition 9, $x' = x^* - s$ when $s \geq x^* - A_l + \sigma$. As the noise vanishes, for $s \geq x^* - A_l$,

$$x' = A'(s,t) = x^* - s. \tag{16}$$

Case 2: $s \leq x^* - A_l$

From Proposition 9, for finite noise ($0 < \sigma < \min\{A_l - \underline{A}, \overline{A} - A_h\}$) and $s \leq x^* - A_l + \sigma$, $\tilde{x}$ is the fixed point of $t \cdot H(x,\sigma) + x^* - s$ where

$$\begin{aligned} H(x,\sigma) &= \frac{1}{2\sigma}\left[\frac{(x+\sigma)^2 - A_0(x,\sigma)^2}{2} - \frac{r(x+\sigma-A_0(x,\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{x-A_0(x,\sigma)}{\sigma}\right)\right)}\right)\right] \\ &= \left(\frac{x - A_0(x,\sigma)}{2\sigma} + \frac{1}{2}\right)\left(\frac{x + A_0(x,\sigma)+\sigma}{2} - \frac{r}{\alpha}\right) - \frac{\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{x-A_0(x,\sigma)}{\sigma}\right)\right)}\right). \end{aligned} \tag{17}$$

As the noise becomes vanishingly small, $\lim\limits_{\sigma\to 0}\tilde{x}$, if it exists, must solve

$$\lim_{\sigma\to 0}\tilde{x} = t \cdot h(\lim_{\sigma\to 0}\tilde{x}) + x^* - s,$$

where

$$h(x) = \lim_{\sigma\to 0} H(x,\sigma).$$

From Proposition 9, $\tilde{x} \in X' := \{x \in \mathbb{R} : A_0(x,\sigma) - \sigma \leq x < A_0(x,\sigma) + \sigma)\}$. Let X be the closure of the set X', i.e., $X := \{x \in \mathbb{R} : A_0(x,\sigma) - \sigma \leq x \leq A_0(x,\sigma) + \sigma\}$. For any $x \in X$,

$$h(x) = \lim_{\sigma\to 0}\left\{\left(\frac{x - A_0(x,\sigma)}{2\sigma} + \frac{1}{2}\right)\left(\frac{x + A_0(x,\sigma)+\sigma}{2} - \frac{r}{\alpha}\right) - \frac{\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{x-A_0(x,\sigma)}{\sigma}\right)\right)}\right)\right\}.$$

For $x \in X$, $\lim\limits_{\sigma\to 0} A_0(x,\sigma) = x$. By L'Hopital's rule, $\lim\limits_{\sigma\to 0}\frac{x-A_0(x,\sigma)}{\sigma} = \lim\limits_{\sigma\to 0} -\frac{\partial}{\partial\sigma}A_0(x,\sigma) = 2\frac{\underline{K}}{\overline{K}} + 1 - \frac{2\delta}{\overline{K}(1-\alpha)^2(x-\frac{r}{\alpha})}$. Then,

$$
\begin{aligned}
h(x) &= \left(\frac{\underline{K}}{\overline{K}} + 1 - \frac{\delta}{\overline{K}(1-\alpha)^2(x-\frac{r}{\alpha})}\right)\left(x - \frac{r}{\alpha}\right) - \frac{\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\frac{\delta}{(1-\alpha)^2(x-\frac{r}{\alpha})}}\right) \\
&= \left(\frac{\underline{K}}{\overline{K}} + 1\right)\left(x - \frac{r}{\alpha}\right) - \frac{\delta}{\overline{K}(1-\alpha)^2}\left(1 + \log\left(\frac{\underline{K}+\overline{K}}{\frac{\delta}{(1-\alpha)^2(x-\frac{r}{\alpha})}}\right)\right). \qquad (18)
\end{aligned}
$$

We show the existence of $\lim_{\sigma\to 0}\tilde{x}$ and compute it by first showing that $H(x,\sigma)$ converges uniformly to $h(x)$ as $\sigma \to 0$. The following lemma states this.

Lemma 3. *For $x \in X$, $H(x,\sigma)$ converges uniformly to $h(x)$ as $\sigma \to 0$.*

Proof. Equation (18) demonstrates that $h(x) = \lim_{\sigma\to 0} H(x,\sigma)$ exists for any $x \in X$ (note that $r/\alpha \notin X$). Also, $H(x,\sigma)$ is differentiable on X for $\sigma > 0$. We then prove that for $x \in X$, $\frac{\partial}{\partial x}H(x,\sigma)$ converges uniformly to a function $d(x)$. Then because $\lim_{\sigma\to 0} H(x,\sigma)$ exists for any $x \in X$, $H(x,\sigma)$ converges uniformly to $h(x)$ and $h'(x) = d(x)$ (Theorem 7.17 (Rudin, 1976)).

For all $x \in X$, $H'(x,\sigma)$ exists and is

$$
\begin{aligned}
H'(x,\sigma) &= \frac{1}{2\sigma}\left[x + \sigma - A_0'(x,\sigma)\left(A_0(x,\sigma) - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K} + \frac{\overline{K}}{2}\left(1 - \left(\frac{x - A_0(x,\sigma)}{\sigma}\right)\right)}\right) - A_0(x,\sigma)\right] \\
&= \frac{1}{2}\left(\frac{x - A_0(x,\sigma)}{\sigma} + 1\right).
\end{aligned}
$$

Substituting the value of $\lim_{\sigma\to 0}\frac{x - A_0(x,\sigma)}{\sigma}$,

$$
d(x) = \lim_{\sigma\to 0} H'(x,\sigma) = \left(\frac{\underline{K}}{\overline{K}} + 1 - \frac{\delta}{\overline{K}(1-\alpha)^2(x-\frac{r}{\alpha})}\right).
$$

We show that for all $x \in X$, $H'(x,\sigma)$ converges uniformly to $d(x)$ as $\sigma \to 0$. Let $\sigma = 1/n$. Denote $H'(x, 1/n)$ as $H_n'(x)$. Then,

$$
\begin{aligned}
H_n'(x) - d(x) &= \frac{1}{2}\left[(n(x - A_0(x,1/n)) + 1) - \left(2\frac{\underline{K}}{\overline{K}} + 2 - \frac{2\delta}{\overline{K}(1-\alpha)^2(x-\frac{r}{\alpha})}\right)\right] \\
&= \frac{1}{2}\left[\frac{\underline{K}}{\overline{K}} + \frac{1}{2} + \frac{nx}{2} - \frac{nr}{2\alpha} + 1 - \sqrt{\left(\frac{nx}{2} - \frac{nr}{2\alpha} - \frac{\underline{K}}{\overline{K}} - \frac{1}{2}\right)^2 + \frac{2\delta n}{\overline{K}(1-\alpha)^2}}\right. \\
&\qquad \left. - \left(2\frac{\underline{K}}{\overline{K}} + 2 - \frac{2\delta}{\overline{K}(1-\alpha)^2(x-\frac{r}{\alpha})}\right)\right] \\
&= -\frac{1}{2}\left[\frac{\underline{K}}{\overline{K}} + \frac{1}{2} - \frac{nx}{2} + \frac{nr}{2\alpha} + \sqrt{\left(\frac{nx}{2} - \frac{nr}{2\alpha} - \frac{\underline{K}}{\overline{K}} - \frac{1}{2}\right)^2 + \frac{2\delta n}{\overline{K}(1-\alpha)^2}} - \frac{2\delta}{\overline{K}(1-\alpha)^2(x-\frac{r}{\alpha})}\right].
\end{aligned}
$$

Denote $\frac{\underline{K}}{\overline{K}} + \frac{1}{2} = k$ and $\frac{2\delta}{\overline{K}(1-\alpha)^2} = d$ and rewrite $x - \frac{r}{\alpha}$ as y. Then,

$$|H_n'(y) - d(y)| = \left| -\frac{1}{2}\left[\left(k - \frac{ny}{2}\right) + \sqrt{\left(\frac{ny}{2} - k\right)^2 + dn} - \frac{d}{y}\right]\right|$$
$$= \left| -\frac{1}{2}\left[\left(k - \frac{ny}{2}\right) + \left(\frac{ny}{2} - k\right)\sqrt{1 + \frac{dn}{\left(\frac{ny}{2} - k\right)^2}} - \frac{d}{y}\right]\right|.$$

Using binomial series expansion to approximate the square root term,

$$|H_n'(y) - d(y)| \approx \left|\frac{1}{2}\left[\left(k - \frac{ny}{2}\right)\left(1 + \frac{dn}{2\left(\frac{ny}{2} - k\right)^2}\right) - \left(k - \frac{ny}{2}\right) + \frac{d}{y}\right]\right| = \left|\frac{1}{2}\left[\frac{d}{\left(y - \frac{2k}{n}\right)} + \frac{d}{y}\right]\right|.$$

Then, for any $\epsilon > 0$, there exists an N such that $|H_n'(y) - d(y)| < \epsilon$ for all $n \geq N$ and for all y. This implies that for $x \in X$, $H_n'(x)$ converges uniformly to $d(x)$, i.e., $H'(x, \sigma)$ converges uniformly to $d(x)$ as $\sigma \to 0$. Furthermore, X is an interval $[A_l - \sigma, A_h + \sigma]$ because $x' \geq A_0(x', \sigma) - \sigma \implies x' \geq A_l - \sigma$. If that were not true, i.e., $x' < A_l - \sigma$, then, because $A_0(x', \sigma) \geq A_l$ (from equation (15)), $x' < A_0(x', \sigma) - \sigma$, which is a contradiction. By similar logic, $x' \leq A_0(x', \sigma) + \sigma \implies x' \leq A_h + \sigma$. Therefore $X = [A_l - \sigma, A_h + \sigma]$. Then, by Theorem 7.17 (Rudin, 1976), $H(x, \sigma)$ converges uniformly on X to $h(x)$ as $\sigma \to 0$, and $h'(x) = d(x)$ for $x \in X$. A direct calculation verifies that $h'(x) = d(x)$ indeed holds. □

For $x \in X'$, the slope of $h(x)$ is

$$\frac{\partial}{\partial x} h(x) = \left(\frac{\underline{\underline{K}}}{\overline{K}} - \frac{\delta}{\overline{K}(1-\alpha)^2(x - \frac{r}{\alpha})} + 1\right).$$

For all $x \in X'$, $\lim\limits_{\sigma \to 0} \frac{x - A_0(x,\sigma)}{\sigma} \in [-1, 1)$. Then $\lim\limits_{\sigma \to 0} \frac{x - A_0(x,\sigma)}{\sigma} = 2\frac{\underline{\underline{K}}}{\overline{K}} + 1 - \frac{2\delta}{\overline{K}(1-\alpha)^2(x - \frac{r}{\alpha})} < 1 \implies \left(\frac{\underline{\underline{K}}}{\overline{K}} - \frac{\delta}{\overline{K}(1-\alpha)^2(x - \frac{r}{\alpha})} + 1\right) < 1$. The slope of $h(x)$ is $\frac{\partial}{\partial x} h(x) < 1$. Therefore, $h(x)$ admits a unique fixed point and because $t \in [0, 1]$, $t \cdot h(x) + x^* - s$ admits a unique fixed point. Denote it by $\tilde{A}$.

