Assignment - 1 LADC

1. a) $x + y + 2z = 4$

$2x - y + 3z = 9$

$3x - y - z = 2$

$$\begin{bmatrix} 1 & 1 & 2 & -4 \\ 2 & -1 & 3 & -9 \\ 3 & -1 & -1 & -2 \end{bmatrix} \Rightarrow R_2 \to R_2 - 2R_1 \begin{bmatrix} 1 & 1 & 2 & -4 \\ 0 & -3 & -1 & -1 \\ 3 & -1 & -1 & -2 \end{bmatrix}$$

$$R_3 \to R_3 - 3R_1 \begin{bmatrix} 1 & 1 & 2 & -4 \\ 0 & -3 & -1 & -1 \\ 0 & -4 & -7 & 10 \end{bmatrix}$$

$$R_3 \to -3R_3 + 4R_2 \begin{bmatrix} 1 & 1 & 2 & -4 \\ 0 & -3 & -1 & -1 \\ 0 & 0 & 17 & -34 \end{bmatrix}$$

$17z = 34 \Rightarrow z = 2$

$-3y - z = +1 \Rightarrow y = -1$

$x - 1 + 4 = 4 \Rightarrow x = 1$

$(x, y, z) = (1, -1, 2)$

b) $x + 2y - z = 3$ — (1)

$3x - y + 2z = -1$ — (2)

$2x - 2y + 3z = 2$ — (3)

$x - y + z = -1$ — (4)

Consider (1), (3), (4)

$$\begin{bmatrix} 1 & 2 & -1 & -3 \\ 2 & -2 & 3 & -2 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

$$R_2 \to R_2 - 2R_1 \Rightarrow \begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -6 & 5 & 4 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

$$R_3 \to R_3 - R_1 \Rightarrow \begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -6 & 5 & 4 \\ 0 & -3 & 2 & 4 \end{bmatrix}$$

$$R_3 \to 2R_3 - R_2 \Rightarrow \begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -6 & 5 & 4 \\ 0 & 0 & -1 & 4 \end{bmatrix}$$

$(x, y, z) = (-1, 4, 4)$ Substitute in (2)

$-3 - 4 + 8 = -1$ $\quad 1 \neq -1$ They are not consistent.

(c) $x+2y-z-3=0$

$3x-y+2z-1=0$

$2x-2y+3z-2=0$

$1x-y+z+1=0$

$$\begin{bmatrix} 1 & 2 & -1 & -3 \\ 3 & -1 & 2 & -1 \\ 2 & -2 & 3 & -2 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -7 & 5 & 8 \\ 0 & -6 & 5 & 4 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -7 & 5 & 8 \\ 0 & -6 & 5 & 4 \\ 0 & -3 & 2 & 2 \end{bmatrix}$$

$R_3 \to 7R_3 - 6R_2$
$R_4 \to 7R_4 - 3R_2$

$$\begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -7 & 5 & 8 \\ 0 & 0 & 5 & -20 \\ 0 & 0 & -1 & -10 \end{bmatrix}$$

$R_4 \to 5R_4 + R_3$

$$\begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -7 & 5 & 8 \\ 0 & 0 & 5 & -20 \\ 0 & 0 & 0 & -30 \end{bmatrix}$$

$z=4$

$-7y = -28 \Rightarrow y=4$

$x+8-4-3=0$

$x=-1$

$(x,y,z) = (-1,4,4)$

d) $\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & -1 & -4 \\ 1 & 1 & -1 & 1 & 4 \\ 1 & -1 & 1 & 1 & -2 \end{bmatrix}$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$
$R_4 \to R_4 - R_1$

$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & -4 \\ 0 & 0 & -2 & 0 & 4 \\ 0 & -2 & 0 & 0 & -2 \end{bmatrix}$

$R_2 \leftrightarrows R_4$

$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 0 & 0 & -2 \\ 0 & 0 & -2 & 0 & 4 \\ 0 & 0 & 0 & -2 & -4 \end{bmatrix}$

$-2x_4 = 4 \Rightarrow x_4 = -2$
$-2x_3 = -4 \Rightarrow x_3 = 2$
$-2x_2 = 2 \Rightarrow x_2 = -1$
$x_1 - 1 + 2 - 2 = 0 \Rightarrow x_1 = 1$

$(x_1, x_2, x_3, x_4) = (1, -1, 2, -2)$

e) $\cancel{x_1 + 2x_2 + x_3 - 2x_4}$ $\begin{bmatrix} 1 & 2 & 1 & -2 & -6 \\ 2 & 3 & 2 & -2 & -8 \\ 3 & 1 & 2 & -1 & -4 \\ 4 & 2 & 2 & -3 & -9 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 4R_1$

$\begin{bmatrix} 1 & 2 & 1 & -2 & -6 \\ 0 & -1 & 0 & 2 & 4 \\ 0 & -5 & -1 & 5 & 14 \\ 0 & -6 & -2 & 5 & 15 \end{bmatrix}$

$R_4 \to R_4 - 6R_1$
$R_3 \to R_3 - 5R_1$

$\begin{bmatrix} 1 & 2 & 1 & -2 & -6 \\ 0 & -1 & 0 & 2 & 4 \\ 0 & 0 & -1 & -5 & -6 \\ 0 & 0 & -2 & -7 & -9 \end{bmatrix}$

