PROBABILITY ASSIGNMENT

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E2EDS: Batch 8

I BASIC PROBABILITY

(1) $P(A \cdot B) = P(A) \cdot P(B)$

Probability of Sum of numbers being even and one of the dice shows 6

$P(6) = 1/6$

$6 \mid 2, 4, 6$

~~P(6 for~~ $P(\text{even} \mid \text{Provided other face} = 6) \Rightarrow 3/6$

$\therefore P(A \cdot B) = 1/6 \cdot 3/6 = 3/36$

(2) Probability ~~of~~ Sum of numbers being less than 7.
(Two dice are rolled.)

(Combinations) Possible

1: 5, 4, 3, 2, 1	2: 4, 3, 2, 1	3: 3, 2, 1	4: 2, 1	5: 1

$P(\Sigma(\text{No})) < 7$

$\therefore P(\text{Event}) = 15/36$

$$\left[(1/6 \cdot 5/6) + (1/6 \cdot 4/6) + (1/6 \cdot 3/6) + (1/6 \cdot 2/6) + (1/6 \cdot 1/6)\right]$$

$$= \left[\frac{5+4+3+2+1}{36}\right] = 15/36$$

(3) Coin Tossed 3 times.
Given that You observed at least one Heads, P( ) observing at least 2 heads?

Let A = at least one head.

$P(A \mid 3 \text{ tosses})$ (at least one H) $= 7/8$

$P(B \mid 3 \text{ tosses})$ (at least 2 H) $= 4/8$

TTT
TTH
THT
THH
HTT
HTH
HHT
HHH

at least (1) H — TTH to HHH

at least (2) H — HTH, HHT, HHH

$$\therefore P(A \mid B) = \frac{P(A \cdot B)}{P(B)} = \frac{4/8}{7/8} = 4/7$$

(4).

P(one of Two kids = girl.

Probability that other kid is also a girl = ?.

~~P(E|F)~~

P(First|Second) = Possible scenarios

(G B, G G, B G, B B)

But since, already one girl is there,

the possible combinations are (G B, G G, B G)

P(G, G) = .

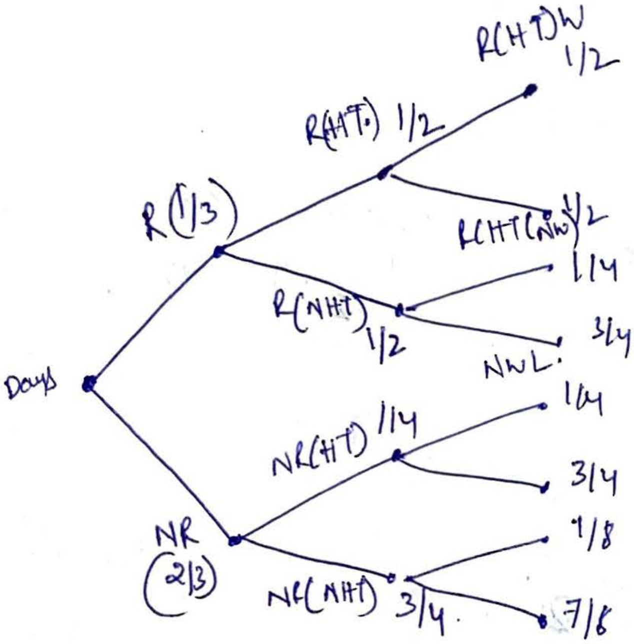
$$P(\text{Both Girls} \mid \text{at least one girl}) = \frac{P(\text{Both Girls})}{P(\text{At least one girl})}$$

$$= \frac{1/2 \cdot 1/2}{1 - P(\text{all boys})} = \frac{1/4}{1-1/4}.$$

$$= \frac{1/4}{3/4} = \underline{\underline{1/3}}$$

II Conditional, Joint & Marginal Probability

5



$P = 1/3 \cdot 1/2 \cdot 1/2 = 1/12$

$P = 1/3 \cdot 1/2 \cdot 1/2 = 1/12$

$P = 1/3 \cdot 1/2 \cdot 1/4 = 1/24$

$P = 1/3 \cdot 1/2 \cdot 3/4 = 3/24$

$P = 2/3 \cdot 1/4 \cdot 1/4 = 2/48 = 1/24$

$P = 2/3 \cdot 1/4 \cdot 3/4 = 6/48 = 1/8$

$P = 2/3 \cdot 3/4 \cdot 1/8 = 6/96 = 1/16$

$P = 2/3 \cdot 3/4 \cdot 7/8 = 7/16$

(a) $P(NR \cap HT \cap \text{Not } L) = P(\cancel{NR}) \cdot$

$P(HT) = P(HT/NR) = 1/4$.