Next, we show that $\lim\limits_{\sigma \to 0} \tilde{x}$ exists and is equal to $\tilde{A}$. From Proposition 9, for any $\sigma \in (0, \min\{A_l - \underline{A}, \overline{A} - A_h\})$, there exists a unique fixed point $\tilde{x}$ (denote it by $\tilde{x}(\sigma)$) that solves

$$\tilde{x}(\sigma) = t \cdot H(\tilde{x}(\sigma), \sigma) + x^* - s.$$

Because we define $H(x, \sigma)$ on $X' \subset [A_l - \sigma, A_h + \sigma]$, the fixed point $\tilde{x}(\sigma) \in [A_l - \sigma, A_h + \sigma]$. Also, $H(x, \sigma)$ is continuous in both x and σ. Then, existence of $\lim\limits_{\sigma \to 0} \tilde{x}(\sigma)$ follows because $h(x)$ admits a unique fixed point. We showed earlier that $t \cdot h(x) + x^* - s$ admits a unique fixed point $\tilde{A}$. Therefore, it must be that $\lim\limits_{\sigma\to 0}\tilde{x}(\sigma) = \lim_{\sigma\to 0}\tilde{x} = \tilde{A}$.

Substituting $\tilde{A}$ for x for $h(\tilde{A})$ (equation (18)), when $s \leq x^* - A_l$, as the noise becomes vanishingly small, $\lim\limits_{\sigma\to 0}\tilde{x} = \tilde{A}$ uniquely solves,

$$\tilde{A} = x^* - s + t\,\frac{\delta}{(1-\alpha)^2\overline{K}}\left\{\frac{\tilde{A} - r/\alpha}{A_l - r/\alpha} - \log\left(\frac{\tilde{A} - r/\alpha}{A_l - r/\alpha}\right) - 1\right\}. \tag{19}$$

Note that there exist s, t such that $\tilde{A} > x^*$ (for example, $s = 0$ & $t = 1$ yields $\tilde{A} > x^*$). However, the lower cutoff will never be greater than the cutoff without the program, x^*. Thus, when $s \leq x^* - A_l$, in the limit, the lower cutoff is

$$A'(s,t) = \lim_{\sigma\to 0} x'(s,t) = \min\{\tilde{A}(s,t), x^*\}. \tag{20}$$

Next, we compute the upper cutoff x''. For a small noise ($0 < \sigma < \min\{A_l - \underline{A}, \overline{A} - A_h\}$), when $s/t \geq \overline{A} - A_l = \pi(\overline{K}, \overline{A})$, $t\pi(\overline{K}, \overline{A}) - s = \pi(2, \overline{K}, \overline{A}) - \pi(1, \overline{K}, \overline{A}) \leq 0$. Therefore, the upper cutoff is $x'' = \overline{A} + \sigma$. As the noise becomes vanishingly small, $x'' = \overline{A}$ in this case.

Finally, consider the case where $s/t < \overline{A} - A_l = \pi(\overline{K}, \overline{A})$. As we show in Proposition 9, the upper cutoff is $x'' = \max\{x', x^\diamond\}$, where $x^\diamond$ is the minimum x that solves

$$\frac{s}{t} = E[\max\{\pi(K(A; x'), A), 0\} | x_i = x]. \tag{21}$$

For $0 < \sigma < \min\{A_l - \underline{A}, \overline{A} - A_h\}$, the two cutoffs $x''(s,t)$ and $x'(s,t)$ induced by the GPS program (s,t) must satisfy either $x''(s,t) > x'(s,t) + 2\sigma$ or $x''(s,t) < x'(s,t) + 2\sigma$. Suppose, it is the first case, i.e., $x''(s,t) > x'(s,t) + 2\sigma$. Then, in this case, from equation (21),

$$\begin{aligned}
\frac{s}{t} &= E\left[\max\left\{\left(A - \frac{\delta}{(1-\alpha)^2}\frac{1}{(\underline{K} + K(A; x'))} - \frac{r}{\alpha}\right), 0\right\}\middle| x_i = x''\right] \\
\frac{s}{t} &= \int_{x''-\sigma}^{x''+\sigma}\left(A - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K} + \overline{K}}\right) f(A|x_i = x'')\, dA \\
x'' &= \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K} + \overline{K}} + \frac{s}{t} = A_l + \frac{s}{t}
\end{aligned}$$

As $\sigma \to 0$, $A''(s,t) > A'(s,t)$ and is,

$$A''(s,t) = \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2(\underline{K} + \overline{K})} + \frac{s}{t}. \tag{22}$$

When $x''(s,t) \leq x'(s,t) + 2\sigma$, as $\sigma \to 0$, $\lim\limits_{\sigma\to 0} x''(s,t) = \lim\limits_{\sigma\to 0} x'(s,t)$. Then,

$$A''(s,t) = A'(s,t). \tag{23}$$

Thus, for any GPS program (s, t), A'' exists and can be characterized by either equation (22) or equation (23), depending on the program parameters s and t. When $A''(s,t) > A'(s,t)$, its value is given by equation (22), which is continuous and increasing in s for a given t, whereas $A'(s,t)$ is continuous and decreasing in s, reaching $A'(s,t) = x^*$. Consequently, by the intermediate value theorem, for any given t, there exists an s such that $A''(s,t) = A_l + \frac{s}{t} = A'(s,t)$. Therefore, as the noise becomes vanishingly small, the upper cutoff $A''(s,t)$ is

$$A''(s,t) = \begin{cases} A_l + s/t & ; s > t\ (A'(s,t) - A_l) \\ A'(s,t) & ; s \leq t\ (A'(s,t) - A_l). \end{cases}$$

This concludes the proof. □

Proof of Proposition 5. The proof consists of three parts:

1. Show that for a given t and a target x', there is a unique guarantee $s(x', t)$ that induces x'.
2. Show that the cost of implementing any given target x' is decreasing t.
3. Compute the expected costs for removing the inefficiency interval with the optimal GPS program as the noise becomes vanishingly small.

The following lemma proves part 1,

Lemma 4. *Under assumptions 1 and 2, for any $x' \in (x^* - A_l + \underline{A} + \sigma, x^*)$, the set of GPS programs $\{(s,t) : s \in [0, \overline{s}], t \in [0,1]\}$ that implement the target x' is non-empty. Furthermore, for a given t, there is a unique s, denoted by $s(x', t)$, that implements x'.*

Proof. Any $x' \in (x^* - A_l + \underline{A} + \sigma, x^*)$ will either satisfy $x' < A_0(x', \sigma) - \sigma$ or $x' \in [A_0(x', \sigma) - \sigma, A_0(x', \sigma) + \sigma)$. This follows because any x' satisfies $x' < A_0(x', \sigma) + \sigma$. If $x' \geq A_0(x', \sigma) + \sigma$, then, from equation (15), $x' \geq A_h + \sigma \implies x' > A_h$ $(\sigma > 0)$, which is a contradiction because $x' \leq x^* \leq A_h$.

Then, consider the following two cases:

Case 1: $x' < A_0(x', \sigma) - \sigma$

From equation (15), $A_0(x', \sigma) = A_l$. In this case, $x' < A_l - \sigma$. It follows from Proposition 9 that any (s, t) that satisfies the following equation implements x',

$$x' = x'(s,t) = \frac{r}{\alpha} + \frac{\delta}{(1-\alpha^2)\overline{K}} \log\left(\frac{\underline{K} + \overline{K}}{\underline{K}}\right) - s.$$

Then, the set of programs that achieves x' when $x' < A_0(x', \sigma) - \sigma$ is $\{(s,t) : t \in [0,1], s = x^* - x'\}$. Therefore, for any given t, there exists a unique guarantee $s(x', t)$ that achieves x' where,

$$s(x', t) = x^* - x'. \tag{24}$$

Case 2: $x' \in [A_0(x', \sigma) - \sigma, A_0(x', \sigma) + \sigma)$

For $x' < x^*$, any (s, t) that satisfies the following implements x' (from Proposition 9), $x' - T(x') = 0$, where

$$T(x') = \frac{t}{2\sigma}\Bigg[\frac{(x'+\sigma)^2 - A_0(x',\sigma)^2}{2} - \frac{r(x'+\sigma - A_0(x',\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K} + \overline{K}}{\underline{K} + \frac{\overline{K}}{2}\left(1 - \left(\frac{x' - A_0(x',\sigma)}{\sigma}\right)\right)}\right)\Bigg] + x^* - s.$$

For any $t \in [0,1]$, $T'(x') = \frac{t}{2\sigma}(x' + \sigma - A_0(x', \sigma)) \in [0, 1)$. Then, by the implicit function formula, $\frac{dx'}{ds} = -\frac{1}{1-T'(x')} < 0$. This implies that for a given t, there is a one-to-one mapping from x' to s. Thus, there exists a set of programs (s, t) that achieves any $x' \in [A_0(x', \sigma) - \sigma, A_0(x', \sigma) + \sigma)$ and for a given t, there is a unique guarantee $s(x', t)$ that achieves x'.