$R_4 \to R_4 - 2R_3$ $\begin{bmatrix} 1 & 2 & 1 & -2 & -6 \\ 0 & -1 & 0 & 2 & 4 \\ 0 & 0 & -1 & -5 & -6 \\ 0 & 0 & 0 & 3 & 3 \end{bmatrix}$

$x_4 = -1$
$-x_3 + 5 = 6 \Rightarrow x_3 = -1$
$-x_2 + 2 = 0 \Rightarrow x_2 = 2$
$x_1 + \cancel{4} - 1 + \cancel{2} - \cancel{6} = 0$
$x_1 = 1$

$(x_1, x_2, x_3, x_4) = (1, 2, -1, -1)$

2. $$\begin{bmatrix} 2 & 3 & 5 & -9 \\ 7 & 3 & 2 & -8 \\ 2 & 3 & p & -q \end{bmatrix}$$

$R_2 \to 2R_2 - 7R_1$
$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 2 & 3 & 5 & -9 \\ 0 & -15 & -31 & 47 \\ 0 & 0 & p-5 & q-9 \end{bmatrix}$$

i) No solution $\Rightarrow$ $p = 5, q \neq 9$
ii) Unique solution $\Rightarrow$ $p \neq 5$.
iii) Infinite solutions $\Rightarrow$ $p = 5, q = 9$

3. i) $$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ \boxed{0} & 1 & 0 & 1 & 0 \end{bmatrix}$$

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$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$R_2 \rightleftarrows R_4$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$(x, y, w, z) = (0, 0, 0, 0)$

$z = 0$
$w = 0$
$y = 0$
$x = 0$

ii) $$\begin{bmatrix} 4 & 2 & 1 & 3 & 0 \\ 6 & 3 & 4 & 7 & 0 \\ 2 & 1 & 0 & 1 & 0 \end{bmatrix}$$

RQ

$R_2 \rightarrow 2R_2 - 3R_1$

$R_3 \rightarrow 2R_3 - R_1$

$$\begin{bmatrix} 4 & 2 & 1 & 3 & 0 \\ 0 & 0 & 5 & 5 & 0 \\ 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

$R_3 \rightarrow 5R_3 + R_2$

$$\begin{bmatrix} 4 & 2 & 1 & 3 & 0 \\ 0 & 0 & 5 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$4x + 2y + z + 3w = 0$

$5z + 5w = 0 \implies z = -w$

$2x + y = -w$

$2x + y = -w = z$

iii)
$$\begin{bmatrix} 1 & 1 & -2 & 3 & 0 \\ 1 & -2 & 1 & -1 & 0 \\ 4 & 1 & -5 & 8 & 0 \\ 5 & -7 & 2 & -1 & 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - 4R_1$

$R_4 \rightarrow R_4 - 5R_1$

$$\begin{bmatrix} 1 & 1 & -2 & 3 & 0 \\ 0 & -3 & 3 & -4 & 0 \\ 0 & -3 & 3 & -4 & 0 \\ 0 & -12 & 12 & -16 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$R_4 \rightarrow R_4 - 4R_2$

$$\begin{bmatrix} 1 & 1 & -2 & 3 & 0 \\ 0 & -3 & 3 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$-3y + 3z - 4w = 0$

$y = \dfrac{3z - 4w}{3}$

$x + y - 2z + 3w = 0$

$x = 2z - 3w - \dfrac{3z}{3} + \dfrac{4w}{3}$

$x = \dfrac{3z - 5w}{3}$

4. $$\begin{bmatrix} 2 & 3K & 3K+4 \\ 1 & K+4 & 4K+2 \\ 1 & 2K+2 & 3K+4 \end{bmatrix}$$

Non trivial solutions

$$\begin{vmatrix} 2 & 3K & 3K+4 \\ 1 & K+4 & 4K+2 \\ 1 & 2K+2 & 3K+4 \end{vmatrix} = 0$$

$$2(3K^2+16K+16-8K^2-12K-4)-3K(2-K)+(3K+4)(K-2)=0$$

$$-10K^2+8K+24+6K^2-8K-8=0$$

$$4K^2=16 \Rightarrow K=\pm 2$$

5. a) $x+2y+3z=\lambda x$
$3x+y+2z=\lambda y$
$2x+3y+z=\lambda z$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(1+\lambda^2-2\lambda-6)-2(3-3\lambda-4)+3(9-2+2\lambda)=0$$

$$\lambda^2+3\lambda-5-\lambda^3+2\lambda^2+6\lambda+2+21+6\lambda=0$$

$$-\lambda^3+3\lambda^2+15\lambda+18=0$$

$$\lambda^3-3\lambda^2-15\lambda-18=0$$

$$\begin{array}{c|cccc} & 1 & -3 & -15 & -18 \\ 6 & 0 & 6 & 18 & 18 \\ \hline & 1 & 3 & 3 & \boxed{0} \end{array}$$

$\lambda=6$ (Remaining two are imaginary)

b) $3x_1 + x_2 - \lambda x_3 = 0$

$4x_1 - 2x_2 - 3x_3 = 0$

$2\lambda x_1 + 4x_2 + \lambda x_3 = 0$

$$\begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix} = 0$$

$3(-2\lambda + 12) - 1(10\lambda) - \lambda(16 + 4\lambda) = 0$

$-4\lambda^2 - 32\lambda + 36 = 0$

$\lambda^2 + 8\lambda - 9 = 0$

$(\lambda - 1)(\lambda + 9) = 0$

$\lambda = 1, -9$