Probability distribution function question

Bernoulli Distribution:

1. A coin is tossed once. If the probability of getting a head is 0.6, find (a) $P(X = 1)$and (b) $P(X = 0)$.
2. A student passes an exam with probability 0.7. Define the Bernoulli random variable and find its mean and variance.
3. If $p = 0.4$, find the probability mass function of a Bernoulli distribution.
4. A machine produces a defective item with probability 0.05. Find the probability that the item is non-defective.
5. For a Bernoulli distribution, find the variance when the probability of success is 0.8.

Binomial Distribution:

1. A coin is tossed 6 times. Find the probability of getting exactly 4 heads.
2. If the probability of success is 0.3 and number of trials is 10, find (a) mean and (b) variance of the distribution.
3. A die is thrown 8 times. Find the probability of getting exactly two 6's.
4. The probability that a bulb is defective is 0.1. Find the probability that exactly 3 bulbs are defective out of 12.
5. In a binomial distribution with $n = 5$and $p = 0.5$, find $P(X \leq 2)$.

Normal (Gaussian) Distribution

1. The mean height of students is 170 cm with standard deviation 5 cm. Find the probability that a student's height is between 165 cm and 175 cm.
2. The marks obtained by students follow a normal distribution with mean 60 and SD 10. Find the probability that a student scores more than 75.
3. If $X \sim N(50,9)$, find $P(47 < X < 53)$.
4. The lifetime of a bulb is normally distributed with mean 1000 hours and SD 100 hours. Find the probability that the bulb lasts more than 1100 hours.
5. Find the probability that a normally distributed variable deviates from its mean by less than one standard deviation.

Poisson Distribution:

1. The average number of phone calls received per hour is 5. Find the probability that exactly 3 calls are received in an hour.
2. Accidents occur on a road at an average rate of 2 per day. Find the probability that no accident occurs on a given day.
3. If the mean number of defects per meter of cloth is 1.5, find the probability of exactly two defects.
4. A shop receives an average of 4 customers every 10 minutes. Find the probability that exactly 6 customers arrive in 10 minutes.
5. Find the mean and variance of a Poisson distribution with parameter $\lambda = 7$.

Log-Normal Distribution:

1. If $\ln(X) \sim N(2,0.25)$, identify the distribution of X.
2. The logarithm of income follows a normal distribution with mean 3 and variance 1. Find the probability that income exceeds e^4.
3. If a variable follows a log-normal distribution, explain how it can be transformed into a normal variable.
4. Given $\ln(X) \sim N(\mu, \sigma^2)$, find the expression for the probability density function of X.

Uniform Distribution:

1. A random variable is uniformly distributed between 2 and 10. Find its mean and variance.
2. If $X \sim U(0,1)$, find $P(0.2 < X < 0.7)$.
3. The time taken by a bus is uniformly distributed between 30 and 50 minutes. Find the probability that it takes more than 40 minutes.
4. Find the probability density function of a uniform distribution on the interval [5, 15].
5. A random variable is uniformly distributed over [–3, 7]. Find $P(X < 2)$.

Standard Normal Distribution:

1. Find $P(Z < 1.25)$where $Z$is a standard normal variable.
2. Find $P(-1 < Z < 2)$using the standard normal table.
3. If $Z$is standard normal, find $P(Z > 1.96)$.
4. Convert $X = 85$into a Z-score if mean = 70 and SD = 10.
5. Find the value of $z$such that $P(Z < z) = 0.975$.