Therefore, for any $x' \in (x^* - A_l + \underline{A} + \sigma, x^*)$, the set of programs that achieves the target x' is non-empty and for a given t, there is a unique $s(x', t)$ that achieves x'. $\square$

The following lemma proves the second part:

Lemma 5. *Under assumptions 1 and 2, the expected cost to the policymaker to implement a target x', where $x' \in (x^* - A_l + \underline{A} + \sigma, x^*)$, is (weakly) decreasing in t.*

Proof. Following Lemma 4, we can now state the GPS program (s, t) in terms of the profit sharing rate t and x' that it induces, i.e., (x', t). Let $C(A, x', t; \sigma)$ denote the cost of inducing target x' for a given t and A. The expected cost of the program (x', t) is,

$$
\begin{aligned}
E_A[C(A,x',t;\sigma)] &= \int_{\underline{A}}^{\overline{A}} \big(s(x',t) - t\cdot \max\{\pi(K(A;x'),A),0\}\big)\cdot \big(F(x''|A) - F(x'|A)\big) f(A)\,dA \\
&= \int_{\underline{A}}^{\overline{A}} [\pi(1,K(A;x'),A) - \pi(2,K(A;x'),A)]\big(F(x''|A) - F(x'|A)\big) f(A)\,dA \\
&= \int_{\underline{A}}^{\overline{A}} [\pi(1,K(A;x'),A) - \pi(2,K(A;x'),A)]\bigg(\int_{x'}^{x''} f(x_i|A)\,dx_i\bigg) f(A)\,dA \\
&= \int_{\underline{A}}^{\overline{A}} \bigg(\int_{x'}^{x''} [\pi(1,K(A;x'),A) - \pi(2,K(A;x'),A)] f(x_i|A)\,dx_i\bigg) f(A)\,dA \\
&= \int_{x'}^{x''}\int_{\underline{A}}^{\overline{A}} [\pi(1,K(A;x'),A) - \pi(2,K(A;x'),A)] f(A|x_i)\,dA f(x_i)\,dx_i \\
&= \int_{x'}^{x''}\Big(E\big[\pi(1,K(A;x'),A)|x_i\big] - E\big[\pi(2,K(A;x'),A)|x_i\big]\Big) f(x_i)\,dx_i \\
&= \int_{x'}^{\overline{A}+\sigma} \max\big\{E\big[\pi(1,K(A;x'),A) - \pi(2,K(A;x'),A)|x_i\big],0\big\} f(x_i)\,dx_i. \quad (25)
\end{aligned}
$$

Because $E\big[\pi(2,K(A;x'),A)|x_i\big]$ does not depend on t, the relationship between the expected cost of implementing x', $E_A[C(A,x',t;\sigma)]$, and t depends on how $E\big[\pi(1,K(A;x'),A)|x_i\big]$ varies with t. Then,

$$
\begin{aligned}
E\big[\pi(1,K(A;x'),A)|x_i\big] &= \int_{\underline{A}}^{\overline{A}} (\pi(K(A;x'),A) - t\max\{\pi(K(A;x'),A),0\} + s) f(A|x_i)\,dA \\
&= \int_{\underline{A}}^{\overline{A}} \pi(2,K(A;x'),A) f(A|x_i)\,dA - t\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\} f(A|x_i)\,dA + s \\
&= E\big[\pi(2,K(A;x'),A)|x_i\big] - t\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\} f(A|x_i)\,dA + s,
\end{aligned}
$$

where $s = s(x',t)$. Then for a fixed x',

$$
\frac{d}{dt}E\big[\pi(1,K(A;x'),A)|x_i\big] = -\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A,x'),A),0\} f(A|x_i)\,dA + \frac{ds(x',t)}{dt}.
$$

Next, as we do in lemma 4, consider the two cases:

Case 1: $x' < A_0(x',\sigma) - \sigma$

In this case, from equation (24), $s(x',t) = x^* - x'$. Then, for a fixed x', $\frac{ds(x',t)}{dt} = 0$ and $\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A,x'),A),0\} f(A|x_i)\,dA \geq 0$ for $x_i > x'$. Then, $\frac{d}{dt}E\big[\pi(1,K(A;x'),A)|x_i\big] \leq 0$ for all $x_i > x'$.

Case 2: $x' \in [A_0(x',\sigma) - \sigma, A_0(x',\sigma) + \sigma)$

For any given x', the marginal investor is indifferent between investing with participation and not investing which gives us the relationship between $s(x',t)$ & t.

$$s(x',t) = t \cdot E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big] - E[\pi(K(A;x'),A)|x_i = x']$$
$$\frac{d}{dt}s(x',t) = E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big] = \int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\} f(A|x')\, dA.$$

In this case,

$$\frac{d}{dt}E\big[\pi(1,K(A;x'),A)|x_i\big] = -\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\} f(A|x_i)\, dA + \frac{ds(x',t)}{dt}$$
$$= -\bigg(\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\} f(A|x_i)\, dA - \int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\} f(A|x')\, dA\bigg).$$

For $A > A_0(x',\sigma)$, $\max\{\pi(K(A;x'),A),0\} > 0$ and is strictly increasing in A. Therefore, $\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\} f(A|x_i)\, dA$ is strictly increasing in x_i for $x_i > A_0(x',\sigma) - \sigma$. Because $x' \geq A_0(x',\sigma) - \sigma$, $\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\} f(A|x_i)\, dA$ is strictly increasing in x_i for $x_i > x'$. Furthermore, we only need to check for $x_i > x'$ since that is the range over which we integrate in equation (25). Therefore, for any $x_i > x'$,

$$\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\} f(A|x_i)\, dA > \int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\} f(A|x')\, dA.$$

This implies that $\frac{d}{dt}E\big[\pi(1,K(A;x'),A)|x_i\big] < 0$. Threfore, in this case, the expected cost of implementing x' is strictly decreasing in t. $\square$

Finally, we compute the expected cost of removing miscoordination as the noise becomes vanishingly small. To remove inefficiency, the target must be $x' = A_l - \sigma$. Because the cost is decreasing in t, we can set the highest profit-sharing rate, $t = 1$. The corresponding compensation, $s(x',1)$, is $s(x',1) = \frac{\delta}{(1-\alpha)^2\overline{K}}\left[\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) - \frac{\overline{K}}{\underline{K}+\overline{K}}\right] + \sigma = x^* - A_l + \sigma$. Thus, the upper cutoff x'' (from Proposition 9) is

$$x'' = A_l + s(x',1) = \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) + \sigma = x^* + \sigma. \tag{26}$$

Thus, the ex-ante cost is

$$
\begin{aligned}
E_A[C(A,x',t=1;\sigma)] &= \int_{\underline{A}}^{\overline{A}} \Big(\big(s(x',1) - 1 \cdot \max\{\pi(K(A;x'),A),0\}\big) \cdot \big(F(x''|A) - F(x'|A)\big) \Big) f(A)\, dA \\
&= 0 + \int_{x'-\sigma}^{x'+\sigma} \left(\big(s(x',1) - \max\{\pi(K(A;x'),A),0\}\big) \cdot \left(1 - \frac{1}{2}\left(\frac{x'-A}{\sigma}+1\right)\right)\right) f(A)\, dA \\
&\quad + \int_{x'+\sigma}^{x''-\sigma} \Big(\big(s(x',1) - \max\{\pi(K(A;x'),A),0\}\big) \cdot (1-0) \Big) f(A)\, dA \\
&\quad + \int_{x''-\sigma}^{x''+\sigma} \left(\big(s(x',1) - \max\{\pi(K(A;x'),A),0\}\big) \cdot \left(\frac{1}{2}\left(\frac{x''-A}{\sigma}+1\right) - 0\right)\right) f(A)\, dA + 0 \\
&= \int_{x'-\sigma}^{x'+\sigma} \frac{s(x',1)}{2}\left(1 - \left(\frac{x'-A}{\sigma}\right)\right) f(A)\, dA + \int_{x'+\sigma}^{x''-\sigma} \big(s(x',1) - \pi(K(A;x'),A)\big) f(A)\, dA \\
&\quad + \int_{x''-\sigma}^{x''+\sigma} \left(\frac{\big(s(x',1) - \pi(K(A;x'),A)\big)}{2}\left(\frac{x''-A}{\sigma}+1\right)\right) f(A)\, dA \\
&= \frac{1}{\overline{A}-\underline{A}} \Bigg\{ s(x',1)\sigma + (x''-x'-2\sigma)\left[s(x',1) - \left(\frac{x''+x'}{2} - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2(\underline{K}+\overline{K})}\right)\right] \\
&\quad + \sigma\left[s(x',1) + \left(\frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2(\underline{K}+\overline{K})}\right) + \frac{\sigma}{3} - x''\right]\Bigg\}
\end{aligned}
$$

As the noise vanishes, the cost converges to

$$
\begin{aligned}
\lim_{\sigma\to 0} E_A[C(A,x',1;\sigma)] &= \lim_{\sigma\to 0}\left\{\frac{(x''-x')}{\overline{A}-\underline{A}}\left[s(x',1) - \left(\frac{x''+x'}{2} - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2(\underline{K}+\overline{K})}\right)\right]\right\} \\
&= \lim_{\sigma\to 0}\left\{\frac{(x''-x')}{\overline{A}-\underline{A}}\left[s(x',1) - \left(\frac{x''+x'}{2} - A_l\right)\right]\right\}.
\end{aligned}
$$

Substituting $\lim_{\sigma\to 0} s(x',1) = x^* - A_l$, $x' = A'(\lim_{\sigma\to 0} s(x',1), t) = A_l$, and $x'' = A''(\lim_{\sigma\to 0} s(x',1), 1) = A_l + s(x',1) = x^*$ (from proposition 4),

$$
\lim_{\sigma\to 0} E_A[C(A,x',1;\sigma)] = \lim_{\sigma\to 0}\left\{\frac{(x^*-A_l)}{\overline{A}-\underline{A}}\left[x^* - A_l - \left(\frac{x^*+A_l}{2} - A_l\right)\right]\right\} = \frac{(x^*-A_l)^2}{2(\overline{A}-\underline{A})}.
$$

Therefore, as the noise becomes vanishingly small, the optimal GPS program that removes miscoordination is $(s,t) = (x^* - A_l, 1)$, and its expected cost is $\frac{1}{2}\frac{(x^*-A_l)^2}{\overline{A}-\underline{A}}$. □

Proof of Proposition 6. The proof follows a similar approach to that of the proof for Proposition 4. Consider the following cases for x':

Case 1: $\frac{s}{1-t^-} \geq x^* - A_l$

For $0 < \sigma < \min\{A_l - \underline{A}, \overline{A} - A_h\}$, $x' = x^* - \frac{s}{1-t^-}$ when $\frac{s}{1-t^-} \geq x^* - A_l + \sigma$ (from