$P(\text{Not Late}) = 3/4 \quad \therefore P(\ ) = 2/3 \cdot 1/4 \cdot 3/4 = 1/8$.

(b) $P(\text{Late}) = P(R, HT, \text{Late}) + P(R, NHT, \text{Late})$
$+ P(NR, HT, \text{Late}) + P(NR, NHT, \text{Late})$

$= 1/12 + 1/24 + 1/24 + 1/16 = \underline{11/48}$

(c) $P(\text{Rainy} \mid \text{Late}) = \dfrac{P(R \cap L)}{P(L)} = \dfrac{P(R, HT, \text{Late}) + P(R, NHT, L)}{P(L)}$

$\Rightarrow \dfrac{1/12 + 1/24}{11/48} = \dfrac{6/48}{11/48}$

$= 6/11$.

(6) Box containing 3 coins.

2 Normal coins. | one Fake 2-Headed coin ($P(H) = 1$) always.

Pick a coin at Random & toss it

(a) Probability that it lands heads.

TH
TH
HH

$P(H) \Rightarrow$

$P(H|NC) = 2/3$, $P(T|NC) = 1$

$P(H|FC) = 1$

$P(H) = P(H|NC) \cdot P(H|FC) = 1/2 \cdot 1 = \underline{1/2}$.

There are two set of coins

(1) Normal, (2) Fake.

NC = Normal
FC = Fake coin

$\therefore P(H) = P(H|NC) + P(H|FC)$

$\Rightarrow P$

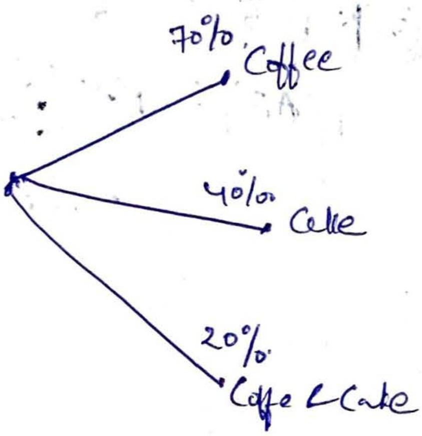
3 coins
- Fair coin (2/3)
  - T (1/2)
  - H (1/2)
- 2-H coin (1/3)
  - H (1)
  - T (0).

$P(H) = 1 \cdot 1/3 + 1/2 \cdot 2/3 = 1/3 + 2/6 = 2/3$.

(b) $= \frac{fav}{Tot} = \frac{1/3 \cdot 1}{1/3 \cdot 1 + 2/3 \cdot 1/2} = \frac{1/3}{1/3 + 1/3} = \frac{1/3}{2/3} = \underline{1/2}$.

(7)

(7)



$P(Cof) = 70/100 = 0.7.$

$P(Cake) = 40/100 = 0.4$

$P(Cof \cap Cake) = 0.2.$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} \quad \frac{P(Cof \cap Cake)}{P(Cake)}$$

$$= \frac{0.2}{0.4} = \underline{1/2}$$

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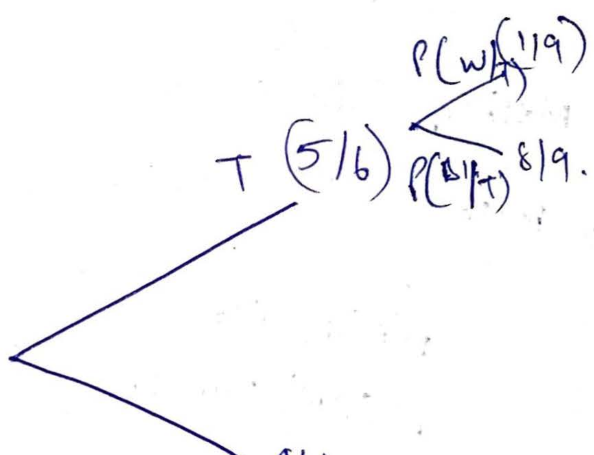
(8)



