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Distribution Assignment

2) Average Monthly Sales of 2000 firms are normally distributed. $\mu = 38000$ $\sigma = 10000$

(i) Firms with sales over 50000.

$$x = 50000 \quad \mu = 38000 \quad \sigma = 10000$$

$$ND \Rightarrow Z = \frac{x - \mu}{\sigma} = \frac{12000}{10000} = 1.2 \quad \therefore Z > 1.2 \text{ or } 2?$$

$$Z > 1.2 \sigma \quad 1.2 \sigma = 0.89 \quad \therefore P(\text{Sales} > 50000) = 1 - 0.89 = 0.11$$

$$\text{No. of firms} = P \cdot 2000 = \underline{\underline{220}}$$

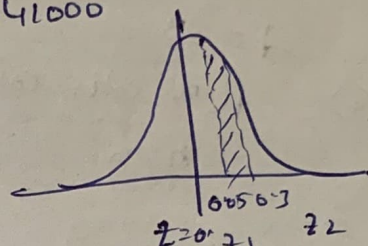
(ii)

% of firms with sales b/w 38500 & 41000

$$Z_1 = \frac{38500 - 38000}{10000} = 0.05$$

$$Z_2 = \frac{41000 - 38000}{10000} = 0.3$$

$$\begin{aligned} P(0.05) &= 0.5199 \\ P(0.3) &= 0.6179 \end{aligned} \quad \left. \begin{aligned} &\text{Area} \\ &(Z_2 - Z_1) \end{aligned} \right\} \begin{aligned} &= 0.6179 - 0.5199 \\ &= 0.098 = \approx 9.8\% \end{aligned}$$



(iii) No. of firms with sales b/w 30000 & 50000

$$Z_1 = \frac{30000 - 38000}{10000} = -0.8$$

$$Z_2 = \frac{50000 - 38000}{10000} = 1.2$$

$$\begin{aligned} P(Z_1) &= 0.2119 \\ P(Z_2) &= 0.8944 \end{aligned}$$

$$\therefore \text{Area} = P(Z_2 - Z_1) = 0.8944 - 0.2119 = 0.6825 \approx 68.25\%$$

$$\therefore \text{No. of firms} = \frac{(68.25) \times 2000}{100} = \underline{\underline{1365}}$$

3) Test = 20 MCQ. (4 option for each Q)

n = Total answers = 20

5 wrong

$\therefore 15$ correct = 1 (Success)

Binomial

$P(5 \text{ wrong answers})$

x = Success

$${}^nC_x P^x q^{n-x}$$

$$P = P(\text{wrong}) = \frac{5}{20} = 1/4$$

$$q = P(\text{right}) = \frac{15}{20} = 3/4$$

$$\therefore P = {}^{20}C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{15}$$

(4) Avg. rate of photons reaching the telescope is 4/sec.

~~1 sec.~~
 $P(\text{no photon reaches the telescope in a given second}) = ?$
 $\lambda = 4$ (Poisson Random Variable, mean),

$$\text{PMF} = P(X=1) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \quad \boxed{x=0}$$

$$x = \text{required no. of times.} \quad \lambda = 4 \quad P = \frac{e^{-4} \cdot 4^0}{0!} = e^{-4}$$

(5) $\lambda = 3$

(a) $P(\text{no calls come in a given 1-minute period})$

$$\lambda = 0 \quad P = \frac{e^{-\lambda} \cdot \lambda^x}{x!} = \frac{e^{-3} \cdot 3^0}{0!} = e^{-3} = 0.0497 = 4.97\%$$

(b) $P(\text{at least two calls will arrive in a given 2-min period}) = ?$

$\lambda = 2$ $\lambda = 3/\text{minute}$

$\left. \begin{array}{l} 1 \text{ min} = 3 \\ 2 \text{ min} = 6 \end{array} \right\}$

$$\therefore \lambda / 2 \text{ min} = 3 \times 2 = 6$$

$$P = \frac{e^{-6} \cdot 6^2}{2!} = 0.046175$$

~~$P(X=2)$~~

Probab(least 2)

$$\therefore P(X \geq 2) = 1 - 0.046175 = 95.38\%$$

(6) 20% defective rate in a production line.