Proposition 10). As the noise becomes vanishingly small, for $\frac{s}{1-t^-} \geq x^* - A_l$,

$$x' = A'(s, t^+, t^-) = x^* - \frac{s}{1-t^-}. \tag{27}$$

Case 2: $\frac{s}{1-t^-} \leq x^* - A_l$

From Proposition 10, the cutoff is $x' = \min\{\tilde{x}(s, t^+, t^-), x^*\}$ where $\tilde{x}$ uniquely solves

$$\begin{aligned}\tilde{x} = &\frac{t^+}{2\sigma}\Bigg[\frac{(\tilde{x}+\sigma)^2 - A_0(\tilde{x},\sigma)^2}{2} - \frac{r(\tilde{x}+\sigma - A_0(\tilde{x},\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\Bigg(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{\tilde{x}-A_0(\tilde{x},\sigma)}{\sigma}\big)\big)}\Bigg)\Bigg]\\ &+\frac{t^-}{2\sigma}\Bigg[\frac{A_0(\tilde{x},\sigma)^2 - (\tilde{x}-\sigma)^2}{2} - \frac{r(A_0(\tilde{x},\sigma) - \tilde{x}+\sigma)}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\Bigg(\frac{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{\tilde{x}-A_0(\tilde{x},\sigma)}{\sigma}\big)\big)}{\underline{K}}\Bigg)\Bigg]\\ &+x^* - s.\end{aligned}$$

Let $t^+ = t^- + \gamma$. We can rewrite the implicit function for $\tilde{x}$ as

$$\begin{aligned}\tilde{x} = &\frac{\gamma}{2\sigma}\Bigg[\frac{(\tilde{x}+\sigma)^2 - A_0(\tilde{x},\sigma)^2}{2} - \frac{r(\tilde{x}+\sigma - A_0(\tilde{x},\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\Bigg(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{\tilde{x}-A_0(\tilde{x},\sigma)}{\sigma}\big)\big)}\Bigg)\Bigg]\\ &+\frac{t^-}{2\sigma}\Bigg[\frac{(\tilde{x}+\sigma)^2 - (\tilde{x}-\sigma)^2}{2} - \frac{r(\tilde{x}+\sigma - \tilde{x}+\sigma)}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\Bigg(\frac{\underline{K}+\overline{K}}{\underline{K}}\Bigg)\Bigg]\\ &+x^* - s\\ \tilde{x} = &\frac{\gamma}{1-t^-}\frac{1}{2\sigma}\Bigg[\frac{(\tilde{x}+\sigma)^2 - A_0(\tilde{x},\sigma)^2}{2} - \frac{r(\tilde{x}+\sigma - A_0(\tilde{x},\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\Bigg(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{\tilde{x}-A_0(\tilde{x},\sigma)}{\sigma}\big)\big)}\Bigg)\Bigg]\\ &+x^* - \frac{s}{1-t^-}.\end{aligned}$$

Therefore, in the GPLS case, $\tilde{x}$ is the fixed point of the function $\frac{\gamma}{1-t^-}H(x,\sigma) + x^* - \frac{s}{1-t^-}$ where $H(x,\sigma)$ is given by equation (17). For $x \in X = \{x \in \mathbb{R} : A_0(x,\sigma) - \sigma \leq x \leq A_0(x,\sigma) + \sigma\}$, $H(x,\sigma)$ converges uniformly to $h(x)$ (Lemma 3). Because $\frac{\gamma}{1-t^-} = \frac{t^+-t^-}{1-t^-} \leq 1$, $\frac{\gamma}{1-t^-}h(x) + x^* - \frac{s}{1-t^-}$ admits a unique fixed point, $\tilde{A}$. Then, using the same arguments as in the proof of Proposition 4, $\lim\limits_{\sigma\to 0}\tilde{x} = \tilde{A}$.

Calculating $h(\tilde{A})$ (equation (18)), when $\frac{s}{1-t^-} \leq x^* - A_l$, as the noise becomes vanishingly small, $\lim\limits_{\sigma\to 0}\tilde{x} = \tilde{A}$ uniquely solves

$$\tilde{A} = x^* - \frac{s}{1-t^-} + \frac{t^+-t^-}{1-t^-}\,\frac{\delta}{(1-\alpha)^2\overline{K}}\left\{\frac{\tilde{A}-r/\alpha}{A_l - r/\alpha} - \log\left(\frac{\tilde{A}-r/\alpha}{A_l-r/\alpha}\right) - 1\right\}. \tag{28}$$

Because $A'(s, t^+, t^-) \leq x^*$,

$$A'(s,t^+,t^-) = \min\{\tilde{A}(s,t^+,t^-), x^*\}.$$

For the upper cutoff x'', for a small noise, $x'' = \overline{A} + \sigma$ for $s/t^+ \geq \overline{A} - A_l = \pi(\overline{K}, \overline{A})$ because $t^+\pi(\overline{K}, \overline{A}) - s = \pi_{gpls}(2, \overline{K}, \overline{A}) - \pi_{gpls}(1, \overline{K}, \overline{A}) \leq 0$. Therefore, as the noise becomes vanishingly small, $x'' = \overline{A}$.

Finally, when $s/t^+ < \overline{A} - A_l = \pi(\overline{K}, \overline{A})$, from Proposition 10, $x'' = \max\{x', x^\diamond\}$ where $x^\diamond$ is the minimum x that solves $s = E\big[\big(t^+ \max\{\pi(K(A;x'),A),0\} + t^- \min\{\pi(K(A;x'),A),0\}\big)|x_i = x\big]$. Under a GPLS program (s,t^+,t^-), the induced $x''(s,t^+,t^-)$ and $x'(s,t^+,t^-)$ either satisfy $x''(s,t^+,t^-) > x'(s,t^+,t^-) + 2\sigma$ or $x''(s,t^+,t^-) \leq x'(s,t^+,t^-) + 2\sigma$. When $x''(s,t^+,t^-) > x'(s,t^+,t^-) + 2\sigma$, $E\big[\min\{\pi(K(A;x'),A),0\}|x_i = x''\big] = 0$. Thus, the upper cutoff x'' solves $s = t^+ E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x''\big] = \int_{x''-\sigma}^{x''+\sigma} \left(A - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K}+\overline{K}}\right) f(A|x_i = x'')\, dA$. Therefore, the upper cutoff is $x'' = A_l + \frac{s}{t^+}$. As $\sigma \to 0$, $A''(s,t^+,t^-) > A'(s,t^+,t^-)$ and is

$$A''(s,t^+,t^-) = \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K}+\overline{K}} + \frac{s}{t^+}. \tag{29}$$

When $x''(s,t^+,t^-) \leq x'(s,t^+,t^-) + 2\sigma$, as $\sigma \to 0$,

$$A''(s,t^+,t^-) = A'(s,t^+,t^-). \tag{30}$$

Thus, as in the GPS program, for any GPLS program (s,t^+,t^-), as the noise vanishes, $A''(s,t^+,t^-)$ is given by either equation (29) or equation (30). By the same arguments used to compute A'' in the GPS case, we obtain

$$A''(s,t^+,t^-) = \begin{cases} A_l + s/t^+ & ; s > t^+\ (A'(s,t) - A_l) \\ A'(s,t^+,t^-) & ; s \leq t^+\ (A'(s,t) - A_l). \end{cases}$$

This concludes the proof. $\square$

Proof of Proposition 7. We prove this in three parts:

1. Show that for a given t^+ and t^-, there is a unique $s(x',t^+,t^-)$ that induces x'.
2. Show that the expected cost of implementing any target x' is decreasing in both t^+ and t^-.
3. Compute the expected cost of resolving miscoordination with the optimal GPLS program as the noise becomes vanishingly small.

Lemma 6. *Under assumptions 1 and 4, for any $x' \in (x^* - A_l + \underline{A} + \sigma, x^*)$, the set of GPLS programs $\{(s, t^+, t^-) : s \in [0, \overline{s}], t^+ \in [0, 1], t^- \in [0, \overline{t}]\}$ that implement the target x' is non-empty. Furthermore, for a given t^+ and t^-, there is a unique s, denoted by $s(x', t^+, t^-)$, that implements x'.*

Proof. The proof for this follows the same approach as the proof for lemma 4 with some modifications. Any $x' \in (x^* - A_l + \underline{A} + \sigma, x^*)$ will either satisfy $x' < A_0(x', \sigma) - \sigma$ or $x' \in [A_0(x', \sigma) - \sigma, A_0(x', \sigma) + \sigma)$. When $x' < A_0(x', \sigma) - \sigma$, $A_0(x', \sigma) = A_l$ (from equation (15)). It follows that $x' < A_l - \sigma$. Then, from Proposition 10, x' is $x' = x'(s, t^+, t^-) = x^* - \frac{s}{1-t^-}$. For any $x' < A_0(x', \sigma) - \sigma$, the set of GPLS program that induces x' is $\{(s, t^+, t^-) : s = (1 - t^-)(x^* - x'), t^+ \in [0, 1], t^- \in [0, \overline{t}]\}$, and for any t^+ and t^-, the guarantee s that induces x' is,

$$s(x', t^+, t^-) = (1 - t^-)(x^* - x'). \tag{31}$$

When $x' \in [A_0(x', \sigma) - \sigma, A_0(x', \sigma) + \sigma)$, for any $x' < x^*$, any (s, t^+, t^-) that satisfies $x' - T_2(x') = 0$ implements x' (Proposition 10), where

$$\begin{aligned}
T_2(x') = {} & \frac{t^+}{2\sigma}\left[\frac{(x' + \sigma)^2 - A_0(x', \sigma)^2}{2} - \frac{r(x' + \sigma - A_0(x', \sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K} + \overline{K}}{\underline{K} + \frac{\overline{K}}{2}\left(1 - \frac{x' - A_0(x', \sigma)}{\sigma}\right)}\right)\right] \\
& + \frac{t^-}{2\sigma}\left[\frac{A_0(x', \sigma)^2 - (x' - \sigma)^2}{2} - \frac{r(A_0(x', \sigma) - x' + \sigma)}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K} + \frac{\overline{K}}{2}\left(1 - \frac{x' - A_0(x', \sigma)}{\sigma}\right)}{\underline{K}}\right)\right] \\
& + x^* - s \\
= {} & \frac{\gamma}{2\sigma}\left[\frac{(x' + \sigma)^2 - A_0(x', \sigma)^2}{2} - \frac{r(x' + \sigma - A_0(x', \sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K} + \overline{K}}{\underline{K} + \frac{\overline{K}}{2}\left(1 - \frac{x' - A_0(x', \sigma)}{\sigma}\right)}\right)\right] \\
& + t^-(x' - x^*) + x^* - s.
\end{aligned}$$