$P(W) = ?$ $P(w)$

$P(w) = 1/9$

$P(T|w) = 5/6.$

$P(T|w') = 1/6$

$P(w') = 8/9.$

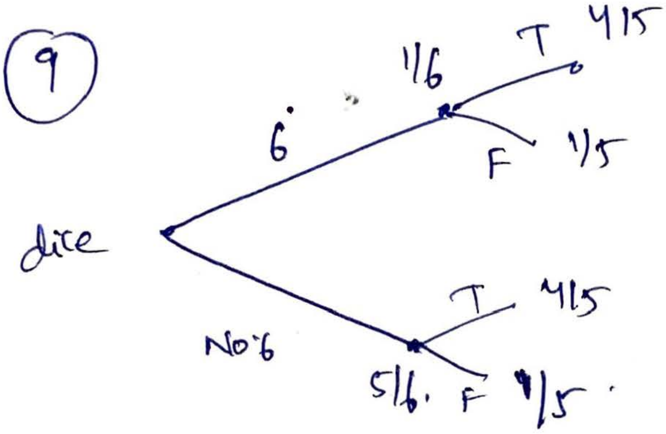
$P(w|T) = \cancel{5/6 \cdot 1/9} = \frac{5}{54}$

$$P(w/T) = \frac{P(T|w) \cdot P(w)}{P(T)} = \frac{5/6 \cdot 1/9}{5/6 \cdot 1/9 + 8/9 \cdot 1/6} = \underline{5/13}$$

$P(T) = P(T|w) \cdot P(w) + P(T|w') \cdot P(w')$

$$\boxed{P(w|T) = 5/13}$$

(9)



$P(6) = 1/6.$

$P(A|T) = 4/5$

$P(A|T') = 1/5$

$P(6') = 5/6.$

$P(A_T) = P(A_T/6')\cdot P(6') + P(A_T/6)\cdot P(6)$

$$P(6/A_6) = \frac{P(A_6/6)\cdot P(6)}{P(A_6)} =$$

$$\Rightarrow \frac{P(A_T/6)\cdot P(6)}{P(A_T/6')\cdot P(6') + P(A_T/6)\cdot P(6)}$$

$$= \frac{4/5 \cdot 1/6}{4/5\cdot 1/6 + 1/5 \cdot 5/6} = \frac{4}{9}$$

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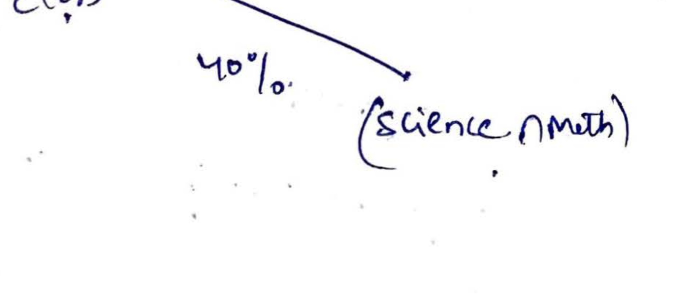
(10) 40% of students study Math & Science

60% of students study Math.



$P(M) = 0.6$

$P(S\cap M) = 0.4.$

$P(S|M) = ?$

$P(S|M)$

$P(S\cap M) = P(M)\cdot P(S/M).$

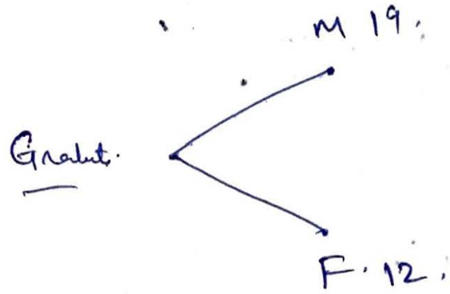
$$\therefore P(S|M) = \frac{P(S\cap M)}{P(M)}$$

$$\boxed{P(S|M) = \frac{0.4}{0.6} = 2/3}$$

(11).

	Graduate	Post Graduate	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

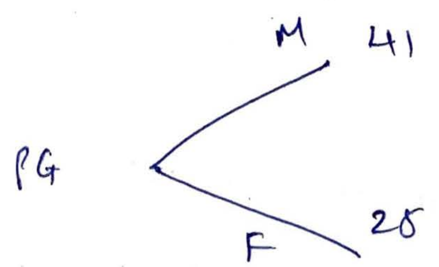
(a).