$P(\text{obtaining the first defective part after three good parts}) = ?$

$$\rightarrow \left. \begin{array}{l} P(\text{defect}) = \frac{20}{100} = \frac{1}{5} = 0.2 \\ P(\text{good}) = \frac{80}{100} = 0.8 \end{array} \right\} \begin{array}{l} P = 0.2 \\ q = 0.8 \end{array}$$

$$P(X=4) = \frac{G \ G \ G \ D}{0.8 \ 0.8 \ 0.8 \ 0.2} = 0.1024 = 10.24\%$$

Avg. No of inspections to obtain the first defect = ?

$$P(d) = \frac{1}{5} \text{ i.e. one in } 5$$

\therefore for every 5 items, we can find one defect

(5)

②

⑦ $P(\text{Student is accepted to a prestigious college}) = 0.3$
 If 5 students apply from the same school,
 $P(\text{at most 2 got accepted}) =$

$$P(0) = 0.3 \quad P(1) = 0.7$$

$$P(X \leq 2) = \sum_{i=0}^2 nC_i p^i q^{n-i} \quad n=5, i \geq 2$$

$$= \binom{5}{0} (0.3)^0 (0.7)^5 + \binom{5}{1} (0.3)^1 (0.7)^4 + \binom{5}{2} (0.3)^2 (0.7)^3$$

$$P(X \leq 2) = 0.8361$$

⑧ Max weight = 800 kg
 $\mu = 70$

$$\sigma^2 = 200, \quad \sigma = \sqrt{200} = 10\sqrt{2}$$

$$n=10, \quad Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{800 - (70 \times 10)}{10\sqrt{2}} = 2.23$$

$$P(Z < 2.23) = 0.9871 = 98.71\%$$

$$n=12, \quad P(12) = \frac{800 - (70 \times 12)}{\frac{(10\sqrt{2}) \cdot 12}{\sqrt{12}}} = \frac{-40}{(14.14)(\sqrt{12})} = -0.82$$

$$P(Z < -0.82) = 0.2061 = 20.61\%$$

⑨

$n=50$ 2 choices each q.

$$① X \geq 20, \quad P = nC_x p^x q^{n-x}$$

$$P(\text{con}) = 1/2, \quad P(\text{un}) = 1/2$$

$$P(X \geq 20) = {}^{50}C_{20} (1/2)^{20} (1/2)^{50-20}$$

$$= 0.04185 = 4.185\%$$

② If ch24
 $p = 1/4$
 $q = 3/4$

$$P(20) = {}^{50}C_{20} (1/4)^{20} (3/4)^{50-20} = 0.76\%$$

(10)

$$P(\text{fault}) = \frac{36}{100} = \underline{0.3}$$

$P(\text{exactly } 2 = \text{defect}), \text{ randomly } \underline{= 6}$

$$P(NF) = \frac{70}{100} = \underline{0.7} \quad \left. \begin{array}{l} n=6 \\ k=2 \end{array} \right\}$$

$$P(X=2) = {}^nC_k (p)^k (q)^{n-k} = {}^6C_2 (0.3)^2 (0.7)^4$$

$$= \underline{0.32418} \quad (\text{--- } k=np)$$

(11)

For a writer, 6 errors per hour enter \rightarrow 1/min

$P(2 \text{ errors in 322 word report}) =$

$$\left. \begin{array}{l} P(e) = 6/60 = 0.1 \\ P(neu) = 1 - 0.1 = 0.9 \end{array} \right\}$$

$$\lambda = 6/\text{hr.} \quad \underline{\underline{\lambda = 2}}$$

$$\underline{\underline{\lambda = 0.1}}$$

$$P(X=2) = \frac{e^{-0.1} \cdot (0.1)^2}{2!}$$