The derivative of $T_2(x')$ w.r.t. x' is $T_2'(x') = \frac{\gamma}{2\sigma}(x' + \sigma - A_0(x', \sigma)) + t^-$. Because $x' \in [A_0(x', \sigma) - \sigma, A_0(x', \sigma) + \sigma)$, $(x' + \sigma - A_0(x', \sigma)) \in [0, 2\sigma)$. Substituting $\gamma = t^+ - t^-$, we have $T_2'(x') \in [0, 1)$. Then, by the implicit function formula, $\frac{dx'}{ds} = -\frac{1}{1 - T_2'(x')} < 0$, which implies that for a given t^+ and t^-, there is a one-to-one mapping from x' to s. Therefore, there exists a set of programs (s, t^+, t^-) that induce any $x' \in (x^* - A_l + \underline{A} + \sigma, x^*)$ and for a given t^+ and t^-, there is a unique $s(x', t^+, t^-)$ that induces x'. $\square$

The following lemma proves the second part:

Lemma 7. *Under assumptions 1 and 4, the expected cost to the policymaker to implement a target x', where $x' \in (x^* - A_l + \underline{A} + \sigma, x^*)$, is (weakly) decreasing in both t^+ and t^-.*

Proof. By lemma 6, we can state the GPLS program (s, t^+, t^-) in terms of x', t^+, and t^-. Let $C(A, x', t^+, t^-; \sigma)$ denote the cost of inducing x' for given t^+, t^-, and A. Using the same approach as in lemma 5, the expected cost of the program (x', t^+, t^-) is

$$
\begin{aligned}
E_A[C(A, x', t^+, t^-; \sigma)] &= \int_{x'}^{x''} \int_{\underline{A}}^{\overline{A}} [\pi_{gpls}(1, K(A; x'), A) - \pi_{gpls}(2, K(A; x'), A)] f(A|x_i)\, dA f(x_i)\, dx_i \\
&= \int_{x'}^{x''} \big(U_1^{gpls}(x', x_i) - U_2^{gpls}(x', x_i)\big) f(x_i)\, dx_i \\
&= \int_{x'}^{\overline{A}+\sigma} \max\{U_1^{gpls}(x', x_i) - U_2^{gpls}(x', x_i), 0\} f(x_i)\, dx_i, \qquad (32)
\end{aligned}
$$

where

$$U_1^{gpls}(x', x_i) = \int_{\underline{A}}^{\overline{A}} \pi_{gpls}(1, K(A; x'), A)\, f(A \mid x_i)\, dA, \quad U_2^{gpls}(x', x_i) = \int_{\underline{A}}^{\overline{A}} \pi_{gpls}(2, K(A; x'), A)\, f(A \mid x_i)\, dA.$$

Because U_2^{gpls} does not depend on t^+ or t^-, the relationship between the cost of implementing x' and t^+, t^- depends on how U_1^{gpls} varies with t^+ and t^- for $x_i > x'$. For a given x',

$$U_1^{gpls}(x', x_i) = U_2^{gpls}(x', x_i) - \int_{\underline{A}}^{\overline{A}} t^+ \max\{\pi(K(A, x'), A), 0\} - t^- \min\{\pi(K(A, x'), A), 0\} f(A|x_i)\, dA + s.$$

Since U_2^{gpls} and π are independent of t^+ and t^-,

$$
\begin{aligned}
\frac{d}{dt^+} U_1^{gpls}(x', x_i) &= -\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A, x'), A), 0\} f(A|x_i)\, dA + \frac{ds(x', t^+, t^-)}{dt^+}, \\
\frac{d}{dt^-} U_1^{gpls}(x', x_i) &= -\int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A, x'), A), 0\} f(A|x_i)\, dA + \frac{ds(x', t^+, t^-)}{dt^-}.
\end{aligned}
$$

Next, consider the two cases:

Case 1: $x' < A_0(x', \sigma) - \sigma$

From equation (31), $s(x', t^+, t^-) = (1 - t^-)(x^* - x')$. Then, for a fixed x', $\frac{ds(x', t^+, t^-)}{dt^-} = -(x^* - x') = x' - x^*$. Then,

$$\frac{d}{dt^-} U_1^{gpls}(x', x) = x' - x^* - \int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A, x'), A), 0\} f(A|x_i)\, dA.$$

Since $\int_{\underline{A}}^{\overline{A}} \pi(K(A,x'),A)f(A|x_i)\,dA$, the expected payoff with x' as the cutoff strategy, is increasing in x_i then for any $x \geq x'$,

$$\begin{aligned}
\int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A,x'),A),0\}f(A|x_i)\,dA &\geq \int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A,x'),A),0\}f(A|x')\,dA \\
&\geq \int_{\underline{A}}^{\overline{A}} \pi(K(A,x'),A)f(A|x')\,dA \quad \because \{x' < A_0(x',\sigma)-\sigma\} \\
&\geq x' - x^*.
\end{aligned}$$

This implies that $\frac{d}{dt^-}U_a^{gpls}(x',x_i) = x' - x^* - \int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A,x'),A),0\}f(A|x_i)\,dA \leq 0$ for all $x_i > x'$. So, the least costly way to implement $x' < A_0(x',\sigma)-\sigma$ is with the highest t^-.

Next, for a fixed x', $\frac{ds(x',t^+,t^-)}{dt^+} = 0$ and $\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A,x'),A),0\}f(A|x_i)\,dA \geq 0$. Then, $\frac{d}{dt^+}U_1^{gpls}(x',x_i) \leq 0$ for all $x > x'$. So, the least costly way to implement $x' < A_0(x',\sigma)-\sigma$ is with the highest t^+.

Case 2: $x' \in [A_0(x',\sigma)-\sigma, A_0(x',\sigma)+\sigma)$

For any given x', when investors receive that signal, their expected payoff must be 0. Then,

$$s = t^+ \cdot E\big[\max\{\pi(K(A;x'),A),0\}|x'\big] + t^- \cdot E\big[\min\{\pi(K(A;x'),A),0\}|x'\big] - E[\pi(K(A;x'),A)|x'].$$

Taking derivatives w.r.t. t^+ & t^-,

$$\frac{ds(x',t^+,t^-)}{dt^+} = E\big[\max\{\pi(K(A;x'),A),0\}|x'\big] = \int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\}f(A|x')\,dA,$$
$$\frac{ds(x',t^+,t^-)}{dt^-} = E\big[\min\{\pi(K(A;x'),A),0\}|x'\big] = \int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A;x'),A),0\}f(A|x')\,dA.$$

First consider t^+. In this case,

$$\begin{aligned}
\frac{d}{dt^+}U_a^{gpls}(x',x_i) &= -\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\}f(A|x_i)\,dA + \frac{ds(x',t^+,t^-)}{dt^+} \\
&= -\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\}f(A|x_i)\,dA + \int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\}f(A|x')\,dA.
\end{aligned}$$

Now, $\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\}f(A|x_i)\,dA$ is weakly increasing in x_i. Furthermore, we only need to check for $x_i > x'$ since that is the range over which we integrate in equation (32). Then, for any $x_i > x'$, $\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\}f(A|x_i)\,dA \geq \int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\}f(A|x')\,dA$.

This implies that

$$\frac{d}{dt^+}U_1^{gpls}(x',x_i) = -\int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\}f(A|x_i)\,dA + \int_{\underline{A}}^{\overline{A}} \max\{\pi(K(A;x'),A),0\}f(A|x')\,dA \leq 0.$$

Therefore, the cost of implementing any $x' \in [A_0(x',\sigma)-\sigma, A_0(x',\sigma)+\sigma)$ is decreasing in t^+. Finally, consider t^-.

$$\begin{aligned}\frac{d}{dt^-}U_1^{gpls}(x',x_i) &= -\int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A;x'),A),0\}f(A|x_i)\,dA + \frac{ds(x',t^+,t^-)}{dt^-} \\ &= -\int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A;x'),A),0\}f(A|x_i)\,dA + \int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A;x'),A),0\}f(A|x')\,dA.\end{aligned}$$

The term $\int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A;x'),A),0\}f(A|x_i)\,dA$ is weakly increasing in x_i. Therefore, for any $x_i > x'$, $\int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A;x'),A),0\}f(A|x_i)\,dA \geq \int_{\underline{A}}^{\overline{A}} \min\{\pi(K(A;x'),A),0\}f(A|x')\,dA$. This implies that $\frac{d}{dt^-}U_1^{gpls}(x',x_i) \leq 0$.

Therefore, the cheapest way to implement any target x' is to set the highest possible value for t^+ & t^-. □

Finally, we compute the expected cost of removing miscoordination. The guarantor must set a target $x' = A_l - \sigma$. Because the cost is decreasing in both t^+ and t^-, she can set $t^+ = 1$ and $t^- = \bar{t}$. The corresponding guarantee is $s(x',1,\bar{t}) = (1-\bar{t})(x^* - A_l + \sigma)$ and the upper cutoff is, $x'' = A_l + (1-\bar{t})(x^* - A_l + \sigma)$. Then, for $x' = A_l - \sigma$, the expected cost is

$$\begin{aligned}E_A[C(A,x',1,\bar{t};\sigma)] &= \int_{\underline{A}}^{\overline{A}} \Big(\big(s(x',1,\bar{t}) - \max\{\pi(K(A;x'),A),0\} \\ &\qquad - \bar{t}\cdot\min\{\pi(K(A;x'),A),0\}\big)\cdot\big(F(x''|A) - F(x'|A)\big)\Big)f(A)\,dA \\ &= \frac{1}{\overline{A}-\underline{A}}\Big\{0 + \int_{x'-\sigma}^{A_0=x'+\sigma} \big(s(x',1,\bar{t}) - \bar{t}\cdot\pi(K(A;x'),A)\big)\left(\frac{1}{2} - \left(\frac{x'-A}{2\sigma}\right)\right)dA \\ &\qquad + \int_{A_0=x'+\sigma}^{x''-\sigma} \big(s(x',1,\bar{t}) - \pi(K(A;x'),A)\big)\,dA \\ &\qquad + \int_{x''-\sigma}^{x''+\sigma} \big(s(x',1,\bar{t}) - \pi(K(A;x'),A)\big)\left(\frac{1}{2}\left(\frac{x''-A}{\sigma}\right)+1\right)dA + 0\Big\}.\end{aligned}$$

We compute each of the three terms individually. Consider the first term,

$$\int_{x'-\sigma}^{x'+\sigma} \big(s(x',1,\bar{t}) - \bar{t}\cdot\pi(K(A;x'),A)\big)\left(\frac{1}{2} - \left(\frac{x'-A}{2\sigma}\right)\right) dA$$
$$= \int_{x'-\sigma}^{x'+\sigma} \left[s(x',1,\bar{t}) - \bar{t}\left(A - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2\big(\underline{K} + \frac{\overline{K}}{2}(1-(\frac{x'-A}{\sigma}))\big)}\right)\right]\left(\frac{1}{2} - \left(\frac{x'-A}{2\sigma}\right)\right) dA$$
$$= \sigma\left[s(x',1,\bar{t}) - \bar{t}\left[x' - \frac{r}{\alpha} + \frac{\sigma}{3} - \frac{2\delta}{(1-\alpha)^2\overline{K}} + \frac{2\delta\underline{K}}{(1-\alpha)^2\overline{K}^2}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right)\right]\right].$$

