Computer Assignment 2

- Find steady-state temperature distribution on a 2D rectangular plate as shown in Figure 1 below by solving Laplace equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

with the specified temperatures at the boundaries as indicated in the figure.

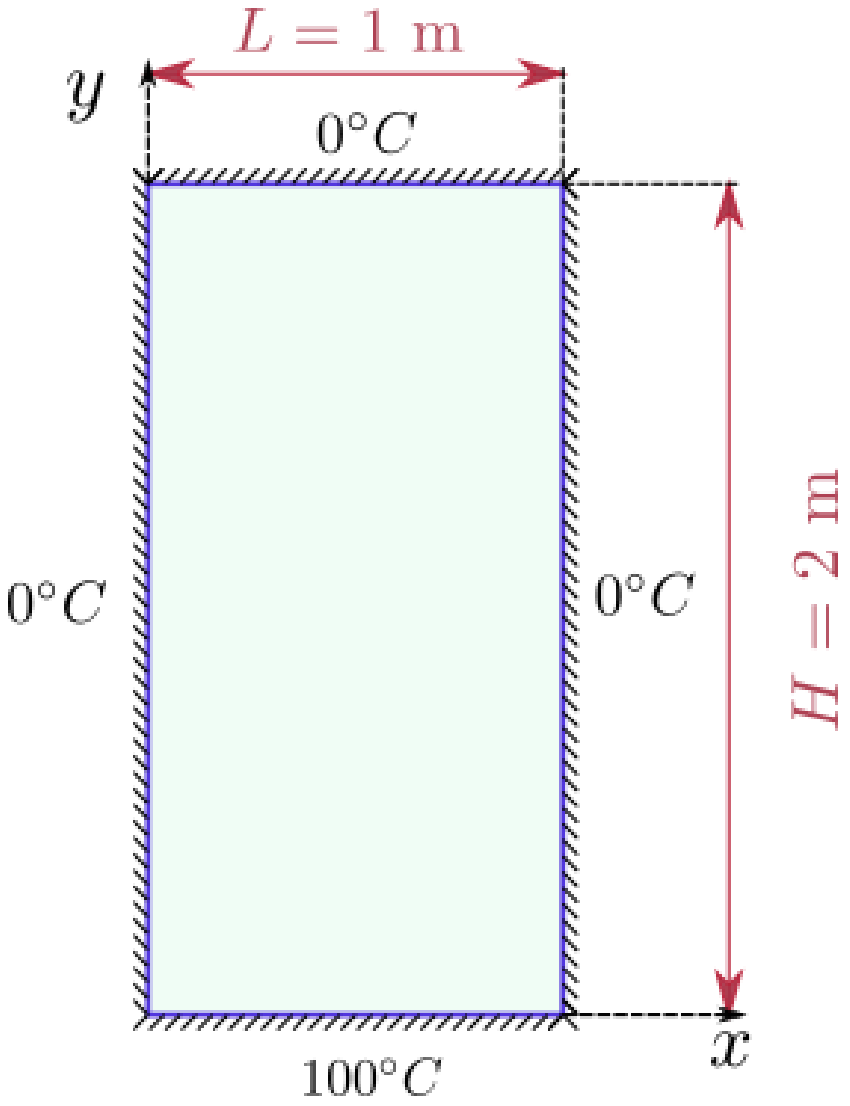

Figure 1

- In order to solve the Laplace equation (1) and present the numerical results, **strictly** follow the instructions given below:

  1. Deiscretize the domain using 21×41 grid points.
  2. Use (i) Point Jacobi (PJ), (ii) Point Gauss-Seidel (PGS) with relaxation parameter $\omega = 1.78$, (iii) Point Succesive Over Relaxation (PSOR) for soving the system of equations obtained from the discretized Laplace equation.
  3. Impose convergence criterion

  $$\text{ERROR} < 10^{-2} \tag{2}$$

  where

  $$\text{ERROR} = \sum_{i,j} |T_{i,j}^{n+1} - T_{i,j}^{n}|$$

  4. Show convergence history as a plot '$log_{10}(ERROR)$' Versus 'Number of iterations' for PJ, PGS, PSOR schemes.
  5. Note down the number of iterations required by PJ, PGS, PSOR schemes to achieve convergence criterion (2). Write your comments on the observation.
  6. Find out optimum ω for PSOR method for the given problem using a plot of 'ω' Versus 'Number of iterations'.
  7. Plot variations of temperature T along the midlines; i.e. (i) along $x = 0.5$ line and (ii) along $y = 1.0$ line. Compare these numerical results with the exact solutions as given by

  $$T(x,y) = (100)\left[2\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n\pi}\frac{Sinh\left(\frac{n\pi(H-y)}{L}\right)Sin\left(\frac{n\pi x}{L}\right)}{Sinh\left(\frac{n\pi H}{L}\right)}\right]$$

- Also attempt the following **Optional** questions:

  1. Find out the spectral radii of PJ, PGS, PSOR schemes for the given problem numerically and justify the above results.
  2. Provide contour plots for the above results.

General Instructions

- **Checklist for submission:**
  - The code (written in C only) with proper inline documentation for each function. (Use meaningful variable names).
  - A "README.txt" file which contains the proper description on how to run the code and get the plots.
  - The plots submitted by you must be reproducible independently by the TA's from your code.
- **Instruction for submission:**
  - Rename your program file as your roll number (example: 184010006.c).
  - Zip the folder (*.zip) which contain the code and plots (See the checklist) and name it as your roll number (example: 194010006.zip).
- **Notes:**
  - Marks will be given only if the program is working and showing correct result. No step marks will be given.
  - Assignment will not be evaluated if "instruction for submission" are not followed properly.
- **Copying program from each other will lead to severe penalty.**

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