$P(M/G) = 19/31$

$P(F/G) = 12/31$

$P(G) = 31/100$ $\quad P(PG) = 69/100$



$P(M/PG) = 41/69$

$P(F/PG) = 28/69$

$P(M \cap G) = ?$ $\quad P(M) \cdot P(M/G)$

$=$ $\quad =$

~~$P(M) = P(M/G) \cdot P(G) + P(M/PG) \cdot P(PG)$~~ ⓪

$= P(M) = 60/100$

$P(F) = 40/100$ $\quad$ (Its Joint Prob)

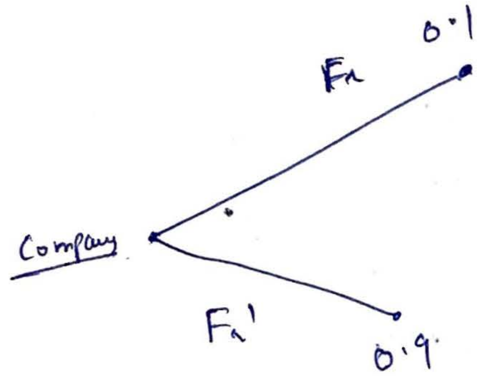
$$\boxed{P(M \cap G) = P(M) \cdot P(M/G) = (60/100) \cdot (19/31)}$$

(b). $P(M) = ? = 60/100 = 0.6$

(c) $P(G) = ? = 31/100$ (M.P)

(d) $P(F/PG) = 28/69$ (C.P)

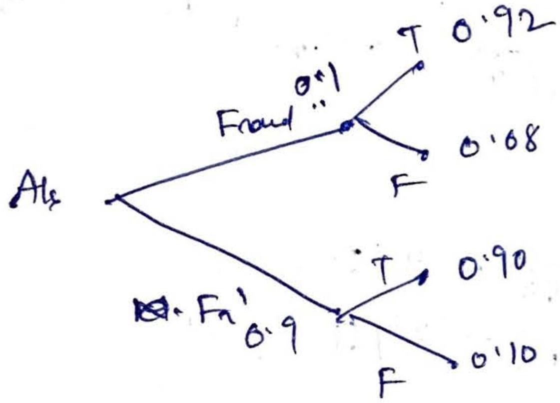
(12)



$P(F_n) = 0.1$
$P(F_n') = 0.9$

$P(Ab|F_n) = 0.92$
$P(Al'|F_n') = 0.90$

$P(Al|F_n') = 0.08$
$P(Al'|F_n) = 0.10$



$P(F_n|A_s)$ ..

Suppose we observe a company for whom the algorithm test returns a fraud result.
Calculate the posterior Prob. that this company truly did fraud in filing?

$P(F_n|A_l) = ?$

$P(Al') = P(Al|F_n')$

$$P(F_n|A_l) = \frac{P(Al|F_n)\cdot P(F_n)}{P(Al)}$$

$$\Rightarrow \frac{P(Al|F_n)\cdot P(F_n)}{P(Al|F_n)\cdot P(F_n) + P(Al|F_n')\cdot P(F_n')}$$

$$= \frac{(0.92)\cdot(0.1)}{(0.92)(0.1) + (0.08)(0.9)} = \frac{0.092}{0.164}$$

$$\boxed{P(F_n|A_l) = 0.56.}$$

(14).

~~PCO~~ .

$P(\text{swineflue} / \text{Positive}) = ?$

$$P(SF/Pos) = \frac{P(Pos/SF) \cdot P(SF)}{P(Pos)}.$$

$$P(Pos) = P(Pos/SF) \cdot P(SF) + P(Pos/NSF) \cdot P(NSF)$$

$$P(SF) = \frac{1}{10000} = 0.0001$$

$$P(NSF) = 1 - 0.0001 = 0.9999.$$

TP ~~B~~FP

TN FN.

$FP = . P(Pos/NSF) = 0.01.$ (a

$FN = P(Neg/NSF)$

$TP = P(Pos/SF) = 1 - P(Neg/NSF).$

$P(Pos/SF) = 1 - 0 = 1.$

$$\therefore P(SF/Pos) = \frac{1 \times (0.0001)}{(1) \times (0.0001) + (0.01)(0.9999)} = 0.01.$$