For the second term,

$$\int_{x'+\sigma}^{x''-\sigma} \big(s(x',1,\bar{t}) - \pi(K(A;x'),A)\big)\, dA = \int_{x'+\sigma}^{x''-\sigma} \left(s(x',1,\bar{t}) - \left(A - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2(\underline{K}+\overline{K})}\right)\right) dA$$
$$= (x'' - x' - 2\sigma)\left[s(x',1,\bar{t}) - \left(\frac{x''+x'}{2} - A_l\right)\right].$$

Finally, consider the third term,

$$\int_{x''-\sigma}^{x''+\sigma} \big(s(x',1,\bar{t}) - \pi(K(A,x'),A)\big)\left(\frac{1}{2}\left(\frac{x''-A}{\sigma}\right) + 1\right) dA$$
$$= \int_{x''-\sigma}^{x''+\sigma} \left(s(x',1,\bar{t}) - \left(A - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2(\underline{K}+\overline{K})}\right)\right)\left(\frac{1}{2}\left(\frac{x''-A}{\sigma}\right) + 1\right) dA$$
$$= 2\sigma\left(s(x',1,\bar{t}) - \left(x'' - A_l - \frac{\sigma}{6}\right)\right).$$

Then, the expected cost is

$$E_A[C(A,x',1,\bar{t};\sigma)] = \frac{1}{\overline{A}-\underline{A}}\Bigg\{\sigma\left[s(x',1,\bar{t}) - \bar{t}\left[x' - \frac{r}{\alpha} + \frac{\sigma}{3} - \frac{2\delta}{(1-\alpha)^2\overline{K}} + \frac{2\delta\underline{K}}{(1-\alpha)^2\overline{K}^2}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right)\right]\right]$$
$$+ (x''-x'-2\sigma)\left[s(x',1,\bar{t}) - \left[\frac{x''+x'}{2} - A_l\right]\right] + 2\sigma\left[s(x',1,\bar{t}) - \left(x'' - A_l - \frac{\sigma}{6}\right)\right]\Bigg\}.$$

where $s(x',1,\bar{t}) = (1-\bar{t})(x^* - A_l + \sigma)$. As the noise goes to 0, the cost is

$$\lim_{\sigma\to 0} E_A[C(A,x',1,\bar{t};\sigma)] = \lim_{\sigma\to 0}\frac{(x''-x')}{\overline{A}-\underline{A}}\left[s(x',1,\bar{t}) - \left(\frac{x''+x'}{2} - A_l\right)\right].$$

As the noise becomes vanishingly small, $x' = A' = A_l, x'' = A'' = A_l + (1-\bar{t})(x^* - A_l)$ and $\lim_{\sigma\to 0} s(x',1,\bar{t}) = (1-\bar{t})(x^* - A_l)$. The expected cost of removing miscoordination is

$$\lim_{\sigma\to 0} E_A[C(A,x',1,\bar{t};\sigma)] = \frac{(A''-A')}{\overline{A}-\underline{A}}\left[(1-\bar{t})(x^*-A_l) - \left(\frac{A''+A'}{2} - A_l\right)\right]$$
$$= \frac{(1-\bar{t})(x^*-A_l)}{\overline{A}-\underline{A}}\left[\frac{(1-\bar{t})(x^*-A_l)}{2}\right] = \frac{(1-\bar{t})^2}{2}\frac{(x^*-A_l)^2}{\overline{A}-\underline{A}}.$$

Therefore, as the noise becomes vanishingly small, the optimal GPLS program that removes miscoordination is $(s, t^+, t^-) = ((1-\bar{t})(x^* - A_l), 1, \bar{t})$, and its expected cost is $\frac{(1-\bar{t})^2}{2}\frac{(x^*-A_l)^2}{\overline{A}-\underline{A}}$.

$\square$

Proof of Proposition 8. The R-GPLS program is a GPLS program with $t^+ = t^-$. Thus, lemma 7 holds for the R-GPLS program which implies that the optimal R-GPLS program sets $t^+ = t^- = \bar{t}$. Therefore, the guarantee s required to remove miscoordination ($x' = A_l - \sigma$) is,

$$s(A_l - \sigma, \bar{t}, \bar{t}) = (1-\bar{t})(x^* - A_l + \sigma). \tag{33}$$

and $x'' = A_l + \frac{s(A_l-\sigma,\bar{t},\bar{t})}{\bar{t}}$. The expected cost calculation is identical to the optimal GPLS case, except that here we set $t^+ = \bar{t}$ (instead of $t^+ = 1$). Then, as the noise becomes vanishingly small, $x' = A' = A_l$, $x'' = A'' = A_l + \frac{(1-\bar{t})}{\bar{t}}(x^* - A_l)$ and $\lim_{\sigma\to 0} s(x', 1, \bar{t}) = (1-\bar{t})(x^* - A_l)$. The expected cost of removing miscoordination is

$$\begin{aligned}\lim_{\sigma\to 0} E_A[C(A, x', 1, \bar{t}; \sigma)] &= \frac{(A'' - A')}{\overline{A} - \underline{A}}\left[(1-\bar{t})(x^* - A_l) - \bar{t}\left(\frac{A'' + A'}{2} - A_l\right)\right] \\ &= \frac{(1-\bar{t})}{\bar{t}}\frac{(x^* - A_l)}{\overline{A} - \underline{A}}\left[\frac{(1-\bar{t})(x^* - A_l)}{2}\right] = \frac{(1-\bar{t})^2}{2\bar{t}}\frac{(x^* - A_l)^2}{\overline{A} - \underline{A}}.\end{aligned}$$

Therefore, as $\sigma \to 0$, the optimal R-GPLS program that removes miscoordination is $(s, t^-, t^-) = ((1-\bar{t})(x^* - A_l), 1, \bar{t})$, and its expected cost is $\frac{(1-\bar{t})^2}{2\bar{t}}\frac{(x^*-A_l)^2}{\overline{A}-\underline{A}}$.

Finally, for $\bar{t} < 1$, $\frac{(1-\bar{t})^2}{2\bar{t}}\frac{(x^*-A_l)^2}{\overline{A}-\underline{A}} > \frac{(1-\bar{t})^2}{2}\frac{(x^*-A_l)^2}{\overline{A}-\underline{A}}$ $\square$

Proof of Corollary 4. Under optimal GPS and GPLS programs, (1) nearly all invest when $A > A_l$ and nearly none do when $A < A_l$, and (2) the normalized net payoff from investing versus not $\pi(\underline{K}, A) < 0$ for $A < A_l$, and $\pi(\overline{K}, A) > 0$ for $A > A_l$. Therefore, from (8) and (3), a participating investor's return under the optimal GPS program is

$$\begin{aligned} r_{gps} = \alpha\ \pi(1, K, A) + r &= \begin{cases}\alpha\ s_{gps} + \alpha\left(A - \frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K}}\right) & ; A < A_l \\ \alpha\ s_{gps} + r & ; A > A_l\end{cases} && (34)\\ &= \begin{cases} r - \alpha\ (A_h - x^*) - \alpha\ (A_l - A) & ; A < A_l \\ r + \alpha\ (x^* - A_l) & ; A > A_l,\end{cases} && (35)\end{aligned}$$

where $s_{gps} = x^* - A_l$ (Proposition 5). For $A < A_l$, we have $r_{gps} < r$, because $x^* \leq A_h$.

Similarly, from (14) and (3), the corresponding return under the optimal GPLS program is

$$r_{gpls} = \alpha\ \pi_{gpls}(1, K, A) + r \ = \ \begin{cases} \alpha s_{gpls} + \bar{t}\ r + \alpha(1-\bar{t})\left(A - \frac{\delta}{(1-\alpha)^2 \underline{K}}\right) & ; A < A_l \\ \alpha\ s_{gpls} + r & ; A > A_l \end{cases} \tag{36}$$

$$= \begin{cases} (1-\bar{t})\ r_{gps} + \bar{t}\ r & ; A < A_l \\ r + (1-\bar{t})\ \alpha\ (x^* - A_l) & ; A > A_l, \end{cases} \tag{37}$$

where the second line is strictly large since $r > r_{gps}$ when $A < A_l$ (the second line can be written as $(1-\bar{t})\ r_{gps} + \bar{t}r + (1-\bar{t})(r - r_{gps} + \alpha\ (x^* - A_l))$).

where we used $s_{gpls} = (1-\bar{t})s_{gps}$ since $s_{gpls} = (1-\bar{t})(x^* - A_l)$ from Proposition 7. The returns for investors who do not participate is similarly

$$r_{np} = \alpha\ \pi(K, A) + r \ = \ \begin{cases} \alpha\left(A - \frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K}}\right) & ; A < A_l \\ \alpha\left(A - \frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K}+\overline{K}}\right) & ; A > A_l. \end{cases} \tag{38}$$

$$= \begin{cases} r - \alpha\,(A_h - A) & ; A < A_l \\ r + \alpha\,(A - A_l) & ; A > A_l. \end{cases} \tag{39}$$

□

Proof of Proposition 9. It follows from Lemma 2 that a cutoff pair (x', x'') is given by equation (11). First, we compute the lower cutoff x'. Consider the two cases:

Case 1: $s > \frac{\delta}{(1-\alpha)^2\overline{K}}\left[\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) - \frac{\overline{K}}{\overline{K}+\underline{K}}\right] + \sigma = x^* - (A_l - \sigma)$

Assumption 2 ensures that the set $\min\{x\ s.t.\ \Delta^1(x; x') \geq 0\}$ is non-empty. From equation (11), x' solves

$$0 = E[\pi(K(A; x'), A)|x_i = x'] - t \cdot E\big[\max\{\pi(K(A; x'), A), 0\}|x_i = x'\big] + s$$

$$s = t \cdot E\big[\max\{\pi(K(A; x'), A), 0\}|x_i = x'\big] - E[\pi(K(A; x'), A)|x_i = x'].$$

Note that $s > x^* - (A_l - \sigma) \iff x' < A_0(x', \sigma) - \sigma$. When $s > x^* - (A_l - \sigma)$,

$$x^* - (A_l - \sigma) < t \cdot E\big[\max\{\pi(K(A; x'), A), 0\}|x_i = x'\big] - E[\pi(K(A; x'), A)|x_i = x']$$

$$x' < A_l - \sigma + t \cdot E\big[\max\{\pi(K(A; x'), A), 0\}|x_i = x'\big]. \tag{40}$$

From equation (15), $x' < A_0(x', \sigma) - \sigma \iff A_0(x', \sigma) = A_l$ and $x' \leq A_0(x', \sigma) - \sigma \iff E\big[\max\{\pi(K(A; x'), A), 0\}|x_i = x'\big] = 0$. Then, $x' \in \big\{x \in \mathbb{R} : x < A_0(x, \sigma) - \sigma\big\}$ satisfies equation 40.

