Computer Assignment 1

(100 marks)

- Consider a linear advection equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

with four different Initial Conditions (i.e. solution at time t=0) as described below. Take the value of wave speed $a = 1.0$ m/s.

1. Discontinuous initial solution as described in Figure 1

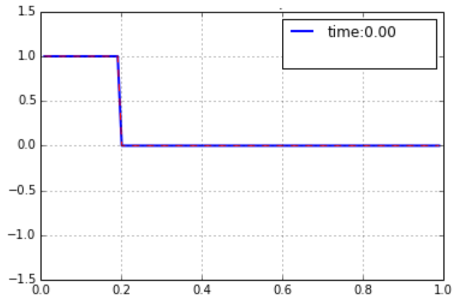

Figure 1: Initial condition 1

$$u(x,0) = \begin{cases} 1 & \text{if} \quad 0.2 \leq x \\ 0 & \text{Otherwise} \end{cases}$$

2. Initial solution having two periods as described in Figure 2

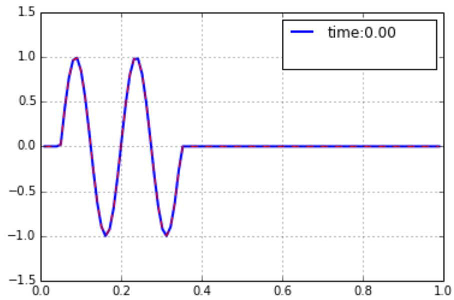

Figure 2: Initial condition 2

$$u(x,0) = \begin{cases} 0 & \text{for } 0 \leq x < 0.05 \\ sin(4\pi(\frac{x-0.05}{0.3})) & \text{for } 0.05 \leq x < 0.35 \\ 0 & \text{for } 0.35 \leq x \leq 1 \end{cases}$$

3. Initial solution having four periods as described in Figure 3

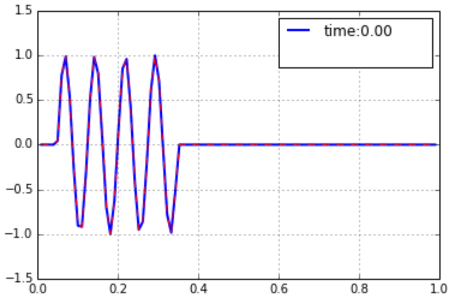

Figure 3: Initial condition 3

$$u(x,0) = \begin{cases} 0 & \text{for } 0 \leq x < 0.05 \\ sin(8\pi(\frac{x-0.05}{0.3})) & \text{for } 0.05 \leq x < 0.35 \\ 0 & \text{for } 0.35 \leq x \leq 1 \end{cases}$$

4. Initial solution having six periods as described in Figure 4

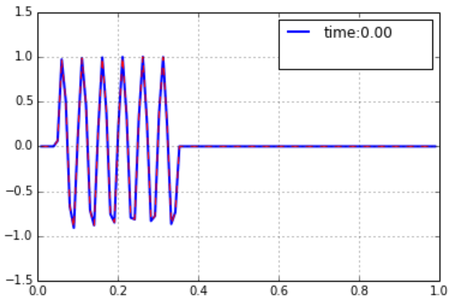

Figure 4: Initial condition 4

$$u(x,0) = \begin{cases} 0 & \text{for } 0 \leq x < 0.05 \\ sin(12\pi(\frac{x-0.05}{0.3})) & \text{for } 0.05 \leq x < 0.35 \\ 0 & \text{for } 0.35 \leq x \leq 1 \end{cases}$$

- Consider 101 grid points for discretizing the computational space $0 \leq x \leq 1$ for all the above four initial conditions described above.
- Apply the following Finite Difference Schemes (derivations of which will be explained in future) to obtain solutions of the linear advection equations at time t>0 as described below. At present, you need not know the background details of these schemes.
  1. Forward Time Forward Space (FTFS) scheme
  $$u_i^{n+1} = u_i^n - \nu(u_{i+1}^n - u_i^n)$$
  2. Forward Time Central Space (FTCS) scheme
  $$u_i^{n+1} = u_i^n - \frac{\nu}{2}(u_{i+1}^n - u_{i-1}^n)$$
  3. Forward Time Backward Space (FTBS) scheme
  $$u_i^{n+1} = u_i^n - \nu(u_i^n - u_{i-1}^n)$$
  4. Lax-Wendroff (LW) scheme
  $$u_i^{n+1} = u_i^n - \frac{\nu}{2}(u_{i+1}^n - u_{i-1}^n) + \frac{\nu^2}{2}(u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$
  5. Beam-Warming (BW) scheme
  $$u_i^{n+1} = u_i^n - \frac{\nu}{2}(3u_i^n - 4u_{i-1}^n + u_{i-2}^n) + \frac{\nu^2}{2}(u_i^n - 2u_{i-1}^n + u_{i-2}^n)$$
  6. Fromm (FR) scheme
  $$\left(u_i^{n+1}\right)_{FR} = \frac{1}{2}\left[\left(u_i^{n+1}\right)_{LW} + \left(u_i^{n+1}\right)_{BW}\right]$$

  where ν is Courant Friedrichs Lewy and it is given by $\nu = \dfrac{a\Delta t}{\Delta x}$, where Δt is the time step by which solution is advanced.
- While computing solutions, consider three values of ν for each scheme; namely $\nu = 0.5$, $\nu = 1.0$, $\nu = 1.5$.
- Compute at least upto 100 time steps and document your observations as to how the solutions are varying with time and how solutions of different schemes differ from each other.

- Write your "critical observations" in a tabular form for ease of comparison between schemes.
- Plot the solution after 40 time steps for all the schemes considering all the initial conditions and all the prescribed values of ν.
- Use the plotting range for x-axis as $0 \leq x \leq 1$ and that for y-axis as $-1.5 \leq u \leq 1.5$ (as shown in the Figures 1-4 for initial conditions) uniformly.

General Instructions

- **Checklist for submission:**
  - The code (written in C only) with proper inline documentation for each function. (Use meaningful variable names).
  - A "README.txt" file which contains the proper description on how to run the code and get the plots.
  - The plots submitted by you must be reproducible independently by the TA's from your code.
- **Instruction for submission:**
  - Rename your program file as your roll number (example: 184010006.c).
  - Zip the folder (*.zip) which contain the code and plots (See the checklist) and name it as your roll number (example: 194010006.zip).
- **Notes:**
  - Marks will be given only if the program is working and showing correct result. No step marks will be given.
  - Assignment will not be evaluated if "instruction for submission" are not followed properly.
- **Copying program from each other will lead to severe penalty.**

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