Also, note that for any $x' \in [A_0(x',\sigma)-\sigma, A_0(x',\sigma)+\sigma)$, $\max\{\pi(K(A;x'),A),0\} \leq (A-A_l)^+$. For $x_i = x'$, $A \leq x' + \sigma$. Then, $t \cdot E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big] \leq t(x'+\sigma-A_l) \leq x'+\sigma-A_l$. That is, $x' \geq A_l - \sigma + t \cdot E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big]$. Thus, $s > x^* - (A_l - \sigma) \iff x' < A_0(x',\sigma) - \sigma$.

The marginal investor gets a payoff of zero. Therefore, when $s > x^* - (A_l - \sigma)$, at $x_i = x'$,

$$E[\pi(1,K(A;x'),A)|x_i = x'] = E[\pi(K(A;x'),A)|x_i = x'] - t \cdot E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big] + s$$
$$0 = E[\pi(K(A;x'),A)|x_i = x'] + s$$
$$x' = \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) - s = x^* - s.$$

Case 2: $s \leq \frac{\delta}{(1-\alpha)^2\overline{K}}\left[\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) - \frac{\overline{K}}{\overline{K}+\underline{K}}\right] + \sigma = x^* - (A_l - \sigma)$

Because $s > x^* - A_l + \sigma \iff x' < A_0(x',\sigma) - \sigma$, $s \leq x^* - A_l + \sigma \implies x' \geq A_0(x',\sigma) - \sigma$. Also, $x' < A_0(x',\sigma) + \sigma$ because $x' \leq x^* \leq A_h$. Therefore, in this case, $x' \in [A_0(x',\sigma) - \sigma, A_0(x',\sigma)+\sigma)$. The lower cutoff x' is bounded at x^*. When $x' < x^*$, the lower cutoff is given by

$$E[\pi(1,K(A;x'),A)|x_i = x'] = E[\pi(K(A;x'),A)|x_i = x'] - t \cdot E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big] + s$$
$$\begin{aligned}
0 &= E\left[\left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right)\Big| x_i = x'\right] \\
&\quad - t \cdot E\left[\max\left\{\left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right), 0\right\}\Big| x_i = x'\right] + s \\
0 &= \int_{x'-\sigma}^{x'+\sigma}\left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right) f(A|x_i = x')\,dA \\
&\quad - t\left\{\int_{x'-\sigma}^{A_0(x',\sigma)} 0 \cdot f(A|x_i = x')\,dA + \int_{A_0(x',\sigma)}^{x'+\sigma}\left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right) f(A|x_i = x')\,dA\right\} + s \\
0 &= \int_{x'-\sigma}^{x'+\sigma}\left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right) f(A|x_i = x')\,dA \\
&\quad - t \cdot \int_{A_0(x',\sigma)}^{x'+\sigma}\left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right) f(A|x_i = x')\,dA + s.
\end{aligned}$$

Solving the above equation,

$$x' - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) - \frac{t}{2\sigma}\Bigg[\frac{(x'+\sigma)^2 - A_0(x',\sigma)^2}{2} - \frac{r(x'+\sigma-A_0(x',\sigma))}{\alpha}$$

$$- \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x'-A_0(x',\sigma)}{\sigma}\big)\big)}\right)\Bigg] = -s$$

$$x' - \frac{t}{2\sigma}\Bigg[\frac{(x'+\sigma)^2 - A_0(x',\sigma)^2}{2} - \frac{r(x'+\sigma-A_0(x',\sigma))}{\alpha}$$

$$- \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x'-A_0(x',\sigma)}{\sigma}\big)\big)}\right)\Bigg] = x^* - s.$$

The lower cutoff x' solves $x' = T(x')$ where

$$T(x') = \frac{t}{2\sigma}\Bigg[\frac{(x'+\sigma)^2 - A_0(x',\sigma)^2}{2} - \frac{r(x'+\sigma-A_0(x',\sigma))}{\alpha}$$

$$- \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x'-A_0(x',\sigma)}{\sigma}\big)\big)}\right)\Bigg] + x^* - s,$$

and $x' \in X' := \{x \in \mathbb{R} : A_0(x,\sigma) - \sigma \leq x < A_0(x,\sigma) + \sigma\}$. $T(x)$ is differentiable over $X' := \{x \in \mathbb{R} : A_0(x,\sigma) - \sigma \leq x < A_0(x,\sigma) + \sigma\}$. Then, the slope $T'(x)$ is

$$T'(x) = \frac{t}{2\sigma}\left[x + \sigma - A_0(x,\sigma)A_0'(x,\sigma) - \frac{r(1-A_0'(x,\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2}\frac{1}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x-A_0(x,\sigma)}{\sigma}\big)\big)}\frac{1-A_0'(x,\sigma)}{2\sigma}\right]$$

$$= \frac{t}{2\sigma}\Bigg[x + \sigma - A_0'(x,\sigma)\left(A_0(x,\sigma) - \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x-A_0(x,\sigma)}{\sigma}\big)\big)}\right)$$

$$- \frac{r}{\alpha} - \frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x-A_0(x,\sigma)}{\sigma}\big)\big)}\Bigg]$$

$$= \frac{t}{2\sigma}\Big[x + \sigma - 0 - A_0(x,\sigma)\Big] \qquad \{\textit{from equation } (15)\}$$

$$= \frac{t}{2\sigma}(x - A_0(x,\sigma) + \sigma).$$

Since $x \in X' := \{x \in \mathbb{R} : A_0(x,\sigma) - \sigma \leq x < A_0(x,\sigma) + \sigma\}$ and $t \in [0,1]$, $T'(x) \in [0,1) \implies T'(x) < 1$. It follows that $T(x)$ admits a unique fixed point. Thus, when $x' < x^*$, x' uniquely solves equation (5).

Since the cutoff with program cannot be greater than the benchmark cutoff x^*, $x' \leq x^*$, the lower cutoff in equilibrium when $s \leq \frac{\delta}{(1-\alpha)^2\overline{K}}\left[\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) - \frac{\overline{K}}{\overline{K}+\underline{K}}\right] + \sigma$ is

$$x' = \min\{\tilde{x}(s,t), x^*\},$$

where $\tilde{x}$ uniquely solves,

$$\tilde{x} = \frac{t}{2\sigma}\left[\frac{(\tilde{x}+\sigma)^2 - A_0(\tilde{x},\sigma)^2}{2} - \frac{r(\tilde{x}+\sigma-A_0(\tilde{x},\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+K(A_0(\tilde{x},\sigma);\tilde{x})}\right)\right] + x^* - s. \tag{41}$$

Thus, there is a unique x' in the equilibrium. Finally, note that for $s = x^* - A_l + \sigma, \tilde{x} = x^* - s$ solves equation (41) implying $x'(s,t) = x^* - s$ in this case.

Next, we compute the upper cutoff x''. When $s \geq t(\overline{A} - A_l) = t\pi(\overline{K}, \overline{A})$, $t\pi(\overline{K}, \overline{A}) - s = \pi(2, \overline{K}, \overline{A}) - \pi(1, \overline{K}, \overline{A}) \leq 0$. Therefore, the upper cutoff is $x'' = \overline{A} + \sigma$ because any investor that invests participates in the program.

For $\frac{s}{t} < \pi(\overline{K}, \overline{A}) = \overline{A} - A_l$, because $\pi(1, K(\overline{A}, x'), \overline{A}) - \pi(2, K(\overline{A}, x'), \overline{A}) < 0$, $x'' < \overline{A} + \sigma$. Consider the indifference condition

$$\frac{s}{t} = E[\max\{\pi(K(A;x'),A),0\}|x_i]. \tag{42}$$

Note that $E[\max\{\pi(K(A;x'),A),0\}|x_i] = E\left[\max\left\{\left(A - \frac{\delta}{(1-\alpha)^2}\frac{1}{(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right), 0\right\}\middle|x_i\right] \geq 0$ and is weakly increasing in x_i. Also, $E\left[\max\left\{\left(A - \frac{\delta}{(1-\alpha)^2}\frac{1}{(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right), 0\right\}\middle|x_i\right]$ is strictly increasing in x_i when the conditional expectation is positive. Consider the following cases:

- If $\frac{s}{t} > 0$,
  In this case, $E\left[\max\left\{\left(A - \frac{\delta}{(1-\alpha)^2}\frac{1}{(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right), 0\right\}\middle|x_i\right] > 0$ and because it is strictly increasing, there is a unique $x^\diamond$ that solves,
  $$\frac{s}{t} = E\left[\max\left\{\left(A - \frac{\delta}{(1-\alpha)^2}\frac{1}{(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right), 0\right\}\middle|x_i = x^\diamond\right].$$

- If $s/t = 0$,
  $x' = x^*$ and equation (42) is satisfied for all $x_i \leq x^* - \sigma$. Then, by the tie breaking rule (we choose the lowest signal), $x^\diamond = \underline{A} - \sigma$.

Finally, because $x'' \geq x'$, the unique upper cutoff is given by

$$x'' = \max\{x', x^\diamond(s,t)\}.$$

□

Proof of Proposition 10. First, consider the lower cutoff x'.

Case 1: $\frac{s}{1-t^-} > \frac{\delta}{(1-\alpha)^2\overline{K}}\left[\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) - \frac{\overline{K}}{\overline{K}+\underline{K}}\right] + \sigma = x^* - A_l + \sigma$

When $\frac{s}{1-t^-} > x^* - A_l + \sigma$, $x' < A_0(x', \sigma) - \sigma$. The arguments for this are similar to that in the proposition for the GPS. Let $t^+ = t^- + \gamma$, then for a GPLS program (s, t^+, t^-), x' solves

$$\begin{aligned}
s &= t^+ E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big] + t^- E\big[\min\{\pi(K(A;x'),A),0\}|x_i = x'\big] \\
&\qquad - E[\pi(K(A;x'),A)|x_i = x'] \\
s &= \gamma E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big] - (1-t^-)E\big[\pi(K(A;x'),A)|x_i = x'\big] \quad \{\because \gamma = t^+ - t^-\} \\
\frac{s}{1-t^-} &= \frac{\gamma}{1-t^-}E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big] - E\big[\pi(K(A;x'),A)|x_i = x'\big] \\
\implies x' &< A_l - \sigma + \frac{\gamma}{1-t^-}E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big]. \qquad (43)
\end{aligned}$$

Note that $x' < A_0(x', \sigma) - \sigma$ satisfies equation (43). Also, for $x' \in [A_0(x', \sigma) - \sigma, A_0(x', \sigma) + \sigma)$, $\max\{\pi(K(A;x')),0\} \leq (A - A_l)^+$. When $x_i = x'$, $A \leq x' + \sigma$ and $\frac{\gamma}{1-t^-}E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big] \leq x' - (A_l - \sigma)$ implies $x' \geq A_l - \sigma + \frac{\gamma}{1-t^-}E\big[\max\{\pi(K(A;x'),A),0\}|x_i = x'\big]$. Therefore, $\frac{s}{1-t^-} > \frac{\delta}{(1-\alpha)^2\overline{K}}\left[\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) - \frac{\overline{K}}{\overline{K}+\underline{K}}\right] + \sigma \iff x' < A_0(x', \sigma) - \sigma$.

The lower cutoff is the signal at which her payoff from investing while participating in the program is 0, i.e., $E[\pi_{gpls}(1, K(A;x'), A)|x_i = x'] = 0$. Therefore,

$$\begin{aligned}
0 &= E[\pi(K(A;x'),A)|x'] - t^+ \cdot E\big[\max\{\pi(K(A;x'),A),0\}|x'\big] - t^- \cdot E\big[\min\{\pi(K(A;x'),A),0\}|x'\big] + s \\
0 &= (1-t^-)E[\pi(K(A;x'),A)|x'] + s \qquad \{\because x' < A_0(x', \sigma) - \sigma\} \\
x' &= \frac{r}{\alpha} + \frac{\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) - \frac{s}{1-t^-}.
\end{aligned}$$

Case 2: $\frac{s}{1-t^-} \leq \frac{\delta}{(1-\alpha)^2\overline{K}}\left[\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right) - \frac{\overline{K}}{\overline{K}+\underline{K}}\right] + \sigma = x^* - A_l + \sigma$

Using the same arguments as in the proof of Proposition 9, in this case, $x' \in [A_0(x', \sigma) - \sigma, A_0(x', \sigma) + \sigma)$. The lower cutoff x' solves $E[\pi_{gpls}(1, K(A;x'), A)|x_i = x'] = 0$. Thus,

$$0 = E[\pi(K(A;x'),A)|x'] - t^+ \cdot E\Big[\max\{\pi(K(A;x'),A),0\}|x'\Big] - t^- \cdot E\Big[\min\{\pi(K(A;x'),A),0\}|x'\Big] + s$$

$$\begin{aligned}
0 = &\int_{x'-\sigma}^{x'+\sigma} \left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right) f(A|x_i = x')\, dA \\
&- t^+ \cdot \int_{A_0(x',\sigma)}^{x'+\sigma} \left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right) f(A|x_i = x')\, dA \\
&- t^- \cdot \int_{x'-\sigma}^{A_0(x',\sigma)} \left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right) f(A|x_i = x')\, dA + s \\
x' = &\, t^+ \cdot \int_{A_0(x',\sigma)}^{x'+\sigma} \left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right) f(A|x_i = x')\, dA \\
&+ t^- \cdot \int_{x'-\sigma}^{A_0(x',\sigma)} \left(A - \frac{\delta}{(1-\alpha)^2(\underline{K}+K(A;x'))} - \frac{r}{\alpha}\right) f(A|x_i = x')\, dA + x^* - s \\
x' = &\, \frac{t^+}{2\sigma}\left[\frac{(x'+\sigma)^2 - A_0(x',\sigma)^2}{2} - \frac{r(x'+\sigma-A_0(x',\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x'-A_0(x',\sigma)}{\sigma}\big)\big)}\right)\right] \\
&+ \frac{t^-}{2\sigma}\left[\frac{A_0(x',\sigma)^2 - (x'-\sigma)^2}{2} - \frac{r(A_0(x',\sigma)-x'+\sigma)}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x'-A_0(x',\sigma)}{\sigma}\big)\big)}{\underline{K}}\right)\right] \\
&+ x^* - s.
\end{aligned}$$

The RHS of the above equation has a slope less than 1. To prove this, let $t^+ = t^- + \gamma$. Then,

$$\begin{aligned}
x' = &\, \frac{\gamma}{2\sigma}\left[\frac{(x'+\sigma)^2 - A_0(x',\sigma)^2}{2} - \frac{r(x'+\sigma-A_0(x',\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x'-A_0(x',\sigma)}{\sigma}\big)\big)}\right)\right] \\
&+ \frac{t^-}{2\sigma}\left[\frac{(x'+\sigma)^2 - (x'-\sigma)^2}{2} - \frac{r(x'+\sigma-x'+\sigma)}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\overline{K}}{\underline{K}}\right)\right] + x^* - s \\
x' = &\, \frac{\gamma}{2\sigma}\left[\frac{(x'+\sigma)^2 - A_0(x',\sigma)^2}{2} - \frac{r(x'+\sigma-A_0(x',\sigma))}{\alpha} - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x'-A_0(x',\sigma)}{\sigma}\big)\big)}\right)\right] \\
&+ t^-\big(x' - x^*\big) + x^* - s \\
x' = &\, \frac{\gamma}{1-t^-}\frac{1}{2\sigma}\left[\frac{(x'+\sigma)^2 - A_0(x',\sigma)^2}{2} - \frac{r(x'+\sigma-A_0(x',\sigma))}{\alpha}\right. \\
&\left. - \frac{2\sigma\delta}{(1-\alpha)^2\overline{K}} \log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\big(1-\big(\frac{x'-A_0(x',\sigma)}{\sigma}\big)\big)}\right)\right] + x^* - \frac{s}{1-t^-}.
\end{aligned}$$

Taking derivative of the RHS w.r.t. x',

$$\left(\frac{\gamma}{1-t^-}\right)\frac{1}{2\sigma}\left[x'+\sigma-A_0(x',\sigma)A_0'(x',\sigma)-\frac{r}{\alpha}+\frac{r}{\alpha}A_0'(x',\sigma)-\frac{\delta}{(1-\alpha)^2}\frac{1-A_0'(x',\sigma)}{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{x'-A_0(x',\sigma)}{\sigma}\right)\right)}\right]$$
$$=\left(\frac{\gamma}{1-t^-}\right)\frac{1}{2\sigma}\left[x'+\sigma-\frac{r}{\alpha}-\frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{x'-A_0(x',\sigma)}{\sigma}\right)\right)}\right.$$
$$\left.-A_0'(x',\sigma)\left(A_0(x',\sigma)-\frac{r}{\alpha}-\frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{x'-A_0(x',\sigma)}{\sigma}\right)\right)}\right)\right]$$
$$=\left(\frac{\gamma}{1-t^-}\right)\frac{1}{2\sigma}\left[x'+\sigma-\frac{r}{\alpha}-\frac{\delta}{(1-\alpha)^2}\frac{1}{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{x'-A_0(x',\sigma)}{\sigma}\right)\right)}\right]$$
$$=\left(\frac{\gamma}{1-t^-}\right)\frac{1}{2\sigma}\left[x'+\sigma-A_0(x',\sigma)\right]<1.$$

The last inequality follows because $\left(\frac{\gamma}{1-t^-}\right)\leq 1$ and $(x'-A_0(x',\sigma)+\sigma)\in[0,2\sigma)$. Thus, there exists a unique x'. Moreover, because x' is bounded above at x^*, the lower cutoff is

$$x'(s,t^+,t^-)=\min\{\tilde{x}(s,t^+,t^-),x^*\},$$

where $\tilde{x}$ uniquely solves

$$\tilde{x}=\frac{t^+}{2\sigma}\left[\frac{(\tilde{x}+\sigma)^2-A_0(\tilde{x},\sigma)^2}{2}-\frac{r(\tilde{x}+\sigma-A_0(\tilde{x},\sigma))}{\alpha}-\frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\overline{K}}{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{\tilde{x}-A_0(\tilde{x},\sigma)}{\sigma}\right)\right)}\right)\right]$$
$$+\frac{t^-}{2\sigma}\left[\frac{A_0(\tilde{x},\sigma)^2-(\tilde{x}-\sigma)^2}{2}-\frac{r(A_0(\tilde{x},\sigma)-\tilde{x}+\sigma)}{\alpha}-\frac{2\sigma\delta}{(1-\alpha)^2\overline{K}}\log\left(\frac{\underline{K}+\frac{\overline{K}}{2}\left(1-\left(\frac{\tilde{x}-A_0(\tilde{x},\sigma)}{\sigma}\right)\right)}{\underline{K}}\right)\right]$$
$$+x^*-s.$$

Finally, note that for $\frac{s}{1-t^-}=x^*-A_l+\sigma,\tilde{x}=x^*-\frac{s}{1-t^-}$ solves the above equation implying $x'(s,t)=x^*-\frac{s}{1-t^-}$ in this case.

Next, for the upper cutoff x'', when $s/t^+\geq\overline{A}-A_l=\pi(\overline{K},\overline{A})$, $\pi_{gpls}(2,\overline{K},\overline{A})-\pi_{gpls}(1,\overline{K},\overline{A})\leq 0$ implying that $x''=\overline{A}+\sigma$. When $s/t^+<\overline{A}-A_l$, $x''<\overline{A}+\sigma$ and x'' solves

$$s=t^+\cdot E\big[\max\{\pi(K(A;x'),A),0\}|x_i=x\big]+t^-\cdot E\big[\min\{\pi(K(A;x'),A),0\}|x_i=x\big].\quad(44)$$

Because $E[\pi(K(A;x'),A)|x_i]$ is strictly increasing in x_i, the RHS of (44) is weakly increasing in x when $s/t^+\leq\pi(\overline{K},\overline{A})$. $x''<\overline{A}+\sigma$ implies existence. Let $x^\diamond(s,t^+,t^-)$ denote the minimum x that solves equation (44). Furthermore, $x''\geq x'$. Therefore, the upper cutoff is

$$x''=\max\{x',x^\diamond(s,t^+,t^-)\}.\qquad\square$$